Transportation
Library
TA
1
.A526
v. 109

TRANSACTIONS

OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 109

1944

Edited by the Secretary, under the direction of the Committee on Publications.

Reprints from this publication, which is copyrighted, may be made on condition that the full title of Paper, name of Author, and page reference are given.

NEW YORK
PUBLISHED BY THE SOCIETY

1945

Copyright, 1945, by the AMERICAN SOCIETY OF CIVIL ENGINEERS.

NOTE.—The Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

PAPERS

No. PAGE

2207 CONFORMITY BETWEEN MODEL AND PROTOTYPE: A SYMPOSIUM.......... 1

FOREWORD........ 2

HYDRAULIC STRUCTURES.

By Jacob E. Warnock and H. G. Dewey, Jr..........

A DRY-DOCK SUCTION CHAMBER.

By C. B. Patterson........ 29

PICKWICK, WHEELER, AND GUNTERSVILLE LOCKS.

By Martin E. Nelson and James J. Hartigan........ 35

BOULDER DAM OUTLET WORKS.

By S. P. Wing........ 59

SPILLWAY COEFFICIENTS, NORRIS DAM.

By G. H. Hickox........ 68

PRESSURE HEADS ON BONNEVILLE DAM.

By J. C. Stevens and R. B. Cochrane........ 77

METER MEASUREMENTS OF DISCHARGE, UNIVERSITY DAM.

By Edward Soucek........ 86

GRAND TOWER, MISSISSIPPI RIVER.

By Frederick R. Brown........ 100

HYDRAULIC CONDITIONS TO LOCK AND DAM NO. 4.

By William J. Hopkins........ 103

BIBLIOGRAPHY AND NOTATION.

By the Symposium Authors........ 112

Discussion:

By A. E. NIEDERHOFF........ 118
C. I. GRIMM........ 119
JOE W. JOHNSON........ 120
V. L. STREETER........ 124
GLEN N. COX........ 127
GRAHAM WALTON........ 131
H. A. EINSTEIN........ 134
K. G. TOWER........ 139
R. J. PAFFORD, JR........ 140
EDWARD H. SCHULZ........ 143
F. T. MAVIS........ 144
D. C. McCONAUGHY........ 148
L. STANDISH HALL........ 150
A. J. GILARDI........ 154
FRED W. BLAISDELL........ 157
A. R. THOMAS........ 167
GILBERT H. DUNSTAN........ 168
ROBERT F. KREISS........ 168
J. H. DOUMA........ 170
MARVIN J. WEBSTER........ 173
MARTIN E. NELSON........ 177
JACOB E. WARNOCK AND H. G. DEWEY, JR........ 181
MARTIN E. NELSON AND JAMES J. HARTIGAN........ 184
J. C. STEVENS AND R. B. COCHRANE........ 185
EDWARD SOUCEK........ 185
FREDERICK R. BROWN........ 191

No. PAGE

2208 SEISMIC SUBSURFACE EXPLORATION ON THE ST. LAWRENCE RIVER PROJECT.

By **E. R. Shepard and Reuben M. Haines** 194

Discussion:

By R. WOODWARD MOORE 213
ARTHUR J. GRIER 214
F. W. LEE 216
L. DON LEET 219
WILLIAM L. SHANNON AND WINTHROP A. WELLS 219
E. R. SHEPARD AND REUBEN M. HAINES 220

2209 DETERMINATION OF KUTTER'S n FOR SEWERS PARTLY FILLED.

By **C. Frank Johnson** 223

Discussion:

By THOMAS R. CAMP 240
RICHARD G. COULTER 243
C. E. RAMSER 245
C. FRANK JOHNSON 247

2210 ORGANIZING AND FINANCING SEWAGE TREATMENT PROJECTS.

By **Samuel A. Greeley** 248

Discussion:

By MILTON P. ADAMS 264
HAL F. SMITH 266
HERBERT MOORE 268
FRANCIS H. KINGSBURY 269
A. BRINTON CARSON 271
SAMUEL A. GREELEY 277

2211 PHYSICAL PROPERTIES THAT AFFECT THE BEHAVIOR OF STRUCTURAL MEMBERS.

By **Wilbur M. Wilson** 279

Discussion:

By JONATHAN JONES 290
A. B. KINZEL 292
ALMON H. FULLER 295
C. F. GOODRICH 296
R. L. MOORE 297
LEON S. MOISSEIFF 299
FRED L. PLUMMER 301
A. A. EREMIN 302
WILBUR M. WILSON 304

2212 PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING.

By **Merrill Bernard** 311

Discussion:

By EDGAR DOW GILMAN 352
CLARENCE S. JARVIS 353
IVAN E. HOUK 359
W. G. HOYT 360
L. R. BEARD 361
ROBERT E. KENNEDY 363
L. K. SHERMAN 365
C. H. EIFFERT 369
GAIL A. HATHAWAY 370
L. STANDISH HALL 376
MERRILL BERNARD 378

No. PAGE

2213 APPLICATION OF SOIL MECHANICS IN DESIGNING BUILDING FOUNDATIONS.
By A. Casagrande and R. E. Fadum 383
Discussion:
By Jacob Feld 417
Leonard Zeevaert 419
Gregory P. Tschebotarioff 422
Guthlac Wilson 424
W. H. McAlpine 426
Karl Terzaghi 427
D. P. Krynine 435
O. G. Julian 438
A. A. Eremin 443
D. M. Burmister 444
Abraham Woolf 448
Lazarus White 452
George L. Freeman 454
Soil Mechanics Committee of the Los Angeles Section—Trent R. Dames, Chairman, Frederick J. Converse, Sterling S. Green, and Paul Baumann 459
A. Casagrande and R. E. Fadum 463

2214 STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS.
By Silas H. Woodard 491
Discussion:
By C. M. Goodrich 501
Maurice P. Van Buren 502
Joseph A. Wise 503
David B. Hall 506
Roberto Contini 509
I. Nelidov 511
Bernard L. Weiner 514
Richard W. Albrecht 517
Duff A. Abrams 520
C. A. P. Turner 529
Mark W. Huggins 533
A. A. Eremin 533
Silas H. Woodard 535

2215 AERATION OF SPILLWAYS.
By G. H. Hickox 537
Discussion:
By Joe W. Johnson 557
J. W. Howe and Claud C. Lomax, Jr. 559
J. C. Stevens 563
G. H. Hickox 565

2216 CONCRETE RESERVOIRS OF THE VERTICAL-BEAM TYPE.
By C. Maxwell Stanley, Jr. 567
Discussion:
By Lewis A. Schmidt, Jr. 577
Victor H. Cochrane 577
R. E. Koon and J. C. Gearhart 580
Gustavo Pérez Guerra 587
C. Maxwell Stanley, Jr. 590

2217 FLOW AROUND BENDS IN STABLE CHANNELS.
By C. A. Mockmore 593
Discussion:
By J. M. Robertson 619
Joe W. Johnson and M. A. Selim 623
I. Nelidov 625
S. M. Woodward, J. W. Howe, and C. J. Posey 626
C. A. Mockmore 628

No. PAGE

2218 EFFECT OF TURBULENCE ON SEDIMENTATION.

By William E. Dobbins 629

Discussion:

By A. M. GAUDIN 657
JOHN S. McNOWN 657
H. A. EINSTEIN 660
THOMAS R. CAMP 660
GARBIS H. KEULEGAN 667
C. W. THORNTHWAITE 669
A. A. KALINSKE 672
E. R. VAN DRIEST 674
R. SCHUHMANN, JR. 676
H. B. SOUTHWOOD 676
L. STANDISH HALL 677
WILLIAM E. DOBBINS 678

2219 THE QUEENS MIDTOWN TUNNEL.

By Ole Singstad 679

Discussion:

By THOMAS W. FLUHR 716
HOWARD L. KING 716
FRANCIS V. WAGNER 717
WILLIAM J. WILGUS 724
S. A. THORESEN 725
H. G. CRUTHERS 727
LEO GEENENS 728
CHARLES E. CONOVER 729
EDWARD LEVY 731
ROBERT F. LEGGET 737
ORRIN L. BRODIE 737
A. A. EREMIN 747
ARTHUR S. TUTTLE 749
OLE SINGSTAD 751

2220 A SYMMETRICALLY LOADED BASE SLAB ON AN ELASTIC FOUNDATION.

By Stanley U. Benscoter 763

Discussion:

By ROBERT O. JAMESON 777
L. J. MENSCH 780
ALFRED L. PARMÉ 784
A. A. EREMIN 785
A. E. CUMMINGS 787
HARRIS EPSTEIN AND JAMES R. AYERS, JR. 793
STANLEY U. BENSCOTER 795

2221 ILLINOIS WATER LITIGATION, 1940–1941.

By Langdon Pearse 799

Discussion:

By WILLEM RUDOLFS 824
L. R. HOWSON 826
A. M. BUSWELL 828
LLOYD M. JOHNSON 830
H. P. RAMEY 831
LANGDON PEARSE 832

No. Page

2222 CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA.

By Luna B. Leopold 837
Discussion:
By Lawrence Pratt 867
L. L. Harrold and A. J. Dickson 868
James Girand 871
Clarence S. Jarvis 874
Paul V. Hodges 874
Edgar E. Foster 876
R. W. Davenport 877
A. K. Showalter 878
Walter B. Langbein 879
Emil P. Schuleen 884
Luna B. Leopold 887

2223 FLOW CHARACTERISTICS AT RECTANGULAR OPEN-CHANNEL JUNCTIONS.

By Edward H. Taylor 893
Discussion:
By Harold K. Palmer 903
G. H. Hickox 904
J. C. Stevens 905
C. J. Posey 906
I. B. Hosig 906
L. Standish Hall 908
Brooks T. Morris 910
Edward H. Taylor 911

2224 SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES.

By Richard M. Hodges 913
Discussion:
By C. D. Geisler 942
Arthur G. Hayden 944
Bernard L. Weiner 946
Alfred L. Parmé 954
J. Charles Rathbun 960
Maurice Barron 962
Phillips H. Lovering 966
Jaroslav Polivka 967
A. A. Eremin 968
Richard M. Hodges 969

2225 SLUDGE-DRYING DEVELOPMENTS AT CHICAGO, ILL.

By Lloyd M. Johnson 979
Discussion:
By C. W Gordon 988
Paul Hansen 989
Langdon Pearse 991
Lloyd M. Johnson 993

2226 DEVELOPMENT OF THE CHICAGO TYPE BASCULE BRIDGE.

By Donald N Becker 995
Discussion:
By Alonzo J. Hammond 1026
Clinton D. Hanover, Jr. 1028
Armour T. Granger 1031
C. B McCullough 1035
Donald N. Becker 1043

No. Page

2227 SEDIMENTATION IN RESERVOIRS.

By Berard J. Witzig 1047

Discussion:

By Joe W. Johnson 1072
John W. Stanley 1075
Stafford C. Happ 1078
Thomas H. Means 1079
Carl B. Brown 1080
C. S. Jarvis 1086
Hugh Stevens Bell 1088
Harry F. Blaney 1089
A. L. Sonderegger 1091
L. Standish Hall 1093
Berard J. Witzig 1098

2228 THE HYDRAULIC JUMP IN SLOPING CHANNELS.

By Carl E. Kindsvater 1107

Discussion:

By Joe W. Johnson 1121
Karl R. Kennison 1123
J. C. Stevens 1125
C. J. Posey 1135
Jerome Fee 1138
Frank S. Bailey 1141
G. H. Hickox 1141
Carl E. Kindsvater 1147

2229 RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING.

By P. C. Rutledge 1155

Discussion:

By Benjamin K. Hough, Jr. 1184
F. M. Van Auken 1186
Jacob Feld 1189
Raymond F. Dawson 1190
Hamilton Gray 1193
Karl Terzaghi 1196
D. P. Krynine 1201
D. M. Burmister 1204
P. C. Rutledge 1208

2230 A METHOD OF COMPUTING URBAN RUNOFF.

By W. I. Hicks 1217

Discussion:

By C. S. Jarvis 1254
S. W. Jens 1255
W. W. Horner 1260
W. I. Hicks 1261

2231 RÔLE OF THE LAND DURING FLOOD PERIODS.

By W. W. Horner 1269

Discussion:

By L. K. Sherman 1295
Raphael G. Kazmann 1298
W. G. Hoyt and W. B. Langbein 1299
Charlie M. Moore 1304
Waldo E. Smith 1305
P. B. Rowe 1310
C. S. Jarvis 1312
W. W. Horner 1313

No. PAGE

2232 STRENGTH OF SLENDER BEAMS.
By George Winter 1321
Discussion:
By T. McLean Jasper 1350
C. M. Goodrich 1350
J. P. Wadleck 1351
H. D. Hussey 1354
George Winter 1356

2233 RIVETED AND PIN-CONNECTED JOINTS OF STEEL AND ALUMINUM ALLOYS.
By Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore 1359
Discussion:
By Jonathan Jones 1397
Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore 1398

2234 PENDLETON LEVEE FAILURE.
By Kenneth E. Fields and William L. Wells 1400
Discussion:
By Ralph B. Peck 1414
K. Terzaghi 1416
T. A. Middlebrooks 1421
D. P. Krynine 1424
Kenneth E. Fields and William L. Wells 1426

2235 CHART FOR THE FUTURE: ADDRESS PREPARED FOR THE ANNUAL CONVENTION, CLEVELAND, OHIO, JULY 19, 1944 1430

MEMOIRS OF DECEASED MEMBERS

PAGE
George Stewart Davison, Past-President and Hon. M. Am. Soc. C. E. 1437
John Frank Stevens, Past-President and Hon. M. Am. Soc. C. E. 1440
John Watson Alvord, Hon. M. Am. Soc. C. E. 1447
John Debo Galloway, Hon. M. Am. Soc. C. E. 1451
Joseph Ruggles Worcester, Hon. M. Am. Soc. C. E. 1456
Thomas David Allin, M. Am. Soc. C. E. 1459
Thomas Ward Bailey, M. Am. Soc. C. E. 1461
Bernard Benfield, M. Am. Soc. C. E. 1464
Reginald Pelham Bolton, M. Am. Soc. C. E. 1465
Charles Edward Bowron, M. Am. Soc. C. E. 1470
Sylvester Quayle Cannon, M. Am. Soc. C. E. 1472
Woolsey Moorman Caye, M. Am. Soc. C. E. 1474
Edward Cartwright Constance, M. Am. Soc. C. E. 1477
Paul Darwin Cook, M. Am. Soc. C. E. 1478
William Otto Galbreath, M. Am. Soc. C. E. 1481
John Philip Hallihan, M. Am. Soc. C. E. 1483
George Prince Hawley, M. Am. Soc. C. E. 1484
John Coffee Hays, M. Am. Soc. C. E. 1486
William Herzberg, M. Am. Soc. C. E. 1489
Robert Leslie Holmes, M. Am. Soc. C. E. 1490
Winfred Dean Hubbard, M. Am. Soc. C. E. 1492
Clarence Walter Hudson, M. Am. Soc. C. E. 1493
David Edward Hughes, M. Am. Soc. C. E. 1495
William Robert Kales, M. Am. Soc. C. E. 1497
David Andrew Keefe, M. Am. Soc. C. E. 1499
Harry Raymond Leach, M. Am. Soc. C. E. 1500
Lloyd Tevis McAfee, M. Am. Soc. C. E. 1502
Charles T. Main, M. Am. Soc. C. E. 1504
John Walter May, M. Am. Soc. C. E. 1507
James Lafayette Parker, M. Am. Soc. C. E. 1509
Harry Tinker Poe, Jr., M. Am. Soc. C. E. 1510
Walter Percival Rice, M. Am. Soc. C. E. 1511
James Walter Rickey, M. Am. Soc. C. E. 1513
John Wesley Ripley, M. Am. Soc. C. E. 1517
Louis Charles Rockett, M. Am. Soc. C. E. 1519
Edward Henry Rockwell, M. Am. Soc. C. E. 1520
Walter Alexander Rogers, M. Am. Soc. C. E. 1522
Harry Robinson Safford, M. Am. Soc. C. E. 1524
Will Joseph Sando, M. Am. Soc. C. E. 1526
Marshall Ney Shoemaker, M. Am. Soc. C. E. 1528
Arthur Gregg Singer, M. Am. Soc. C. E. 1529
Charles Sleeper Sterling, M. Am. Soc. C. E. 1532
George Henry Tinker, M. Am. Soc. C. E. 1533
Enrique Augusto Touceda, M. Am. Soc. C. E. 1534
Thomas Thomson Towles, M. Am. Soc. C. E. 1536
Paul Anthony Trost, M. Am. Soc. C. E. 1538
Henry Aldis Varney, M. Am. Soc. C. E. 1539
William Wesley Varney, M. Am. Soc. C. E. 1541
Robert Spurr Weston, M. Am. Soc. C. E. 1542
Daniel Merrick Wheeler, M. Am. Soc. C. E. 1544
James Gilbert White, M. Am. Soc. C. E. 1548
Harry James Wight, M. Am. Soc. C. E. 1549
Edgar Kennard Wilson, M. Am. Soc. C. E. 1551
Samuel Lamson Wonson, M. Am. Soc. C. E. 1552
Edward Lowrey Woodruff, M. Am. Soc. C. E. 1554
Edward Russell Allen, Assoc. M. Am. Soc. C. E. 1558
Harold James Manning Baker, Assoc. M. Am. Soc. C. E. 1561
Edward Crosby Bebb, Assoc. M. Am. Soc. C. E. 1563
Julius Montgomery Bischoff, Assoc. M. Am. Soc. C. E. 1563

Page
Arthur Boniface, Assoc. M. Am. Soc. C. E. 1564
Dalton Munroe Brown, Assoc. M. Am. Soc. C. E. 1567
Knox Folsom Burnett, Assoc. M. Am. Soc. C. E. 1568
DeWitt Clinton Fenstermaker, Assoc. M. Am. Soc. C. E. 1570
George Addison Field, Assoc. M. Am. Soc. C. E. 1571
Irvin Gittleson, Assoc. M. Am. Soc. C. E. 1572
Dan A. Hulick, Assoc. M. Am. Soc. C. E. 1573
Alexander Andrew Laird, Assoc. M. Am. Soc. C. E. 1574
James Henry Miner, Assoc. M. Am. Soc. C. E. 1576
Arthur Leland Richardson, Assoc. M. Am. Soc. C. E. 1577
William Sherman Van Loan, Assoc. M. Am. Soc. C. E. 1579
Albert Willard Walker, Assoc. M. Am. Soc. C. E. 1580
Carl Robert Weidner, Assoc. M. Am. Soc. C. E. 1582
Robert Lee Whitehead, Jr., Assoc. M. Am. Soc. C. E 1585
Guy Smith Wood, Assoc. M. Am. Soc. C. E. 1586
Edwin Park Arneson, Jun. Am. Soc. C. E. 1587
Richard Alester Coleman, Jun. Am. Soc. C. E. 1588
Nathan Elias Elkin, Jun. Am. Soc. C. E. 1589
Rolland Francis Lang, Jun. Am. Soc. C. E. 1590
Thomas Albert McTeer, Jun. Am. Soc. C. E. 1591
Ernest Frederick Hartman, Affiliate Am. Soc. C. E. 1592

ERRATA

Transactions, Vol. 108 (1943)

Page 283, lines 24 and 36, and page 284, line 1, change "Sante Fe" to "Santa Fe."

Page 284, line 11, change "railroad" to "zeolite," and line 19, change "inorganic" to "organic."

PAPERS AND REPORTS OF COMMITTEES PUBLISHED IN PROCEEDINGS, AM. SOC. C. E., JANUARY, 1943, TO SEPTEMBER, 1944, INCLUSIVE, NOT PUBLISHED IN TRANSACTIONS, VOL. 109

	Number of *Proceedings.*
DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE: A SYMPOSIUM	**January, 1943**
RAINBOW ARCH BRIDGE OVER NIAGARA GORGE: A SYMPOSIUM	**October, 1943**
STORAGE AND THE UNIT HYDROGRAPH. **By C. O. Clark**	**November, 1943**
RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES. **By D. B. Steinman**	**November, 1943**
UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY: A SYMPOSIUM	**November, 1943**
SHEAR EFFECT ON THE STRENGTH OF STRUTS. **By S. Sergev**	**November, 1943**
GEOLOGY IN HIGHWAY ENGINEERING. **By Marshall T. Huntting**	**December, 1943**
ORIGINAL AND READJUSTED REPAYMENT CONTRACTS, BOULDER CANYON PROJECT. **By Clay C. Elder**	**December, 1943**
FAILURE OF THE TACOMA NARROWS BRIDGE: REPORT OF THE SPECIAL COMMITTEE OF THE BOARD OF DIRECTION	**December, 1943**
SURGES IN PANAMA CANAL REPRODUCED IN MODEL. **By F. W. Edwards and Edward Soucek**	**January, 1944**
COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW. **By William S. Eisenlohr, Jr.**	**January, 1944**
MILITARY AIRFIELDS: A SYMPOSIUM	**January, 1944**
ANALYSIS OF RIGID FRAMES BY SUPERPOSITION. **By David M. Wilson**	**February, 1944**
SIMULTANEOUS CONSOLIDATION OF CONTIGUOUS LAYERS OF UNLIKE COMPRESSIBLE SOILS. **By Hamilton Gray**	**February, 1944**
FLOODS: PROGRESS REPORT OF THE CHAIRMAN, JOINT DIVISION COMMITTEE ON FLOODS	**February, 1944**
ERECTION OF HYDRAULIC TURBINES AND GENERATORS. **By Andrew M. Komora**	**March, 1944**
FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS. **By H. B. Kinnison and B. R. Colby**	**March, 1944**
SHEAR AND BOND STRESSES IN REINFORCED CONCRETE. **By Stanley U. Benscoter and Samuel T. Logan**	**March, 1944**

Number of *Proceedings.*

HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION.
By Maurice N. Amster.. March, 1944

BIBLIOGRAPHY ON TIMBER STRUCTURES: PROGRESS REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON TIMBER STRUCTURES.. March, 1944

ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK.
By Yves Nubar.. April, 1944

ADVANCES IN SEWAGE TREATMENT AND PRESENT STATUS OF THE ART: SECOND PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SEWERAGE AND SEWAGE TREATMENT.. April, 1944

ECONOMICAL CANAL CROSS SECTIONS
By Victor L. Streeter.. May, 1944

PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS.
By D. P. Krynine.. May, 1944

CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN.
By H. Alden Foster.. May, 1944

DESIGN AND CONSTRUCTION OF AIRPORT RUNWAYS AND FLIGHT STRIPS: PROGRESS REPORT OF A COMMITTEE OF THE HIGHWAY DIVISION.. May, 1944

TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER.
By Vito A. Vanoni.. June, 1944

CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES.
By John B. Wilbur.. June, 1944

TRANSLATORY WAVES IN NATURAL CHANNELS.
By J. H. Wilkinson.. June, 1944

WEDGE-SHAPED STRUCTURAL MEMBERS UNDER DIRECT STRESS AND BENDING.
By Robert B. B. Moorman.. June, 1944

PRINCIPLES OF DEPRECIATION: REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION.. June, 1944

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS.
By Karl Terzaghi.. September, 1944

AMPLIFIED SLOPE DEFLECTION.
By Ralph W. Stewart.. September, 1944

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2207

CONFORMITY BETWEEN MODEL AND PROTOTYPE
A SYMPOSIUM

Page
Foreword 2
Hydraulic Structures.
By Jacob E. Warnock, M. Am. Soc. C. E., and H. G. Dewey, Jr., Jun. Am. Soc. C. E. 3
A Dry-Dock Suction Chamber.
By C. B. Patterson, Jun. Am. Soc. C. E. 29
Pickwick, Wheeler, and Guntersville Locks.
By Martin E. Nelson, M. Am. Soc. C. E., and James J. Hartigan, Esq. 35
Boulder Dam Outlet Works.
By S. P. Wing, M. Am. Soc. C. E. 59
Spillway Coefficients, Norris Dam.
By G. H. Hickox, M. Am. Soc. C. E. 68
Pressure Heads on Bonneville Dam.
By J. C. Stevens, M. Am. Soc. C. E., and R. B. Cochrane, Assoc. M. Am. Soc. C. E. 77
Meter Measurements of Discharge, University Dam.
By Edward Soucek, Assoc. M. Am. Soc. C. E. 86
Grand Tower, Mississippi River.
By Frederick R. Brown, Jun. Am. Soc. C. E. 100
Hydraulic Conditions to Lock and Dam No. 4.
By William J. Hopkins, Assoc. M. Am. Soc. C. E. 103
Bibliography and Notation.
By the Symposium Authors 112

With Discussion by Messrs. A. E. Niederhoff, C. I. Grimm, Joe W. Johnson, V. L. Streeter, Glen N. Cox, Graham Walton, H. A. Einstein, K. G. Tower, R. J. Pafford, Jr., Edward H. Schulz, F. T. Mavis, D. C. McConaughy, L. Standish Hall, A. J. Gilardi, Fred W. Blaisdell, A. R. Thomas, Gilbert H. Dunstan, Robert F. Kreiss, J. H. Douma, Marvin J. Webster, Martin E. Nelson, and Jacob E. Warnock and H. G. Dewey, Jr., Martin E. Nelson and James J. Hartigan, J. C. Stevens and R. B. Cochrane, Edward Soucek, and Frederick R. Brown.

Note.—Published in October, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

FOREWORD

A comprehensive Symposium on the conformity of model to prototype behavior has been in the minds of the Executive Committee of the Hydraulics Division for a number of years. The Committee on Hydraulic Research (J. C. Stevens, M. Am. Soc. C. E., Chairman), appointed in April, 1934, has announced this subject as one of its major interests (1),[1] and the present Symposium is the result of the efforts of Subcommittee 2, comprising Clarence E. Bardsley, M. Am. Soc. C. E., and Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., with E. W. Lane, M. Am. Soc. C. E., as Chairman. Credit should also be given to Paul W. Thompson, Assoc. M. Am. Soc. C. E., and Herbert D. Vogel, M. Am. Soc. C. E. Lieutenant-Colonel Thompson succeeded Colonel Vogel on the committee and was in turn succeeded by Mr. Tiffany.

Two of the papers were first presented at the San Diego, Calif., meeting of the Division in July, 1941; one was presented at the Denver, Colo., meeting of the Division in July, 1940; the remaining papers have not been presented publicly in any form. Some papers on this subject have been published separately and these are included in the Bibliography (2) (3) (24). Each author is outstanding in the art of model experimentation and in the analysis of results, and has had an opportunity to determine how nearly the model tests conformed to the prototype observations.

The objective of this research can be stated as a single question: "Does the prototype act as the model predicted?" It is rather difficult to obtain complete conformity for two or three reasons. For example, working conditions may not even approach the load limit that was used in the model; and the prototype rarely is built precisely in accord with the model. The purpose of the Symposium is to assemble the results of a wide variety of studies in which actual comparisons have been made so that hydraulic engineers may judge the sufficiency of the model as a tool in design.

[A rigorous and careful wartime censorship was necessary when this Symposium was first published (October, 1942). In this final printing it has been possible to restore the identity of many of the objects mentioned. This has been done, in some cases, by inserting an explanatory footnote as necessary. —Ed.]

---

[1] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography, which appears as the last unit of the Symposium.

HYDRAULIC STRUCTURES

BY JACOB E. WARNOCK,[2] M. AM. SOC. C. E., AND H. G. DEWEY, JR.,[3]
JUN. AM. SOC. C. E.

SYNOPSIS

Some of the general factors involved in making model-prototype comparisons of hydraulic structures are mentioned in this paper. Reasons for desiring comparisons, the types made, the difficulties encountered, and the measurements and instruments required are discussed briefly. Citing two of the comparisons made by the Bureau of Reclamation as examples, an account is given of a small irrigation structure, and of the pressure and discharge tests on the 102-in. river outlets of the Grand Coulee Dam in the State of Washington.

INTRODUCTION

Although great progress was made during the 1932–1942 decade in the use of hydraulic models to aid in the design of hydraulic structures, there has been but little progress in comparing the performance of a model and its prototype. It is of considerable interest and importance to make such a comparison in order to determine the degree with which the model may be relied upon for quantitative results even if the laws of similitude have been rigidly followed. In the field of hydraulic structures this is of particular value when applied to discharge coefficients, water-surface profiles, erosion of river beds, and pressures on boundary surfaces, among others. Of nearly equal importance is the fact that reliable prototype measurements will furnish accurate design data enabling more economical designs in the future.

The reasons why little progress has been made in comparing the hydraulic performance of a model and its prototype may be explained by a lack of interest, particularly after the prototype is in satisfactory operation; a natural reliance on model theory; and the difficulties of making prototype measurements. Fortunately, considerable interest is now being shown for making these comparisons and it is believed that this Symposium not only will assist materially in helping to create more interest but, perhaps, from the exchange of ideas, the difficulties involved in making the comparison will be lessened.

Accordingly, this paper will treat some of the recognized difficulties in procuring prototype measurements, together with the measurements and instruments required. Two types of comparisons between model and prototype will follow, one qualitative and the other quantitative. The former is the easiest and the most frequently made. It consists of describing, for example, the observed performance of some structure for several flows, and then comparing these observations with those made of similar flows in the model. It is recognized that this type of comparison is of particular value only to those most familiar with the structure, is not too convincing to others, and leads the engineer no nearer to actually verifying the quantitative results of the model. Obviously, a quantitative comparison is the most valuable since it not only will permit direct comparison between several quantitative factors of the model and prototype, but it will also begin to establish the degree with which the model can be relied upon. The hydraulic model is reliable to a high degree, because the laws of similitude in themselves are fundamentally sound when used correctly; but it is of importance to obtain concrete evidence and thereby recognize the influence of any digressions made in conducting the model experiments.

---

[2] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[3] 1st Lt., Field Artillery, U. S. Army, Camp Roberts, Calif.

FACTORS MAKING COMPARISONS DIFFICULT

Prototype Measurements.—The difficulty of obtaining accurate prototype data is the principal reason why correlation work has been delayed. Although several methods are available for measuring the flow, the difficulty lies not only in actually being able to procure the measurements, but also in being able to obtain data applicable to the particular feature of the prototype for which a comparison is to be made. In these respects the discharge, velocity, and water-surface measurements are the most troublesome.

Ordinarily the discharge passing a structure can be measured at a gaging station downstream. During flood flows, however, it is frequently impossible to obtain an accurate measurement because of a change in river section due to bed movement, or because the peak flow may pass before a complete measurement is recorded. Occasionally a flood will occur at night, whereupon an automatic gage, if installed, must be relied upon or the measurement is lost. On the other hand, it is sometimes necessary to wait many years before a structure operates, and even then a sufficient range of head and discharge may not be obtained. Frequently the discharge passing a structure may be derived from the simultaneous operation of a spillway, outlet works, and power house. In such an event it is usually necessary to wait until the flow is derived from only one source, but by that time the range of measurements desired may have passed. Similarly, if a confluence of streams exists between the structure and gaging station, or if heavy rainfall occurs, a discharge measurement adjusted to be applicable to the structure may be of questionable value.

Perhaps the most trouble is encountered when it is desired to obtain velocity and water-surface measurements, not only for comparison with the model, but also for procuring fundamental data. If the flow travels at slow velocities and is confined in narrow channels, these measurements may be made with a minimum of difficulty. If the flow travels at high velocities, however, the measurements are extremely troublesome, regardless of the size of channel. For example, how would one measure accurately the velocity and water surface of the flow in the spillway section of a dam in a western state or of the dam in question here[3a] where the velocity is in excess of 100 ft per sec, and the water surface is affected by air entrainment? It is impracticable to use current meters or pitot tubes to measure the velocity, or point gages to obtain the water surface.

[3a] Of Shasta Dam in California or of Grand Coulee Dam.

The futility of using pitot tubes was realized at Grand Coulee Dam, more from the trouble of maintaining the equipment than from the magnitude of the quantities to be measured. During construction three streamlined, bronze piers, about 21 in. high, were fastened to bronze base plates at the entrance and on the lip of the spillway bucket. Each bronze pier supports five stainless-steel pitot tubes whose static and dynamic openings are connected within the piers to hydrostatic pressure cells similar to those used for measuring seepage pressures in earth dams. Each cell is connected to an instrument panel in a gallery at a higher elevation by copper tubing extending through the dam. To provide check measurements, a by-pass is formed at each cell and connected by copper tubing to manometers in the gallery. In spite of all the precautions made, many of the copper tubes leading to the gallery became plugged and all attempts to clear them failed. Some of the diaphragms of the pressure cells were ruptured during the attempts to clear them. It is also feared that debris in the tailwater has plugged or coated some of the pitot-tube openings and may have damaged the entire assembly from impact, which may be even more severe since the start of spillway operation in June, 1942. As a result, it is doubtful that any reliable measurements can be taken with this equipment, even though carefully planned and all precautions taken to assure an efficient arrangement. It was realized that the pitot-tube assembly might be inadequate, so the salt-injection method will be attempted making use of electrode plates, which were also installed at intervals down the spillway section of the dam. As yet no suitable methods have been developed to measure the water surface for the high-velocity flow occurring in spillways at high dams or in other open channels.

Difficulties are also encountered when measuring velocities in large outlets under high heads because of the tremendous forces involved, which either necessitate extremely rigid instruments or prohibit the use of instruments entirely. Lack of access also eliminates any measurements, particularly if previous preparations have not been made for installing the necessary equipment during construction.

The success with which other measurements can be made in the field, such as pressures on boundary surfaces, vibrations, and air-demand, depends more on the careful planning of the instruments, the ability to foresee their workability, and their installation, than upon the difficulties of actually being able to obtain the measurements.

Planning and Cost.—Since the success of any verification depends upon the flow measurements made on a model and its prototype, it is compulsory to formulate plans for the comparison during the laboratory experiments and during the design of the structure. Such plans should provide for quantitative model data covering several flows, not just the maximum, and for the installation of piezometers, gages, and special instruments in the prototype design as dictated by the model tests. As a result of careful planning, the unfortunate condition of having prototype data in one range and model data in another will be avoided, even though it might be possible to regulate the prototype tests to conform to similar ones of the model—a condition which rarely exists.

The success of any planning, moreover, depends upon the circumstances under which the laboratory tests are made. Usually tests are performed under considerable strain because of the limited time available after the preliminary design is submitted for testing and before the final design is prepared for construction. Discovery of faults in design during construction also requires model experiments to be made quickly so as not to interfere with the progress of construction. Because of this constant rush, the model data usually do not cover the desired range, thorough checks on the data are not possible, and the adequacy of the prototype measuring equipment may suffer.

If some flow measurements have been made over a period of years at a structure never previously tested by a model, it is of considerable interest to construct a model and secure the data necessary for a comparison. This reversed procedure has been tried at the University of Iowa, at Iowa City, Iowa (see paper by Mr. Soucek in this Symposium), as an academic study, and was used recently at the Bureau of Reclamation, but for the primary purpose of revising hydraulic conditions at an existing structure.

The cost of taking field measurements will be greatly reduced if plans provide for the installation of equipment during construction Once the proper arrangements have been made, the actual cost of making the measurements will be small in comparison with the entire cost of the structure itself. The cost of instrumentation, on the other hand, is relatively high and in many cases prohibits any field measurements.

Properties of the Model.—Not all of the difficulties in making comparisons are attributed to the prototype tests; in fact, most of them must be traced directly to the model experiments, assuming, of course, that the prototype data are reasonably accurate. This is evident since any comparison actually is a check on the model tests, not on the prototype. In open-channel studies a small error is introduced because it is usually impossible to select a scale ratio that will permit rigorous similitude between model and prototype roughness, the model surfaces being too rough. During experiments on different scale models of sluices in the Assuan Dam (4), in Egypt, it was found that models having a surface roughness commonly used today agreed with the prototype discharge within 2%. When the model surfaces were roughened artificially with sand, however, the model discharge decreased 5%. From measurements on five geometrically similar models of spillways, F. Eisner (5) showed that there was a definite variation in the coefficient of discharge with the scale ratio. The relative roughness of the boundary surfaces was believed to be the important factor, the effects of viscosity being almost negligible. Because of the uniform variation between the coefficients of discharge and the relative roughness, Eisner predicted the prototype coefficients reasonably well, in comparison to extrapolating by the Froude number.

A much greater error is introduced in model studies of closed conduits because it is practically always impossible to operate the model at Reynolds numbers as high as those obtained at the prototype. It is important to realize, therefore, that the model friction losses are too large, and that a correction should be made to the model data before estimating any corresponding prototype values. Other errors introduced by such fluid forces as capillarity and elasticity are negligible in the problems usually studied in hydraulic laboratories. In any event it is well to know the requirements for similitude, how to apply them, and their effects on the results.

In regard to the model data, they are usually more accurate than the prototype measurements, since errors in the field are not controlled as easily as they are in the laboratory. In this regard it should be determined if the model and prototype dimensions are geometrically similar, because frequently the field structure may not be constructed in accordance with the dimensions used for the model.

PROTOTYPE FLOW MEASUREMENTS

Since it is necessary to take nearly the same measurements in the field as in the laboratory, instruments should be available for measuring the discharge, pressure on boundary surfaces, water-surface elevations and profiles, velocity, vibration, air-demand, bed movement, and for photography. Keeping in mind the difficulties mentioned in obtaining prototype measurements, several well-known methods will be outlined. Although the instruments used in the laboratory and in the field are basically similar, the prototype equipment must be designed to withstand tremendous forces and be convenient for use. With little precedent to follow, one must rely on good judgment for the design of the instruments and, at the same time, be able to foresee their efficacy.

Discharge.—Open-channel flows can usually be measured by current meters, pitot-tube or pitometer traverses, weirs, Parshall flumes, floats, and the salt-injection method. The particular method used depends on the accuracy required, the size and type of channel, quantity of water, and its velocity. Accordingly, a current meter is used most frequently in rivers, lined channels, and in canals. Smaller quantities in canals are measured with weirs or Parshall flumes. For high-velocity flow in lined channels, the salt-injection method is used unless the discharge measurement can be made at some other place where a more convenient method could be adapted.

Discharges in closed conduits are obtained by pitot-tube traverses, the salt-velocity method, the Gibson method, and by venturi meters when installed in the conduit. These methods are adaptable only if plans have been made for their use, since it has been pointed out that it is often impossible to gain access to many conduits in dams. If access is impossible, the discharge probably could be obtained downstream at a gaging station.

In most of these methods a calibration of the instruments is required, and it is debatable whether the calibration of some of them will be the same at the test site as it is at the place of calibration. Using a rating curve at the prototype which has been established from model tests obviously defeats the purpose of making a comparison.

Pressure.—Piezometers, which are used almost exclusively to obtain pressures on boundary surfaces, should be installed on the prototype in positions similar to those on the model. The installation of piezometers in two of the 102-in. river outlets of the dam[3b] is offered as an example. Piezometers in these outlets have been placed on the vertical and horizontal diameters. Each opening consists of a high-tension bronze plug drilled with a $\frac{1}{4}$-in.

[3b] Grand Coulee Dam.

hole and screwed into the steel plate lining of the conduits, with the orifice end projecting about $\frac{1}{16}$ in. past the interior surface. After this projection had been ground flush with the surface lining, a $\frac{1}{16}$-in. radius was formed in the orifice end and all burrs were removed. A short length of $\frac{1}{8}$-in. standard brass pipe connects each plug to a steel junction box, where a reducing coupling is used to connect each piezometer plug to $\frac{3}{8}$-in. outside diameter by 0.035-in. wall, soft copper tubing. Junction boxes placed opposite each plug are connected by 2-in. galvanized conduit, through which the soft copper tubing is collected, and lead to six headers recessed in an inspection gallery below the outlets. The headers, one placed above the other, are made of 2-in., extra-strong brass pipe, with $\frac{1}{8}$-in. brass air cocks placed in rows along the top of each header to make connection with each piezometer line. Each header is connected to a high-pressure line at one end and to a bleeder pipe at the other end, which, in turn, is connected to a pressure measuring instrument through a by-pass.

A similar method could be used for measuring pressures on concrete surfaces, except the piezometers would be attached to a metal plate set in the concrete. In any event the installation must be made during construction.

To measure the pressures, it is common practice to use water or mercury manometers, and dial gages. Manometers are not always satisfactory because of their fragility, the fluctuations of the mercury columns, and the magnitude of the pressures frequently encountered. Commercial-type dial gages are usually not precise enough. A fluid-pressure scale of the type used for turbine testing proved satisfactory in high-head tests at the two dams.[3c] Pressures as great as 300 lb per sq in. can be recorded within 0.25 lb per sq in. to pressures of 100 lb, and within 0.25% at any pressure greater than 100 lb with this instrument, which is compact and of simple construction. When reading piezometer pressures, the pressure is transmitted to a copper tube grid mounted in gimbals in a weighing beam in the base of the instrument. The weighing piston, which is lapped fit in the cylinder, is restrained from being forced upward. The resulting downward reaction on the weighing cylinder is transmitted, therefore, to the lower weighing beam, and measured on an upper scale beam mounted above the base. The weighing piston is rotated by a small electric motor while readings are being made to avoid static friction with the walls of the cylinder.

Pressure cells used in combination with a group of piezometers and an oscillograph may be used, but the cost of such equipment is prohibitive in many cases unless it is available from previous experiments. This method has one advantage in that a continuous record of the pressure fluctuations is recorded on the film or sensitized paper in the oscillograph.

In making any pressure measurements either on the model or prototype, the care with which the piezometer openings are installed cannot be overemphasized. Any burr will affect the reading seriously, and excessive surface roughness either upstream or downstream from the piezometer opening will influence the measurements. It is essential to have a high-pressure water line for flushing the piezometer lines clear of air bubbles, and the tubing from the piezometer to the measuring instrument should be alined, if possible, to eliminate any bends or sags that might trap air.

[3c] Boulder Dam (Arizona-Nevada) and Grand Coulee.

Water Surface.—Although these measurements are not as important as the discharge and pressure, nevertheless a check is desirable on such design assumptions as freeboard and roughness coefficients. In connection with the work of the Society's Special Committee on Hydraulic Research, the problem of air entrainment in high-velocity flow requires, in part, a measurement of the water surface in open channels at field structures. In comparison to the low-velocity flow in models where the water surfaces are relatively smooth and well-defined, there is actually no definite water surface for the high-velocity flow at field structures, but only a mass of spray and aerated water. Efforts to point-gage such a water surface in a narrow wasteway in the Yakima project, Washington, were successful in a measure, but only an "average" water surface was actually obtained.

Although point gages may be useful for measuring water surfaces in narrow channels, it might be more feasible at wide channels to use a system of cables to manipulate a heavy target, which could be located by transits. A profile of the water surface could also be obtained by suspending a plumb bob from the training walls, or from a grid painted on the training walls; but such measurements would be affected by the wall friction.

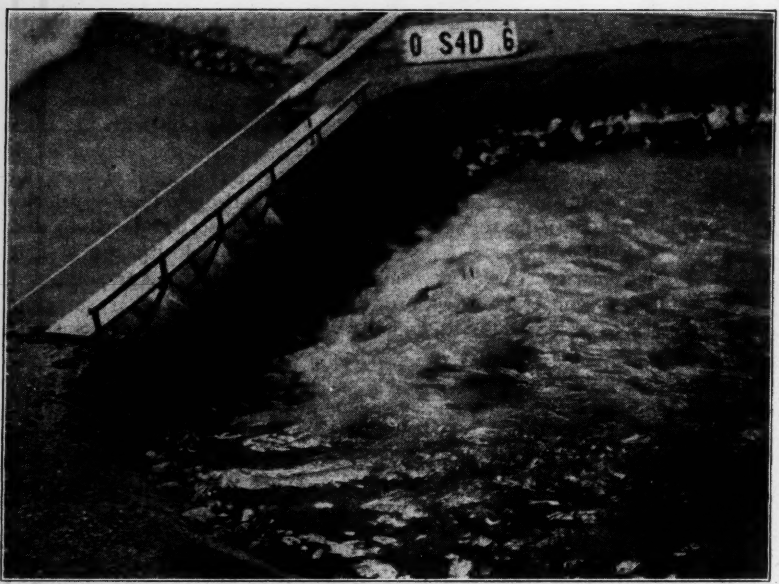
For measuring water-surface elevations in a reservoir, use has been made of the familiar staff gage and of several commercial instruments. Those based on the principle of a long air column transmitting the pressure should be avoided because of the effect of temperature change on the air column, and because of possible air leaks in the tubing.

Velocity.—The methods available are directly associated with discharge measurements, so use is made of current meters, pitot tubes, salt-injection method, and floats. Although somewhat unsuccessful at Grand Coulee Dam, the installation of pitot tubes, as discussed herein, may prove successful under different circumstances. The accuracy required for model-prototype comparisons eliminates the use of surface and subsurface floats.

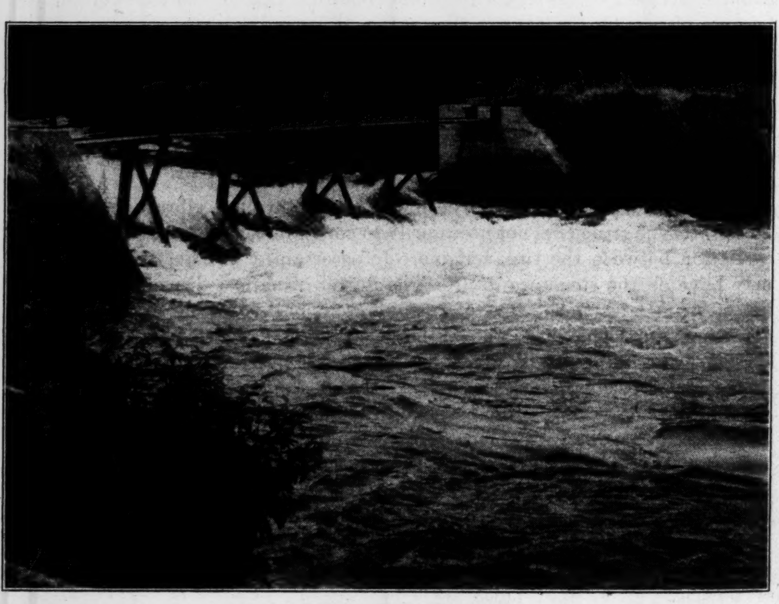
Vibration.—It is frequently desirable to measure the vibrations occurring at a prototype structure, not necessarily to compare with the model to establish a relation between the two, but more to ascertain what effect the vibrations may have on the structure itself. The flutter of nappes discharging over the lips of drum gates on the crest of movable weirs or dams is one example of such vibration (6). The discharge at large valves and regulating gates also is frequently accompanied by vibrations. Accelerometers and pressure cells were used with an oscillograph to measure vibrations due to fluttering nappes at a dam in another western state.[3d] The instruments were designed on the principle that a change of resistance occurs with a change in pressure on a pile of carbon disks. The accelerometers measured the accelerations in three directions in the dam, and the pressure cells were used to measure the amplitude and frequency of the inducing forces exerted on the dam by the flutter of the nappes spilling over the lip of drum gates. The instruments were calibrated by noting the deflections produced on the moving elements of the oscillograph under

---

[3d] The Black Canyon Dam in Idaho.

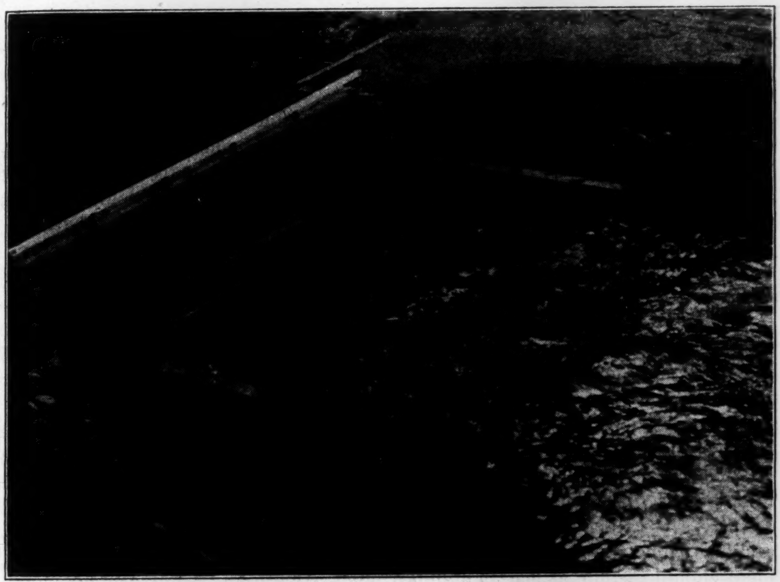


(a) 1 : 15 Scale Model, Before Revision

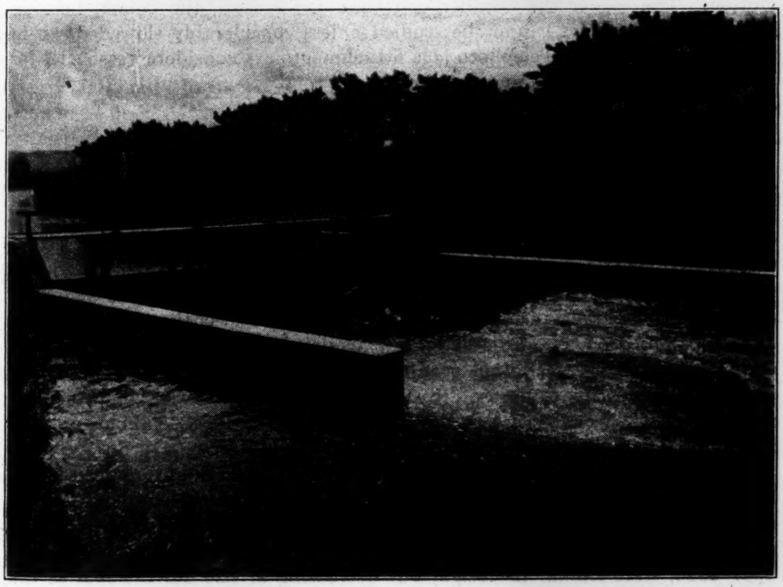


(b) Prototype Before Revision

FIG. 1.—CHECK DROP NO. 4, SUNNYSIDE MAIN CANAL,



(*c*) **1 : 15 Scale Model, After Revision**



(*d*) **Prototype After Revision**

Yakima Project, Washington (Discharge, 1,300 Cu Ft per Sec)

various conditions created on a shaking table. Less complicated instruments, such as vibrating reeds, which give only the frequency, have been used elsewhere, the amplitude being determined by magnifying the vibration and using an oscillograph.

Air-Demand.—It is often necessary and desirable to measure the quantity of air required on a prototype to relieve sub-atmospheric pressures under nappes, in conduits downstream from needle valves and regulating gates, and in regulating gates and valves themselves. It is a difficult problem to determine the size of air ducts required on the prototype by model tests, because of the uncertainty of the similitude involved; thus any check on the prototype will aid considerably in establishing the transference relations. To measure air quantities on the prototype (and in the model) an anemometer, orifice plate, or pitot tube could be installed in the air ducts. Although not entirely suitable for prototype measurements because of the large quantities involved, a rotameter may be used for model tests.

Bed Movement.—The scour to river channels downstream from dams and spillways is obtained by surveys or by soundings. These measurements, however, can be used only for a qualitative comparison.

Photography.—Still and motion pictures of all flow conditions observed both in laboratory and field work are invaluable for purposes of comparison. Frequently a startling similarity is seen between a model and prototype photograph of some flow phenomenon. For still pictures, the photographic equipment should include at least a standard view camera and a miniature camera, both with fast lens. The motion-picture camera should be equipped for slow-motion. The higher the number of frames per second, the better, because many prototype flows cannot be studied unless considerably slowed down in the motion pictures. Telephoto lens attachments are considered essential for both the still and motion-picture cameras.

EXAMPLES OF QUALITATIVE COMPARISONS

Irrigation Structures.—Such relatively small irrigation structures as diversion dams, check drops, wasteways, and turnouts offer opportunities for making model-prototype comparisons because of their continuous operation at various discharges during an irrigation season. The primary concern at these structures is to prevent scouring of the canal downstream by using a hydraulic-jump stilling basin wherever possible. Consequently, it is sufficient to make only a qualitative comparison of any erosion that occurs. Most of the small irrigation structures in use today are based on designs developed over a period of years, so that model studies have not been utilized except for revising some existing structure of inadequate design. Such a condition required a laboratory investigation of Check Drop 4 in the Sunnyside Main Canal, Yakima project, Washington.

The Prototype.—Because of a gradual growth in irrigation, the Main Canal had to be increased in capacity. To maintain a proper gradient in the canal, twenty-three check drops were installed at intervals of about one and one-half miles. These drop structures, which were built during the period 1907–1916, were similar in design, but decreasing in size downstream as the discharge in the canal became less due to deliveries. Each structure consists of a rectangular concrete basin divided into bays by concrete piers which are surmounted by steel brackets. By placing flashboards between each pair of brackets, the depth of flow between successive drops can be regulated. Wing walls at each end of the concrete basin extend into the canal banks at right angles. The drop in water surface at these structures varies from 30 in. for the maximum to 7¼ in. for the minimum, with a maximum discharge of 1,300 cu ft per sec at the first drop gradually decreasing to 520 cu ft per sec at the last one.

Not many irrigation seasons had passed before excessive scouring developed in the canal immediately downstream from the drops, particularly at the larger ones. All attempts to check the erosion failed, until finally the width of the canal below the drops was increased as much as 50% in some cases. In 1938 the problem of eliminating this scour was assigned to the hydraulic laboratory of the Bureau of Reclamation, Denver, Colo.

Model Tests.—Since the drops were similar in design, Check Drop 4 was selected for study, and a 1 : 15 scale model was constructed. The initial tests in the laboratory demonstrated that the model would reproduce the flow conditions and scour existing in the field (Figs. 1(*a*) and 1(*b*)). Further analysis disclosed that, because of the small drop in water surface, standing waves developed instead of a more efficient energy dissipator, such as the hydraulic jump. As a result, high surface velocities proceeded downstream in the center of the canal, thereby producing large return eddies along the canal banks which eroded them severely.

A satisfactory solution to this problem was not readily apparent, for not until many designs had been tested was a solution found which would eliminate the unfavorable conditions completely. This solution consisted of adding curved training walls in the entrance to the drop, extending vertical walls and a concrete floor downstream to form a stilling pool, and placing a deflector between these walls directly above the concrete basin to force the flow under the tailwater. These revisions created a more uniform flow distribution through the structure, and provided energy dissipation sufficient to eliminate the high surface velocities which had caused side eddies to scour the canal banks.

Comparison.—After the prototype had been revised, observations of its performance showed that it was acting in a manner similar to that predicted by the model tests. Figs. 1(*c*) and 1(*d*) show the revised design of the model and prototype. The collection of debris at the prototype structure is evidence of roller action in the stilling pool. This is indicative of energy dissipation throughout the entire depth, instead of only on the surface as noticed in Figs. 1(*a*) and 1(*b*).

During the laboratory studies a similar collection of debris was noticed, but it was given little consideration since it interfered with photographing the action of the model. In the field, however, it has been of considerable value in clearing the canal of large quantities of debris, conveniently removed at the drop structure. Three miles downstream at Drop 6 (which was also revised), there is little trash remaining in the canal.

The results of this comparison are of considerable satisfaction to those familiar with the problem, but as previously mentioned such a correlation is more convincing to those directly concerned. Nevertheless, this study is offered as convincing proof that model tests are an excellent tool for the hydraulic engineer, and that they can be relied upon to a high degree.

Examples of Quantitative Comparisons

The Prototype.—Sixty 102-in. outlets were provided in three tiers of 20 each at the dam[3b] to release a total discharge of 225,000 cu ft per sec (Fig. 2). Each outlet has a bellmouth entrance followed by a length of straight conduit, which terminates in an elbow and cone at the downstream face of the dam. Each outlet is closed by a service gate and an emergency gate, partial gate openings never being used with ring-seal or paradox gates. Except for a part of the lower tier, each outlet has a steel-plate lining. The three tiers of outlets at El. 1136.67, El. 1036.67, and El. 935.76 are designated as the upper, intermediate, and lower outlets, respectively. Since the outlets occur in pairs in each tier, 15 ft on centers, further designation is given by east (E) and west (W), and by the block number in which they occur. Fig. 3, for example, shows upper outlet 63W discharging down the face of the dam.

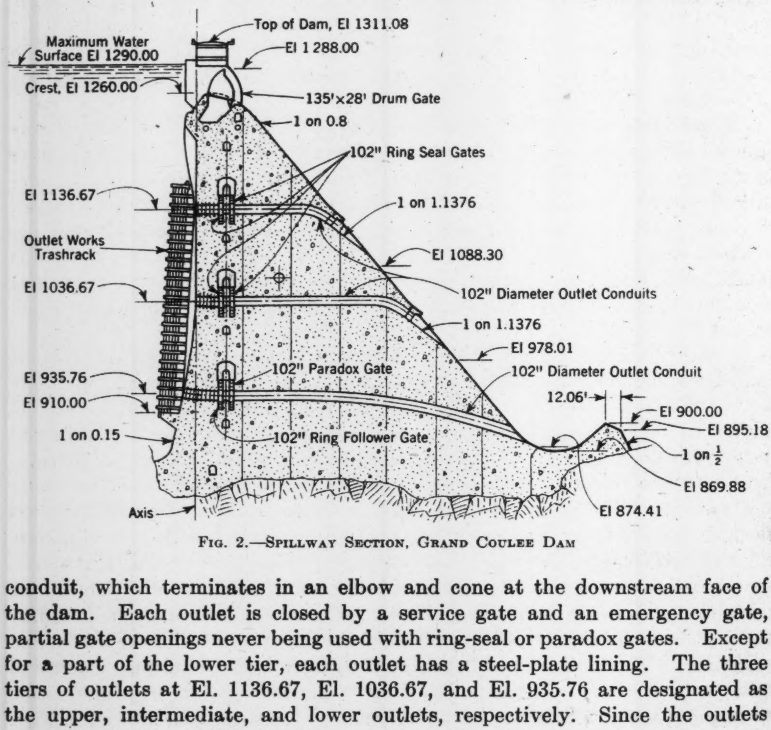

Top of Dam, El 1311.08
Maximum Water Surface El 1290.00
El 1 288.00
Crest, El 1260.00
135'×28' Drum Gate
1 on 0.8
102" Ring Seal Gates
El 1136.67
1 on 1.1376
Outlet Works Trashrack
El 1088.30
El 1036.67
102" Diameter Outlet Conduits
1 on 1.1376
El 978.01
102" Paradox Gate
102" Diameter Outlet Conduit
El 935.76
12.06'
El 910.00
El 900.00
El 895.18
1 on 0.15
102" Ring Follower Gate
1 on 1/2
El 869.88
Axis
El 874.41

Fig. 2.—Spillway Section, Grand Coulee Dam

During the design of the outlets, plans were formulated for installing piezometers in one intermediate outlet, 51E, and one upper outlet, 51E, 100 ft immediately above. Piezometers were not placed in the lower outlets because they were not completely steel-lined and because detailed model studies were not made of them. In locations similar to those in the model, a series of eleven piezometers have been placed in the bellmouth entrances along both the invert and crown (vertical center line), and on the sides (horizontal center line). Similarly, the horizontal part of the two conduits contains twelve piezometer rings in the intermediate outlet, and ten in the upper outlet; both have ten rings in the elbow and cone. The installation methods have been described and the vertical section of an intermediate outlet shown in Fig. 4 indicates the distribution of piezometers.

[3b] Grand Coulee.

Model Tests.—A résumé of the 1 : 17 scale model tests on the outlets of the dam[3b] has been given in another paper (7). The tests were concerned with the bellmouth entrance, the profile of the outlets, the elbow and cone, and the deflectors placed in the spillway immediately above the exit of the upper and intermediate outlets, which can be seen in Figs. 3 and 4(*a*). Measurements on the model were concerned primarily with the pressures on the boundary surfaces, measured with water and mercury manometers, and the discharge for several heads, obtained from weir measurements.

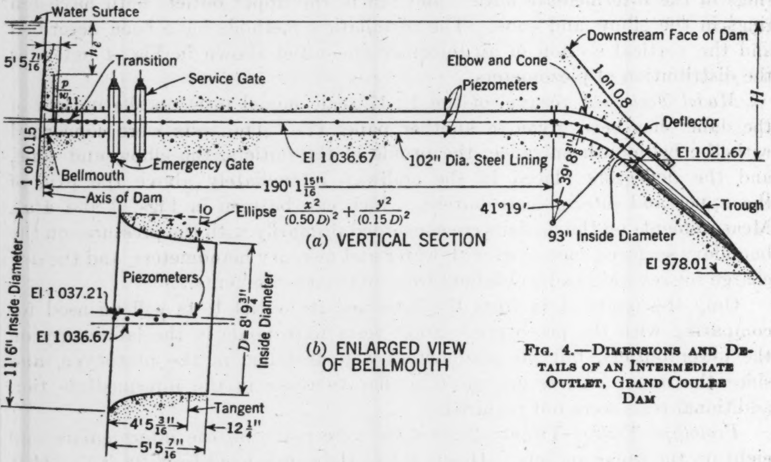
Only the model data from the intermediate outlet tests will be used for comparing with the prototype. Some tests were made in the laboratory on the lower outlets, but no piezometers were installed in the prototype, and since the upper outlets are quite similar to those in the intermediate tier, additional tests were not required.

Prototype Tests.—Twenty-three tests were made on the intermediate and eight on the upper outlets. Heads above the exit ranged from 28 ft to 234 ft

FIG. 3.—UPPER OUTLET 63W; GRAND COULEE DAM, DISCHARGE 2,400 CU FT PER SEC

for the intermediate outlet, and from 33 ft to 133 ft for the upper outlet. In addition to measuring pressures with the fluid pressure scale, the reservoir elevation, water temperature, tailwater elevation, discharge, and the number of outlets open in each tier were recorded. To make comparisons with the model of the intermediate outlet, it was possible to use only those tests in which the adjacent outlet, 51W, and the upper outlet, 51E, were closed. This not only conformed with the laboratory arrangement, but it also eliminated from consideration any secondary effects on the pressures. These effects were discovered first during a separate model investigation of the bellmouth, and later during the prototype tests when it was noticed that pressures in the bellmouth were slightly lowered when the adjacent outlet 51W was operating, while pressures in the elbow and cone were raised when the upper outlet 51E was operating.

[3b] Grand Coulee.


Water Surface
5' 5 7/16"
h_e
Transition
Service Gate
$\frac{p}{w}$ 11
1 on 0.15
Bellmouth
Emergency Gate
El 1 036.67
102" Dia. Steel Lining
190' 1 15/16"
Axis of Dam
Downstream Face of Dam
Elbow and Cone
Piezometers
1 on 0.8
Deflector
El 1021.67
H
39' 8 3/8"
41°19'
93" Inside Diameter
Trough
El 978.01
Ellipse $\frac{x^2}{(0.50\,D)^2}+\frac{y^2}{(0.15\,D)^2}$
10
x
y
Piezometers
El 1 037.21
El 1 036.67
D = 8' 9 3/4" Inside Diameter
11' 6" Inside Diameter
Tangent
4' 5 3/16"
12 1/4"
5' 5 7/16"

(a) VERTICAL SECTION

(b) ENLARGED VIEW OF BELLMOUTH

FIG. 4.—DIMENSIONS AND DETAILS OF AN INTERMEDIATE OUTLET, GRAND COULEE DAM

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

Model-Prototype Pressure Characteristics.—The first comparison given in Fig. 5 is intended to show the characteristics of the hydraulic gradient. Because the field tests could not always be made when the reservoir reached a corresponding elevation, the model and prototype heads shown are not comparable. Upon first consideration it would seem desirable for the heads to agree, but even if they did, an agreement between the model and prototype pressures would be lacking. The reason for this is evident from Fig. 6, where it may be seen that the model tests were made at much lower Reynolds numbers (R); hence the friction losses

$$f = \frac{h_f}{\frac{L}{D}\,\frac{V^2}{2\,g}} \qquad \text{(1)}$$

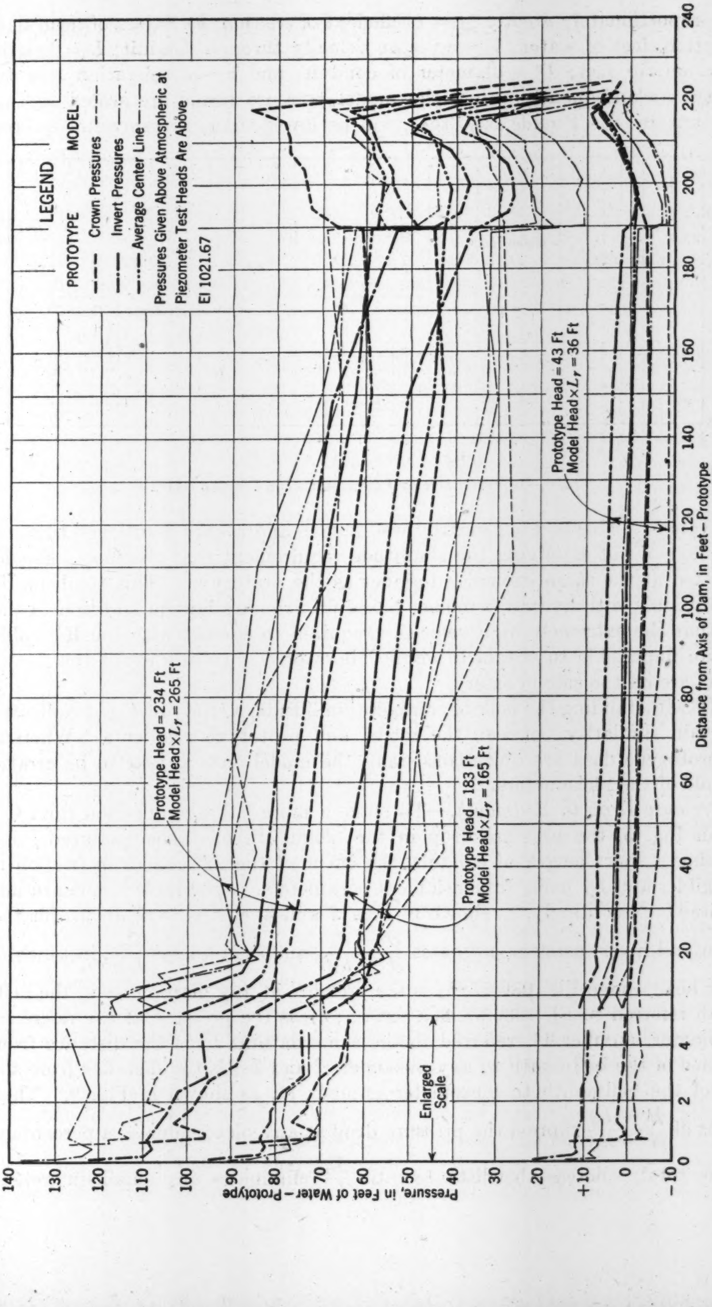


Fig. 5.—Pressure Characteristics, Model and Prototype, Grand Coulee Dam

were proportionately greater (f = coefficient of friction; h_f = loss of head due to friction, feet of water; V = average velocity through conduit; L = length of section, in feet; D = diameter of conduit; and g = acceleration due to gravity). Consequently, when the model pressure heads are scaled to the prototype by the Froude law, they will be lower than corresponding prototype values. This fact is generally well-known and is only mentioned to show the effect of not satisfying the similitude requirement that the model should be tested at the same Reynolds number as the prototype. This condition is incompatible with hydraulic models based on Froude's law, in addition to the fact that the extremely high velocities required to comply with the Reynolds law are impossible to obtain in many laboratories. In any event the model results are on the side of safety.

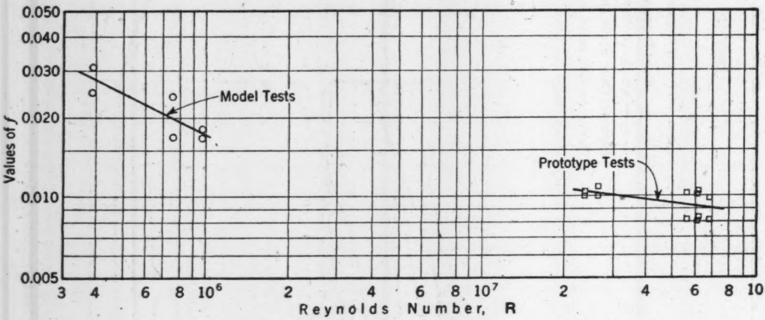


FIG. 6.—COEFFICIENTS OF FRICTION, GRAND COULEE DAM

Notwithstanding the lack of complete similitude, a study of Fig. 5 indicates a certain similarity between the model and prototype gradients. Whereas the prototype data are quite consistent, the model data appear to be erratic at some of the piezometers.

Pressure Drop in Bellmouth.—To make a more valid comparison than the one in Fig. 5, the pressure drop in the bellmouth has been analyzed. By choosing a short length of conduit, the effect of the differences in friction is negligible, and by using dimensionless parameters, comparable heads are not required. Accordingly, as shown in Figs. 7 and 8, curves were drawn for the crown and invert using as ordinates $\frac{H - h_x}{H - h_{11}}$, and as abscissas $\frac{L_x}{L_{11}}$, in which H is the head above El. 1036.67; h_x is the pressure at any piezometer in the bellmouth referred to El. 1036.67 as a datum; h_{11} is the pressure at the reference piezometer, number 11, referred to the same datum; L_x is the distance from the face of the bellmouth to any piezometer; and L_{11} is the distance from the face of the bellmouth to piezometer number 11, as shown in Fig. 9. Then values of $\frac{H - h_x}{H - h_{11}}$ express the pressure drop at any piezometer as a percentage of the total, whereas the distance ratio $\frac{L_x}{L_{11}}$ eliminates any small differences between the location of piezometers in the model or prototype. Two of the prototype tests were omitted from the diagrams so as not to confuse them with more superimposition of the points, but all of the model values are included. The actual pressures at the piezometers in the bellmouth were all positive (above atmospheric) except for low heads, and these did not exceed a vacuum of 10 ft of water.

The prototype curves illustrate that the percentage pressure drop is practically constant at nearly all the piezometers regardless of the head. This is not true for the model tests, even though the average values of the model are within approximately 5% of the prototype. Reasons for the dispersion of the model data may be explained by experimental error, different approach conditions, and unstable flow in the model at lower Reynolds numbers, although above critical Reynolds numbers.

Experimental errors are probably due to the installation of the model piezometers, even though great care was taken to install them properly. Nevertheless, any minute obstructions on the boundary surfaces and in the piezometer openings have serious effects. The laboratory experiments, moreover, were made under considerable duress to enable the design to keep ahead of construction in the field. Consequently, the customary close checks on the data could not be made as complete as they would have been under a more normal procedure. The different approach conditions refer to the absence in the laboratory of a trashrack upstream from the bellmouth. However, it was observed from separate tests on the bellmouth that the trashrack effects were negligible.

Evidently the differences in friction play a small part even though a comparison is confined to the bellmouth alone. No estimate can be made readily of the relative roughness of either the model or prototype surfaces, but, since the type of surface in each is nearly the same, the relative roughness of the model would be greater. Consequently at lower Reynolds numbers, the resistance factor for the model will vary considerably more within its range of tests. It can be assumed further that the model tests were for rough flow, whereas the prototype tests were more nearly smooth flow. Finally, it might be added that a transition from a laminar sublayer to a turbulent sublayer will develop less rapidly in the model, thus influencing the velocity distribution and pressures. It is reasonable to assume, therefore, that the model flow is not sufficiently stable to give the consistent data obtained for the prototype. Accordingly, it is believed that the laboratory tests should have been made at greater velocities (higher Reynolds numbers), disregarding heads based on the scale ratio of the model. Had this been done, the model data as analyzed in Figs. 7 and 8 would have been more consistent and would have approached the prototype values more closely, although 5% is certainly good agreement for the average curves.

Pressure Drop in Elbow and Cone.—Dimensionless curves have been drawn in Figs. 10 and 11 to show the pressure drop in the elbow and cone in a manner similar to that described for the bellmouth comparison. To eliminate the differences in friction up to the elbow, the pressure drop has been expressed as

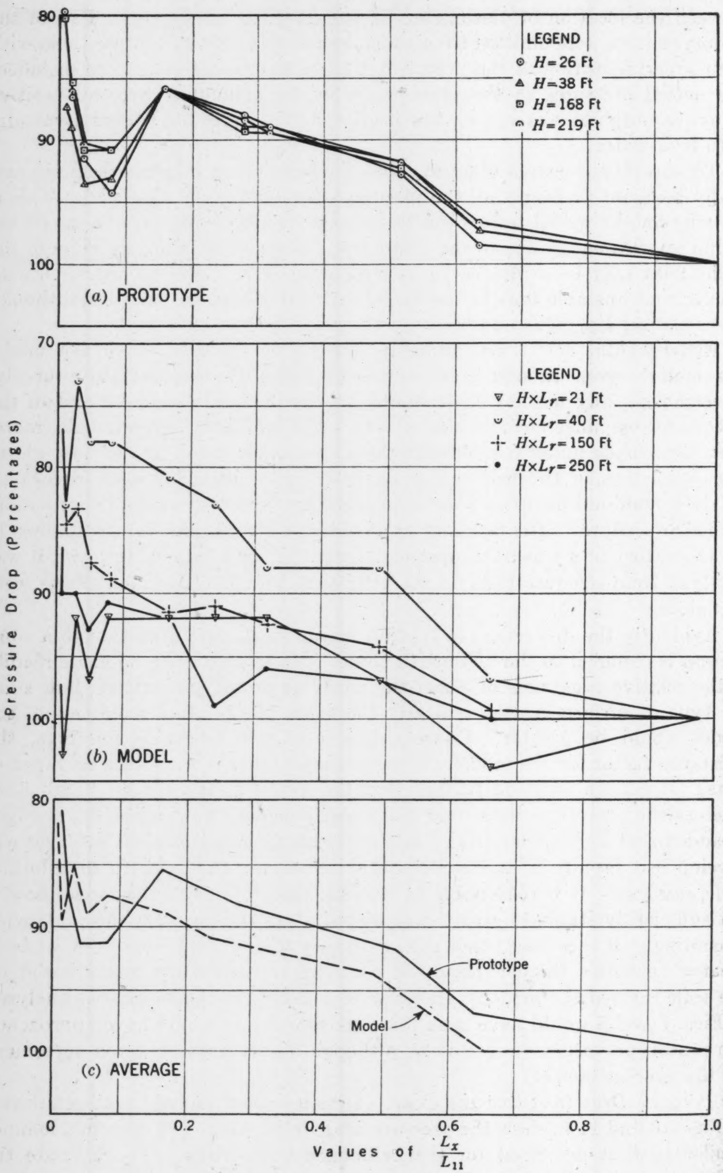


FIG. 7.—COMPARISON OF PRESSURE DROPS ON CROWN OF BELLMOUTH

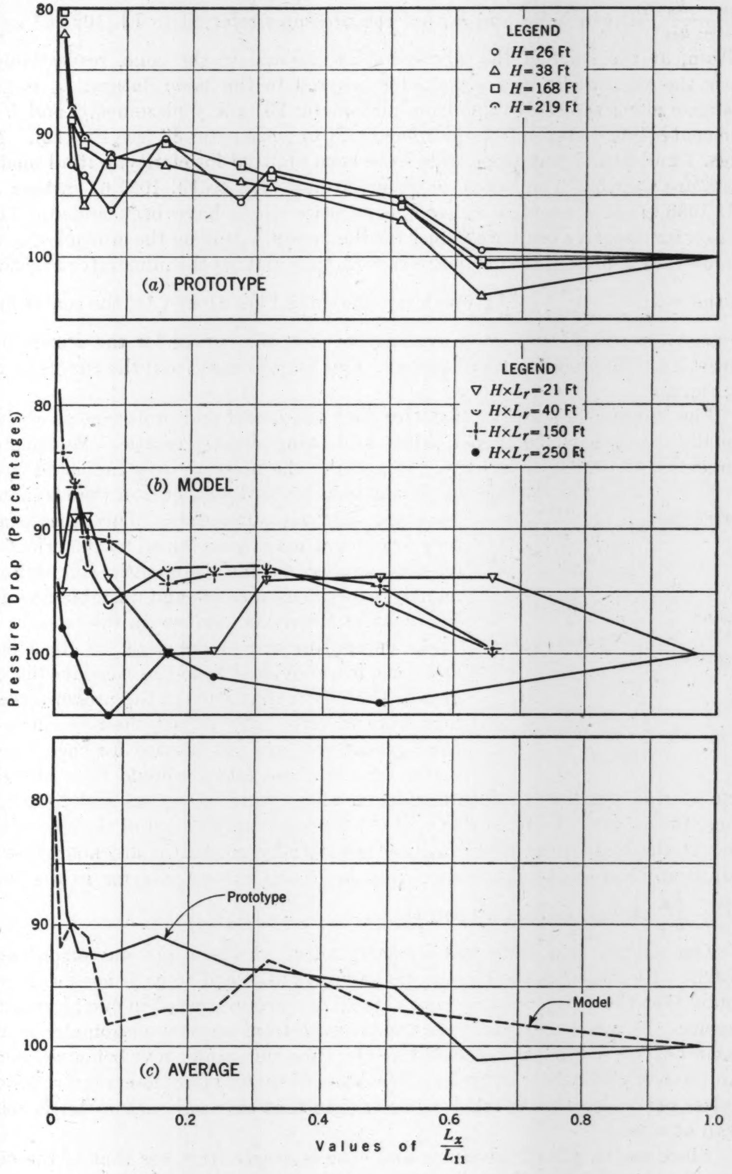


Fig. 8.—Comparison of Pressure Drops on Invert of Bellmouth

$\frac{h_{23} - h_x}{h_{23} - h_{33}}$, in which h_{23} and h_{33} are the pressures referred to El. 1021.67 as a datum, at the start of the elbow and at the end of the cone, respectively; h_x is the pressure at any piezometer referred to the same datum; L_x is the distance along the center line from piezometer 23 to any piezometer; and L is the center-line distance from piezometer 23 to piezometer 33 (see Fig. 12). In Figs. 7 and 8, two prototype tests have been omitted for clarity, but all model tests are shown. The heads, however, are referred to El. 1021.67 instead of El. 1036.67, and average center-line pressure drops have been added. The reason for negative ordinate values for the pressure drop on the crown is due to an increase in pressure from piezometer 23, thus making the numerator negative in the ratio $\frac{h_{23} - h_x}{h_{23} - h_{33}}$. Although not shown in Figs. 10 and 11, the center-line pressure drop falls as a good average between the curves for the crown and invert for both model and prototype. This may be seen from the curves given in Fig. 5.

Figs. 10 and 11 show the data for both the model and prototype as almost equally consistent, the model values still being slightly erratic. Reasonable agreement, however, is lacking. In general, the pressure drop indicated from the model tests is about 30% greater than is shown from the field measurements. This unsatisfactory agreement may be explained by the principal reasons applied to the bellmouth comparison; namely, experimental errors and unstable flow for the range of Reynolds numbers in the model.

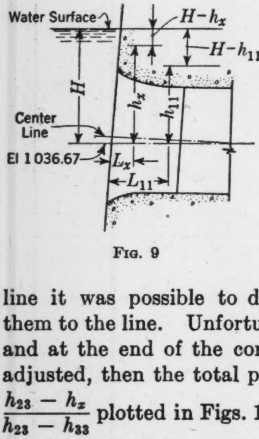


FIG. 9

In computing the model data for the elbow and cone, it was evident from the excessive dispersion of the points that some of the pressure readings were in error. To adjust these, a curve of head versus pressure was plotted for each piezometer. Since these curves should be a straight line it was possible to detect which readings were erroneous and to adjust them to the line. Unfortunately, if the pressures at the start of the elbow, h_{23}, and at the end of the cone, h_{33}, are not initially correct or are not properly adjusted, then the total pressure drop $h_{23} - h_{33}$ causes an error in the ratio $\frac{h_{23} - h_x}{h_{23} - h_{33}}$ plotted in Figs. 10 and 11.

One of the field tests was similarly adjusted where obvious errors were noted. By checking the piezometer numbers attached to each header, it was found that the erroneous readings were all in a group located in one piezometer header. It was concluded that the pressure from another piezometer in the same header was being registered at the time this group was being recorded. An error of this nature is easily made when so many piezometers are required, unless particular care is taken to be certain that only one piezometer is being read at a time.

Since the length of the elbow and cone is nearly six times that of the bellmouth, it is evident that the differences between the model and prototype

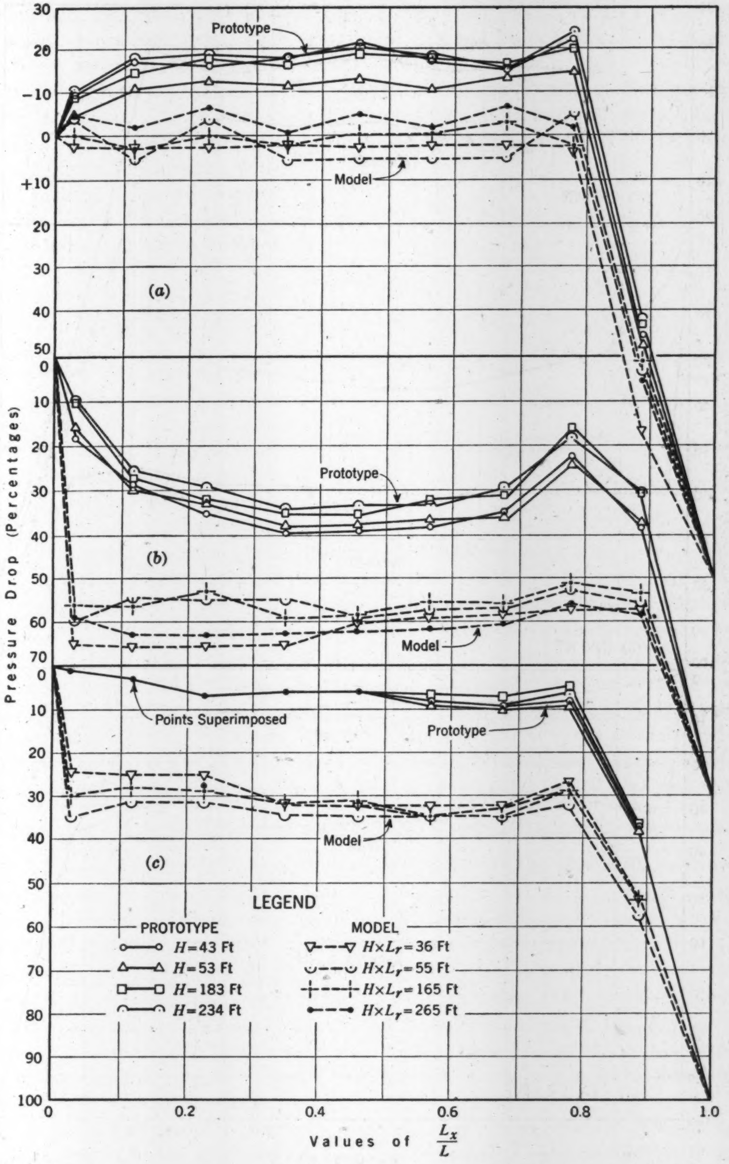


FIG. 10.—COMPARISON OF PRESSURE DROPS IN ELBOW: (*a*) CROWN; (*b*) INVERT; AND (*c*) CENTER LINE

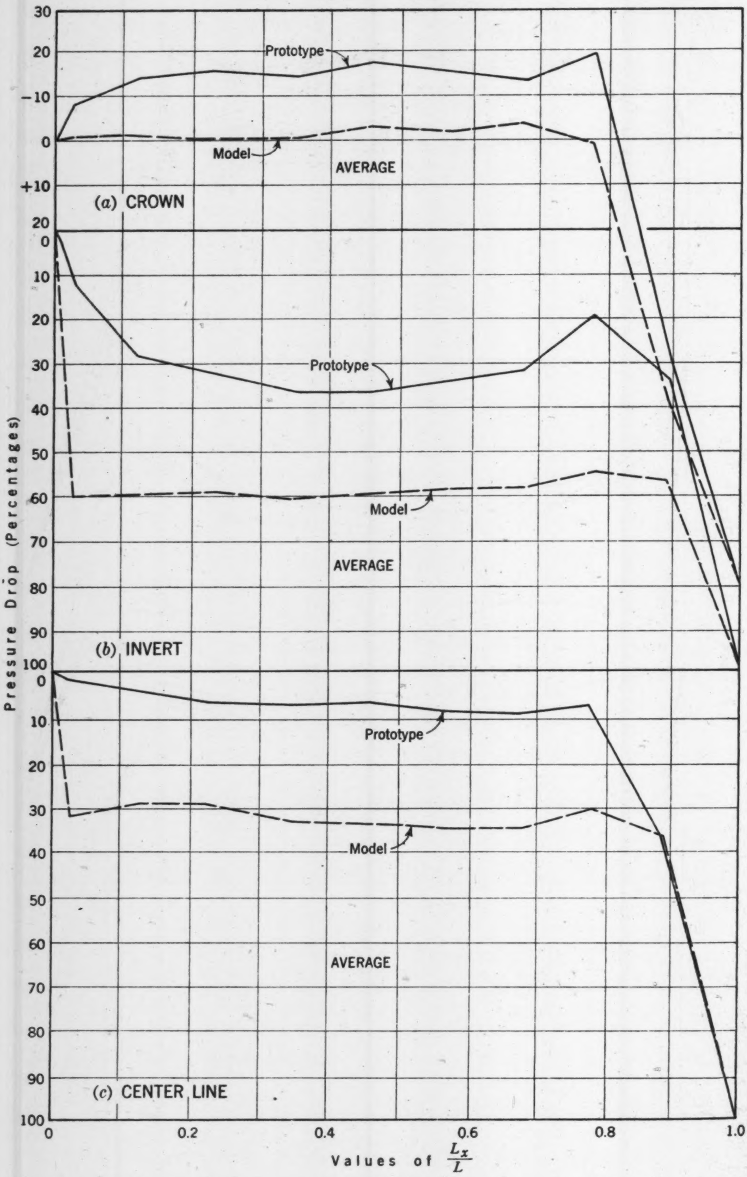


FIG. 11.—COMPARISON OF PRESSURE DROPS IN CONE

friction will have still more effect on the comparison, even though a study of the losses shows the bend loss or centrifugal effect to be considerably greater. The latter seems to be noticeable in the prototype curves in Figs. 10 and 11, where the points are more scattered than for the bellmouth measurements shown in Fig. 9. From tests on other structures, it is known that a slight change of diameter of the cone exit will cause appreciable changes in the pressure gradients through the penstock, making comparison more difficult. Time did not permit a more complete examination of the data for the elbow and cone, which might disclose a more valid method for comparing the pressure drop.

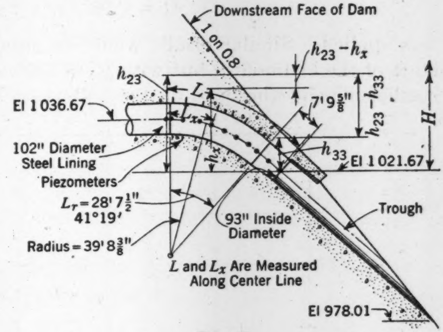


FIG. 12

Discharge Measurements.—The model and prototype discharge comparison is indicated in Fig. 13. Curve 1 is based on the 1 : 17 scale model tests for which $Q_p = Q_m \cdot L^{2.5}{}_r$, whereas Curve 2 is the model rating curve corrected to take into account the difference between model and prototype friction. To make this correction, it was first necessary to estimate an average friction coefficient for the prototype outlet. This was not readily accomplished because of a lack of reliable friction-coefficient data at Reynolds numbers as high as 5×10^7. Nevertheless, it was estimated that a value of 0.0085 for the friction factor would be reasonably accurate. The model friction coefficients and Reynolds numbers were then computed from the laboratory pressure tests and plotted in Fig. 6. Since the model friction is too large, the model heads for given discharges would be too high. Accordingly, assuming that all losses may be scaled to the prototype except friction, the model heads were reduced by the value $(f_m - f_p)\dfrac{L}{D} \times \dfrac{V^2}{2g}$, in which f_m was obtained from Fig. 6 and $f_p = 0.0085$ as estimated. After the pressure data for the prototype became available, the friction coefficients and Reynolds numbers were computed and plotted in Fig. 6. The average value for f_p was 0.0095 as compared to an estimate of 0.0085. Two 40-ft reaches were used for the prototype computations, starting at a distance of approximately eight diameters downstream from the entrance of the outlet, corresponding reaches being used for the model computations. That this is probably not a sufficient length to develop a normal velocity distribution is seen in Fig. 6, where the upper points were obtained using the first 40-ft reach, whereas the lower points were obtained from the following 40-ft reach.

The discharges given by Curve 3, Fig. 13, which were used for computing the prototype friction factors and Reynolds numbers, were developed from prototype pressure measurements, treating the bellmouth as a submerged orifice. The average pressure heads measured at piezometer ring 11 enabled the values of h_e to be computed (see Fig. 4(*a*)). The four piezometers at this point are located 4 ft $5\frac{3}{16}$ in. from the entrance and at the tangent between the bellmouth and transition immediately downstream, as shown in section in Fig. 4. Using the area of the bellmouth at this point, and assuming a coefficient of discharge, the nozzle formula

$$Q = 0.98\, A_{11} \sqrt{2\, g\, h_e} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

was applied. Similar results would be obtained using h_e and the area at the start of the bellmouth, but with $C_q = 0.60$, or by applying the same area and coefficient with the head on the bellmouth reduced by the losses in the bellmouth instead of h_e. These computations are based on the premise that the bellmouth fits the jet from a sharp-edged orifice placed in the plane of the upstream face of the dam.

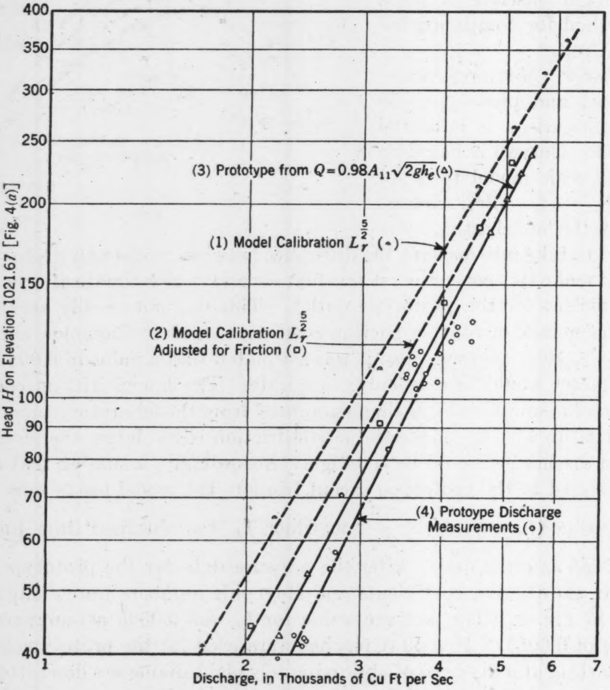


Fig. 13.—Comparison of Discharges

Curve 4, Fig. 13, was obtained from discharge measurements at a river gaging station one-half mile downstream from the dam. Since it is impossible to pass the flow of the river[3e] through only one outlet, the discharge applicable to a single outlet had to be computed by an indirect method. First, a rating curve was obtained for one lower outlet before the water surface in the reservoir reached the intermediate outlets. This was accomplished by plotting effective head (reservoir elevation minus tailwater elevation at the gaging station) against the discharge for one lower outlet (total measured discharge divided by the number of lower outlets operating). As soon as the intermediate outlets began to operate, the total river discharge increased accordingly, but knowing the effective head on the lower outlets and the number operating, it was possible to obtain the discharge per outlet from their rating curve, and thus the total discharge contributed by the lower outlets. Subtracting this value from the measured flow in the river, and dividing by the number of intermediate outlets operating, gave the discharge for one outlet as shown by Curve 4.

[3e] Columbia River.

This method assumes that each outlet will discharge the same amount, and that the jets from the intermediate outlets will not affect the effective head on the lower outlets. Although these assumptions are not strictly correct, it is believed that Curve 4, Fig. 13, and certainly Curve 3, would agree within 5% of any rating curve obtained by measuring the discharge of only one outlet.

In comparing the four rating curves over a range of head of from 230 ft to 50 ft, the uncorrected model discharge (Curve 1) is from 10% to 21% less than the prototype discharge computed at the bellmouth (Curve 3), whereas the corrected model discharge (Curve 2) is only 3% to 8% less. Similarly, the uncorrected model discharge (Curve 1) is from 11% to 27% less than the measured prototype discharge (Curve 4), whereas the corrected model discharge (Curve 2) is only 5% to 15% less.

Summary.—The agreement between the model and prototype in this quantitative comparison is reasonably good except in one instance. Considering experimental errors and lack of similitude relative to Reynolds' number, the pressure drop in the model bellmouth agrees with the prototype within 5%; the pressure drop in the elbow and cone of the model, on the other hand, is 30% greater than in the prototype; and, finally, the model estimate of the prototype discharge is, on an average, 8% low. Even though complete accord could not be obtained, the 1 : 17 scale model estimates are on the safe side and the field measurements show that an excellent prototype design has been evolved. It follows, therefore, although complete similitude was lacking, that the model could be relied upon to prove not only the inadequacy of several proposed designs, but also to develop the satisfactory design finally adopted.

CONCLUSIONS

To make better model-prototype comparisons, it is evident that plans should be prepared for procuring field measurements during the model tests and while the prototype is being designed. This not only enables a sufficient range of model data to be obtained, but it also makes certain that the instruments required at the prototype will be located properly and incorporated as a part of the design of the structure. Even these precautions may not yield the desired results because of the difficulties in actually obtaining prototype flow measurements. Thus, the reasons for so few quantitative comparisons are traced to the necessity of waiting years before sufficient water is available, or to the difficulties in measuring large discharges and high velocities, or to the fact that the measured discharge is derived from several sources instead of the one to be used in a comparison. Finally, the model experiments themselves introduce objectionable factors influencing comparisons, the chief one being the inability to test the model of conduits at the correct range of Reynolds numbers. In any event it is important to build hydraulic models to as large a scale ratio as possible.

The qualitative and quantitative examples of model-prototype comparisons given in this paper reaffirm the high degree to which the model can be relied upon. Although the avowed purpose of any comparison, particularly the quantitative type, should be to affirm the theory of hydraulic models or to modify the transference relations if necessary, it cannot be accomplished from only a few comparisons. At present, the emphasis should be laid on the limitations and accuracy of quantitative estimates based on model tests. In this regard, it is equally important to show both inferior and good model-prototype comparisons. The lessons learned therefrom will certainly demonstrate the adequacy of present-day model research.

ACKNOWLEDGMENTS

The material used for this paper represents the efforts of many individuals in the laboratory and field. The 1 : 17 scale model tests were conducted by D. M. Lancaster, Jun. Am. Soc. C. E. The planning of the prototype piezometer installations and field tests was done by C. W. Thomas and J. W. Ball, Assoc. Members, Am. Soc. C. E. The field tests were made by L. J. Snyder and A. F. Myers. D. J. Hebert contributed valuable suggestions for presenting the data. All tests were under the direction of the senior author in the materials, testing, and control division which is supervised by R. F. Blanks and Arthur Ruettgers, Members, Am. Soc. C. E., senior engineers. Design studies and investigations are under the direction of J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer. S. O. Harper, M. Am. Soc. C. E., is chief engineer of the Bureau of Reclamation, and J. C. Page, M. Am. Soc. C. E., is commissioner of Reclamation. E. W. Lane, M. Am. Soc. C. E., at the University of Iowa, and the staff of the U. S. Waterways Experiment Station, Vicksburg, Miss., contributed many of the references that comprise the Bibliography at the end of this Symposium.

A DRY-DOCK SUCTION CHAMBER

BY C. B. PATTERSON,[4] JUN. AM. SOC. C. E.

SYNOPSIS

Other papers of this Symposium describe studies in which the analyses of model and prototype data could be based upon direct quantitative comparisons. This paper will be devoted to a study wherein, due to the lack of certain measurements in the prototype, the comparison of model and prototype performance was approached from an inductive standpoint. The problem under investigation was the unsatisfactory operation of pumps used to drain a large dry dock; the medium of study was a 1:10-scale model constructed at the Waterways Experiment Station.

The dry dock, together with its culvert system and suction chamber, is shown by the general plan drawing in Fig. 14. (Interconnecting passages to the suction chamber from adjacent dry docks are not shown.) The suction chamber is shown in greater detail in Fig. 15, and the arrangement of pump intakes (1, 2, and 3) and sluice gates (3, 4, and 5) is indicated. This arrangement, the high rate of discharge (130,000 gal per min maximum per pump), and the limited dimensions of the suction chamber, gave rise to unstable and sometimes violent flow conditions, which were manifested at the pumps by vibration, loss of capacity, and symptoms of cavitation. (All such objectionable features will be covered by the general term "rough operation.") Test runs in the prototype yielded notes and observations as to the character of pump performance, but did not provide any numerical data or evidence as to the character of flow within the suction chamber.

In constructing a model for the investigation, it was essential that the proper distribution of flow through the three sluice gates be assured. Accordingly, the entire system of side culverts and inlets was reproduced in the model. It was not feasible to reproduce the prototype pumps by means of miniature pumps of a homologous series; however, the proper metered discharges were induced through the intakes by ordinary centrifugal pumps. For purposes of visual observation, the suction chamber and pump intakes were made of transparent pyralin.

To gain some idea as to flow conditions within the suction chamber, compressed air was injected into the model under operating conditions similar to those known to give unsatisfactory pump operation in the prototype. From observations of the air bubbles in these preliminary tests it was learned (*a*) that flow entering the suction chamber from the sluice gates created initial rotation about one or another of the pump intakes (depending upon the particular conditions of operation), and (*b*) that initial vortices so formed were accentuated as the water was drawn upward into the bellmouth intakes. Under such conditions it was reasonable to expect that the pressure at the axis of any intake so

---

[4] Asst. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

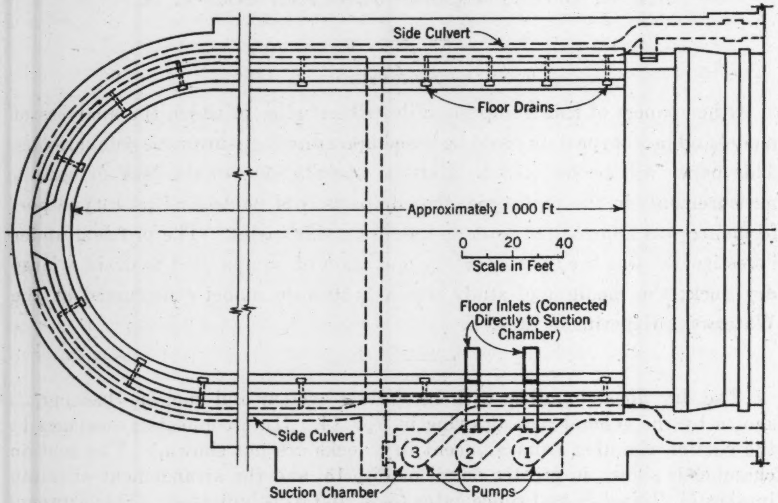

Side Culvert
Floor Drains
Approximately 1 000 Ft
0 20 40
Scale in Feet
Floor Inlets (Connected Directly to Suction Chamber)
Side Culvert
3
2
1
Suction Chamber
Pumps

Fig. 14.—General Plan of Dry Dock Showing Culvert System and Suction Chamber

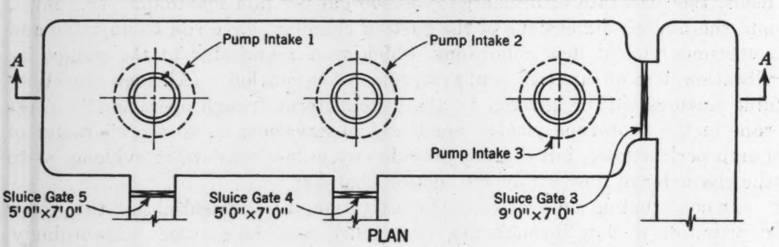

Pump Intake 1
Pump Intake 2
A
A
Pump Intake 3
Sluice Gate 5
5'0"×7'0"
Sluice Gate 4
5'0"×7'0"
Sluice Gate 3
9'0"×7'0"
PLAN
0 5 10 15
Scale in Feet
Outline of Pump Casing
LONGITUDINAL SECTION A-A

Fig. 15.—Arrangement of Elements Within a Pump Suction Chamber

affected (that is, at the center of the vortex) would frequently fall below the suction lift capability of the pump. It was concluded that the rapid and violent fluctuations of pressure attending vortex action within the intakes were responsible for an alternate breaking and restoring of suction head at the pumps, resulting in the aforementioned symptoms.

To support this conclusion, and to provide a basis for the comparison of model and prototype performance, it was desired that a record be made of the amplitudes and frequencies of pressure fluctuations within the pump intakes. It was believed that such data would permit the establishment of a criterion for pump operation, whereby rough pump operation would be revealed by a record showing fluctuations of great amplitude and frequency, and whereby smooth operation would be indicated by slight variations of pressure. From such a criterion, the model reproduction of prototype performance could be confirmed, and the effectiveness of any corrective measures under trial could be evaluated.

To obtain a record of pressures within the suction bells, the instrument shown in Fig. 16 was devised. The operation of this instrument is very simple: Pressure impulses originating at the pitot-tube opening are transmitted through the confined liquid in the tubing to the face of the pressure cell; the resulting deflections of this diaphragm are recorded electrically by an oscillograph to give a continuous time-record of the instantaneous values of pressure.

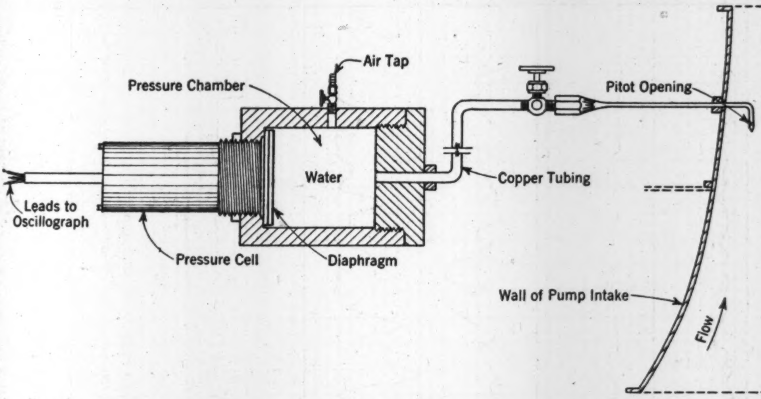


Fig. 16.—Device for Measuring Rapid Fluctuations of Pressure

Using this instrument the model was run under each operating condition—combination of pumps and sluice gates in operation—for which observations had been made in the prototype. Oscillograph records taken during a few such tests are plotted in Fig. 17; and for each of the test conditions the observations made under comparable conditions in the prototype are recorded in Table 1. (In Fig. 17, and all other figures, dimensions are expressed in terms of prototype equivalents.) By comparison of model and prototype observations it may be seen that they are consistent in all cases. Furthermore, the criterion for rough and smooth pump performance is definitely established. From such a model-prototype comparison it was safe to assume that predictions as to the effectiveness of corrective measures could be relied upon.

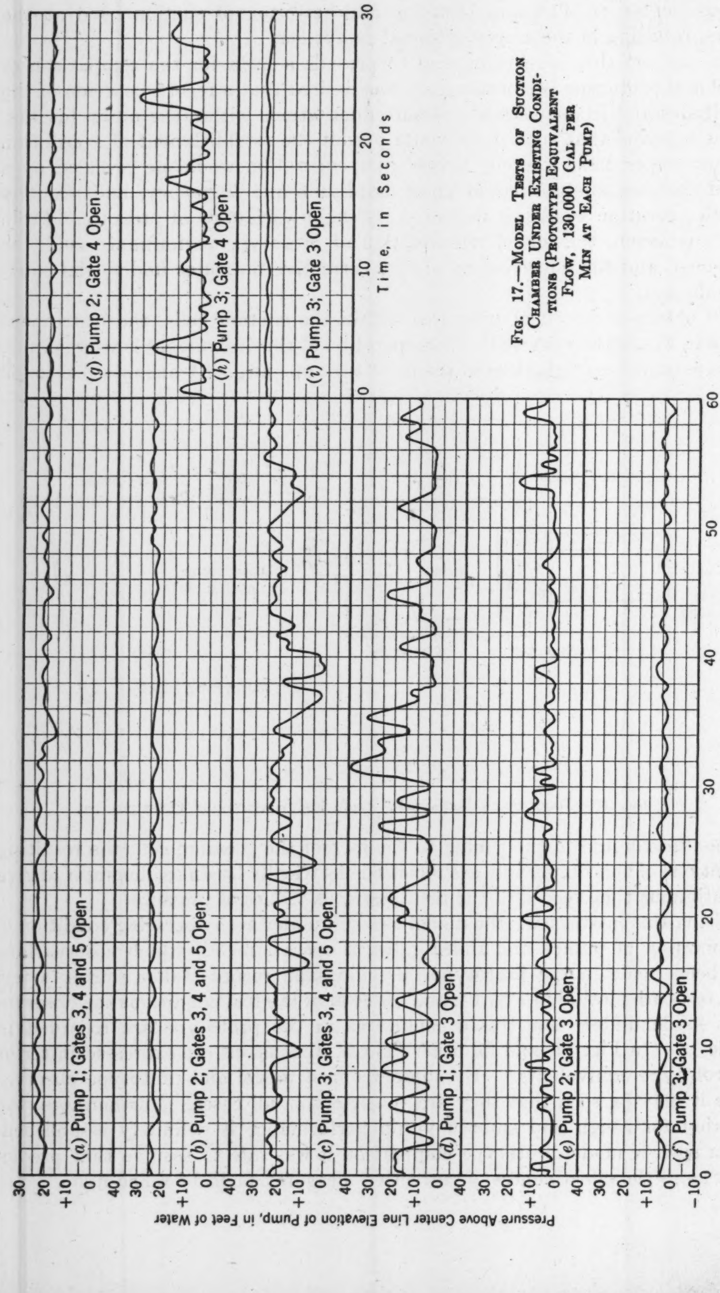


FIG. 17.—MODEL TESTS OF SUCTION CHAMBER UNDER EXISTING CONDITIONS (PROTOTYPE EQUIVALENT FLOW, 130,000 GAL PER MIN AT EACH PUMP)

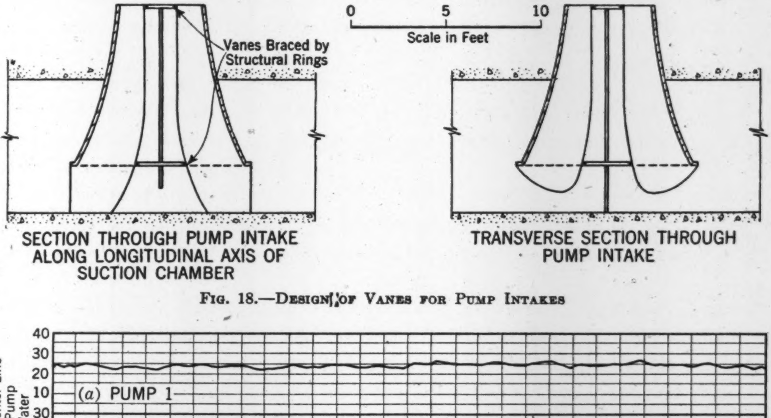
It was considered that the unsatisfactory performance of the pumps would be improved if the vortices in the intakes could be broken up. For this purpose, vertical vanes of the design shown by Fig. 18 were installed in the model pump intakes. In order that flow in a longitudinal direction be unobstructed, transverse vanes were not extended to the floor of the model suction chamber. In the prototype, however, all vanes were extended to the floor. The results

TABLE 1.—Test Conditions for Model Observations in Fig. 17

(Discharge 130,000 Gal per Min)

Fig. 17	Sluice gates open	Pump operating	Corresponding prototype observation
a	3, 4, and 5	1	....
b	3, 4, and 5	2	....
c	3, 4, and 5	3	Operation very rough
d	3	1	Extremely bad operation
e	3	2	Generally unsatisfactory operation
f	3	3	With more than one pump operating on Sluice Gate 3 the farthest unit could not operate; generally unsatisfactory operation
g	4	2	Under initial conditions Pump 2 operated satisfactorily
h	4	3	Operation very rough
i[a]	3	3	When Pump 2 was shut down, power input to Pump 3 decreased. When Sluice Gate 3 was opened power input to Pump 3 decreased again and its operation was good.

[a] Initially (see Figs. 17(*g*) and 17(*h*)) pumps 2 and 3 were each discharging 130,000 gal per min with Sluice Gate 4 open. Then (see Fig. 17(*i*)) Pump 2 was shut down and Sluice Gate 3 was opened.



SECTION THROUGH PUMP INTAKE ALONG LONGITUDINAL AXIS OF SUCTION CHAMBER

TRANSVERSE SECTION THROUGH PUMP INTAKE

Fig. 18.—Design of Vanes for Pump Intakes

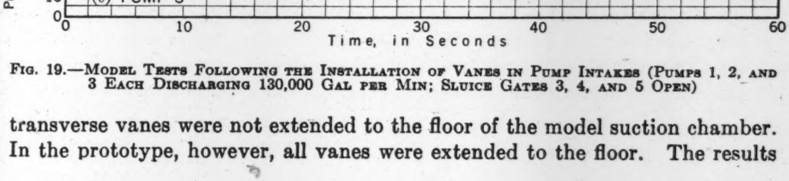


Fig. 19.—Model Tests Following the Installation of Vanes in Pump Intakes (Pumps 1, 2, and 3 Each Discharging 130,000 Gal per Min; Sluice Gates 3, 4, and 5 Open)

of a model test run with these vanes in place are shown in Fig. 19. By comparing these results with those of Fig. 17, the improvement is evident, and a smooth operation of the pumps is indicated. Modifications of the basic design vanes were also tested, but no improvement over the basic design was found.

Accordingly, vanes of the basic design were adopted for installation in the prototype. As a final step in the analysis, the model predictions were thoroughly substantiated by the prototype; it is reported that operation of the pumps, following installation of vertical vanes in the intakes, has been entirely satisfactory. A complete model-prototype confirmation would include the measurement of pressure fluctuations in the prototype but thus far such measurements have not been practicable.

PICKWICK, WHEELER, AND GUNTERSVILLE LOCKS

BY MARTIN E. NELSON,[5] M. AM. SOC. C. E., AND
JAMES J. HARTIGAN,[6] ESQ.

SYNOPSIS

A lock and dam,[6a] one of the major projects constructed on Tennessee River by the Tennessee Valley Authority (TVA), was designed in 1934. A model of the proposed lock was built and tested at the U. S. Engineer Sub-Office, Iowa City, Iowa, for the purpose of aiding in the hydraulic design of the filling and emptying system. As the Tennessee development continued, other locks were designed and models were used to study special problems and to check the final plans.

The question of the reliability of hydraulic lock models in simulating conditions which would exist in the full-size structures developed with the increased use of this method of design. Accordingly, steps were initiated to test a model of an existing lock in which exact geometric similarity could be obtained. The project chosen for the correlation was a lock,[6b] the first project completed in the river by the TVA. A model of the lock was built in December, 1936, and comparisons were made between model tests and a limited amount of prototype test data available at that time. The results of these comparisons were satisfactory, but the prototype data available were not adequate to warrant drawing definite conclusions.

The division engineer of the Ohio River Division initiated steps for testing a number of prototype locks in the Tennessee and Ohio rivers to verify the model tests and to study, under actual operating conditions, the hydraulics of various modifications of the port and culvert system. Tests were made on Pickwick, Wheeler, Wilson, and Guntersville locks on the Tennessee River; and No. 41 and Montgomery Island locks on the Ohio River. Model tests had been made on only Pickwick, Wheeler, and Guntersville locks, and this paper will be confined to a discussion of tests made on these three locks and their models.

GENERAL DESCRIPTION OF LOCK HYDRAULICS SYSTEMS

The locks in question are all equipped with the same general type of filling and emptying system. Longitudinal culverts located in the side-walls of the lock open into the upper pool through gallery intakes and into the lower pool through similar discharge sections. Short lateral ports connect the culverts with the lock chamber near the floor. The flow in each culvert is controlled

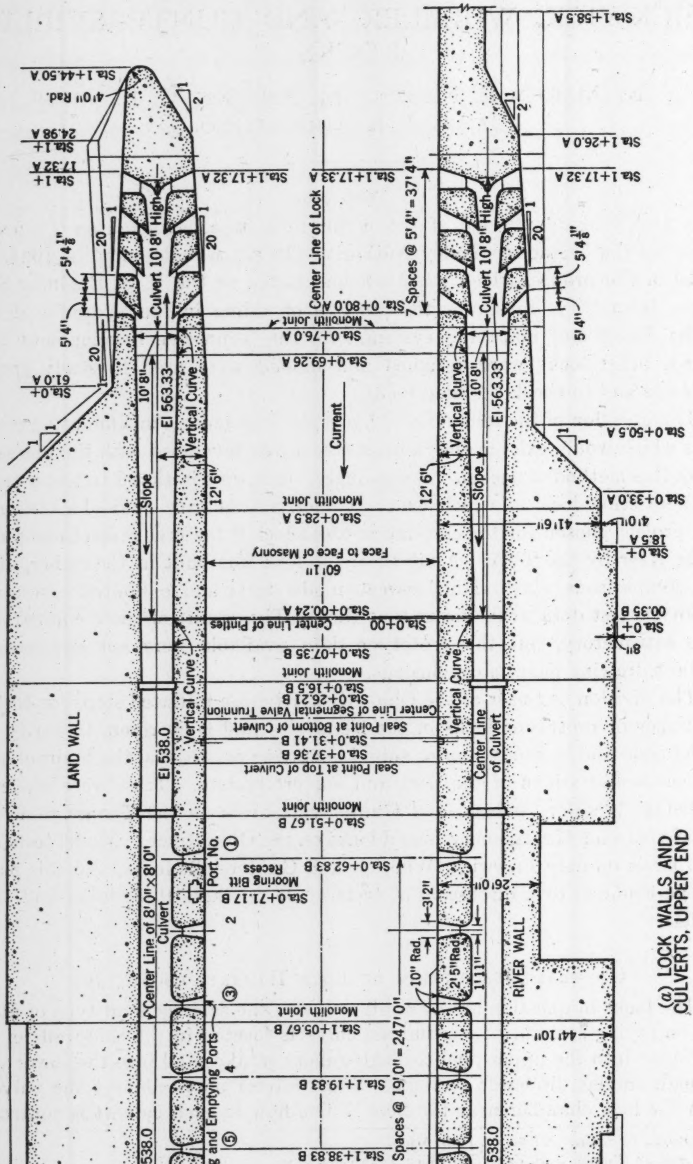
---

[5] Engr., U. S. Engr. Office, St. Paul, Minn.

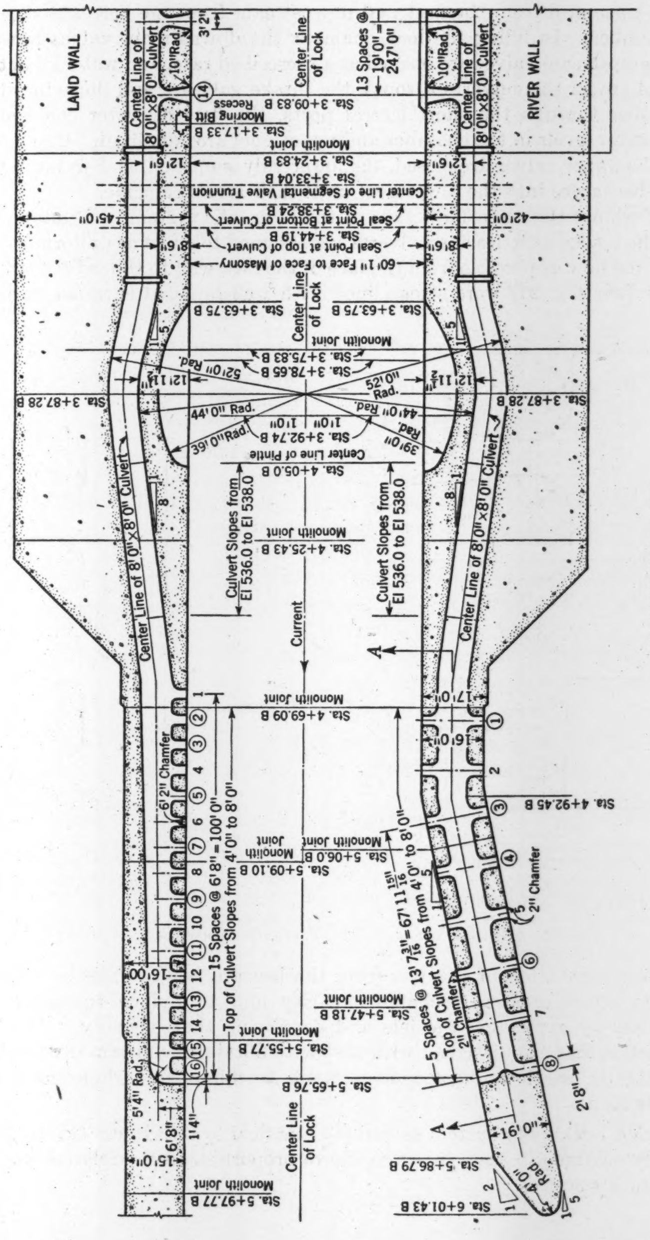
[6] Asst. in Charge, U. S. Engr. Sub-Office, Iowa City, Iowa.

[6a] Pickwick Lock and Dam.

[6b] Wheeler Lock.


LAND WALL
RIVER WALL
Center Line of Lock
Current
Face to Face of Masonry
60'1"
Center Line of 8'0"×8'0" Culvert
Center Line of Culvert
Filling and Emptying Ports
Port No. ①
2
③
4
⑤
6
13 Spaces @ 19'0" = 247'0"
7 Spaces @ 5'4" = 37'4"
Culvert 10'8" High
Vertical Curve
Slope
El 563.33
El 538.0
10'8"
12'6"
5'4"
5'4⅛"
20
1
2
4'0" Rad.
10" Rad.
2'5" Rad.
3'2"
1'11"
2'0"
45'0"
42'0"
44'10"
26'0"
41'6"
4'0"
8"
Mooring Bitt Recess
Monolith Joint
Center Line of Pintles
Center Line of Segmental Valve Trunnion
Seal Point at Bottom of Culvert
Seal Point at Top of Culvert
Sta.1+44.50 A
Sta.1+ 24.98 A
Sta.1+ 17.32 A
Sta.1+17.32 A
Sta.1+17.33 A
Sta.1+58.5 A
Sta.1+26.0 A
Sta.0+80.0 A
Sta.0+76.0 A
Sta.0+69.26 A
Sta.0+ 61.0 A
Sta.0+50.5 A
Sta.0+33.0 A
Sta.0+28.5 A
Sta.0+ 18.5 A
Sta.0+00.24 A
Sta.0+00
Sta.0+ 00.35 B
Sta.0+07.35 B
Sta.0+16.5 B
Sta.0+26.21 B
Sta.0+31.41 B
Sta.0+37.36 B
Sta.0+51.67 B
Sta.0+62.83 B
Sta.0+71.17 B
Sta.1+05.67 B
Sta.1+19.83 B
Sta.1+38.83 B
Sta. 1+53.67 B

(*a*) LOCK WALLS AND CULVERTS, UPPER END

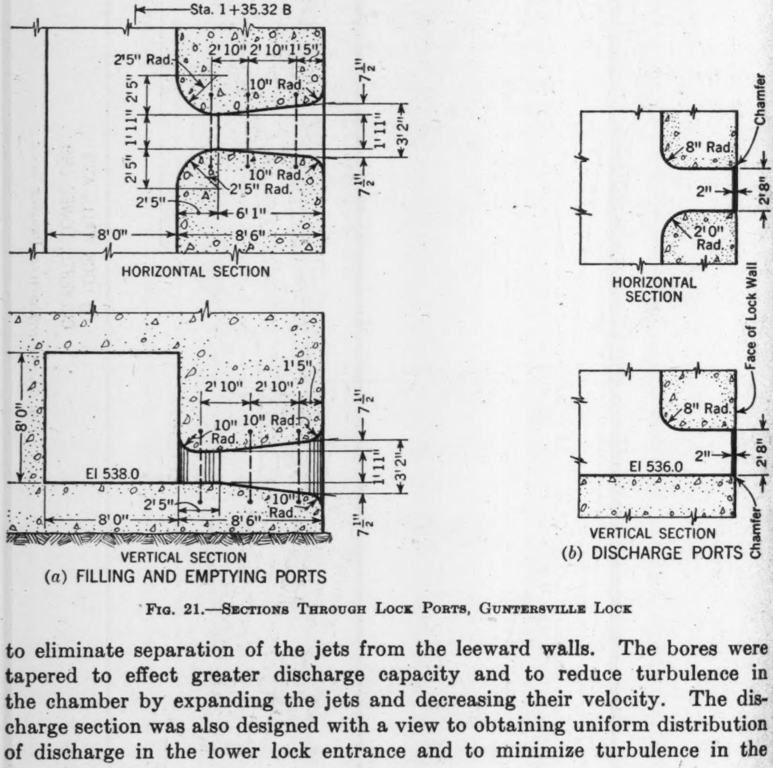

LAND WALL
RIVER WALL
Center Line of 8'0"×8'0" Culvert
Center Line of Lock
13 Spaces @ 19'0" = 247'0"
10"Rad.
3'2"
Sta. 3+09.83 B
Recess
Mooring Bitt
Sta. 3+17.33 B
Monolith Joint
Sta. 3+24.83 B
12'6"
Sta. 3+33.04 B
Center Line of Segmental Valve Trunnion
Sta. 3+38.24 B
Seal Point at Bottom of Culvert
45'0"
42'0"
Sta. 3+44.19 B
Seal Point at Top of Culvert
8'6"
60'1" Face to Face of Masonry
Sta. 3+63.75 B
Sta. 3+75.83 B
Sta. 3+78.65 B
52'0" Rad.
Sta. 3+87.28 B
44'0" Rad.
1'0"
Sta. 3+92.74 B
39'0" Rad.
Center Line of Pintle
Sta. 4+05.0 B
Culvert Slopes from El 536.0 to El 538.0
Sta. 4+25.43 B
Current
A
Sta. 4+69.09 B
17'0"
16'0"
6'2" Chamfer
15 Spaces @ 6'8" = 100'0"
Top of Culvert Slopes from 4'0" to 8'0"
Sta. 4+92.45 B
Sta. 5+06.0 B
Sta. 5+09.10 B
5 Spaces @ 13'7 3/16" = 67'11 15/16"
2" Chamfer
Sta. 5+47.18 B
Sta. 5+55.77 B
Sta. 5+65.76 B
5'4" Rad.
6'8"
1'4"
15'0"
2'8"
4'0" Rad.
Sta. 5+86.79 B
Sta. 5+97.77 B
Sta. 6+01.43 B

(b) LOCK WALLS AND CULVERTS; LOWER END

FIG. 20.—GUNTERSVILLE PROJECT LOCK

by valves upstream from the first and downstream from the last lateral port to the chamber. In filling the lock chamber the downstream valves remain closed, the upstream valves are opened at a prescribed rate, and water from the upper pool enters the culverts through the intake galleries and flows into the lock chamber through the short lateral ports, the flow of water continuing until the water levels in the chamber and upper pool are equalized. To empty the lock the upper valves are closed, the lower valves opened, and water in the chamber discharges into the lower pool until their levels coincide.

Fig. 20 shows the essential details of the hydraulic system of Guntersville Lock. The intake galleries and culverts are designed to effect as uniform acceleration of the flow as possible from the face of the lock wall to the valve section. The ports (see Fig. 21) were streamlined with well-rounded entrance corners to eliminate separation of the jets from the leeward walls. The bores were tapered to effect greater discharge capacity and to reduce turbulence in the chamber by expanding the jets and decreasing their velocity. The discharge section was also designed with a view to obtaining uniform distribution of discharge in the lower lock entrance and to minimize turbulence in the immediate area.


Sta. 1+35.32 B
2'5" Rad.
2'10" 2'10" 1'5"
10" Rad.
2'5"
1'11"
2'5"
7½"
1'11"
3'2"
7½"
10" Rad.
2'5" Rad.
2'5"
6'1"
8'0"
8'6"
1'5"
2'10" 2'10"
10" Rad.
10" Rad.
8'0"
El 538.0
2'5"
8'0"
8'6"
10" Rad.
7½"
1'11"
3'2"
7½"
Chamfer
8" Rad.
2"
2'8"
2'0" Rad.
Face of Lock Wall
8" Rad.
El 536.0
2"
2'8"
Chamfer

HORIZONTAL SECTION

VERTICAL SECTION

(a) FILLING AND EMPTYING PORTS

HORIZONTAL SECTION

VERTICAL SECTION

(b) DISCHARGE PORTS

Fig. 21.—Sections Through Lock Ports, Guntersville Lock

Pickwick Lock has a system essentially identical to that in the Guntersville Lock, only enlarged in scale in approximate proportion to the relative dimensions of the structures.

The hydraulic system of Wheeler Lock is typical of the designs used in inland waterways locks prior to the use of models in analyzing the hydraulics involved in their operation. Deviation from perpendicular lines to effect better hydraulic efficiency was practically avoided. Simplified construction was given primary consideration at the expense of efficiency throughout a lifetime of operation.

Scope of the Tests

The parallel tests that were made in corresponding model and prototype locks will be described in general terms in order to eliminate the necessity of discussing the experimental work performed on each individual structure. Each type of test considered was performed on at least one pair of similar locks and in most cases on all three pairs considered in this paper:

(*a*) The stages near both ends of the lock chamber were observed with respect to time in filling and emptying operation. From these data were plotted filling or emptying curves, rate-of-rise or rate-of-fall curves, and water-surface slope curves, all with respect to time. Comparisons of model and prototype were made on the basis of total time required to complete the operation, the coefficients of discharge for the entire hydraulic system, the maximum value of the rate of change in stage in the lock chamber, and the rate of decrease in the flow after the maximum had been reached as indicated by the slopes of the rate of rise or fall curves.

(*b*) Velocities were observed in the ports of the intake section, the manifold section, and the discharge section, and from these data comparisons of the distribution of discharge in the respective sections were made.

(*c*) Pressure measurements observed at the ends of the culvert transition sections were used in conjunction with volumetric discharge measurements in the lock chamber to determine discharge calibrations and venturi coefficients.

The primary use for hydraulic models is to provide data for design. As such, the functions of a model are generally completed before the prototype is built, and not infrequently the final design is modified from that used in the model, due to contingencies which had not arisen until the final stages of the design were reached and which were considered too insignificant to warrant the expense of checking in the model. As a result, very few prototypes are built exactly like the models from which their designs were developed. Such differences were found in the locks covered by this study, but wherever possible, corrections will be applied or dimensionless parameters used to validate the desired comparisons.

Comparison of Prototype and Model Tests, Pickwick Lock

Intake Ports.—A primary consideration in the design of the intake sections of one lock[6c] was a uniform distribution of flow in the ports. With flow in one direction only, a tapered culvert section could be used. That the design successfully approximated a uniform distribution in the four ports on one side of the culvert, and that agreement existed between model and prototype distributions, are indicated by the graph in Fig. 22. The discharge in the model intake ports varied uniformly from 21.5% to 28.5% and in the corresponding prototype ports from 22% to 27%.

[6c] Pickwick Lock.

Transition Section.—The rating equation for a culvert transition section, expressed in terms of culvert discharge Q and pressure head differential between two sections $h^{0.5}_p$, is based on the Bernoulli theorem; thus:

$$Q = K\,h^{0.5}_p \qquad (3)$$

The coefficient of discharge for the transition section meter is equal to the ratio of the observed to the theoretical rating. A comparison of prototype and

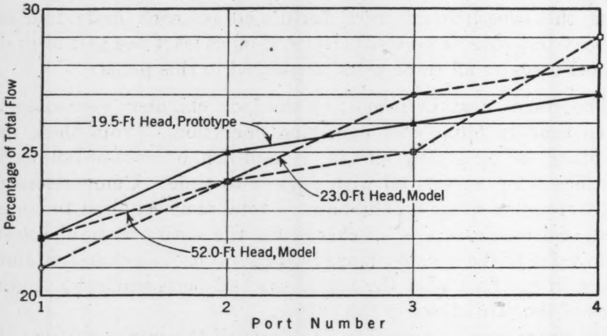


FIG. 22.—DISTRIBUTION OF FLOW THROUGH INTAKE PORTS, PICKWICK LOCK (CONSTANT FLOW)

model locks is made, as shown in Table 2, on the basis of the performance of their culvert transition sections as flow meters. The lack of exact geometric similarity in the transition sections of model and prototype accounts for the small difference in the constants of the theoretical rating equations; however corresponding equations and coefficients for model and prototype are in virtual agreement.

TABLE 2.—COMPARISON OF MODEL AND PROTOTYPE LOCKS

Description	Rating Equation (Eq. 3)		Coefficient of discharge C
	Observed K_o	Theoretical K_t	
Model	1,400	1,500	0.93
Prototype	1,410	1,525	0.92

Lock Stage Tests.—The levels of the water surface in the lock chamber at respective time intervals during filling operations in the model and prototype are shown in Fig. 23. The points defining the filling curves are averages of two stage measurements, one near each end of the lock chamber, observed at each time interval.

The filling curves for the model and prototype locks indicate a considerable difference in the total time of operation and in the form of the filling curves. There were two factors which materially affected the results of these tests—one a difference in construction of the lock-chamber ports (Fig. 24), and the other a difference in the time of opening the culvert valves. In the model the total cross-sectional area of the lock-chamber ports at the smallest section was 125.0 sq ft per culvert, whereas in the prototype the corresponding areas were 145.0 sq ft in the river wall and 147.8 sq ft in the land wall. The culvert valves in the model were opened in a time corresponding to 2 min in the prototype, whereas in the filling test shown the right valve of the prototype required 5 min 4 sec to open, and the left valve 6 min 37 sec. A third factor which also affected the filling time was a difference in the lifts. In the model a lift of 61 ft was simulated, whereas in the prototype 55 ft was the maximum difference in water levels existing during the period of the tests.

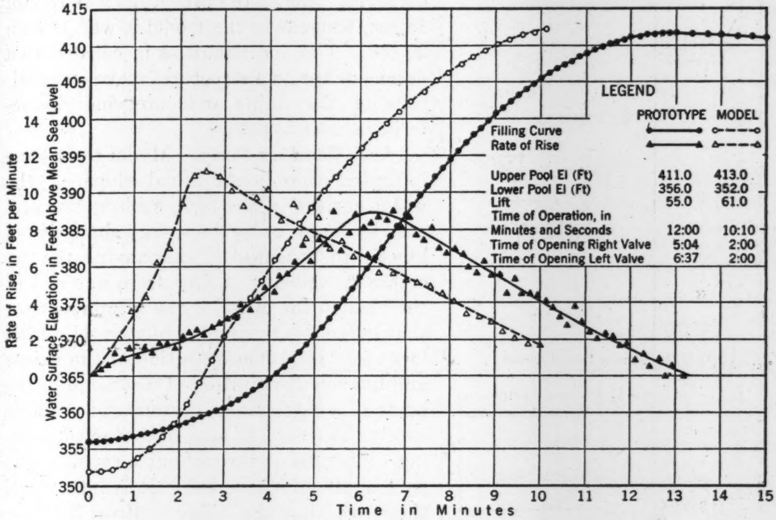


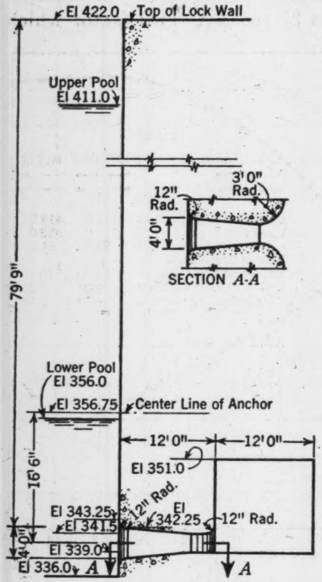
FIG. 23.—FILLING CHARACTERISTICS, PICKWICK LOCK

If the two filling curves shown in Fig. 23 were matched at the instant when the corresponding locks were full, it is noted that they would follow approximately the same path for the period after the valves were completely open. The larger throat area in the prototype lock-chamber ports, which tended to increase the efficiency of the prototype over that of the model, appears to have been offset for the most part by the larger and more efficient divergence angle in the model ports.

The maximum rate of rise or fall of the water level in a lock occurs shortly after the culvert valves are fully open. Because of the difference in time of valve operation, the rate-of-rise and rate-of-fall curves for the model and prototype of one of the locks[6c] differ considerably. The maximum rate of rise in the model was 11.1 ft per min at 2 min 35 sec, and in the prototype it reached 9.0 ft per min at 5 min 25 sec. In a manner similar to the filling curves, the rate-of-rise curves for model and prototype follow nearly parallel paths after the peak rate has been reached; the average slope of the curve for the prototype is 1.40 ft per min^2, while that for the model is 1.24 ft per min^2.

[6c] Pickwick Lock.

The differences in construction details and valve operation, noted in the discussion of the filling tests, apply also with respect to the emptying tests. However, the larger port area and smaller lift existing in the prototype, tending to reduce the emptying time, were practically offset by the longer valve period. The emptying time in the prototype was 13 min 18 sec, whereas in the model it was 13 min 29 sec. The dissimilarities in construction details of the two structures make comparisons of the filling and emptying characteristics unsatisfactory.



SECTIONAL ELEVATION

FIG. 24.—SECTIONS THROUGH CENTER LINE OF LOCK-CHAMBER PORT (20 PORTS IN EACH WALL)

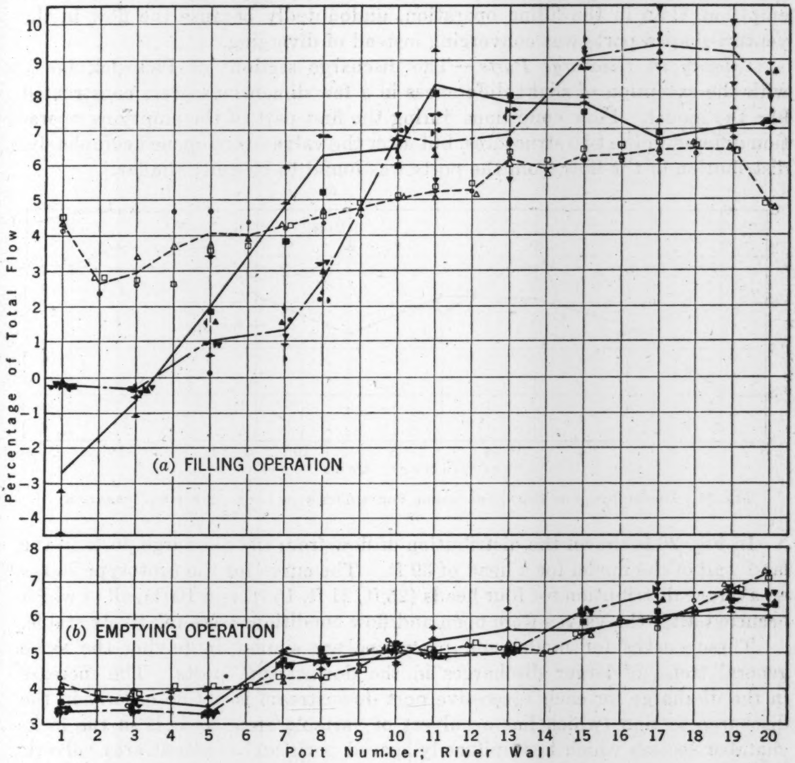
Lock-Chamber Ports.—Model tests have indicated that longitudinal slope of the water surface in the lock, such as was created by surges or by the energy change from kinetic to potential accompanying the decrease in velocity of flow from one end of the lock to the other, is the primary factor that influences movement of a vessel during lockage. Improper operation of the valves and uneven distribution of discharge in the ports of a side culvert system will promote surging in the lock; and, once started, surging will continue throughout a considerable part of the filling period. It is important, therefore, that the filling controls be operated so as to eliminate a sudden inrush of water at the beginning of the filling period, and that the system be designed to distribute the lockage inflow evenly among the lock-chamber ports.

From the port velocity tests made in the model and prototype locks,[6e] distribution-of-flow graphs for the ports in the right or river wall for the filling operation were plotted as shown in Fig. 25(*a*). The flow distributions in the model and prototype differed considerably, due largely to the dissimilarity of the ports. In earlier laboratory experiments it was found that the distribution of discharge from the ports during a filling operation was influenced to a large extent by the value of the cumulative cross-sectional area of the lock-chamber ports at the smallest section. When the port area exceeded a certain value, the shape of the hydraulic gradient was changed so that during part or all of the filling period water would flow from the lock chamber into the culvert through a number of the ports at the upstream end of the manifold. The prototype port area exceeds that of the model by 16% in the river wall and 18.2% in the land wall. Excess area in the prototype is equivalent to that of three ports. The effect of the excess port area in the prototype is indicated in Fig. 25(*a*). Discharge through ports 1 and 3 was reversed, flowing from the lock chamber into the culvert during the entire period of the filling

---

[6e] Pickwick Lock.

operation, and, although the flow in port 2 was not measured, it undoubtedly was negative also. Therefore, the distribution patterns for the filling operation in the model and prototype manifolds cannot be correlated.



Description	Average of Three Heads[b]			Average of Six Heads[b]						Average of Three Heads[c]		
	-●-	◓	■	◒	▲	◖	●	◗	▼	□	△	○
Head, in feet	50	45	40	35	30	25	20	15	10	40	30	22
Discharge,[a] in Cubic Feet per Second:												
(*a*) Filling operation	..	..	..	6,080	5,630	5,200	4,650	4,030	3,320	6,310	5,290	4,640
(*b*) Emptying operation	..	..	..	4,370	4,430	4,220	3,770	3,270	2,670	4,670	4,050	3,470

[a] Discharge refers to flow into, and out of, lock chamber through the ports in the river wall. [b] Prototype. [c] Model.

FIG. 25.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS, PICKWICK LOCK

A very close similarity in the flow distribution in the model and in the prototype during emptying operations is shown in Fig. 25(*b*). The distribution of flow through the ports was only slightly affected by variations in head, and the discrepancies in port dimensions in the prototype had less effect in the emptying than in the filling operation, undoubtedly because the flow in the venturi-shaped ports was converging instead of diverging.

Velocity in Discharge Ports.—The discharge sections of Pickwick Lock, with the exception of slight differences in a few dimensions, were constructed like the model. Flow conditions during the first part of the emptying operation differed in the two structures, but after the valves were opened completely, distribution of the flow from the ports was found to be quite similar.

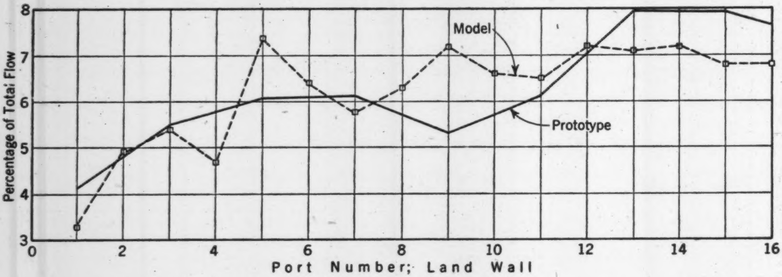
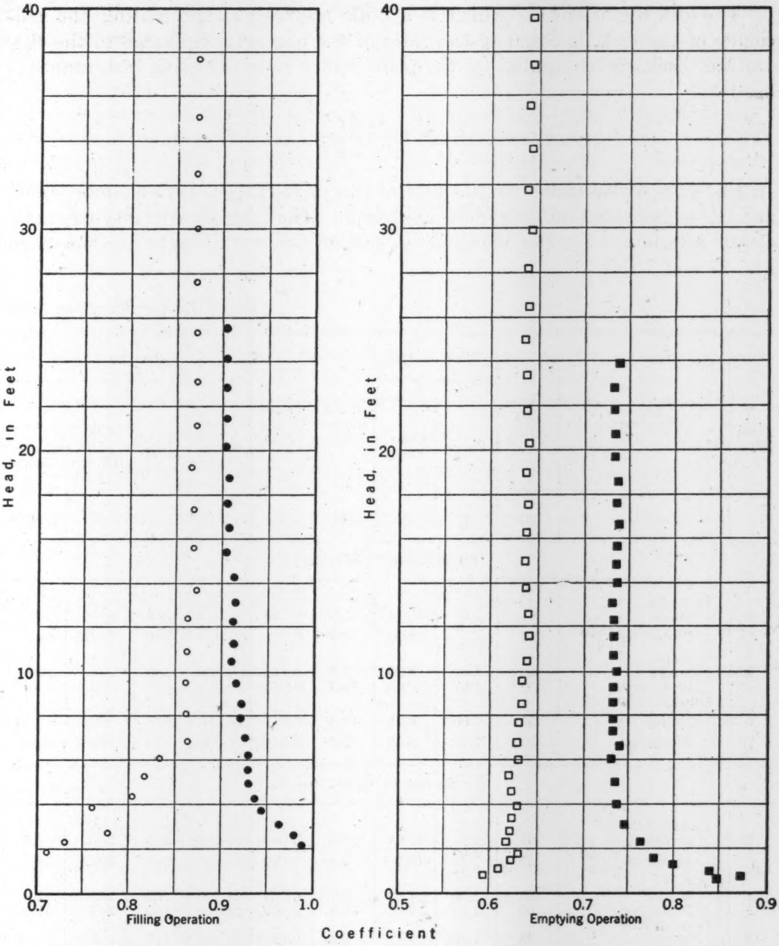


FIG. 26.—DISTRIBUTION OF FLOW, DISCHARGE PORTS, PICKWICK LOCK (EMPTYING OPERATION)

In Fig. 26 is shown the distribution of flow from the discharge ports in the land wall of the model for a head of 39 ft. The curve for the prototype shows an average distribution for four heads (25 ft, 21 ft, 15 ft, and 10 ft), all of which occurred after the valves were open and flow conditions were relatively stable.

These curves for model and prototype are similar in having the same general trend of larger discharges in the downstream ports. The increase in the discharge for each successive port downstream is not so orderly in the discharge section (which has a culvert of variable area) as it is in the lock-chamber section which has uniformly spaced ports and constant area culvert. The percentages of culvert flow carried by the eight upstream and eight downstream ports of the discharge section were 44.4% and 55.6% for both model and prototype, variations between model and prototye having balanced in each half of the section.

Lock Coefficients.—The coefficients for lock hydraulic systems presented in this paper were computed for the period after the valves were fully opened and were based on the minimum culvert area. In the prototype lock system the section having the least area through which the water passes, both in filling and emptying, is the 12-ft by 12-ft culvert below the transition section. In the model the section of minimum area was at the throat of the lock-chamber ports, this area being 250 sq ft as compared with the minimum culvert area of 288 sq ft. The culvert area in the prototype was the same as in the model, but the cumulative port area at the minimum section was 292.8 sq ft.



Description	Legend:			
	●	○	■	□
Filling operation	Prototype	Model	....	....
Emptying operation	....	....	Prototype	Model
Average coefficient C (Eq. 4)[a]	0.91	0.87	0.73	0.64

[a] For heads greater than 6 ft.

FIG. 27.—LOCK COEFFICIENT, PICKWICK LOCK

The lock coefficient C_l, which is a bulk coefficient representing the efficiency of the lock, is equal to the ratio of the observed discharge to the theoretical discharge computed by the plain orifice equation using instantaneous heads:

$$C_l = \frac{v_l A_l}{A_c \sqrt{2 g h}} \tag{4}$$

In Eq. 4: v_l = the lock-chamber rate of rise or fall; A_l = lock-chamber area; and A_c = cross-section area of two culverts. The filling and emptying coefficients determined for the Pickwick prototype and model locks are shown in Fig. 27 and Table 3.

TABLE 3.—COMPARISON

Item No.	Structure	Lift (ft)	TIME (MIN)		Rate[a] (ft per min)	Slope[b] (ft per 100 ft)	AREA (SQ FT)			
			Valve	Operation			Lock chamber	Culvert	Port	
									Throat end	Lock-chamber end
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(a) FILLING OPERATION									
	Pickwick									
1	Model	61	2.0	10.17	11.1	0.148	74,250	288	250	640
2	Prototype right	55	5.07	12.00	8.9	0.062	74,250	288	292.8	640
3	Prototype left		6.62							
	Wheeler									
4	Model	50	1.00	7.20	12.6	0.148	24,300	160	240	240
5	Prototype	50	1.07	7.08	12.4	0.273	24,300	160	240	240
	Guntersville									
6	Model	39	1.00	6.15	12.5	0.117	24,300	128	102.8	211.8
7	Prototype	39	1.12	5.50	12.7	0.118	24,300	128	102.8	280.8
	(b) EMPTYING OPERATION									
	Pickwick									
8	Model	61	2.0	13.48	8.6	0.045	74,250	288	250	640
9	Prototype right	55	3.97	13.30	7.3	0.030	74,250	288	292.8	640
10	Prototype left		5.12							
	Wheeler									
11	Model	50	1.00	8.03	11.7	0.059	24,300	160	240	240
12	Prototype	50	1.07	7.37	11.8	0.106	24,300	160	240	240
	Guntersville									
13	Model	39	1.00	7.60	9.3	0.029	24,300	128	102.8	211.8
14	Prototype	39	1.12	7.60	8.8	0.052	24,300	128	102.8	280.8

[a] Maximum rate of rise or fall. [b] Maximum water surface slope. [c] Distribution based on equal number

The average coefficient for filling the prototype lock based on the culvert area, which is the smallest section in the course of flow, was 0.91 as shown in the legend. In the model the coefficient based on the culvert area was 0.87. However, the smallest area in the course of flow in the model was at the throat of the ports, and when the model coefficient is correlated to the prototype value by applying the ratio of culvert to port area, 1.15, it is increased to

1.00, which is not in agreement with the prototype value. It would appear that the venturi type flow in the ports during filling makes a comparison of the lock coefficients unsatisfactory because the divergence effect in the model and prototype was not the same.

The average coefficients for the emptying operation, based on the least culvert or port area, are for the prototype 0.73 and for the model 0.64 × 1.15, or 0.74, which are in agreement within 2%. The flow through the lock-chamber ports during an emptying operation is a converging flow, and the decrease in the efficiency of the model as indicated by the lock coefficient is directly proportional to the reduction in the area of the controlling section.

OF MODEL AND PROTOTYPE

Coefficients		Distribution of Flow (Percentages)[c]							
		Intake Ports		Lock-Chamber Ports		Discharge Ports			
Lock hydraulic system (11)	Transition section (12)	Up-stream half (13)	Down-stream half (14)	Up-stream half (15)	Down-stream half (16)	Up-stream half (17)	Down-stream half (18)	Scale ratio (19)	Item No.
				(a) Filling Operation					
0.87	0.93	45.5	54.5	40.4	59.6	....	....	1	1
0.91	0.92	47.0	53.0	17.3	82.7	....	....	30	2 3
0.62	....	....	....	31.3	68.7	....	....	1	4
0.61	....	....	....	32.4	67.6	....	....	20	5
0.88	0.93	45.5	54.5	47.0[d] 46.5	53.0[d] 53.5	....	....	1	6
0.95	0.95	46.0	54.0	40.8	59.2	....	....	20	7
				(b) Emptying Operation					
0.64	....	....	....	42.1	57.9	44.4	55.6	1	8
0.73	....	....	....	40.5	59.5	44.4	55.6	30	9 10
0.56	....	....	....	30.2	69.8	....	....	1	11
0.58	....	....	....	35.0	65.0	....	....	20	12
0.63	....	....	....	44.3	55.7	44.4	55.6	1	13
0.63	....	....	....	46.5	53.5	43.6	56.4	20	14

of ports upstream and downstream. [d] Model test at Norris Laboratory.

In the orifice equation, used in computing lock-chamber discharge,

$$Q = C A \sqrt{2 g h} \qquad (5)$$

the head, h, for filling, is the upper pool elevation minus the lock-chamber elevation, and, for emptying, the lock-chamber elevation minus the lower pool elevation. This head is simply the hydrostatic head and is not the complete effective head acting on the column of water in the culverts. Considering the filling operation, the lock-chamber water surface does not stop when it reaches upper pool elevation but rises above it. Therefore, at the instant when the lock is full the effective head is not zero but has some positive value. When the dynamic head factor is included in the equation for discharge the resulting coefficients for decreasing heads become smaller instead of larger. Thus, in the prototype tests, as shown in Fig. 27, the ascending value of the coefficient as the head decreases can be attributed to the fact that the effective head was not used in the computation, whereas in the model tests the decrease in the coefficients for low heads may indicate that the hydrostatic head is more nearly equal to the effective head in the model than in the prototype.

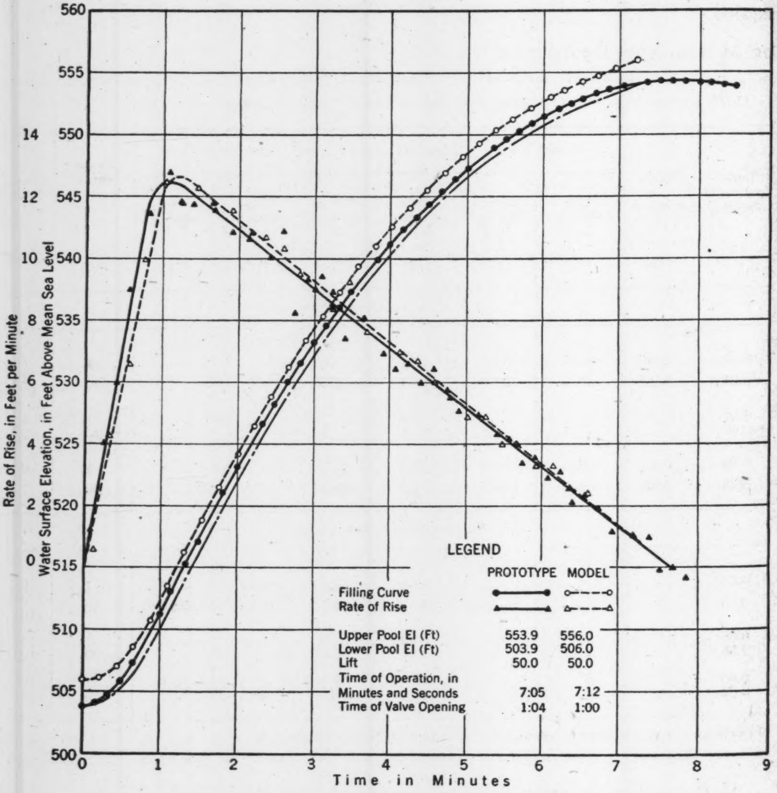


FIG. 28.—FILLING CHARACTERISTICS, WHEELER LOCK

Coefficients based on the effective head (that is, static plus dynamic) and coefficients based on static head alone have the same adverse characteristic of not being entirely constant throughout the lock operation. On the other hand, the effective head is not so easily measured as the static head. In view of the difficulties of establishing the exact value of the effective head and of the questionable advantage, in the comparison of model and prototype, of the coefficient thus obtained over that based on the static head, all lock coefficients presented in this paper are based on the static head alone.

WHEELER LOCK—COMPARISON OF PROTOTYPE AND MODEL TESTS

Lock Stage Tests.—Stages measured with respect to time during filling operations in the Wheeler model and prototype lock chambers are shown in Fig. 28. The points which define the curves are averages of two stage measurements, one near each end of the lock chamber. Typical sections through the lock-chamber ports are shown in Fig. 29.

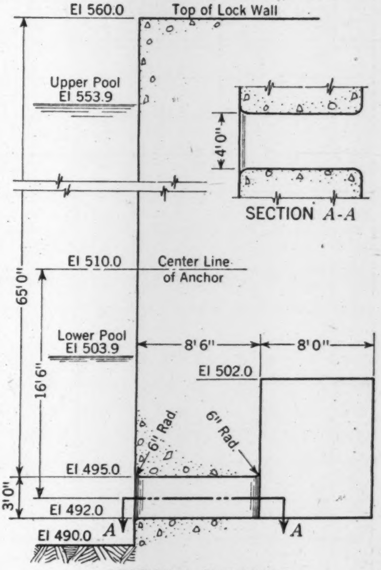


FIG. 29.—SECTIONS THROUGH CENTER LINE OF LOCK-CHAMBER PORT (10 PORTS IN EACH WALL)

In the prototype, it was necessary to make the tests with existing elevations of 553.9 for the upper pool and 503.9 for the lower pool; tests had been made previously in the model with elevations of 556.0 and 506.0, respectively. In each case the head was 50 ft, but the pool levels used in the model were 2.1 ft higher than in the prototype. In Fig. 28 the filling curve of the model is shown as observed and also adjusted to agree with the lower pool elevation of the prototype. The prototype lock filled in 7 min 5 sec, whereas the model filled in 7 min 12 sec. Maximum rates of rise were 12.4 and 12.6 ft per min, and the slopes of the rate of rise curves were 1.90 and 1.94 ft per min^2, respectively.

From emptying tests in the model and prototype locks there was observed about the same degree of agreement in prototype and model emptying characteristics as that shown in Fig. 28 for filling characteristics. Emptying time in the prototype was 7 min 22 sec, and in the model was 8 min 2 sec. Maximum rates of fall of the water surface in the chambers were 11.8 and 11.7 ft per min, respectively, and corresponding slopes of the rate of fall curves were 1.9 and 1.6 ft per min^2.

The valve opening periods were nearly identical in the prototype and model, and the crests of the water surface change curves occurred at about the same time in the two structures. In the filling tests the model was slightly more efficient than the prototype, whereas in the emptying tests the reverse was true.

Velocity in Lock-Chamber Ports.—Distribution of flow in the lock-chamber ports for the left or river wall of the prototype and for the right wall of the

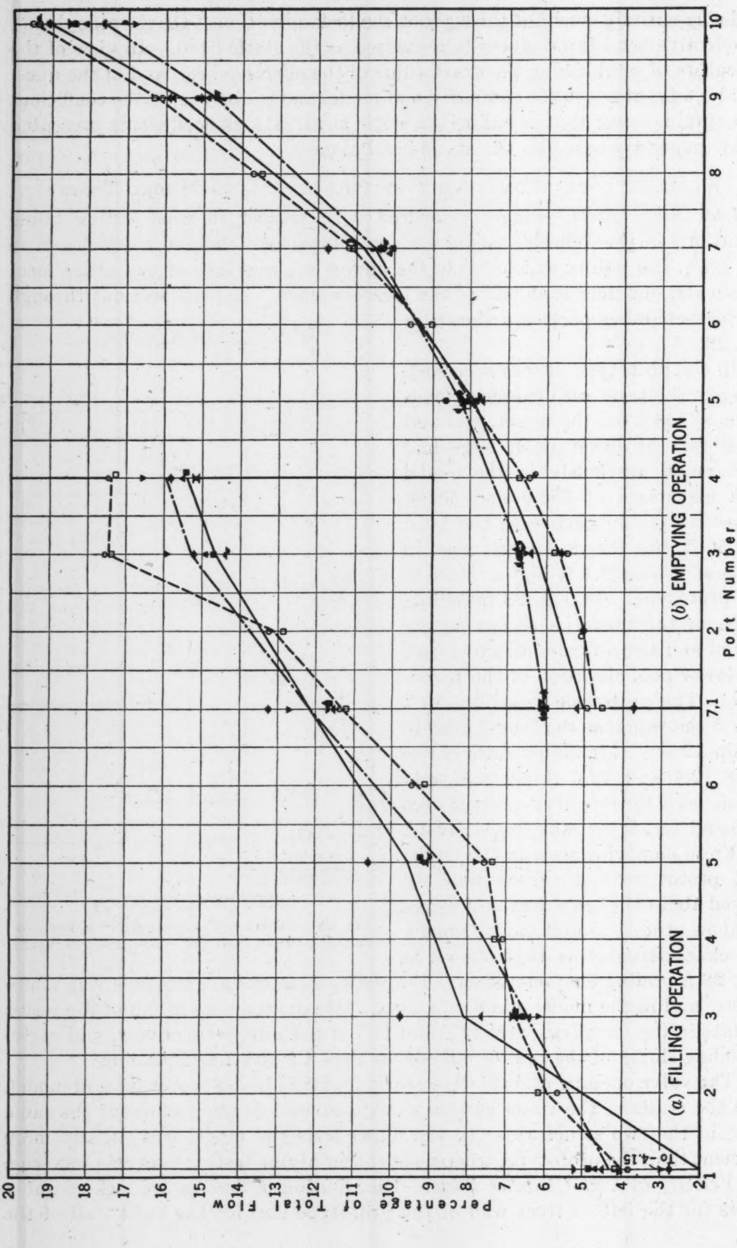


FIG. 30.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS, WHEELER LOCK (RIVER WALL) (SEE TABLE 3A)

model is shown in Fig. 30(*a*) for the filling operation. The curves for the model were based on velocity tests with steady flow through the system, whereas the curves for the prototype were based on velocity measurements made during filling operations. The distribution of flow observed in the lock-chamber ports of the model is in good agreement with that of the prototype (see Table 3*A*). Inflow into the lock through the upstream five ports in the prototype was 32.4% of the total, and in the model 31.3%. The corresponding values for the downstream five ports were 67.6% and 68.7%, respectively.

TABLE 3*A*.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS (RIVER WALL) (SEE FIG. 30)

Description	AVERAGE OF THREE HEADS[b]			AVERAGE OF SIX HEADS[b]						AVERAGE OF TWO HEADS[d]	
	-●-	◓	◒	■	▲	●	◖	◗	▼	□	○
Head, in feet	48	45	40	35	30	24	20	15	10	35	24
Discharge,[a] in Cubic Feet per Second:											
(*a*) Filling operation	..	..	2,430	2,280	2,110	1,920	1,750	1,540	1,260	2,370	1,960
(*b*) Emptying operation	..	..	2,390	2,240	2,040	1,810	1,690	1,460[c]	1,180	2,130	1,760

[a] Discharge refers to flow into, and out of, lock chamber through ports in river wall. [b] Prototype. [c] 16-ft head. [d] Model.

A good agreement is also noted in the distributions of discharge in the lock-chamber ports in emptying operations in the Wheeler model and prototype locks which are shown in Fig. 30(*b*).

Lock Coefficients.—Filling and emptying coefficients for the Wheeler Lock hydraulic system of the prototype and model are shown in Fig. 31. These coefficients are based on observed discharges computed from the average rate-of-rise and rate-of-fall curves and on theoretical discharge computed from stage records (see Eq. 4). The average filling coefficients for heads above 5 ft are 0.61 and 0.62 for the prototype and model, respectively. Corresponding emptying coefficients are 0.58 and 0.56. The lock coefficient varies as the square root of the slope of the rate-of-rise or rate-of-fall curves. For that part of the filling operation for which coefficients were computed, the model had the steeper rate-of-rise curve and the larger filling coefficient, and the prototype had the steeper rate-of-fall curve and the larger emptying coefficient.

As in Pickwick Lock, the pattern of the Wheeler coefficients indicated that there was less difference between the static and effective heads in the model than in the prototype.

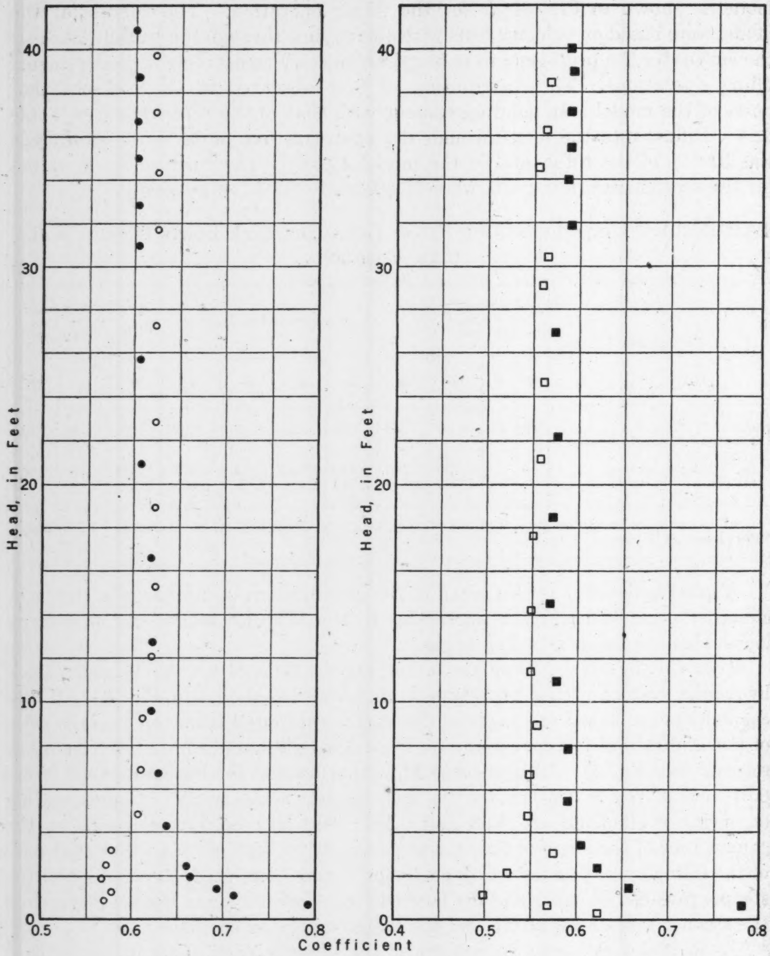
GUNTERSVILLE LOCK—COMPARISON OF PROTOTYPE AND MODEL TESTS

Intake Ports.—In the model tests on one lock[6c] it was found that the distribution of flow through the intake ports was fairly uniform and that it was affected only slightly by variations in the head. The intakes used in one model[6c] were also used in another model,[6d] and no further tests were made

---

[6c] Pickwick Lock.

[6d] Guntersville Lock.



Description	Legend:			
	●	○	■	□
Filling operation	Prototype	Model	....	....
Emptying operation	....	....	Prototype	Model
Average coefficient[a]	0.61	0.62	0.58	0.56

[a] For heads greater than 5 ft.

FIG. 31.—LOCK COEFFICIENT, WHEELER LOCK

in the latter model to determine the distribution of flow through intake ports. In prototype tests of flow distribution at one lock[6d] it was found that the distribution of flow in the four ports on one side of the intake section varied from 20.5% to 28% and that the pattern of flow distribution was similar to that for the Pickwick intake ports.

Transition Section.—The prototype and model transition sections are correlated by using the dimensionless coefficient of discharge because corresponding dimensions of the culvert at which pressure measurements were taken were not similar. Coefficients for the prototype and model transition section meter —0.95 and 0.93, respectively—are in good agreement.

Lock Stage Tests.—Stages with respect to time in the lock chambers (see Fig. 21) during filling operations in the model and prototype are shown in Fig. 32. The time required to fill the lock chamber with a 39.2-ft head in the prototype (5 min 30 sec) was 39 sec, or about 11%, less than the time required to fill the lock chamber in the model. The rate-of-rise curves indicate that the water surface rose faster in the prototype than in the model during the time the valves were opening. It is believed that this difference is due largely to a personal factor introduced in the manual operation of the valves in the model. After the valves are open the rate-of-rise curves are in better agreement, having slopes of 2.77 and 2.57 ft per min^2 for prototype and model, respectively. The maximum rates of rise were 12.7 and 12.5 ft per min in the prototype and model, respectively.

Emptying curves are shown in Fig. 33. The same period of time, 7 min 36 sec, was required to empty the model and prototype lock chambers with a head of 39.0 ft, and the emptying curves were practically identical. Maximum rates of fall in the prototype and model were 8.8 and 9.3 ft per min, respectively, and corresponding slopes of the descending rate-of-fall curves were 1.22 and 1.27 ft per min^2.

Lock-Chamber Ports.—The distribution of discharge in the lock-chamber ports observed in filling operations on the prototype, on the Iowa City model, and on a model of the Guntersville Lock tested in the TVA Hydraulic Laboratory at Norris, Tenn., are shown in Fig. 34.

Velocities measured in the lock-chamber ports of the models with steady flow are shown in the distribution of flow curves as percentages of the total flow. Velocities measured in the lock-chamber ports of the prototype lock during filling operations were plotted against time, and from these graphs the port velocity distribution for a given static head was obtained, using the time interval at which the given static head occurred.

The distribution of flow during the filling operation was better in the models than in the prototype. Percentages of total flow carried by the seven ports in the upstream half of the chamber and by the seven ports in the downstream half of the chamber are: For one model[6e] (average of four heads), 47% and 53%; for another model[6f] (25-ft head), 46.5% and 53.5%; and for the prototype (average of six heads), 40.8% and 59.2%. Patterns of flow distribution in the two models are very similar, and they agree more nearly with that

---

[6d] Guntersville Lock.

[6e] Norris Dam.

[6f] Iowa City model.

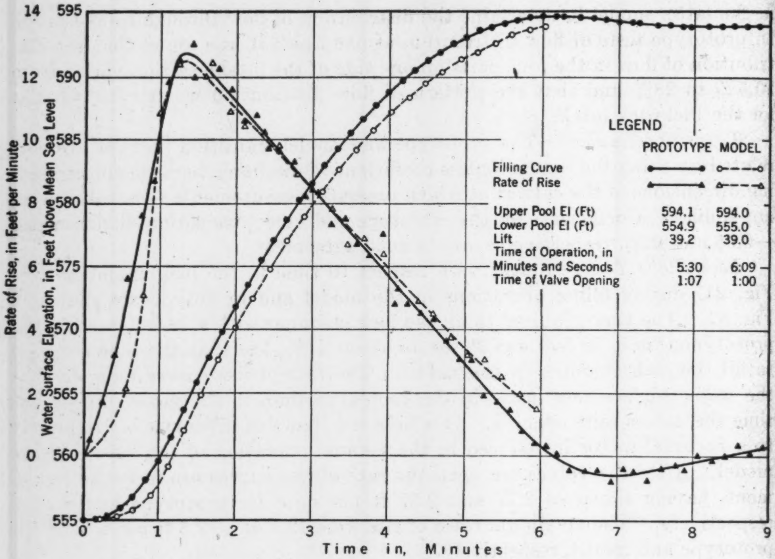


FIG. 32.—FILLING CHARACTERISTICS, GUNTERSVILLE LOCK

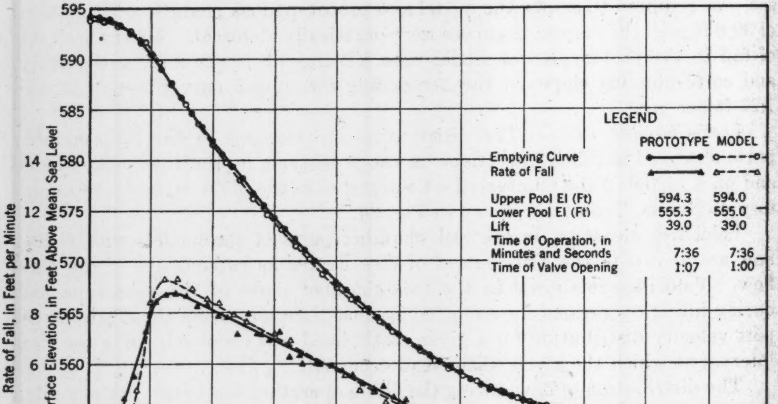
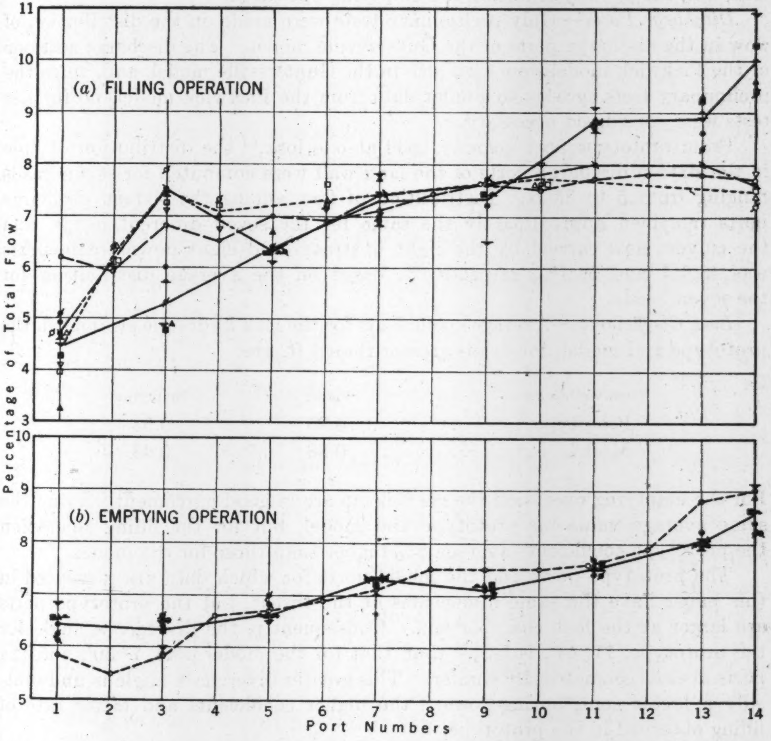


FIG. 33.—EMPTYING CHARACTERISTICS, GUNTERSVILLE LOCK

of the prototype at a 35-ft head than with the average prototype distribution after the valves are open.

The distribution of flow in the lock-chamber ports during the emptying operation in the model and prototype was in better agreement than the dis-



Description	Average of Six and Seven Heads[c]							One Head[d]	Average of Four Heads[d]			
	⦸	⊘	●	◓	■	▲	◒	○	ø	△	□	▼
Head, in feet.............	35[b]	30	25	20	15	10	5	25	33	25	17	7
Discharge,[a] in Cubic Feet per Second:												
(a) Filling operation.....	..	2,590	2,410	2,180	1,890	1,550	1,120	2,260	2,380	2,060	1,690	1,130
(b) Emptying operation..	..	1,740	1,600	1,430	1,240	1,020	730	1,640	....	....	....	....

[a] Discharge refers to the flow into the lock chamber through the ports in the land wall. [b] The curve for the 35-ft head is plotted as a separate line in Fig. 34(*a*), the remaining six in this group being averaged. In Fig. 34(*b*), all seven are averaged. [c] Prototype. [d] Model.

FIG. 34.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS (LAND WALL)

tribution during filling. Percentages of total flow carried by the seven ports in the upstream half, and in the downstream half of the chamber are, for one model[6f] (25-ft head), 44.3% and 55.7%, and for the prototype, 46.5% and 53.5%. The models had the more even distribution during the filling operation, and the prototype during the emptying operation.

Discharge Ports.—Only preliminary tests were made on the distribution of flow in the discharge ports of the Guntersville model. The discharge sections of the Pickwick model were used also in the Guntersville model, and, since the preliminary tests agree with similar data from the Pickwick model, no further tests were considered necessary.

From prototype port velocity tests at one lock,[6d] the distribution of flow in the sixteen discharge ports of the land wall were computed for seven heads ranging from 5 to 35 ft. Distribution of flow among the sixteen discharge ports remained approximately the same for the seven different heads, and the culvert flow carried by the eight upstream and eight downstream ports was 43.6% and 56.4%, respectively, based on the average distribution for the seven heads.

Lock Coefficients.—Average coefficients for the lock hydraulic systems in the prototype and model, for heads greater than 5 ft, are:

Observations on:	Filling	Emptying
Prototype	0.95	0.63
Model	0.88	0.63

For the emptying operation the coefficients are in good agreement, having the same average value for prototype and model, but for the filling operation the prototype coefficients average 8% higher than those for the model.

The prototype ports and the model ports for which data are presented in this paper have the same dimensions at the throat, but the prototype ports are larger at the lock-chamber end. Consequently, the divergence angle for the prototype, 11° 44′, is larger than that for the model port, 7° 50′, and the ports are not geometrically similar. This greater divergence angle is undoubtedly a factor contributing toward the higher coefficients and faster rate of filling observed in the prototype.

Although the area of the culverts was used in computing coefficients for the lock hydraulic system, the minimum area in the prototype and in the model is the cumulative area at the throats of the lock-chamber ports. If, instead of the culvert area, the minimum port area was used, the average coefficient for emptying would be 0.78. For emptying Pickwick Lock the average coefficients based on least port or culvert area were 0.73 and 0.74 for prototype and model, respectively.

The coefficients for a prototype lock[6d] based on static head gradually increase as the head decreases, and for the model they gradually decrease as the head decreases, until just before the end of the lock operation where values of the coefficient increase sharply.

[6f] Iowa City model.

[6d] Guntersville Lock.

SUMMARY AND CONCLUSION

In Table 3 the results of model and prototype studies of Pickwick, Wheeler, and Guntersville locks are summarized. It includes operation data and areas of critical sections of the lock hydraulic systems.

Tests on one lock[6b] offered the best opportunity for comparing model and prototype performance because the corresponding structures were geometrically similar and lock operation data were in agreement. Moreover, it will be noted that this lock[6b] had the most consistent agreement between model and prototype of the three locks tested, the only material difference in the performance of the corresponding structures being in the longitudinal slope of the water surface for which readings in the prototype lock were larger than in the model.

For model and prototype locks of Guntersville there was agreement in lock operation data, and the two structures were geometrically similar except at the lock end of the lock-chamber ports. A more efficient filling operation was observed in the prototype than in the model, the former having the greater divergence angle in the lock-chamber ports. Furthermore, during the filling operation, the less efficient model had the more even distribution of flow in the lock-chamber ports. It is of interest to note the performance of Guntersville model and prototype locks during the emptying operation. There was agreement in operation time, lock coefficient, and the distribution of flow in the lock-chamber ports irrespective of the difference in lock-chamber port divergence angles. The throat of the lock-chamber port, for which the cumulative area was the minimum in the system, appeared to be effective as the control section of the system during the emptying operation.

There was less agreement in the performance of one model and prototype locks[6c] because, in addition to dissimilarity in the throat area of the lock-chamber ports, lift and valve time in the two structures were not in agreement. However, the performance of this model and prototype[6c] during the emptying operation, similar to that of other locks,[6d] showed agreement in the lock coefficients based on the minimum areas in the respective systems and in the distribution of flow in the lock-chamber ports.

When geometric similarity does not exist in the important elements of two hydraulic structures it is not expected that their operations should conform to the basic laws of similitude. However, where geometric similarity did obtain, and where analogous functions could be correlated by means of the basic laws, this study substantiates the theory upon which the use of hydraulic models is founded and, it is believed, tends to justify the confidence engineers have placed upon this method of analyzing difficult problems in design and operation within the past few years.

ACKNOWLEDGMENTS

Col. R. G. Powell, then division engineer, Ohio River Division, Cincinnati, Ohio, sponsored the project and authorized the tests on the prototype locks

---

[6b] Wheeler Lock.

[6c] Pickwick Lock.

[6d] Guntersville Lock.

in December, 1937. The tests were supervised by personnel of the U. S. Engineer Sub-Office, Iowa City, under the direction of the district engineer, U. S. Engineer Office, St. Paul, Minn.

The prototype lock tests were conducted by Marvin J. Webster, Assoc. M. Am. Soc. C. E., of the Iowa City Sub-Office, with the assistance of personnel from the District Engineer Offices in whose areas the respective locks are located. Some of the material presented in this paper has been drawn from a report on the project prepared by Mr. Webster for which acknowledgment is made. Permission to use the data obtained in the prototype lock tests is acknowledged to Col. J. W. Moreland, M. Am. Soc. C. E., district engineer of St. Paul.

BOULDER DAM OUTLET WORKS

BY S. P. WING,[7] M. AM. SOC. C. E.

SYNOPSIS

The principal hydraulic model tests of the dam outlet[7a] works (34), which included flow through the intake tower, penstock turbine, and outlet-valve manifolds, were made on a 5⅝-in. (1 : 64 scale), extremely smooth, sheetmetal model which had a friction coefficient of $f = 0.021$ at a Reynolds number **R** of 80,000 and used a maximum of 2 cu ft per sec of water. Performance and efficiency tests of the 30-ft diameter prototype, utilizing a maximum discharge of 20,000 cu ft per sec with measured velocities up to 90 ft per sec and at a Reynolds number of from 40 to 70 $\times 10^6$, made it possible to obtain data from which various hydraulic features of the model and prototype could be compared. These included loss coefficients for the trashrack, bends, branches, valves, and straight-pipe friction. A full discussion of the data was presented before the hydraulic section at the Annual Convention of the Society in July, 1940 (35). This paper is a summary of such of the results as may be generally useful in hydraulic design, together with suggestions concerning the technique of field measurements which, if found practicable, may make possible better comparisons between models and prototypes in future tests.

The letters in Figs. 35 and 36 indicate points at which measurements were taken. Distances are given in units of pipe diameters, that of the 25-ft section being converted into its 30-ft hydraulic equivalent. Three series of comprehensive tests were run from 1937 to 1939 with heads varying from 213 to 337 ft. During the tests the turbines were shut down, varying discharges being obtained by manipulating the 84-in. outlet valves. Pressures were measured with calibrated Bourdon gages and differential mercury manometers, utilizing the dead-ended penstocks for piezometric connections. Tests showed that such large connections taking off at an angle resulted in the gages registering an excess pressure of 0.04 h_v.

Discharges were obtained by impact-pressure tube traverses at right angles to one another in a 96-in. section of pipe located immediately downstream from a reducer and in front of valve A_1, point K. Utilizing these measurements and the simultaneously measured heads on the other valves, the total discharge for the entire battery of valves was computed. The coefficient of the pitot tube in the formula,

$$Q = \frac{A}{C} \sqrt{2\,g\,H} \qquad \text{(6)}$$

---

[7] Civ. Engr., Bureau of Reclamation, Denver, Colo.

[7a] Boulder Dam.

obtained from a still-water rating, was corrected for wall proximity effects and for the effects due to curved streamlines caused by the valve and the obstruction of the streamlined struts used to support the instrument (see Fig. 37). These latter corrections were determined by the photoelastic laboratory of the Bureau of Reclamation, using the electric-analogy method (see Fig. 38). Fig. 39 shows the results found by the laboratory, the close correspondence between the laboratory result and that obtained in the rating channel for 2% obstruction being of interest. Since the obstruction offered by the struts is a variable percentage in each circular lamina of the pipe, separate coefficients were applied to each individual reading of the pitot traverse.

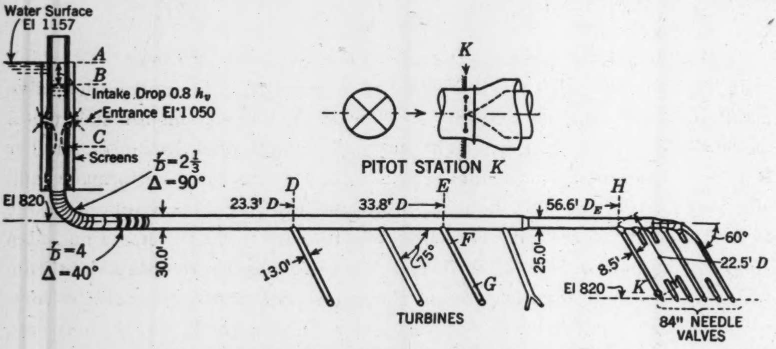

Water Surface
El 1157
A
B
Intake Drop 0.8 h_v
Entrance El 1 050
C
Screens
$\frac{r}{D}=2\frac{1}{3}$
$\Delta=90°$
El 820
$\frac{r}{D}=4$
$\Delta=40°$
30.0'
K
PITOT STATION K
D
23.3' D
E
33.8' D
H
56.6' D_E
13.0'
75°
F
G
25.0'
TURBINES
8.5'
El 820
K
60°
22.5' D
84" NEEDLE VALVES

FIG. 35.—SCHEMATIC VIEW OF OUTLET WORKS, BOULDER DAM

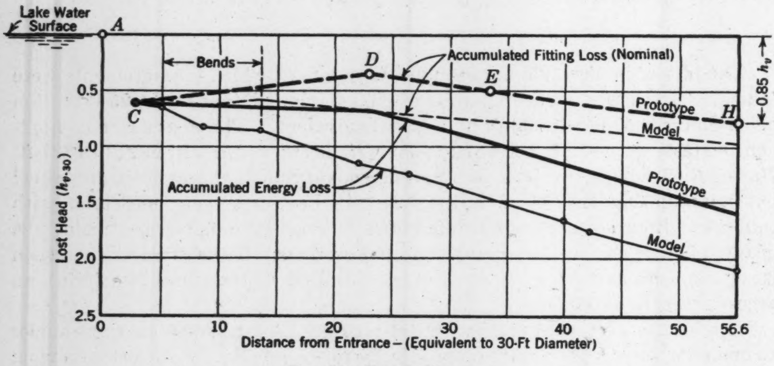


FIG. 36.—ENERGY LOSSES

Plots of the velocity and pressure distribution in the cross section indicated that upstream from the pitot tube the friction in 22.5 diameters of pipe plus a 15% reducer did not introduce enough resistance to normalize the upset velocity distribution due to branching, the velocity contours showing the crescent-like form (typical of flow below a bend) superimposed on the upstream cone of pressure caused by the downstream valve. The latter caused differences in pressure up to 40 ft of water, or nearly 0.5 h_v, between the pipe wall and the center of the pipe. Worth considering in valve design is the fact that the energy of flow was also unsymmetrical, the half of the pipe away from the center of curvature of the branch carrying 10% to 15% h_v more energy than the inside, with the possibility that pressure unbalance existed within the valve.

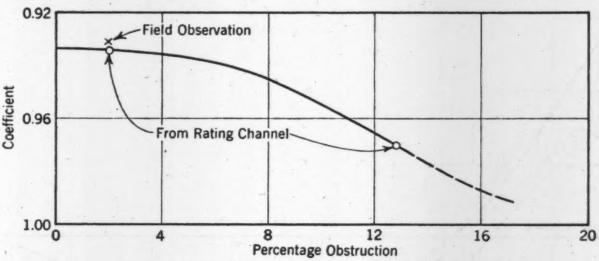


FIG. 37.—PITOT-TUBE COEFFICIENT VERSUS PERCENTAGE OBSTRUCTION

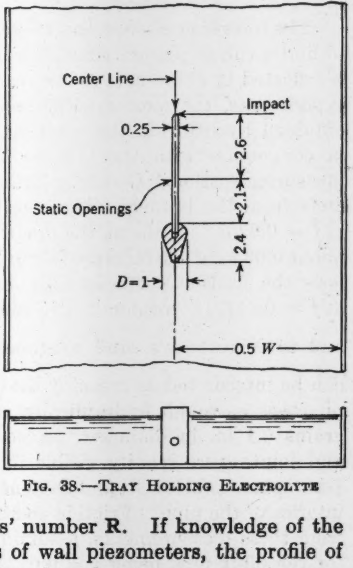


FIG. 38.—TRAY HOLDING ELECTROLYTE

On the whole, the pitot-tube gagings were reasonably self-consistent. An estimate of their errors and a comparison with current-meter gagings in the river indicated a standard error for the largest discharges of the order of 2%.

The interpretation of losses in a penstock, with fittings but relatively few diameters apart, depends upon an ability to separate fitting losses from those due to straight pipe. A pipe entrance or fitting tends to result in a contracted jet of high-velocity water which changes in 20 to 200 diameters downstream to a normal velocity distribution, the length required being dependent on the roughness of the pipe wall and on Reynolds' number **R**. If knowledge of the energy grade is based only on the readings of wall piezometers, the profile of the energy grade characteristically will be found to show an apparent increase in energy in the first 3 or 4 diameters following a fitting, pass through a point of inflection, and then will become concave upward, approaching asymptotically the friction gradient. The model test, moreover, indicated the possibility that nodes of pressure of about 0.03 h_v, located 3 to 4 diameters apart possibly caused by the expansion and contraction of the high-velocity jet, were superimposed on the profile immediately downstream from the branch fitting. This possibility requires more investigation before it can be taken as a fact.

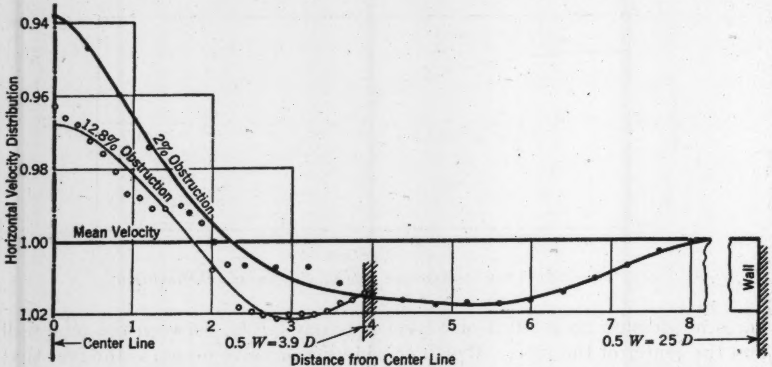


FIG. 39.—VELOCITY DISTRIBUTION VERSUS WIDTH IN THE PLANE OF THE STATIC OPENINGS

The foregoing phenomena make it almost impossible in a power installation to find a run of pipe in which it can be certain that only straight-pipe friction is reflected in differences in the readings of wall piezometers. For the present experiments, the most reliable result estimates of straight-pipe friction were obtained by utilizing the measurements made in the 13-ft turbine penstocks in connection with the Gibson (36) efficiency tests of the turbines. The measuring section F G of Fig. 35 included 8 diameters of pipe, located 8 diameters from the branch. The results, Fig. 40, at $\mathbf{R} = 20 \times 10^6$, gave a value of $f = 0.0207$. Utilizing the model tests, which showed that in such a location about 0.03 h_v of the measured drop should be ascribed to the upstream diversion loss, the friction coefficient for 13-ft diameter straight pipe was estimated at $f = 0.0177$, corresponding to Manning's $n = 0.015$, to Scobey's $K_s = 0.475$, and to Nikuradse's sand roughness factor $\frac{K}{\mathbf{R}} = 0.00134$. This latter factor can be interpreted as meaning that the bitumastic-covered surface of the 13-ft pipe was as rough hydraulically as if it had been coated with uniform sand grains 0.1 in. in diameter. According to the field observers, broom marks and joints were readily visible in the bitumen, a 20-ft straightedge showed $\frac{1}{8}$-in. openings below it, and flattened bolt heads projected $\frac{1}{8}$ in. from the welded interior of the pipe. Friction coefficients for other sizes of pipe were computed from these measurements, assuming them to vary inversely as $D^{1/3}$, the value for the 30-ft pipe being $f = 0.0134$. These unexpectedly high values can be compared with the results obtained from concrete pipe at comparable values of **R** plotted on the graph of Fig. 40. Also on the same graph are plotted overall losses per diameter for each of the sections measured—A D, A E, A H, etc. (see Fig. 35). Since the values are approximately constant at 0.03 h_v per

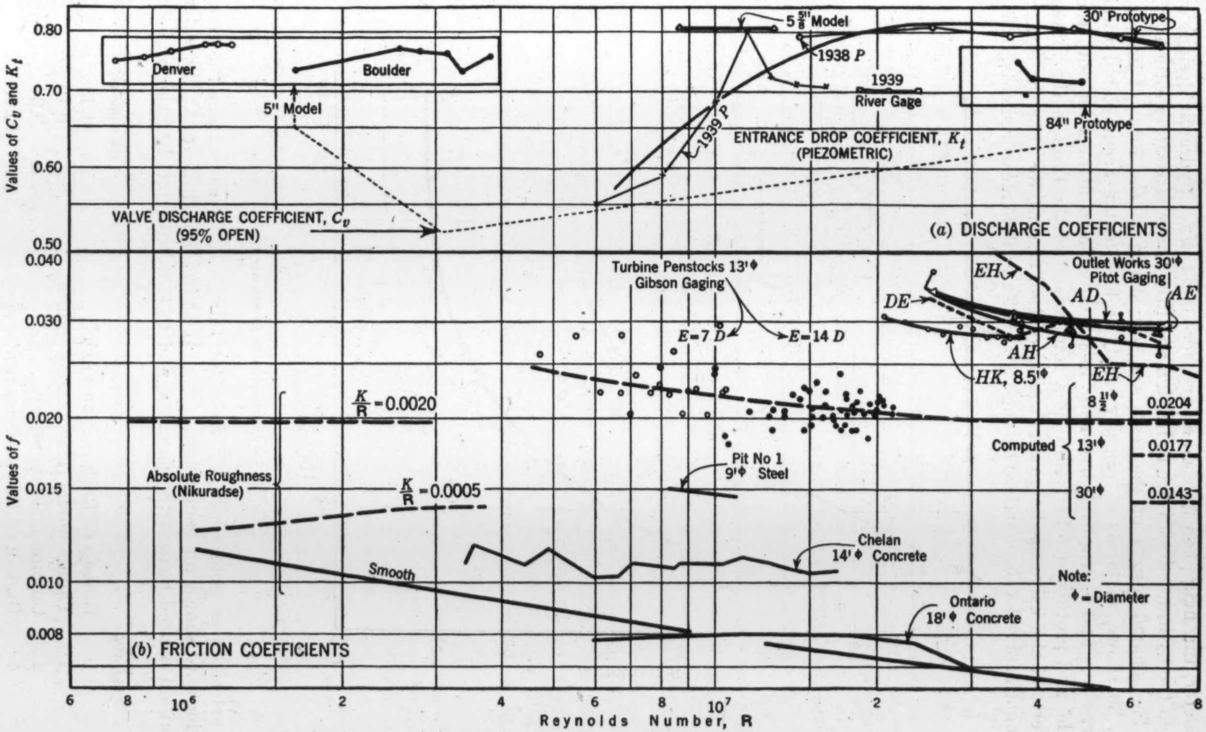


FIG. 40.—COMPARISON OF FRICTION FACTORS

diameter, this figure can be used as an over-all check for hydraulic designs in which the hydraulic losses due to friction and fittings are separately estimated.

The general results of the tests are best shown in the form of graphs. Fig. 36 shows, in terms of velocity head, profiles of the accumulated total energy loss and of the total hydraulic loss ascribable to fittings for both the model and prototype. Fig. 40(*a*) is a comparison of the drop in the intake tower and the valve discharge coefficients plotted against **R**. For the former a smooth curve has been passed through the 1939 and 1938 measurements, based on velocities as determined by the pitot tube. The results from the river gagings are shown for comparison. Fig. 41 shows branch losses at the valve manifold plotted against the percentage of total flow diverted to the branch:

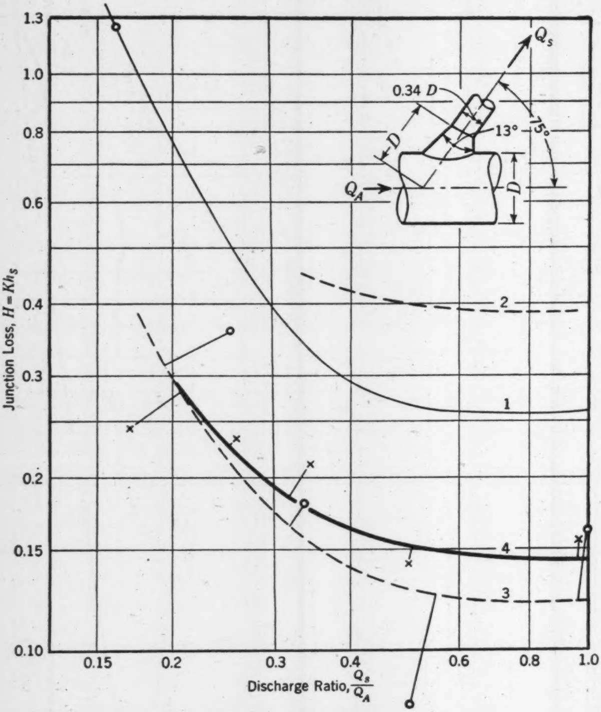


FIG. 41.—JUNCTION LOSS COEFFICIENTS

Curve (Fig. 41)	Structure	Reynolds' number (in millions)
1	Bureau of Reclamation model	0.05
2	Thoma model (interpolated values)	0.10
3	Commercial laboratory	0.30
4	Bureau of Reclamation prototype	40.00

Table 4 compares the various loss coefficients for the entire structure as originally estimated, as found from the model, and as estimated from the prototype tests.

Quantitative correspondence between model and prototype can be expected only where Reynolds' criteria can be satisfied and where absolute roughness in the model can be scaled down in accordance with model laws. This was not possible in the Boulder model which, while as smooth as possible, was materially rougher than demanded by similitude. Referring to the fitting loss of Fig. 36, it can be seen that in the prototype at point D, 23 diameters from the entrance, there was an indicated loss of about one half that measured in the model and at point H, 57 diameters downstream, a value 85% as large. It will be noted, moreover, that at point D the accumulated fitting loss in the prototype is shown as only one half that observed 20 diameters upstream immediately downstream from the intake. It will be realized at once that this decrease in accumulated loss downstream as measured by wall piezometers is due to the fact that the profiles are only nominal energy grades and include no corrections for actual velocity distributions. It appears probable from these tests that the prototype, with relatively less friction than the model, requires a greater length, measured in diameters, to regain its pressure head following the contraction below the intake. The required length may be analogous to the distance required to produce normal distribution below an intake in a smooth pipe, for which S. Goldstein (37) gives the formula

$$L = 0.7\, D\, \mathbf{R}^{0.25} \qquad (7)$$

Applied to the present tests, four times the length required in the model, measured in diameters, would be required to produce normal velocity distribution in the prototype. Whatever the reason, it is believed that the over-all measured prototype fitting loss, 0.85 h_v (which includes entrance, two bends, the disturbances caused by openings for four branches, together with their eight tie rods, and a 30 by 25 reducer), provides a satisfactory check on the model results. In Table 4 this over-all loss has been broken down into its

TABLE 4.—COMPARISON OF LOSS COEFFICIENTS

No.	Description	Loss Coefficient in Terms of Velocity Head of 30-Ft Diameter Pipe		
		Preliminary design	Model	Prototype
1	Screens, h_v of 30-ft pipe	....	0.10	0.01
2	Intake drop from the lake to within the tower	....	0.81	0.76 ±
3	Estimated entrance loss 57 diameters from intake	1.10	0.68	0.63
4	Estimated vertical and horizontal bend loss	0.40	0.11	0.08
5	Estimated losses for eight 12-in. tie rods and four 13-ft branch openings	0.20	0.17	0.12
6	Estimated 30 by 25 reducer loss	0.05	0.04	0.02
7	Measured total entrance and fitting losses at 57 diameters	1.75	1.00	0.85
8	Branch loss with 100% diversion, measured 30 diameters from branch	....	0.26	0.14
9	Valve coefficient	0.72	0.76	0.72
10	Friction coefficient f for 30-ft pipe	0.012	0.021	0.0134

components and compared with the standard loss coefficient originally used for the preliminary design of the outlet works[7a] and with those later obtained from the model. Although the breakdowns are only estimates based on a consideration of all the data, these fitting coefficients should be helpful to designers until better values become available. It is estimated that the values have a standard error of the order of 15%.

The entrance loss into the tower through its radial ports, 0.63 h_v as given from the foregoing breakdown, is sometimes confused with the piezometric drop which occurs from the lake to the water surface within the tower. The latter (as is shown in the bulletin[7a] (34) on the model tests) involves a reaction head due to the water turning downward within the tower and is not an absolute measure of entrance losses. This coefficient of tower drop, however, as obtained from various gagings, forms a good means of comparing performance of model and prototype. The results are shown in Fig. 40. Although at first sight the variations in the different tests may seem large, the 12% range in the experimental values is believed comparable to the precision of the majority of prototype tests. The increase in the coefficient with increasing **R**, as indicated by the curve averaging the 1939 and 1938 prototype results, may, perhaps, be partly accounted for by the formation of vortices at the intake which were observed in the 1938 measurements. At that time the head on the intake was only from four to six times the mean velocity head. Since the velocity heads for some of the stream filaments passing around certain unavoidable sharp edges of the intake, measured in terms of the mean velocity, may have been even a larger multiple of the mean head with the corresponding absolute pressures approaching zero, this is a condition which should be taken into account by designers, as the vacuum may cause cavitation and the entraining by vortices of large quantities of air. Without complete exploration of pressures in a model, it is a condition that is difficult to predict.

The model showed a loss through the screens of 0.10 h_v. In the prototype the loss was less than one tenth of this value. This undoubtedly was due to the fact that velocity through the model screens was below Reynolds' critical value. A model should not reproduce from the prototype details within which subcritical velocities may exist during the model tests.

Of considerable interest to designers is the loss due to a closely connected 90° vertical and 40° horizontal bend. Although the computed loss coefficient for the prototype of 0.08 h_v may have a considerable percentage error, the total measured loss for all the fittings was such that the bend loss, regardless of assumption, must have been small. To the writer's knowledge this is one of the few prototype tests that has been made on bends.

The junction loss for the branch from the manifold is shown in Fig. 41. Curve 1 for the model is interpolated from a very complete series of tests described in the bulletin[7a] (34). Curve 2 was likewise interpolated from D. Thoma's well-known tests (38). Curve 3 was obtained from the results of a commercial laboratory in connection with the design of the turbine manifold. It will be noted that the observations defining its averaging curve are widely scattered. Any one who has examined critically the technique by which fitting losses must be obtained, as set forth in the bulletin[7a] (34), will appreciate how difficult it is to obtain reliable results and will know that only in the exceptional test can the absolute values of the results be accepted. It is enough to state that coefficients of fitting loss are not fixed quantities depending merely on shape. They tend to decrease with increasing smoothness and with increase of Reynolds' number, and to be greater the greater the distance below the fitting at which they are measured. Also, it is to be noted that with multiple similar branches from a manifold, the loss will be different at each successive offtake due to change in the value of the percentage of the main flow diverted.

[7a] Boulder Dam.

The needle valve discharge coefficients are shown in Fig. 40. It will be noted that in the model with increasing **R** the coefficient increased, whereas in the prototype it decreased. The first case undoubtedly reflects the decrease in friction through the valve with increase in **R**. In the second case an analysis of the velocity distribution due to a curved streamline within the valve indicated the probability that at certain points vacuum was approached at high values of **R**. Once this point was reached, the effective area of the waterway would be reduced, cavitation would commence, and the discharge coefficient would decrease. The performance of the prototype was in line with these predictions.

It is believed that on the whole the use of a model for predicting quantitative results in the outlet works[7a] was pre-eminently successful, nearly halving the error in design that would have existed without its use. On the other hand, much more experimental data and development of theory are needed if predictions of losses within 10% are desired in the prototype. Particularly required are methods for forecasting pipe friction in terms of a quantitative pipe roughness and for estimating fitting losses at various distances downstream in the stretch before velocity distribution has become normal. Finally, it must not be forgotten that there is no way in which a factor of safety can be applied in the economic layout of the flow line of a hydroelectric development. An overestimate of hydraulic losses leads to the overdimensioning of the conduit with excess interest charges in perpetuity, whereas an underestimate leads to continuous waste of power in excess friction losses.

ACKNOWLEDGMENT

The field work was directed by John Parmekian and N. G. Noonan. Special appreciation is due R. E. Kennedy, M. Am. Soc. C. E., who made the computations.

The hydraulic laboratory of the Bureau of Reclamation is under the direct charge of Jacob E. Warnock, M. Am. Soc. C. E. All design and research work is under J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer. S. O. Harper, M. Am. Soc. C. E., is chief engineer, and the Bureau is directed by John C. Page, M. Am. Soc. C. E., commissioner.

[7a] Boulder Dam.

SPILLWAY COEFFICIENTS, NORRIS DAM

By G. H. Hickox,[8] M. Am. Soc. C. E.

Synopsis

The determination of discharge coefficients for the dam spillway crest gates[8a] is described in this paper. Actual coefficients calculated for the gates from records of their performance during the flood of January and February, 1937, are compared with the coefficients predicted from tests made on a 1 : 72 scale model by the Bureau of Reclamation. This comparison indicates that the coefficients predicted by the model average 3.9% larger than those actually measured on the dam during this period.

General Conditions

Norris Dam was built by the TVA as a part of its program of navigation, flood control, and power development of the Tennessee River Valley. It is situated on the Clinch River about 25 miles northwest of Knoxville, Tenn., and controls the discharge of 2,912 sq miles of land drained by the Clinch and Powell rivers.

It is a concrete gravity structure having a total length of 1,860 ft and a maximum height, to the roadway on the top at El. 1061, of 265 ft. The spillway crest is at El. 1020 and is 300 ft long, divided into three bays each 100 ft long. Three hydraulically operated steel drum gates are installed on the concrete spillway crest, and the crests of the gates may be raised to any elevation between 1020 and 1034 to retain flood peaks. Eight outlet conduits permit the discharge of water when the reservoir level is below the spillway crest. The maximum discharge of these conduits with reservoir level at El. 1020 is approximately 37,000 cu ft per sec. The power house contains two generating units, each having a capacity of 56,000 kva.

Fig. 42 shows a general plan and elevation of the dam and a section through the spillway. Fig. 43 gives the dimensions of the spillway crest in detail.

Flood of January, 1937.—Unusually heavy rains on the Ohio River drainage basin in December, 1936, and January, 1937, resulted in flood stages all along the Ohio River. During this period the entire flow of the Clinch and Powell rivers above Norris Dam was stored in Norris Lake. This necessitated raising the spillway drum gates to hold water a maximum of about 11 ft above the concrete spillway crest. Late in January it was seen that moderate discharges at Norris Dam would not cause further rises on the Ohio River, and accordingly water was released from Norris Lake to provide storage space for additional runoff. Much of the water was released by lowering the crest gates and allowing it to discharge over the spillway. Advantage was taken of this opportunity to obtain the necessary measurements for determining the discharge coefficients of the spillway for heads on the gates as great as 9.8 ft and for discharges as great as 30,000 cu ft per sec.

---

[8] Senior Hydr. Engr., TVA, Norris, Tenn.

[8a] Norris Dam.

DATA AVAILABLE

Gate Operating Records.—The program of water release at Norris Dam required the discharge to be held approximately constant for considerable periods of time. It was necessary to change the elevation of the crest gates, since the water level in the reservoir was changing. Accordingly, the gates were reset about every two hours. A complete record of the operation of the spillway crest gates, giving the time at which changes were made in the gate

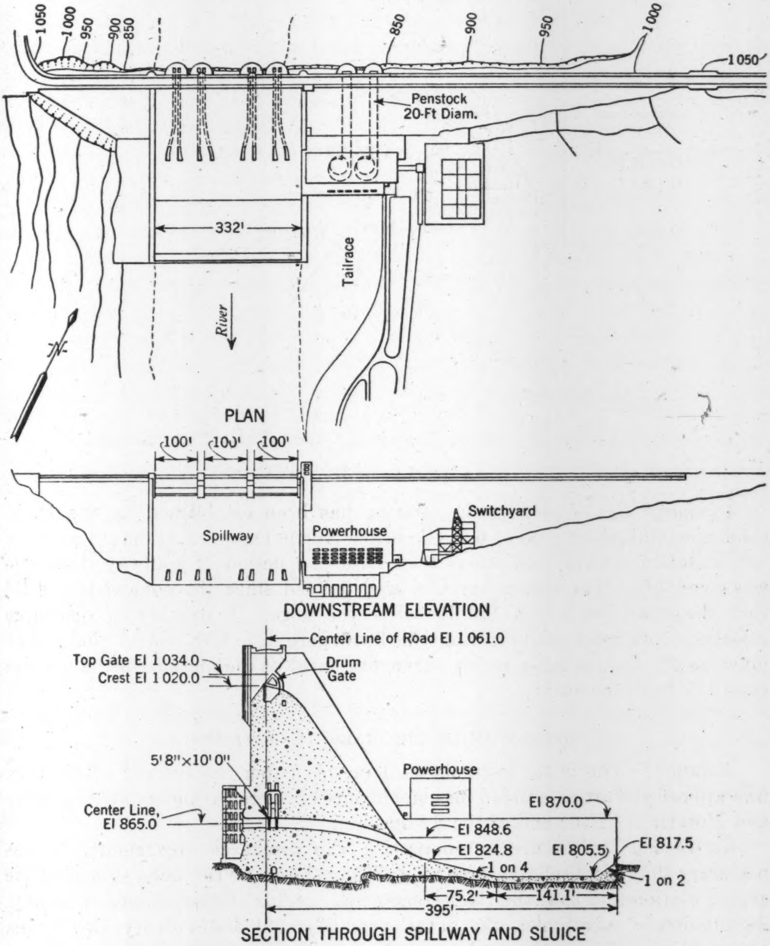

1050
1000
950
900
850
850
900
950
1000
1050
Penstock
20-Ft Diam.
332'
Tailrace
River
N
PLAN
100'
100'
100'
Spillway
Powerhouse
Switchyard
DOWNSTREAM ELEVATION
Center Line of Road El 1 061.0
Top Gate El 1 034.0
Crest El 1 020.0
Drum
Gate
5' 8"×10' 0"
Powerhouse
Center Line,
El 865.0
El 870.0
El 848.6
El 824.8
El 805.5
El 817.5
1 on 4
1 on 2
75.2'
141.1'
395'
SECTION THROUGH SPILLWAY AND SLUICE

FIG. 42.—PLAN, ELEVATION, AND SECTION OF NORRIS DAM

settings and the elevations to which the gates were set, was available from the records kept at the power house. Records of reservoir water-surface elevations were also available.

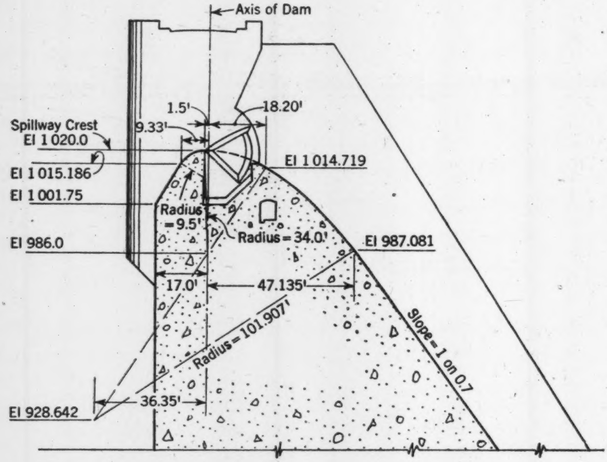


FIG. 43.—DIMENSIONS OF SPILLWAY CREST

Discharge Records.—A gaging station has been established by the U. S. Geological Survey at a point 0.9 mile below Norris Dam. A standard recorder was installed, and a continuous record for the period of spillway discharge was available. The rating curve is well defined since the control is a solid rock diagonal reef about 150 ft below the gage. A number of discharge measurements were made during the flood period. Only one of these is as much as 4% from a mean rating curve, and most of the measurements are less than 1% from the curve.

CALCULATION OF DISCHARGE COEFFICIENTS

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

Correlating the Data.—In computing the discharge coefficients, it was necessary that the discharge over the spillway should be the same as that at the gaging station and that the river stage at the gaging station should be steady. As previously mentioned, the gates were adjusted about every two hours, causing fluctuations in the discharge. In most cases a 2-hr period was sufficient to allow the stage at the gage to become steady, since the changes in discharge were usually small. When observations were used which were only two hours apart, the elevations of the gate crest and the reservoir level were taken as of immediately before the change—that is, at the end of the 2-hr period. The gage height at the gaging station was also taken at the end of the period.

When large changes in discharge occurred, a greater time was required for the river stage to become steady. When it appeared from the recorder chart that a steady condition had not been established at the gage, the data were not used. No data were used to determine discharge coefficients if there was any doubt as to their value, either because of unsteady flow or for any other reason. Use of river discharge at the gaging station to represent the discharge over the spillway at the same time is justified because of the care in rejecting data which did not represent steady flow and because of the relatively small storage between the dam and the gaging station.

Table 5, Cols. 1 to 8, inclusive, lists the data regarding reservoir and gate-crest elevations obtained from the power-house records. Col. 9, Table 5, gives the gage height, in feet, at the gaging station, corrected from the continuous chart.

Calculation of Discharge.—A rating curve for the gaging station was constructed from the discharge measurements by plotting the discharges against gage heights and drawing a curve through them so as to give greatest weight to those measurements that were made during the period of observation.

There is practically no inflow to the river between the dam and the gaging station. The gaging station records indicate that, just before spillway discharge began, the combined inflow and leakage through the dam was about 50 cu ft per sec. Col. 10, Table 5, gives discharges taken from the rating curve, less 50 cu ft per sec for all discharges less than 10,000 cu ft per sec. For discharges greater than 10,000 cu ft per sec, the correction was neglected since it amounted to less than 0.5%. The discharges of Col. 10, determined as described, are considered to be the spillway discharges for the corresponding reservoir and gate-crest elevations of Cols. 3 to 7.

Calculation of Discharge Coefficient C.—The discharge coefficient, C, was computed by means of the customary weir equation,

$$Q = C\,b\,H^{1.5} \quad \text{.........................} (8)$$

in which: Q = discharge, in cubic feet per second; C = discharge coefficient; b = length of weir crest, in feet (for one dam, 300 ft); and H = head on weir crest, in feet.

Cols. 4, 5, and 6, Table 5, show that the crest gates were not all at the same elevation but varied somewhat, giving different heads on the gates. The discharge coefficient varies slightly with both the elevation of the gate crest and the head, but the variation is not perceptible for the differences observed. The maximum variation of head between gates is 5.7% for Observation No. 1. The results of the first six observations are quite widely scattered, and, if these are omitted from consideration, the maximum variation of head is 1.5% in Observation No. 7. Variation of the discharge coefficient due to this change in head may be neglected. A slight error is also introduced if the average head is used in the weir formula, but with a maximum head difference of 1.5%, this error also may be safely neglected. Col. 8, Table 5, gives the average head on the three gates for each observation. Values of C calculated from

TABLE 5.—Discharge Coef-
(Calculation of Prototype Coefficients

Observation No.	Time	Reservoir elevation[a]	Elevation[a] of Crest			Average elevation[a] of crest
			Gate 1	Gate 2	Gate 3	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	January 31					
1	11:05 a.m.	30.35	29.28	29.28	29.34	29.30
2	1:10 p.m.	30.41	28.68	28.68	28.74	28.70
3	3:10 p.m.	30.45	28.27	28.27	28.33	28.29
4	5:00 p.m.	30.47	27.86	27.86	27.93	27.88
	February 1					
5	8:30 a.m.	30.70	27.56	27.56	27.63	27.58
6	11:05 a.m.	30.73	27.05	27.05	27.12	27.07
7	7:00 p.m.	30.78	26.10	26.10	26.17	26.12
8	9:00 p.m.	30.78	25.74	25.74	25.82	25.77
9	11:00 p.m.	30.78	25.54	25.54	25.62	25.57
	February 2					
10	1:00 a.m.	30.77	25.20	25.20	25.27	25.22
11	3:00 a.m.	30.77	24.90	24.90	24.97	24.92
12	5:00 a.m.	30.75	24.55	24.55	24.62	24.57
13	7:15 p.m.	30.63	24.55	24.55	24.62	24.57
	February 3					
14	8:50 a.m.	30.49	24.44	24.44	24.52	24.47
15	5:00 p.m.	30.37	24.34	24.34	24.42	24.37
16	10:15 p.m.	30.32	24.24	24.24	24.32	24.27
	February 4					
17	8:25 a.m.	30.17	24.04	24.04	24.12	24.07
18	12:45 p.m.	30.08	23.74	23.74	23.82	23.77
19	1:45 p.m.	30.03	23.54	23.54	23.62	23.57
20	2:45 p.m.	30.01	23.13	23.13	23.21	23.16
21	4:45 p.m.	29.98	22.53	22.53	22.61	22.56
22	8:45 p.m.	29.86	22.33	22.33	22.41	22.36
	February 5					
23	2:40 a.m.	29.73	22.12	22.12	22.21	22.15
24	10:00 a.m.	29.54	21.92	21.92	22.00	21.95
25	1:00 p.m.	29.45	21.82	21.82	21.90	21.85
26	2:00 p.m.	29.49	21.72	21.72	21.80	21.75
27	11:45 p.m.	29.13	21.52	21.52	21.60	21.55
	February 6					
28	3:15 a.m.	29.03	21.32	21.32	21.40	21.35
29	12:00 m.	28.77	21.12	21.12	21.20	21.15
30	2:00 p.m.	28.68	21.02	21.02	21.10	21.05
31	4:30 p.m.	28.61	20.00	20.00	20.00	20.00
	February 7					
32	12:05 a.m.	28.37	20.71	20.71	20.79	20.74
33	1:30 a.m.	28.33	20.61	20.61	20.69	20.64
34	4:30 a.m.	28.23	20.51	20.51	20.60	20.54
35	8:00 a.m.	28.16	20.41	20.41	20.50	20.44
36	10:00 a.m.	28.17	20.00	....	20.00	20.00
37	12:00 m.	28.13	....	....	20.00	20.00
38	2:05 p.m.	28.10	....	20.11	....	20.11
39	4:10 p.m.	28.07	20.11	....	....	20.11
40	7:25 p.m.	28.05	20.31	20.31	20.40	20.34
41	10:10 p.m.	27.95	20.21	20.21	20.30	20.24
	February 11					
42	6:00 a.m.	31.05	22.63	22.63	22.71	22.66
43	9:00 a.m.	31.09	22.28	22.28	22.36	22.31
44	12:00 m.	31.10	21.72	21.72	21.80	21.75
45	12:00 mid.	30.96	21.12	21.12	21.20	21.15
	February 12					
46	4:30 a.m.	30.86	21.02	21.02	21.10	21.05
47	8:00 a.m.	30.74	20.91	20.91	21.00	20.94
48	10:00 a.m.	30.67	20.81	20.81	20.89	20.84
49	2:00 p.m.	30.58	20.71	20.71	20.79	20.74
50	7:00 p.m.	30.43	20.61	20.61	20.69	20.64
51	9:30 p.m.	30.35	20.51	20.51	20.60	20.54
	February 13					
52	2:30 a.m.	30.16	20.31	20.31	20.40	20.34
53	5:00 a.m.	30.07	20.21	20.21	20.30	20.24
54	10:50 a.m.	29.82	20.11	20.11	20.20	20.14
55	1:45 p.m.	29.72	20.00	20.00	20.00	20.00
56	2:15 p.m.	29.70	20.00	20.00	20.00	20.00
57	3:15 p.m.	29.65	20.00	20.00	20.00	20.00

[a] Add 1,000 to Cols. 3 to 7, inclusive, to obtain actual elevations.

FICIENTS, NORRIS SPILLWAY
and Comparison with Model Tests)

Average head, *H*, in ft (8)	Gage height, in ft (9)	Corrected discharge, *Q*, in cu ft per sec (10)	*C* (11)	*K* (12)	*n* (13)	C_m (14)	% difference (15)	Observation No. (1)
1.05	2.34	1,150	3.56	3.82	1.506	3.82	+ 7.3	1
1.71	3.35	2,800	4.17	3.77	1.510	3.81	− 8.6	2
2.16	3.87	3,760	3.96	3.74	1.512	3.78	− 4.5	3
2.59	4.56	5,200	4.16	3.70	1.516	3.77	− 9.4	4
3.12	4.71	5,570	3.37	3.66	1.519	3.73	+10.7	5
3.66	5.29	7,050	3.36	3.59	1.523	3.70	+10.1	6
4.66	6.41	10,300	3.40	3.44	1.533	3.61	+ 6.2	7
5.01	6.80	11,500	3.43	3.37	1.538	3.57	+ 4.1	8
5.21	7.00	12,100	3.39	3.33	1.541	3.56	+ 5.0	9
5.55	7.36	13,200	3.36	3.27	1.545	3.53	+ 5.1	10
5.85	7.77	14,500	3.41	3.21	1.549	3.50	+ 2.6	11
6.18	8.06	15,300	3.31	3.13	1.555	3.47	+ 4.8	12
6.06	7.69	14,200	3.18	3.13	1.555	3.44	+ 8.2	13
6.02	8.03	15,200	3.43	3.11	1.557	3.45	+ 0.6	14
6.00	7.74	14,400	3.27	3.08	1.559	3.42	+ 4.6	15
6.05	7.65	14,100	3.16	3.06	1.561	3.43	+ 8.5	16
6.10	7.55	13,800	3.05	3.02	1.565	3.38	+10.8	17
6.31	7.87	14,800	3.12	2.95	1.571	3.36	+ 7.7	18
6.46	8.11	15,500	3.15	2.91	1.575	3.35	+ 6.4	19
6.85	8.55	16,800	3.13	2.84	1.582	3.32	+ 6.1	20
7.42	9.12	18,500	3.06	2.75	1.591	3.30	+ 7.8	21
7.50	9.54	19,800	3.22	2.73	1.593	3.30	+ 2.5	22
7.58	9.53	19,800	3.16	2.71	1.596	3.28	+ 3.8	23
7.59	9.46	19,600	3.13	2.69	1.598	3.28	+ 4.8	24
7.60	9.52	19,700	3.13	2.68	1.599	3.27	+ 4.5	25
7.74	9.59	20,000	3.10	2.67	1.600	3.28	+ 5.8	26
7.58	9.56	19,900	3.18	2.66	1.602	3.27	+ 2.8	27
7.68	9.52	19,700	3.08	2.64	1.604	3.27	+ 6.2	28
7.62	9.66	20,200	3.21	2.62	1.607	3.25	+ 1.2	29
7.63	9.69	20,300	3.21	2.62	1.608	3.28	+ 2.2	30
8.61	10.95	24,100	3.18	2.57	1.617	3.32	+ 4.4	31
7.63	9.68	20,200	3.19	2.60	1.611	3.25	+ 1.9	32
7.69	9.70	20,300	3.18	2.60	1.612	3.28	+ 3.1	33
7.69	9.73	20,400	3.20	2.59	1.613	3.26	+ 1.9	34
7.72	9.73	20,400	3.18	2.59	1.613	3.26	+ 2.5	35
8.17	7.93	14,900	3.19	2.57	1.617	3.29	+ 3.1	36
8.13	5.23	6,880	2.96	2.57	1.617	3.29	+11.2	37
7.99	5.23	6,880	3.04	2.57	1.617	3.29	+ 8.2	38
7.96	5.16	6,670	2.97	2.57	1.617	3.26	+ 9.8	39
7.71	9.58	19,900	3.10	2.58	1.614	3.25	+ 4.8	40
7.71	9.76	20,500	3.19	2.58	1.614	3.25	+ 1.9	41
8.39	10.75	23,500	3.23	2.75	1.589	3.33	+ 3.1	42
8.78	11.19	24,800	3.18	2.72	1.592	3.32	+ 4.4	43
9.35	12.10	27,500	3.21	2.67	1.600	3.34	+ 4.1	44
9.81	13.02	30,300	3.29	2.63	1.607	3.37	+ 2.4	45
9.81	12.95	30,100	3.27	2.62	1.607	3.35	+ 2.4	46
9.80	12.92	30,000	3.26	2.61	1.608	3.34	+ 2.5	47
9.83	13.10	30,600	3.31	2.61	1.610	3.37	+ 1.8	48
9.84	13.06	30,400	3.28	2.60	1.611	3.35	+ 2.1	49
9.79	12.91	30,000	3.27	2.60	1.612	3.35	+ 2.4	50
9.81	12.88	29,900	3.25	2.59	1.612	3.34	+ 2.8	51
9.82	12.83	29,700	3.22	2.58	1.614	3.35	+ 4.0	52
9.83	12.88	29,900	3.24	2.58	1.615	3.35	+ 3.4	53
9.68	12.68	29,300	3.25	2.57	1.617	3.34	+ 2.8	54
9.72	12.79	29,600	3.26	2.57	1.617	3.34	+ 2.5	55
9.70	12.86	29,800	3.29	2.57	1.617	3.34	+ 1.5	56
9.65	12.87	29,800	3.31	2.57	1.617	3.34	+ 0.9	57
Total, Nos. 7 to 36 and 40 to 57, + 189.0; average, $\frac{189.0}{48}$							+ 3.9	

the formula,

$$C = \frac{Q}{b\,H^{1.5}} \qquad (9)$$

are given in Col. 11.

COMPARISON WITH MODEL COEFFICIENTS

Determination of Model Discharge Coefficients.—Discharge coefficients for the dam spillway were measured by the Bureau of Reclamation on a 1 : 72 scale model (39). It was found that because of the shape of the drum-gate crest, which changed with the elevation of the gate, C, in Eq. 8 was neither a constant nor a function of the head. The Bureau found that the tests made on the model could be represented by the equation,

$$Q = K\,b\,H^{n} \qquad (10)$$

in which K and n are constants which vary only with the elevation of the gate crest. Values of K and n for each elevation of the drum-gate crest are given (39*a*) in Fig. 44.

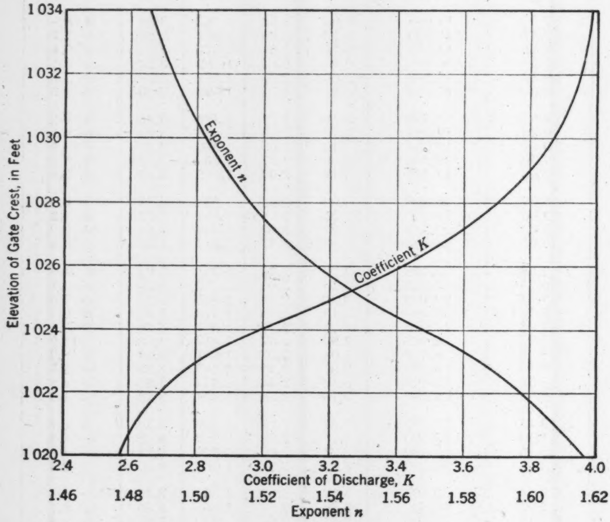


FIG. 44.—VALUES OF K AND n IN EQ. 10

To compare the model tests with the results of observations on the prototype, it was desirable that coefficients of discharge be expressed in the same terms for each. The data obtained on the prototype do not cover a sufficient range to permit the determination of K and n. However, values of C can be computed from the Bureau's tests. Equating the two expressions for Q, Eqs. 8 and 10,

$$K\,b\,H^{n} = C\,b\,H^{1.5} \qquad (11)$$

From this,

$$C = K\,H^{n-1.5} \qquad (12)$$

Values of K and n from Fig. 44 are given in Cols. 12 and 13 of Table 5, and the corresponding value of C, computed by Eq. 12, is given in Col. 14. This column has been headed C_m in order to distinguish the coefficients computed on the basis of the model tests from those determined on the prototype. Col. 15, Table 5, compares the two values of C by showing the percentage by which C_m is in error when compared to C.

A few of these values should not be used for drawing general conclusions. The first six observations are for relatively small discharges and low heads on the crest for which errors of observation cause large percentages of variation. Observations Nos. 37 to 39 give discharge coefficients for a single gate. They are probably correct but should not be compared with the coefficients determined for three gates, since the discharge over a single gate is subject to much greater end contractions. If these nine observations are eliminated, the average of the forty-eight remaining values is +3.9%, indicating that the model tests predicted discharges which average 3.9% greater than those actually obtained. The reason for this is not known, although it is possible that the model coefficients are not accurately known in this range. It is generally believed that, other things being equal, small-scale models will give discharge coefficients somewhat less than the prototype. There is no reason to discard any prototype observations other than those referred to, as all questionable observations were omitted before calculations were made. It is believed that the values of C determined for the prototype are reasonably accurate.

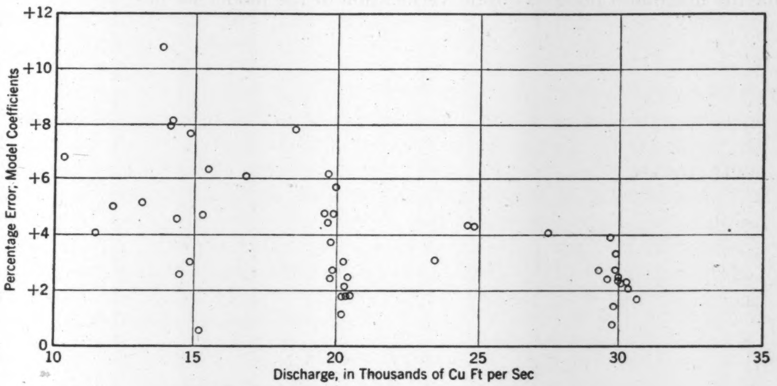


FIG. 45.—PERCENTAGE OF ERROR IN MODEL COEFFICIENTS

Fig. 45 shows the percentage difference of the forty-eight observations considered, plotted against the discharge. There is a definite tendency for the difference to decrease with increasing discharge. In this connection, it is interesting to note that, of the tests performed by the Bureau of Reclamation, only five actually fell within the range of operating conditions represented by the observations on the prototype. In other words, the model discharge coefficients for this range have been determined largely by extrapolation. As the region of more model data is approached, the difference becomes less. It is possible that the difference may decrease still further for higher discharges. Observations for discharges in the range of 60,000 to 100,000 cu ft per sec will be necessary before a more definite comparison of the model and its prototype can be made.

CONCLUSION

The comparison shows that model tests may be relied on to predict prototype discharge coefficients with an accuracy at least equal to that of streamflow measurements. The average discrepancy of 3.9% is not serious and is certainly less than might be expected without the benefit of model tests. It is quite possible that this difference might have been much less if the measurements on the prototype had been made in a range of discharges that were more adequately covered by model tests. This seems to be indicated by the trend of Fig. 45.

Accurate quantitative comparisons of model and prototype discharge coefficients are scarce because the model tests are generally made in the region of maximum discharge which seldom, if ever, occurs in the prototype. Many more comparisons could be made if the model were tested not only for the maximum discharge but also for the smaller discharges that occur more often. It is suggested that future tests on spillway coefficients be made to include the range of frequent prototype operation so that it will not be necessary to wait for the maximum flood to obtain verification of the model studies.

PRESSURE HEADS ON BONNEVILLE DAM

BY J. C. STEVENS,[9] M. AM. SOC. C. E., AND R. B. COCHRANE,[10]
ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The main hydraulic features pertaining to the design of Bonneville Dam, in Washington and Oregon, were studied by models in the Bonneville Hydraulic Laboratory. (This laboratory was located at Government Moorings near Linnton, within the City of Portland, Ore., from December, 1933, to January, 1937; hence, it is sometimes called the Linnton Hydraulic Laboratory. It was later moved to the Bonneville Dam site.) Among these features were the hydraulic characteristics of the 50-ft by 50-ft crest gates with water flowing beneath them (40), and the hydraulic effect of departures of the profile of the constructed dam from the design profile. During the latter studies the pressures on the model crest were determined for various gate openings.

A series of piezometer taps constructed in the crest of the prototype dam, corresponding to those in the model, were used to observe pressures on the crest for various gate openings.

This paper presents the results of the pressure-head profiles on the underface of the overflowing nappe in both model and prototype. When the gate was in the downstream position, the pressure profiles of model and prototype checked fairly well; when the gate was in the upstream position, however, anomalies developed that may serve as the subject for some interesting speculations.

THE MODEL

A replica of a 10-ft length of the crest and gate was made to a scale of 1 : 5 in a flume 2 ft wide, 10 ft deep, and 10 ft long (Fig. 46). In the prototype piers there are two gate grooves; in one the gate seal is 9 ft upstream from the center line of the crest and in the other it is 8 ft downstream. To simulate these positions the entire model crest block could be shifted to either position. The gate was held by stay rods so that it moved parallel to itself, and the rods were so long that this movement virtually simulated the prototype. The effect of the gate grooves was absent in the model, watertightness being secured by stanching strips of rubber.

MODEL TESTS

The model tests were made primarily to determine the effect of slight departures of the constructed crest from the design crest, and to establish the tolerances that might be permitted in the crest and ogee profile. These experiments consisted essentially of observing pressures at the contact surface between the concrete crest and the overflowing nappe. Three distorted crests were studied for gate openings of 0.25, 0.5, 1.0, 2.0, and 3.0 ft with the gate in both the upstream and the downstream positions. Fig. 47 shows the design crest and the three types of distortions used in the experiments.

---

[9] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[10] Capt., Corps of Engrs., U. S. Engr. Office, Bonneville, Ore.; formerly, Director, Bonneville Hydr. Laboratory.

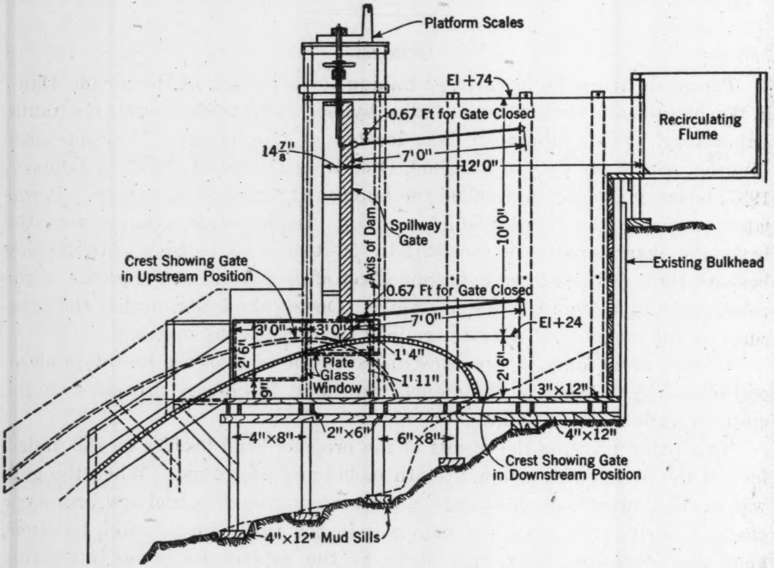

Platform Scales
El +74
0.67 Ft for Gate Closed
Recirculating Flume
$14\frac{7}{8}$"
7'0"
12'0"
10'0"
Axis of Dam
Spillway Gate
Existing Bulkhead
Crest Showing Gate in Upstream Position
0.67 Ft for Gate Closed
3'0"
3'0"
7'0"
El+24
2'6"
1'4"
Plate Glass Window
1'11"
2'6"
9"
3"×12"
4"×8"
2"×6"
6"×8"
4"×12"
Crest Showing Gate in Downstream Position
4"×12" Mud Sills

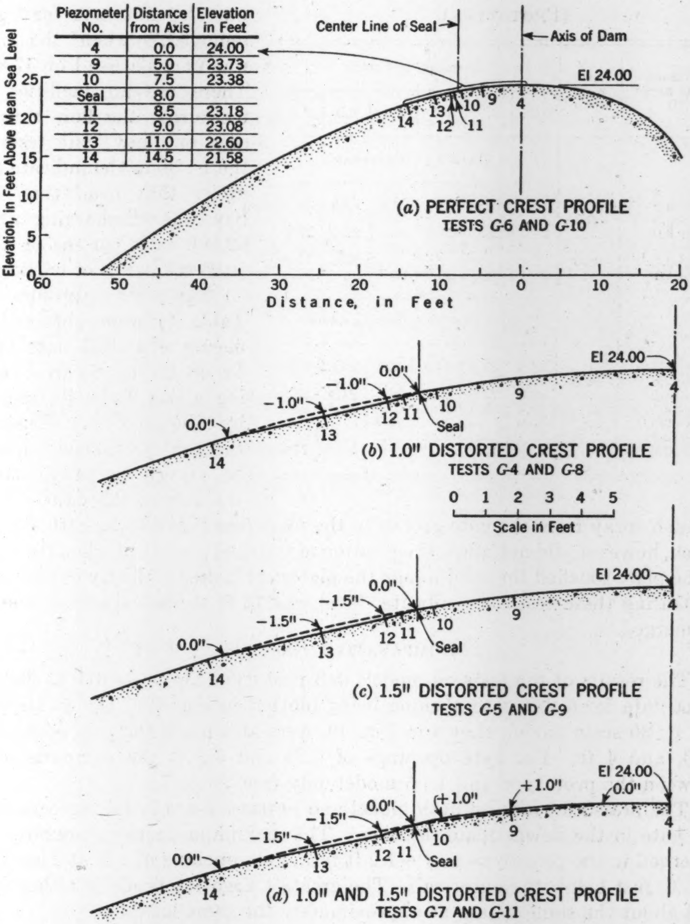
Fig. 46.—Sectional Elevation of Gate Model, Bonneville Dam; Scale, 1 : 5

In addition to tests on the 1 : 5 model, pressure on the crest was observed for a large range of gate openings in the middle one of three bays of a 1 : 36 model. The gate was in the downstream position only, and the piezometer taps were at 5-ft intervals downstream from the center line of the crest. Since the prototype gate openings were 4 ft and less, comparisons up to that value only could be made.

Prototype Tests

Prototype tests were made in Bay No. 13 of the completed dam in October, 1939. The crest profile in this bay was without appreciable departure from the design crest. The piezometer taps were spaced to conform to those of the 1 : 5 model except that in the prototype, Piezometer No. 1, 4 ft upstream from the axis, was substituted for model Piezometer No. 8, 9.58 ft upstream from the axis. The prototype also had, in addition to those in the model, Piezometers Nos. 10 and 12 located 27.5 ft and 42.5 ft downstream from the axis.

The piezometer taps were made in a steel plate 10 in. wide set flush with the concrete and bent to the shape of the design crest. From each tap a ½-in. iron pipe led through a stopcock to a manifold in an inspection gallery in the dam. A 60-lb Bourdon pressure gage, reading positive pressures above atmospheric, was connected to the manifold at El. −28.89 ft, so that the pressure at each piezometer could be read in turn.



Piezometer No.	Distance from Axis	Elevation in Feet
4	0.0	24.00
9	5.0	23.73
10	7.5	23.38
Seal	8.0	
11	8.5	23.18
12	9.0	23.08
13	11.0	22.60
14	14.5	21.58

Center Line of Seal
Axis of Dam
El 24.00
Elevation, in Feet Above Mean Sea Level
0
5
10
15
20
25
60
50
40
30
20
10
0
10
20
Distance, in Feet
14
13
12
10
9
11
4
(a) PERFECT CREST PROFILE
TESTS G-6 AND G-10
El 24.00
0.0"
−1.0"
−1.0"
0.0"
14
13
12 11
10
9
4
Seal
(b) 1.0" DISTORTED CREST PROFILE
TESTS G-4 AND G-8
0 1 2 3 4 5
Scale in Feet
El 24.00
0.0"
−1.5"
−1.5"
0.0"
14
13
12 11
10
9
4
Seal
(c) 1.5" DISTORTED CREST PROFILE
TESTS G-5 AND G-9
El 24.00
+1.0"
0.0"
+1.0"
0.0"
−1.5"
−1.5"
0.0"
14
13
12 11
10
9
4
Seal
(d) 1.0" AND 1.5" DISTORTED CREST PROFILE
TESTS G-7 AND G-11

FIG. 47.—CREST PROFILES, MODEL OF BONNEVILLE DAM

Two series of tests were made, one with the gate in the downstream groove —its normal position—and the other in the upstream groove—an emergency position. In addition to observing the pressures at the underface of the nappe, the surface profile of the nappe was obtained for a distance of 24.5 ft downstream from the gate seal when in its downstream position and 13 ft downstream from the gate seal when in its upstream position.

TABLE 6.—WATER SURFACE ELEVATION (PROTOTYPE)

Distance from axis (ft)	Gate wide open	Opening, in Feet:					
		0.25	0.5	1.0	2.0	3.0	4.0
		(a) GATE 9 FT UPSTREAM					
−9.0	....	23.2	23.4	23.9	24.9	25.9	26.9
−3.3	59.2	....	....	....	....	....	....
−3.0	....	24.7	25.0	25.1	25.7	26.2	27.2
0	....	....	....	....	....	....	....
+3.8	56.0	....	....	....	....	27.0	27.8
+4.1	....	26.1	26.0	26.0	26.1	....	....
		(b) GATE 8 FT DOWNSTREAM					
+ 8.0	....	23.6	23.8	24.3	25.3	26.3	27.3
+12.8	50.8	....	....	....	....	....	....
+13.2	....	22.8	22.6	23.3	24.1	24.4	25.2
+23.1	....	19.6	19.5	20.2	20.6	21.5	21.2
+32.4	....	15.9	....	....	16.2	....	....
+32.5	....	....	....	15.6	....	....	....
+32.6	....	....	15.3	....	....	17.0	17.4

The surface profile for a length of 16 ft near the crest was also obtained for "gate removed," when the crest was under a head of 47.8 ft. There are no facilities for measuring the flow through any one bay, but tests on the 1 : 36 model indicate that under that head this 50-ft bay was discharging about 62,000 cu ft per sec or 1,240 cu ft sec per ft of crest.

The surface profiles (see Table 6) were obtained by means of a steel plate 48 in. by 48 in. by 1½ in., weighing about 1,000 lb, handled by the auxiliary hoist on the gantry crane which could be moved perpendicular to the axis of the dam. There is much spray from the gate groove in the piers (see Fig. 48) beneath the gate which, however, did not affect the position of the steel plate; but when the corner of the plate touched the solid nappe the plate was kicked violently downstream.

During these tests the tailwater level was 12 ft so that the crest was not submerged.

COMPARATIVE RESULTS

The results of the tests on models and prototype are presented in Table 7, other data for a 2-ft gate opening being plotted in Fig. 49. Observations on the 1 : 36 scale model, shown in Fig. 49, were also made for gate openings of 1, 3, and 4 ft. For gate openings of 0.25 and 0.5 ft the comparison was between the prototype and 1 : 5 model only (see Table 7).

The pressure heads on both model and prototype are in fair agreement for the gate in the downstream position. The maximum negative pressure head observed in the prototype was −1.2 ft for a gate opening of 1 ft at Piezometer No. 6, just below the gate seal. The greatest negative heads in either model had about the same values at approximately the same localities.

For the gate in the upstream position there are some interesting departures of the prototype pressure heads from those of the model. Table 7(*b*) shows the comparative pressure heads (see also Fig. 49(*b*), for a 2-ft opening). Unfortunately, prototype Piezometers Nos. 3 and 8 were clogged. Every effort to free them with compressed air failed.

If the prototype pressure heads for Piezometer No. 3 could have been taken for gate openings of 0.25 and 0.5 ft, one might assume that conformity could have been demonstrated, but one dares not go any farther with that assumption. For gate openings of 1, 2, and 3 ft, the anomalies between prototype piezometer heads Nos. 2 and 3, and the corresponding model piezometer heads Nos. 4 and 9, became increasingly obvious.

In drawing the line of prototype pressure heads between Piezometers Nos. 7 and 9 for the five gate openings, one might have "upped" the curve to conform to the line obtained in the model, but without the model results the prototype line would not have been so drawn. Moreover, to have drawn it otherwise than indicated in the example (Fig. 49), particularly for the gate openings of 3 and 4 ft, would have made it a rather "goofy" curve.

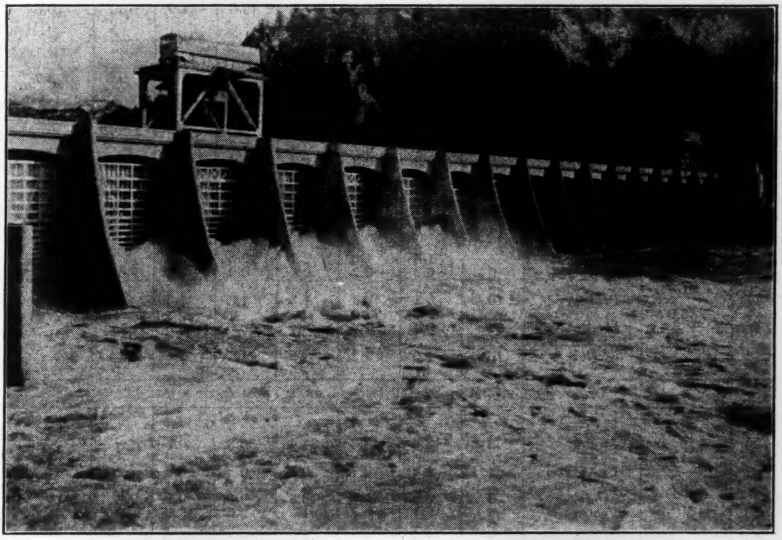


FIG. 48.—VIEW OF BONNEVILLE DAM SHOWING SPRAY FROM GATE GROOVE IN PIERS

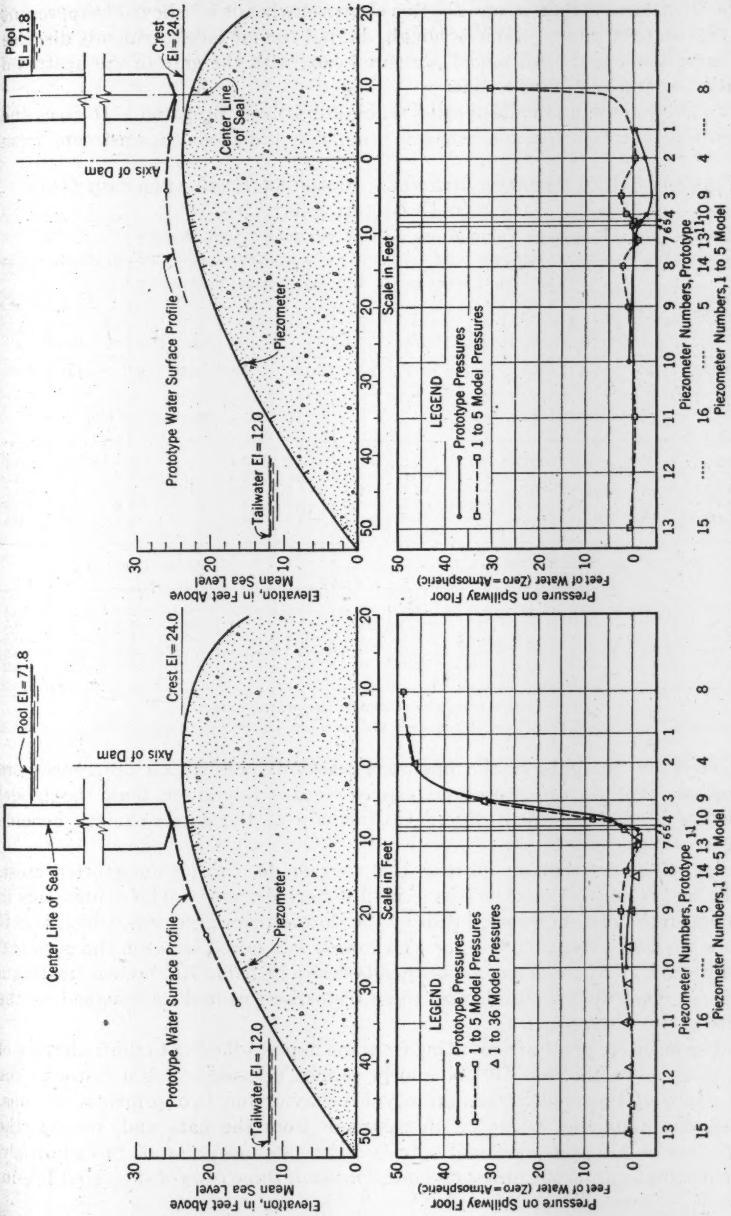
The explanation for the discordant results undoubtedly lies in the fact that with the gate in the upstream position there is a tendency for the jet to spring free from the crest. This tendency creates a zone of subatmospheric pressure just beyond the axis or crest of the spillway. The maximum observed negative pressure head in the prototype in this zone is -5.0 ft at Piezometer No. 4 (Table 7(*b*), item 26), whereas the projected pressure head at Piezometer No. 3 (Fig. 49) might have reached -6.0 ft.

These negative pressure heads appear to reach a maximum about 5 ft beyond the crest and gradually diminish to positive pressures about 15 ft beyond the crest (see Table 7(*b*), 4-ft opening). This effect doubtless would become more marked and reach a maximum at some larger gate opening, and thereafter gradually diminish, as evidenced in the last column of Table 7 where all pressure heads are definitely positive. For the prototype this is a

TABLE 7.—COMPARISON OF PRESSURE (IN FEET[a]) ON SPILLWAY FACE, MODEL AND PROTOTYPE

No.	Distance from axis (ft)	Structure	Piezometer No.[b]	GATE OPENING, IN FEET: 0.25	0.5	1.0	2.0	3.0	4.0	Wide open
		(a) CENTER OF SEAL 8 FT DOWNSTREAM FROM AXIS								
1	−9.6	Model	8	49.3	49.3	49.3	49.3	49.3	....	....
2	−4.0	Prototype	1	48.5	48.5	48.5	47.9	47.0	46.2	15.4
3	0	Model	4	48.0	48.0	47.6	46.6	46.8	....	....
4		Prototype	2	48.3	48.3	47.7	46.6	44.9	43.7	10.6
5	5.0	Model	9[c]	47.5	45.1	39.8	31.4	31.9	....	....
6	7.5	Model	10	22.8	14.9	9.6	8.1	11.7	....	....
7		Prototype	4	17.8	9.8	5.8	4.7	7.3	9.5	3.7
8	8.5	Model	11	0.9	1.3	1.8	2.7	5.4	....	....
9		Prototype	5	−0.1	0.5	1.4	2.5	4.9	6.0	5.2
10	9.0	Model	12	0.7	0.7	0.9	1.9	4.0	....	....
11		Prototype	6	0.3	0.3	−1.2	−0.9	0.6	0.6	5.0
12	11.0	Model	13	−0.8	−0.9	−1.0	−1.0	−0.4	....	....
13		Prototype	7	0.4	−0.1	0.1	−0.5	−0.2	0.7	5.4
14	14.5	Model	14[c]	1.5	1.6	1.6	1.4	1.3	....	....
15	20.0	Model	5	1.3	1.6	2.1	2.5	2.6	....	....
16		Prototype	9	−0.6	−0.2	0.6	0.9	1.5	0.9	8.0
17	27.5	Prototype	10	0.1	0.4	1.3	1.3	1.3	1.6	9.0
18	35.0	Model	16[c]	0.4	0.4	0.4	0.5	0.5	....	....
19	50.0	Model	15[c]	1.0	1.2	1.2	1.1	1.1	....	....
		(b) CENTER OF SEAL 9 FT UPSTREAM FROM AXIS								
20	−9.6	Model	8	43.6	43.5	34.4	30.3	30.9	....	....
21	−4.0	Prototype	1	1.2	0.7	0.1	−0.5	−0.5	1.2	15.4
22	0	Model	4	0.2	0.4	0.2	−0.2	−0.2	....	....
23		Prototype	2	−0.1	−0.7	−1.3	−2.5	−3.5	−3.7	10.6
24	5.0	Model	9[c]	1.2	2.1	2.9	2.5	2.4	....	....
25	7.5	Model	10	0.4	0.9	1.5	1.4	1.2	....	....
26		Prototype	4	−0.3	0.0	−1.2	−3.0	−4.1	−5.0	3.7
27	8.5	Model	11	0.1	0.1	0.2	−0.1	−0.4	....	....
28		Prototype	5	0.2	0.2	−0.1	−1.6	−2.8	−2.8	5.2
29	9.0	Model	12	0.1	0.1	0.2	0.1	−0.1	....	....
30		Prototype	6	−0.3	−0.3	0.0	−1.2	−2.5	−2.7	5.0
31	11.0	Model	13	−0.1	−0.2	−0.4	−0.5	−0.8	....	....
32		Prototype	7	0.1	−0.2	0.1	−0.5	−1.7	−1.7	5.4
33	14.5	Model	14[c]	0.2	0.3	0.7	1.9	2.1	....	....
34	20.0	Model	5	0.2	0.3	0.4	0.9	0.8	....	....
35		Prototype	9	−0.2	−0.2	0.0	0.3	0.3	0.9	8.0
36	27.5	Prototype	10	−0.2	0.1	0.1	0.7	1.0	1.3	9.0
37	35.0	Model	16[c]	0.0	−0.1	−0.2	−0.4	−0.4	....	....
38	50.0	Model	15[c]	0.3	0.3	0.3	0.4	0.4	....	....

[a] Pressures are given in feet of water on the spillway floor (zero feet of water equals one atmosphere).
[b] See Fig. 49. [c] Corresponding prototype piezometer clogged.



(a) Gate in Downstream Position

(b) Gate in Upstream Position

FIG. 49.—SPILLWAY PRESSURE HEADS; 2-FT OPENING

perfectly normal behavior, and the line of pressure heads is believed to represent the true state of affairs. Why, though, did not the model experiments disclose the same tendency? All model pressure heads with the gate in the upstream position are on the positive side.

To show these anomalies better, Table 8 has been prepared. It gives the difference in pressure heads between prototype and model downstream from the crest for the gate in the upstream position. Nearly all differences are negative; that is, the prototype pressure heads are lower than the model pressure heads, and these differences increase as the gate openings become larger.

TABLE 8.—DIFFERENCE BETWEEN PRESSURE HEADS FOR THE GATE IN THE UPSTREAM POSITION

(Prototype Value Minus Value Determined from Model)

Gate opening (ft)	PROTOTYPE (UPPER) AND 1 : 5 MODEL (LOWER) PIEZOMETER NOS.:											
	2 4	4 10	5 11	6 12	7 13	9 5	2 4	4 10	5 11	6 12	7 13	9 5
Distance[a]	0.0	7.5	8.5	9.0	11.0	20.0	0.0	7.5	8.5	9.0	11.0	20.0
	(*a*) MODEL TEST G-4						(*c*) MODEL TEST G-6					
0.25	−0.4	−0.7	0.1	−0.4	−0.3	−0.6	−0.3	−0.7	0.1	−0.4	0.2	−0.4
0.50	−1.1	−0.5	0.1	−0.4	−0.6	−1.1	−1.1	−0.9	0.1	−0.4	0.0	−0.5
1.00	−1.5	−2.1	−0.3	−0.4	−1.0	−0.7	−1.5	−2.7	−0.3	−0.2	0.5	−0.4
2.00	−2.2	−3.0	−1.2	−2.8	−2.3	−1.3	−2.3	−4.4	−1.5	−1.3	0.0	−0.6
3.00	−3.3	−3.5	−1.8	−4.1	−3.6	−0.7	−3.3	−5.3	−2.4	−2.4	−0.9	−0.5
	(*b*) MODEL TEST G-5						(*d*) MODEL TEST G-7					
0.25	−0.4	−0.7	0.1	−0.2	−0.1	−0.4	−0.5	−0.3	0.1	−0.3	−0.2	−0.4
0.50	−1.2	−0.6	0.1	−0.1	−0.4	−0.5	−1.2	0.0	0.1	−0.3	−0.6	−0.5
1.00	−1.6	−2.3	−0.3	0.2	−0.6	−0.4	−1.6	−1.3	−0.3	0.0	−1.0	−0.5
2.00	−2.3	−3.4	−1.3	−2.0	−2.2	−0.5	−2.4	−3.1	−1.9	−2.5	−2.7	−0.9
3.00	−3.3	−3.3	−2.0	−3.5	−3.8	−0.6	−3.3	−3.2	−2.8	−4.4	−4.6	−0.9

[a] Distance from axis of dam to piezometer, in feet.

Fig. 47 shows that model tests G-6 were made with an undistorted crest, while tests G-4, G-5, and G-7 were with distorted crests. The depressions in the concrete crest, however, begin at the downstream gate seal, which is 8 ft below the axis except for G-7 for which crest was raised between the gate seal and the axis. Anomalous pressure heads shown in Table 7(*b*) appear upstream of these crest distortions; hence, these distortions cannot be assigned as the cause of the discrepancies.

The negative pressures resulting from the nappe adhering to the underface of the gate below the seal (40) obviously cannot be assigned as a cause of the departure of the model from prototype behavior for two principal reasons: First, the anomalies occur far downstream from the gate and, second, the underface of both the model and the prototype gate was perforated to admit air. The perforations on the prototype gate consist of three rows of staggered $1\frac{1}{16}$-in.

holes, so spaced as to be the equivalent to a continuous $\frac{7}{8}$-in. slot parallel to the rubber seal strip and communicating with the atmosphere on the downstream side of the gate. The first row of holes is 5 in. downstream from the center line of the seal strip. The perforations through the underface of the 1 : 5 model gate simulated those of the prototype as nearly as was practicable.

CONCLUSIONS

The writers can offer no valid explanation for the anomalous results between model and prototype with the gate in the upstream position.

Under such high velocities—45 ft per sec for the prototype and 20 ft per sec for the model—the difficulty of obtaining accurate pressure heads through piezometers was fully recognized and every precaution was taken to eliminate such sources of error. The departures from conformity are not of this nature. The model consistently refused to show negative pressures where such pressures should have obtained just below the crest.

METER MEASUREMENTS OF DISCHARGE, UNIVERSITY DAM

By Edward Soucek,[11] Assoc. M. Am. Soc. C. E.

Synopsis

A comparison between a series of current-meter measurements of discharge over a dam on a midwestern river[11a] and tests of a 1 : 12 scale model of a section of the dam is presented in this paper. Prototype heads covered by the measurements ranged from 1.7 to 6.3 ft, and discharges ranged from less than 2,000 to nearly 16,000 cu ft per sec. Eleven of the twenty-three discharge measurements were made while the dam was submerged. Submergences ranged from less than 12% to nearly 90%.

Model discharges averaged approximately 5% less than comparable discharges obtained for the prototype. The comparison would appear somewhat more favorable if made in terms of head. It is by no means certain that the discrepancy is chargeable to the model.

Both model and prototype exhibit a considerably smaller diminution of discharge capacity, due to submergence, than was anticipated. The model tests demonstrate the inadequacy of single-valued relations between submergence and reduction in discharge capacity. Observations on the behavior of the nappe are presented.

Introduction

Submerged flow, defined as the condition that exists when the tailwater at a dam is at an elevation greater than that of the crest, often occurs during floods at low-head dams. Dams designed to remain submerged at all times are used to increase depths in lakes and rivers. Despite the considerable importance of the problem, published data on submerged flow over dams are exceedingly rare. The writer knows of no previous model-prototype comparison involving this phenomenon. These facts motivated the investigation described in this paper.

Prototype

Measurement of the dam at five cross sections revealed that it was built to close agreement with construction drawings. The river bottom upstream from the dam, although somewhat variable transversely, can be represented fairly well by the broken line in Fig. 50. The dam is founded on rock which, at the toe, has been excavated and scoured to a maximum depth approximately 20 ft lower than the crest.

The spillway is 273.50 ft long, composed of a main section 269.50 ft long and a section 4.00 ft long apparently intended for use as a fishway. These sections are separated by a wall 1.48 ft wide that has its top 1.41 ft higher than the spillway crest. In profile, the top of the dividing wall roughly parallels the spillway. The wall is subject to overflow when the head on the dam exceeds 1.41 ft. An additional length of 6.75 ft is subject to overflow when the head exceeds 3.10 ft on the spillway proper. This additional length is a flat-crest section, 2.00 ft wide, at one end of the dam.

---

[11] Senior Civ. Engr., U. S. Engr. Office, Wilmington, N. C.

[11a] University Dam on the Iowa River at Iowa City, Iowa.

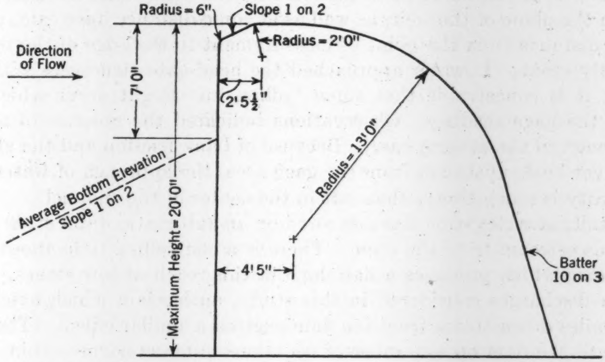


FIG. 50.—SPILLWAY CROSS SECTION, UNIVERSITY DAM

In comparing discharges of model and prototype, the measured prototype discharge was reduced by the estimated flow over the odd sections—namely, the dividing wall and the flat crest. Owing to the short length of overflow for the wall section and the low head on the high flat-crest sections, these corrections were small, never exceeding 1% of the measured discharge. Since any reasonable weir coefficient used in making the adjustment would give substantially the same results, the computation is not explained in detail, but both measured and corrected discharges are given.

The laboratory head-gate structures act as a non-overflow section of the dam 30 ft long. The gate pier adjacent to the spillway is well rounded, and no contraction of the nappe can be observed at the laboratory end of the dam. The power house forms a short non-overflow section of the other end of the dam. The intake crib, trashracks of which are parallel to the river bank, tends to reduce end contraction here also. Since even the Francis correction for a full contraction would have reduced the effective length of the spillway by only about 0.2% at the maximum head observed, no adjustment for end effects was considered necessary.

All data pertaining to the prototype are based on field measurements made during the course of the study.

OBSERVATIONS ON PROTOTYPE

The head on the dam was read on a staff gage mounted on a retaining wall on the laboratory bank of the river. The gage is only 6 ft upstream from a line projected along the spillway crest, but its sheltered position, 30 ft transversely from the overflow section, appears to keep the readings from being affected by drop-down. Some concern was felt over this apparently unfavorable location, but simultaneous observations of the water surface with a surveyor's level at the gage and at points clearly beyond any possible drop-down effect indicated that the location was satisfactory. The situation seems somewhat analogous to the measurement of head for a contracted weir in the side of a tank. There seems to be no reason why the head could not be measured nearly in the plane of the weir, as well as at some distance upstream, provided that the distance from the point of measurement to the edge of the weir were sufficiently great. If water approached the head-gate structures with a high velocity, it is conceivable that some "piling up" might occur which would increase the gage reading. Observations indicated the absence of any such phenomenon in the present case. Because of bank friction and the alinement of the river bank upstream from the gage area, the approach of water toward this vicinity is much slower than out in the center of the channel.

The tailwater elevation was measured by an automatic water stage recorder 350 ft downstream from the dam. There is a controlling riffle about 1,200 ft downstream which produces a flat slope in this reach at low stages. During the high discharges considered in this study, an ice jam which extended for several miles downstream from the dam exerted a similar effect. The ice jam built up to the dam on a number of occasions but was never within 500 ft of the dam during any of the measurements. These data on downstream conditions are cited because it is desired to show that the tailwater level would have been affected very little by changing the measuring point, provided that the point was clearly beyond the immediate effects of the dam, as was the case.

Discharge measurements were made from a cable station 470 ft downstream from the dam. Price current meters in good condition were used. Twenty of the measurements were taken by the 0.2 and 0.8 depth method. The remaining three were based on surface velocity alone. For these, a reduction coefficient of 0.9 was applied to determine the mean velocity in the vertical. (Some of the measurements made during the course of the study were used in making up the station record during the period of ice effect.) With due regard for the difficulty of making accurate measurements under adverse conditions, the measurements are considered reliable. A number of attempted measurements had to be abandoned because of floating ice.

MODEL TESTS

The model was a 1 : 12 scale replica of the cross section of the prototype but did not reproduce the structure as regards length, end effects, or bottom irregularities. The model was made by stretching a galvanized iron skin plate tightly over a welded steel frame. The surface presented to the flowing water was very smooth.

The model was operated in a glass-sided steel flume 26 ft long, 2.5 ft deep, and 2.56 ft wide, glass to glass. The model was bolted to the floor of the flume 15 ft from its downstream end. End seals were of pitch and marine putty, smoothed to the profile of the model. Bottom seals were of portland cement mortar. The silt against the upstream side of the prototype was simulated by an inclined plane of wood. Wooden baffles upstream from the model were used to produce uniform approach conditions.

Water was supplied to the flume at constant rates from the laboratory circulating system and was measured on a sharp-crested, fully suppressed and aerated, rectangular weir 2.51 ft long and 1.29 ft high. To insure an accurate rating, the weir was calibrated in place. This was considered especially necessary in view of the high discharges which taxed the capacity of the weir. A larger weir would have been preferable. For the calibration, a by-pass flume leading to a 20,000-lb capacity weighing tank was installed. An attempt was made to keep the condition of the weir crest the same during the model tests as during the calibration.

Prior to the tests, it was determined that the friction slope in the flume at the depths and discharges which were to be used would not exceed approximately 0.0001. Accordingly, the effect of friction in the flume could be neglected. This made it unnecessary to attain exact correspondence of points at which heads were measured on model and prototype, again provided that measurements were beyond the immediate effects of the dam. The head was measured 3.1 ft upstream from the back of the dam and the tailwater elevation 11.3 ft downstream from the same point. These points were determined experimentally as being out of the drop-down curve and beyond the rising and irregular water surface downstream from the dam, respectively. Both water levels were observed with point gages of the rack and pinion type mounted along the center line of the flume.

The discharge over the model was controlled with a gate valve in a 10-in. supply line leading to the weir tank. The tailwater elevation was regulated with a hinged flap gate at the downstream end of the flume.

NOTATION

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

RESULTS OF MODEL TESTS

Fig. 51 shows the relation observed for unsubmerged or free discharge between the coefficient C and the energy head for the model in the formula:

$$Q = C\,L\left(H + \frac{V^2}{2\,g}\right)^{1.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

In this weir formula: Q is the time rate of discharge, in cubic feet per second; C is a coefficient, dimensionally the square root of an acceleration; L is the length of a weir or dam crest, in feet; H is the measured head on a dam, referred to the elevation of the crest, in feet; V = average velocity of flow; and g = acceleration due to gravity. The head corresponding to the average velocity of approach, in feet, is expressed as $\frac{V^2}{2\,g}$.

Eq. 13 is used to characterize the flow whether free or submerged. The ratio of the discharge which occurs when the flow is submerged to that which would occur under the same energy head for free flow conditions is used as a measure of the diminution of the discharge capacity caused by submergence.

The results of model tests for submerged conditions are shown in Fig. 52. Following usual practice, the diminution in discharge capacity due to submergence is indicated by curves in which the submerged discharge, expressed as a percentage of the unsubmerged discharge which would obtain under the same energy head, is plotted against the submergence. For submergences lower than 45%, the relation between discharge and energy head was not affected by the submergence so the curves are not carried below a 45% submergence. The total model discharge and the discharge per foot of crest length are both given on these curves, the latter item being more significant since the length of the model bears only an incidental relation to the length of the prototype.

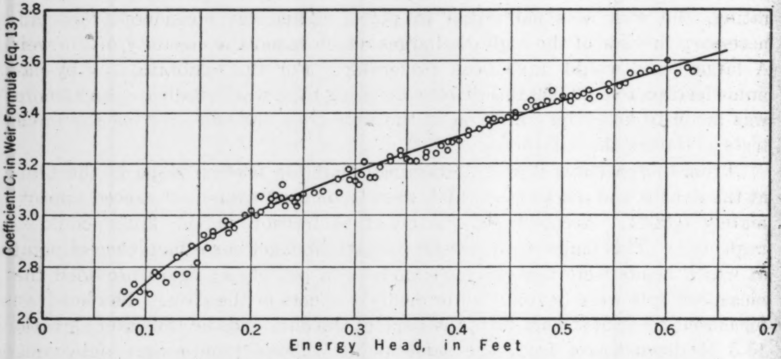


FIG. 51.—HEAD-COEFFICIENT RELATION FOR A MODEL WHEN NOT SUBMERGED

In Fig. 53, the curves of Fig. 52 are compared to a larger scale. An interesting relation exists between the curves for the five higher discharges. Given any two of the curves, or one curve and a well-defined point on another, the remaining three or four curves could be fixed by linear interpolation or extrapolation in terms of the discharge along the scale of relative discharges. Perhaps a slightly better fit to the plotted points in Fig. 52 could have been obtained if the curves had been drawn without regard for this property. It is seen that the curve for the lowest discharge does not correspond to the other five. The energy head for this discharge is less than 0.1 ft on the model. The matter is of great interest in connection with model-prototype relations at low heads because there is some question as to the behavior of the prototype for a corresponding discharge. It is regretted that this question could not be thoroughly investigated.

When water flows over a submerged spillway, the nappe may either plunge downward along the face of the spillway (as when the flow is unsubmerged) or it may flow away from the dam at or near the surface of the tailwater. The flowing nappe condition might be further subdivided according to the

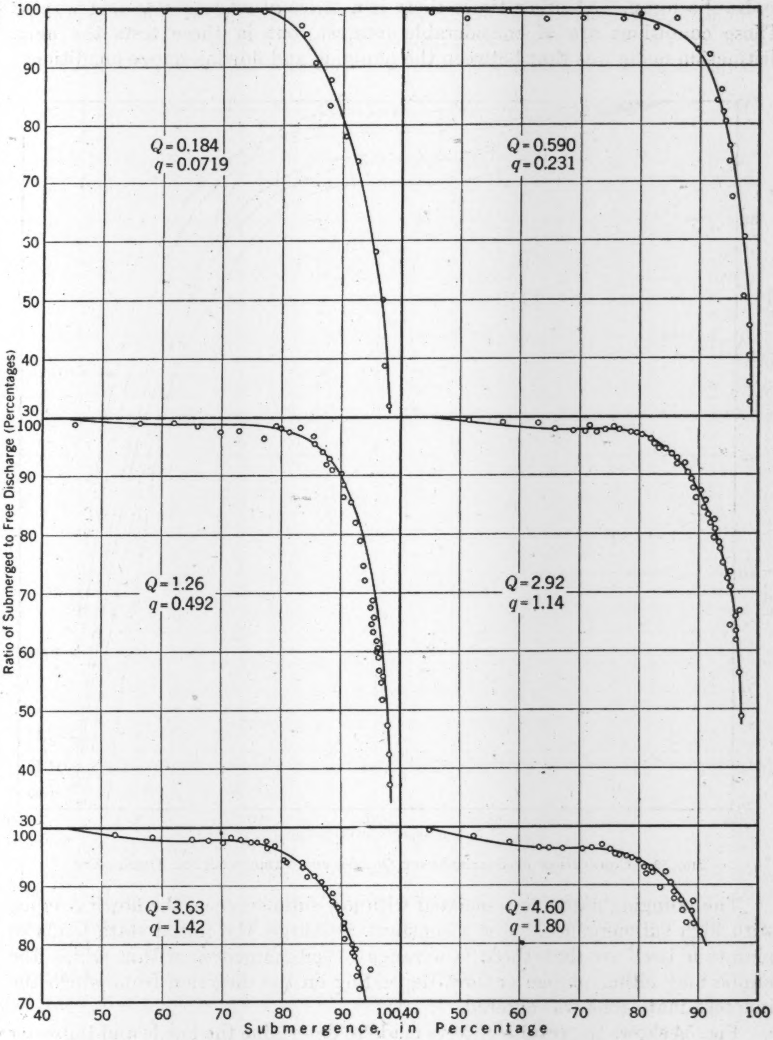


FIG. 52.—EFFECT OF SUBMERGENCE UPON MODEL DISCHARGE

appearance of the surface. Sometimes the surface has the appearance of a hydraulic jump. At other times there is a series of smooth standing waves. These conditions are of considerable interest, but in these tests the only distinction made was that between the plunging and flowing nappe conditions.

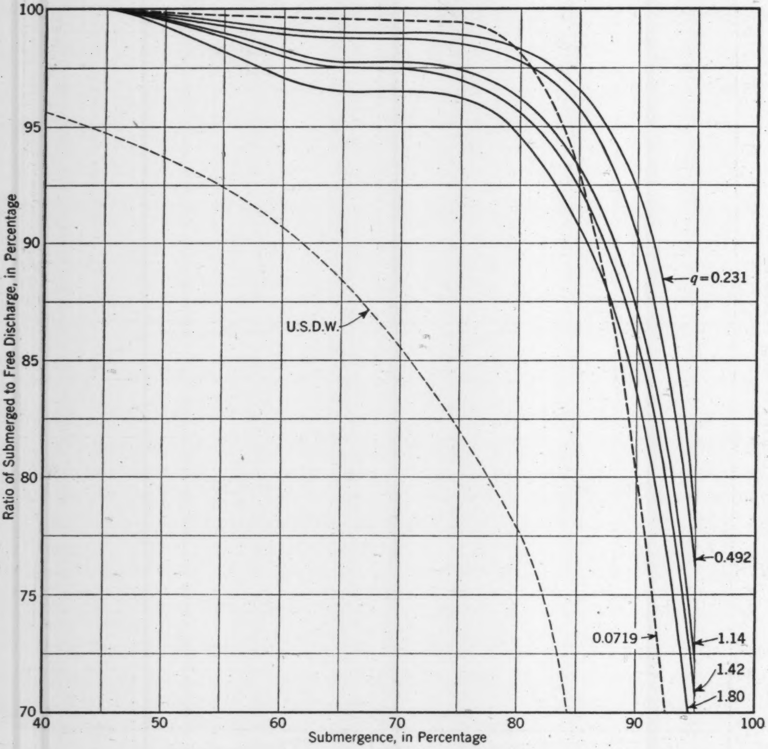


Fig. 53.—Comparison of Submergence Curves for Various Model Discharges

The plunging nappe is associated with low submergence, the flowing nappe with high submergence. For a constant discharge, the initial state tends to maintain itself so that there is a range of submergences within which the nappe may either plunge or flow, depending on the direction from which the indeterminate zone was entered.

Fig. 54 shows the results of tests made to determine the heads and tailwater elevations at which the changes from one nappe condition to the other occurred. The observational error was unavoidably high in making this determination. The results are self-explanatory. The lines drawn are not intended to represent the points but merely to give a better idea of the submergences at which these changes occurred.

Special tests were conducted to determine whether or not the behavior of the nappe had to be considered in studying the effect of submergence upon the discharge. This was done by manipulating the tailwater so as to create alternately plunging and flowing nappes for the same submergence and discharge. The nappe condition had no apparent effect on the relation between submergence and discharge.

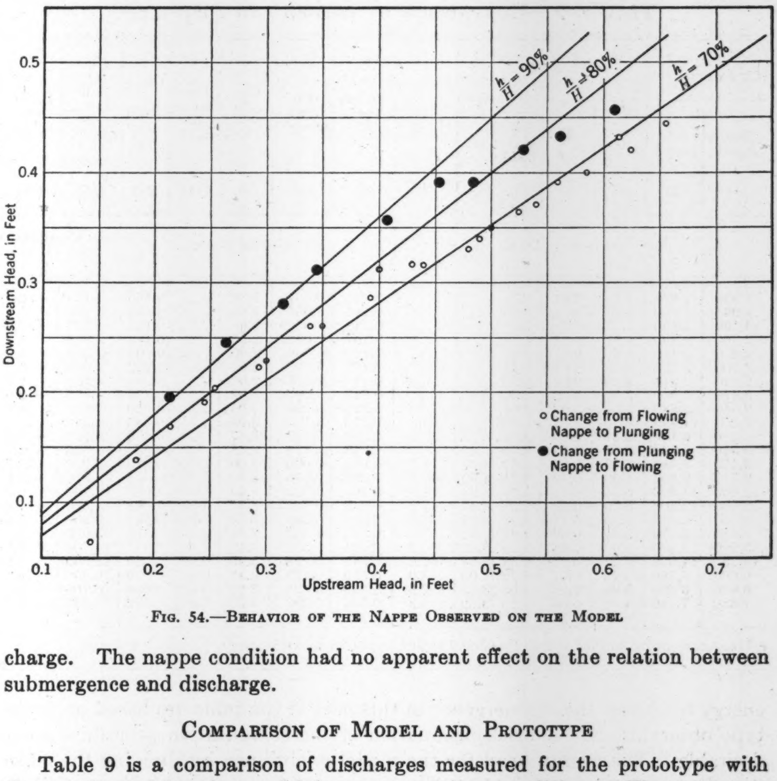


FIG. 54.—BEHAVIOR OF THE NAPPE OBSERVED ON THE MODEL

COMPARISON OF MODEL AND PROTOTYPE

Table 9 is a comparison of discharges measured for the prototype with those indicated by the model for corresponding conditions. The prototype measurements are arranged in ascending order of the discharge for free flow observations and in ascending order of the submergence for the submerged observations. The comparison could be made in various ways, the method adopted being one which gives the result in terms of discharge.

As previously explained, the length of the prototype spillway is 273.50 ft. A model 22.8 ft long $\left(\frac{273.50}{12}\right)$ would have been necessary to simulate the prototype as regards length. The model discharges per foot of length, therefore, must be multiplied by 22.8 as a length adjustment before being "scaled up" by Froude's law.

Table 9(*a*) gives prototype data which appear to require no explanation. Since model and prototype data are separated in the table, no subscripts are necessary to differentiate between them. Table 9(*b*) gives steps in the determination of the spillway discharge indicated by the model. Only the energy head and the submergence in this part of the table are based on prototype observations. A brief explanation of the computation procedure seems desirable. The energy head for the model is obtained by dividing the corresponding prototype item by 12. The submergence, being a ratio, is numerically equal for model and prototype.

TABLE 9.—COMPARISON OF MODEL AND PROTOTYPE

Flow, Q (Cu Ft per Sec)		Head (Ft)				$\frac{H_t}{H}$ (%)	Energy head (ft)	(Energy head)$^{1.5}$	Coefficient, C	Discharge ratio (%)	Flow, q (cu ft per sec)	Flow, Q, indicated by model (cu ft per sec)	Difference (%)
Measured	Adjusted	H	$\frac{V^2}{2g}$	Energy head	H_t								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(*a*) PROTOTYPE DATA							(*b*) MODEL DATA					(*c*) COMPARATIVE DATA	
1,920	1,920	1.70	0.01	1.71	....	....	0.143	0.0541	2.86	....	0.155	1,760	− 8.3
1,960	1,960	1.74	0.01	1.75	....	....	0.146	0.0558	2.87	....	0.160	1,820	− 7.1
2,190	2,190	1.77	0.01	1.78	....	....	0.148	0.0569	2.88	....	0.164	1,860	−15.0
2,260	2,260	1.87	0.01	1.88	....	....	0.157	0.0622	2.90	....	0.180	2,050	− 9.2
2,410	2,410	1.96	0.01	1.97	....	....	0.164	0.0664	2.92	....	0.194	2,210	− 8.3
2,700	2,700	2.13	0.01	2.14	....	....	0.178	0.0751	2.95	....	0.222	2,530	− 6.3
2,910	2,910	2.18	0.02	2.20	....	....	0.183	0.0783	2.96	....	0.232	2,650	− 8.9
3,470	3,460	2.52	0.02	2.54	....	....	0.212	0.0976	3.02	....	0.295	3,360	− 2.9
3,840	3,830	2.64	0.03	2.67	....	....	0.223	0.105	3.04	....	0.320	3,640	− 5.0
4,500	4,490	2.96	0.04	3.00	....	....	0.250	0.125	3.08	....	0.385	4,380	− 2.4
6,180	6,160	3.57	0.06	3.63	....	....	0.303	0.167	3.17	....	0.529	6,020	− 2.3
6,580	6,560	3.69	0.07	3.76	....	....	0.313	0.175	3.18	....	0.557	6,340	− 3.4
3,450	3,440	2.39	0.02	2.41	0.28	11.7	0.201	0.090	3.00	1.000	0.270	3,070	−10.7
9,710	9,650	4.59	0.12	4.71	1.92	40.8	0.393	0.246	3.30	1.000	0.814	9,270	− 3.9
12,600	12,500	5.44	0.18	5.62	2.37	43.6	0.468	0.320	3.40	1.000	1.089	12,400	− 0.7
12,600[a]	12,500	5.19	0.19	5.38	2.45	47.2	0.448	0.300	3.37	0.998	1.008	11,500	− 7.9[a]
9,740	9,680	4.69	0.12	4.81	2.57	54.8	0.401	0.254	3.31	0.992	0.834	9,500	− 1.9
3,480	3,470	2.65	0.02	2.67	1.52	57.4	0.223	0.105	3.04	0.993	0.318	3,620	+ 4.3
15,900[a]	15,800	6.03	0.26	6.29	3.62	60.0	0.524	0.379	3.47	0.980	1.289	14,700	− 6.9[a]
14,750[a]	14,650	6.28	0.21	6.49	3.84	61.2	0.541	0.398	3.49	0.980	1.359	15,500	+ 5.9[a]
6,600	6,580	3.65	0.07	3.72	3.03	83.0	0.310	0.173	3.18	0.965	0.530	6,030	− 8.4
6,930	6,890	3.98	0.07	4.05	3.48	87.5	0.338	0.196	3.22	0.930	0.589	6,700	− 2.7
7,380	7,340	4.06	0.07	4.13	3.62	89.2	0.344	0.202	3.22	0.912	0.592	6,740	− 8.1

[a] Measurement by surface velocity with 0.9 reduction; other measurements, 0.2 and 0.8 depth.

For free flow, or when the submergence is less than 45% (below which value the model indicated that submergence had no effect), q, the discharge per foot length of the model, is computed from the formula,

$$q = \frac{Q}{L} = C\left(H + \frac{V^2}{2g}\right)^{1.5} \qquad (14)$$

the coefficient being taken from the curve of Fig. 51.

When the submergence exceeds 45%, the discharge computed by Eq. 14 must be reduced by the curves of Fig. 53. The model shows, however, that the ratio of submerged to free discharge at the same energy head (hereinafter called simply the "discharge ratio") is a function not only of the submergence but also of the discharge, which is as yet unknown. Accordingly, the final value of the discharge ratio cannot be found until the discharge is known and vice versa. A solution can be found by a few easy approximations.

A simple procedure is to assume that the discharge ratio has the value of 100%. A first trial value of q can now be found, and a second trial value of the discharge ratio can be read from Fig. 53 by interpolation. This gives a second trial value of q. The process is repeated until q remains unchanged by further approximation, which it will after a very few trials. Obviously, the procedure can be shortened if a close estimate of either q or the discharge ratio is made initially. No trial computations are shown in Table 9, but the correctness of the results can be verified without approximation. An easier method of calculation could be derived if a large number of discharges were to be determined.

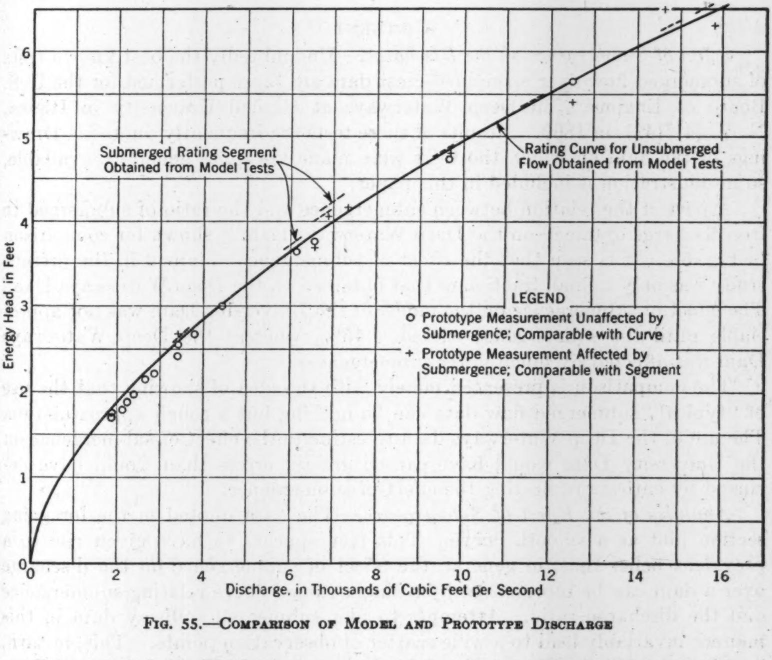


FIG. 55.—COMPARISON OF MODEL AND PROTOTYPE DISCHARGE

Cols. 13 and 14, Table 9, are comparative data. The spillway discharge indicated by the model is the product of three factors—the discharge per foot of length of the model, the length adjustment as previously explained, and the five-halves power of the scale ratio. Col. 14, Table 9, gives the percentage difference between the measured net spillway discharges and the spillway discharges indicated by the model for corresponding energy heads and submergences. The difference is considered negative when the indicated discharge is less than the measured discharge.

In Fig. 55 the performance of the model is compared graphically with prototype measurements. The continuous curve is an energy-head-discharge relation for free flow over the spillway, established from the model tests. Observations on the prototype when unsubmerged, or when the submergence did not exceed 45%, are comparable directly with this curve.

To avoid confusing the diagram, continuous curves were not drawn for submerged flow. For each submergence at which a measurement was made, a short segment of the submerged rating indicated by the model tests was drawn at the corresponding energy head. Since submergence reduces the discharge for a given energy head, the submerged rating segments all lie above and to the left of the curve for free flow. It should be noticed that the three measurements which show the greatest discrepancies are based only on surface velocity. No close agreement can be expected for these three measurements.

COMMENT

Effect of Submergence on the Discharge.—Undoubtedly the best known tests of submerged flow over a rounded-crest dam are those performed for the U. S. Board of Engineers on Deep Waterways at Cornell University in Ithaca, N. Y. (41)(42), in 1899. Results of these tests are frequently quoted. Drawings of the dam on which the tests were made are quite generally available, so no illustration is included in this paper.

A part of the relation between submergence and the ratio of submerged to free discharge obtained on the Deep Waterways Dam is shown for comparison in Fig. 53. It is seen that the effect of submergence obtained in the present study was only a small fraction of that obtained on the Deep Waterways Dam. The effect of submergence on the model of the University Dam was not appreciable until the submergence exceeded 45%, whereas the Deep Waterways Dam was affected by much lower submergences.

The comparison is presented merely with the idea of showing that the use of "typical" submerged flow data can be nothing but a rough approximation. The use of the Deep Waterways data to estimate the effect of submergence on the University Dam would have caused greater errors than would have occurred by entirely neglecting the effect of submergence.

Analysis of the Effect of Submergence.—The data quoted in the foregoing section plot as a smooth curve. This fact appears to have given rise to a prevalent belief that, in general, the effect of submergence on the discharge over a dam can be represented by a single-valued curve relating submergence and the discharge ratio. Attempts to plot submerged spillway data in this manner invariably lead to a wide scatter of observation points. This, in turn, leads to the opinion that submerged flow over dams is a far more unstable phenomenon than is really the case. The submerged flow tests of the first dam[11b] were made at a constant head of 6.6 ft. This made the tailwater elevation and the submergence functions of the discharge, which had to be decreased as the tailwater was raised to keep the measured head constant. Study of

---

[11b] Deep Waterways Dam.

Fig. 53 shows that, if a set of observations were plotted, in which the submergence was functionally related to the discharge, a single curve rather than a family of curves would express the results adequately.

If a stable control exists downstream from a dam, the submergence will indeed be related to the discharge in this special way. If model tests are made for such a case, the tailwater must be regulated to represent the prototype condition if a single-valued relation between submergence and its effect on the discharge is desired. This procedure is not recommended. If unwilling to vary the tailwater independently of the discharge as a matter of curiosity, the experimenter should consider the possibility that the control might change and invalidate the results if these results were so restricted in scope. The ice effect which caused the unusual degree of submergence observed in this study is a pertinent example.

Measurement of Tailwater Elevation.—The Society's *Manual of Engineering Practice No. 11*, entitled "Letter Symbols and Glossary for Hydraulics," defines submergence as follows:

> "The ratio of the tail-water elevation to the head-water elevation, when both are higher than the crest, the overflow crest of the structure being the datum of reference. The distances up stream or down stream from the crest at which head-water and tail-water elevations are measured are important, but have not been standardized."

In connection with measurement of headwater elevation, there seems to be no reason for departing from the procedure commonly used for unsubmerged weirs and dams—namely, the measurement at a point sufficiently distant from the overflow to be safely out of the drop-down curve.

G. N. Cox, M. Am. Soc. C. E. (45), measured tailwater elevation very near the dam. At this point, especially when the nappe plunges, there is a decided depression in the water surface. Even for the flowing nappe condition, the water level near the dam is at a lower elevation than farther downstream. The reason for a negative slope in this reach is a readjustment of velocity distribution which has an effect that is similar, as regards energy transfer, to the recovery of pressure head which occurs in the diffusing cone of a venturi meter. The investigation by Professor Cox dealt primarily with the use of the submerged weir as a measuring device. For this purpose, the use of a tailwater measuring point very near the dam is apparently satisfactory.

For most purposes, however, it is believed that a tailwater measuring point clearly beyond the rising water surface is decidedly preferable. Professor Cox found it necessary to consider the behavior of the nappe and gave a different formula for each condition. This is a complication that should be avoided if possible. If recording gages are used, the behavior of the nappe cannot be determined definitely from gage readings alone if the submergence is in the critical zone. At times during observations on the second dam,[11a] a plunging nappe existed over part of the spillway length and a flowing nappe over the remainder.

The model tests indicated that, if the tailwater level is measured at a sufficient distance from the dam, the behavior of the nappe need not be considered. This is just another way of stating that the over-all loss of head in flow over the model did not depend to any measurable extent on the behavior of the nappe.

[11a] University Dam.

It seems best, also, to consider the rising water surface downstream from the dam as a part of the phenomenon of submerged flow. Otherwise, the magnitude of the rise in water level must be estimated in fitting results of model tests to a stage-discharge relation downstream from the dam site. No satisfactory method of making this adjustment is available. It is probable that model tests indicate its magnitude more accurately than it can be computed.

Comparison of Model and Prototype.—On the whole, the agreement between model and prototype is considered satisfactory. Not much weight can be given to the three measurements made by surface velocity. If the coefficient 0.85 had been used for these measurements instead of 0.90, two of them would agree very well with the model. A coefficient of 0.85 is well within the range of possibility, but the originally estimated value of 0.90 has been retained. As may be seen from Fig. 55, the agreement between model and prototype except for the three surface velocity measurements is about as good as that between the discharge measurements themselves.

For estimating flood discharges from water level observations at dams, it would appear that the use of small-scale sectional models must be regarded as being as satisfactory as the current meter—from the standpoint of efficiency, far more so. In no case would an estimate of the backwater due to the dam, based on the model tests, have been in error by more than a few inches.

For all but two of the twenty-three measurements, the model indicated a discharge smaller than that measured on the prototype. To what extent this difference is chargeable to disproportionately large viscosity, surface tension, friction, and end effects is not known. Overregistration of the current meters may have accounted for some of the difference.

As previously suggested, the nonconformity of the curve for the lowest discharge in Fig. 53 to the general pattern for higher discharges is of special interest. It seems possible that the model fails for a discharge as low as this, but it cannot be disproved or substantiated from data now available.

CONCLUSIONS

The conclusions presented are necessarily tentative. However, they are considered to constitute a logical interpretation of the results of model tests and measurements of the University Dam.

(*a*) The effect of submergence on the discharge is affected by the shape of the spillway and perhaps by other factors. Data at present available are insufficient for generalization of the effect of submergence in terms of spillway shape. The use of "typical" data cannot be recommended.

(*b*) In general, it is not possible to express the effect of submergence upon the discharge by a single-valued relation between the submergence and the discharge ratio. In the range covered by these tests, the effect of submergence is greater for high discharges.

(*c*) If the tailwater elevation is measured at a sufficient distance from the dam, it is not necessary to consider the behavior of the nappe in determining the effect of submergence upon the discharge. For this and other reasons, the tailwater elevation should be measured beyond the immediate effects of the dam.

(*d*) Reasonably good agreement may be expected between spillway models and prototypes of the sizes used in these tests under submerged as well as free flow conditions.

Certain other observations might be presented as secondary or incidental conclusions, but these appear to have been covered to the extent that their importance justifies in the body of the paper.

ACKNOWLEDGMENT

Acknowledgment must be made of the assistance of many engineers. The possibilities of this study were recognized by E. W. Lane, M. Am. Soc. C. E., when an ice jam below the dam[11a] created unprecedented tailwater conditions. The prototype measurements were conducted by the writer under his direction. Some discharge measurements were made by the staff of the U. S. Geological Survey, under the direction of R. G. Kasel, M. Am. Soc. C. E., in connection with its regular stream-gaging work. The experiments on the model were conducted by C. L. Morgan, Jun. Am. Soc. C. E., under the joint direction of F. T. Mavis, M. Am. Soc. C. E., and the writer. The manuscript has been reviewed by F. W. Edwards, M. Am. Soc. C. E., and C. J. Posey, Assoc. M. Am. Soc. C. E., both of whom made valuable suggestions. Others who assisted in the project are: E. R. Van Driest, J. S. McNown, C. K. Willey, H. D. Brice, Juniors, Am. Soc. C. E., and G. L. Whitaker and B. D. Lind.

[11a] University Dam.

GRAND TOWER, MISSISSIPPI RIVER

By Frederick R. Brown,[12] Jun. Am. Soc. C. E.

Synopsis

The U. S. Waterways Experiment Station recently received from the U. S. Engineer Office, St. Louis (Mo.) District, field data that served to confirm the results obtained from a hydraulic model study (53) (54) conducted at the Experiment Station during 1936 and 1937 on a reach of the Mississippi River.[12a] A careful analysis of these data has been made and a comparison of the action of the prototype with the indications of the model furnishes the basis for this paper.

The Problem

The model study involved the development of an effective plan for improving the navigation channel of the Mississippi River between Mile 75 and Mile 80 above Cairo, Ill. At Mile 84.0 the Mississippi River enters a rock gorge, the bluffs of which confine the flow of the river to Mile 79.5. At the latter point the rock bluffs on the left bank of the Mississippi River recede, allowing the river to widen and to divide its flow between Grand Tower Island chute and the river channel along the right bluff line. The natural tendency of the river is to follow the chute channel. However, because of the more satisfactory navigation conditions that would prevail if the river could be made to develop the crossing to the right bluff line at Mile 79, two pile dikes were constructed in 1939 from the left bank at Mile 79.5 and Mile 79.2. These dikes effected considerable improvement in the navigation channel, but did not eliminate the need for periodic maintenance dredging through this crossing. The material dredged from the crossing was spoiled in front of, and off the end of, Dike 79.2 L and in the chute behind this dike in an attempt to confine the river flow to the crossing. Samples obtained after the completion of the model study indicated that the river-bed material in the vicinity was composed of sand, gravel, and coarse rock fragments.

The Model

The scale ratios, model to prototype, were as follows: Horizontal dimensions, 1 : 600; and vertical dimensions, 1 : 150. The model was of the movable bed type, with banks and overbank topography molded in concrete and the movable channel section molded in crushed coal. In addition to the slope scale of 4 : 1 induced by the difference in scale ratios, the model was given a slight additional slope sufficient to provide adequate movement of the bed material used. The scales used were within the range in which it has been found that velocity distributions and current directions can be reproduced accurately in models of wide, shallow rivers such as the Mississippi.

---

[12] Asst. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[12a] Adjacent to Grand Tower, Ill.

After a thorough adjustment and verification of the model, in which operating conditions were developed such that the movement of bed material in the model was consistent with that in the prototype, tests of plans were begun The testing procedure consisted of installing the elements of a proposed improvement plan in the model, the bed of which had been molded to conform to the 1936 prototype conditions, and operating the model in accordance with the stages of a modified form of the 1935 hydrograph (Fig. 56). The 1935 hydrograph included within its range an adequate distribution of critical high stages as well as normal and low stages. Therefore, since it was believed to embrace all conditions of flow likely to occur in the Grand Tower reach in the future, this hydrograph was selected for use in the model study. Both the dredging proposed in the improvement plan and the necessary maintenance dredging were simulated in the model.

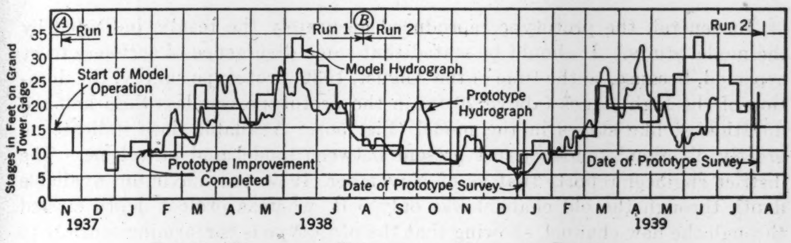


FIG. 56.—COMPARATIVE HYDROGRAPHS, MODEL AND PROTOTYPE

It was thought advisable to begin testing the improvement plan at a stage comparable to that expected in the prototype at the time of completion of the prototype improvement. It was believed that this completion would occur in November, 1937. Consequently, operation of the model was started with the 15-ft stage in November (modified 1935 hydrograph). Although dredging of the initial pilot channel in the prototype was not actually completed until January 30, 1938, the effect of the difference in time was not considered to be important.

MODEL AND PROTOTYPE IMPROVEMENT PLANS

The plan included the shortening of Dike 79.2 L and the addition of a 1,500-ft trail dike downstream from this dike, a dredge cut 250 ft wide to a depth of − 13 ft (low-water reference plane) along the right bank of Grand Tower Island, use of the dredge spoil chiefly to build up the left bank of the dredge cut, and riprap and mattress protection for the left bank of the dredge cut and for the trail dike.

In the prototype, the outer 600 ft of Dike 79.2 L was removed, the 1,500-ft trail dike was constructed, and revetments were placed along the left bank of the new channel during the period from August, 1937, to January, 1938. The pilot cut for the re-alined channel was made during several dredging periods—namely, September 28, 1937, to January 30, 1938; March 17 to May 28, 1938; and August 20 to October 31, 1938. This cut was more than 400 ft wide in the prototype in contrast to the 250-ft wide pilot cut in the model. It is understood that the dredge spoil (about 3,400,000 cu yd) was disposed generally in accordance with plans followed in the model test. In addition to the works scheduled in the improvement plan, prior to the August, 1939, prototype survey, the dike near Mile 79.1 on the right bank below Tower Rock was extended about 500 ft across the old channel. This extension was intended to aid the filling of the old channel.

COMPARISON OF PROTOTYPE AND MODEL RESULTS

In general, the prototype reproduced accurately the results indicated by the model study. It should be stated that some divergence of action is to be expected, because, to the time of the August, 1939, prototype survey, the duration of the high stages (above 20 ft) in the prototype was less than half the duration of like stages in the model (Fig. 56). Available data indicate no greatly divergent tendencies at present between model and prototype. The district engineer reports that as of November, 1939, the maximum available depth through the old channel was only 5 ft, whereas project depth existed through the new channel, showing that the prototype is conforming sensibly to the indications of the model. Careful examination of prototype and model data reveals the following interesting comparisons:

(*a*) Until the time of maintenance dredging in January, 1939, the re-alined channel in the prototype exhibited a persistent tendency to shoal. This agrees with the model results, since by the end of Run 1 the model channel had shoaled to a controlling depth of 5 ft below the low-water reference plane.

(*b*) Since the prototype maintenance dredging of January, 1939, the scour in the re-alined channel indicated in the model has occurred also in the prototype, although at a somewhat slower rate. This difference in rate of scour may be explained by the fact that the plan developed in the model was based on the assumption that the river bed in this vicinity was composed of average river sand containing only a small amount of larger material, whereas actual spoil samples (obtained when the actual improvement was undertaken) were found to contain 40% rock fragments and large gravel.

(*c*) A tendency to scour was noted in the model at the upper end of the old channel along the right bank, and tests indicated that this channel would continue to fill after each high stage. Thus far, alternate scouring and filling has also occurred in the prototype, with scouring predominating. It is to be noted that development of the re-alined prototype channel is not yet complete. It is believed probable, however, that, once this new channel has fully developed, the old channel will become filled substantially as predicted in the model.

The comparison of model and prototype behavior in the Grand Tower reach of the Mississippi River will be supplemented from time to time as additional prototype data are made available.

HYDRAULIC CONDITIONS TO LOCK AND DAM NO. 4

By William J. Hopkins,[13] Assoc. M. Am. Soc. C. E.

Synopsis

The new lock and dam here considered is located on the Monongahela River.[13a] Because of local conditions, the structure, as originally designed, was subjected to adverse currents that had an effect on the maneuverability of river vessels in the upper approach to the locks. At moderately high stages, the main currents, after sweeping around two comparatively sharp bends upstream, followed the lock side of the river, increased in velocity as they reached the upper approach, and then swung sharply across the lock entrance toward the center of the stream before discharging over the spillway section. Examination of the site and of the current conditions disclosed that the flow characteristics might have been due to several factors, or to a combination of any of these—namely, (*a*) the effect of the upstream bends on velocity and current distribution; (*b*) the effect of the hydrography in the vicinity of the approach on velocities and currents; and (*c*) the effect of a contraction in the stream cross section due to the lock and to the existence of a large bar along the shore opposite the lock chamber.

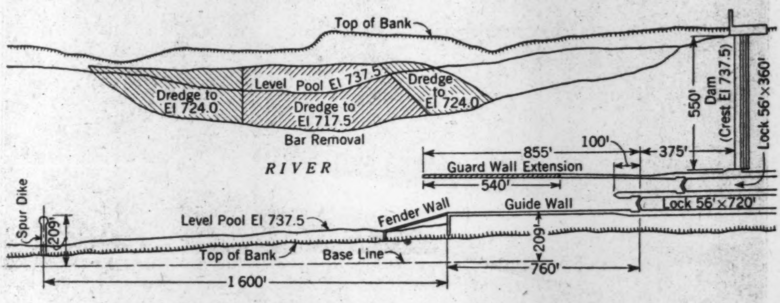

Top of Bank
Level Pool El 737.5
Dredge to El 724.0
Dredge to El 717.5
Dredge to El 724.0
Bar Removal
RIVER
855'
100'
375'
550'
Dam (Crest El 737.5)
Lock 56'×360'
Guard Wall Extension
540'
Guide Wall
Lock 56'×720'
Fender Wall
Spur Dike
209'
Level Pool El 737.5
Top of Bank
Base Line
209'
760'
1 600'

Fig. 57.—Modified Design of Upper Lock Approach

The existence of the foregoing condition prompted an investigation to be made by means of hydraulic model tests to study the causes of the current phenomena at the upper approach and to determine the most suitable means of improving the entrance conditions. On the basis of these studies, necessary

---

[13] Engr., Engr. Corps, U. S. Army, U. S. Engr. Office, Pittsburgh, Pa.

[13a] Lock and Dam No. 4, at Charleroi, Pa., about 40 river miles above the mouth of the stream, which is at Pittsburgh, Pa.

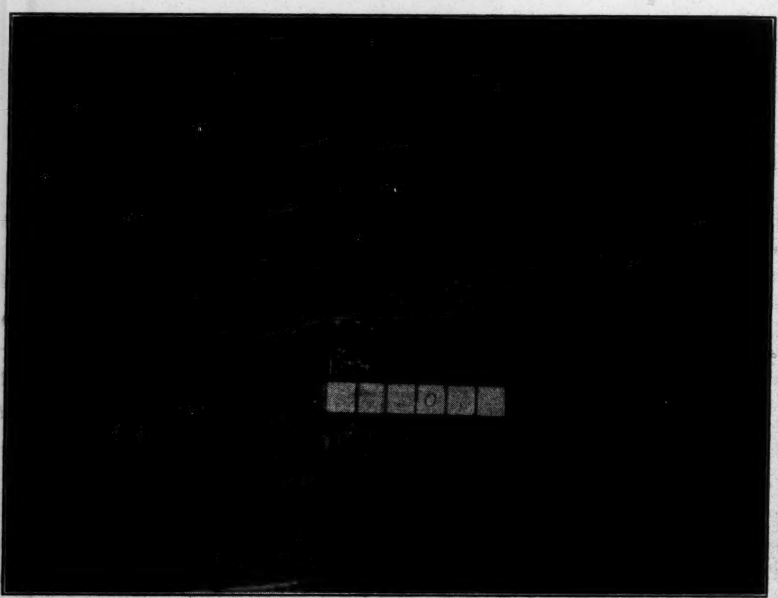


FIG. 58.—CONFETTI MOVEMENTS SHOWING CURRENT DIRECTIONS AND VELOCITIES; ORIGINAL DESIGN

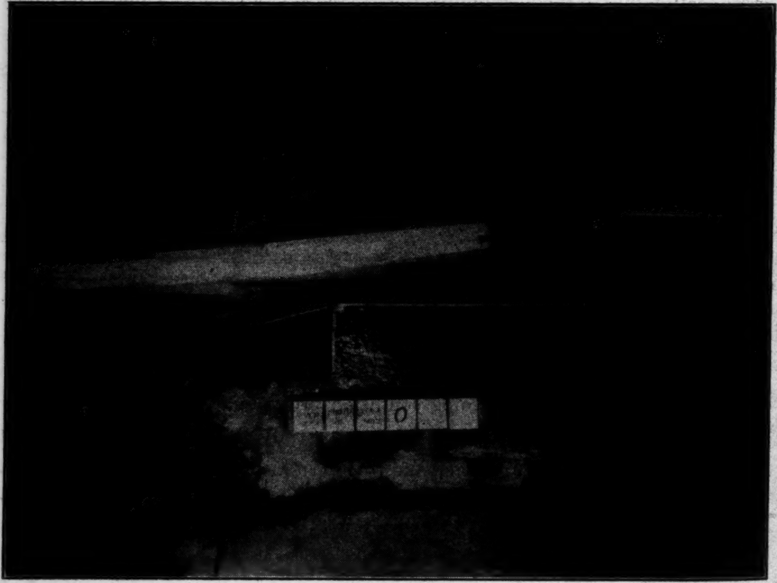


FIG. 59.—SHIFT OF MODEL TOW DUE TO CURRENTS IN THE ORIGINAL DESIGN

revisions were made in the upper approach to the lock and in the stream channel above the dam. These involved an extension to the guard wall, the construction of a spur dike upstream from the lock, and the removal of a portion of the bar opposite the lock chamber.

To determine the accuracy and the fidelity with which the model simulated the prototype conditions, certain field tests were conducted. The results of these investigations showed that, generally, hydraulic similarity between the model and prototype existed. As far as can be ascertained from the experience of navigators in traversing the upper approach to the lock chamber, the flow conditions have been considerably improved since the modifications in the upper pool were effected.

DESCRIPTION OF THE STRUCTURE

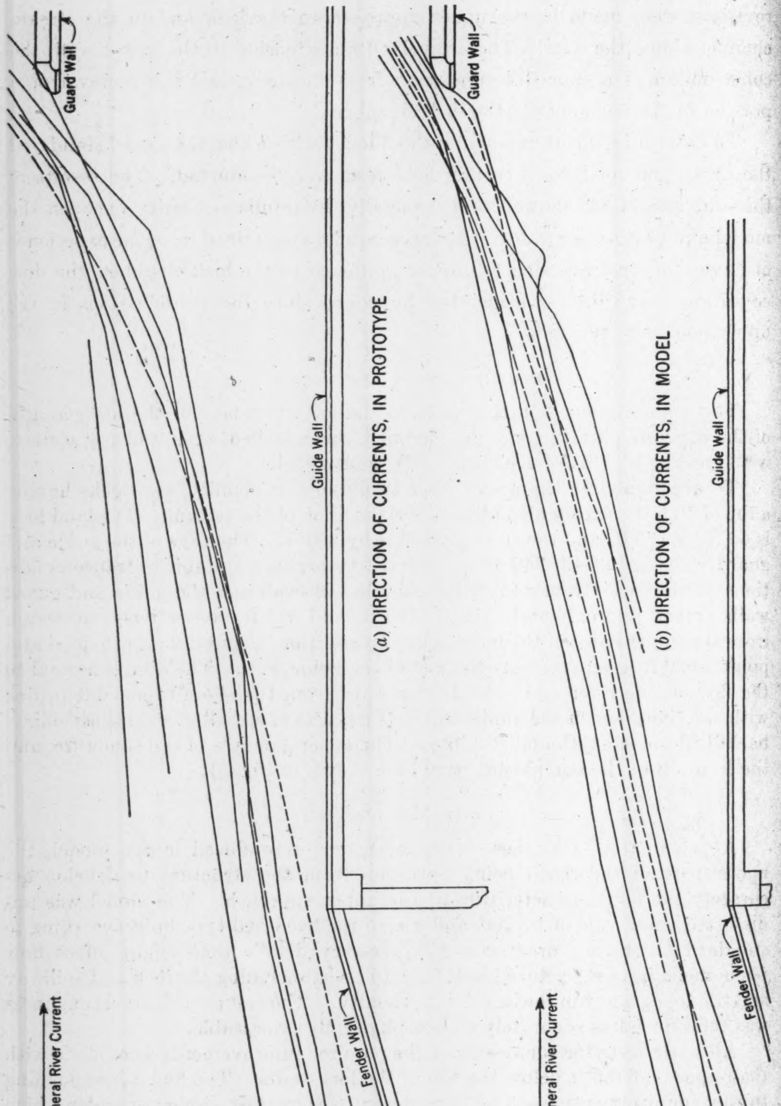
The dam is about 550 ft long from the left abutment to the lock, and is of the concrete gravity type. A modified, uncontrolled, ogee spillway section, with crest at El. 737.5 (mean sea level), is provided.

To accommodate the heavy river traffic in this vicinity, two locks having a lift of 10.6 ft are provided along the right bank of the stream. The land lock is 56 ft by 720 ft, and the river lock 56 ft by 360 ft. The tops of the guide and guard walls are at El. 749.0. As altered to produce suitable entrance conditions to the locks from above, the middle lock-wall and the guide and guard walls extend approximately 100 ft, 760 ft, and 855 ft, respectively, upstream from the upper gates of the land lock. In addition, a spur dike is provided at a point 1,600 ft upstream from the end of the guide wall. The dike is normal to the flow of the river and extends riverward from the shore to a point in line with the river face of the guide wall. It consists of a shell of steel sheet piling, backfilled for strength and stability. The general details of the structure, and the location of the component parts, are shown in Fig. 57.

THE MODEL TESTS

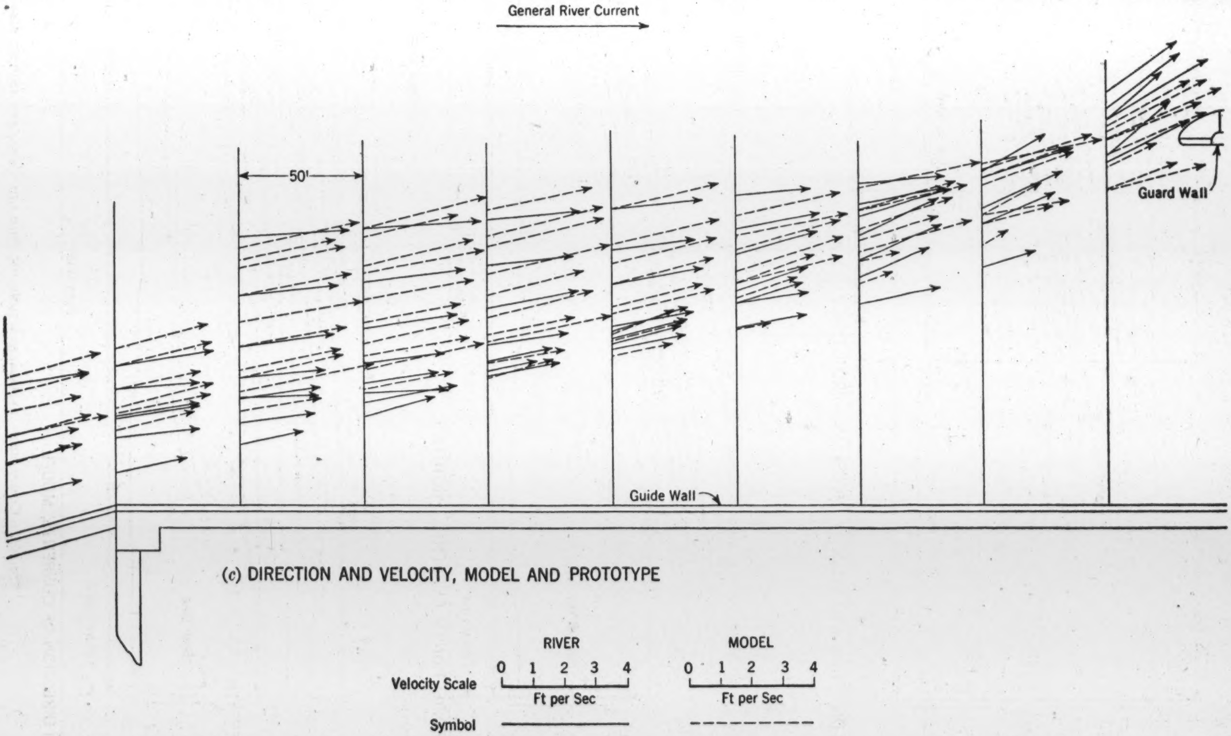
Approximately 2.4 miles of the river were reproduced in the model, the major part of the reach being upstream from the structure to develop accurately the flow characteristics at the upper approach. The model was undistorted, on a scale of 1 : 200, and was of the fixed-bed type built according to standard laboratory practice. All necessary details that might affect flow phenomena were reproduced faithfully to scale, including the lock and spillway and the topography in the immediate vicinity. The assumed channel roughness was reproduced as accurately as was physically practicable.

All basic tests for the design of the required improvements were made with the upper pool 0.5 ft below the top of the lock-walls. The flow corresponding to this condition was assumed to represent the greatest discharge under which the navigation facilities could be used. The tests consisted of analyses of the resulting velocities and currents, and of their effects on tows entering the locks from above. The investigations involved a determination of the flow condi-


Guard Wall
Guide Wall
Fender Wall
General River Current
Guard Wall
Guide Wall
Fender Wall
General River Current

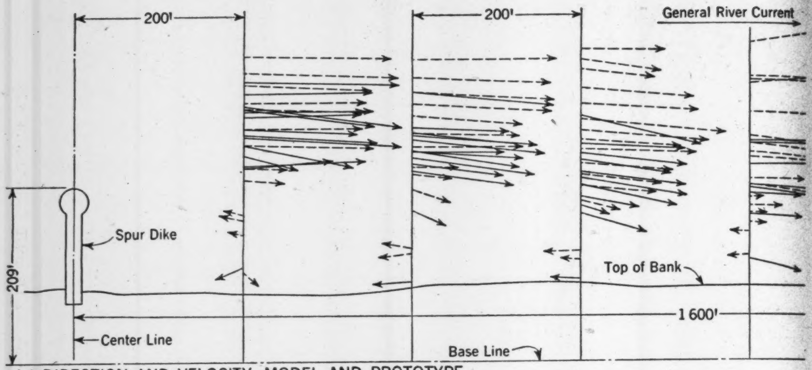
(*a*) DIRECTION OF CURRENTS, IN PROTOTYPE

(*b*) DIRECTION OF CURRENTS, IN MODEL

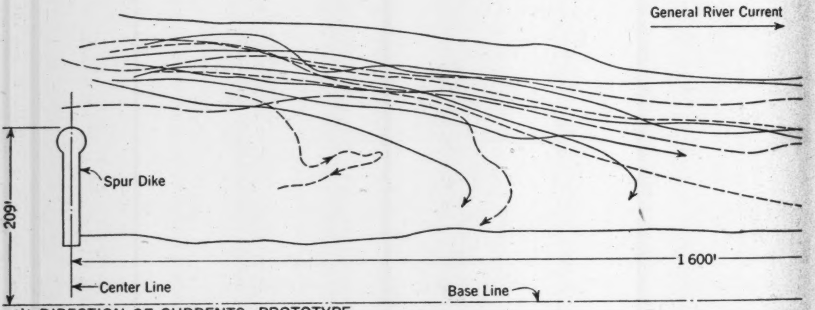


(c) DIRECTION AND VELOCITY, MODEL AND PROTOTYPE

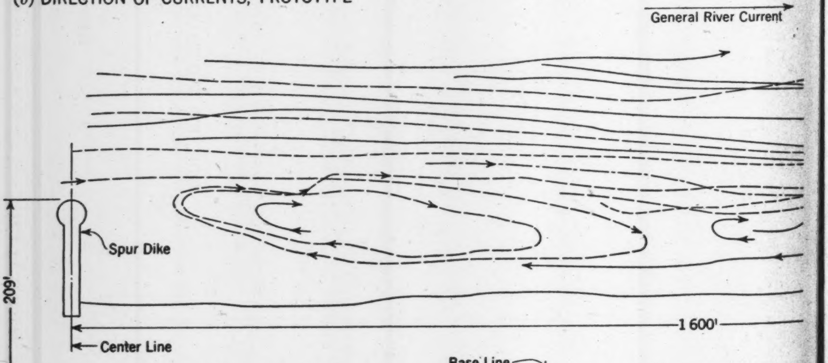
FIG. 60.—COMPARISON OF OBSERVATIONS ON SURFACE CURRENTS IN THE MODEL AND THE PROTOTYPE, FOR THE ORIGINAL CONDITION OF THE RIVER AND UPPER LOCK APPROACH


200'
200'
General River Current
Spur Dike
209'
Top of Bank
1 600'
Center Line
Base Line

(*a*) DIRECTION AND VELOCITY, MODEL AND PROTOTYPE

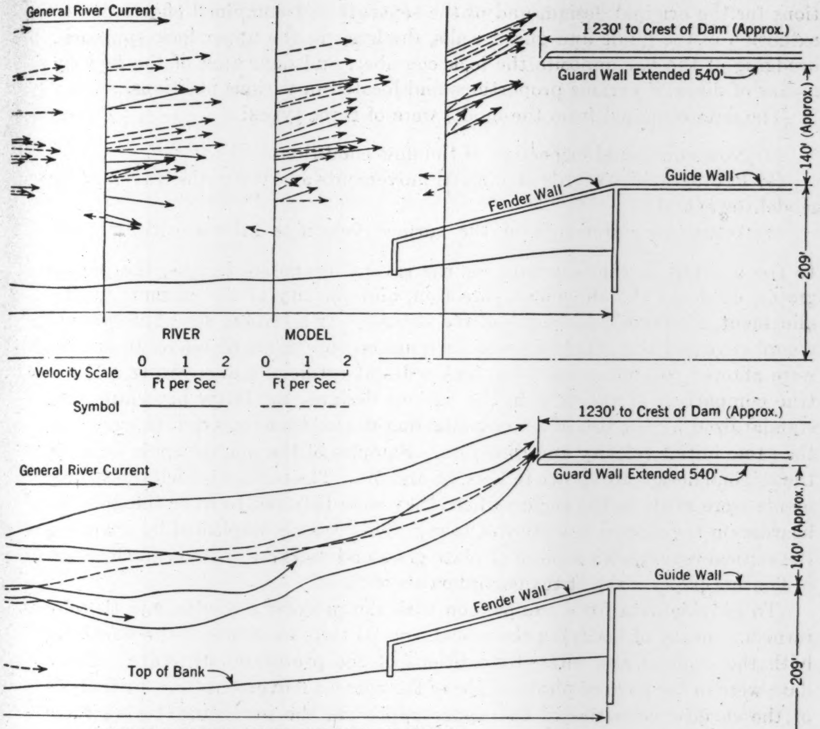

General River Current
Spur Dike
209'
1 600'
Center Line
Base Line

(*b*) DIRECTION OF CURRENTS, PROTOTYPE

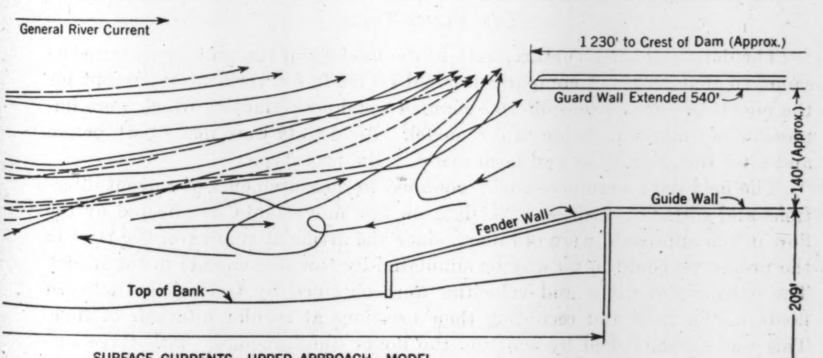

General River Current
Spur Dike
209'
1 600'
Center Line
Base Line

(*c*) DIRECTION OF CURRENTS, MODEL

Fig. 61.—Comparison of Observations on Surface Currents in the Model and


General River Current
1 230' to Crest of Dam (Approx.)
Guard Wall Extended 540'
140' (Approx.)
Guide Wall
Fender Wall
209'
RIVER
MODEL
Velocity Scale
0 1 2
Ft per Sec
0 1 2
Ft per Sec
Symbol
1230' to Crest of Dam (Approx.)
General River Current
Guard Wall Extended 540'
140' (Approx.)
Guide Wall
Fender Wall
Top of Bank
209'

SURFACE CURRENTS – UPPER APPROACH – PROTOTYPE


General River Current
1 230' to Crest of Dam (Approx.)
Guard Wall Extended 540'
140' (Approx.)
Guide Wall
Fender Wall
Top of Bank
209'

SURFACE CURRENTS – UPPER APPROACH – MODEL

Prototype for the Altered Conditions of the River and Upper Lock Approach

tions for the original design, and of the separate and combined effects due to extension of the guide and guard walls, dredging in the upper lock approach, dredging of the bar opposite the lock chamber, and deflection of the flow by means of dikes of various proportions and located in various positions.

The data obtained from the model were of three types:

(*a*) Notes on visual inspection of the flow conditions;

(*b*) Photographic records of confetti movements and the paths traversed by model tows; and

(*c*) Actual measurements of the surface velocities in the critical areas.

In the analysis of the test data for the several proposed designs, the photographs disclosed the alinement, direction, and velocity of the currents by the alinement, direction, and length of the streaks. In addition, the photographic record revealed the effect of these currents on the travel of model tows that were allowed to glide toward the lock without artificial maneuvering. For a true comparison of the data in the various designs, the latter procedure was standardized by the use of a tow-projecting device that imparted to each tow the same initial velocity and direction. Samples of the photographic data for the original design are shown in Figs. 58 and 59. The surface velocity measurements were made in the region where they were believed to have the greatest bearing on the maneuverability of tows. This was accomplished by spanning the critical region with a piece of plate glass and marking thereon the location of floating paper disks at regular intervals of time.

To provide data for a comparison with the prototype results, and thereby furnish a means of verifying the model, special tests were run on the model for both the original and altered conditions of the prototype structure. These data were in the form of photographs of the current movements and an analysis of the surface velocities in the upper approach, the procedure during these tests being the same as the foregoing.

THE FIELD TESTS

The details of the structure, both in the model and the prototype, were the same, so that an exact comparison could be made for verification, except for the effects of surface tension and channel roughness—factors which were impossible of exact simulation in the model. Field tests were made both before and after the alterations had been made to the prototype.

The field data were necessarily confined to measurements of current directions and surface velocities. No data on tow movements, as affected by the flow in the approach, were obtained, since the artificial guiding of the tows in the prototype could in no way be simulated by tow movements in the model. The current directions and velocities were obtained by tracing the paths of floats in the river and recording their locations at regular intervals of time. This was accomplished by spotting the floats simultaneously with three surveyor's transits located at strategic points. Corrections were applied to the measurements for wind velocity by noting the wind effect on floats in the lock chamber.

VERIFICATION OF RESULTS AND CONCLUSIONS

A comparison of the data obtained on the model and in the field, in the form of current alinement and velocity vectors, is presented in Figs. 60 and 61. For the original design of the approach (Fig. 60), the operating conditions in the model and prototype were identical, corresponding to a head of 5.6 ft on the crest of the dam and to a discharge of 30,000 cu ft per sec (headwater elevation, 743.1). Operation for the revised design of the upper approach was not identical in the model and prototype. The head on the crest of the dam for this condition was 6.6 ft in the prototype as against 6.0 ft in the model, corresponding to flows of 40,000 and and 35,000 cu ft per sec, respectively (headwater elevations were 744.1 and 743.5 ft, respectively). The model tests had been conducted before an opportunity had presented itself for obtaining field measurements approximating the conditions tested in the model for the altered design, so that identical operation was not possible. However, the differences in head and discharge are not believed to be of sufficient magnitude to invalidate the results of the verification.

The degree of similarity between the model and prototype results is apparent from an examination of the data presented. Exact simulation by the model of the prototype phenomena was not attained; nor was it expected. Generally, the model data reveal close approximations for the magnitude of the prototype velocities, for the alinement of the currents, and for the nature and location of the quiescent pool formed downstream from the spur dike. A comparison of the data also shows a considerable reduction in the velocities of flow across the lock entrance and at the head of the guard wall, this condition existing in both the model and prototype.

In the course of the model and field tests, it was noted that slight surges in the flow occurred, especially in regions where an attempt was made to alter the flow characteristics, or where an obstruction to flow existed. Because of these surges, the resulting path of flow through any definite point for successive intervals of time was not identical, although the general pattern of flow for any area was approximately the same. A slight variation in the data should be expected, therefore, both in the model and prototype, and a comparison of the data should be made on the basis of general values and flow patterns rather than on the specific analysis of a particular flow line. Generally, therefore, the velocities, currents, and eddies in the model and prototype were similar.

CONCLUSION

It can be concluded that satisfactory verification of the model tests for the entrance conditions to the upper approach of the new lock and dam resulted. Subsequent experience in the maneuvering of tows in the reach of the stream in question has indicated it to be devoid of any serious hazards and to be satisfactory in every respect, which further attests to the success of the plan of alteration proposed as a result of the model tests. As in many other instances, model analysis in this case provided a direct approach to a problem that could not be solved by purely analytical methods.

BIBLIOGRAPHY AND NOTATION

By the Symposium Authors

I. Bibliography

Those who are closely associated with model-prototype work realize the few references that are available on the subject. The following list, including references in the text of the various papers, is probably far from complete, but is offered as a representative beginning:

(1) **"First Annual Report of Special Committee on Hydraulic Research,"** *Civil Engineering*, March, 1935, p. 169.

(2) **"Superelevation and Bridge Pier Losses in Model and Field,"** by J. G. Jobes and J. H. Douma, *ibid.*, January, 1943, p. 16.

(3) **"Hydraulic Design of Drop Structures for Gully Control,"** by B. T. Morris and D. C. Johnson, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 887.

(4) **"The Similarity of Motion of Water Through Sluices and Through Scale Models: Experiments with Models of Sluices of the Assuan Dam,"** by H. E. Hurst and D. A. F. Watt, *Proceedings*, Inst. C. E., London, Vol. 218, 1923–1924, pp. 72–112.

(5) **"Uberfallversuche in verschiedener Modellgrösse,"** by F. Eisner, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 11, 1931, p. 416.

(6) **"Über Schwingungen von Wehren,"** by R. Seifert, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 84, No. 7, February 17, 1940, pp. 105–110.

(7) **"Distinctive Features of Grand Coulee Outlets,"** by Jacob E. Warnock, *Civil Engineering*, December, 1940, pp. 779–782.

(8) **"Models Predict Behavior of Falling Dam,"** by A. J. Ackerman, *ibid.*, July, 1932, p. 415.

(9) Discussion by T. T. Knappen of **"Construction and Testing of Hydraulic Models, Muskingum Water-Shed Project,"** by G. E. Barnes and J. G. Jobes, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 249 *et seq.*

(10) **"Comparison of Sluice-Gate Discharge in Model and Prototype,"** by F. W. Blaisdell, *ibid.*, Vol. 102 (1937), p. 544.

(11) **"Report on Experiments with a Model of Khesano Regulator on the Rohri Canal, Mile No. 194,"** by W. E. Bushby and Hiranandanio, *Technical Paper No. 1*, 1937, Government of Sind, Public Works Dept., India.

(12) **"Rapport sur les Travaux Effectués Pendant les Années 1932–35,"** by C. Camichel, *Travaux Scientifiques*, August, 1935.

(13) **Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View of Ascertaining How Far the Law of Similitude Is True,"** by C. Camichel and L. Escande, *Paper No. 57*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

(14) **"La Similitude des ouvrages courts; experiences sur la grande forme de radoub du port du Havre,"** by C. Camichel, F. Bear, and L. Escande, Académie des Sciences, Seance du Lundi 29, August, 1932.

(15) **"Verification of Drop-Inlet Hydraulic-Model Studies by Field Tests,"** by E. R. Dodge, *Civil Engineering*, August, 1941, p. 496.

(16) **"Observational Hydraulics in Miami Conservancy District,"** by C. H. Eiffert, *ibid.*, November, 1939, p. 672.

(17) **"Über die Ähnlichkeit bei flussbaulichen Modellversuchen,"** by H. A. Einstein and R. Müller, *Schweizer Archiv für angewandte Wissenschaft und Technik*, Vol. 8.

(18) **"L'Étude sur modéle réduit des ouvrages de rupture de charge,"** by L. Escande, *Genie Civil*, December 16, 1939.

(19) **"Performance of Norris Stilling Basin During 1937 Flood,"** by G. H. Hickox, TVA, Knoxville, Tenn., July, 1938.

(20) **"Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View to Ascertaining How Far the Law of Similitude Is True,"** by Giulio de Marchi, *Paper No. 59*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

(21) **"Experiments on Discharge Over Spillways and Models, Keokuk Dam,"** by F. A. Nagler and A. Davis, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 777. (a) p. 810.

(22) **"Observed Effects of Geometric Distortion in Hydraulic Models,"** by K. D. Nichols, *ibid.*, Vol. 104 (1939), p. 1488.

(23) **"Checks on the Model Law for Hydraulic Structures,"** by M. P. O'Brien, *Transactions*, Am. Geophysical Union, National Research Council, April 28–29, 1932, pp. 390–394.

(24) **"Conformity Between Model and Prototype Tests—Madden Dam Spillway,"** by P. S. O'Shaughnessy, *Civil Engineering*, August, 1941, p. 491.

(25) **"Hydraulic Tests on the Spillway of the Madden Dam,"** by R. R. Randolph, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1080.

(26) **"Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View to Ascertaining How Far the Law of Similitude Is True,"** by Rudolf Seifert, *Paper No. 53*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

(27) **"Hydraulic Laboratory Results and Their Verification in Nature,"** by H. D. Vogel, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 597.

(28) **"Annual Report of the Work of the Central Board of Irrigation, India, 1936–37,"** Simla, April 1, 1938, p. 67.

(29) **"Central Board of Irrigation Report of Proceedings of Research Commission,"** India, 1939, pp. 32 *et seq.*

(30) **"Model and Full Size Surge Tests on Large Branched Tunnel System of Detroit Water Supply,"** by A. C. Michael, *Journal*, A. W. W. A., Vol. 28, March, 1936, p. 295.

(31) **"Performance of Prototype During Floods of October and December 1937 Compared with Flow Conditions Observed in the Models; Supplemental Report—Hydraulic Model Tests—Spillway and Outlet Works, Tygart River, Reservoir Dam, Grafton, W. Va.,"** U. S. Engr. Office, Pittsburgh, Pa., April 21, 1938.

(32) **"Report on Investigation of Artificial Stream-Control Structures,"** National Bureau of Standards, Washington, D. C., May 1, 1939, p. 13.

(33) **The Experiment Station Bulletin (Hydraulics),** U. S. Waterways Experiment Station, Vicksburg, Miss., Vol. 2, No. 3, July 1, 1939, pp. 5 and 17; Vol. 3, No. 1, February 1, 1940, pp. 1–6; Vol. 3, No. 2, June 15, 1940, pp. 1–6.

(34) **"Model Studies of Penstocks and Outlet Works,"** *Boulder Canyon Project Final Reports,* Bulletin 2, Pt. IV, Hydraulic Investigations, Denver, Colo., U. S. Dept. of the Interior, Bureau of Reclamation, 1938.

(35) **"Comparison of Field and Model Hydraulic Experiments, Boulder Outlet Works,"** *Technical Memorandum No. 610,* Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., July 22, 1940.

(36) **"Experimental and Practical Experience with the Gibson Method of Water Measurement,"** by N. R. Gibson and E. B. Strowger, *Transactions,* A. S. M. E., Vol. 57, 1935, p. 213.

(37) **"Modern Developments in Fluid Dynamics,"** by S. Goldstein, Clarendon Press, Oxford, Vol. II, 1938, p. 360.

(38) **"Loss in Oblique-Angled Pipe Branches,"** by D. Thoma, *Bulletin 3, Transactions of the Hydraulic Institute of the Munich Technical Institute,* A. S. M. E., New York, N. Y., 1935.

(39) **"Hydraulic Model Experiments for the Design of the Norris Dam,"** *Technical Memorandum No. 406,* Bureau of Reclamation. (*a*) p. 121.

(40) **"Models Cut Costs and Speed Construction,"** by J. C. Stevens, *Civil Engineering,* October, 1936, p. 674.

(41) **"Report of the Board of Engineers on Deep Waterways,"** Pt. I, 1900, p. 291.

(42) **"Weir Experiments, Coefficients, and Formulas,"** by R. E. Horton, *Water Supply and Irrigation Paper No. 200,* U. S. Geological Survey, 1907. (Reference to formulas and experiments of Bazin, Dubuat, Francis, Fteley and Stearns, Nelles, and others may be found in this paper.) (a) (Original Source "Report of the U. S. Board of Engineers on Deep Waterways," 1900, Pt. I, p. 291.)

(43) **"Experiments on Small Weirs and Measuring Modules,"** by J. C. Stevens, *Engineering News,* Vol. 64, No. 7, August 18, 1910, p. 171.

(44) **"Report on Weir in the Niagara River,"** *Engineering Record,* Vol. 68, No. 8, p. 206. (Digest of a report of the International Waterways Commission, June 20, 1913. Digest includes all significant data and is more generally available.)

(45) **"The Submerged Weir as a Measuring Device,"** by G. N. Cox, *Bulletin No. 67,* Univ. of Wisconsin Eng. Experiment Station, Madison, Wis., 1928.

(46) **"Flow of Flood Water Over Railway and Highway Embankments,"** by D. L. Yarnell and F. A. Nagler, *Public Roads,* Vol. 11, No. 2, 1930, p. 30.

(47) **"Tests of Broad-Crested Weirs,"** by James G. Woodburn, *Transactions,* Am. Soc. C. E., Vol. 96 (1932), p. 387.

(48) **"Supplemental Tests—Weirs with Aprons Inclined Up Stream and Down Stream,"** by A. R. Webb, *ibid.*, p. 408.

(49) **"Recreation Dam Built in Miami Reservoir,"** by C. H. Eiffert, *Engineering News-Record*, Vol. 120, No. 4, January 27, 1938, p. 137.

(50) **"Submerged Weir Formula Verified,"** by G. N. Cox, *ibid.*, Vol. 123, No. 9, August 31, 1939, p. 52.

(51) **"Concrete Controls for Stream-Gaging Stations,"** by O. W. Hartwell, *Civil Engineering*, February, 1939, p. 121.

(52) **"Hydraulic Studies of a Model of the University Dam at Iowa City,"** by C. L. Morgan, June, 1938 (graduate thesis, Dept. of Mechanics and Hydraulics, State Univ. of Iowa).

(53) **"Model Study of Plans for Channel Improvement at Grand Tower Reach, Mississippi River,"** *Technical Memorandum No. 114-1*, U. S. Waterways Experiment Station, Vicksburg, Miss., August 10, 1937.

(54) **The Experiment Station Bulletin (Hydraulics)**, U. S. Waterways Experiment Station, Vicksburg, Miss., Vol. 1, No. 2, October 1, 1938, pp. 5–6.

(55) **"Translatory Waves in Open Channels,"** by Horace W. King, *Civil Engineering*, June, 1933, pp. 319–321.

(56) **"Flow of Water in Tidal Canals,"** by Earl I. Brown, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 749–834.

(57) **"Fluid Mechanics for Hydraulic Engineers,"** by H. Rouse, *Engineering Societies Monograph*, 1938, pp. 353–402. (*a*) Chapter I.

Miscellaneous references, mentioned by discussers, are as follows:

(58) **"Models of Upper Narrows Dam Spillway,"** by W. M. Stubblefield and J. P. Henck, thesis presented to the University of California in December, 1939, in partial fulfilment of the requirements for the degree of Bachelor of Science (unpublished).

(59) **"Gesetzmässigkeiten der turbulenten strömung in glatten röhren,"** by J. Nikuradse, Verein Deutscher Ingenieur, Forschungsheft, No. 356, Berlin, Germany, 1932.

(60) **"The Kinetic Energy and Momentum Correction Factors for Pipes and Open Channels of Great Width,"** by Victor L. Streeter, *Civil Engineering*, April, 1942, p. 212.

(61) **Technical Report for 1941**, Central Board of Irrigation, *Publication No. 27*, India: (*a*) "A Comparison of the Results of Model Experiments and Observations on the Prototype," by Harbans Lal Uppal, p. 64. (*b*) "Working of the Armoured Spurs on the Beas River during the First Year and Comparisons with the Results Obtained on the Model," by J. L. Roy, p. 68. (*c*) Discussion of Model-Prototype Relations, Research Committee, p. 69.

(62) **"Die Internationale Rheinregulierung von der Illmündung bis zum Bodensee,"** by Karl Böhi, *Schweizerische Bauzeitung*, Vol. 109–110, 1937, p. 187.

(63) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 561.

(64) **"Beitraege zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen fuer Stroeme, Kanaele und geschlossene Leitungen,"** by A.

Strickler, Mitt. des eidg. Amtes fuer Wasserwirtschaft, No. 16, Bern, 1913.

(65) **"Hydraulic-Fill Dams,"** by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1713.

(66) **"Reconstructing a Spillway,"** by R. C. Kennedy, *Western Construction News*, April, 1942, p. 158.

(67) **"Side Channel Spillways: Hydraulic Theory, Economic Factors, and Experimental Determination of Losses,"** by Julian Hinds, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 881.

(68) **"The Control of Water,"** by Phillip à Morley Parker, George Routledge & Sons, Ltd., London, 1913, p. 839.

(69) **"Hydraulic Jump in Enclosed Conduits,"** by E. W. Lane and C. E. Kindsvater, *Engineering News-Record*, December 29, 1938, p. 815.

(70) **"Entrainment of Air in Flowing Water: Closed Conduit Flow,"** by A. A. Kalinske and James M. Robertson, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1435.

(71) **"Report on Tests of Models of Rate Measuring Installations,"** by Fred W. Blaisdell, 1940 (unpublished).

(72) **"The Parshall Measuring Flume,"** by Ralph L. Parshall, *Bulletin No. 423*, Colorado Experiment Station, Fort Collins, Colo., March, 1936. (*a*) p. 54.

(73) **"Parshall Flumes of Large Size,"** by Ralph L. Parshall, *Bulletin No. 386*, Colorado Experiment Station, Fort Collins, Colo., May, 1932. (*a*) p. 48.

(74) **"Annual Reports,"** Central Irrigation and Hydrodynamic Research Station, Poona, India, 1937–1938, p. 39; 1939–1940, p. 45; 1940–1941, p. 42.

(75) **"Hydrodynamic Models as an Aid to Engineering Skill,"** by C. C. Inglis, Presidential Address, Section of Eng., Indian Science Cong., 1941.

(76) **"Laboratory Tests on Hydraulic Model of Filling and Emptying System for Proposed Watts Bar Project Lock,"** U. S. Engr. Sub-office, Iowa City, Iowa, December, 1937.

(77) **"Studies of Crests for Overfall Dams,"** *Bulletin No. 3*, **Pt. VI, Hydraulic Investigations, Boulder Canyon Final Repts., U. S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo., December 31, 1942** (unpublished).

(78) **"Hydraulic Models,"** *Manual of Engineering Practice No. 25*, Am. Soc. C. E., 1942, p. 39, Fig. 2. (*a*) Section V, p. 36.

(79) **"Annual Reports"** (Technical), Central Board of Irrig., Simla, Ind, 1939–1940, p. 176; 1941, p. 64; 1942, p. 44.

II. NOTATION

The following symbols, used in the Symposium, conform essentially with "Letter Symbols for Hydraulics," prepared by a Committee of the American Standards Association with Society representation and approved by the Association in 1942.[14]

A = area of cross section, a subscript denoting the section location:

A_c = area of culverts;

A_l = area of lock chamber;

---

[14] ASA—Z10.2—1942.

b = length of weir crest;
C = discharge coefficient:
  C_l = lock coefficient;
  C_m = coefficient determined by model tests;
  C_q = coefficient of discharge;
D = diameter;
f = friction coefficient, subscripts m and p referring to model and prototype, respectively;
g = acceleration due to gravity;
H = head above a given datum plane: H_t = measured tailwater elevation, referred to the elevation of the crest;
h = pressure heads at selected sections indicated by appropriate subscripts:
  h_e = drop in pressure at entrance;
  h_f = loss of head due to friction;
  h_p = pressure head differential between two sections;
  h_v = head, velocity;
  h_x = pressure at any piezometer in the bellmouth;
K = a discharge coefficient defined by Eq. 11; also a constant in Eq. 3;
  K_o = observed constant in the rating equation (Table 2) for a flow meter;
  K_s = Scobey's friction coefficient;
  K_t = theoretical constant in the rating equation (Table 2) for a flow meter;
L = lengths, such as center-line distances from the bellmouth, with subscripts to denote section locations; the distance required to produce normal distribution below an intake in a smooth pipe:
  L_r = geometric scale ratio, $\frac{L_p}{L_m}$;
  L_x = a length factor;
n = constant exponent defined by Eq. 11;
p = pressure, in pounds per square foot; $\frac{p}{w}$ = pressure head, feet of water;
Q = time rate of discharge, subscripts m and p denoting model and prototype, respectively;
q = discharge per unit width;
$\mathbf{R}$ = Reynolds' number;
r = radius;
V = average velocity at a given cross section;
V_l = lock-chamber rate of rise or fall;
$\mathbf{W}$ = Weber number;
w = specific weight of water in pounds per cubic foot; $\frac{p}{w}$ = pressure head, feet of water;
y = depth of flow;
Δ = central angle.

DISCUSSION

A. E. **Niederhoff**,[15] Assoc. M. Am. Soc. C. E.—About the only thing that can be added to the thorough and fine paper by Messrs. Nelson and Hartigan is to call attention to corroborative evidence found on the West Coast. This evidence concerns two high lift locks with lifts of 66 ft and 45 ft, respectively.

The authors have revealed the fact that, where geometric similarity does not exist between model and prototype, correlation of test results is difficult. Of this there can be no doubt but, since total similarity, geometric and dynamic, can never be obtained when the same liquid is used in both model and prototype, one should not be too disturbed at failure to obtain exact quantitative results. After all, a model of any kind is not the answer to all questions on a subject. At the risk of seeming irreverent, the writer calls attention to the model husband who has been defined as "a small imitation of the real thing." Hydraulic models fall in the same category, and engineers do well to observe trends and characteristics in a model that are duplicated in the prototype. Actual measurements of time, velocity, forces, or discharges are done best on a full-size lock for possible design use on a similar structure.

Fig. 62(*a*) shows the filling of a model lock, utilizing a floor culvert and port system patterned after one of the locks discussed herein (66-ft lift). The same water disturbance in the model chamber will be observed in the prototype (see Fig. 62(*b*)) especially at the far end, near the lower miter gates. Also, when emptying the model chamber, the fountain effect below the lower miter gates is duplicated every time the 66-ft lock is emptied. It is interesting to note that, in spite of lack of exact similarity, the coefficient of filling for the model and prototype agreed within 1%. An average coefficient of filling was 0.83, and, for emptying, this coefficient dropped to 0.72.

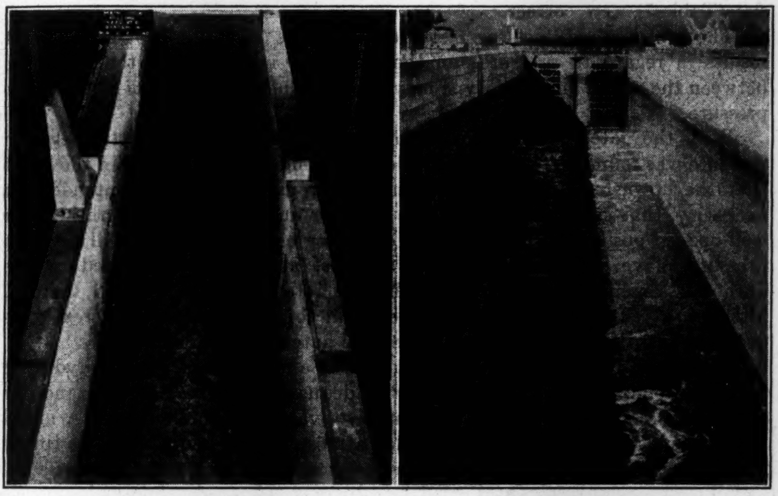
Hawser stresses in model lock chambers have been measured as a criterion for proper functioning for several years. However, to the writer's knowledge, only one attempt to measure hawser stresses has been made on a high lift lock. In 1940 a government derrick boat was moored by four lines to floating mooring bits in the 66-ft lock. Dynamometers were placed in these lines and readings were taken simultaneously at each 5 ft of lift. When plotted as a curve, the results showed that the maximum pull occurred when the valves were fully open and when discharge into the lock was a maximum. The curves showed the same family characteristics as those obtained in a model even if the actual magnitude of the pull on the lines was considerably greater than that shown in the model.

The distribution of water from ports in a lock chamber has been ably described by Messrs. Nelson and Hartigan, and it is only in deference to Southern California that the writer wishes to point to the full-scale results of the "manifold problem" investigated at the second high lock mentioned herein

---

[15] Senior Structural Engr., U. S. Engr. Dept.

(45-ft lift). Southern California was a factor in this test because the grapefruit used as floaters came from Coachella Valley. These juicy, yellow spheres were placed just upstream from the filling valves, and timed and counted as they emerged from the several chamber ports. In this particular case, most of the grapefruit, and therefore most of the water, came from the port farthest upstream in the chamber.



(a) Model (Head, 47.6 Ft) (b) Prototype (Head, 49.0 Ft)

FIG. 62.—DISTURBED WATER CONDITION WHEN LOCK CHAMBER IS BEING FILLED

Model tests have shown the necessity for relieving entrapped air in a culvert system if the lock is to be filled efficiently. The conditions at the 45-ft lock at the time of conversion from a tandem lock to a single lift of 45 ft were such that only one large vent hole per culvert could reasonably be drilled. This hole was 20 in. in diameter. When completed, it worked beautifully, sending a geyser of air and water 60 ft into the air above the top of the lock-wall and adding another spectacle to the scenic beauty of the region.

When the diameter of the vent hole was reduced to 10 in. by placing a pierced diaphragm over the end of the vent pipe, the water no longer shot out of the vent pipe but entrapped air exploded into the lock chamber. Cutting the effective vent hole to a diameter of only 4 in. merely made the chamber explosions more violent and more prolonged.

C. I. GRIMM,[16] M. AM. SOC. C. E.—A number of acceptable formulas have been developed for use in determining the crest profile of an overflow concrete spillway. The profile of the crest of the spillway reported by Messrs. Stevens and Cochrane comes within the limits of these formulas and, therefore, may be considered as a conventional shape. Computations for a thin jet were made as a guide to design. These showed that, with the gate in the downstream position, the jet followed quite close to the crest profile, but that with the gate in the upstream position it departed considerably. It was not considered practicable, however, to design the profile so that there would be no departure in this case. These calculations would raise doubt as to whether the positive pressures indicated by the model with the gate in the upstream slot (Table 7) were correct.

---

[16] Formerly with U. S. Engr. Dept., Portland, Ore. (Mr. Grimm died on December 1, 1942.)

The prototype studies seem to prove that the model did not give correct results in regard to pressures on the crest and indicate that the difference between the model and prototype may be due to non-similitude of atmospheric pressure.

With the gate in the downstream slot on the prototype, little negative pressure was found. The upstream slot is for emergency use only in the event that the downstream slot is out of use for repairs. It is not intended, or necessary, that gates in these slots be used for fine regulation, but rather in a well-opened position. As a practical operating condition, only small negative pressures have existed on the spillway crest at the dam discussed by the authors. Examination has shown the concrete to be undamaged thereby, although extensive damage to the piers and some damage to the crest have occurred just downstream from the vertical gate slots.

Comparison of prototypes with models is an important step in hydraulic investigations that has been entirely inadequate in scope, to date, and the study by Messrs. Stevens and Cochrane is a valuable contribution. A model study of spillway crests under controlled atmospheric pressure would be timely and of decided value as a research project to supplement present knowledge.

JOE W. JOHNSON,[17] ASSOC. M. AM. SOC. C. E.—The various papers included in this Symposium indicate that a wide variety of hydraulic problems is susceptible to investigation by means of models. All of the papers are important contributions to a field in which the literature is rather meager.

Of particular interest to the writer were the papers by Mr. Hickox and by Mr. Soucek on spillway coefficients. In the investigations described in these two papers, a period of five years was required to obtain the various data. Mr. Hickox describes prototype observations and compares them with model studies that were made when the structure was in the design stage. On the other hand, Mr. Soucek describes model studies and compares the data with past observations on an existing structure. The sequence of the observations is unimportant for giving valuable information on conformity between model and prototype. Model studies made prior to the construction of a prototype give valuable assistance in obtaining the best design, but model studies made of existing structures are more likely to be academic in character. Somewhat between these two limits was a model study conducted at the University of California at Berkeley, Calif., on the spillway crest of the Upper Narrows Dam on the Yuba River. This investigation (58) was intended primarily to provide (1) a check on the assumed discharge curve of the spillway, and (2)

---

[17] Asst. Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

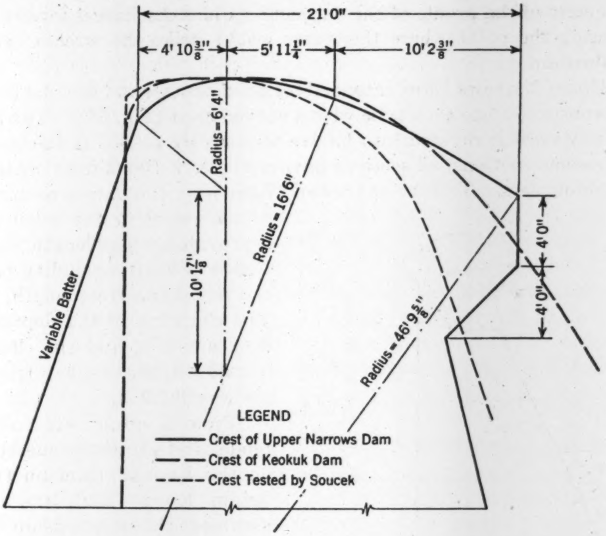

21' 0"
4' 10 3/8"
5' 11 1/4"
10' 2 3/8"
Radius = 6' 4"
Radius = 16' 6"
10' 1 7/8"
Variable Batter
Radius = 46' 9 5/8"
4' 0"
4' 0"
LEGEND
Crest of Upper Narrows Dam
Crest of Keokuk Dam
Crest Tested by Soucek

FIG. 63.—COMPARATIVE CREST DESIGNS

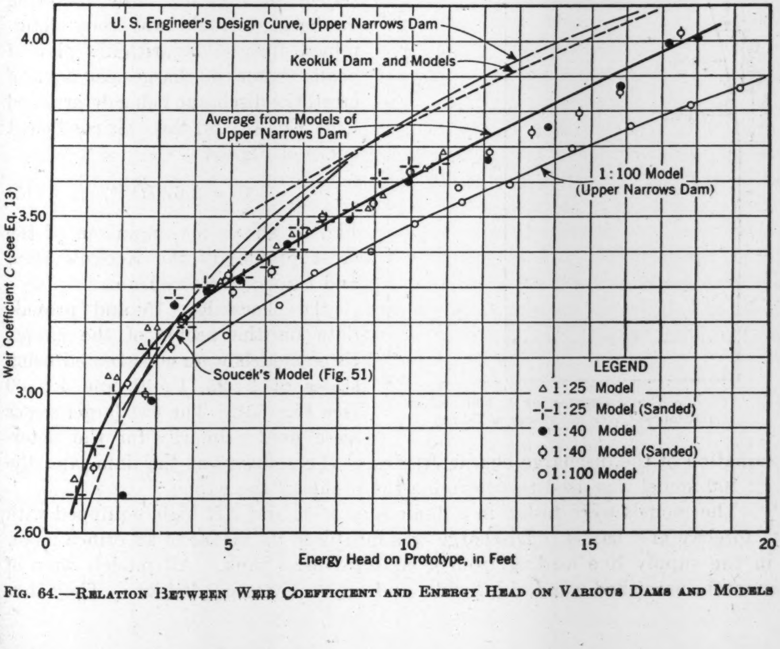


FIG. 64.—RELATION BETWEEN WEIR COEFFICIENT AND ENERGY HEAD ON VARIOUS DAMS AND MODELS

measurements of the profile of the nappe at various discharges for the purpose of predicting the point where the water would strike the stream bed at the base of the dam.

The Upper Narrows Dam, intended for flood control and as a debris barrier, is of the constant-angle arch type with a curved crest (Fig. 63). The length of the spillway crest is divided into twelve sections by piers 5 ft thick. Except for the two end sections, all sections have a length of 100 ft from center line to center line of piers. To take the normal flows over the dam a center section, 4 ft lower than the other sections, is provided. The length of this section is 189.4 ft, excluding piers, and the remaining crest length, 908.4 ft. The elevation of the depressed section is 527 ft, and the elevation of low flow in the original stream channel was 291.2 ft.

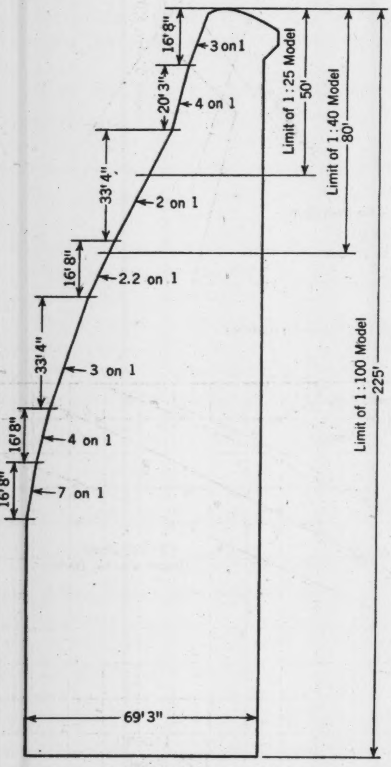


FIG. 65.—CROSS SECTION OF 1 : 100 MODEL (Distances Given in Prototype Values)

From a consideration of the model and prototype measurements on the Keokuk Dam on the Mississippi River (21*a*), the designers assumed the relationship between the weir coefficient C (computed from Eq. 13) and the energy head shown in Fig. 64. The following discharge formula was then determined from a logarithmic plot of head versus discharge per foot of crest, the discharge being determined from Eq. 13 and the weir coefficient curve of Fig. 64:

$$Q = 2.62\, H^{1.66} \ldots\ldots (15)$$

Fig. 63 shows a comparison of the crest profiles of the Keokuk Dam and Upper Narrows Dam.

To check Eq. 15 and provide data on the profile of the nappe, three models were constructed using scales of 1 : 25, 1 : 40, and 1 : 100 (see Fig. 65). The two larger scales were used primarily for the determination of the discharge characteristics of the spillway at the dam, and the 1 : 100 model was used to determine the profile of the nappe.

The models were tested in a flume 6 in. wide and 3 ft high, equipped with a forebay and baffles. Discharge was measured by means of an orifice meter in the supply line leading from a constant-head tank. All models were of smooth, varnished wood, 6 in. wide, and approximately 2 ft high. They thus represented only the upper part of the dam (depending on the scale), as shown in Fig. 65. The heads used in the analysis included the measured depth above the crest and the velocity head in the approach channel and, therefore, represented the energy head. Observations were taken with heads varying from 0.030 ft to 0.450 ft. Because of the relatively strong surface tension forces at the heads below 0.03 ft, the nappe would cling to the face of the model—a condition which probably would not exist on the prototype at a corresponding head. A high degree of turbulence in the approach channel prevented accurate readings of the head for heads greater than 0.450. All runs were made with the nappe completely aerated by means of a 1-in. pipe through which air could flow to the underside of the nappe.

To determine whether frictional forces had any significant effect on the weir characteristics over the range of flow under investigation, tests were conducted on the 1 : 25 and 1 : 40 models with a smooth, varnished surface and with a sand-rough surface. The roughened surface was obtained by sprinkling sand, which passed the 100-mesh sieve and was retained on the 200-mesh sieve, on the freshly varnished surface of the model.

Fig. 64 shows the relation between the weir coefficient and the energy head for the various models. The values for the 1 : 25 and 1 : 40 models, whether with a smooth or sand surface, agree very well and can be represented by an average curve. In addition, the results from the 1 : 100 model, which are consistently lower than the larger models, are shown in Fig. 64.

By using the average curve drawn through the experimental points for the 1 : 25 and 1 : 40 models (Fig. 64), values of discharge were computed for various assumed values of head. A plot of discharge versus head on logarithmic paper gave the relationship:

$$Q = 2.79\,H^{1.62} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

For comparative purposes, the data of Mr. Soucek are also of interest. Thus, the mean curve through the experimental points in his head-coefficient relation obtained by model studies (Fig. 51) is also plotted in Fig. 64. This curve follows quite closely the designer's curve based on the Keokuk experiments, and such agreement is to be expected when it is considered how similar the profiles of Keokuk Dam and the dam discussed by Mr. Soucek are to each other (Fig. 63). A more definite measure of the agreement between the various studies is obtained if the head-discharge relation shown in Fig. 55 is plotted on logarithmic paper. If the discharge per foot is considered instead of the total discharge, this plot yields the equation

$$Q = 2.85\,H^{1.62} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Comparing Eq. 17 with Eqs. 15 and 16, it is found that below a head of about 3 ft the various equations give practically the same discharge. For a head of 10 ft, Eqs. 15 and 17 give discharges that are about 3% greater than the Upper Narrows Dam (Eq. 16). For almost all design problems this agreement can be considered as well within the limits of error to which certain basic design data, such as the "maximum" spillway discharge, can be estimated.

V. L. Streeter,[18] **Assoc. M. Am. Soc. C. E.**—Those in charge of hydraulic laboratories are inclined to look to model studies for the answer to all hydraulic questions, when the application of fundamental hydraulic design principles will frequently give more reliable information at a small fraction of the cost. In many cases, a more detailed and searching analysis of the problem before the model is constructed would show that a complete model study of all hydraulic features is unnecessary. To be specific, in the case of pressure and discharge tests on the 102-in. river outlets described by Messrs. Warnock and Dewey, a more reliable discharge may be obtained by computation than by scaling up the model discharge. Although Froude's number must be used to scale the discharge, the value of friction factor, which depends on the Reynolds number, cannot be determined from the model.

The writer, while working in the hydraulic laboratory of the U. S. Bureau of Reclamation, prepared a prototype discharge curve for the intermediate outlets before the model was constructed. For values of the head, H, greater than 40 ft, one outlet has the discharge

$$Q = 340\,H^{0.5} \qquad (18)$$

in which, supplementing the Symposium nomenclature, H is the total head above the center of the exit, in feet. Eq. 18 is based on:

(1) A loss at the entrance and gate section of one tenth the velocity head in the pipe;

(2) A friction factor $f = 0.0073$ for the 225 ft of pipe; and

(3) A loss of six hundredths of the velocity head for elbow and converging section. The friction factor was determined by a consideration of smooth pipe friction factors for the same Reynolds numbers, as given by the Nikuradse (59) formula. The curve resulting from this discharge equation is plotted in Fig. 66 with those given in Fig. 13. This computed discharge curve seems more reliable than the curve through the points obtained from the prototype discharge measurements. Curve 4, representing the prototype discharge measurements, has a loss of head proportional to $V^{2.13}$, in which V is the average velocity in the 102-in. outlet. It is significant that Messrs. Warnock and Dewey did not use the discharge measurements as a basis for computing the prototype friction factor.

The prototype friction factors as shown in Fig. 6 may easily be 25% too high. To determine accurately the value of f for a pipe with small length-diameter ratio, velocity distributions should be obtained at each end of the test section. At the downstream end of the bellmouth the velocity distribution would be practically uniform, but about fifty diameters farther downstream the velocity would assume its characteristic distribution for the particular Reynolds number. The value of the kinetic energy correction factor for a friction factor 0.0073 would be about 1.02 (60). For a head of 150 ft, the velocity head is about 83.6 ft. Neglecting the kinetic energy correction factor could change the head loss by two hundredths of the velocity head, or 1.67 ft. The head loss due to friction in 40 ft of outlet should be about 2.87 ft. Hence a change in kinetic energy correction factor of 0.011 in the reach would change the friction factor by 25%.

---

[18] Associate Prof., Hydraulics, Illinois Inst. of Technology, Chicago, Ill.

In many cases the best method of predicting prototype performance is first to formulate analytical relationships for model performance. When relationships are established so that model performance may be predicted and verified by tests, the analytical relationships may be altered to fit the prototype. The discharge curve for the outlet works is an example of this method. Using the best values of f, and minor losses, the discharge for the model may be obtained in equation form, and the various losses may then be adjusted to agree with model experimental data. When this is accomplished, the value of f should be changed to that anticipated in the prototype. As the minor losses are more dependent on Froude's number, they should not require alteration. This method permits an evaluation of model performance in terms of the prototype when the Reynolds number effects are important.

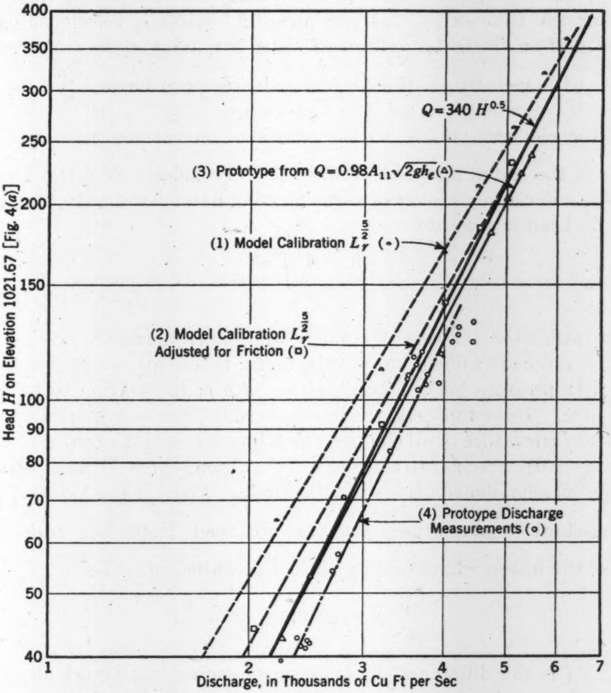


Fig. 66.—Comparison of Discharges (See Fig. 13)

The authors' explanation of the discrepancy in pressures between the model and prototype—that is, due to unstable flow—is not clear (heading:

"Examples of Quantitative Comparisons: Pressure Drop in Elbow and Cone"). For a scale of 1 : 17 the Reynolds number for the model would be about one seventieth as much as for the prototype. For the outlet model the Reynolds number would be between 300,000 and 1,000,000—a range in which there is no reason to expect unstable flow.

The observed pressure distributions through the elbow and cone, as given in Fig. 5, do not appear susceptible to analysis. Forgetting the observed results, for the time being, the fundamental hydraulic relationships will be examined. If an elemental volume of water is moving along a curved path of radius, r, with velocity, v, the difference in pressure heads, $\frac{dp}{w}$, on the two sides of the element normal to the radius of curvature, must be $\frac{v^2}{g\,r}\,dr$, in which dr is the radial distance between the two sides, and g the acceleration of gravity. For a finite layer of water moving in a curved path, the difference in pressure head is given by

$$\frac{\Delta p}{w} = \frac{1}{g}\int \frac{v^2}{r}\,dr \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which is integrated between the outer and inner filaments of flow. If the values of v and r are known for each filament, this relation will give the proper difference in pressure head, assuming the path of integration to be in a horizontal plane. Unfortunately these relationships are not known. Assuming the center of curvature of all elements on a line through the center of curvature of the elbow perpendicular to the plane of the elbow, three different assumptions regarding velocity distributions will be made. First, if the velocity is everywhere constant in magnitude, the pressure head drop, $\frac{\Delta p}{w}$, from crown to invert for the intermediate outlets, in feet of water would be

$$\frac{\Delta p}{w} = 0.00000206\,Q^2 - \Delta z \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

in which Δz is the difference in elevation of crown and invert along a radial line from the center of curvature. The term Δz corrects the formula for the case in which the filaments are not in a horizontal plane. If, instead of the foregoing velocity assumption, it is assumed that the magnitude of the velocity varies inversely as the distance from the center of curvature, as with the "free vortex," the pressure head drop in feet of water becomes

$$\frac{\Delta p}{w} = 0.00000216\,Q^2 - \Delta z \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

Finally, assuming that the velocity magnitude varies directly as the distance from center of curvature, as with the "forced vortex," the pressure head drop becomes

$$\frac{\Delta p}{w} = 0.00000205\,Q^2 - \Delta z \ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

It is surprising how well Eqs. 20 agree. One is led to think the velocity distribution is not too important. Approximately the pressure drop is found to vary inversely with the radius of curvature for each assumption regarding velocity distribution. If the elbow turned through an angle of, say, 180°, there would be no question as to whether the center of curvature used was substantially correct. For an angle 41° 19′, however, the radius of curvature of the filaments may for the most part be greater than that of the elbow. Using the results of the model studies (Fig. 5) to determine this radius for the assumption of constant velocity, the radius of curvature at the center line is found to be 52.8 ft. This is based on an average of eight values taken from the two higher model heads shown in Fig. 5 for distances from the axis of 195, 200, 205, and 210 ft. The spread was from 48 ft to 60.5 ft. Using the value 52.8 ft, the formula becomes

$$\frac{\Delta p}{w} = 0.00000155\, Q^2 - \Delta z \ldots\ldots\ldots\ldots\ldots\ldots (20d)$$

Checking Eq. 20*d* against the prototype measurements in Fig. 5 corresponding to those for the model, the pressure head drops are found to agree on the average within 2.4%, with a spread from − 11% to + 4.5%.

This is a method, therefore, of predicting the difference in pressure between prototype crown and invert from model studies. From considerations of equilibrium in the elbow, the pressure rise at the crown may be shown to equal the pressure drop at the invert, assuming a "forced vortex" velocity distribution. Approximately the same results will be obtained for the other two assumptions. The pressure at the center line would not change because of the centrifugal forces set up in the elbow. The poor agreement shown in Fig. 5 would appear due to errors in model piezometer pressures.

This discussion has been submitted in order to emphasize the importance of careful analytical study of hydraulic design features before construction of a model and the continued analytical attitude during model testing and evaluation of results.

Glen N. Cox,[19] M. Am. Soc. C. E.—The various papers of this Symposium are a valuable addition to the literature on the operation and reliability of hydraulic models.

The paper by Mr. Soucek is of special interest to the writer since it deals with a dam operating as a submerged weir. The definition of submergence quoted by Mr. Soucek infers that a weir can only be submerged if the elevation of the tailwater is above the crest of the weir. This assumption is satisfactory for any weir not having a sharp crest. However, a sharp-crested weir behaves as a submerged weir when the tailwater elevation is materially lower than the crest elevation. This is true because it is impossible to ventilate the underside of the nappe properly from the sharp-crested weir when the tailwater elevation approaches the crest elevation. A desirable definition of a submerged weir is as follows:

---

[19] Prof., Mechanics and Hydraulics, Louisiana State Univ., Baton Rouge, La.

> A weir is said to be submerged whenever the tailwater rises to such an elevation that the discharge over the weir is affected.

Mr. Soucek is correct when he states that the discharge over an ogee crest is not materially affected when the submergence is less than 45%.

Mr. Soucek chose appropriate words in the statement (see heading, "Comment: Measurement of Tailwater Elevation")—

> "* * * the water level near the dam is at a lower elevation than farther downstream. The reason for a negative slope in this reach is a readjustment of velocity distribution which has an effect that is similar, as regards energy transfer, to the recovery of pressure head which occurs in the diffusing cone of a venturi meter."

The downstream pressure should be measured at the point of minimum pressure, namely, at the throat of the venturi tube. The head recovered downstream from the throat of the venturi meter varies between rather wide limits and depends upon the original turbulence, the angle of divergence, the viscosity of the fluid, the roughness of the surface, and possibly upon other factors. Although it would be possible to calibrate a given tube and thus obtain a relationship between the discharge and over-all pressure drop for a given fluid, it would not be reasonable to expect that the same relationship would hold for the same tube with a different piping arrangement or in transporting a fluid of decidedly different physical properties. Neither could the calibration be used for tubes of other proportions and angles of divergence. On the other hand, calibrations that are based upon a downstream pressure reading taken at the throat can be used for predicting the discharge from other tubes and for other fluids flowing in the calibrated tube.

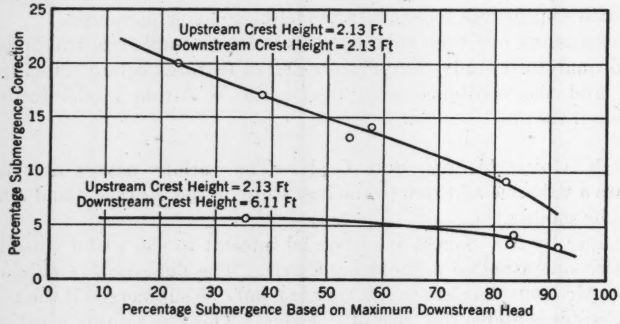


Fig. 67.—Correction Curve for Obtaining Percentage Submergence at Low Point Near Crest

The same reasoning applies to the submerged weir. The magnitude of the recovery head downstream from the low pressure point depends upon the proportions and roughness of the channel downstream from the dam. No two natural channels could be expected to be similar in this reach, nor would the reaches upstream and downstream from the crest be similar. Fig. 67 shows the difference in percentage submergence when the tailwater elevation is measured near the dam as compared to the value when the tailwater elevation is measured at the point of maximum recovery. These values were measured upon a suppressed weir, there being no lateral expansion downstream from the crest. The transfer of energy is more efficient when the proportionate amount of the expansion is reduced. For these reasons, a consideration of model laws would reveal that it would be poor practice to include any more variables than necessary in the test arrangement and that, therefore, the tailwater elevation should be measured near the dam.

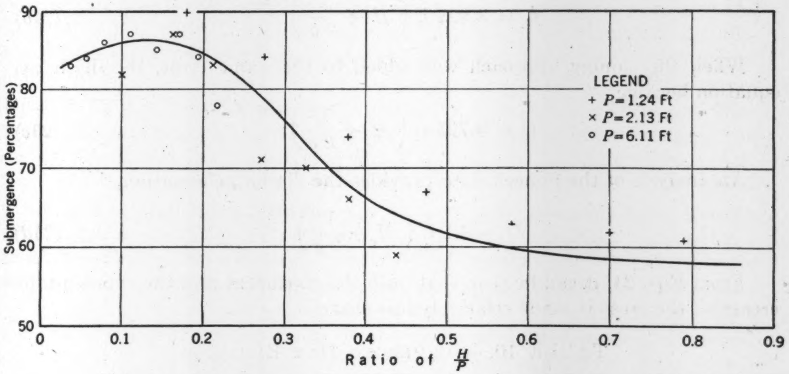


FIG. 68.—MAXIMUM SUBMERGENCE FOR PLUNGING NAPPE FLOW

Mr. Soucek referred to the maximum submergence for which the plunging nappe condition might be maintained. Fig. 68 shows a different method of presenting this subject. The values used in Fig. 68 were obtained from the Wisconsin study (45).

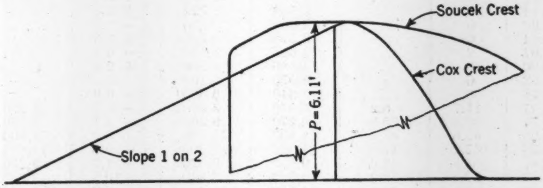


FIG. 69.—COMPARATIVE CREST SECTIONS

The discharge over an ogee, submerged weir can be represented by the formula

$$Q = C\,L\left(H + \frac{V^2}{2\,g}\right)^n \qquad (21a)$$

in which Q is the time rate of discharge, in cubic feet per second; L is the length of the crest, in feet; H is the measured head on the dam referred to the crest, in feet; V is the average upstream velocity of flow; g is the acceleration due to gravity; C is a coefficient; and n is an exponent. Both C and n depend upon the section of the crest, the type of flow, the submergence, and possibly other factors. The section of the ogee crest used for the Wisconsin tests is shown in Fig. 69, in which the section of the Soucek crest has been added for comparative purposes. The discharge equation for the Cox crest with the vertical back was

$$Q = 3.895\, L \left(H + \frac{V^2}{2\, g} \right)^{1.625} \qquad \text{(21b)}$$

When the sloping approach was added to this same crest, the discharge equation became

$$Q = 3.725\, L \left(H + \frac{V^2}{2\, g} \right)^{1.596} \qquad \text{(21c)}$$

An analysis of the Soucek data provides the discharge equation,

$$Q = 3.1\, L \left(H + \frac{V^2}{2\, g} \right)^{1.55} \qquad \text{(21d)}$$

From Eqs. 21, it can be seen that both the coefficient and the exponent decrease as the crest is made relatively less sharp.

TABLE 10.—SUBMERGED DAM BEHAVIOR

Adjusted Q (cu ft per sec) (1)	Energy H (ft) (2)	S (PERCENTAGES)		Indicated Q; model (cu ft per sec) (5)	Q, Eq. 21d (cu ft per sec) (6)	Difference, Cols. 1 and 6 (%) (7)	Q, Eqs. 21a and 23 (cu ft per sec) (8)	Difference, Cols. 5 and 8 (%) (9)
		Observed (3)	Corrected (4)					
1,920	1.71	0	....	1,760	1,920	0	....	....
1,960	1.75	0	....	1,820	1,990	+ 1.5	....	....
2,190	1.78	0	....	1,860	2,040	− 6.9	....	....
2,260	1.88	0	....	2,050	2,220	− 1.8	....	....
2,410	1.97	0	....	2,210	2,390	− 0.8	....	....
2,700	2.14	0	....	2,530	2,720	+ 0.7	....	....
2,910	2.20	0	....	2,650	2,830	− 2.8	....	....
3,460	2.54	0	....	3,360	3,540	+ 2.3	....	....
3,830	2.67	0	....	3,640	3,820	− 0.3	....	....
4,490	3.00	0	....	4,380	4,580	+ 2.4	....	....
6,160	3.63	0	....	6,020	6,160	0	....	....
6,560	3.76	0	....	6,340	6,500	− 0.9	....	....
3,440	2.41	11.7	6.2	3,070	3,260	− 5.2	3,270	+6.5
9,650	4.71	40.8	35.3	9,270	9,200	− 4.7	9,090	−1.9
12,500	5.62	43.6	38.1	12,400	12,100	− 3.2	11,900	−4.0
12,500	5.38	47.2	41.8	11,500	11,300	− 9.6	11,100	−3.5
9,680	4.81	54.8	49.6	9,500	9,520	− 1.7	9,250	−2.6
3,470	2.67	57.4	52.2	3,620	3,840	+10.7	3,690	+1.9
15,800	6.29	60.0	54.9	14,700	14,400	− 8.9	13,800	−6.1
14,650	6.49	61.2	56.2	15,500	15,100	+ 3.1	14,400	−7.1
6,580	3.72	83.0	79.0	6,030	6,400	− 2.7	5,580	−7.5
6,890	4.05	87.5	84.0	6,700	7,300	+ 6.0	6,080	−9.3
7,340	4.13	89.2	86.0	6,740	7,520	+ 2.5	6,130	−9.1

The submerged discharge equation (plunging nappe) for the Cox crest having the sloping approach was given by Eq. 21a, in which

$$C = 3.725 - \frac{0.193\, S^2}{1.15 - S} \qquad \text{(22a)}$$

$$n = 1.596 - \frac{0.035\, S^2}{2 - S} \qquad \text{(22b)}$$

and S is the submergence ratio based upon a tailwater reading taken near the crest. When $S = 0$, the values of C and n given by Eqs. 22 become the same as those given in Eq. 21*c*. A trial solution also shows only slight variation in C and n for values of S less than 0.5, which supports Mr. Soucek's conclusion that the submergence need not be considered for values of S less than 0.45.

The data contained in Table 9 have been computed to furnish Table 10. The discharges in Col. 6 have been obtained by use of Eq. 21*d*. Judging by the values in Col. 6, the Soucek crest experienced little, or no, diminution of discharge because of submergence. Also, the measured discharges for the higher submergences are in excess of the true discharges and to find a satisfying explanation for this conclusion becomes necessary. During the testing at Wisconsin (45), the writer observed that there was an upstream current near the floor of the channel on the upstream side of the weir for all flow conditions and that there was an upstream current near the floor of the channel on the downstream side for the flowing nappe type of flow. Mr. Soucek states that the gaging station was 470 ft downstream from the dam and that no gaging was taken when the ice jam was closer than 500 ft of the dam (see heading, "Observations on Prototype"). From this statement, it must be concluded that one or more gagings were made within 30 ft of the ice jam. The proximity of this barrier and the depth of the water on the upstream side of it certainly would indicate that an upstream current existed near the bed of the river at the gaging station. Such an upstream current could not have been detected by means of the Price current meter, which was used in making the gaging, and a computed discharge in excess of the true discharge would result from such an upstream current.

Should the same reducing factors for C and n be assumed to apply for the Soucek crest as did apply for the Cox crest with the approach, the following equations would result:

$$C = 3.1 - \frac{0.193\,S^2}{1.15 - S} \qquad \text{(23a)}$$

and

$$n = 1.55 - \frac{0.035\,S^2}{2 - S} \qquad \text{(23b)}$$

Eqs. 21*a* and 23 have been used to compute the discharge over the Soucek crest and the results appear in Table 10, Col. 8. These values have been compared with the discharges which were indicated by the model (Col. 5) to give Col. 9, Table 10. Considering the rather great difference in the section of the crests, the agreement between the values in Cols. 5 and 8 is remarkable, and it must be concluded that the model tests were conducted very carefully by Mr. Soucek.

GRAHAM WALTON,[20] ASSOC. M. AM. SOC. C. E.—The information on submerged weir discharge in Mr. Soucek's paper is of particular interest. Weirs are frequently installed in previously constructed channels to measure sewage or industrial wastes. Restrictions in available head loss may require submerged flow conditions, at least during periods of peak flow.

---

[20] Instr., Civ. Eng., Univ. of Wisconsin, Madison, Wis.

In 1936 a weir of trapezoidal cross section (Fig. 70), with a surface of smooth cement-sand mortar, was calibrated at the hydraulic laboratory of the University of Wisconsin, at Madison, for the express purpose of measuring waste discharge at a paper mill. Special attention was given to the investigation of the effect of submergence on the discharge. The heads were measured, 3 and 5 ft, respectively, upstream and downstream from the upstream edge of the weir crest.

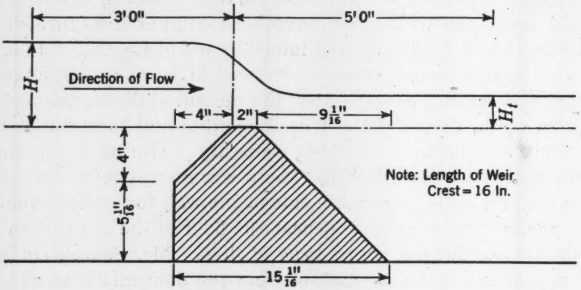


FIG. 70.—WEIR CROSS SECTION

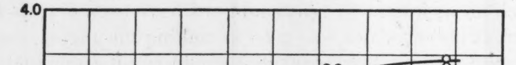


FIG. 71.—COEFFICIENT *C* FOR WEIR WITH FREE DISCHARGE

Test results are summarized in Fig. 71, which gives the coefficient of discharge *C*, as determined for

$$Q = C\,L\left(H + 1.2\,\frac{V^2}{2\,g}\right)^{\frac{3}{2}} \quad \text{(24)}$$

Fig. 72 shows the ratio of submerged to free discharge for various submergences. The latter is plotted as a single-valued function and is surprising in its agreement with U. S. Deep Waterways tests (*42a*). There is some indication, however, that, had it been possible to increase the distance at which the head

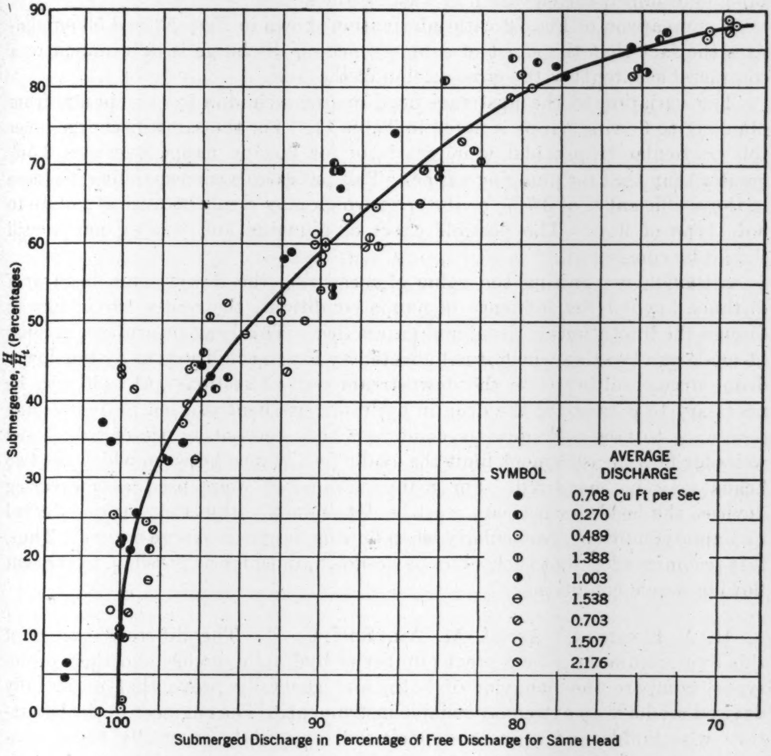


FIG. 72

TABLE 11.—VARIATION OF HEAD ON A WEIR WITH THE TRANSITION FROM A PLUNGING TO A FLOWING NAPPE

Description	DISCHARGE, IN CUBIC FEET PER SECOND:								
	0.268	0.490	0.704	0.707	0.999	1.389	1.509	1.536	2.16
Discharge for flowing nappe conditions (cu ft per sec)			0.702	....	....	....	1.502	1.524	2.14
Head on Weir (Ft):									
Plunging nappe	0.153	0.220	0.266	0.264	0.329	0.413	0.425	0.443	0.538
Flowing nappe	0.156	0.228	0.273	0.274	0.346	0.424	0.432	0.442	0.546
Weir Submergence (%) for:									
Plunging nappe	21.2	44.3	46.1	36.0	32.4	48.4	52.0	48.2	59.6
Flowing nappe	42.2	60.7	48.0	43.8	54.2	60.1	59.6	52.7	63.8
Transition (±)	36	39?	42?	....	47	51	53	....	60

downstream was measured, a family of curves might have been obtained. Careful examination of the original data shows abnormal change in downstream head with transition from plunging to flowing nappe. This should not be the case with constant discharge if the control section downstream from the weir alone determined the elevation of water surface.

A comparison of Fig. 72 with information shown in Figs. 52 and 53 emphasizes the fact that the effect of submergence on discharge is determined to a considerable extent by the cross section of the weir.

The variation of the upstream head on the weir due to the change from plunging to flowing nappe is noted in Table 11. For the same discharge, over this particular trapezoidal weir, the head for flowing nappe averages 2.5% greater than that for plunging nappe. This indicates a corresponding decrease in the coefficient *C* of 3.7%, or the error which may result because of failure to note type of flow. The possible effect of plunging and flowing nappe still should be considered.

Although recognizing the value of measuring the downstream head at a distance beyond the influence of nappe conditions, the writer would like to discuss the limitations encountered in practice. This head measurement must be made at a location which is not affected by nappe conditions or by the downdrop curve resulting from the downstream control section. Also, it may be necessary to correct for the drop in hydraulic gradient utilized in overcoming resistance to flow. Frequently conditions that necessitate use of submerged weirs for flow measurement limit the available distance between which the two heads may be measured. For many submerged weirs used as measuring devices, the head downstream must be determined within that region affected by nappe conditions, particularly when flowing nappe discharge occurs. Thus, it is recommended that such weirs be designed, in so far as possible, to prevent flowing nappe conditions.

H. A. Einstein,[21] Assoc. M. Am. Soc. C. E.—The different papers of this Symposium describe a great number of hydraulic models and their prototypes, compare the behavior of both, and generally prove that a carefully devised model is an extremely reliable instrument. The engineer in the laboratory who builds and operates models will appreciate especially these comparisons because, more than anybody else, he knows the possible reasons why similitude may be disturbed.

In several of the papers, friction has been given as the cause for deviation between model and prototype. It is known that friction along rough walls will conform, in different scales, according to Froude's law if the roughness is reduced in the model according to the scale of all other lengths and if this reduced roughness combined with the reduced Reynold's number of the flow still represents a rough wall. If this is not the case, disagreement is inevitable. This case occurs most frequently in connection with hydraulic structures, which are generally rather smooth in the prototype to reduce friction. Usually, this friction is unimportant or may be corrected by calculation. There are cases, however, in which the true reproduction of this friction is essential.

---

[21] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

In these cases a method of distortion may be used which proves very effective for river models, many of which must be distorted to keep the flow turbulent. While visiting the hydraulic laboratory in Vicksburg, Miss., where such models usually can be found, the writer was astonished to see how much the natural roughness of the model must be increased to counteract the effect of the distortion on the friction. It is clear, therefore, that a comparatively slight distortion of any hydraulic model will allow an effective increase of the wall roughness which may just make the difference between rough and smooth.

This raises the question of the reliability of distorted hydraulic models in general. It was somewhat disappointing that no quantitative comparison of any such model with its prototype could be offered in this Symposium. Elsewhere (17), the writer, with R. Müller, has described the conditions under which a distorted model with movable bed will conform with the prototype, giving the results of the application of the method to a river model.

The model in this case was that of the Rhine River above the Lake of Constance (62). Within the first 20-mile stretch above the lake the stream loses the greater part of its bed load by sedimentation in its own bed. This gradual decrease of the rate of bed-load transport down this stretch is parallel with a gradual decrease of the slope. The alarming speed with which the bed was filling up forced the responsible authorities to study all possible ways of relieving the dangerous situation. The model in question was intended to represent a section of the river in the upper part of the 20 miles and to reveal constructive measures which would increase the river's capacity for bed load without increased slope. The rules for transportation in the model had to be found empirically because no theory of any kind existed at the time. Therefore, a series of trials was made to duplicate, in the model, the changes of the river bed that had taken place during past years using Froude's law for the flow. When agreement was satisfactory, the main experiments were started. However, only the preliminary experiments can be used in comparing the model with the prototype, because the corresponding measurements in the prototype applied to them only.

The measurements in the river, in addition to data not used in this connection, include (*a*) several sets of cross sections, (*b*) the hydrographs for the time intervals between these surveys, and (*c*) velocity and slope measurements at different stages. Mechanical analyses of the bed material were available for the entire 20-mile stretch. A set of bed-load measurements using a calibrated trap gave the rates of bed-load movement at one section for different stages and checked very well with calculations using the bed-load formula, commonly referred to as the Swiss formula. This same formula was used to calculate the transportation at other sections of the stretch, and the difference of transport during certain periods of time was checked against the deposition between the sections and in the lake. The measurements within this rather intricate system agreed astonishingly well, thus introducing the rates of transportation into the list of variables measured in the prototype.

The model was built entirely empirically. Originally it was supposed to be geometrically undistorted except for the grain size in the movable bed which was composed of coal. The low specific gravity of the grains should counteract the effect of the large grain sizes as far as transportation is concerned. However, it was known at the time from similar models that the time ratio chosen for transportation must be entirely different from the time ratio of the flow proper. By trial and error a ratio of 360 gave, very closely, the same configuration of the model bed as was found in the prototype and was used thereafter throughout the entire study.

All the evidence seemed to indicate that the model was similar to the prototype, but the quantitative rules of this similarity were not discovered until more than a year later, when Froude's law was abandoned as a basis of comparison and was replaced by a new system of equations describing similarity in a distorted model.

Instead of a general ratio for all different variables of the dimension of length, four different ratios were used:

(1) λ for lengths in direction of the main flow;
(2) ϕ for the width, a horizontal length perpendicular to the main flow;
(3) χ for the height, any vertical length; and
(4) δ for the grain diameters.

An additional distortion was allowed by the separate introduction of

(5) ξ for the ratio of slopes, independent from λ and χ.

Each common Greek letter represents the ratio of a certain measure in nature over the corresponding measure in the model. Also, for time, a distortion is allowed by the separate introduction of:

(6) τ for any time concerning the movement of water; and
(7) θ for any time concerning the movement of sediment.

A number of constants enter any hydraulic problem involving bed-load movement. The acceleration of gravity and the density of the water are assumed to be equal in model and prototype:

(8) η may be the ratio of the effective specific gravities of the sediments under water. Their specific pore volumes in the bed are assumed to be equal;

(9) ζ is the ratio of the constants in the Manning-Strickler formula as explained subsequently; and

(10) σ is the ratio of the constants F introduced in the bed-load formula (63).

For convenience, three more ratios of composite variables are introduced and defined by the three equations:

(11) γ as the ratio of the specific bed-load transport defined by

$$\gamma \theta = \eta \lambda \chi \ldots\ldots (25a)$$

(12) μ as the ratio of corresponding masses of water defined by

$$\mu = \lambda \phi \chi \ldots\ldots (25b)$$

and

(13) ψ as the ratio of corresponding discharges, defined by

$$\psi = \mu \, \tau^{-1} \qquad (25c)$$

Four more equations can be introduced describing similarity between model and prototype. For the distorted model the general Froude's law reduces to

$$\chi = \lambda^2 \, \tau^{-2} \qquad (25d)$$

expressing the fact that any heights are correlated like values of $\frac{v^2}{2g}$. The next equation expresses the similarity of corresponding bed friction. Friction in a stream with bed-load movement is usually of the rough type, especially if the bed material is as coarse as that in the Rhine River. Therefore, friction may be described by Manning's formula. The same formula must be used in both model and prototype; otherwise the principal characteristic of similitude is lost, because corresponding values of the same type must be correlated by a constant factor. The constant n in Manning's formula can be expressed for a granular bed, according to A. Strickler (64) as the sixth root of a representative grain diameter of the bed divided by another constant. This new constant will vary only slightly for extremely low Reynolds numbers and with the intensity Φ of bed-load transportation. In ratios the Manning-Strickler formula, therefore, reads

$$\lambda \, \tau^{-1} = \zeta \, \delta^{-1/6} \, \xi^{1/2} \, \chi^{2/3} \qquad (25e)$$

The two remaining equations are derived from the postulated identity of the transportation formula. Using the $(\Psi$–$\Phi)$-relationship, the values of both Ψ and Φ must be the same for corresponding points in model and prototype as long as the same $(\Psi$–$\Phi)$-curve holds in both scales. If the Reynolds number of the flow in the model is extremely low (as was the case for the Rhine River model), a slightly different $(\Psi$–$\Phi)$-curve applies. This difference of curve is taken care of by the introduction of ratios (14–15) α and β for corresponding Φ and Ψ values, respectively:

$$\alpha = \gamma \, \sigma^{-1} \, \eta^{-3/2} \, \delta^{-3/2} \qquad (25f)$$

and

$$\beta = \eta \, \delta \, \xi^{-1} \, \chi^{-1} \qquad (25g)$$

Flume experiments with the model sediment under flow conditions similar to those in the model resulted in a $(\Psi$–$\Phi)$-curve that is lower and flatter than normal, suggesting that both α and β are smaller than unity.

Even when this method of comparison was used, the similitude did not seem satisfactory. Only when the model was interpreted as duplication of the flow at another section of the river farther downstream in the 20-mile stretch was satisfactory similitude achieved.

Table 12 shows a comparison of the values for the fifteen ratios as derived from direct measurements in model and prototype (Col. 6) with those computed according to Froude's law (Col. 3), with those using Eqs. 25 for a distorted model introducing α and β equal to 1 (Col. 4) and for a distorted model choosing α and β according to special flume studies (Col. 5). Only normal flows were studied in the model. The measured length ratio λ as well as the χ values, therefore, were obtained from the comparison of the characteristic average form of the sediment bars in model and prototype. The model appears to be almost undistorted geometrically except for being tilted downstream, as shown in Table 12.

Parameters ψ, η, and ϕ have been chosen; θ was determined by trial and error; and α, β, ζ, and σ are measured independent of the model. A comparison of the values in Cols. 5 and 6 shows remarkably close agreement of all the seven remaining calculated ratios with the measurement. Col. 4, Table 12, may illustrate how easily the entire system can be disrupted by the wrong assumption of any constants in the equations involved. These data may prove that a quantitatively similar river model with lightweight material in the bed is possible and that its behavior can be predicted. A certain distortion, especially of the time scale, is unavoidable. Because of these distortions Froude's law cannot be applied. Instead, a system of empirical formulas with their constants describing bed friction and bed movement determines the different ratios. The same equations must be applicable to both model and prototype if similarity is to prevail. All constants in these equations must be known for both scales if the different ratios are to be predicted.

TABLE 12.—COMPUTED AND MEASURED RATIOS FOR THE RHINE RIVER MODEL

Ratio	Symbol	Calculated			Measured
		Froude	Distorted		
(1)	(2)	(3)	(4)	(5)	(6)
Φ-values	α	1	1	0.595	0.5–0.7
Ψ-values	β	1	1	0.838	0.8–1.0
Width	ϕ	100	100	100	100
Time (sediment)	θ	10	360	360	360
Strickler-constant	ζ	1	0.80	0.80	0.80
Density (sediment)	η	1	7.7	7.7	7.7
Settling-constant	σ	1	1	1	1
Discharges	ψ	10^5	10^5	10^5	10^5
Length	$\lambda=\alpha\,\beta^{9/4}\,\phi^{-1/3}\,\theta\,\zeta^{-9/2}\,\eta^{-7/4}\,\sigma\,\psi^{1/3}$	100	276	110	100–130
Height	$\chi=\phi^{-2/3}\,\psi^{2/3}$	100	100	100	95–104
Slope	$\xi=\beta^{1/2}\,\zeta^{-3}\,\eta^{-1/2}$	1	0.704	0.645	0.6–0.65
Time (hydraulic)	$\tau=\alpha\,\beta^{9/4}\,\theta\,\zeta^{-9/2}\,\eta^{-7/4}\,\sigma$	10	27.6	11.0	9.3–13.7
Specific transportation	$\gamma=\alpha\,\beta^{9/4}\,\phi^{-1}\,\zeta^{-9/2}\,\eta^{-3/4}\,\sigma\,\psi$	10^3	591	236	250
Hydraulic masses	$\mu=\alpha\,\beta^{9/4}\,\theta\,\zeta^{-9/2}\,\eta^{-7/4}\,\sigma\,\psi$	10^6	2.76×10^6	1.1×10^6	$(0.95–1.35)\times10^6$
Grain diameter	$\delta=\beta^{3/2}\,\phi^{-2/3}\,\zeta^{-3}\,\eta^{-3/2}\,\psi^{2/3}$	100	9.14	7.0	6.8–7.2

Unfortunately, knowledge of the fundamental laws of bed movement in large streams is still unsatisfactory. Therefore, it is not possible to build quantitative models for large streams with fine sediment. In this case, only qualitative models producing qualitative results can be built; but even those qualitative results cannot be trusted unless it is possible to find in each model a number of qualitative checks such as those described by Mr. Brown in his excellent paper about "Development of Mississippi River Channel." The care with which statements in general are made in Mr. Brown's paper illustrates the unreliability of such models and the great danger of drawing erroneous conclusions from them. It is hoped that as a result of basic research investigators soon will find the fundamental laws needed for quantitative model studies in this field.

K. G. TOWER,[22] JUN. AM. SOC. C. E.—The "Foreword" to the admirable series of papers appearing in this Symposium could advantageously have been more lengthy and explicit. In explanation of the objective of the research the rather bald question is stated: "Does the prototype act as the model predicted?" This question will cause the uninitiated to query: How, then, if at this late date such a question must be asked, has the art and science of hydraulic model testing reached its present prominence? For all practical purposes, of course, with the exception of the relatively infrequent case where air entrainment or the pressure of the atmosphere (or a combination of both) may be critical factors, models operated at atmospheric pressures will give a thoroughly reliable solution to problems which, in many cases, if there is an alternative, are susceptible to but questionable mathematical analyses.

With perhaps a few exceptions, those engineers who have had recourse to the hydraulic laboratory for the solution of a specific problem have been "sold" on the idea of model testing. They have seen the solution obtained in the laboratory verified by the prototype. So far as has been observed and checked by rough field measurements, qualitatively and quantitatively, for all practical purposes the prototype has performed as predicted by the model. The irrigation-canal check-drop investigation presented by Messrs. Warnock and Dewey is typical of the type of problem that has been solved repeatedly in the past to the entire satisfaction of all concerned, and upon which successful solutions the present excellent reputation and reliance in hydraulic model testing have been founded.

Thus, qualitatively at least, with the exceptions noted, the question of the "Foreword" need not be asked of hydraulic laboratory personnel. The question should be reworded to read: "To what degree do quantitative measurements recorded in the laboratory differ from those obtained in the field?" Viewed in the light of the restated objective of this research, the Symposium is seen to represent the first organized public expression of the manner in which hydraulic designers and laboratory technicians are striving toward the achievement of perfection. Those who have participated in developing the technique and interpretation of hydraulic model studies from an awkward adolescence to a confident maturity within the past decade can consider themselves as having engaged in a job well done.

The attainment of perfection in any activity is highly desirable, and is always praiseworthy, even in those instances where the activity itself may be of doubtful merit. It is hoped, however, that before those engaged in the direction of hydraulic laboratories definitely conclude to spend all of their available spare time and funds in an effort to perfect the interpretation of results obtained from model studies that thought first be given to furnishing the hydraulic designer with reliable basic data which will permit final and economic solutions of his everyday problems. In metaphor, before worrying about the twine in the binder, first learn how to plow.

---

[22] Associate Engr., U. S. Engr. Dept., Portland, Ore.

An important task of the future is the elimination of the need for a number of the hydraulic tests of models accepted by many agencies at present as standard practice. Typical structures frequently constructed should be analyzed in the hydraulic laboratory in such a manner that the data obtained therein, when correlated with other modifying or confirming data from prototype measurements, would constitute a reliable guide to the hydraulic designer and eliminate the necessity for the continual repetition year after year of the construction and testing of models of structures which are nearly identical.

Spillway crests furnish possibly the best example of the case in point. It is a recognized fact that the highest discharge coefficients are obtained with crests properly designed and shaped to fit the lower nappe curve of a discharge sheet over a sharp-crested weir. Hydraulic laboratories should endeavor, at the earliest opportunity, to supplement information already available on the curve of the sharp-crested weir lower nappe and the entire matter should be placed on record in such a manner that no hydraulic designer in the future, by reason of the lack of adequate information, need be found recommending a spillway shaped, for example, similar to the one examined by Mr. Soucek and illustrated in Fig. 50 in his study of submerged flow, "Meter Measurements of Dam Discharge." Stilling basins and transitions at the entrances to conduits are additional structural features deserving critical and intensive examination for the purpose of design standardization and the elimination of routine hydraulic model testing. The use of the three structural features mentioned requires the greater part of the work involved in the hydraulic design of dams and control works, and more comprehensive information on them is needed daily at the drafting boards of organizations engaged in this work.

The suggested research program will have to be conducted principally in laboratories as there are insufficient suitable existing full-scale structures, even if all were investigated, to contribute an appreciable quantity of the necessary basic data. Those existing structures that are suitable, however, well may be examined for data to be used as a check of the models at one or more stages of the variations in them which will be required.

The fact that at this late date the meager store of basic hydraulic information makes the proposal contained herein necessary is sufficient evidence that considerable fortitude, initiative, and skill will be demanded of those participating in the program. The magnitude of the work required is such that the program will have to be deferred until after the war. In the meantime, the practicing hydraulic engineer will welcome any prototype study that contributes to his working knowledge regardless of whether or not the intent of such study is merely to confirm or deny data obtained from a particular hydraulic model.

R. J. Pafford, Jr.,[23] Esq.—Individually the papers are an interesting and enlightening addition to the literature on hydraulic model practice and hydraulic design, for which the various authors are to be commended. Collected together in this Symposium, they represent important contributions toward more effective and more comprehensive use of the hydraulic model as an instrument of design, through the illustration of the wide field of problems susceptible to model analyses, and toward better general understanding of how this tool can be improved further by the illustrations of both its capabilities and its present limitations.

---

[23] Engr. (Hydr.), U. S. Engr. Office, Omaha, Nebr.

Considering the subject matter of the Symposium as a whole, the paper by Messrs. Warnock and Dewey is particularly thought provoking. There is urgent need for much more prototype data, both for evaluation of accuracy of the model and for direct use in design. This calls for the creation of greater interest on the part of all concerned, and for still greater cooperation in overcoming the difficulties of making prototype measurements. Past lack of general interest undoubtedly has been contributed to by both the model worker and the designer of hydraulic structures. Too great enthusiasm of the model worker arising from his intense interest in his tool in many cases has tended to "oversell" the capabilities of the model; and also, in many cases, there has been a tendency of the designer to present his finished product as if it were the best and final answer to a given set of conditions rather than just a good design which will perform satisfactorily, but which undoubtedly could be improved either in performance characteristics, economy, or both, if better information were at his disposal. A more realistic attitude on the part of both can contribute materially to greater general interest and, in turn, to greater opportunity for the collection of needed prototype data, even where the expenses of observations may be high.

The numerous difficulties involved in obtaining accurate prototype data make this indeed an exasperating and expensive field, yet fascinating, in the writer's experience at least, because of these very difficulties. The utmost in ingenuity and the fullest possible interchange in ideas are essential. With reference to difficulties with equipment such as that used in measuring spillway bucket velocities and water-surface elevations over spillways as mentioned by Messrs. Warnock and Dewey, the following suggestions are offered for general consideration in future cases of this nature. Tubes leading to observation galleries, in many cases, can be kept open by regular flushing with clear water at frequent intervals from the time of installation until all desired performance data are collected. This often will be a nuisance to operating personnel, and of course in some instances will be impracticable; but then, why install the tubes if they are to become unserviceable before data can be collected? There also seems a possibility that small tubes placed without sharp bends, with a flexible corrosion-resisting wire to be removed only during tests extending their full length (same idea as is used in an automobile hand-throttle control except that the wire would be pulled entirely from the tube during tests) might solve this clogging problem at some locations. The problem of coating or plugging of pitot-tube openings is not necessarily beyond solution either. It would seem that some kind of protective cover which might be removed either directly or by remote control just prior to a test could be devised for many installations. Turning to the problem of measurements of water surface of the flow over spillway sections of a dam, it appears that for at least some cases not complicated by spray from gates at partial opening, some relatively inexpensive method employing a narrow intense light beam and transit observations of the "spot" projected on the water surface (possibly coupled in some cases with measurement of reflection angles) could be utilized. Developments along the lines of sonic or electronic soundings also would seem to hold considerable promise for the future, perhaps especially in cases complicated by spray and foam on top of the "solid" jet.

As mentioned in the Warnock-Dewey paper, still and motion pictures of all flow conditions observed both in laboratory and field are indeed invaluable for purposes of comparison of model and prototype performance. On this score, the writer desires to emphasize the desirability of complete pictorial records of model performance from viewpoints that can be duplicated in the field as well as from vantage points possible in the laboratory which may better illustrate performances of certain features of the model but which are impractical in the field. It has been the writer's unfortunate experience, which is feared not unique, to fail on several occasions in attempts to compile fully satisfactory comparative pictorial records because the only laboratory pictures were taken through the sides of glass-walled flumes or from points which would require that the prototype photographer be suspended in mid-air over the stream below a dam. On the other hand, really excellent photographic comparisons are possible, as illustrated by Fig. 1, when model photographs from proper angles are available. For most effective photographic comparison of model and prototype performance, attention to relative focal lengths of camera lenses used in the laboratory and in the field, to insure similar perspectives of the photographs, is almost as important as the position of the camera.

In 1942 the writer had a convincing demonstration of the qualitative accuracy of the hydraulic model in a field not covered by the papers comprising the Symposium. In connection with design of a spillway stilling basin utilizing baffle piers for energy dissipation, there was some concern for a possible cavitation hazard, and extensive model investigations were made under the writer's general supervision in two leading hydraulic laboratories. The test method in one involved the measurement and analysis of pressures that occurred on baffle surfaces in a model under atmospheric pressure; the other involved visual inspection and interpretation of the performance of a model erected in a vacuum tank apparatus which enabled the reproduction of pressures in the correct scale ratio. To enable the best possible interpretation of the tests for the structure being designed, both laboratories first conducted tests on models of an existing stilling basin also dependent on baffle piers for energy dissipation where some pitting in certain areas along the sides of the baffle piers had actually occurred during the passage of an extreme flood. So far as the main test program was concerned, it is sufficient to state that the results of independently conducted tests at each laboratory were in reasonable agreement with conditions observed at the existing prototype and with conditions indicated at the other laboratory, and that a design involving partly streamlined baffle piers was evolved. The original descriptions and photographs of conditions at the existing structure had indicated pitting only along certain areas on the sides of the baffle piers. During testing of the model of this structure in the vacuum tank, it was seen that conditions along the stilling basin floor in certain areas between the baffle piers were apparently nearly as severe as along the sides of the baffle piers where pitting had occurred and there were also indications of incipience of cavitation at the extreme upstream edge of the tops of the baffles. Incidentally, indications of areas on the sides of the baffles in which cavitation might be suspected in the model were in excellent general agreement with pitted areas in the prototype. Some time after observing the foregoing phenomena at the laboratory, the writer visited the existing prototype for the specific purpose of further investigation of the extent of pitting in the stilling basin area. Visual inspection revealed a minor degree of pitting at the leading edge of the tops of the baffle piers in the areas indicated by the model. The stilling basin floor was covered by a thin layer of mud and 4 or 5 ft of murky water, making visual inspection there impracticable at the time. Careful probing, however, definitely established that there was considerable pitting of the apron floor in the very areas indicated by the model. As nearly as could be determined by "feel" through the sticks used for probing, the degree and depth of pitting were very similar to the pitting on the sides of the baffle piers where the depth ranged up to 2 or 3 in. below the original concrete surface.

EDWARD H. SCHULZ,[24] M. AM. SOC. C. E.—The physical comparisons of models in this Symposium cover a wide range of structures, including irrigation works, discharge outlets, dry dock suction chambers, lock construction, dam outlets, discharge and pressure heads, spillway coefficients, meter measurements, river improvement, and lock approaches.

There are discussions of qualitative and quantitative comparisons, the latter of which were found to be the more valuable and important. The ratio of prototype to model has a wide range from 1 : 5 to 1 : 600. Hydraulic models should be on as large a scale as reasonable. Perhaps there has been too little interest in comparing performance of the prototype with the model, partly due to the difficulty of making measurements. The accuracy of measurements is important in the prototype as well as in the model. Some of the more difficult determinations are slope, velocity, discharge, pressure, friction, etc., and these are more difficult in closed circuits than in open ones, and more important for the prototype than for model studies. Studies as to erosion, bar movement, etc., may be necessary.

Geometric similarity is very important. The comparisons should cover the range of hydraulic conditions. The cost of prototype field measurements may be a deterrent.

Conclusions in the Symposium, based on the tests of various structures, show that performances generally have been in line with the model tests. Such tests are an excellent help to the engineer. Model conditions, in general, were substantiated by the prototypes, especially where geometric similarity existed. Discrepancies were within 5%, and probably less than would have occurred without such model tests. There were some anomalous results, however, such as pressure head comparisons. The meter measurements of dam discharge of the model were about 5% less than those of the prototype. The general agreement was satisfactory. On the river improvement the prototype reproduced the results indicated by the model. Some diversion was to be expected. On the lock approaches there was satisfactory verification.

---

[24] Col., Corps of Engrs., U. S. Army (*Retired*), Berkeley, Calif.

In conclusion, the writer considers model tests very useful in the study and correction of hydraulic relations and conditions. In building any structure where such studies and comparisons are likely to be needed and where the expense is justified, provision should be made to incorporate appropriate accessories and equipment in the prototype during construction. Measurements can then be made more readily and tests can be conducted at any time within the limits of the structure and the range of the hydraulic conditions to be expected.

F. T. Mavis,[25] M. Am. Soc. C. E.—In a singularly complete manner Mr. Soucek has presented the results of his laboratory studies and their comparisons with observations of a prototype. The systematic relationship between relative discharge (submerged to free) and submergence shown in Figs. 52 and 53 reflects the precision of the laboratory measurements. It is clear from Fig. 53, as Mr. Soucek states in his first conclusion, that generalizations of the effect of spillway profile and absolute discharge should be accepted with caution as a basis for estimating precisely the relationship between headwater head, submergence ratio, and discharge over a spillway.

Profile of Model *B*, in Inches

x	*y* Below Crest	
0"	11.25" and 1.45"	
2	0.25	
4	0.00	Crest
6	0.30	
8	0.75	
10	1.85	
12	3.00	
14	4.30	
16	5.75	
18	7.55	
20	9.95	
21	11.25	Toe

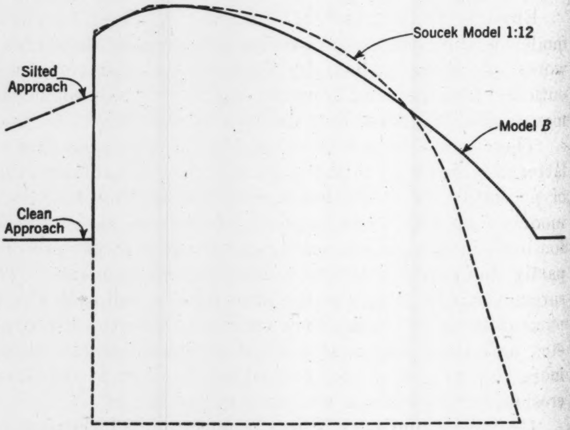


Fig. 73.—Comparison of Models

The verification by laboratory test that the relation between submergence and discharge was unaffected by the condition of the nappe, whether plunging or flowing, should dispel misgivings for one shape of spillway at least. An interesting problem of hydromechanics may lie behind the data plotted in Fig. 54.

---

[25] Head and Prof., Civ. Eng. Dept., and Acting Head, Dept. of Eng. Mechanics, The Pennsylvania State College, State College, Pa.

In 1934 the writer conducted a brief series of tests at the University of Iowa, Iowa City, Iowa, on a model 11.25 in. high, designated Model B, in Fig. 73. The model profile discussed by Mr. Soucek is shown for comparison.

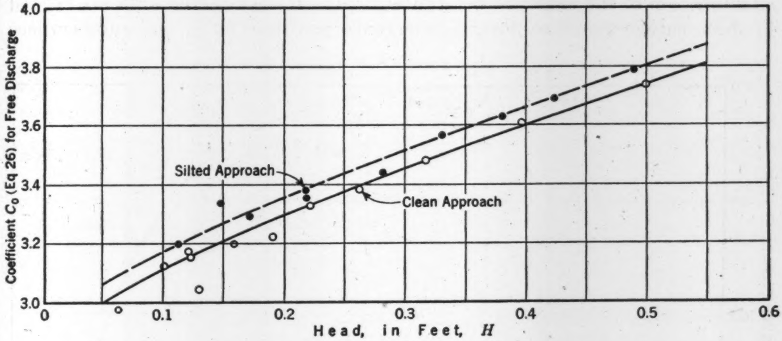


FIG. 74.—TESTS OF MODEL B; FREE FLOW

Model B was tested in a glass-walled flume 12 in. wide under two conditions—"clean approach" with a level approach channel; and "silted approach" with a sand fill 1 on 2.5 intersecting the upstream face of the model 4 in. below the crest.

Fig. 74 shows the relation between coefficient for free discharge

$$C_0 = \frac{Q_0}{L\,H^{1.5}} \qquad (26)$$

and head for both "clean approach" and "silted approach." Analysis of data on this basis is simpler than analysis based on Eq. 13. If a model is truly representative of its prototype, either analysis leads precisely to the same conclusion. Used discretely either analysis leads to a valid engineering conclusion.

Fig. 75 shows the relationship between submergence percentage, p, and the ratio of submerged to free discharge as expressed by the formula

$$\frac{C}{C_0} = 0.01\,[100 - 2^{0.2(p-70)}] \qquad (27)$$

Opposite each plotted point is a figure representing the corresponding discharge, q, in cubic feet per second per linear foot of crest. The absolute rate of discharge, q, may be recognized in Fig. 75 as an independent variable. Although the data plotted in Fig. 75 might be presented in support of Mr. Soucek's discussion of Fig. 53, one may ask whether the ultimate uses of the data warrant the refinement of analysis in this instance. To explore this point the generalized analysis of data from tests of Model B were compared with data reported by Mr. Soucek in Cols. 2 and 3 of Table 9.

Fig. 76 shows the coefficient of discharge for the prototype:

$$C = \frac{Q}{L\,H^{1.5}} \qquad (28)$$

as a function of the observed headwater depth, H. Observed data are classed in three categories—free flow, submergence less than 60%, and submergence greater than 60%. Fig. 76 shows also the curves based on tests of Model B for clean approach and silted approach. The conversion from model to prototype was based on Froude's law. Note that the scatter of points in the first category (free flow) is of the order of 5% of the coefficient of discharge. According to Fig. 53, the submergence would have to exceed 80% before the actual discharge would be reduced correspondingly below that calculated for free discharge. Hence, as a tentative premise one may assume that only the last three field observations in the third category (Table 9) are significantly below a valid general curve relating coefficient of discharge and head on prototype as shown in Fig. 76. Tentatively, then, the relationship between head and free discharge for Model B and the prototype seems verified by Fig. 76.

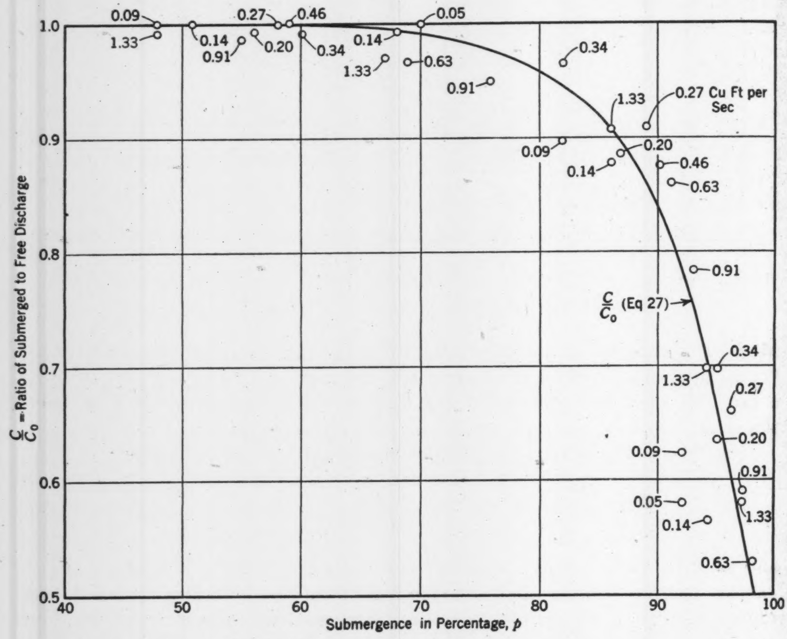


FIG. 75.—EFFECTS OF SUBMERGENCE; MODEL B

As stated, Eq. 27 in Fig. 75 shows an average curve relating the ratio of submerged to free discharge $\frac{C}{C_0}$, and percentage submergence, p. Although this equation appears complex, it means that the ratio of submerged to free discharge, $\frac{C}{C_0}$, is reduced from unity by 1.0% if the submergence is 70%;

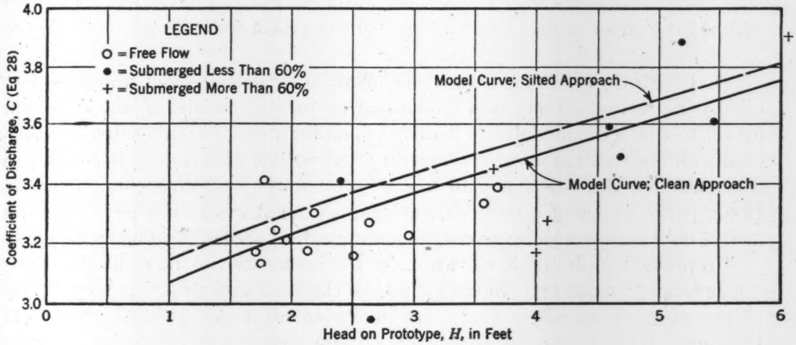


FIG. 76.—OBSERVATIONS OF PROTOTYPE

this percentage reduction is successively doubled for each additional 5% of submergence. For example,

For a submergence ratio (%) of:	the reduction of $\frac{C}{C_0}$ is	and the ratio $\frac{C}{C_0}$ is
70	0.01	0.99
75	0.02	0.98
80	0.04	0.96
85	0.08	0.92
90	0.16	0.84
95	0.32	0.68

As a final step, the writer compared observed data on the prototype reported in Table 9 with corresponding data computed on the basis of tests of Model B for both free and silted approaches. Adjusted discharge (Col. 2, Table 9), headwater depth (Col. 3), and tailwater depth (Col. 6), were used as the basis for a spillway 273.50 ft long. The maximum difference between field observations and corresponding calculations based on tests of Model B with the generalized submergence correction (Fig. 75) was 11% for the model with clean approach and 13% for the model with silted approach. These values correspond to a maximum difference of 15% in Table 9. The mean absolute percentage difference between Model B and the prototype was 3.8% for the clean approach and 4.2% for the silted approach. These data correspond to a mean absolute percentage difference of 6.0% in Table 9.

The conclusions seem justified, therefore, that—

(*a*) The differences in model spillway profiles shown in Fig. 73 are not significant in reflecting the behavior of the prototype under conditions of free and submerged discharge; and

(*b*) The generalized relationship between submergence percentage, p, and the ratio of submerged to free discharge, $\frac{C}{C_0}$, as expressed by Eq. 27, is valid within the scatter of field observations on the prototype.

D. C. McConaughy,[26] Esq.—Mr. Warnock mentions the importance of comparing the performance of model and prototype in order to determine the extent to which the model may be relied upon for quantitative results, and comments on the small amount of progress made in this direction. As bearing on this question, the writer should like to present the following comparison between performance of a river channel improvement model having a discharge of 1.75 cu ft per sec and its prototype having a discharge of 276,000 cu ft per sec.

To prevent undesirable elevations of the backwater in the Columbia River above Grand Coulee Dam in the vicinity of the International Boundary, studies of the possibilities of river channel improvement were undertaken in 1940. Of course, these required a knowledge of backwater conditions over the entire distance of nearly 150 miles, which was obtained by computation. For improvement work, attention was concentrated on a large rapids known as "The Little Dalles," about 16 miles below the International Boundary line. Here, for a distance of about 2,500 ft, the normal channel width of about 1,000 ft is constricted to a width of about 300 ft, with a depth of about 200 ft. The flow at normal flood discharges was characterized by huge boils and eddies, the loss in head through the reach varying from a few feet for low discharge to an estimated 32 ft for the probable maximum discharge of about 650,000 cu ft per sec.

The right side of the channel at "The Little Dalles" rises abruptly; the left side levels out to a broad shelf. A large rock island, submerged at high discharges, obstructed the entrance. Near the lower end a large pinnacle of rock rose above the water surface, and at the right side a spur of rock projected into the channel, apparently causing considerable loss of head. On account of the topography and inaccessibility of the right bank, the aim was to confine excavation to the left side and to keep the bottom of the excavation above low water. The resulting enlarged channel thus would have a comparatively shallow bench, of varying width, at one side of the natural deep narrow channel. It was desired to know the effect of removal of the island to varying elevations, the effect of removal of the pinnacle and spur previously mentioned, and the effect of these on the amount of excavation required on the left bank, as well as the effect of different amounts of excavation on the losses. Some of the excavation had to be wasted in the natural river channel, if possible, to save the expense of handling. As the bottom was irregular, it was hoped this might be done without appreciable effect.

---

[26] Senior Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

The stretch of river immediately below the rapids acted as a stilling basin for the high velocities through the narrow section, and the value of the roughness coefficient to be used under altered conditions was uncertain. Also it was desired to have a check on the validity of certain assumptions as to the relation between the roughness coefficient for the channel under natural and under backwater conditions, used in computations.

A model offered the best, if not the only, means of obtaining the information desired. The channel was stable, and water-surface elevations above and below the reach were available up to a discharge of 420,000 cu ft per sec, so that the gage-discharge curves could be accepted with confidence, at least up to that discharge. Since data for discharges up to 650,000 cu ft per sec were required, it was necessary to extend the curves to that figure. The upstream curve had certain peculiarities, probably due to the effect of the island, and extensions failed to agree by several feet with an apparently reliable high-water mark. Computed water surfaces, starting from the downstream station, checked one method of extension, and the model later checked this extension to within about a foot. This elevation was very important, on account of its nearness to the boundary. There was also a gage-discharge record up to 361,000 cu ft per sec at a point slightly above the lower end of the constricted section and about halfway between the two stations, which were about a mile apart. Topography had been taken above the low-water surface, and the low-water channel had been cross-sectioned at an average interval of about 200 ft, the sections being closer together at the upper end where the section was changing fast.

Space limited the scale of the model to 1 : 120, although a larger scale would have been better. After the model was built, it was "calibrated" by roughening the sides until the model water surface agreed with the field data for the three known points. This roughening was done by adding lumps of concrete to the sides and bottom of the channel and by placing somewhat larger pieces in the shape of needles in the channel, the cross sections indicating that such formations were present in the prototype. Photographs of the river were of considerable assistance in this work. After the calibration was completed in the spring of 1940, profiles of the water surface were obtained from the field during the annual flood for discharges between 62,000 and 240,000 cu ft per sec. The agreement between these profiles and those obtained from the model was remarkable, such differences as existed being easily attributable to the effects of local conditions at the points where the field measurements were made. After the "calibration" was completed, studies of the losses through the reach for varying amounts of backwater and discharge were made for a variety of channel improvements.

As a result of the model studies, it was found that excavation of the island at the entrance for any practicable distance below El. 1,255 was ineffective; that the removal of the spur on the right side at the outlet was comparatively ineffective; that equally good results could be obtained by excavation on the left bank; that the removal of the needle was effective; that some further decrease in the loss could be effected by proper disposition of the spoil from the excavation; and that none of the spoil could be deposited in the channel without affecting the losses. It was possible to lower the water surface upstream by 7 ft at the maximum discharge contemplated (650,000 cu ft per sec), by excavation estimated at 300,000 cu yd, mostly in the left bank. Comparison of relative effects of varying amounts of excavation indicated that this was about the limit of what could be accomplished, great further increases in the amount of excavation resulting in little corresponding benefit.

Unfortunately for the purposes of this discussion, there have been no large discharges since the work was completed. However, it seems timely to give such data as have been obtained (see Table 13).

TABLE 13.—COMPARISON OF ACTUAL AND PREDICTED LOSSES, THE LITTLE DALLES CHANNEL IMPROVEMENT—COLUMBIA RIVER

Discharge, in cu ft per sec	Water-surface elevation downstream	LOSSES, IN FT		Error (ft)
		Predicted	Prototype	
94,700	1,289.93	0.90	0.83	0.07
127,000	1,287.63	1.40	1.62	0.22
144,000	1,261.93	8.00	8.06	0.06
146,000	1,288.53	1.60	1.87	0.27
188,000	1,290.53	2.45	2.91	0.46
215,000	1,287.48	3.70	4.25	0.55
217,000	1,291.89	3.10	3.62	0.52
225,000	1,270.17	9.90	10.30	0.40
235,000	1,291.96	3.70	3.81	0.11
246,000	1,273.25	9.70	10.19	0.49
247,000	1,291.06	4.30	4.70	0.40
252,000	1,277.04	8.25	8.73	0.48
255,000	1,280.62	7.05	7.29	0.24
269,000	1,285.78	6.30	6.66	0.36
273,000	1,287.90	5.90	6.60	0.70
276,000	1,288.14	5.95	6.53	0.58

The actual losses are greater than those predicted from the model, except in one case, and the difference seems to increase with increasing discharge. The variation in differences for nearly similar conditions leads to the conjecture that the average of a greater number of field observations might agree more closely with the predictions. Although, as previously stated, the model showed that no spoil should be deposited in the channel, it was not possible to keep out all the excavation. An unknown amount (probably not large, comparatively) reached the channel and, to some extent at least, the discrepancies may be caused by this fact. Model observations were made in discharge increments of 100,000 cu ft per sec, and there may be small errors due to interpolation for other discharges.

L. STANDISH HALL,[27] M. AM. SOC. C. E.—The subject covered by the Symposium is very timely and indicates the need for further study on the conformity between model and prototype. As a matter of fact, much can be learned by studying the actual operation of structures already built. The reasons for the apathy in checking the performance of hydraulic structures have been excellently presented in the paper by Messrs. Warnock and Dewey. Nevertheless, many factors entering into hydraulic design are observed best in full-size structures. These factors include air entrainment, air locking, vibration, cavitation, standing waves, wave trains, bed movement, and scour below structures. The value of photography in studying many of the phenomena needs to be stressed particularly.

---

[27] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

Frequently papers are prepared describing in detail new construction shortly after the completion of the work, but before an opportunity has been provided to test out their actual operation. If, at a later date, it is found that changes are necessary in order to improve the operation, very often no publicity is given to these alterations, with the result that some future designer searching engineering literature for ideas may duplicate the same errors by repeating the original unsatisfactory designs. For this reason, the writer wishes to call attention to two unsatisfactory designs that have required modification.

The San Pablo Dam near Oakland, Calif., was constructed in 1917–1920 (65) and was provided with both an open-channel spillway and an auxiliary vertical shaft spillway equipped with outlet gates below full reservoir level. The horizontal leg of the shaft spillway was laid on a re-entrant grade, and a weir was built below the outlet to assure submergence of the tunnel. The idea behind this design was that a water cushion always would be provided by the 24-ft column of water impounded at the bottom of the vertical shaft. The problem of disposal of air carried by the falling water into the horizontal leg, if considered, was dismissed as not being of serious importance. In operation, it was found that small bubbles were carried down the vertical shaft and into the horizontal leg, where they accumulated in large bubbles along the roof. These bubbles moved slowly along the grade of the tunnel toward the outlet and discharged periodically with explosive violence, throwing water as spray to a height of 40 to 50 ft in the air. The vibration caused in the tunnel following each of these air discharges was also a serious problem.

A model was built of this shaft by the engineering division of the East Bay Municipal Utility District in Oakland to test the operation with valves placed at the bottom of the vertical leg which would control the opening and keep the proposed discharge outlet into the horizontal leg submerged at all times. This alteration reduced the discharge capacity of the shaft spillway, but the open spillway is of adequate capacity to carry the largest flood (66). The model was built on a 1-to-14 scale and it was found that, if the submergence of the valves was not greater than 6 ft, air bubbles, caused by the impact of the stream falling from the top of the shaft, could be carried through the water cushion into the horizontal leg of the spillway. In the prototype, this would be equivalent to a submergence of 84 ft, although it is not known whether this proportional relation between model and prototype is applicable with reference to these air bubbles. However, a submergence in the 84 ft of the prototype was entirely feasible. In the model test, air bubbles discharging freely at the tunnel outlet through the horizontal leg threw spray to a height of about 1 ft. This would be equivalent to 14 ft in the prototype, although, as previously stated, the spray actually was thrown to a height of between 40 to 50 ft in the prototype. Likewise, the mild vibrations in the model would be difficult to translate into the violent temblors of the prototype. The alteration on this structure apparently will remedy the previous faulty operation, but the tests indicate that strict conformity between model and prototype cannot be obtained under conditions of air entrainment in closed conduits.

The second structure to be considered is the tunnel spillway at the Chabot Dam near Oakland, which is one of the two spillways at this dam. This particular structure, constructed of masonry in 1889, has a side-channel spillway discharging into a tunnel. The weir at the inlet consists of three openings, each 13 ft wide, with the tunnel portal located only 10 ft from the downstream opening. The water drops 8 ft to the level floor of the channel, which merges into a transition section built on a grade of 6.1%. The tunnel is a 10-ft by 10-ft horseshoe section 1,438 ft long. Except for the entrance, the grade is 2.5%. At the tunnel outlet the waterway expands abruptly into an open masonry channel about 17 ft wide. This channel has a length of 662 ft from the tunnel outlet to the vertical overfall to the creek, and has a horizontal curve near the tunnel outlet. From this description it can be gathered that the structure violates practically every principle of modern hydraulic design for streamline flow.

The operation of the tunnel spillway under full discharge has never been satisfactory because of the poor hydraulic properties of the tunnel inlet and outlet. The weir has a potential capacity of 4,000 to 5,000 ft, but this cannot be utilized fully as the carrying capacity of the tunnel is only 2,000 cu ft per sec.

As the maximum carrying capacity is approached, the tunnel mouth is submerged, and air entrained in the water at the drop over the weir is carried into the tunnel. This air rises to the roof of the tunnel within 400 ft of the inlet, and a portion escapes through the downstream portal. A section of the tunnel, 100 to 200 ft long below the inlet, is under pressure of the air and water mixture, and, when sufficient pressure is created, a portion of the air accumulated along the roof discharges violently back through the inlet, throwing spray into the air. The water, rushing in to replace the discharging air, causes severe vibrations in the masonry of the tunnel.

Flow conditions at the outlet, at maximum discharge, also are not satisfactory. The sudden expansion of the cross section creates cross waves which overtop the walls. The horizontal curve farther downstream augments the cross waves and causes an overtopping of the walls for a second time. Water overtopping the walls of the channel does no particular damage, since the top of the wall is below natural ground surface and the water returns to the channel. Furthermore, floods of sufficient magnitude to cause overtopping rarely occur.

Normal consideration of the hydraulic conditions at the tunnel entrance would not reveal the cause of the pressure built up by entrained air at this point. With the contraction of the stream and loss of head at the portal, it would be impossible for the tunnel to flow at full capacity below the inlet, and, therefore, the air carried into the tunnel should be able to discharge freely from the lower end. The most probable explanation of the phenomenon is as follows:

1. Air is entrained by the water dropping over the steps of the entrance weir. Model experiments (67) by Julian Hinds, M. Am. Soc. C. E., on side-channel spillways indicate that a considerable volume of air may be entrained in the water below a weir. Even with the small heads used in his experiments (not more than 1 ft), the volume of entrained air was as high as 20% and averaged at least 4%. With large spillways, the entrainment naturally would be much greater than the average determined from model tests.

2. Air entrained in the water by the initial drop tends to escape, provided the velocity of the water is not sufficient to maintain the insufflation created by the drop over the weir. This would be the case above the Chabot tunnel portal, as a large part of the velocity is lost by the change in the direction of flow. However, much of the entrained air would be carried forward into the tunnel, as time would not permit the escape of much air in the short distance between the weir and the portal. In the tunnel section under capacity discharge, air entrainment or insufflation is reduced because of the small area exposed to the air. Hence, air entrained in the water tends to escape. As the water entering the tunnel will contain a considerable proportion of air in the form of bubbles, these will rise to the surface and with maximum flow will collect along the roof of the tunnel, since as soon as the mouth or portal is submerged, the upper end of the tunnel actually will be flowing under pressure. Experiments (68) by J. P. Frizell indicate the rate of rise of small bubbles in still water to be approximately 9 in. per sec.

3. The uncertainty as to the amount of air entrained in the pool below the entrance weir precludes a precise calculation of the volume of air carried into the tunnel with the water. Study of photographs taken during the flood periods in the past leads to the belief that the amount was large enough to obstruct the tunnel seriously. Assuming a 10% entrainment in the tunnel entrance, there would be a volume of 200 cu ft of air per sec at a discharge of 2,000 cu ft per sec. Air bubbles in the pool above the tunnel portal are compressed in direct proportion to their depth of submergence. Those near the bottom naturally have the greatest compression. After passing through the tunnel portal, a portion of the pressure head is converted into velocity head, and the reduction in pressure causes the air bubbles to expand, thereby increasing the air bulking. Hence, contrary to normal hydraulics the air-water mixture fills the tunnel, except for an air pocket formed below the tunnel mouth by rising air bubbles. The mixture of air and water must cause a pressure within the tunnel well past the break in grade located 50 ft from the tunnel mouth, since, as soon as a clear space is formed continuously at the tunnel roof, the air should escape at the downstream end. The air locking at the tunnel mouth must be caused by conditions in the first 50 ft or 100 ft of tunnel, since it does not seem likely, under reasonable assumption of entrance losses and tunnel friction losses, that the tunnel could flow full for its entire length even under the maximum possible reservoir elevations. As soon as sufficient air pressure has been built up in this part of the tunnel to exceed the water pressure at the portal, air escapes explosively through the tunnel entrance. The water rushing in to replace the escaped air in the tunnel creates a severe vibration of the tunnel lining. The vibration possibly could

be caused by a hydraulic jump within the tunnel. If an air bubble formed along the roof 100 ft or more from the portal, the velocity in passing this obstruction might be great enough to cause a jump to form downstream. The slope of the tunnel at Chabot Reservoir is so steep (2.5%) that under flow conditions without air entrainment a jump could not occur. The references (69)(70) for a jump in closed conduits describe tests made on conduits for slight slope or with the outlet end submerged.

4. The maximum distance that air bubbles can be carried into the tunnel before rising to the roof can be computed. The velocity of the water in the tunnel is slightly more than 30 ft per sec; hence, assuming a uniform distribution of entrained air in the tunnel cross section and assuming an average rate of rise of bubbles in water of 0.75 ft per sec, a bubble on the floor of the tunnel at the entrance will travel a distance of 430 ft from the inlet before reaching the roof. Turbulent flow existing in the water actually will vary this distance of travel, but the computation serves to indicate the approximate length of tunnel required to accumulate all of the entrained air at the roof. The free air space downstream from this point will permit escape of this air through the outlet.

The remedy directed toward the cure of the conditions of flow caused by submergence of the tunnel portal is relatively simple (although not yet effected because of the infrequency of major floods), as in this instance it is only necessary to provide gates on the entrance weir in order to preclude a discharge of water over the weir in excess of the capacity of the tunnel with free flow. However, to study more fully all of the flow problems of this structure, a model test would be very desirable.

Undoubtedly, many other structures exist which have exhibited unsatisfactory flow conditions. A study of these structures by means of models would provide better knowledge of the conditions leading to nonconformity between model and prototype. At the same time more confidence would be developed in the conditions under which model tests may be relied on for quantitative results. Finally, a better interpolation of tendencies would be indicated in the model for those factors in which the attainment of exact similitude is impossible.

A. J. GILARDI,[28] M. AM. SOC. C. E.—In a discussion of conformity between model and prototype, it is well to differentiate between generic conformity and specific conformity. The first, generic conformity, refers to the similarity of behavior, indications, and tendencies between model and prototype, whereas the second, specific conformity, refers to agreement of measured quantities and derived coefficients in accordance with the laws of hydraulic similitude. The first also might be called qualitative conformity, and, the second, quantitative conformity.

Specific conformity between model and prototype often is not ascertainable with accuracy. For instance, the prototype or improvement works may have been built differently from the model; or the conditions tested in the model may never occur in the prototype. On the other hand, accurate field measurements in the prototype may be too expensive or too impracticable to secure, and often less desirable methods or curtailed observations are substituted.

---

[28] Seattle, Wash.

In one particular experimental program, the preliminary tests of a large pump indicated that, with a special type of suction line, the output could be increased materially, especially if the diameter of the suction nipple were increased substantially. The prototype pump was built accordingly; but, unfortunately, the larger nipple was never tested in the model, and thus an excellent chance was lost to determine the specific conformity. The prototype results were very satisfactory and demonstrated the generic conformity. Incidentally, the preliminary tests alone were sufficient to secure the adoption of the formerly controversial type of suction line. In addition, the operational savings brought about by the increased diameter of the suction nipple in all probability paid for the cost of the entire experimental program in a short time.

In a broad sense, generic conformity is much easier to ascertain than specific conformity, mostly because a reasonable similarity of construction and of conditions is usually sufficient. For this reason, statements are much more frequent on generic conformity than on specific conformity, and would be still more frequent, were it not for the following peculiarity of human nature.

Men—and engineers—can learn as much by failures as they can by success. From the broad viewpoint of the engineering profession as a whole, how desirable it would be if the veil of secrecy surrounding some engineering mistakes were to be lifted just enough to prevent the recurrence thereof! Unfortunately, not much printer's ink ever will be needed for such confessions.

One or two examples will be sufficient to illustrate the point. In the case of an overflow dam, some of the experiments showed that baffle piers would be subjected to the impact of debris and to much erosion on account of the presence of cobbles and other sediment in the water. These warning indications were disregarded and the baffle piers were built, with the result that considerable damage took place in a short time.

On another project, experiments showed that a proposed layout would bring about uneven and rough filling conditions and therefore that other layouts which had performed more satisfactorily should be considered instead. However, the first-mentioned layout was adopted, and there has been trouble ever since. Again, a faulty decision!

The Symposium is a very helpful contribution to the engineering literature on the subject of hydraulic models, and it is to be hoped that many other observations on the conformity between model and prototype will be made available to the profession in the course of time. However, the writer is inclined to think that what engineers need mostly is a more discriminating attitude toward hydraulic model experiments in general.

There was a time when hydraulic experimentation was looked upon by many as mere play and when securing of funds for such work was full of endless difficulties. The pendulum has swung too far in the opposite direction since those days, and in more recent years some involved experimental programs have been conducted which either could have been omitted entirely or could have been curtailed very materially by adequate investigation and research.

Also, there was a time when the results of hydraulic experimentation were looked upon with some suspicion, but here again the pendulum has swung too far in the opposite direction; and, particularly during the last ten or fifteen years, a definite tendency to place a blind reliance on hydraulic experimentation seems to have developed. In other words, although at one time reliability was doubted by many engineers, an even greater number now take such reliability for granted.

The foregoing circumstances place a great responsibility upon hydraulic laboratories to undertake model studies only after thorough investigation and understanding of the prototype conditions and of the possibilities of reproducing them with accuracy in the model. The experimental program then should be selected and planned for the sole purpose of securing the required results with a minimum of time and expenditure and with elimination of all superfluous or duplicating data. Finally, great care should be exercised in the analysis of the results, formulation of conclusions, and presentation of recommendations. The most important phases of this work should be handled by engineers with practical experience and mature judgment.

The project engineers, or consulting engineers as the case may be, should never forget that the ultimate responsibility for the success or failure of the project is theirs, and should not permit their judgment to be swayed by pet ideas of their own or of some influential fellow engineers. These men should have a good working knowledge of hydraulic models and a realization of the limitations of such work, and should retain at all times the perspective of the ultimate goal to be attained. They should make sure that the hydraulic laboratory has understood and reproduced the prototype conditions and analyzed the results of the tests properly. Such warning indications as might have turned up during the experimental work should be taken into consideration.

Some of the remarks in the preceding paragraph may seem quite obvious and somewhat superfluous, but engineers with extensive practical experience and with understanding of human nature probably will recall examples to which some of the foregoing statements apply readily.

The field of hydraulic experimentation is very broad and extremely varied; as such, it has many pitfalls. In some unusually complex situations, the laboratories at which work was conducted failed to understand the prototype conditions, then the laboratories proceeded to formulate and conduct the experimental programs under conditions different from the significant ones, which of course led to wrong conclusions. In view of the aforementioned tendency to take the laboratory data at face value, satisfactory solution of such situations has been delayed by many years—if not prevented entirely—through the confusing and harmful influence of such hydraulic experimental work.

Hydraulic experimentation is an excellent tool for the solution of problems that cannot be approached satisfactorily in any other manner. However, it should be undertaken only after careful preliminary research, and then it should be prosecuted with meticulous care to its completion and should not be stopped before the best results are secured.

Experience with hydraulic models shows that conformity between model and prototype already is well demonstrated and that engineers should focus their attention much more on both careful planning and execution of hydraulic experimentation and correct application of its results to the prototype.

Fred W. Blaisdell,[29] **Jun. Am. Soc. C. E.**—This Symposium on the conformity between model and prototype will serve partly to fill the need for quantitative data on this subject. As stated by Messrs. Warnock and Dewey (see heading, "Introduction"), qualitative data, even though the comparisons are excellent, do not have quite the value to persons not associated closely with a specific project as do quantitative data. It is to be regretted that several of the prototypes discussed in the Symposium were not geometrically similar to their models. However, the results presented for even these structures are of considerable value in spite of the fact that, in some cases, the geometrical discrepancies are large enough to account for the lack of conformity.

In 1939, at the Hydraulic Laboratory of the National Bureau of Standards, Washington, D. C., the writer tested several models of combination Parshall flume and shallow Columbus-notch measuring devices installed by the Soil Conservation Service on the Blacklands Experimental Watershed near Waco, Tex. (71). In this discussion, the results of the tests on Parshall flumes are compared with tests (72)(73) made by R. L. Parshall, Assoc. M. Am. Soc. C. E. The only comparisons possible for the Columbus notches are between the full-size and half-size models.

Description of Parshall Flume Models.—The models (see Fig. 77 for drawings of a typical model) were made of concrete, the surface of which was painted with neat cement mortar after stripping the plywood forms. All inside surfaces were polished with a carborundum brick after the mortar had hardened. The same Parshall flume floor, crest, and false approach floor were used for all models, new walls being cast to give the desired model dimensions. Dimensions of the models tested, corresponding to those shown in Fig. 77, are presented in Table 14.

Head and Discharge Measurements.—Head measurements, at the points indicated in Fig. 77 and Table 14, were made to 0.0005 ft with point gages. In addition, water-level recorder data ordinarily were obtained for each run. The recorder charts were read to 0.001 ft, and the readings were corrected for instrumental errors, line shift, and paper expansion before being compared with the point gage readings. The agreement between point gages and water-level recorders was ordinarily better than 0.005 ft. However, the water-level recorder readings were used mainly to determine average heads when erratic or long-period fluctuations of the water surface occurred.

Discharge measurements were made—

With a:	For a discharge (in cu ft per sec) between:
⅝-in. water meter	0.007 and 0.06
Venturi meter in a 1.5-in. line	0.06 and 0.17
Venturi meter in a 4-in. line	0.06 and 0.57
Venturi meter in an 8-in. line	0.57 and 3.5
8-ft rectangular weir	0.43 and 35

---

[29] Associate Hydr. Engr., SCS, U. S. Dept. of Agriculture, St. Anthony Falls Hydr. Laboratory, Univ. of Minnesota, Minneapolis, Minn.

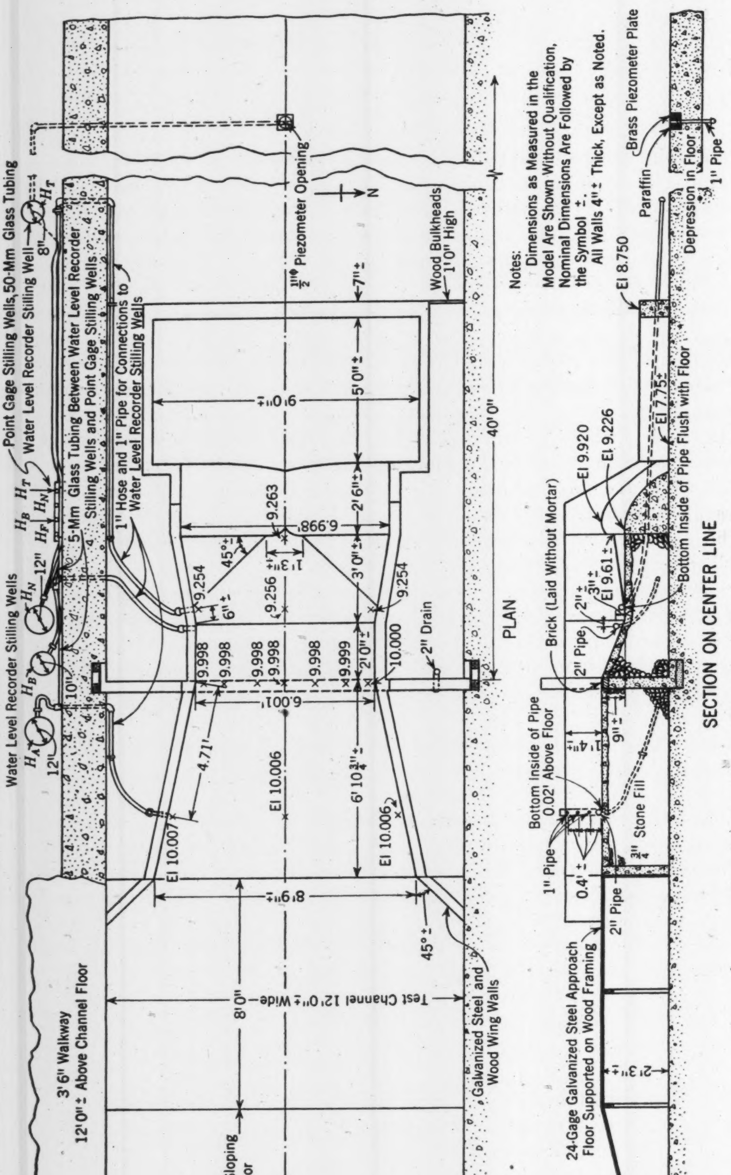

Water Level Recorder Stilling Wells
Point Gage Stilling Wells, 50-Mm Glass Tubing
Water Level Recorder Stilling Well
5-Mm Glass Tubing Between Water Level Recorder Stilling Wells and Point Gage Stilling Wells
1" Hose and 1" Pipe for Connections to Water Level Recorder Stilling Wells
H_A
H_B
H_N
H_F
H_T
12"
10"
8"
$\frac{1}{2}$"ϕ Piezometer Opening
N
Wood Bulkheads 1'0" High
7"±
9'0"±
5'0"±
2'6"±
3'0"±
2'0"±
6'10$\frac{3}{4}$"±
6.998'
6.001'
4.71'
1'3"±
6"±
45°±
9.263
9.254
9.256
9.998
9.999
10.000
10.006
10.007
El 10.006
El 10.007
2" Drain
40'0"
PLAN
3'6" Walkway 12'0"± Above Channel Floor
8'0" Sloping Floor
8'9"±
8'0"
Test Channel 12'0"± Wide
Galvanized Steel and Wood Wing Walls
45°±
Notes:
Dimensions as Measured in the Model Are Shown Without Qualification, Nominal Dimensions Are Followed by the Symbol ±.
All Walls 4"± Thick, Except as Noted.
Brass Piezometer Plate
Paraffin
Depression in Floor
1" Pipe
El 8.750
El 7.75±
Bottom Inside of Pipe Flush with Floor
El 9.920
El 9.226
El 9.61±
Brick (Laid Without Mortar)
2" Pipe
2"±
3"±
9"±
1'4"±
Bottom Inside of Pipe 0.02' Above Floor
1" Pipe
Stone Fill
$\frac{3}{4}$"
0.4'±
2" Pipe
2'3"±
24-Gage Galvanized Steel Approach Floor Supported on Wood Framing
SECTION ON CENTER LINE

FIG. 77.—FULL-SCALE MODEL OF TYPICAL RATE MEASURING INSTALLATION

—the combined capacity of the available pumps. Water passing through the 1.5-in. venturi meter also passed through the 4-in. venturi meter; all water passed over the 8-ft weir. When possible, rates of flow were frequently measured with two measuring devices. All measuring devices were calibrated in place several times, the water meter and venturi meters being accurate to ± 1% and the weir to ± 1.5%.

TABLE 14.—Prototype Dimensions (in Feet) of Parshall Flumes Tested

Crest length, throat width	Scale ratio 1 to	Width, upstream end (nominal)	Flume length (nominal)	Crest to piezometer along wall	Approach Floor Elevation Opposite Piezometer		
					South	Center line	North
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
6.001	1	8.75	6.86	4.71	10.007	10.006	10.006
6.006	1	8.75	6.86	4.71	....	....	....
6.004	2	8.75	6.86	4.33	10.002	10.002	10.000
9.990	3	15.62	14.00	6.00	10.000	10.000	9.999
14.985	5	25.00	25.00	7.67	9.997	....	....

TABLE 14.—(*Continued*)

Crest length	Crest Elevations (Uniform Spacing, South to North)						
	South			Center line	North		
(1)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
6.001	9.998	9.998	9.998	9.998	9.998	9.999	10.000
6.006	....	....	....	....	....	....	....
6.004	10.001	10.000	9.999	10.001	9.999	10.002	10.009
9.990	10.001	9.999		10.000	9.999		9.999
14.985	9.998	10.001		10.001	10.001		9.998

Tests on Parshall Flumes.—Two full-scale models of the 6-ft Parshall flume were tested; 97 tests were made on the first model and 38 tests at low heads on the second model. The capacity of the laboratory pumps would not permit testing the full-scale structure at its maximum design head, and half-scale models were tested principally to obtain the upper end of the rating curve. The full-scale tests were made to define the rating curve for the lower heads where viscous and surface tension forces have the greatest influence. The number of tests run on the half-scale model of the 6-ft Parshall flume was 255. The number of tests made on all three models of the 6-ft Parshall flume was 440. A total of 131 tests was made on a 1 : 3 scale model of a 10-ft flume, and 127 tests on a 1 : 5 model of a 15-ft flume. Mr. Parshall has reported in the Bibliography (72)(73) the number of free-flow tests, for these flume sizes, as 35, 19, and 4, respectively.

Parshall Flume Rating Curves.—The results of the tests by both Mr. Parshall and the writer are presented in Fig. 78. Data points for many tests on the 6-ft Parshall flume were omitted for the sake of clarity, as noted in Fig. 78(*a*).

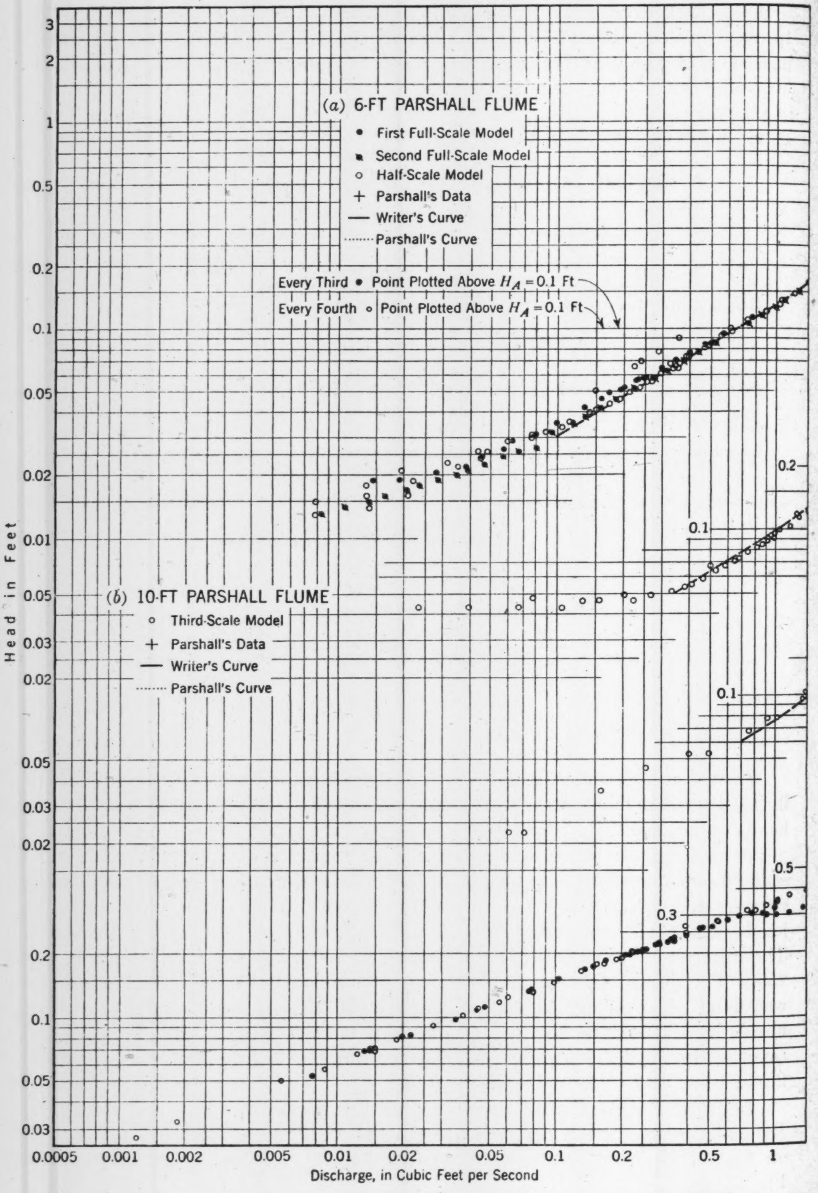


FIG. 78.—COMPARISON

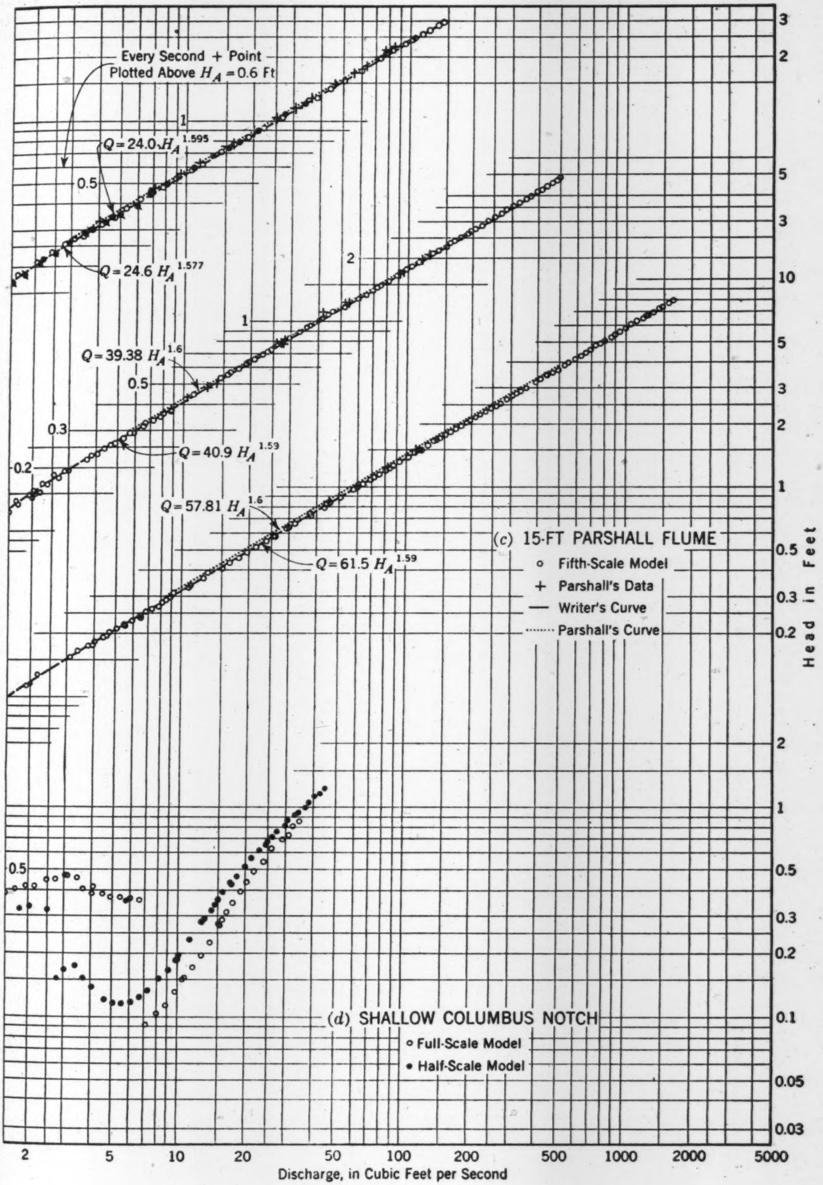

Every Second + Point Plotted Above $H_A = 0.6$ Ft
$Q = 24.0 H_A^{1.595}$
$Q = 24.6 H_A^{1.577}$
$Q = 39.38 H_A^{1.6}$
$Q = 40.9 H_A^{1.59}$
$Q = 57.81 H_A^{1.6}$
$Q = 61.5 H_A^{1.59}$
(c) 15-FT PARSHALL FLUME
○ Fifth-Scale Model
+ Parshall's Data
— Writer's Curve
······ Parshall's Curve
(d) SHALLOW COLUMBUS NOTCH
○ Full-Scale Model
● Half-Scale Model
1
0.5
2
1
0.5
0.3
0.2
0.5
3
2
5
3
10
5
3
2
1
0.5
0.3
0.2
2
1
0.5
0.3
0.2
0.1
0.05
0.03
2
5
10
20
50
100
200
500
1000
2000
5000
Discharge, in Cubic Feet per Second
Head in Feet

TABLE 15.—Comparison of Observed Parshall Flume Discharges

Parshall's Published Data						Writer's Tests	
Heads		Ratio $\frac{H_B}{H_A}$ (%)	Discharge (Cu Ft per Sec)		Deviation (%)	Discharge computed (cu ft per sec)	Deviation (%)
H_A (ft)	H_B (ft)		Observed	Computed			
(*a*) 6-Ft Flume, Original Tests, 1923							
1.550	0.757	0.489	49.29	48.28	−2.1	49.08	−0.4
1.812	1.090	0.601	62.54	61.94	−1.0	62.83	+0.5
1.458	0.640	0.439	44.59	43.80	−1.8	44.59	0.0
1.368	0.518	0.379	40.46	39.57	−2.2	40.33	−0.3
1.260	0.379	0.301	35.33	34.70	−1.8	35.42	+0.3
1.139	0.208	0.183	30.14	29.54	−2.0	30.21	+0.2
1.005	0.019	0.019	24.68	24.19	−2.0	24.80	+0.5
0.892	−0.134	....	20.24	20.00	−1.2	20.54	+1.5
0.737	−0.332	....	14.83	14.75	−0.5	15.20	+2.5
0.570	−0.501	....	9.79	9.79	0.0	10.14	+3.6
0.465	−0.013	....	7.03	7.08	+0.7	7.36	+4.7
0.382	−0.035	....	5.12	5.17	+1.0	5.40	+5.5
2.158	1.476	0.684	80.81	81.85	+1.3	82.73	+2.4
2.017	1.321	0.655	73.31	73.49	+0.2	74.37	+1.4
1.844	1.105	0.599	63.70	63.68	−0.3	64.58	+1.4
1.660	0.858	0.517	54.76	53.86	−1.6	54.71	−0.1
1.498	0.685	0.457	46.64	45.72	−2.0	46.52	−0.3
1.090	0.121	0.111	28.10	27.53	−2.0	28.18	+0.3
1.678	0.870	0.518	55.15	54.80	−0.6	55.65	+0.9
1.511	0.663	0.438	45.55	46.36	+1.8	47.16	+3.5
(*b*) 6-Ft Flume, Check Tests, 1926							
0.326	....	....	4.04	4.02	−0.5	4.21	+4.2
0.628	....	....	11.45	11.43	−0.2	11.83	+3.3
0.742	....	....	14.88	14.91	+0.2	15.38	+3.4
0.755	0.524	0.694	14.83	15.33	+3.4	15.79	+6.5
0.756	....	....	15.48	15.36	−0.8	15.84	+2.3
0.899	....	....	20.33	20.25	−0.4	20.80	+2.3
1.023	....	....	25.11	24.89	−0.9	25.50	+1.6
1.151	....	....	30.34	30.04	−1.0	30.71	+1.2
2.239	1.556	0.695	85.29	86.81	+1.8	87.58	+2.7
2.142	1.445	0.675	79.92	80.87	+1.2	81.80	+2.4
2.114	1.475	0.698	77.36	79.21	+2.4	80.12	+3.6
1.975	1.262	0.639	70.66	71.07	+0.6	71.98	+1.9
1.792	1.041	0.581	60.72	60.85	+0.2	61.75	+1.7
1.570	....	....	50.39	49.28	−2.2	50.08	−0.6
1.375	....	....	40.63	39.89	−1.8	40.64	0.0
(*c*) 10-Ft Flume in Fort Bent Canal							
0.78	....	....	27.1	26.5	−2.3	27.6	+1.8
0.79	....	....	27.8	27.0	−3.0	28.1	+1.1
0.79	....	....	27.7	27.0	−2.6	28.1	+1.4
0.83	....	....	29.6	29.2	−1.4	30.4	+2.7
0.83	....	....	28.7	29.2	+1.7	30.4	+5.9
0.82	....	....	28.7	28.7	0.0	29.8	+3.8
(*d*) 10-Ft Flume in Las Animas Consolidated Canal							
1.15	....	....	49.5	49.3	−0.4	51.1	+3.2
1.71	....	....	96.1	92.9	−3.4	96.0	−0.1
1.99	....	....	120.4	118.4	−1.7	122.2	+1.5
1.16	....	....	50.2	49.9	−0.6	51.8	+3.2
0.48	....	....	13.0	12.2	−6.6	12.7	−2.3
0.51	....	....	14.3	13.4	−6.7	14.0	−2.1
0.43	....	....	10.5	10.2	−2.9	10.7	+1.9
2.05	....	....	127.6	124.2	−2.7	28.0	+0.3
1.18	....	....	51.4	51.3	−0.2	53.2	+3.5
1.22	....	....	54.2	54.1	−0.2	56.1	+3.5
1.09	....	....	42.9	45.3	+5.3	46.9	+9.3

TABLE 15.—(*Continued*)

Parshall's Published Data						Writer's Tests	
Heads		Ratio $\frac{H_B}{H_A}$ (%)	Discharge (Cu Ft per Sec)		Deviation (%)	Discharge computed (cu ft per sec)	Deviation (%)
H_A (ft)	H_B (ft)		Observed	Computed			
(*e*) 10-Ft Flume in Pine River Canal							
0.78	....	....	25.8	26.5	+2.6	27.5	+6.6
1.65	....	....	92.4	87.8	−5.2	90.6	−1.9
(*f*) 15-Ft Flume in Lamar Canal							
1.25	....	....	83.0	82.6	−0.5	87.7	+5.7
1.50	....	....	111.8	110.6	−1.1	117.2	+4.8
(*g*) 15-Ft Flume in Rocky Ford Highline Canal							
0.85	....	....	45.5	44.6	−2.0	47.5	+4.4
1.39	0.26	19.0	100.8	97.9	−3.0	103.8	+3.0

The Parshall flume curves drawn in Fig. 78 and based on the model experiments have the equations:

For the 6-ft Parshall flume,

$$Q = 24.6\ (H_A)^{1.577} \ldots\ldots (29a)$$

for the 10-ft Parshall flume,

$$Q = 40.9\ (H_A)^{1.59} \ldots\ldots (29b)$$

and, for the 15-ft Parshall flume,

$$Q = 61.5\ (H_A)^{1.59} \ldots\ldots (29c)$$

in which Q is the discharge in cu ft per sec and H_A is the free-flow head measured in the stilling well in feet. These equations represent the experimental data, with few exceptions, to within ± 1% above heads of 0.05 ft. Corresponding equations developed by Mr. Parshall from his tests are, respectively:

$$Q = 24.0\ (H_A)^{1.595} \ldots\ldots (30a)$$

$$Q = 39.38\ (H_A)^{1.6} \ldots\ldots (30b)$$

and

$$Q = 57.81\ (H_A)^{1.6} \ldots\ldots (30c)$$

The curves represented by these equations are also shown in Fig. 78.

The agreement between the model tests made by the writer and the data obtained by Mr. Parshall is presented in Table 15. The agreement of Mr. Parshall's data with his equations developed therefrom also is presented for purposes of comparison.

Several interesting speculations can be made in regard to the data presented in Table 15. In regard to the observed discharges, Mr. Parshall states that current meters were used for the 10-ft and 15-ft flumes and standard 10-ft or 15-ft rectangular weirs for the 6-ft flume. Mr. Parshall also states (73*a*) that "The probable error of individual current-meter measurements * * * is from 2 to 3 percent." It is probable that no greater accuracy can be expected from standard weirs that have not been calibrated. Errors, probably systematic, of this magnitude therefore must be expected in the equations presented by Mr. Parshall. On the other hand, discharge measurements by the writer were made by means of devices calibrated by weight measurement of the discharge and are accurate to within ± 1.5%. It appears, therefore, that the average differences between Mr. Parshall's data and the equations developed by the writer are not greater than reasonably can be expected.

The test data obtained by the writer are plotted in Fig. 78. The excellent agreement between the plotted data and the equations determined therefrom is readily apparent, even at prototype heads as low as 0.05 ft for the 6-ft flume (half-scale model head = 0.025 ft) and 0.06 ft for the 10-ft and 15-ft flumes (10-ft flume, third-scale model head = 0.02 ft; 15-ft flume, fifth-scale model head = 0.012 ft).

Data for both full-scale and half-scale models of the 6-ft flume are presented in Fig. 78(*a*). It will be noted that both full-scale and half-scale model data plot on a single curve even at prototype heads as low as 0.013 ft, although the scatter is high at the lower heads. The agreement between full-model and half-model tests and the equation of the average curve, with few exceptions, is better than ± 1% for heads greater than 0.05 ft.

It is interesting to speculate on the comparison of the writer's model tests, made in a laboratory, where carefully calibrated measuring devices were available, with Mr. Parshall's tests, which were made in the field with less accurate measuring devices. It seems reasonable to assume that in this instance the model tests, made in the laboratory where a high degree of precision is obtainable, give better results than prototype field tests made under conditions permitting less precision in the measurements. This same thought has been stated by Messrs. Warnock and Dewey in this Symposium (see heading, "Factors Making Comparisons Difficult").

Mr. Parshall mentions (72*a*) a depression in the water surface over the stilling-well entrance for the 6-ft flume, caused by water flowing past the contractions at the upstream end of the converging section, which he estimates to be 3 in. for discharges of 75 to 85 cu ft per sec. This phenomenon was observed for the half-scale model and, for a discharge of 139 cu ft per sec, caused a depression amounting to 2.5 in. in the prototype. However, the depression in the head observed in the stilling well was only 0.051 ft.

Description of Columbus-Notch Models.—The shallow Columbus notches are used in the field for measuring discharges less than those that can be measured accurately by the Parshall flumes. A typical installation is shown in Fig. 77, this particular installation being the only one on which both full-scale and half-scale model tests were made. The notches were pre-cast and stored until needed in the model. After curing, the notches were polished with a carborundum brick. Wall and approach floor construction was similar to that for the flume. Reference is made to Fig. 77 for details of the installation.

Tests on Columbus Notches.—In all, 73 free-flow tests were made on the full-scale Columbus-notch model and 140 tests on the half-scale model. Because of the sloping floor of the Parshall flume throat and the short distance between this throat and the Columbus notch, there were intense eddies which produced poor flow conditions approaching the notch. Thus, the heads

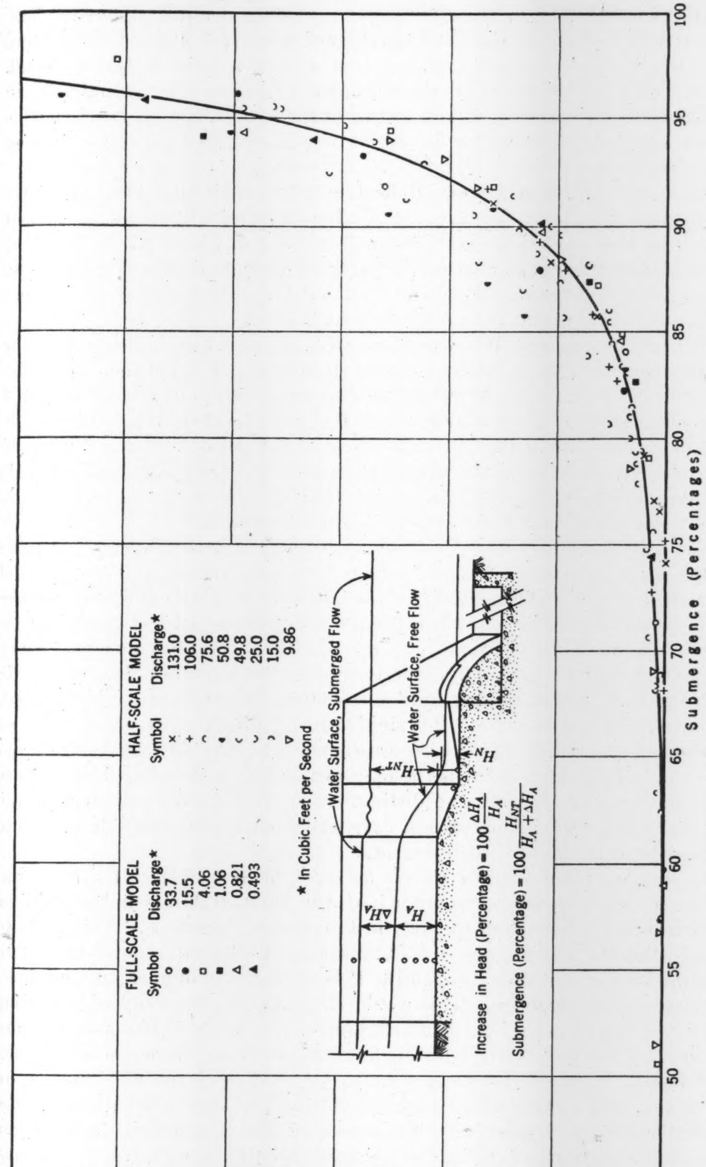


FIG. 79.—PARSHALL FLUME SUBMERGENCE CURVE FOR 6-FT PARSHALL FLUME

fluctuated considerably in the stilling wells, making readings difficult. The piezometer for measuring the head on the notch is very close to the sloping floor of the Parshall flume throat and just around a bend in the side-wall. The water at this piezometer is very turbulent and any slight dissimilarity in the positions of the standing waves, whirls, or the hydraulic jump, between the full-scale and half-scale models would have a large effect on the measured head at the notch.

Columbus-Notch Rating Curves.—The data obtained on these two Columbus-notch models are presented in Fig. 78(*d*). It will be noted that the agreement between the two models is excellent up to a head of about 0.3 ft, but that beyond that point the agreement is poor. Although it is not possible to explain all of the differences fully and with certainty, data were obtained that provide a plausible explanation for some of them.

Observations made at Waco on the prototype structure indicate that the pool upstream from the notch washed out (that is, the flow between the notch and the flume throat changed from streaming to shooting) at a head of 0.45 ft on the rising stage and filled in again at the same head on the falling stage. The pool washed out in the full-scale model at a head of 0.48 ft, which is in reasonable agreement with the value observed in the field. In the half-scale model the corresponding prototype head is 0.34 ft. The following explanation may have some bearing on the reason for this discrepancy:

From a study of the water-level recorder charts, it is possible to determine that the discharge was increased from test to test for the full-scale model and decreased, with one exception, for the half-scale model. The fact that the flow was increased from test to test for the full-scale model and decreased for the half-scale model may explain why the two curves do not coincide. Additional weight is given to this hypothesis because, for one run on the half-scale model, the flow was increasing until just before the readings were obtained. The datum point plotted from this single run (Q = 5.82 cu ft per sec) agrees with the full-scale data, which also were made with the flow increasing from run to run. This single point is the only available evidence on which to base the conclusion that separate and distinct rating curves exist over part of the range for rising and falling stages. No other data are available either to substantiate or to refute this statement.

Comparison of Submergence Curves for 6-Ft Parshall Flume Models.—Continuous records of stage are made for both the Parshall flumes and the shallow Columbus notches in each of the field installations of this type. This leads to the suggestion that the stage records obtained for the Columbus notch be used to determine whether the flow through the Parshall flume is submerged, and, if so, the degree of submergence. This would eliminate the necessity of providing a third recording gage at each installation to record Parshall's submergence head H_B. Therefore, tests with high tailwater levels were made on each model tested. Since full-scale and half-scale models of the 6-ft flume were constructed and tested under submerged conditions, a comparison of the submergence curves is possible. The results of 30 full-scale and 78 half-scale model tests are presented in Fig. 79. Special attention is called to the method of computing the percentage increase in head, which can result in increases in head greater than 100%. This method was used to simplify the computations for the submergence correction curves prepared for each installation tested. It will be noted that there is considerable scatter to the data, but that data obtained from either model can be used to define the curve shown.

Discussion of Discharge Computations for Submerged Flow Conditions.—The method of computing the discharge for submerged flow presented by Mr. Soucek (see heading, "Comparison of Model and Prototype") can be shortened considerably and all trial computations eliminated if his family of submergence curves is replaced by an average submergence curve. The maximum indicated error in discharge as a result of this substitution is only ± 3% up to a submergence of 85%. Above this submergence, the submergence curves are steep, and small errors in determining the submergence result in large errors in the submerged discharge correction. This makes the increase in accuracy as a result of rising multiple submergence curves rather fictitious, from a practical standpoint at least. The resulting simplification in computing the discharge is considerable, since trial computations are unnecessary and it is unnecessary to estimate the discharge before beginning the computations. This method was used by the writer in computing submergence correction curves for the Parshall flumes discussed herein.

A. R. THOMAS,[30] ESQ.—This subject has received considerable attention in India from the Central Board of Irrigation and its Research Committee. There is general agreement on the great value of model experiments, but it is recognized that, from the aspect of accuracy of reproduction of prototype conditions, models are of different types.

Models of rigid structures, without erodible boundaries and in the absence of sediment transportation, can give highly accurate results although there are cases in which allowance is required for scale effect. For example, experiments with models of high-coefficient spillways (74) showed considerable scale effect despite moderately high Reynolds numbers, presumably because of the influence of viscous shear in the sharply curved non-turbulent stream. Expanding flow is another example—an expansion which is satisfactory in a model is not necessarily so in the prototype.

In cases of scour due to flow patterned mainly on the shape of rigid structures (for example, scour downstream of weirs), the results obtained have been sufficiently accurate to have comparative and quantitative value.

River models are in another category, however, for two reasons: (1) General quantitative accuracy is unattainable with present technique, and (2) because of the complexity of conditions (for example, the variation of quantity of transported material with discharge and time), the accuracy of results obtained depends largely on the design of the model and the way in which the experiments are conducted. C. C. Inglis, M. Am. Soc. C. E., has referred to mobile models (75) as "a valuable aid to engineering skill, which however they can never replace." The value of such models has been proved by the number of successful investigations in which river models played a major part.

It is not possible to give details of results obtained, but papers and discussions on the subject have been published in the Central Board of Irrigation Annual Reports for the years 1939–1940, 1941, and 1942 (79).

---

[30] Secy., Central Board of Irrigation, Simla, India.

Gilbert H. Dunstan,[31] **Assoc. M. Am. Soc. C. E.**—The various papers of this Symposium provide a valuable contribution to the literature of model testing. Time limitations in the teaching of hydraulics in most colleges, however, prevent more than a passing mention of the subject, except in advanced courses. Nevertheless, the writer believes that it is desirable to introduce model testing to students taking hydraulic laboratory work, and to call their attention to the various papers that have been published.

Since the value of a model depends largely upon the use that can be made of the results obtained from it, a practical demonstration will do much to convince students of the importance, validity, and limitations of model studies. This may not always be possible in a college laboratory, although the following arrangement has provided one solution at low cost.

Several years ago a standard 1-ft Parshall flume was constructed in the Hydraulics Laboratory at the University of Alabama, at University. It was made of timber, lined with sheet metal. The water supply available does not permit the calibration of the flume at rates of flow greater than about 1.5 cu ft per sec. It would be desirable to have at least twice this amount for the highest point on the calibration curve for this flume.

Subsequently, a small sheet metal flume was made with a 2-in. throat, or one sixth the prototype size. This can be calibrated over a wide range of flows. Students calibrate both the prototype and model flume. From the model data and the geometric scale ratio, the head for the prototype is obtained, and the prototype discharge is obtained from the model discharge by multiplying by $(L_r)^{2.5}$. The measured prototype data are plotted on a graph sheet, after which the prototype values computed from the model points are plotted.

Results obtained to date have shown good conformity, some students drawing only a single line to represent all points, whereas others represent the results by parallel lines, close together. In general, one or more model points, for the lowest flows, will be completely out of line. Computations show that, in the actual model test, the flow for these cases was laminar, whereas in the prototype it would be turbulent. This study, therefore, has been of value in demonstrating to students that a model may be used to predict prototype performance, provided that model laws and limitations are applied properly.

Robert F. Kreiss,[32] **Jun. Am. Soc. C. E.**—The data from model and prototype on the locks discussed in this Symposium not only offer model-prototype comparisons but are useful to designers of navigation locks. The writer regrets the brief mention of dynamic head and overtravel and the omission of any description of apparatus for testing prototype locks.

The difficulty of comparing culvert systems in which port designs differ is appreciated. Lock coefficients vary not only with different port designs but with aggregate port area per culvert. Differences in coefficients for model and prototype are not unexpected for the first locks and the third locks (see Table 3), inasmuch as, in the first locks, the throat area of most of the prototype ports is symmetrically larger than in the model and, in the third locks, the area of the lock chamber end of each prototype port is larger. Coefficients are in good agreement in the single comparison having geometric similitude of manifold systems.

---

[31] Associate Prof., San. Eng., Univ. of Alabama, University, Ala.

[32] Omaha, Nebr.

The effect of difference in port design on discharge capacity is clarified to some extent by the realization that lock coefficients for a single port design vary with the ratio of cumulative port area to culvert area. A relationship has been reported (76) wherein the lock coefficient, determined either as described in the paper or as an average of ten short intervals of a lockage, varies linearly on a semilogarithmic graph with the area ratio in the range from 0.89 to 2.38. The relationship is

$$C_l = \frac{\beta}{10^{\mu x}} \qquad (31)$$

in which C_l is the lock coefficient based on the exit area of all ports, x is the ratio of cumulative port exit-end area to culvert area, and β and μ, equaling 1.0 and 0.15, respectively, are constants for a lock-filling operation. The lock coefficient decreases from 0.73 to 0.44 over the range of area ratio. Differentiation gives

$$\frac{dC_l}{C_l} = \mu \log_e 10 \, dx \qquad (32)$$

This expression would be of use not only in evaluating changes in C_l resulting from small changes in area ratio for a given port design but also in approximating changes in C_l resulting from minor changes in port design if, by additional tests, the μ-values for each new design were found not to depart greatly from 0.15.

Eq. 31 is useful in a second and more important analysis. The lock coefficient, when computed from test data of short time intervals following the culvert valve opening, substantially eliminates the dynamic head effect. Consequently, Eq. 31 can be introduced into a steady flow discharge equation, as follows:

$$Q = \frac{\beta}{10^{\mu x}} A_p \sqrt{2gh} \qquad (33)$$

in which A_p is the summation of port area. From Eq. 33 it can be shown that the area ratio, at which percentage changes in either port area or culvert area are equally effective in changing the discharge capacity, is

$$x = \frac{1}{2 \mu \log_e 10} \qquad (34)$$

From Eq. 34, note that the area ratio is 1.45 when μ is 0.15. A percentage change in culvert area becomes increasingly more effective on discharge capacity as the area ratio progressively increases above 1.45. For example, the culvert area is twice as effective as the port area when the area ratio is 1.93. With area ratios less than 1.45, the culvert is less effective than the ports.

The venturi-type port design, to which Eq. 31 applies, has a throat area 51.8% as large as the exit-end area. The ratio of total throat area to culvert area is 0.75 when the area ratio x is 1.45. The area ratio is 1.93 when the total throat area is the same as the culvert area.

Eq. 34, which is applicable to manifold systems satisfying Eq. 31, is sufficient proof that the minimum flow area in a manifold system need not be the major control area for discharge. Furthermore, the shifting from port area to culvert area, as the minimum flow area passes from ports to culvert or at any area ratio, is not advisable in computing coefficients, since each area—culvert, total port throat, or total port exit (for flared ports)—provides some degree of control on discharge. Coefficients throughout the range of area ratio should be expressed in terms of one area, say that of the culvert, with a qualifying expression regarding area ratio, as $C_l = 0.62$ at $x = 1.50$.

The development of expressions comparable to Eq. 31 is needed. The exponential function is suitable probably for venturi-type ports in a manifold system but may not be applicable for square-cornered ports. For systems satisfying Eq. 31, the constants β and μ are of greater interest than the variable C_l. Their determination is difficult in prototype lock tests because of the requirement that the area ratio be changed. In model tests their measurement is made more easily.

The writer agrees with the Symposium authors that inclusion of dynamic head effects in discharge coefficients is satisfactory in model-prototype comparisons. For design purposes coefficients eliminating dynamic head effects are preferred.

J. H. DOUMA,[33] JUN. AM. SOC. C. E.—Interesting results and useful hydraulic data on problems in the field of model-prototype conformity that are not clearly understood generally by hydraulic engineers are presented in the papers of this Symposium. The first paper, entitled "Hydraulic Structures," is especially valuable to future conformity studies because of its comprehensive outline of the factors that make satisfactory model-prototype comparisons difficult. Future investigators will do well to use this outline as a guide in developing testing techniques and in eliminating experimental errors.

The results of the qualitative model-prototype comparison, described by Messrs. Warnock and Dewey, together with results of other similar comparisons appear to support the conclusion that qualitative model studies, conducted under the present-day technique, are reliable in predicting many prototype conditions with a satisfactory degree of accuracy. Quantitative comparisons, however, have frequently shown wide differences in model and prototype results. The explanation undoubtedly is that satisfactory quantitative comparisons require more exact reproduction of the hydraulic phenomena involved to make possible accurate predictions of pressures, velocities, water-surface profiles, depths of scour, and discharges.

Discrepancies in quantitative model-prototype comparisons often have been explained as due to the lack of strict dynamic similarity, when usually other factors are the chief sources of error. Frequently, the statement is made that Reynolds' criterion has not been satisfied; and that, therefore, prototype quantities are not predicted accurately by the model. Messrs. Warnock and Dewey use this reason to explain the differences in model and prototype pressures and discharges of the 102-in. outlet conduit.

Many large-scale models do not require strict compliance with Reynolds' law. Under conditions of fully developed turbulent flow in a model, or prototype, the friction coefficient is constant for any one roughness value and independent of viscosity. Therefore, model-prototype comparisons do not require compliance with Reynolds' criterion when the model scale and roughness are selected so that the friction coefficient in the model and prototype is equal and so that fully developed turbulent flow occurs in both model and prototype. A brief review of the laws of similitude will assist in clarifying the limitations of model-prototype comparisons.

---

[33] Hydr. Engr., U. S. Engr. Office, Los Angeles, Calif.

For a model to be a true representation of its prototype in every sense, both geometric and dynamic similarity must exist at the same time. Geometric similarity of the solid boundaries, including the roughness, sets the conditions for geometric similarity of the flow stream lines. Dynamic similarity of the exerting forces, and thus similarity of the fluid properties, sets the conditions for similarity of the flow characteristics. Geometric similarity is obtained by fixing all model dimensions in some direct scale ratio to corresponding prototype dimensions. Strict dynamic similarity, however, is not obtained so easily as every dimensionless parameter referring to the flow characteristics in the model must have the same numerical magnitude as the corresponding parameter referring to the prototype. In other words, the flow characteristics will be similar if the fluid properties are such that the gravitational, viscous, capillary, and elastic forces bear the required relationship of model to prototype (57*a*).

The basic dimensionless parameters pertaining to the action of weight, viscosity, surface tension, and elasticity are known, respectively, as the Froude, Reynolds, Weber, and Cauchy numbers:

$$\mathbf{F} = \frac{\rho\, V^2}{\gamma\, L}\,;\ \mathbf{R} = \frac{V\, L}{\mu\, \rho}\,;\ \mathbf{W} = \frac{\rho\, V^2\, L}{\sigma}\,;\ \mathbf{C} = \frac{\rho\, V^2}{E}$$

in which $\mathbf{F}$ = Froude's number; $\mathbf{R}$ = Reynolds' number; $\mathbf{W}$ = Weber's number; $\mathbf{C}$ = Cauchy's number; V = mean velocity; L = a length (diameter for pipes and hydraulic radius for open channels); γ = specific gravity; ρ = density; μ = viscosity; σ = surface tension; and E = elastic modulus. The magnitude of these numbers, assuming geometric similarity, must be equal in model and prototype to attain complete similarity.

When more than two of the force properties of a fluid appreciably influence some phenomenon of flow it is impossible to obtain significant quantitative observations from a model. Therefore, it is necessary to eliminate at least all but two of the force properties in order to make a model test.

For all practical purposes, elastic forces may be ignored in the usual model study, since both model and prototype are assumed to be rigid and the elastic properties of the model and prototype fluids are identical when the same fluid occurs in the model and prototype. Also, capillary forces need not be considered unless the model is extremely small and surface tension forces are appreciable.

The Froude and Reynolds criteria for similitude remain to be considered. Although both criteria cannot be satisfied when the same fluid (water) occurs in the model and prototype, it is possible to select a different model fluid with kinematic viscosity such as to satisfy both criteria at the same time. In practice, however, it is desirable to use water as the model fluid. Satisfactory model tests can be made with water when either the viscous or the gravitational forces are predominant and all other forces are of secondary importance. The proper law for similitude would be that corresponding to the predominant force.

A criterion from which to determine when Reynolds' law must be satisfied can be developed from the experiments of J. Nikuradse (59) on pipes. In his experiments the relationship between the friction factor f, as defined by the Weisback formula

$$h_f = \frac{f\,L\,V^2}{2\,g\,D} \qquad (35)$$

and the Reynolds number was determined for flow in pipes. Typical plots of the relationship are shown in Fig. 80. It is noted that for laminar flow a small change in Reynolds' number results in a large change in the friction factor, whereas for large Reynolds numbers with fully developed turbulence and for a constant boundary roughness there is no appreciable change in the friction factor for a wide range in Reynolds' number. Thus, when laminar flow exists in either the model or the prototype, Reynolds' law must be satisfied. On the other hand, when fully developed turbulent flow exists in both the model and prototype and the model scale and roughness are such that the friction coefficients are nearly equal, then the viscous forces would have a minor influence on similitude and Reynolds' law need not be satisfied.

Nikuradse's experiments on pipes revealed that the lower limit of Reynolds' number for which fully developed turbulent flow occurred varies with the boundary roughness. Fully developed turbulence existed for all of the roughness values tested when the Reynolds number exceeded one million. For the maximum roughness tested, fully developed turbulence existed for a Reynolds number of 13,000.

Based on the results of Nikuradse's experiments, a straight line, representing the lower limit of Reynolds' number at which fully developed turbulence occurs in pipes, was drawn in Fig. 80. For purposes of discussion, it is assumed that the lower-limit line extends to an f value of 0.005 as shown in Fig. 80. Additional tests on larger and smoother pipes than those tested by Nikuradse would be required to verify the assumed extension.

By plotting, in Fig. 80, the model and prototype test results for the 102-in. outlet conduits reported in the paper, it is shown that fully developed turbulence existed for all of the model and prototype tests, except for two of the model tests which plot in the transition region from laminar flow to fully developed turbulent flow. Furthermore, the values of the friction coefficients in the model tests were two to three times those in the prototype. Under these conditions, viscous forces should not be neglected since they affect similitude appreciably. Better agreement between model and prototype tests for the 102-in. outlet conduits probably would have been obtained had the model scale been larger and the model surfaces smoother.

Based on the satisfactory model-prototype comparisons for the outlet conduits reported in the paper, it appears that, for purposes of design, strict compliance with Reynolds' law is not required. Strict compliance is probably required, however, when a discharge rating curve is to be predicted accurately by a model. Study of additional model-prototype comparisons will be required to determine the limitations as to when and to what degree Reynolds' law need be satisfied.

For comparison with the lower limits of Reynolds' numbers for fully developed turbulence in pipes a similar curve for open channels is shown dotted in Fig. 80. Until more reliable limits are established, these limiting lines may be used by model designers to check model requirements with regard to Reynolds' law. For example, if a proposed model is so small that test points would plot to the left of the appropriate limiting line, then Reynolds' law should be considered.

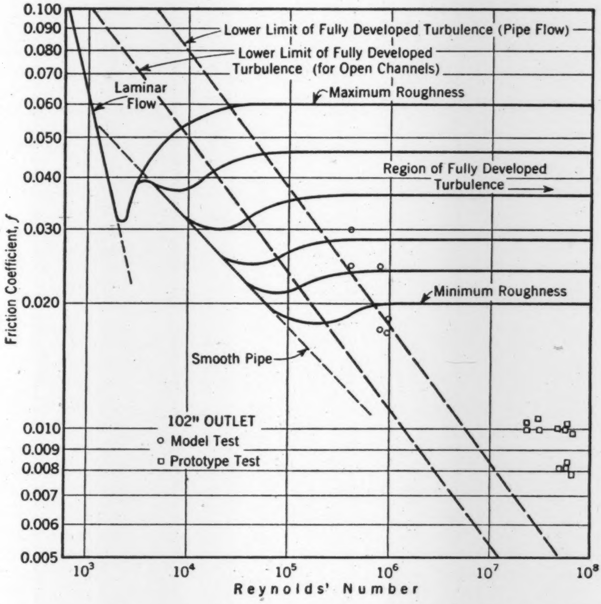


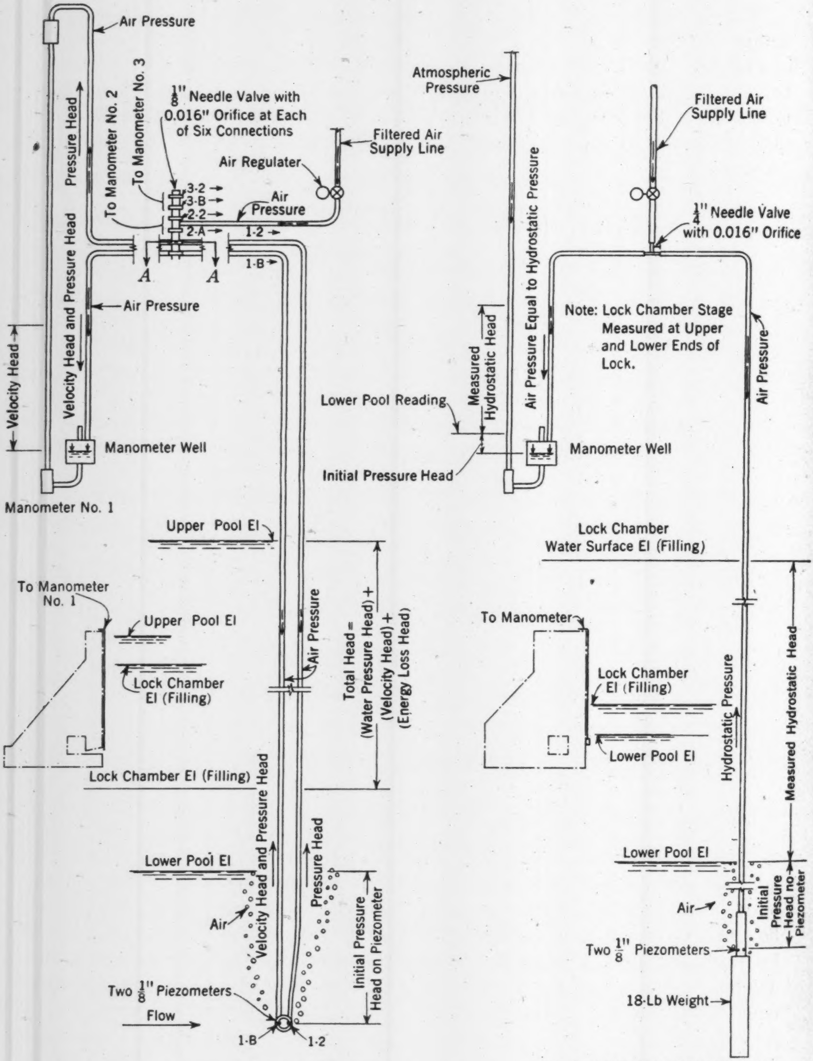
Fig. 80.—Reynolds' Number Vs. Friction Coefficient—Typical Curves for Pipe Flow

With reference to open channels, the laws pertaining to viscous forces are not well defined, and it is necessary to base an analysis of open-channel flow on the established laws for pipe flow. The Reynolds number for open-channel flow is determined by the same expression as that for pipe flow except that the hydraulic radius is used as the linear dimension instead of the pipe diameter. Therefore, Reynolds' numbers for flow in open channels, under conditions of equal velocities and hydraulic radii, will be one fourth the Reynolds numbers for pipes, since the hydraulic radius of a pipe is one fourth the pipe diameter. Accordingly, the limiting line for fully developed turbulence in open channels was drawn with values of Reynolds' numbers equal to one fourth the limiting values for pipes.

Marvin J. Webster,[34] Assoc. M. Am. Soc. C. E.—The design of apparatus for correlation tests between models and their prototypes often presents

---

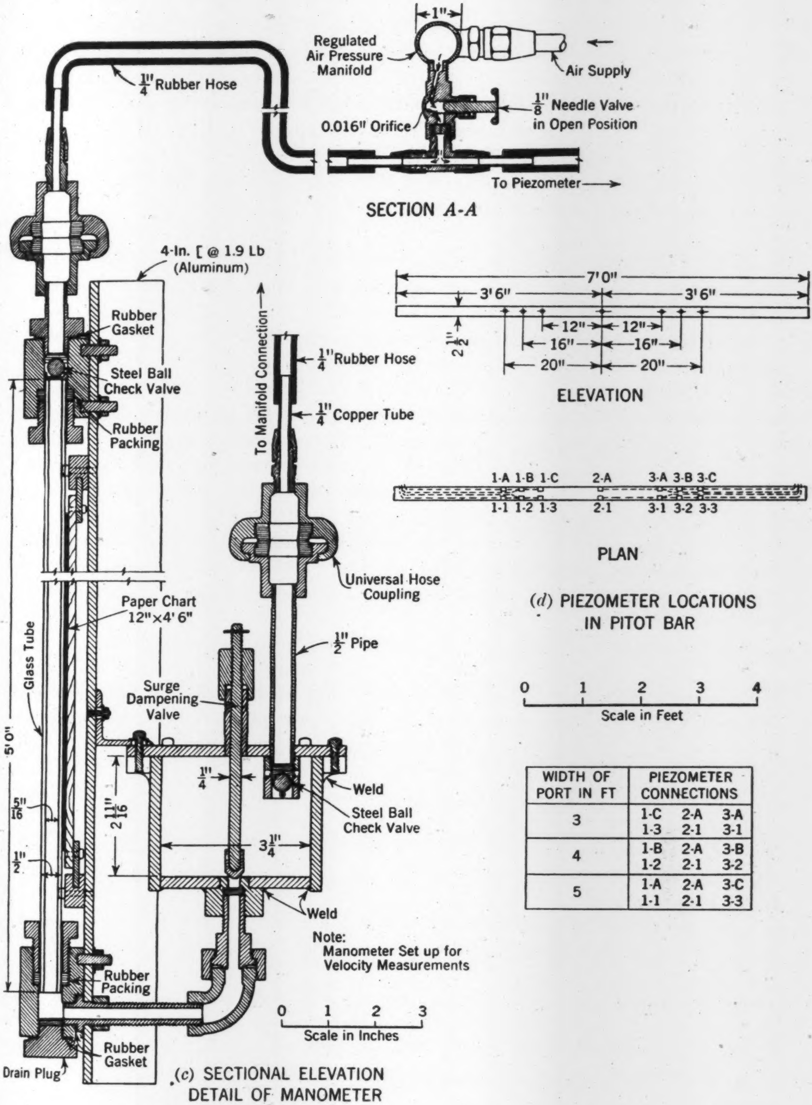
[34] St. Paul, Minn.


Air Pressure
Pressure Head
To Manometer No. 2
To Manometer No. 3
$\frac{1}{8}$" Needle Valve with 0.016" Orifice at Each of Six Connections
Filtered Air Supply Line
Air Regulater
3-2
3-B
2-2
2-A
Air Pressure
1-2
1-B
A
A
Velocity Head and Pressure Head
Air Pressure
Velocity Head
Manometer Well
Manometer No. 1
Upper Pool El
To Manometer No. 1
Upper Pool El
Lock Chamber El (Filling)
Lock Chamber El (Filling)
Air Pressure
Total Head = (Water Pressure Head) + (Velocity Head) + (Energy Loss Head)
Lower Pool El
Velocity Head and Pressure Head
Pressure Head
Air
Initial Pressure Head on Piezometer
Two $\frac{1}{8}$" Piezometers
Flow
1-B
1-2
Atmospheric Pressure
Filtered Air Supply Line
$\frac{1}{4}$" Needle Valve with 0.016" Orifice
Air Pressure Equal to Hydrostatic Pressure
Measured Hydrostatic Head
Note: Lock Chamber Stage Measured at Upper and Lower Ends of Lock.
Air Pressure
Lower Pool Reading
Manometer Well
Initial Pressure Head
Lock Chamber Water Surface El (Filling)
To Manometer
Lock Chamber El (Filling)
Lower Pool El
Hydrostatic Pressure
Measured Hydrostatic Head
Lower Pool El
Air
Initial Pressure Head no Piezometer
Two $\frac{1}{8}$" Piezometers
18-Lb Weight

(*a*) FOR PORT VELOCITY MEASUREMENTS

(*b*) PNEUMATIC SOUNDER FOR MEASUREMENT OF LOCK CHAMBER STAGE

FIG. 81.—DIAGRAMMATIC


1"
Regulated Air Pressure Manifold
Air Supply
$\frac{1}{4}$" Rubber Hose
$\frac{1}{8}$" Needle Valve in Open Position
0.016" Orifice
To Piezometer
SECTION A-A
4-In. [@ 1.9 Lb (Aluminum)
Rubber Gasket
Steel Ball Check Valve
Rubber Packing
To Manifold Connection
$\frac{1}{4}$" Rubber Hose
$\frac{1}{4}$" Copper Tube
Universal Hose Coupling
Paper Chart 12"×4' 6"
$\frac{1}{2}$" Pipe
Glass Tube
Surge Dampening Valve
5'0"
$\frac{1}{4}$"
$2\frac{11}{16}$"
$3\frac{1}{4}$"
$\frac{5}{16}$"
$\frac{1}{2}$"
Weld
Steel Ball Check Valve
Weld
Note: Manometer Set up for Velocity Measurements
Rubber Packing
Rubber Gasket
Drain Plug
0 1 2 3
Scale in Inches
7'0"
3'6"
3'6"
$2\frac{1}{2}$"
12"
12"
16"
16"
20"
20"
ELEVATION
1-A 1-B 1-C 2-A 3-A 3-B 3-C
1-1 1-2 1-3 2-1 3-1 3-2 3-3
PLAN
0 1 2 3 4
Scale in Feet

(c) SECTIONAL ELEVATION DETAIL OF MANOMETER

(d) PIEZOMETER LOCATIONS IN PITOT BAR

WIDTH OF PORT IN FT	PIEZOMETER CONNECTIONS		
3	1-C 1-3	2-A 2-1	3-A 3-1
4	1-B 1-2	2-A 2-1	3-B 3-2
5	1-A 1-1	2-A 2-1	3-C 3-3

SKETCHES OF INSTRUMENTS

unusual difficulties. This was especially true of the apparatus used in the prototype tests discussed in the paper by Messrs. Nelson and Hartigan, and it is regretted that their paper did not include a brief description of the apparatus.

The authors have based their comparisons on results of stage tests and port velocity tests. Stages in a lock chamber during filling and emptying operations can be measured with apparatus of relatively simple design. However, accurate measurement of velocities of the magnitude estimated and under the conditions prevailing in lock chambers with extremely high lifts was without known precedent. Although other methods of discharge measurement were considered, the selection was reduced to the current meter and pitot tube.

In their discussion on port velocity tests on the first lock, the authors state that during a filling operation discharge through ports 1 and 3 was reversed, flowing from the lock chamber into the culvert. During the operation of the tests on some of the other locks, it was observed that the direction of flow changed frequently and rapidly during the filling operation. This condition was anticipated and was one of the principal reasons why current meters were not adopted; namely, they would not respond quickly to velocities changing rapidly in both direction and intensity. Also, current meters would offer considerable restriction to the flow if velocities were measured simultaneously at several points in the port.

Because the distance from the point of measurement to the point of observation greatly exceeded the barometric head, the pitot tube could not be used in its usual form. Since compressed air was a suitable medium and since all locks were equipped with compressed air, it was selected as the transmitting medium.

Diagrammatic sketches of the instruments used are shown in Fig. 81. Velocity heads were observed on manometers placed on top of the lock-wall. A manometer with connections for port velocity tests is shown in Fig. 81(*c*). Mercury was used in the manometers for the higher range of velocities; bromoform was used for consistently low velocities where deflection of a mercury column was insufficient for accurate readings.

The pitot bar (Fig. 81(*c*)) was equipped with piezometers for simultaneous readings at three points in the flow area. The bar offered a minimum of restriction to the flow through the lock-chamber ports and could be supported rigidly by an aluminum frame anchored to the lock-chamber wall above the lower pool water line.

The pitot bar was not calibrated before the tests were made. A coefficient of unity was used until all tests were completed. Then the coefficient of the bar for each lock was computed by correlating port velocity and lock-chamber stage measurements. The average coefficient of 0.84 was obtained from the data of all locks tested for both filling and emptying operations.

The manometers, with slight changes in connections and arrangement (Fig. 81(*b*)), were used for measuring stages in the lock chamber. If current meters had been used, one set of instruments would have been required for the port velocity tests and an entirely different set for the stage tests.

One major change in the apparatus was found necessary after the tests were started: Proper adjustment of the air supply could not be obtained with the $\frac{1}{8}$-in. needle valves. The difficulty was overcome by the installation of a 0.016-in. orifice in each connection between the air-pressure manifold and the individual manometers. This arrangement was very satisfactory and permitted considerable latitude in air pressure without affecting the readings observed on the manometers.

MARTIN E. NELSON,[35] M. AM. SOC. C. E.—The conclusions reached by Mr. Soucek, in his study of the effect of submergence on the flow of water over a spillway and in his comparison of discharge over a model crest and its prototype, are verified substantially by similar tests conducted under the supervision of the writer in 1932. The model tests in question were made in connection with the design of the spillway described in the paper by Mr. Hopkins. The tests were made primarily for the purpose of developing a stilling basin to protect the downstream toe of the dam. Two model spillway crests with cross sections as shown in Fig. 82 were constructed of concrete to dimensions $\frac{1}{20}$ as large as the proposed prototype. Model crest No. 1, simulating the original proposed design, was equipped with several pressure piezometers along its center line. On this model it was observed that, under certain conditions of discharge and head, considerable negative pressure formed under the nappe. A model of crest No. 2 was tested to determine the effect of the indicated modification in shape on the pressure gradient under the nappe. Although the pressure condition was considerably improved, for economic reasons and because the condition under which the negative pressures in the prototype might be serious would be of short duration, this design was not adopted for the prototype. However, the calibrations of model crest No. 2 will be introduced in this discussion to indicate the change in discharge coefficients for free and submerged conditions with respect to crest profile. The prototype spillway was constructed to the section designated as crest No. 1 in Fig. 82, except that the crest elevation is 737.5 instead of 736.0. The spillway as constructed is 551 ft long.

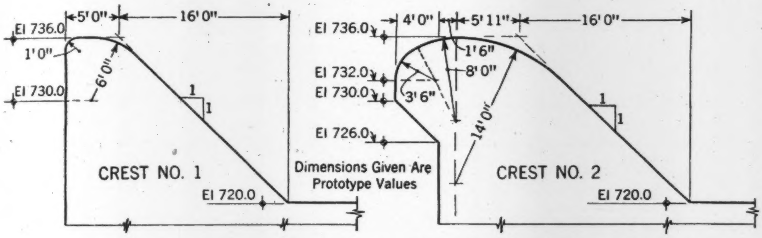


FIG. 82.—HYDRAULIC MODEL TESTS, 1 : 20 SCALE MODEL

The models, 2.53 ft long, representing spillway lengths of 50.6 ft, were tested in a glass-walled flume. The floor of the upstream approach was fixed at El. 718.0 and that of the downstream approach at El. 719.0. The water supply was measured over a V-notch weir, and tailwater levels were regulated by means of an adjustable gate at the lower end of the testing flume. Water-surface elevations in the upper pool were observed by means of a piezometer in the flume wall, together with a stilling well and hook gage, at a point corresponding to 142 ft upstream from the crest, and similar readings in the lower

---

[35] Senior Engr., U. S. Engr. Office, St. Paul, Minn.

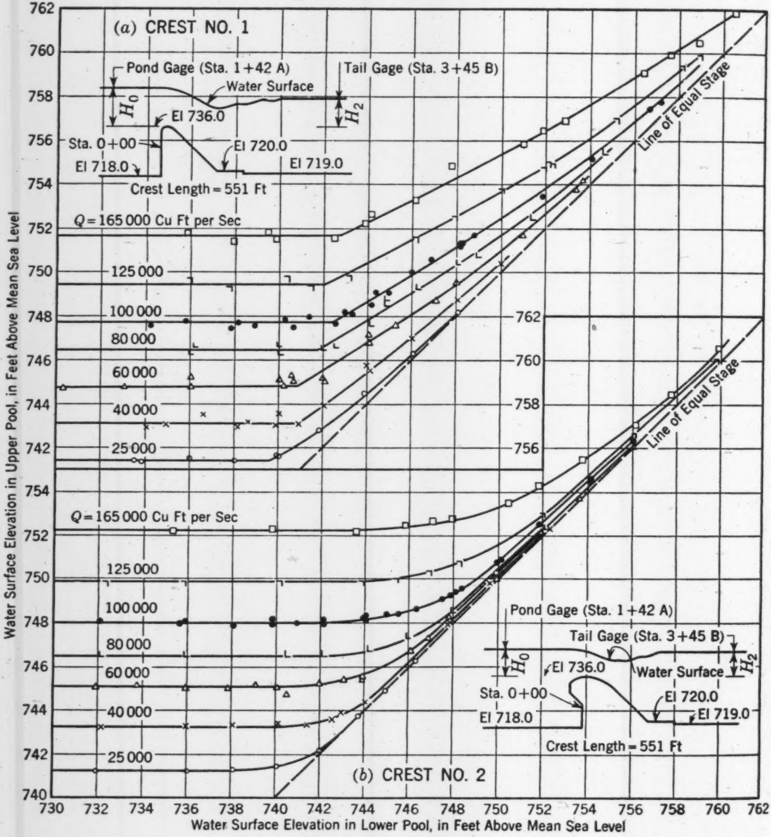


FIG. 83.—STAGE RELATION CURVES

pool were made at a distance of 345 ft downstream from the crest. In a typical test the water supply in the model was adjusted to provide a discharge corresponding to $\frac{50.6}{551}$ times an arbitrarily selected quantity of river discharge. When the flow had become steady and the tailwater level had been adjusted to a desired elevation, several readings were made on the weir gage and on the

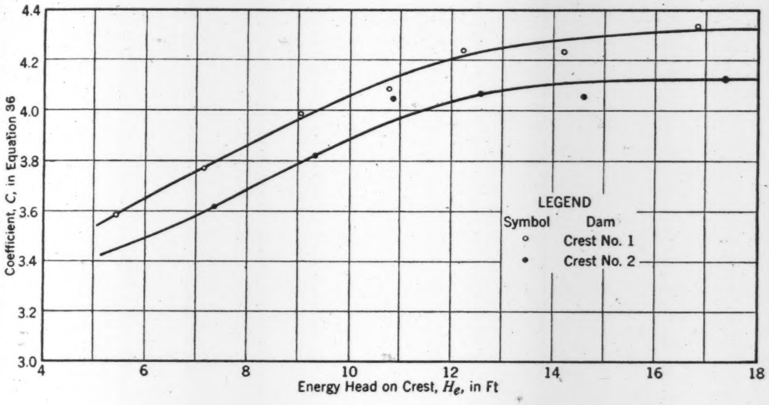


FIG. 84.—COEFFICIENTS FOR FREE DISCHARGE

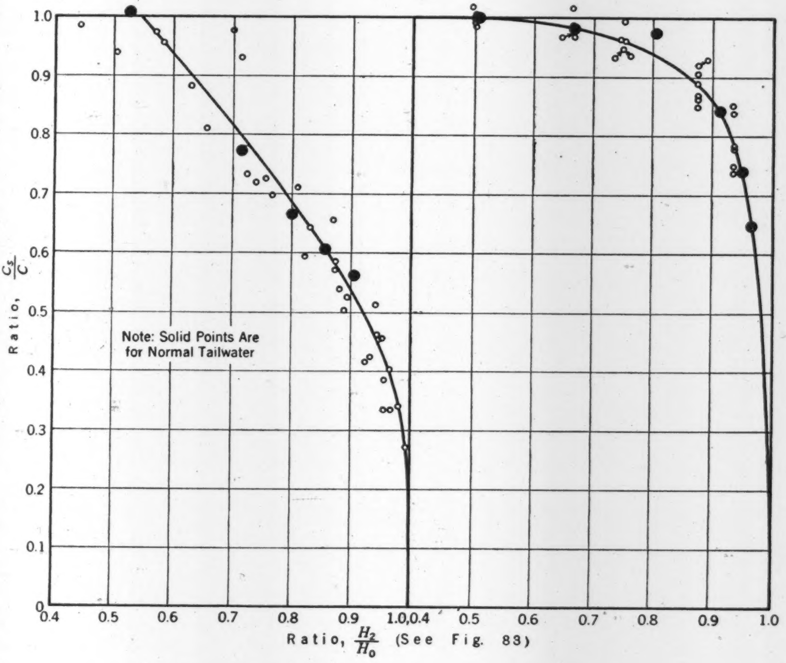
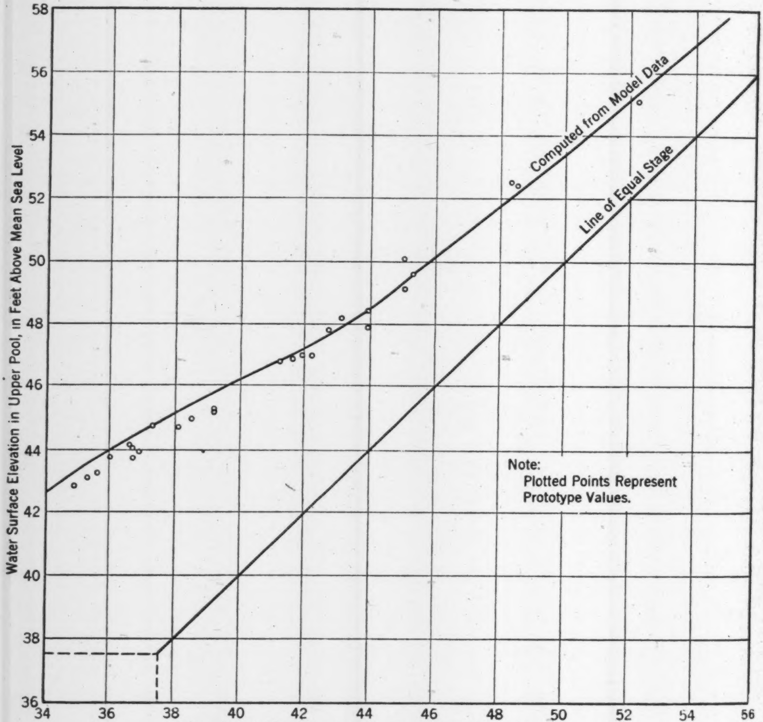


FIG. 85.—COEFFICIENTS FOR SUBMERGED DISCHARGE

upper and lower pool gages. While the water supply was maintained at the same rate, another water level was simulated in the lower pool, and the gages were read again. This procedure was followed until a sufficient number of points had been obtained to describe the headwater-tailwater relation for the given discharge. Then, a different rate of flow was established and the tests were repeated. Seven discharge rates ranging from 25,000 to 165,000 cu ft per sec were simulated.



Note:
Plotted Points Represent
Prototype Values.

FIG. 86.—STAGE RELATION CURVES—MODEL AND PROTOTYPE

The headwater-tailwater relations observed on the models of crest No. 1 and crest No. 2 are shown in Fig. 83. The coefficient C in the formula (see Eq. 13),

$$Q_p = C\,L\,H^{1.5}_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

computed for each discharge rate in the region of free flow is shown graphically with respect to total head in Fig. 84. Coefficients of discharge under submerged flow conditions were also computed by Eq. 13 (or Eq. 36) for each crest at several stages of submergence. Fig. 85 contains the ratios of coefficients for submerged to free flow in relation to the ratio of upstream to downstream water levels above the crest of the dam for crests No. 1 and No. 2, respectively. The tests indicate two marked distinctions in the characteristics of the respective crests. Crest No. 1 is shown in Fig. 84 to be more efficient in the region of free flow, whereas crest No. 2 is decidedly more efficient under submerged flow conditions. The change takes place at approximately $\frac{H_2}{H_0} = 0.50$, the point below which the submergence has no apparent effect on the coefficient ratio. These results verify conclusion (*a*) reached by Mr. Soucek: "The effect of submergence on the discharge is affected by the shape of the spillway * * *." However, contrary to Mr. Soucek's findings, the tests on crest No. 1 and crest No. 2 showed no consistent relation between the rate of discharge and the coefficient pattern.

Comparison of Model and Prototype.—After the prototype had been constructed, the water-surface elevations in the upper and lower pools were observed on staff gages at frequent intervals during the passage of several peak rates of discharge over the crest. From these readings, hydrographs were constructed from which the simultaneous elevations of the upper and lower pools were read at the peak of the hydrograph when the flow could be assumed to be approximately constant. The relation of the water levels observed during several freshets is shown in Fig. 86. In the same figure is shown this relation computed from the discharge coefficients determined in the model. The pool relation curves shown in Fig. 83(*a*) could not be compared directly with the data observed in the prototype because the prototype crest was constructed to a higher elevation than the model crest and, furthermore, because the tailwater rating curve existing in the prototype was different from that used in the model. This correlation between the model and prototype is considered to be very satisfactory and substantiates Mr. Soucek's conclusion (*d*)—"Reasonably good agreement may be expected between spillway models and prototypes of the sizes used in these tests under submerged as well as free flow conditions."

JACOB E. WARNOCK,[36] M. AM. SOC. C. E., AND H. G. DEWEY, JR.,[37] JUN. AM. SOC. C. E.—It is most gratifying to note the number of excellent discussions provoked by this Symposium. No doubt the wartime duties of many other interested engineers have prevented them from submitting a discussion or concrete evidences of model-prototype conformity. The excellent examples of conformity submitted by Messrs. McConaughy, Hall, and Blaisdell add much to the value of the Symposium.

Professor Streeter has stated that personnel of hydraulic laboratories are inclined to rely upon the model for the answer to all hydraulic questions instead of applying fundamental principles. It is true, of course, that in recent years hydraulic engineers have relied heavily upon hydraulic model studies to solve many unprecedented problems. Professor Streeter knows from his experience that many of these problems are not susceptible to fundamental analysis, and, even if an application of fundamental principles were possible, those responsible for the design of a structure would not be satisfied until a model test had been made. On the other hand, many hydraulic problems have been analyzed in laboratories by application of fundamental principles, resulting in a limited amount of testing or no testing at all. Eventually, hydraulic engineers should become more adept at preparing satisfactory designs of structures by using knowledge gained from previous model tests of similar structures and from checks on the prototype performance. As soon as the modern theories of fluid mechanics become more practical and are shown to be entirely adequate, the need for model tests will probably decrease and the general application of fundamental principles should be predominant. Professor Streeter believes that the application of fundamental principles will yield more reliable information at a smaller cost. This might be quite true, provided that the reliability can be established to the satisfaction of all concerned. Unfortunately, this is not generally the case, since some designs so analyzed have either yielded uneconomical solutions or inadequate designs as shown by subsequent model studies. An inspection of some of the older hydraulic structures in the United States also will show this to be true, since model studies were not used so extensively at the time of their design, but fundamental principles were relied upon almost entirely.

---

[36] Engr., Hydr. Testing Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

[37] Capt., Corps of Engrs., U. S. Army, U. S. Waterways Experiment Station, Vicksburg, Miss.

Eq. 18, computed by Professor Streeter, is in good agreement with Curve 3, Fig. 13. It will be recalled that Curve 3 was obtained by treating the bellmouth entrance as an orifice, using the pressure drop as measured in the bellmouth of the prototype (Fig. 4). Either Eq. 18 or Curve 3 is more reliable than Curve 4, which was derived from prototype discharge measurements. Since these discharges were obtained by a rather indirect method, they were not used to compute the prototype friction factor.

In regard to neglecting the kinetic energy correction factor, Professor Streeter suggests that, in using $K_e = 1.00$ instead of 1.02, the prototype friction factors in Fig. 6 may be 25% too high. If, in 40 ft of conduit, the measured friction loss is assumed as 2.87 ft for a head of 150 ft, the friction factor would be 0.0073; if $K_e = 1.02$, the friction factor would be 0.00715, or 2% less than the former. Neglecting the kinetic energy correction factor would not change the measured friction loss, of course, but would make the computed friction factors too high (although only about 2% and not 25%). It was previously recognized that the velocity distribution in so short a conduit would not be sufficiently developed to allow a good determination of the friction factor. As Professor Streeter suggests, velocity distributions should have been obtained in the test section, but this was impossible since no provisions were made on the prototype for velocity measurements—another example of the difficulty of obtaining proper measurements and correlation.

In regard to the statement that the discrepancy between model and prototype pressures was partly due to unstable flow in the model, it was felt that viscous effects in the model were more predominant than in the prototype. That this may be true is demonstrated by Mr. Douma (see Fig. 80).

Professor Streeter's demonstration of estimating the prototype pressure drop from crown to invert from the model pressures is particularly interesting. Although the difference in pressure between the crown and invert is of interest, the actual pressures along the crown and invert are of more concern.

Messrs. Tower and Pafford request that hydraulic laboratories compile data from model experiments and prototype measurements in a form suitable for design purposes, thereby aiding designing engineers and reducing the number of model tests of nearly identical structures. This is a good request and should be given more attention. Unfortunately, those who control the functions of many hydraulic laboratories are primarily interested in the solution of immediate problems and, as yet, have allowed those in the laboratory but little time for compiling data or for conducting research which would eventually furnish design data. In regard to data on the curve of the lower nappe of the sharp-crested weir, the laboratory of the Bureau of Reclamation, in 1942, completed a comprehensive research program on the nappes of such weirs (77).

Mr. Pafford's suggestion of keeping piezometers open in the prototype by frequent flushing with clear water is recommended, but, as he suggests, it will be a nuisance to the operating personnel. It might be added that it would be more of a nuisance during construction when the danger of plugging is probably the greatest. The use of removable, flexible, corrosion-resisting wire would be excellent for short piezometer tubes, but for long tubing this method would probably be impracticable.

Mr. Gilardi wishes to lift the veil of secrecy surrounding some engineering mistakes, and cites as examples two cases where solutions by model studies were not followed, causing inadequate prototype designs. He seems to blame both the laboratory and project, or consulting engineers. Obviously, if the latter group has a poor working knowledge of model studies and little faith in the recommendations obtained from them, and wishes to adhere strictly to its own preconceived ideas, then unfortunate results may occur. On the other hand, there should be little excuse for well-trained and thoroughly experienced laboratory personnel arriving at erroneous conclusions, provided they receive full instructions relative to the prototype conditions from the project engineers. In this regard it is believed that better results can be obtained to the satisfaction of all concerned if the designing engineers are able to keep in close contact with the laboratory through frequent visits.

Mr. Douma's discussion of similitude is quite appropriate. It is agreed that better results could have been obtained had the model of the outlets been built to a larger scale—the surfaces, however, were about as smooth as could be obtained. Had the tests been run at higher heads, disregarding the scale ratio, it is believed that the pressure analysis would have been more consistent using the dimensionless parameters shown in Figs. 7, 8, 10, and 11. The use of outlet models of sufficient size to obtain a larger Reynolds number so as to be somewhat closer to the prototype values of friction factors would have been impossible because the discharge facilities of the laboratory would have been exceeded. On problems where the prototype Reynolds numbers are not too large, it may be easy to close the gap between model and prototype; generally, however, the value of the Reynolds number in the model would have to be increased considerably to reach the point where the variation between the friction factor and the Reynolds number is negligible and at the same time of the same order of magnitude as the prototype friction factor for the entire range of discharges to be studied.

MARTIN E. NELSON,[38] M. AM. SOC. C. E., AND JAMES J. HARTIGAN,[39] ESQ. —An interesting analysis of the relative effectiveness on lock efficiency of area changes in culverts and ports has been developed by Mr. Kreiss. From Eq. 33, he has derived an expression for the area ratio which satisfies the condition of equal percentage changes in port area and culvert area being equally effective in changing the discharge capacity. When μ is 0.15, this area ratio of 1.45 is numerically about the same as the area ratio for square-cornered ports used in early lock designs.

Also, it can be shown from Eq. 33 that the upper limit of the area ratio, at which percentage increase in port area affects the discharge capacity, is

$$x = \frac{1}{\mu \log_e{}^{10}} \dots\dots\dots\dots\dots\dots (37)$$

This limiting area ratio is 2.89 when μ equals 0.15.

For the venturi-shaped ports of the first[6c] and third[6d] models, it is obvious from Figs. 25(*a*) and 34(*a*) that the upper limit for effective increase in area ratio has not been reached in either model test. Area ratios for these two models are 2.22 and 1.65, respectively. Similarly, in the third lock[6d] prototype, having an area ratio of 2.19, all lock-chamber ports are used effectively. However, in the first lock[6c] prototype the distribution of flow curve of Fig. 25(*a*) shows an effective use of only 17 of the 20 ports, corresponding to an area ratio of 1.9. The μ-value satisfying Eq. 37 is 0.23 when the area ratio is 1.9. It would be desirable to have series of tests on venturi-shaped ports, having different divergence angles, to determine whether μ in Eq. 31 remains constant.

Tests have been made on a lock model equipped with ports similar at the culvert end to the bellmouth entrance of the lock-chamber ports of the third lock[6d] (Fig. 24), but having parallel instead of divergent walls. This type of port has a larger discharge capacity than the venturi-shaped port for corresponding area ratios. The tests showed that increase in area ratio above 1.8 did not change the discharge capacity of the lock, area ratios of 1.0, 1.4, 1.8, 2.2, and 2.8 having been tested.

The authors are very grateful to Mr. Webster for his description of the testing apparatus used in obtaining performance data in the prototype locks. This apparatus was developed after a considerable investigation of commercial hydraulic testing equipment failed to bring to light any instruments suitably adaptable to the tests in view. The equipment proved to be remarkably well suited to the severe conditions under which it was necessary to operate.

The coordination and timing of tests performed simultaneously and at great distances on the lock chambers were facilitated greatly by using two sets of sending-receiving short-wave radios to transmit signals from the central operating station to the observation stations. For instance, in making simultaneous lock-stage tests at the two ends of a lock, time signals, originated by an automatic electric timer, were picked up on the radio receiving sets at the observation stations, indicating to the observers when to record the stage on the manometer charts. Likewise, the observers could be kept informed of the start, end, and intermediate progress of the valve operation, and other pertinent information could be transmitted readily from one station to another.

---

[38] Senior Engr., U. S. Engr. Office, St. Paul, Minn.

[39] Asst. Engr., U. S. Engr. Office, Iowa City, Iowa.

[6c] Pickwick Lock.

[6d] Guntersville Lock.

J. C. Stevens,[40] **M. Am. Soc. C. E., and R. B. Cochrane,**[41] **Assoc. M. Am. Soc. C. E.**—From the standpoint of design of the crest profile, Mr. Grimm's discussion of the writers' paper was enlightening. The writers realized that operation of the dam with the gates in the upstream slots would be abnormal and that such gate operation was for emergency purposes only. Since such operation data were procured with the model, a splendid opportunity to obtain corresponding prototype data was presented. The fact that the prototype pressures obtained with the gate in the upstream slot did not corroborate the model pressures constitutes certain "negative" information which at times is just as valuable as "positive" information. The writers agree most heartily with Mr. Grimm in his statement that "A model study of spillway crests under controlled atmospheric pressure would be timely and of decided value as a research project to supplement present knowledge."

Edward Soucek,[42] **Assoc. M. Am. Soc. C. E.**—The interest shown in the paper is most gratifying; particular thanks are due to those who have presented previously unpublished data.

Professor Johnson's remarks deal primarily with unsubmerged spillways, but the tests of the Upper Narrows Dam are of much interest. It is regretted that the model details were not illustrated or described. Since the prototype is curved in plan, similarity demands convergent flow over the spillway and radial approach velocity; these conditions are difficult to obtain on a sectional model in a flume only 6 in. wide. End effects sometimes are feared in so narrow a flume, even when a straight spillway section is represented. The agreement between discharge curves of the 1 : 25 and 1 : 40 models seems reassuring in this connection; certainly the agreement tends to confirm the principle that a spillway model need not exceed a reasonable size. The small effect of surface roughness upon the discharge of the two larger models is highly significant. The curve for the 1 : 100 model shows that it is too small for rating the prototype; it may be too small also for accurate determination of the nappe profile. In connection with that phase of the work especially, but also for the discharge measurements, supplementary measurements were undoubtedly necessary to determine that a 1-in. pipe provided complete aeration of the nappe. The writer has used a small U-tube, partly filled with water and connected to the space beneath the nappe, to indicate the subatmospheric and fluctuating pressures that accompany incomplete aeration. This check is recommended strongly as surprisingly large vents have been required in some cases.

---

[40] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[41] Capt., Corps of Engrs., U. S. Army, Res. Engr., U. S. Engrs., Bonneville, Ore.

[42] Senior Civ. Engr., U. S. Engr. Office, Wilmington, N. C.

It is doubtful that a "more definite measure of the agreement between the various studies" is gained by use of the exponential equations derived by Professor Johnson. For that purpose, the curves of Fig. 64 leave nothing to be desired. It appears that use of the conventional exponent with a varying coefficient is more convenient for nearly all purposes, cases in which the discharge equation must be integrated being an important exception.

Professor Johnson's statement that "model studies made of existing structures are more likely to be academic in character" is a well-meant generalization; it is questioned only because it is not entirely harmless. In the writer's experience, exceptions have been so frequent that they cannot be said to prove the rule. A few cases will be mentioned: (1) A large spillway was tested to determine a rating which was not otherwise obtainable and for study of the effect of proposed modifications; (2) a large waterway was studied to verify the ability of the model to reproduce wave phenomena previously observed on the prototype—then the model was used not only for the study of proposed modifications but also for investigation of extreme conditions which it was unsafe to create and impracticable to observe on the prototype; and (3) a lock was tested to verify the subsequent use of models for study of proposed structures and for the development of apparatus and technique—an inexpensive, practical, and important modification, which probably will be applied to the existing structure, was discovered as an incidental by-product. It is true that models of existing structures frequently are tested in college laboratories. Such tests may indicate a feasible modification for improved performance or may shed light on a poorly-understood phenomenon which has been observed on the prototype. A useful by-product of the tests described in the paper was their immediate local application for stream gaging under flood conditions. Possibilities for valuable model-prototype comparisons are almost unlimited. A student can make a far worse choice in research problems than a study of an existing structure. Believing that such studies should be encouraged, the writer hopes and believes that their character will be judged by the care with which the subject is selected, the manner in which the experimental work is performed, and the skill with which the test results are interpreted.

Professor Cox points out that a sharp-crested weir is affected by the tailwater elevation when submergence, as defined by the writer, does not exist. The point is well taken; nevertheless, the writer prefers the original definition to the one proposed on the basis that the name of a phenomenon should be descriptive rather than indicative of its effects. The more general term "affected by tailwater" would cover the situation.

The writer does not advocate any significant departure from geometric similarity; accordingly, he agrees with many of the statements by Professor Cox regarding the flow downstream from a submerged dam. He still believes, however, that the tailwater should be measured beyond the effects of the dam whenever possible and that this is possible in most cases. The channels downstream from low dams can be represented reasonably well by a flume, when necessary, by molding the channel bottom to represent the variation in depth with distance from the dam. Only if such representation is impossible would the writer adopt the expedient of measuring the tailwater within the effects of the dam. In such a case, the problem of relating the observation to the backwater curve downstream from the dam remains unless discharge measurement is the only objective. The error possible in making this estimate is believed at least to equal the effect of a reasonable approximation to the channel cross section in the model. The several advantages of measuring the tailwater beyond the effects of the dam were stated in the paper and need not be reiterated.

Fig. 68 exhibits a tendency similar to that revealed by Fig. 54—the greatest submergence at which the plunging nappe condition can be maintained decreases as the head increases. Professor Cox' correction curve for adjusting the observed submergence would be expected to exhibit a loop or other discontinuity corresponding to the changes in behavior of the nappe. By the use of this curve and the equations which he derived, Professor Cox obtained computed discharges in good agreement with the observations. In at least one other case (50) Professor Cox has used his method with good results. The writer does not know the scope and limitations of the method. Dimensionally, it appears subject to question but it has been used very successfully by Professor Cox.

In stating that no gagings were made when the ice jam was within 500 ft of the dam, the writer was trying to show only that the jam had no effect except the raising of the tailwater to a level somewhat higher than would occur normally at the same range of discharge. It is regretted that this statement led Professor Cox to speculate upon the existence of a reverse bottom current. It is believed that such a current would have been detected from the angle assumed by the cable which supported the meter and the weights. During all except one of the measurements, the distance to the ice jam was much greater. Angularity of subsurface velocities and turbulence may have affected the measurements, producing results similar to those suggested by Professor Cox.

The data presented by Mr. Walton, as he states, are surprising in their close agreement to the U. S. Deep Waterways tests. Probable reasons for this correspondence are considered subsequently. On the basis of Mr. Walton's observations, the writer agrees that the behavior of the nappe requires consideration in some cases. It might be inferred that this effect is present or perceptible only when the spillway is particularly "sensitive" to submergence. The writer's opinions concerning the measurement of heads coincide with those of Mr. Walton; certainly the tailwater may be measured within the region affected by nappe conditions when no alternative exists.

Mr. Tower expresses the now general acceptance of the nappe-shaped profile. It is interesting to point out, however, that the shape of the dam tested was established by an eminent hydraulic engineer, not "by reason of the lack of adequate information" but in order to modify an existing structure most economically, so as to accommodate a large tunnel for steam pipes and other utilities. The dam always is submerged when a major flood is in progress. The writer knows of no evidence that the spillway is not as efficient as a conventional nappe-shaped profile when the submergence is high; there is no deterioration that could be attributed to the shape of the spillway. This exception to a perfectly valid general rule illustrates the need for complete data before passing judgment on a design.

Professor Mavis presents a useful formula and valuable data. His qualified acceptance of the writer's statements regarding the effect of nappe conditions upon discharge is especially pertinent in view of Mr. Walton's observations. The equation presented is equally applicable to both sections shown in Fig. 73. Since the effect of submergence doubles for each 5% increment in the submergence, the relationship is remembered easily. For a rounded cross section which supports the nappe, the equation is certainly more correct than the widely-quoted and much-used curve based on the U. S. Deep Waterways tests. Further evidence to this effect appears in another discussion which is considered later.

Professor Mavis' conclusion (b) that "the generalized relationship between submergence percentage, *p*, and the ratio of submerged to free discharge, * * * is valid within the scatter of field observations on the prototype" is accepted but the qualification should not be ignored. For preliminary designs or for estimates of the effect of submergence upon the discharge over well-rounded crests when model data are not available, the equation may be used to good advantage. For analysis of test data and their application to geometrically similar prototypes, retention of the discharge as an independent variable is believed to justify fully the minor complications necessitated.

Mr. Nelson kindly has devoted a large part of his discussion to the writer's paper. This presentation of new data leaves little opportunity for comment in addition to a statement of approval. Attention is called to the better collocation of those observation points which represent normal tailwater in Fig. 85. These data support the contention that discharge should be treated as an independent variable. The agreement of the "effect of submergence curve" for the well-rounded crest with the equation suggested by Professor Mavis confirms the applicability of the relationship.

Mr. Nelson measured the water levels beyond the immediate effects of the dam as the writer recommends. The agreement between model data and prototype observation is highly satisfactory, particularly in view of the minor departures from complete similarity. In one respect, these tests were contrary to the writer's findings; the tests did not reveal a consistent relationship between the discharge and the effect of submergence. In this connection, the comments accompanying Figs. 52 and 53 should be emphasized. The tendency for the effect of submergence to increase with the discharge is clearly evident but the consistency of the data is exaggerated to some extent. When the curves were sketched originally, all differed in shape; some actually intersected. By a process of repeated adjustment, all of the curves except the one for the lowest discharge were drawn to conform to an average pattern. It seems possible that a similar smoothing process would reveal a consistent tendency if applied to the data of Mr. Nelson. Fig. 83 shows an excellent method of representing submerged spillway data graphically.

The last paragraph of Mr. Blaisdell's discussion deals with the writer's use of the discharge as an independent variable; he recommends that this variable be ignored in determining the effect of submergence. The writer emphatically disagrees with the suggestion and the implied ideas.

At a submergence of 85%, the reduction in discharge produced by submergence may be as little as 7%. The 3% error which Mr. Blaisdell considers tolerable thus amounts to more than 40% of the effect being investigated. For high submergences, the absolute error in discharge can be at least as great as 10%, and much greater if the average curve is based on a small range of discharge. It is true that for submergence exceeding 85% "the submergence curves are steep, and small errors in determining the submergence result in large errors in the submerged discharge correction." However, water levels are observed so easily and accurately that there is no reason whatever for any significant error in the determination of submergence. An average submergence curve is also steep. The remote possibility of a small error in the submergence by no means justifies the deliberate introduction of a large error, particularly when the only possible gain is the avoidance of a simple computation requiring at most a few minutes' time.

The data were presented in a manner intended to expedite comparison with published material. It was stated that a direct method of making the calculation could be derived easily if a large number of calculations were necessary. The data were so arranged that no trial computations were required in order to check the results. The measurements on the model were intended for comparison with prototype measurements; an average difference of a few per cent, probably caused by errors inherent in the method of gaging the prototype, was anticipated and realized. The possibility exists that, in the future, the model measurements will be compared with more accurate prototype observations. Under these conditions, use of Mr. Blaisdell's method would have been improper. If (as Mr. Blaisdell states) the result of a refinement which avoids an error of 10% or more is a fictitious accuracy, literature records many unwarranted model studies.

In the application of submerged spillway data to a particular problem, the user is best qualified to judge the magnitude of tolerable errors. For estimating a flood-crest elevation, precision might be quite unnecessary; in designing a submerged sill to regulate the Great Lakes, accuracy of the highest order might be essential. It is believed that the experimenter is under obligation to present data with meticulous precision, if possible in a form which enables the designer to make such short cuts as the specific application may justify. A statement to the effect that most uses do not justify great precision is true but hardly necessary—this fact is well known and universally accepted. Objection to an accurate analysis which permits the designer to make an application consistent with his purpose, and without disadvantage, is not considered indicative of a "practical standpoint."

A careful distinction should be made between a legitimate simplification and a rough approximation adopted only to avoid computation. A simplification is proper when the materials dealt with have physical properties which are so complex or variable that an involved equation is unwarranted for physical reasons and undesirable because it obscures significant relationships. Mr. Blaisdell's suggestion is not based on physical reasons; nor does it result in a simplification warranted by the loss of accuracy involved. Its proposal can be based only upon an unwarranted mistrust of the reliability of hydraulic models or of the accuracy obtainable in the design of hydraulic structures.

A number of the discussers have commented favorably upon various indications of accuracy in the experimental work. The writer wishes to emphasize, as stated in "Acknowledgment," that the model tests were conducted by Mr. Morgan.

There were no comments on the writer's statement that the discrepancy between model and prototype is probably chargeable to the latter. Five years have passed since, in 1938, this statement was made in the original draft of the paper. The writer's present belief is that the discharge indicated by a well-designed spillway model, when the model head exceeds 3 in. or 4 in., is fully as accurate as that obtainable by any field method in current use.

The data presented in the discussions fully confirm the conclusion that the effect of submergence upon the discharge is a function of spillway shape. This fact has not always been as evident as it now appears; once the writer was advised to check some submerged spillway data very carefully because they did not agree with the U. S. Deep Waterways curve. This suggestion was influenced by the agreement between Mr. Walton's data and the U. S. Deep Waterways tests. Although Mr. Walton's spillway and the U. S. Deep Waterways spillway differ in shape, they have a common characteristic. This is the factor believed to control the effect of submergence upon the discharge—namely, the distribution of pressure within the nappe at or near the spillway crest. Since the nappe springs free of these spillways or tends to do so, the occurrence of subatmospheric pressure beneath and within the nappe affects the head-discharge relationship. Submergence of these spillways tends to modify pressure conditions radically at the crest, making them "submergence sensitive." When the nappe is in contact with the spillway at all points with atmospheric pressure at the water-masonry interface, or particularly when the nappe exerts a pressure against the spillway, submergence has a smaller effect upon the pressure distribution at the crest. For this reason, a dam with a stable nappe is less affected by submergence. This is the case on the spillways tested by Professor Cox, Professor Mavis, Mr. Nelson, and the writer. The difference in the submergence effect in the two crests tested by Messrs. Nelson and Hartigan confirms this idea. A comparison of the several spillway sections reported indicates that this explanation is reasonable. (It should be observed that the vertical face of the U. S. Deep Waterways section is on its downstream side.)

If (as the writer believes) the data brought to light in the discussions is indicative of the wealth of information available from the personal files of engineers, the mere announcement, for discussion, of subjects carefully chosen by the appropriate committees may be a more effective method of developing references upon suitable problems than waiting—sometimes many years—for an author to introduce a subject concerning which information is needed badly. In the present case, the prevailing method of computing the effect of submergence upon the discharge actually produced errors, when applied to modern spillways, greater than the effect of completely ignoring the submergence.

That this observation can now be made as an established fact rather than as an isolated opinion is due to the efforts of those who presented discussions. The writer is pleased in having had a small part in correcting this situation and in making more reliable quantitative data generally available.

FREDERICK R. BROWN,[43] JUN. AM. SOC. C. E.—Most of the papers and discussions on model-prototype conformity have dealt with hydraulic structures and tests on models constructed geometrically similar to their prototype counterparts. There are other types of model studies, however, which were given little emphasis in the Symposium. This is probably due to the fact that few engineers have tried to study the model-prototype conformity that exists for geometrically distorted models constructed for the solution of problems on channel capacity and bed movement.

Models of hydraulic structures practically always are constructed to the same horizontal and vertical scales, and dynamic similitude is attained in so far as possible by operating these models in accordance with the Froudian scale relationships. In most cases a large model scale must be used to satisfy the roughness ratio requirement which can be shown to be equal to $(L_r)^{1/6}$; thus, as the model scale decreases, the smoothness of the model must be increased. Geometric similitude must be maintained because it is important to the success of the study that the magnitude and direction of prototype forces and flow paths be reproduced accurately.

In the case of some river models, however, where the area to be reproduced is large and it is necessary that the vertical scale be larger than the horizontal scale to obtain measurable quantities on the model or where it is necessary to add an additional slope distortion for movement of bed material, neither geometric similitude nor correct dynamic similitude is obtained. Complete similitude is not essential if care is taken in selecting the model scales for solution of the problem at hand. If the problem is one involving channel capacities or study of flood-crest profiles, the model scales can be distorted considerably inasmuch as there is no need for the exact reproduction of velocity distribution, paths of flow, etc. All that is necessary to insure reliable model results in the foregoing case is to adjust the model roughness in order that a discharge and velocity scale will be obtained that will permit the reproduction of the desired stage-discharge relations and the average change in potential and kinetic energy at varying cross sections. If the problem involves the movement of bed material, the distortion of model scales must not be too large. The success of this type of model depends upon the reproduction of the proper velocity distribution for movement of the bed material. If the horizontal and vertical scales selected are such that the distortion is low and the stream is wide with respect to the depth, the distortion will not alter the general shape of the channel, and close similarity of velocity distribution will exist.

In the case of the Mississippi River channel model described in the Symposium, the horizontal scale was 1 : 600 and the vertical scale was 1 : 150

---

[43] Associate Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

which gives a distortion of 4. This distortion was small enough so that the velocity distribution and current directions were reproduced accurately. No tests of improvement plans were undertaken, however, until the accuracy of the model was insured. The accuracy of the model was established by a verification procedure in which model operating conditions were developed such that the movement of bed material in the model was consistent with that in the prototype. In order to provide adequate movement of the coal bed material, it was found necessary to provide a slight additional slope distortion. Instead of altering the slope, either the discharge scales or the time scales or both could have been adjusted to reproduce the prototype bed movement—that is, the discharge scale or the length of run at low stages could be increased over that used for high stages.

Mr. Einstein states that, during a visit to the U. S. Waterways Experiment Station at Vicksburg, he was astonished to see how much the natural roughness of a model must be increased to counteract the effect of the distortion on friction. This raised the question in his mind of the reliability of distorted hydraulic models.

The actual roughness used in distorted channel models at the Experiment Station is placed in accordance with accepted model practice and can be shown to be mathematically correct. If the linear scale and the distortion is small, however, it may become impossible to make the model smooth enough. On the other hand, if the distortion is too large, the model cannot be made rough enough. Between the two aforementioned extremes, there are model scales that will permit the use of the theoretical roughness ratio [$n_r = (R_r)^{2/3} (L_r)^{-1/2}$], in which R_r is ratio of the hydraulic radii for the model (R_m) to the prototype (R_p). It is not necessary, however, to use the theoretical roughness values inasmuch as a number of discharge scales with their corresponding roughness values will provide identical flow lines.

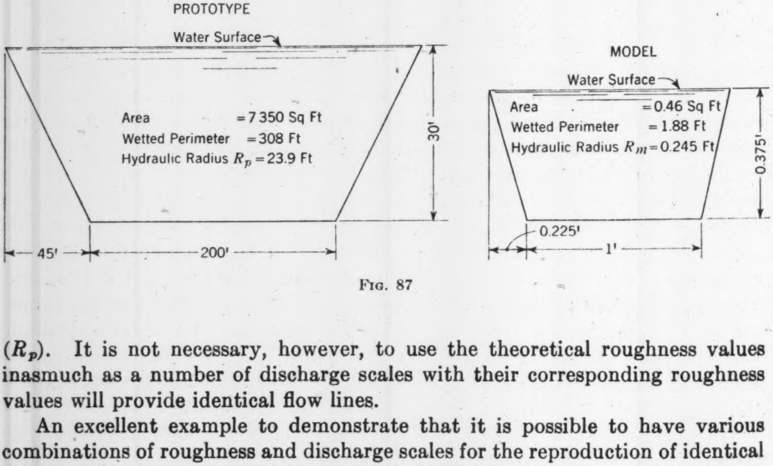

PROTOTYPE
Water Surface
Area = 7 350 Sq Ft
Wetted Perimeter = 308 Ft
Hydraulic Radius R_p = 23.9 Ft
30'
45'
200'
MODEL
Water Surface
Area = 0.46 Sq Ft
Wetted Perimeter = 1.88 Ft
Hydraulic Radius R_m = 0.245 Ft
0.375'
0.225'
1'

FIG. 87

An excellent example to demonstrate that it is possible to have various combinations of roughness and discharge scales for the reproduction of identical flow lines was computed by Lieutenant Tiffany (78). It was shown that, with a model constructed to a horizontal scale ratio of 1 : 200 and a vertical scale ratio of 1 : 80, the following relationships would obtain for the channel whose cross section is shown in Fig. 87: $R_r = \frac{R_m}{R_p} = \frac{0.245}{23.9} = \frac{1}{97.5}$; and

$$Q_r = (L\,y\,V)_r = \left(\frac{R^{2/3}\,L^{1/2}\,y^{3/2}}{n}\right)_r = \frac{1}{n_r} \times \left(\frac{1}{97.5}\right)^{2/3} \times \left(\frac{1}{200}\right)^{1/2} \times \left(\frac{1}{80}\right)^{3/2} = \frac{1}{n_r} \times \frac{1}{215{,}000}.$$

Thus, from the foregoing computations it can be seen that the discharge and roughness ratios are dependent upon each other. Assuming the prototype roughness to be about 0.024, the following conditions would obtain:

Q_r	n_r	n_m
1 : 120,000	1 : 1.79	0.0134
1 : 128,000	1 : 1.68	0.0143
1 : 143,000	1 : 1.50	0.0160
1 : 150,000	1 : 1.43	0.0168

In this case the theoretical discharge scale of 1 : 143,000 [$Q_r = L_r\,(y_r)^{3/2}$] could have been used in the model since a model roughness of 0.0160 is possible of attainment. However, in the event that the model roughness indicated by the theoretical relationship was too rough, the model could have been roughened as much as practical and the discharge scale changed accordingly to reproduce the prototype flow lines.

For further detailed information regarding the accuracy of distorted models and the verification procedure, for movable bed models, reference is made to the Society's Manual on "Hydraulic Models" (78*a*). It is recommended that any one connected with model investigations familiarize himself with this excellent dissertation on both distorted and undistorted models.

All in all, the distorted model provides a valuable tool for the solution of difficult problems. If the tool is handled correctly, a quantitative interpretation of the results pertaining to the problem for which the model was designed is possible. The departure from conditions of similitude, however, makes it necessary to qualify other results or information. Experienced model technicians realize the true value of distorted models and can be depended upon to interpret the model results correctly.

Comparable model and prototype data are difficult to obtain for channel models inasmuch as the structures or plans developed through model investigation often are not installed or have not had time since construction to indicate changes caused by their installation. Of the amount of information available, however, the prototype performance is bearing out the predictions of the distorted scale models, in practically every case.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2208

SEISMIC SUBSURFACE EXPLORATION ON THE ST. LAWRENCE RIVER PROJECT

BY E. R. SHEPARD,[1] ESQ., AND REUBEN M. HAINES,[2]
ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. WOODWARD MOORE, ARTHUR J. GRIER, F. W. LEE, L. DON LEET, WILLIAM L. SHANNON AND WINTHROP A. WELLS, AND E. R. SHEPARD AND REUBEN M. HAINES.

SYNOPSIS

The seismic method of subsurface exploration was used extensively by the Corps of Engineers, U. S. Army, in preliminary studies of the St. Lawrence River Project in the International Rapids Section. A well-defined interface between the glaciated rock and the overlying till made possible the determination of the depth of overburden with satisfactory accuracy. Considerable success was also attained in determining changes in the overburden. The program included tests in both quiet and swift water, for which special apparatus and methods were devised. The use of the seismic data enabled the design of concrete structures, navigation channels, and hydraulic cuts to proceed more rapidly than if exploration consisted only of drill holes. By correlation with drill hole data and the general geology of the region, the results aided in determining the extent and depth of various deposits of glacial till and other types of overburden and also their condition of density and compaction. The subaqueous tests are believed to be unique for obtaining information of the type required for this project. As the results of the subaqueous exploration were satisfactory for this project, the use of the seismic method should be broadened in future investigations to include similar conditions.

INTRODUCTION

Late in 1940 a program of subsurface investigations was inaugurated to obtain data for the design of structures for the St. Lawrence River Project by the Corps of Engineers, U. S. Army, with a district office at Massena, N. Y.

NOTE.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Senior Physicist, Corps of Engrs., U. S. Army, Washington, D. C.

[2] Engr., Corps of Engrs., U. S. Army, Massena, N. Y.

The project, extending a distance of approximately 45 miles, is located in what is known as the International Rapids Section of the St. Lawrence River. Because of the large number of concrete structures, dikes, navigation channels, and hydraulic cuts, the problem of making adequate subsurface explorations assumed major proportions. Prior to 1940, sufficient studies and investigations had been made by the United States and Canadian governments, and various private companies, to determine the general plan of the project and the approximate location of each element. The data, however, were far short of those which would be required for determining the most feasible and economical locations and designs for the various elements and for making cost estimates.

At the outset of the present exploratory program it was recognized that subsurface conditions were favorable to the seismic method of exploration, that the interface between the hard rock and the overburden could be determined with fair accuracy by that method, and that some success might be achieved in determining changes in the overburden. From the previous subsurface and geological surveys, it was known that the bedrock in the region is of sedimentary origin, consisting mainly of dolomite and limestone. Glacial action had removed the residual soft, weathered rock, leaving a generally smooth, undulating, hard rock surface of low relief, broken by occasional valleys, ridges, and faults. The overburden consists of deposits of variable glacial till which appears in the form of elongated ridges at the surface and deposits of clays, silts, and uniform sands in the valleys between the till ridges.

The purpose of the seismic program was twofold: (1) To guide the drilling program and thereby limit it to obtaining necessary and essential data; and (2) to augment the drilling program with a large number of less expensive tests in areas where the required information could be obtained by that method. Extensive tests were made at sites where very little prior information was available, particularly in the numerous areas where cuts, channels, and canals were projected. Tests were also made to obtain data between drill holes. Considerable work was done in the river channel as well as on land. At the Long Sault Dam site, seismic tests were made in very swift water where drilling would have been exceptionally expensive and hazardous.

Except during March and April, when the ground was frozen, the explorations were conducted almost continuously from November, 1940, to October, 1941. During this time 408 lines were explored on land, 38 in quiet water, and 11 single-shot determinations were made in very swift water. Of this number 300 lines were explored to obtain information for the layout and estimates of canals, cuts, and channels. The seismic field party consisted of an instrument operator, one authorized dynamite man, and six helpers. The field crew completed from 2 to 6 lines per day according to weather conditions and the accessibility of the locations. Film records were developed daily by a local commercial photographer.

THEORY AND APPLICATION OF SEISMIC METHOD OF EXPLORATION

Principles Involved.—A comprehensive discussion of the principles involved in seismic exploration will not be given, as this subject is treated adequately in a voluminous literature on the subject. A brief description of the theory is necessary, however, if the significance of the data presented herein is to be understood.

The seismic method of exploration is based on the fact that the velocity of wave propagation in the earth's crust differs greatly in different substances. Granular and plastic materials, such as sand, clay, and gravel, transmit wave disturbances at velocities of roughly 800 to 8,000 ft per sec, whereas rigid rock transmits such disturbances at 10,000 to 20,000 ft per sec. This wide range in velocities in different kinds of soils and rocks is largely dependent on their elastic properties, a factor closely related to rigidity and hardness. To what extent other physical properties, as moisture content, texture, compaction, void space, cementation, and homogeneity, may be determined from velocity measurements is not fully known, although they do enter into the problem.

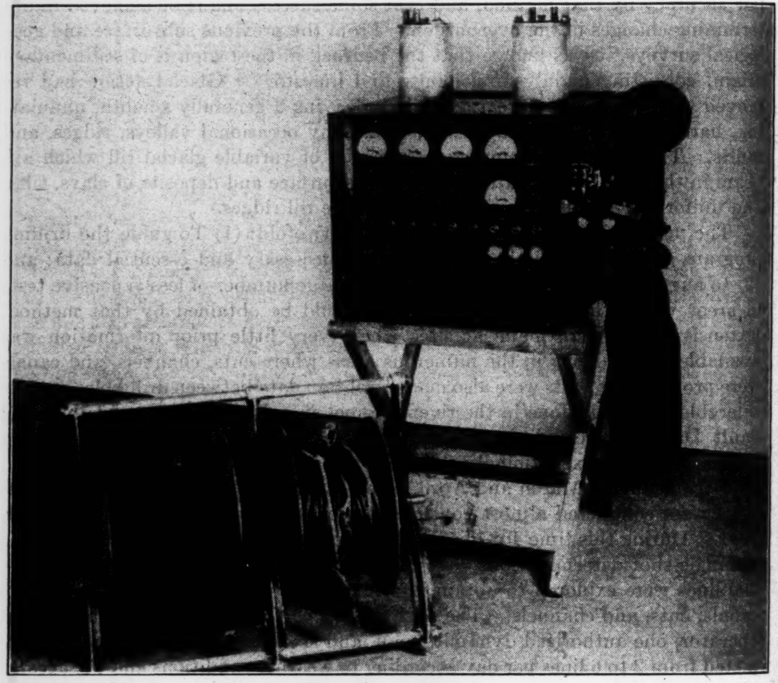


FIG. 1.—PORTABLE SEISMOGRAPH

Apparatus.—The seismic apparatus used for subsurface exploration on engineering projects consists essentially of one or more detectors or geophones and an oscillograph recorder. The detectors are connected electrically with the oscillograph so that the time of arrival of a wave disturbance at each detector may be recorded photographically. The disturbance is created by exploding a charge of dynamite in the ground at a known distance from the detectors. An electrical connection between the oscillograph and the detonating circuit records the shot instant. A time scale, made by an electrically driven tuning fork, is recorded on the oscillograph film. The seismic apparatus used in this investigation, shown in Fig. 1, was of a simple, portable type developed by the Public Roads Administration.

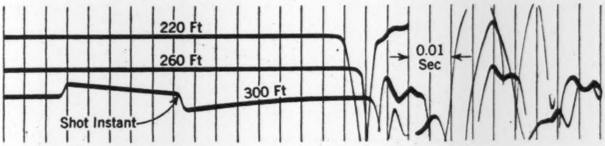


Fig. 2.—Typical Three-Channel Seismic Record

Application of Method.—With the type of apparatus used on the St. Lawrence River Project the usual field procedure is to place three detectors on the ground in a line, at intervals of 40 ft. Dynamite charges varying from 0.25 to 2.5 lb are buried to a depth of 3 ft and fired successively and at increasing distances along the detector line, beginning at 10 ft from the center detector and extending the shooting distance by intervals of about 50 ft to such lengths as may be required. A maximum shooting distance of three to four times the depth to which information is desired is usually necessary. Where possible the seismic lines are laid out along a surface contour in an effort to stay on one type and a nearly uniform depth of overburden. Although these conditions are not required for interpretation of the data, more accurate interpretations can be made if they exist.

From the film records, a sample of which is shown in Fig. 2, the time of the first arrival of wave travel from each shot to each of the three detectors is read and the results plotted in the form of a time-distance graph. In a homogeneous material the time-distance relation will be a straight line through the origin. This is evident from the fact that in such a medium the velocity of wave propagation is constant, or the time of travel is proportional to the shooting distance. It is also evident that the reciprocal of the slope of such a graph is a measure of the velocity in the medium.

When a layer of homogeneous soil is underlain by one through which waves travel at a higher velocity, such as that designated as clay in Fig. 3, there will be a critical distance, OF, for which the time of travel through the upper medium is just equal to the time of travel over the longer path which penetrates the lower medium. At this critical distance there will be a break in the time-distance graph to a different slope, CD, which represents the velocity of wave propagation in the second medium. In like manner, when the critical shooting distance, OG, is passed, the first arrivals will be through the high-velocity rock and the time-distance graph will assume the new trend, DW, the slope of which determines the velocity in the rock stratum. From the ordinates OA and AB on the time axis and the velocities indicated by the slopes of the several elements of the graph, it is possible to calculate the thickness of each stratum of overburden.

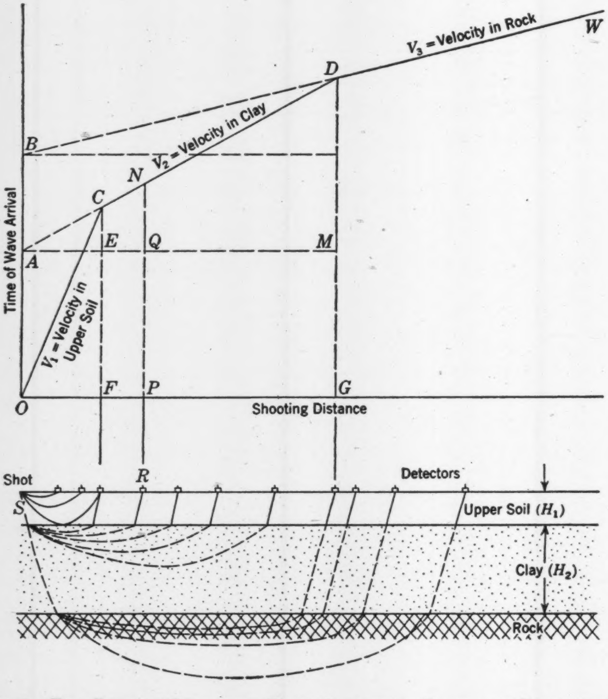

V_3 = Velocity in Rock
W
D
V_2 = Velocity in Clay
B
N
C
E
Q
M
A
Time of Wave Arrival
V_1 = Velocity in Upper Soil
F
P
G
O
Shooting Distance
Shot
R
Detectors
S
Upper Soil (H_1)
Clay (H_2)
Rock

FIG. 3.—TIME-DISTANCE GRAPH FROM WHICH OVERBURDEN DEPTHS ARE DETERMINED

Determination of Overburden Depth by Two-Way Shooting.—In determining the thickness of overburden by the seismic method used on this project, it was necessary to measure the velocities of wave propagation in the several strata of which the overburden is composed. Because of sloping interfaces between different strata and also velocity irregularities in the upper soil, only apparent velocities are obtained from shots along one end of the seismic line. To obtain true velocities, therefore, it was necessary to shoot against the detector setup from opposite directions. The true velocities in the deeper strata were then found by taking the harmonic mean of the respective apparent velocities. When shooting "up-dip" along a sloping interface the apparent velocity in the deeper medium is higher than the true velocity, and when shooting "down-dip" the apparent velocity is less than the true velocity. Variations in moisture content, compaction, and other physical properties of the soil immediately beneath the detectors give rise to similar effects which might easily be interpreted as indicating a sloping interface. Where erratic and divergent data are obtained as a result of such conditions, it is only by a careful analysis of all data pertaining to a given area that errors of interpretation can be avoided. Considerable experience, good judgment, a knowledge of overburden conditions as they exist in the field, and a thorough understanding of the principles involved are required if satisfactory results are to be obtained by the seismic method of exploration.

After the velocities have been determined the actual computations of the depths of the several strata are relatively simple. The formulas by which depths are computed are given by Maurice Ewing, A. P. Crary, and H. M. Rutherford.[3] These formulas in the forms most frequently used are as follows:

$$H_1 = \frac{T_1 V_1}{2 \cos \alpha} \qquad \text{(1a)}$$

$$H_2 = \frac{T_2 V_2}{2 \cos \beta_2} - \frac{H_1 V_2 \cos \beta_1}{V_1 \cos \beta_2} \qquad \text{(1b)}$$

and

$$H_3 = \frac{T_3 V_3}{2 \cos \gamma_3} - \frac{H_1 V_3 \cos \gamma_1}{V_1 \cos \gamma_3} - \frac{H_2 V_3 \cos \gamma_2}{V_2 \cos \gamma_3} \qquad \text{(1c)}$$

in which (see Fig. 3): H_1, H_2, and H_3 = thicknesses of various layers; V_1, V_2, and V_3 = wave velocities in corresponding layers; T_1, T_2, and T_3 = the time intercepts on the Y-axis, OA, OB, etc.; α, β, and γ = angles of refraction; $\sin \alpha = \frac{V_1}{V_2}$, $\sin \beta_1 = \frac{V_1}{V_3}$, $\sin \beta_2 = \frac{V_2}{V_3}$, $\sin \gamma_1 = \frac{V_1}{V_4}$, $\sin \gamma_2 = \frac{V_2}{V_4}$, and $\sin \gamma_3 = \frac{V_3}{V_4}$.

In some instances, particularly for two or three layer formations, it is possible to use a simpler form of computation which gives values closely approximating those derived by the standard method.[4] In Fig. 3 the shorter formulas become:

$$H_1 = \frac{\overline{\mathrm{OA}} \times V_1}{2 \cos \alpha} \qquad \text{(2a)}$$

$$H_2 = \frac{\overline{\mathrm{AB}} \times V_2}{2 \cos \beta_2} \qquad \text{(2b)}$$

Results of Explorations on Land

Typical Time-Distance Graph.—The exploration program on the St. Lawrence River Project included seismic determinations in the river channels as well as on land. The work on land was done in accordance with the aforementioned methods of field procedure and analysis of data. A typical time-distance graph, derived from data obtained on the right abutment of the Long Sault Dam site, is shown in Fig. 4 and illustrates the method of plotting the data. With three detectors spaced at 40-ft intervals along the shot line, three points on the graph were obtained from each shot, as shown in Fig. 4. Shots ahead of the center are shown by open characters, and shots back of the center are shown by corresponding closed characters. From a study of the graph it will be seen that for shooting distances as great as 15 ft, first arrivals were through the upper soil, in which the velocity was 1,430 ft per sec. For shooting distances between 15 and 250 ft first arrivals were through a compact material in which the velocity was found to be 6,200 ft per sec. For shooting distances greater than 250 ft, the arrival times were shorter by way of the deep rock than through the overburden. The apparent velocity in the rock, V_u, when shooting from the "ahead" end of the line, or up-dip, is just twice as great as the apparent velocity, V_d, when shooting from the "back" end of the line, or down-dip. The true velocity in the rock, V_r, was determined by taking the harmonic mean of the apparent velocities and was found to be 16,400 ft per sec.

---

[3] "Geophysical Studies in the Atlantic Coastal Plain," *Bulletin of the Geophysical Society of America*, Vol. 48, June 1, 1937, pp. 753–802; see also "The Seismic Method of Exploration Applied to Construction Projects," by E. R. Shepard, *The Military Engineer*, September–October, 1939.

[4] "Application of the Seismic Refraction Method of Subsurface Exploration to Flood Control Projects," by E. R. Shepard and A. E. Wood, *Technical Publication No. 1219*, Am. Inst. of Mining and Metallurgical Engrs., June, 1940.

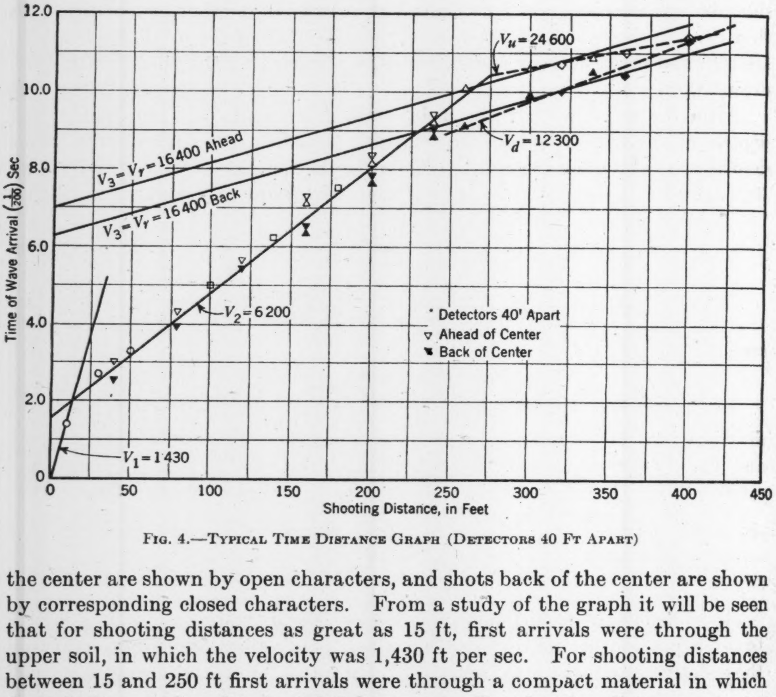


FIG. 4.—TYPICAL TIME DISTANCE GRAPH (DETECTORS 40 FT APART)

The depth of overburden at the center of the line was computed as follows, using the velocities previously determined and the approximate method for which Eqs. 2 are employed. The overburden depths are computed as follows: $V_r = \dfrac{2 \times 24{,}600 \times 12{,}300}{24{,}600 + 12{,}300} = 16{,}400$; $\sin \alpha = \dfrac{V_1}{V_2}$, and $\cos \alpha = \sqrt{1 - \left(\dfrac{V_1}{V_2}\right)^2} = 0.97$; and $\sin \beta_2 = \dfrac{V_2}{V_r}$, and $\cos \beta_2 = \sqrt{1 - \left(\dfrac{V_2}{V_r}\right)^2} = 0.925$. Then:

Computation	Feet
By Eq. 2*a*, $H_1 = \frac{1.6 \times 1,430}{400 \times 0.97} =$	5.9
One half the shot depth $= \frac{3}{2} =$	1.5
Depth of upper soil =	7.4
By Eq. 2*b*, the depth of till, H_2 (ahead) $= \frac{5.34 \times 6,200}{400 \times 0.925} =$	89.5
Depth to rock (ahead) =	96.9
Similarly, since H_2 (back) = 77.5, the depth to rock (back) $= H_{1+2}$ (back) =	84.9
The average depth to rock at the center $= \frac{96.9 + 84.9}{2} =$	90.9
Elevation, top of ground =	238.1
Elevation, top of rock =	147.2

From the computations it will be seen that the depth of the upper-soil layer, H_1, in which the velocity of wave propagation was 1,430 ft per sec, is 7.4 ft; T_1 in this instance is the interval on the time axis between the origin and its point of intersection with the V_2 limb of the graph, or 1.6; H_2 (ahead) or the thickness of the glacial till under the "ahead" end of the line was found to be 89.5 ft; and T_2 in this case is the difference between the intercepts of V_3 (ahead) and V_2 on the time axis, or 5.34. Similarly, H_2 for the back half of the line was found to be 77.5 ft. The total depths of overburden ahead and back are 96.9 and 84.9 ft, respectively. The depth at the center is assumed to be the average of these two values, or 90.9 ft. Theoretically any depth is determined by taking one half the sum of the depths at the shot point and the detector or receiving point, neither of which depth is known. The recorded depth for any line was the average of the mean or normal depths so determined for the two ends of the line. Because of the prevalent variation between the apparent and true velocities in most of the areas investigated, it was believed unwise to attempt to determine slopes in the rock surface from apparent velocities or to predict multiple rock elevations along a shooting line, and only one rock elevation on each line has been recorded in the final analysis of the data. The recorded elevation was designated as that at the center of the line.

Determination of Overburden Conditions.—In most locations considerable success was attained in classifying the overburden with respect to hardness and compaction as well as in determining the elevation of the top of rock. In areas where the characteristics of the overburden were generally known, the seismic data were useful in estimating the depths of individual strata. The seismic data have been used in studies of overburden conditions at structure sites and where excavations will be made for proposed hydraulic cuts and navigation channels.

From a correlation of drill records and seismic data over the entire project, the range of velocities for the various types of overburden was determined. In the areas investigated, velocities of 1,000 to 2,000 ft per sec generally indicated very loose material; velocities of 3,500 ft to 5,000 ft per sec usually were indicative of relatively soft material such as silt or clay; but in some instances velocities from 4,000 to 5,000 ft per sec did indicate fairly loose deposits of glacial till. Velocities in excess of 5,000 ft per sec usually indicated the presence of compact glacial tills.

In a large proportion of the test locations, two types or conditions of overburden exist as shown in Fig. 4. On the till ridges, velocities usually ranged from 1,000 to 2,000 ft per sec, to depths of from 5 to 8 ft. This zone of loose material is indicated by the steep portion of the graph (Fig. 4) through the origin. Beneath this zone of loose material, the velocity usually changed to a much higher value, the magnitude of which depended on the character of the material. Along the margin of the river and also in the bed of the river, overburden of a single velocity was often found. In some locations, three types of overburden were found, as shown in Fig. 5:

Soil type	Feet
Upper soil, H_1	5.1
Clay, H_2	12.5
Till, H_3	71.7
Depth to rock	89.3

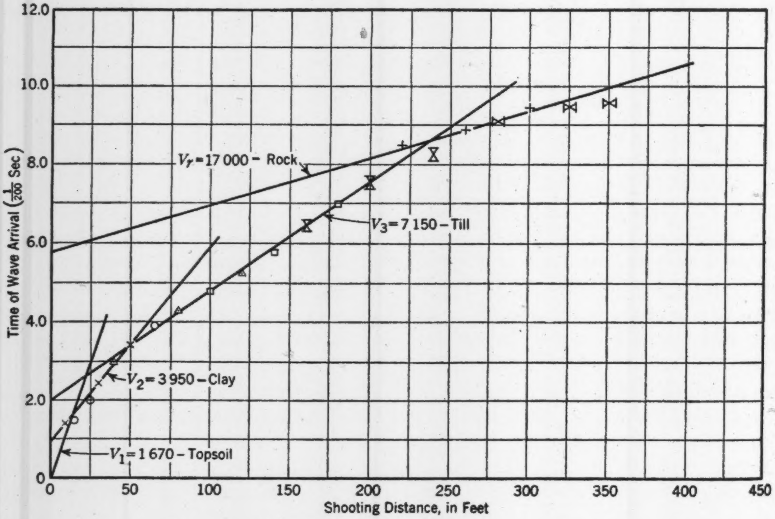


FIG. 5.—TYPICAL TIME-DISTANCE GRAPH SHOWING THREE TYPES OF OVERBURDEN

The soil profile as indicated by such graphs may not always be recognized in drill logs or by visual inspection, as velocity characteristics are determined by the depth of frost action, compaction, and other factors which may not appear from sampling.

Failure to Identify Shallow Rock.—In one section of the Point Rockway Canal area difficulty was encountered in identifying, properly, the hard rock that is within a few feet of the ground surface. In the analysis of the observed data, the intermediate material ($V_2 = 5{,}000$ ft per sec) was believed to be clay or till, although the irregularity of the data threw some doubt on this interpretation, and indicated the possible presence of a zone of seamed or fractured rock. Upon checking with the drill, relatively sound cores were obtained in this zone. A comparatively low velocity of 5,000 ft per sec for shallow and fractured rock is not uncommon in other formations but because of the unusually sound character of the rock previously encountered in the St. Lawrence River investigations, this condition was not anticipated. The low velocity was probably the result of seams opened by frost action and the absence of a heavy overburden load. Under a heavy overburden it is believed that this rock would have exhibited a much higher velocity.

To further investigate and study this irregularity found in the Point Rockway Canal, experimental seismic tests were made in the vicinity of the Northern Quarries near Norfolk, N. Y. At the quarries, where the rock face was exposed, a fractured and weathered zone could be seen to a depth of several feet. This rock was under a shallow overburden and was of the same origin as that in the Rockway Canal. The seismic tests adjacent to the quarries produced graphs very similar to those obtained from the data in the canal and showed that, under some conditions of fracturing, very shallow rock cannot always be identified by the seismic method.

Effect of Frozen Ground.—As early as the middle of December, 1940, frost in the ground began to affect results in some areas, and by February it had penetrated to such a depth, in areas unprotected by deep snow, as to interfere seriously with the accuracy of interpretations. Accordingly, field work was suspended during March and April. As the velocity of wave propagation in frozen soil is much higher than that in normal or unfrozen upper soil, this

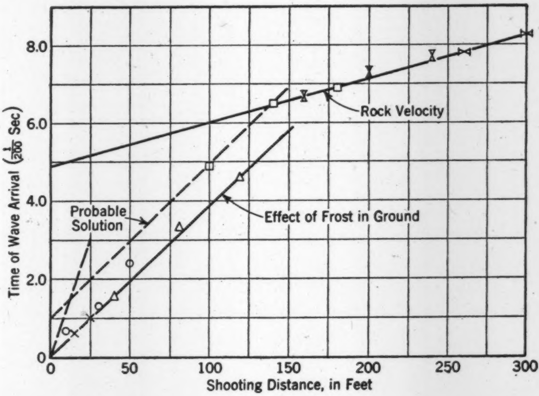


FIG. 6.—TYPICAL TIME-DISTANCE GRAPH SHOWING EFFECT OF FROZEN GROUND

condition leads to uncertainties in the interpretation of seismic data. The effect of 6 in. of frozen ground is shown in Fig. 6. Velocities of 5,000 to 7,000 ft per sec, values abnormally high for upper soil, were obtained for shooting distances as great as 100 ft. For longer shooting distances the crust of frozen ground did not carry sufficient energy to register on the film. Probable solutions were developed for many of the lines affected by frost. These were believed to be reasonably accurate, in regard to total depth of overburden, but their graphs are of little value for indicating the soil profile or character of overburden. Unfortunately many of the lines affected by frost were in areas of deep overburden where a knowledge of the overburden properties was of more importance than the depth to rock.

Fig. 7.—Waterproof Detector Housings

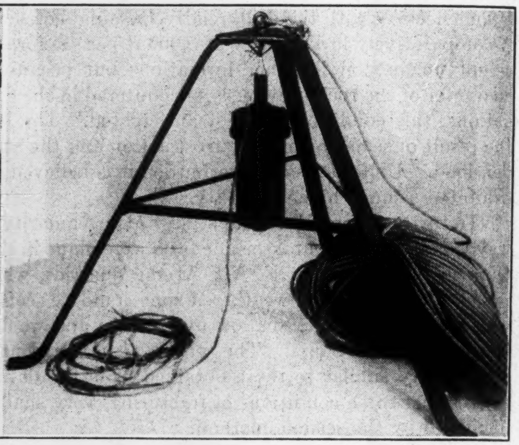


Fig. 8.—Tripod Detector Mounting for Use in Deep Water

Subaqueous Exploration

Tests in Quiet or Slowly Moving Water.—The need for information on the character and depth of overburden in areas of the river channel where the water was quiet, or was moving slowly, called for the development of special equipment and a revised field procedure for under-water exploration. One of the essential requirements in making seismic tests on the bed of a river or other bodies of water is to provide means for placing detectors and shots with reasonable accuracy at predetermined locations. Another requirement is to waterproof the detectors and so mount them that they will assume a vertical position when lowered to the bed of the stream.

To place the shots and detectors accurately a float or marker line was constructed by attaching wooden blocks at designated points on a rope approximately 600 ft long. Measured from the center of the line, floats were attached at the following points: 0, 10, 40, 80, 130, 180, and 230 ft ahead and 0, 25, 40, 80, 130, 180, and 230 ft back. The center float, and those at 40 ft on either side of the center, marked the detector positions, and the other blocks marked the shot points as on a normal land line.

One end of the float line was anchored on the bed of the river at the upper end of the seismic line along which information was desired. A heavy stone usually served for this purpose. The line was then allowed to float downstream until it assumed a stable position. Where the current was not sufficient to stretch the rope, both ends were anchored after maneuvering the line into the desired position. In some instances it was necessary to anchor the center of the line also to prevent it from drifting with the wind or current. Buoys were attached at anchor points, when necessary, to keep the markers afloat. Empty oil cans of 2-gal or 5-gal capacity were found satisfactory for this purpose.

Two methods were used to keep the detectors in a vertical position on the bed of the stream. For depths of water not in excess of 10 ft the mounting shown in Fig. 7 was used. Each detector was provided with a waterproof housing made from a 10-in length of 3.5-in. pipe. The lower end was closed with a steel plate and the upper end with a cap containing a ¾-in. pipe nipple. In assembling the system the detector was placed in the housing and a pair of rubber covered auxiliary leads about 30 ft long were attached to the terminals. Cotton waste was packed around the sides of the detector and over the terminals, after which the leads were passed through the nipple and the cap tightly screwed on. It is important, of course, that the detector rest firmly on the bottom of the housing.

After stretching and anchoring the float line the three detectors were set at designated points. A pipe or rod was driven at each detector position and the detector assembly lashed to it in a vertical position. A boat containing the receiving apparatus was anchored about 50 ft from the center of the line and the three pairs of detector leads used in land work were connected to the auxiliary leads. A rock was tied to a piece of flagging or burlap to act as a sinker for the charge, and the cap wires were spliced to the firing line, which, in turn, terminated at the receiving instrument. The charge was then lowered to the bottom, after which it was detonated as in land practice.

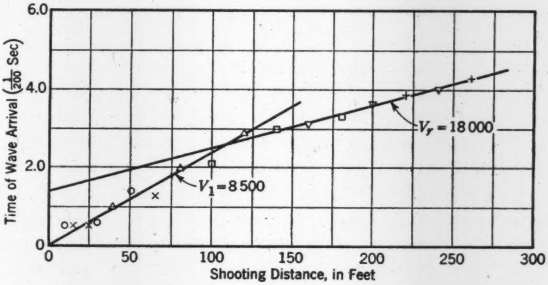


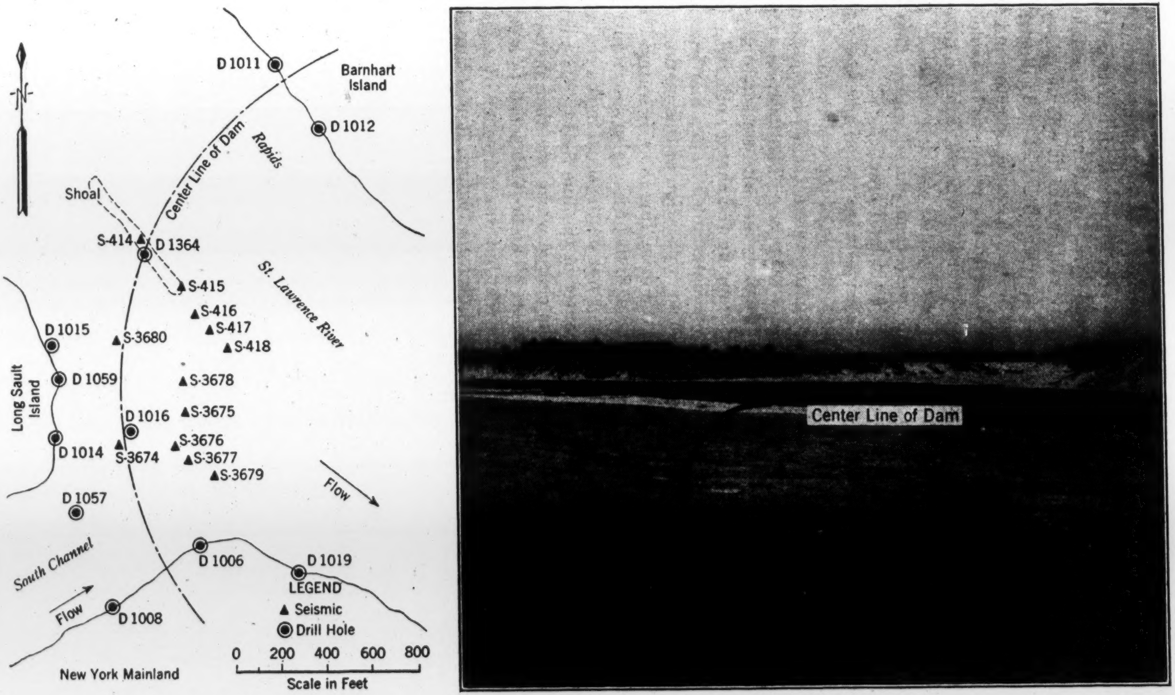
FIG. 9.—TYPICAL TIME-DISTANCE GRAPH OBTAINED WITH THE TRIPOD DETECTOR MOUNTING UNDER WATER

Where the water was more than 10 ft deep, the detector, in the same waterproof housing as that described, was suspended from the apex of a heavy iron tripod, as shown in Fig. 8. After sealing the nipple against the entrance of water the entire assembly was lowered to the bed of the river by a rope. The detector assumed a vertical position irrespective of the configuration of the river bed. Although it did not rest directly on the bottom it received the shock from the explosion through the legs of the tripod and the suspension wire. No difficulty was encountered in obtaining satisfactory records with either type of detector mounting.

A typical graph obtained by under-water exploration is shown in Fig. 9, the depth to sound rock in this case being $H = \frac{1.4 \times 8{,}500}{400 \times 0.88} = 33.8$ ft. This is an excellent graph and typical of most of those obtained in quiet or slowly moving water. In fact, the results are considered more accurate than those obtained on land, since, due to erosion, the overburden was more uniform in character, and usually a two-layer graph was obtained. The main source of error compared with the procedure used on land is in the placement of the charges in deep or moving water. It was found that under these conditions, the charge might drift a considerable distance before reaching the bottom, and allowance had to be made for this effect. Large errors in the placement of the shot, especially for the shorter shooting distances, will result in inaccurate time-distance graphs. Fortunately, where allowances had to be made for the drift of the charges, the overburden conditions were quite uniform so that the results from the shorter shooting distances were not needed for accurate interpretation.

Tests in Swift Water.—Seismic tests were made in the river channel area at the Long Sault Dam site where swift water made drilling too expensive to be included in the present program of exploration. The Long Sault Dam site is located at the foot of the Long Sault Rapids and the proposed dam site extends from the New York mainland to the head of Barnhart Island. Drillings, as shown in Fig. 10(*a*), were made on the shores of the mainland, on Barnhart and Long Sault islands, in the south channel where floats could be used, and on the shoal in the middle of the channel. The information obtained by drilling was very limited along the full extent of the axis of the dam. Soundings in the river channel made several years ago indicated that a rock gorge might exist at the site, especially between the shoal and Barnhart Island. As further drilling operations along the proposed axis of the dam would require large floating equipment and would be very expensive, seismic explorations were made wherever possible. It was found impracticable to make tests between the shoal and Barnhart Island where the velocity of the river is approximately 12 miles per hr and power boats cannot be maneuvered satisfactorily. A limited number of tests were made below the shoal and in the vicinity of the axis of the dam between the shoal and the mainland where the velocity of the river is not so great. Fig. 10(*b*) is a photograph of the river, taken from the New York mainland looking toward Barnhart Island, which shows the condition of the water in the area investigated.

In undertaking seismic exploration in the channel area it was realized that the usual accuracy could not be obtained and that the exploration would not


N
Barnhart Island
D 1011
D 1012
Rapids
Center Line of Dam
Shoal
S-414
D 1364
St. Lawrence River
S-415
S-416
S-417
S-418
D 1015
S-3680
Long Sault Island
D 1059
S-3678
S-3675
D 1016
D 1014
S-3674
S-3676
S-3677
S-3679
Flow
D 1057
South Channel
D 1006
D 1019
LEGEND
▲ Seismic
◉ Drill Hole
Flow
D 1008
New York Mainland
0 200 400 600 800
Scale in Feet
Center Line of Dam

(*a*) Location of Drillings

(*b*) View of the Site

FIG. 10.—LONG SAULT DAM SITE

cover the area between the shoal and Barnhart Island. Nevertheless, it was believed that results could be obtained which would give the approximate elevation and configuration of the bedrock in the area explored, and even indicate the trend of the rock floor in the unexplored area. As it was impossible to place detectors on the river bed in excessively swift water, it was necessary to develop a procedure different from that used in quiet water. Fig. 11 illustrates the arrangement of apparatus used in these tests. A single detector and the oscillograph recorder were placed on shore close to the region to be explored. The detector was placed near a drill hole or seismic line where the depth to rock was known. The velocity in the adjacent overburden was determined in the usual manner. Single charges of dynamite were then placed with a float or boat at points on the river bottom and fired as in land practice.

One group of shots (S-3674 to S-3680, inclusive) were placed and fired while the detector was at drill hole D-1059. The locations of these shots are shown in Fig. 10(*a*). The firing line and a heavy sinker were attached to the charge which was then carried out in the river and dropped. The location of the charge, when dropped, was determined by two observers with transits. If possible, soundings were taken at the time of dropping the charge; otherwise the approximate elevation of the river bottom was determined from a map showing the subaqueous contours in the vicinity. The application of this method was limited, owing to the difficulty of handling the firing line in swift water.

Another group of single shots, S-415 to S-418, inclusive (also shown in Fig. 10(*a*)) were placed and fired while the detector was on the shoal in the channel, 120 ft from drill hole D-1364 and at the center of the line S-414. The firing line and a heavy sinker were attached to the charge which was then placed on a float made of two oil drums lashed together. The float attached to a cable was allowed to drift downstream with the current but was controlled by a winch mounted on the shoal. To keep the firing line from fouling, it was threaded through rings on the cable. When the float was at the desired location, a boat came alongside and the charge was released and dropped into the river. The location of the charge was determined by observers on shore. The float was then pulled 50 to 100 ft upstream and the shot fired. The depth of the river at the location of the shot was determined by actual soundings or from the map showing the subaqueous contours.

The method of analyzing the data obtained in the swift water at the Long Sault Dam site differed from the usual method of analysis as the knowns and unknowns were different and the necessary assumptions more radical. Referring to Fig. 12, the only information obtained from the film record was the total time, T, which elapsed from the shot instant to the arrival of the shock at the detector, D, Fig. 11. This total time T includes: t_1, the time of transit through the overburden at the shot S; t_2, the time of transit through the rock from B to C, Fig. 11; and t_3, the time of transit through the overburden CD. The value T was plotted on the time-distance coordinates at Q for the shooting distance OR, as shown in Fig. 12. As the velocity in the rock was known from previous tests on shore, it was possible to draw the line QM which gives PM or t_2 as the time of transit in the rock. As the depth CD was known from boring and seismic records, and the velocity, V_1, of wave propagation in the overburden from seismic tests on land, it was possible to draw line ON as t_3, the time of transit in the overburden at CD. This left MN, or t_1, as the time consumed in the overburden SB. To compute the depth H, or SB, it was necessary to assume the same overburden velocity there as at CD.

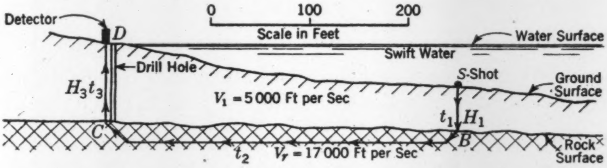


FIG. 11.—ARRANGEMENT OF APPARATUS FOR DETERMINING DEPTH OF OVERBURDEN IN SWIFT WATER

The results, as obtained from shooting in swift water, contain certain probable inherent inaccuracies. The assumption that the velocities in the overburden and rock in the river channel are the same as those determined on the adjacent land could be in error. However, any error resulting from this assumption will be only in proportion to velocity variations from place to place in the respective media. A more likely source of error is that in obtaining the true value of t_1. Where the depths to be measured are not great, the value of t_1 is small in comparison with T, and any errors in determining t_2 and t_3 will be carried into t_1 and may seriously affect its accuracy. Another source of error is in the determination of the shooting distance. With the velocity ratio of 17,000 to 5,000 ft per sec for rock and overburden as shown, an error of 3.4 ft in the shooting distance will result in a 1-ft error in H_1. The accuracy in determining the elevation of the rock also depends on the accuracy of the sounding of the river bottom. For large values of H_1 these errors are relatively small, but where attempts are made to measure shallow overburdens in this manner the indications may be entirely misleading. The data and conditions of the tests in the rapids have been analyzed and studied carefully and, although

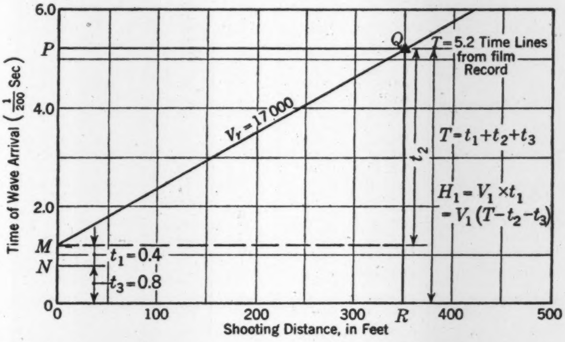


FIG. 12.—ANALYSIS OF OBSERVATIONS FOR DETERMINING DEPTH OF OVERBURDEN IN SWIFT WATER

limited in accuracy as to the actual thickness of the relatively shallow overburden in the bed of the river, it is believed that the results are reliable as indicating the trend of the rock contours and in precluding the possibility of a deep gorge in the area explored.

CORRELATION OF SEISMIC AND DRILLING RESULTS

The correlation between seismic information and drilling data has been thoroughly investigated over the entire project. Table 1 contains the seismic and drilling data at all locations where a drill hole was close enough to the center of a seismic line for a significant comparison to be made. In a few locations drill holes were spotted at or near the centers of seismic lines for the express purpose of checking the seismic predictions. This is true for lines 203 and 246 on Galop Island, lines 318, 320, 323, and 325 in the Rockway Canal area, and line 412 on Long Sault Island. On Galop Island the checks were remarkably close, being 1 ft in a depth of 42.3 ft and 1.5 ft in a depth of 54.0 ft, respectively. At line 412 there was a discrepancy of 11 ft in a total depth of 88.6 ft, which is believed to be the result of a very compact overburden condition immediately above the bedrock surface. Correlations in the Rockway Canal area were not entirely satisfactory, owing to the conditions previously described. At line 269, near the Massena Power Canal intake, there was an error of 9.9 ft in a

TABLE 1.—COMPARISON OF SEISMIC INTERPRETATIONS WITH DRILLING RECORDS, ST. LAWRENCE RIVER PROJECT

Seismic line No.	Location	Drill hole	Distance from drill hole to seismic center (ft)	Seismic			Drill Hole		
				Ground elevation	Depth (ft)	Rock elevation	Rock elevation	Depth (ft)	Ground elevation
S-2	Long Sault Dam	D-1019	20	174.8	29.7	145.1	144.0	28.5	172.5
S-11	Long Sault Dam	D-1007	70	212.7	56.6	156.1	160.1	53.0	213.1
S-12	Long Sault Dam	D-1007	120	212.5	52.0	160.5	160.1	53.0	213.1
S-34	Power house	D-1029	57	185.6	43.6	142.0	146.9	38.7	185.6
S-35	Power house	D-1136	90	191.9	58.7	133.2	132.5	60.5	193.0
S-38	New Cornwall Canal	D-1121	140	211.8	47.4	164.4	162.6	49.0	211.6
S-50	Iroquois Dam	D-1046	27	231.5	25.0	206.5	206.4	25.9	232.3
S-66	Grass River Lock	D-1368	100	184.0	74.3	109.7	104.4	77.5	181.9
S-68	Robinson Bay Lock	D-1070	126	199.7	65.2	134.5	138.2	61.8	200.0
S-74	Ogden Island	D-1083	116	232.1	53.0	179.1	183.5	50.6	234.1
S-88	Rockway Canal	D-1264	125	228.5	26.8	201.7	202.1	24.8	226.9
S-89	Point Rockway Canal	D-1073	115	229.5	30.0	199.5	198.1	34.2	232.3
S-158	Massena Canal Intake Works	D-1260	43	218.4	41.6	176.8	171.3	47.3	218.6
S-164	Point Rockway Canal	D-1281	39	228.0	21.1	206.9	200.5	27.0	227.5
S-175	Point Rockway Canal	D-1283	89	241.1	9.0	232.1	233.8	4.1	237.9
S-203	Galop Island Channel	D-1110	0	247.7	41.3	206.4	205.4	42.3	247.7
S-207	Lalone Island Channel	D-1102	60	236.5	15.0	221.5	221.8	26.0	247.8
S-225	Galop Island Channel	D-1109	173	272.5	75.0	197.5	198.6	70.6	269.2
S-246	Galop Island Channel	D-1106	0	250.2	52.5	197.7	196.2	54.0	250.2
S-299	Iroquois Dam	D-1045	100	233.5	31.6	201.9	208.1	23.5	231.6
S-318	Point Rockway Canal	D-1341	0	240.1	30.4	209.7	209.6	30.5	240.1
S-320	Point Rockway Canal	D-1379	0	237.1	4.5[a] 14.6[b]	232.6 222.5	232.6	5.0	237.6
S-323	Point Rockway Canal	D-1343	0	244.6	4.3[a] 20.7[b]	240.3 223.9	243.8	0.8	244.6
S-325	Point Rockway Canal	D-1378	40	233.2	3.7[a] 6.7[b]	229.5 226.5	228.4	4.3	232.7
S-408	Long Sault Island	D-1391	150	249.0	72.4	176.6	172.4	83.3	255.7
S-412	Long Sault Island	D-1390	0	257.5	77.5	180.0	168.9	88.6	257.5

[a] To top of fractured rock. [b] To top of sound rock.

depth of 67.2 ft. Failure at this location to determine the depth of rock more accurately was the result of an artificial fill of varying depth created by the dumping of spoil from the excavation of the power canal. It should be noted that in all three of these locations, where the correlations are not satisfactory, the drillings were made to check apparent inconsistencies in the seismic data. With the exception of lines 269, 412, and a few of those in the Rockway Canal area, the correlations in Table 1 are quite satisfactory and indeed much better than obtained ordinarily in geophysical explorations.

The differences in depths of overburden, as determined by the two methods, are with few exceptions within the limit of the relief in the rock surface. For the greater depths the differences are relatively small, while for the shallower overburdens—such as at lines 89, 164, 175, 323, and 325—the actual differences, although only a few feet, may represent a large percentage of error. It should be remembered, however, that in all cases the drill record gives the depth of the overburden at one precise location, whereas the seismograph records an average depth over a considerable length of line.

Significance of Investigations

Heretofore the seismic method of exploration usually has been considered primarily applicable for making preliminary or reconnaissance investigations and has not been used extensively for making more detailed analyses. As a guide to drilling and for obtaining comparative data it has been of considerable value on alternate dam and spillway sites investigated by the U. S. Corps of Engineers. After a site for a structure has been definitely selected, it is customary to obtain detailed information by drilling. On the St. Lawrence River Project, the seismic data have been used more extensively than is generally feasible. All proposed cuts and channels were explored primarily by seismic methods, a few drill holes having been added to determine overburden conditions and to verify seismic results where bedrock occurred above or near excavation grade. The results have been used to a large extent in fixing locations and grades for the proposed cuts and channels in an effort to minimize the excavation of ledge rock. Other sites were explored with a small number of seismic lines to supplement the drilling program.

The use of the seismic data has enabled the design of concrete structures, navigation channels and hydraulic cuts to proceed more rapidly under the present program than if exploration consisted only of drill holes. A total of 355 satisfactory seismic determinations, and 47 of fair reliability, were made on land and in quiet water. To make the same number of rock elevation determinations by drilling, it would have been necessary to drill approximately 15,600 ft. The cost of such drilling would have been about four times that of the seismic investigations, an amount too great to have been included in the exploration program. Also the cost of additional drilling in the channel at the Long Sault Dam site would have been prohibitive. Although seismic tests could not be conducted over this entire swift-water area, the few results obtained were useful for design and estimate purposes.

Seismic determinations are valuable depending upon their accuracy in comparison to that required for the study of each feature. The accuracy, in turn, depends upon correct interpretation of the data obtained in the field. Reliable results can be obtained only by an experienced interpreter who understands the geological and surface conditions affecting the data. In general, the subsurface conditions were ideal for the seismic method and the results could be used with greater assurance, therefore, than in areas where conditions are more variable. The results of the seismic and drilling exploration were plotted on the various site maps, after which rock-surface contours were drawn. Although only one rock determination was computed for each line, the graphs often could be used to indicate the direction of slope of the bedrock surface. The results of the determinations at Long Sault Dam site, although not considered as accurate as other measurements, definitely indicated that if a bedrock gorge does exist, it is confined to the narrow area between the shoal and Barnhart Island.

The results of the investigation were very useful in interpreting overburden conditions, particularly in the areas of channels, cuts, and canals. By correlation with drill-hole data and the general geology of the region, the results aided in determining the extent and depth of various deposits of glacial till and other types of overburden and also their condition of density and compaction.

The subaqueous tests are believed to be unique for obtaining information of the type required for this project. Reliable results were obtained where detectors could be placed on the river bottom and sufficiently accurate results for required purposes were obtained in part of the channel area at the Long Sault Dam site where the water is swift. As the results of subaqueous exploration were satisfactory on this project, the use of the seismic method should be broadened in future investigations to include similar conditions.

Acknowledgments

The seismic investigations were made under the direction of Col. A. B. Jones, M. Am. Soc. C. E., district engineer of the St. Lawrence River District. Credit is due to F. P. Fifer, M. Am. Soc. C. E., head engineer on the project, for helpful and encouraging advice, particularly in regard to the subaqueous tests; A. E. Wood, associate geologist in the Binghamton District, supervised the work and analyzed the data during the early period of the investigation; G. R. Butterfield, Jr., Jun. Am. Soc. C. E., assistant engineer, rendered valuable assistance in laying out the seismic program and correlating it with the drilling program, and also in the final preparation of this paper; W. A. Wells assisted throughout the investigation in the interpretation and analysis of the field data and also in the preparation of the final report.

DISCUSSION

R. Woodward Moore,[5] Esq.—The application of refraction seismic tests to shallow explorations in water-covered areas constitutes a distinct forward step in the use of geophysics on civil engineering projects. This method of test already has proved to be a useful tool in connection with subsurface explorations for dam sites, highway grading projects, rock quarry sites, and tunnel locations. Data concerning foundation conditions at bridge abutment locations have been obtained with seismic tests, and, by means of subaqueous tests such as are described by the authors, it should be possible to investigate pier locations as well. The value of the application of the seismograph to bridge site explorations should not be underestimated, particularly in preliminary surveys for highways through undeveloped areas not adequately served by existing roads or trails. At times, a rapid survey of subsurface conditions using the seismograph may eliminate the need for further check tests using the more costly and time-consuming core drill or test pit. Although the exploration costs for a bridge structure may be relatively small, the ratio of cost of exploration to total cost of the structure may be relatively large in comparison with that for a large dam or reservoir.

In connection with the tests on land in the Point Rockway Canal area where the drill located a solid rock formation with a shallow overburden and where the seismic data were inconclusive, it would have been informative to have made additional seismic tests with the detectors and shot lines at right angles to the lines that were used. In one or two instances in the writer's experience appreciable differences in wave velocity (3,000 to 4,000 ft per sec) have been obtained when shooting two lines of tests oriented 90° with respect to each other. One location where such differences in wave velocity were found was near the bank of a small river. It is possible that early stream action exposed the ledge rock to the elements, which resulted in weathering characterized by vertical jointing or fracture zones. This is entirely suppositional since no rock was exposed at the surface at this location. Concealed by a mantle of soil deposited during subsequent periods of erosion, these fracture zones would tend to lower the wave velocity in the rock for seismic tests made along lines transverse to the weathered zones. Tests along lines parallel to the fracture zones would result in wave velocities characteristic of a fairly solid, continuous mass of rock.

With respect to seismic tests made on frozen ground, probably a carbon-button type detector such as was used in the tests on the St. Lawrence River Project, and which has been used by the writer, could be adjusted to such a degree of sensitivity that the first impulse or wave through the frozen ground would produce a relatively small break in the oscillograph trace. The slower but stronger impulse passing through the normal soil below the frozen zone then would be present on the oscillograph record as a "second event" which could be interpreted as the true time of wave travel from the shot to the detector.

---

[5] Associate Civ. Engr., FWA, U. S. Public Roads Administration, Washington, D. C.

These "second events" have been used by the writer in the interpretation of seismic records which were obtained in areas where a low-velocity layer was underlain by a rather thin stratum having an intermediate wave velocity which rested upon solid rock. Fig. 13 shows a film record obtained with the carbon-button detector in which "second events" are shown rather clearly.

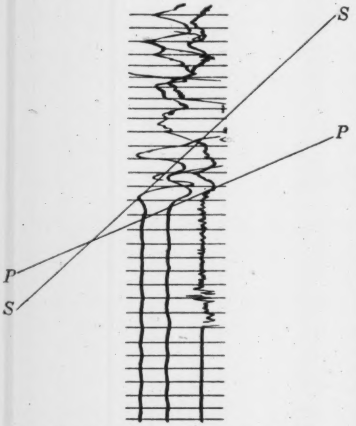


FIG. 13.—THREE-CHANNEL SEISMIC RECORD SHOWING PRIMARY (*P*–*P*) AND SECONDARY (*S*–*S*) WAVE ARRIVALS

Referring to the tests made "in quiet or slowly moving water," it would be interesting to know the actual velocity of the slowly moving water since there is some likelihood of water currents disturbing the detectors and causing troublesome vibrations which might mask the seismic wave coming from the shot. Research work may be required to determine the effect of various water velocities upon the submerged detector units, particularly where pipes or rods which extend to the surface of the stream are used. The effect of the flow conditions at the bottom of swift but deep water courses may require some remedial measures to assure a "quiet" detector should a satisfactory means of placing the units in swift water be developed.

In their analysis of the data obtained from the tests in swift water, the authors point out the probable sources of error in determining the elevation of the rock at the shot point in the stream bed. The average of the data from several shots at a particular point would tend to reduce the error in obtaining t_1 for the time of wave travel in the material overlying the rock. However, rock elevations obtained in the manner described should be useful in the design of dams and bridge piers.

In general, the accuracy indicated in a comparison of the rock elevations predicted from seismic data and those obtained from direct drilling, as given in Table 1, is in close agreement with the degree of accuracy obtained in similarly correlated tests made by the U. S. Public Roads Administration during the eight years since 1935. Usually the rock surface could be predicted within 5% to 10% of the true depth.

ARTHUR J. GRIER,[6] M. AM. SOC. C. E.—Considering how much has been written about geophysical exploration in the fifteen years since 1928, civil engineering literature is notably barren of references to the subject. It is difficult to explain why this laboratory technique, applied in the wide-open spaces, has had so little consideration by civil engineers and why they have made so few contributions to its development.

---

[6] Cons. Engr., Geophysical Exploration, Oakland, Calif.

Perhaps, too often, successfully applied geophysics has been publicized without due emphasis on the advisability of confirming favorable implications by more common and costly exploratory methods before undertaking large expenditures. Geophysics is essentially a technique of elimination by comparatively inexpensive methods to assure the largest amount of information at the least cost in time and money. Its results are highly indicative but should not be accepted as determinative. It is a most useful tool in the hands of those who hold that engineering is not averse to making a dollar earn the most interest.

The authors frankly state that the seismic program of the Corps of Engineers was intended to guide and augment the drilling program and that all was to be interpreted by the observed geology of the region. The physical data of the shock wave were to be correlated with and confirmed by the more common physical tests of applied optics and mechanics. This sequence is just as profitable in civil engineering as it is in mining.

Seismic geophysics is specially applicable to the study of slightly dipping formations such as those for foundations and oil structures. Steeply dipping contacts of quartz veins, dikes, and faults, found in mining and water supply problems, may be delineated rapidly and accurately by electrical methods based on a difference of resistance.

Since 1936 the writer has used an instrument which is simple, sensitive, and portable, based on an electrical circuit which he devised and has never seen described elsewhere. The fundamental circuit without refinements and reversing switches is shown in Fig. 14. This method employs a unidirectional electric field set up by means of common B batteries and two metal current electrodes marked C_1 and C_2. Within this imposed field are three equally spaced nonpolarizing porous cups containing copper sulfate solution in which are the potential electrodes P_1, P_2, and P_3. These cups establish in the ground the two arms of a Wheatstone bridge, the other two arms being within the instrument and composed of the two segments of the potentiometer rheostat P_R with a dial graduated 0 to 100. The field is made strong enough that the potential at P_2 is intermediate between that at P_1 and P_3.

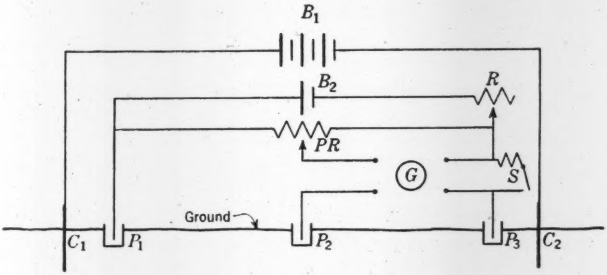


FIG. 14.—FUNDAMENTAL CIRCUIT

Without further provision a current would flow through these cups and the instrument, and the apparent resistance of the ground, in reality, would be the sum of the ground resistance plus the contact resistance at the potential electrodes, which might vary with each setup over the same ground. So it becomes necessary to employ the principle of the compensation bridge, or potentiometer, and neutralize the potential drop between P_1 and P_3 with an equivalent electromotive force from the battery B_2, adjusted by means of the rheostat R. Neutralization is indicated when the galvanometer G (which has been cut into that circuit) reads null, and since there then is no current through P_1 and P_3, there can be no contact resistance.

With the compensation bridge established, the Wheatstone bridge is effected by cutting the galvanometer over into the circuit P_2-P_R and closing the switch S with included resistance equal to that of the galvanometer. If the contactor is moved along the graduated meter wire P_R until the bridge is in balance, no current flows through the potential electrode P_2 and likewise there is no contact resistance at that point.

It remains only to read and note the two segments of the graduated dial P_R which accurately reflect the relative potential drop, and hence the relative ground resistance, of the two adjacent stations under consideration. There is but one instrument, the galvanometer, to be observed, and no mathematical calculations whatever are required.

The diagram does not show reversing switches, but it is necessary to reverse polarities and repeat the operation. The average of the readings will balance out the natural earth potential which usually will be encountered. That is the entire operation.

All electrodes, both current and potential, are moved ahead one station, and new readings taken as before. Upon the completion of the traverse, the notes plot a profile of relative earth resistance to a depth depending upon the spacing of the electrodes. Any anomalies become conspicuous, and such areas can be explored further by rerunning with different electrode spacing and by probing for depth at critical spots. It is observed that depth readings are much sharper when using the bridge circuit than when calculated and plotted in a curved line by the usual Wenner[7] formula. No attempt is made to determine specific earth resistance, and it is questionable if there is any advantage in doing so. Both methods are based on the same recognized laws pertaining to electric currents in a semi-infinite medium.

The entire subject of geophysics, including the several approved methods, may be regarded as an expansion into the realm of the unseen of the ancient science of surveying. It derives from a better knowledge of physical laws and is made possible by instruments of sensitivity and precision.

The economic value to civil engineering of this surveying of the unseen will be recognized as more field engineers with training in the physical sciences apply their skill to just such problems as the St. Lawrence River Project under discussion.

F.W. LEE,[8] ESQ.—A careful explanation has been presented in this excellent paper of the technique as well as of the results that can be obtained by the application of seismic refraction methods for securing subsurface information regarding depth to bedrock. Little can be added other than to compliment the authors for the accurate manner in which they have executed this project.

---

[7] "A Method of Measuring Earth Resistivity," by F. Wenner, *Bulletin No. 258*, U. S. Bureau of Standards.

[8] Chf., Div. of Geophysical Exploration, Bureau of Mines, U. S. Dept. of the Interior, Baltimore, Md.

Field observations made on Governors Island in the Boston (Mass.) Harbor have showed that, even under very simple geological conditions involving glacial drift, the speed in the drift is not uniform and that certain average values can be made to apply in each interval of observation. An attempt was made to chart such velocities in the glacial drift on the island.[9] Apparently, the ratio of gravel to clay in the drift is a factor. In water-sorted gravel beds, particularly where the individual stones all were worn into ellipsoidal bodies and deposited with their minor axes in a vertical position, the vertical and the transverse seismic velocities were not the same. However, the differences in boulder content overshadowed such observations.

In presenting this paper the Society has taken a very definite forward step in making surveys in three dimensions, combining parts under water with nonsubmersed areas. The good old transit which serves so well above ground has added a brother for looking down into the earth, which at present has the title of seismic methods of exploration. Of these the refraction seismic method has proved particularly helpful for the measurement of depths to bedrock, as can be noted from Table 1, which shows that the seismic depth determinations agreed with the drill-hole record to within a few feet.

The following information is given as a matter of caution for executives who may be planning other applications of seismic refraction methods. A controlling feature of seismic methods depends upon the depth to which measurements are made, in conjunction with the accuracy of the final information expected. These various projects may be divided according to the depths of observation into the following classes:

Class	Approximate depths, in feet	Description
1	100	Depth to bedrock required for dam foundations, bridgeheads, road construction, water sources, and excavations.
2	400 to 500	Depths to preglacial channels, required for some foundation work and for sources of water supply for cities and regions.
3	800 to 1,000	Geological stratigraphy related to the planning of subsurface tunnels such as would be required for dewatering mines and mining areas.
4	3,000	Identification of large geologic bodies such as salt domes in southern Texas and Louisiana, which are of economic importance because of the oil generally found on the top and on the flanks of such domes.
5	> 300,000	Depths to regions of change in magmatic material, useful for earthquake studies and in the detection of major geologic changes.

---

[9] *Bulletin No. 8*, Dept. of Public Works, Boston, Mass.

The criterion of a seismic analysis is the degree to which a seismic impulse may be traced from the shot point to the detectors for the proper speeds as well as for the distances traversed in each medium. Mathematically, the formulas for refraction seismic surveys are very similar to those for geometric optics. They differ from those for optics in that they are used to analyze the material traversed instead of to design special surfaces of lenses for certain critical characteristics.

Often, even for very simple field cases, the solution becomes complicated since there is generally a weathered zone near the ground surface which has a comparatively small seismic speed of the order of 1,000 ft per sec. Directly under this weathered zone is a much faster unweathered zone having a seismic speed of the order of 5,000 ft per sec. Below it are the various geological beds each having a characteristic velocity—in granite and dense limestones reaching velocities of the order of 20,000 ft per sec. Therefore small differences in the thicknesses of the slow beds can introduce large time errors which, when applied to the same ray path in the fast beds, will introduce large errors in depths. To these differences must be added corrections for elevations for the various detectors on the shot line as well as for the variations of the topography of the bedrock. The cost of a project often depends to a large degree on the necessity of making supplementary seismic records for determining such irregularities singularly.

When making seismic surveys to a deeper horizon, the distances between the shot point and the detectors become increasingly greater, and the simple assumption that the segments of geologic beds over which the measurements are made may be considered as plane surfaces no longer can be maintained. The thickening and thinning of the intermediary beds make the analysis very slow and tedious. On encountering such difficulties, the field engineer making the seismic surveys uses overlapping and abutting traverse lines of observation for clearing out such singularities. After the field work is done, the results may be likened to a picture puzzle, each piece representing a seismic profile. These pieces must be fitted together into such an integrated whole as would be produced by the geological conditions in question. From this point the procedure becomes increasingly more complex with the addition of each intervening bed which must be taken into consideration. Also, the geological "quirks" which might be associated with each bed must be considered. In deep refraction surveys of the order of 1,000 ft, one often encounters as many as four or five characteristic beds, each of which must be defined and differentiated separately.

From the foregoing, it is evident that the path of a seismic survey engineer is a hard one. Nevertheless, such geophysical methods may be considered the first rays of a new branch in civil engineering for studying hitherto unsounded depths on a scale which may be called planet engineering. Upon the foundations established by such studies it is hoped that in the end mankind will be provided with a power of knowledge that will permit him to make this planet perform according to his will, in much the same manner as a violin in the hands of a master.

L. Don Leet,[10] Esq.—Considerable success in operations under sometimes rather difficult field conditions has been achieved by the authors. They are to be congratulated upon this application of the seismic method, which has established itself rather widely in recent years but is often anything but simple and straightforward in its execution.

In connection with the detailed discussion of methods perhaps it should be emphasized that the authors' system of shooting does not yield what are known as "reverse profiles." In other words, the observations along a line "ahead" of the detectors are based upon elastic waves which do not travel over a common path with those along the line "back" from the detector location. Structurally, the significance of this is that interpretations assume continuity and uniformity of the high velocity rock surface over a distance which represents the combined length of "ahead" and "back" profiles. In some regions such an assumption would lead to serious errors, but in the area covered by the authors' report this apparently was not the case.

Perhaps it is a small point to mention in connection with so excellent a paper, but in the interests of accuracy there seems to be no harm in pointing out that the formulas by which depths were computed are credited to Messrs. Ewing, Crary, and Rutherford (1937),[3] as they have been on previous occasions by Mr. Shepard. Actually, those formulas had been in use for approximately ten years before publication of the paper by Mr. Ewing and his assistants, and it is not correct to imply that they were developed first in 1937. At least one place where they had been published in 1931 was in the Bulletin of the French Bureau of Mines.

William L. Shannon,[11] Jun. Am. Soc. C. E., and Winthrop A. Wells,[12] Esq.—In connection with preliminary foundation investigation for flood control structures in the Merrimack Valley watershed, the District Office of the Corps of Engineers, Boston, Mass., made use of the Shepard type of seismograph to determine the elevation of bedrock. During the autumn of 1938, the proposed Riverhill Dam site[13] was investigated extensively and some work was done at the Franklin Falls Dam site. In the summer of 1939, seventeen proposed dam sites were investigated by the seismic method. At some of these sites, at a later date, drill holes were located and drilled at or adjacent to seismic lines. The results of both seismic and drill hole data are summarized in Table 2. The method used in these investigations is similar to that described by the authors under the heading: "Determination of Overburden Depth by Two-Way Shooting."

The seismic investigations at each site immediately followed a thorough geologic reconnaissance. The results of the geologic reconnaissance and seismic data were then used as a basis for selection of sites for more detailed study. At three of the proposed sites, detailed explorations of some areas by seismograph were made for final explorations to supplement drilling and aid in exact location of structures.

---

[10] Associate Prof. of Geology, Harvard Univ.; and Seismologist in Charge, Harvard Seismograph Station, Cambridge, Mass.

[11] Associate Engr., U. S. Engr. Office, Boston, Mass.

[12] Asst. Engr., U. S. Engr. Office, Boston, Mass.

[13] "Damsite Surveying by Seismograph," by Albert E. Wood, *Engineering News-Record,* March 28, 1940.

In general, the agreement between seismic and drill-hole determinations, as shown in Table 2, is not as good as that obtained at the St. Lawrence River Project. It is believed that, to a large extent, the greater disagreement is due to the presence in this region of one or both of the following: (1) A variable, relatively high velocity weathered rock above sound rock; and (2) a highly irregular rock surface.

TABLE 2.—COMPARISON OF SEISMIC INTERPRETATIONS WITH DRILLING RECORDS, MERRIMACK VALLEY WATERSHED

Location	Seismic line No.	Drill hole No.	Distance,[a] in ft	Seismic			Drill Hole		
				Ground elevation	Depth, in ft	Rock elevation	Rock elevation	Depth, in ft	Ground elevation
Riverhill Dam site	SS-23	D-1	20	422	>126	<296	<301.1	>121.0	422.1
	SS-12	D-8	95	425	62	363	362.4	73.8	436.2
Franklin Falls Dam	18		30	398	15	382.7			
	..	D-107	...	...	...	...	376.2	14.1	390.3
	19		30	384	20	364.7			
West Hopkinton Dam Site No. 1	110	D-4	130	365	82	283	<292.7	> 73.4	366.1
	52	D-12	110	362	78	284	277.0	85.0	362.0
	113	D-10	100	359	73	286	<283.1	> 75.0	358.1
Canal between W. Hopkinton and Everett Reservoir	142	D-14	0	415.5	8.6	406.9	409.9	5.6	415.5
	40	D-7	0	394	39	355	<377.7	> 20.0	397.7
	151	D-3	0	418.3	28.9	389.4	<379.4	> 20.0	399.4
	159	D-4	0	404.0	23.3	380.7	<378.5	> 25.5	404.0
Everett Dam site	47	D-2	40	328.5	33.5	295	303.4	24.0	327.4
	84	D-7	20	407.8	10.8	397	384.3	13.3	397.6
West Peterboro Dam site	S-2	D-7	45	945	40	905	914.8	43.5	958.3
	S-1	D-5	190	945	90	855	<851.4	>100.0	951.4
	18	D-1	100	978	52	926	928.4	44.0	972.4
	S-7	D-11	60	985	28	957	943.8	23.2	967.0
	S-6	D-10	60	925	35	890	885.1	42.7	927.8
West Henniker Dam site	4	D-9	0	540	137	403	461.9	73.1	535.0
	7	D-2	15	612	154	458	<533.0	> 75.2	608.2

[a] Distance from drill hole to seismic center.

The writers agree with the conclusions presented by the authors. Particularly is it desired to emphasize the usefulness of the seismograph for preliminary investigations for dam sites on the basis of relative cost, time required, and accuracy when compared with drilling.

E. R. SHEPARD,[14] ESQ., AND REUBEN M. HAINES,[15] ASSOC. M. AM. SOC. C. E.—The question of disturbance of detectors in moving water has been raised by Mr. Moore. Some disturbance was observed in relatively still water where the detectors were supported by iron pipes projecting above the water surface. This appeared to be the result of surface wind ripples against the supporting pipes. With the tripod type of detector support, where the detector and supporting structure were entirely submerged, the disturbance, even in flowing water, was surprisingly small.

The writers are indebted to Professor Leet for clarifying the record with respect to credit for the equations used in computing overburden depths. The paper does not refer specifically to the origin of the formulas but states that they are given by Messrs. Ewing, Crary, and Rutherford. They, in turn, do not claim originality but refer to the formulas as the standard method of interpreting refraction data. Reference was made to this article because the equations there appear in a simple and usable form and are readily available for reference.

---

[14] Senior Physicist, Corps of Engrs., U. S. Army, Washington, D. C

[15] Engr., Corps of Engrs., U. S. Army, Washington, D. C.

Reverse profiles, as described by Professor Leet, were actually obtained over the 80-ft distance between outside detectors. This 80-ft spread was a common path for waves through the rock from reverse shooting distances greater than the critical distance, beyond which the first arrivals are through the rock stratum. From such shots it was possible to compute apparent and true velocities in the rock directly under the 80-ft detector spread. It is true that the local velocity so obtained was assumed to prevail throughout the entire length of the seismic line. The small errors that might result from this assumption are usually minor in comparison with those referred to by Mr. Lee, involving small differences in the thicknesses of the slow beds which can introduce large time errors when applied to the same ray path in the fast beds. Because of this almost universally prevalent condition only one depth of overburden was recorded for each seismic line. The recorded depth was designated as that at the center of the line where the velocity profile in the overburden was determined most accurately.

In refraction shooting where a plurality of detectors is employed and a relatively few shots placed within the detector spread, the reverse profile through the fast beds is obtained only over a limited central portion of the line, the length of the common path depending on the depth of the overburden. If the overburden is deeper than a critical value depending on the shooting distances and detector spacings, it is possible that no common path will exist, and in such an event it would be impossible to compute the true velocity in the rock from the observed apparent velocities.

Mr. Grier describes a new and interesting modification of the well-known four-terminal electrical method of exploration. From a study of his circuit it appears that what the instrument actually does is to compare the potential drop $P_1 P_2$ with that of $P_2 P_3$. With a symmetrical distribution of the field current between C_1 and C_2, the potential ratio will be a measure also of the ratio of resistances $P_1 P_2$ and $P_2 P_3$. Mr. Grier does not say how far he places the current electrodes from the potential cups, but it is evident that, to insure a reasonably symmetrical distribution of the current in the ground, the current electrodes should be spaced equally from the respective potential cups P_1 and P_3 and at a sufficient distance from them to avoid the distorting effects on the field current of local variations in the resistivity of the top soil. These conditions are usually met, with the standard four-terminal arrangement of electrodes.

The writers are familiar with the seismic programs described by Messrs. Shannon and Wells. In Table 2, showing comparisons of seismic interpretations with drilling records, eight of the drill holes listed were not extended to rock. The only point at which the seismic and drill data appear to be seriously out of agreement is at the West Henniker Dam site where the drill reached the top of rock at a depth of 73.1 ft, and computations from seismic data gave a depth of 137 ft. The original records on this project have been consulted. The drill log for hole No. 9 reveals a soil profile as follows:

Depth, in ft	Soil
0 to 1	Top soil
1 to 11	Light gray, compact, silty fine to medium sand, with a trace of gravel
11 to 53	Grayish brown, compact, slightly gravelly, silty sand (glacial till)
53 to 70	Brown, compact, slightly silty fine to medium sand
70 to 72.6	Dark gray, compact, gravelly sand
72.6 to 89.3	(Bottom of boring) mica-schist, badly weathered to bottom of boring, over-all core recovery 32.5%

The seismic data at line 4 show the following velocity profile:

Depth, in ft	Wave velocity, in ft per sec
0 to 4.5	1,180
4.5 to 17.5	2,240
17.5 to 137	7,000
>137	16,550

It is evident that the velocity of wave propagation in the till and that in the badly weathered mica-shist were so nearly alike that it was impossible to discriminate between the two formations. No doubt other minor discrepancies shown in the table can be attributed to the same condition. It can be said for the seismic method that the weathered mica-shist, which is obviously unfit as foundation material for a masonry dam, did not register as sound rock. This instance is an excellent example of the limitations in geophysical methods of exploration and emphasizes the need for experience and sound judgment in the interpretation of all geophysical data.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2209

DETERMINATION OF KUTTER'S *n* FOR SEWERS PARTLY FILLED

By C. Frank Johnson,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Thomas R. Camp, Richard G. Coulter, C. E. Ramser, and C. Frank Johnson.

Synopsis

Kutter's formula is widely used in computations involving the hydraulics of sewers. The value of the coefficient of roughness, *n*, in this formula can be determined for sewers by only one method—making actual measurements of velocities (or discharges) and depths of flow in the sewers. The results of measurements made in pipes and conduits carrying clean water cannot be used for sewers without modification; however, they do afford a valuable guide. Unfortunately, records of determinations of the value of *n* for sewers are all too scarce.

This paper presents the results of certain determinations, based on field measurements, of the value of Kutter's *n* for sewers partly filled. The technique followed in making the field measurements is described, and a comparison is made between the various methods used for measuring the velocities of flow in the sewers.

Introduction

During extensive gagings of sewage flows in Louisville, Ky., in 1939–1940, field determinations were made of the velocities of flow and corresponding discharges in four sewers at various depths of flow ranging in different sewers from 0.03 D to about 0.4 D. From the results of these determinations Kutter's n was computed for each test run, and an average value determined for each sewer. Several conclusions based on the results of these determinations are given at the close of this paper. The one of most general interest, probably, indicates that the values of n for low flows in sewers are higher than the values for flows of greater depth. The fact that higher values of n obtain for pipes not substantially filled has been recognized for some time;[2,3] however, there is a scarcity of experimental data showing the actual values of n for sewers having low flows.

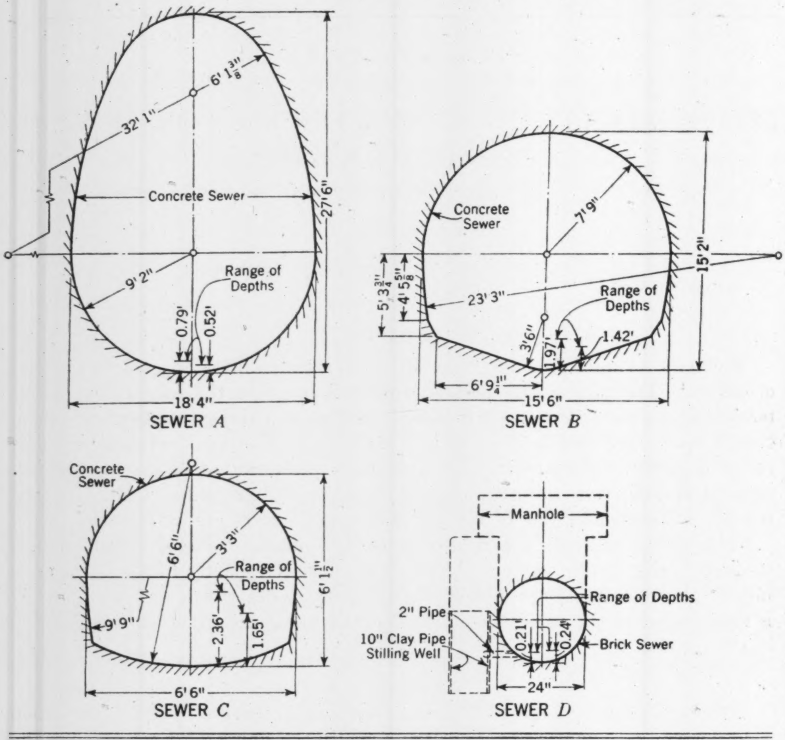
Note.—Published in February, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Chf. Engr., Commrs. of Sewerage of Louisville, Louisville, Ky.

[2] "On the Proper Value of Kutter's *n* for Sewer Computations," by Charles W. Sherman, *Engineering News-Record*, Vol. 88, No. 23, June 8, 1922, p. 948.

[3] "Hydraulic Formulas Incorrect for Partly Filled Pipes," by Charles W. Sherman, *ibid*, Vol. 103, No. 7, August 15, 1929, p. 253.

The sewers in which the measurements were made are shown in Fig. 1. These sewers were considered to be representative of average city sewers with respect to cleanliness; they had never been cleaned by city forces, so far as was known, nor was there any indication that they particularly needed cleaning.


6' 1 3/8"
32' 1"
Concrete Sewer
27' 6"
9' 2"
Range of Depths
0.79'
0.52'
18' 4"
Concrete Sewer
7' 9"
15' 2"
5' 3 3/4"
4' 5 5/8"
23' 3"
Range of Depths
3' 6"
1.97'
1.42'
6' 9 1/4"
15' 6"
Concrete Sewer
6' 6"
3' 3"
Range of Depths
6' 1 1/2"
9' 9"
2.36'
1.65'
6' 6"
Manhole
Range of Depths
2" Pipe
10" Clay Pipe Stilling Well
0.21'
0.24'
Brick Sewer
24"

SEWER *A* SEWER *B* SEWER *C* SEWER *D*

Sewer	Interior Dimensions (Ft)		Shape	Age, in years	Nominal slope	Length, in feet [a]
	Horizontal	Vertical				
A	18.33	27.50	Inverted egg	7	0.00087	1,550
B	15.50	15.17	Horseshoe	30	0.0005	890
C[b]	6.50	6.13	Horseshoe	31	0.0005	1,060
D[c]	2.00	2.00	Circular	66	0.0324	260

[a] Approximate length of test reach. [b] An intercepting sewer. The others are combined sewers. [c] A brick sewer. The others are concrete sewers.

FIG. 1.—CHARACTERISTICS OF SEWERS IN WHICH MEASUREMENTS WERE MADE

The measurements of velocities and discharges described herein were made on various dates during dry weather, when the sewage presumably contained no storm water; nor was there any ground water entering the sewers within the test reaches, according to available data. Although Sewer A (Fig. 1) was partly below the ground-water level, lead-strip water stops incorporated in the transverse construction joints prevented the entrance of ground water. The other sewers were above the ground-water level. It was assumed that the flows were not increased by ground water entering within the test reaches.

GENERAL TECHNIQUE

Velocities were measured by the dye-velocity (or color) and the salt-velocity methods. Each measurement was made by noting the time of travel of the dye or salt solution from the point of application to a downstream point of observation. The solution, after thorough mixing, was poured into the flowing sewage as rapidly as possible, and the average time of the application noted either on a stop watch or by the use of the second hand on an ordinary watch. Observations at the downstream point were made by the use of the same watch or another watch synchronized with the one used at the upstream point.

Dye-Velocity Method.—When using dye, the time of its appearance at the downstream station was noted visually, as was also the approximate time of the most intense color and the time when the dye could last be detected. The exact time of the appearance of the dye was easily determined, but the exact times of the most intense color and the last appearance of the dye were difficult to determine, because of the gradual changing or fading of the color at these times. Also, the natural discoloration of the sewage and the necessity for using artificial light in the sewers hampered these visual determinations somewhat.

The dye-velocity method has previously been utilized extensively for measuring velocities of both sewage and clean water and has been shown[4] to give results averaging within about 3% of those obtained by the use of carefully constructed weirs. The time of travel of the dye is generally considered to be the interval between the instant of introduction of the solution and the average time of the first and last appearance of the color at the downstream station.[4,5]

Salt-Velocity Method.—When using salt, the time of passage of the salt at the downstream station was determined either by the chemical method—that is, by titrating small samples of sewage taken at definite times—or by the electrical-resistance method, using electrodes immersed in the flowing sewage.

For the chemical or titration method, samples were dipped from the flowing sewage near the center of the sewer at a point just below the surface. Except on two runs, the samples were taken at 10-sec intervals, starting 1 or 2 min before the computed time when a pilot dye test indicated that the first salt should reach the downstream station and continuing until 1 or 2 min after the like computed time of the last appearance of the salt. The samples were titrated for chloride content, using standard methods with silver nitrate as a reagent and potassium chromate as an indicator. A graph of the chloride content of the sewage was then plotted, using time as abscissas and chloride content as ordinates. The center of gravity of the area included between the line of normal chloride content extended and the line showing increased chloride content was found by cutting out the area from a piece of cardboard and suspending it from first one point and then another, marking the center of gravity at the intersection of the directions of the suspending cord. Then, on the graph the position of the vertical through this center of gravity, called the centroid vertical, was plotted, the location of the centroid vertical being considered to represent the mean time of passage of the salt particles.[6,7]

---

[4] "The Flow of Water in Concrete Pipe," by Fred C. Scobey, *Bulletin No. 852*, U.S.D.A., 1920, p. 19.

[5] "Handbook of Hydraulics," by Horace W. King, 2d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1929, pp. 372 and 373.

On the later runs, when using salt, the electrical-resistance method was used and found to be much easier, faster, and more satisfactory than titration of samples. Copper electrodes of 24-gage metal, 2 in. square, were mounted on a lead-weighted wooden strip at a spacing of about 1 in. apart. The electrodes were connected to one or more small dry cells, with a variable resistance in the line. Meters reading to 300 milliamperes and to 8 volts were used. The electrodes were suspended in the sewage, all other equipment and the observers being above ground. Readings of the milliammeter were taken at 2-sec or 5-sec intervals and were plotted in the same manner as the results of the titrations, previously described. The centroid vertical of the area of increased current resulting from the passage of the salt charge was taken to indicate the time of passage of the salt.[6]

The salt-velocity method has previously been used extensively for measuring the velocity of clean water in pipes under pressure and has been shown to give results, under such conditions, averaging only 0.32% in excess of a weir.[6] This method has been utilized for testing large pumps, after its accuracy had been verified by volumetric check runs.[8] However, the salt-velocity method does not appear to have been applied to a great extent for measuring velocities in sewers partly filled.

Depth-of-Flow Measurements.—For each test the depth of flow of the sewage was ascertained by measuring from a previously established reference mark or grade nail, set above the water at a known short distance above the sewer invert, to the water surface. This measurement was subtracted from the height of the reference mark above the invert. The height above the invert of each reference mark was determined in advance by measuring the flow depths of the sewage at several points in the vicinity, by averaging these, and by adding to the average depth the distance from the reference mark to the water surface at the time.

In Sewers A, C, and D (Fig. 1), the depths for the tests were measured at the downstream end of the reach, check measurements having indicated that the depths so determined represented the mean depth in the reach. In Sewer B, however, it was necessary to measure the depths at the upstream and downstream ends of the reach, as these depths varied as much as 0.07 ft; in this case, the average areas and average wetted perimeters were used.

Determination of Slopes.—The slope of the water surface in Sewers A, C, and D was considered to be the same as that of the sewer, because the check measurements previously mentioned had indicated that this was substantially correct. After they were checked by levels run in the field, the nominal slopes of these three sewers were used. However, in the case of Sewer B, the measured slope of the water surface (corrected to the slope of the energy gradient on two runs where the correction was as much as 0.01 ft) was utilized.

---

[6] "The Salt Velocity Method of Water Measurement," by C. M. Allen and E. A. Taylor, *Transactions*, A.S.M.E., Vol. 45, 1923, p. 285.

[7] "Study of Sewage Settling Tank Design," by C. H. Capen, Jr., *Engineering News-Record*, Vol. 99, No. 21, November 24, 1927, p. 833.

[8] "Building Big Pumps Better Earns A Bonus," *ibid.*, Vol. 125, No. 11, September 12, 1940, p. 85.

Determination of n.—Based on the velocities determined in the sewers, computations were made of the value of C in the Chézy formula:

$$V = C\sqrt{RS} \qquad (1)$$

in which V = velocity, C = coefficient, R = hydraulic radius, and S = slope of the energy gradient (or water surface). The results of these computations furnished data for the determination of Kutter's n by the use of tables in standard handbooks.

FIG. 2.—SEWER A, BEFORE BEING PUT INTO SERVICE

SEWER A

Description and Condition of Sewer.—Built in 1932, Sewer A is part of the Southwestern Outfall. Within the test reach the sewer is straight except for one curve which has a radius of 100 ft and which is 14 ft long. There are no "live" inlets within the reach. The invert is unlined concrete except for three stretches each 13.5 ft long, which are bricklined; and the concrete of the invert (up to the springing line) was originally poured against steel forms.

At the time of the tests the condition of this sewer was excellent in every way (see Fig. 2). The concrete of the invert was very smooth, being considerably better than average in this respect, and there was no deposit on the invert. The evenness of the grade was shown by a profile of the invert, and by measured depths of flow, which had a maximum variation of only 0.10 ft within the test reach at any one time. This was not considered excessive for such a large sewer.

Parshall Measuring Flume.—The dye-velocity and salt-velocity methods were checked in this sewer by use of a Parshall measuring flume[9] which was in place at the upstream end of the test reach. The check was made to ascertain just what stage or time of the passage of the dye or salt should be used for determining the mean velocity for these particular tests and to check the accuracy of the methods used.

The flume was so constructed that "free-flow" conditions were secured, necessitating the measurement of only one head, H_a, to determine the discharge. For each test, the head was measured immediately after pouring the solution into the sewage.

TABLE 1.—COMPARISON OF CHEMICAL METHOD WITH CORRESPONDING VALUES

Test No.[a]	(a) DETERMINATIONS BY USING DYE AND SALT										
	Chemical Used		Water (gal)[d]	Depth[e] d (ft)	Ratio[f] $\frac{d}{D}$	Section areas[g] A (sq ft)	Wet perimeter[g] P (ft)	Hydraulic radius[g] R (ft)	Velocity[g] (Ft Per Sec) Based on:		
	Kind[b]	Lb[c]							Peak content V_p	Centroid vertical[h] V_c	Time of appearance[i] V_m
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A-1	Dye	1.0	6	0.76	0.0415	3.73	7.54	0.495	1.92	....	1.80
A-2	Dye	4.5	15	0.78	0.0426	3.88	7.67	0.506	1.91	....	1.84
A-3	Salt	15.0	10	0.52	0.0284	2.12	6.25	0.339	1.50	1.44	1.36
A-4	Salt	30.0	15	0.77	0.0421	3.81	7.60	0.501	1.94	1.91	1.86
A-5	Salt	55.0	18	0.79	0.0431	3.96	7.73	0.512	1.99	1.95	1.87
Average		....	....	....	....	....	....	....	....	....	....

[a] The length of reach was 1,553 ft in each test except No. A-3, in which it was 1,592 ft. The slope of the water sodium chloride. [c] Approximate weight. [d] Dissolving water required. [e] Mean depth of flow, *d*. [f] Col. appearance for dye tests, and on centroid vertical of area of increased chloride content for salt tests. [j] From "The equals Q/A, or Col. 18 ÷ Col. 7. [l] Based on hydraulic elements in Cols. 5 to 9, with $S = 0.00087$. $= \frac{V_p - V_f}{V_f}$

General Results.—The results of the titration of the sewage samples taken in Sewer A are shown graphically in Fig. 3. The principal data and the results of these tests, as well as of those made by use of the dye, are given in Table 1(*a*).

---

[9] "The Parshall Measuring Flume," by Ralph L. Parshall, *Bulletin No. 423*, Colorado State Coll., Colorado Experiment Station, Fort Collins, Colo., 1936.

The velocities determined by use of the flume, and the corresponding values of Kutter's *n*, are shown in Table 1(*b*). A comparison of these velocities with those determined by use of dye and salt is given in Table 1(*c*). (In Fig. 3, Test No. A-4: B = beginning of increased chloride content; P = peak of salt content; E = end of increased chloride content; *M* = mean time of salt appearance; and the line BE is the line of normal chloride content.)

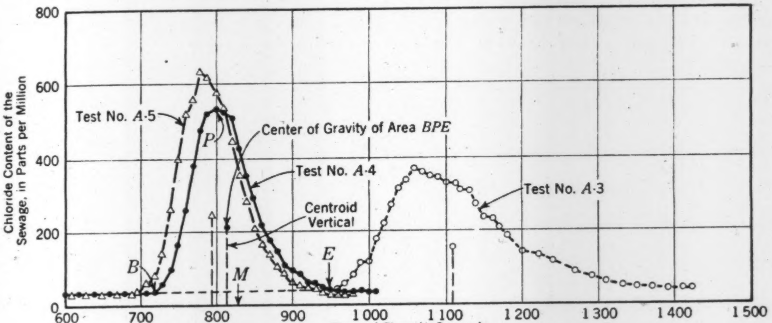


FIG. 3.—TITRATION OF SEWAGE SAMPLES FROM SEWER A

MEASURING FLUME, FOR DETERMINING VELOCITIES, AND THE
OF *n*, FOR SEWER A

n, in Kutter's Formula, Based on:				(*b*) FIVE-FOOT PARSHALL FLUME				(*c*) COMPARISON (%)			
Peak content	Centroid vertical[h]	Time of appearance[g]	Q (cu ft per sec)[i]	Observed head H_a (ft)	Q (cu ft per sec)[j]	V_f (ft per sec)[g,k]	Kutter's[l] n	Peak content[m]	Centroid vertical[n]	Mean time[o]	Test No.[a]
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(1)
0.0138	....	0.0146	6.71	0.48	6.24	1.67	0.0154	15.0	....	7.8	A-1
0.0142	....	0.0146	7.14	0.52	7.09	1.83	0.0147	4.4	....	0.5	A-2
0.0135	0.0140	0.0146	3.05	0.28	2.65	1.25	0.0156	....	....	....	A-3
0.0139	0.0141	0.0144	7.28	0.52	7.09	1.86	0.0143	4.3	2.7	0.0	A-4
0.0138	0.0140	0.0144	7.72	0.53	7.30	1.84	0.0146	8.2	6.0	1.6	A-5
0.0138	0.0140	0.0145	....	....	....	....	0.0149	8.0	4.4	2.5	Average

surface was 0.00087. [b] "Dye" denotes soluble Prussian blue dye (sodium ferri-ferrocyanide). "Salt" denotes 5 + 18.33 ft. [g] Mean values. [h] Area of increased chloride content. [i] Discharge based on the mean time of Parshall Measuring Flume," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E.[9] [k] Velocity of flow in the sewer, $= \frac{\text{Col. 10} - \text{Col. 19}}{\text{Col. 19}}$. [n] $\frac{V_c - V_f}{V_f} = \frac{\text{Col. 11} - \text{Col. 19}}{\text{Col. 19}}$. [o] $\frac{V_m - V_f}{V_f} = \frac{\text{Col. 12} - \text{Col. 19}}{\text{Col. 19}}$.

It will be observed that in Table 1(*c*) no comparison is shown for Test No. A-3. The results of this test are viewed with some question for two reasons: (1) The head on the flume (0.28 ft) was very near the recommended minimum[9] for a 5-ft flume; and (2) it appeared that some of the salt solution might have been retained unduly long at the point of application. This point was located at the downstream end of the flume, where some of the dye on a preceding test (which was rejected) had been retained for a short time, by eddies in the sewage flow. Furthermore, the chloride-content graph for Test No. A-3 (Fig. 3) is very flat after passing the peak, apparently from the same cause. This difficulty was obviated on the other tests by pouring the solution into the sewage at a point approximately 40 ft downstream from the flume.

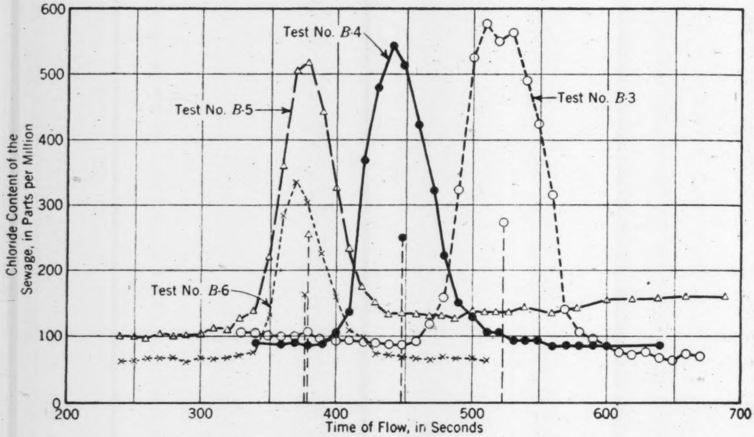


FIG. 4.—TITRATION OF SEWAGE SAMPLES FROM SEWER B

The comparison shown in Table 1(*c*) indicates a satisfactorily close check for the purposes of the gagings being made, the use of the centroid vertical on Tests Nos. A-4 and A-5 indicating velocities averaging 4.4% in excess of those obtained by use of the flume. According to Mr. Parshall,[9] the accuracy of discharge measurements made with the Parshall measuring flume is within 2% to 5%, under normal operating conditions. A closer check between the flume measurements and the dye and salt measurements might have been secured during the tests by taking the sewage samples for titration from some point in the cross section other than near the center of the sewer, say, at or near the quarter point. As the samples were taken near the center of the sewer, however, the computed velocities may be as much as 2% to 5% in excess of the respective true mean velocities, because there is some chance that only the fast water was thus timed. Such difference has been found when using the salt-velocity method in this way for measuring velocities in pipes entirely filled.[6] For these reasons, the velocities determined by use of the centroid vertical of the salt tests, made as described herein, seemed slightly above the true average velocities, and the resulting values of *n* were therefore slightly low. It was thought that these values of *n* were as much as, say, 2% to 5% low. (This possibility should be kept in mind when studying the results of the tests.) The use of a slightly low value of *n* was not objectionable from the standpoint of the purpose of the sewage gagings being made—to obtain data upon which to base the design of intercepting sewers and a sewage-treatment plant—as the computed discharges would then be slightly, but not significantly, on the high or safe side.

In addition to the other considerations, the results of the salt-velocity tests, rather than those of the dye-velocity tests, were used for the determinations of the mean velocities and the values of *n*, because the salt method affords more tangible data and gives more definite and satisfactory results than the dye method. For this reason, and, as a result of the comparisons made and discussed in the preceding paragraph, these determinations were based on the centroid vertical of the salt tests.

The dye tests were necessary in some instances as pilot tests, however, particularly when using titration of samples, and were of value in all instances as check observations. When the color method is used exclusively, the mean time of the dye appearance would probably indicate the mean velocity most satisfactorily.

Use of the centroid vertical for the salt tests and the mean time of the dye appearance for the color tests is in accord with the recorded practice of others.

Value of n.—Based on the centroid vertical, the average value of Kutter's *n* for Sewer A, at the extremely low depths of flow encountered, is 0.0140. The value of *n* for this sewer when substantially filled is probably 0.0130 or lower.

SEWER B

Description and Condition of Sewer.—Sewer B is part of the Southern Outfall, built in 1909. Within the test reach the sewer is straight, with one 15-in. inlet. However, no water seemed to be discharged by this inlet during the tests. The entire inside surface of the sewer is unlined concrete and, according to a report of the commission supervising its construction, "steel forms were used [on the side-walls] * * *, the invert having a troweled finish equivalent to that ordinarily given sidewalks."

At the time of the tests the condition of this sewer was good and the concrete surface fairly smooth, but not as smooth as the concrete of Sewer A. An inspection of the entire stretch, made about two years prior to the time when the velocity measurements were made, revealed that "most of the invert is free from deposit and is slippery"; however, a deposit about ½ in. deep was reported at the center of the sewer at three points within the stretch. At the time of the tests, conditions appeared to be substantially the same as when the inspection was made.

General Results.—The results of the titration of the sewage samples taken in Sewer B are shown graphically in Fig. 4, and in Table 2(*a*) are given the principal data and the results of all the tests conducted in this sewer.

The values of *n* based on the individual tests in this sewer are not so consistent as might be desired. This may be the result of a considerable deposit on the invert of the sewer at a point approximately 900 ft downstream from the test reach. The deposit acted as a low dam, causing the previously mentioned variations in the depth of flow within the test reach.

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer B at the depths encountered is 0.0144. The value of *n* for this sewer when substantially filled is probably about 0.0130.

SEWER C

Description and Condition of Sewer.—Built in 1908, this sewer is part of the Beargrass Intercepter. Within the test reach the sewer is straight for only about one third of its length. However, the curves are of such long radii (more than 500 ft) that the curvature seems to have had little or no effect on the value of *n*. There is one 8-in. inlet and one 20-in. lateral sewer within the stretch. It is not known whether the 8-in. inlet was discharging any water during the tests, but the 20-in. lateral sewer was discharging a volume equal probably to about 10% of the total volume flowing in Sewer C. However, this lateral is situated so near the upper end of the test reach (71 ft downstream) that the resulting error due to the addition of this flow within the reach should be slight. The entire surface of the sewer is unlined concrete. The concrete of the sidewalls was originally poured against wooden forms and the invert was probably given a troweled finish similar to that given Sewer B.

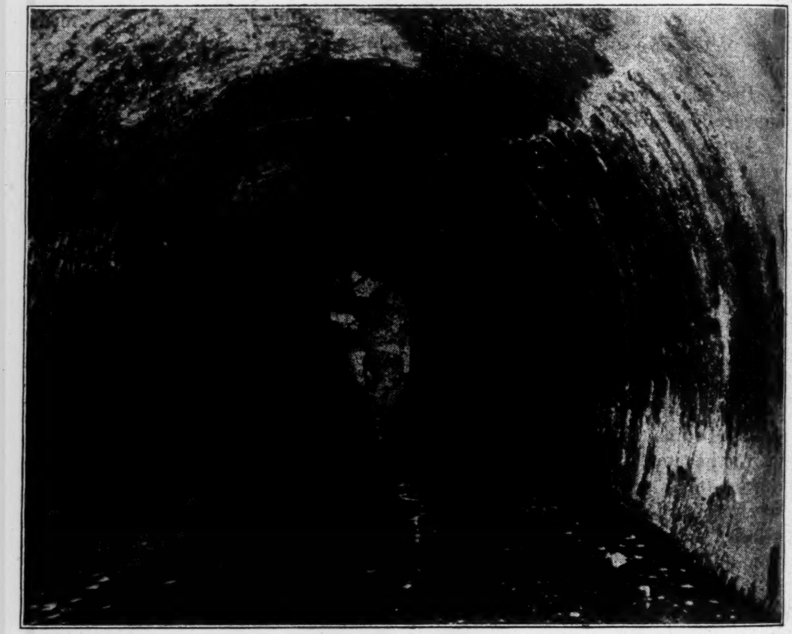


FIG. 5.—A VIEW IN SEWER C, WITH A PORTION OF THE FLOW DIVERTED TEMPORARILY

At the time of the tests the condition of this sewer was good, and the concrete surface fairly smooth (see Fig. 5)—about as smooth as the concrete of Sewer B. An inspection, made with most of the flow temporarily diverted,

TABLE 2.—DETERMINATIONS OF VELOCITIES AND CORRESPONDING VALUES OF *n* IN SEWERS B, C, AND D

Test No.	CHEMICAL USED		Water (gal)[c]	Length of reach (ft)	Slope[d] S	Depth[d,e] d (ft)	Ratio[f] $\frac{d}{D}$	Section areas[e] A (sq ft)	Wet perimeter[e] P (ft)	Hydraulic radius[e] R (ft)	VELOCITY[g] V (FT PER SEC) BASED ON:		*n*, IN KUTTER'S FORMULA, BASED ON:		Q (cu ft per sec)[h]
	Kind[a]	Lb[b]									V_c	V_m	Centroid vertical	Time of appearance[e]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(*a*) SEWER B (15 FT 6 IN. BY 15 FT 2 IN.—CONCRETE)															
B-1	Blue	0.8	5	892	0.00049	1.86	0.123	12.19	12.64	0.964	....	2.10	....	0.0152	25.6
B-2	Blue	2.0	15	892	0.00051	1.97	0.130	13.62	13.36	1.019	....	2.25	....	0.0150	30.6
B-3	Salt[i]	40	14	892	0.00049	1.42	0.094	7.51	9.94	0.756	1.70	1.67	0.0155	0.0159	12.8
B-4	Salt[i]	45	14	892	0.00049	1.58	0.104	9.07	10.92	0.831	1.99	1.88	0.0144	0.0152	18.1
B-5	Salt[i]	55	16	892	0.00052	1.82	0.120	11.74	12.40	0.947	2.34	2.26	0.0139	0.0144	27.5
B-6	Salt[i]	60	18	892	0.00052	1.84	0.121	11.98	12.52	0.956	2.36	2.29	0.0139	0.0143	28.3
(*b*) SEWER C (6 FT 6 IN. BY 6 FT 1½ IN.—CONCRETE)															
C-1	Fluor	0.13	2	1,057	0.0005	2.34	0.382	13.06	9.50	1.375	....	2.77	....	0.0148	36.2
C-2	Fluor	0.13	2	1,057	0.0005	1.68	0.274	8.82	8.18	1.078	....	2.44	....	0.0143	21.5
C-3	Salt[i]	40	17	1,057	0.0005	2.36	0.385	13.18	9.54	1.382	2.89	2.78	0.0144	0.0148	38.1
C-4	Salt[i]	40	17	1,057	0.0005	1.70	0.277	8.95	8.22	1.089	2.44	2.46	0.0145	0.0144	21.8
C-5	Salt	30	15	1,060	0.0005	1.65	0.269	8.63	8.12	1.063	2.66	2.60	0.0131	0.0134	23.0
C-6	Salt	30	15	1,060	0.0005	1.67	0.273	8.76	8.16	1.074	2.61	2.60	0.0135	0.0135	22.9
(*c*) SEWER D (24 IN.—BRICK)															
D-1	Blue	0.13	1	261	0.0324	0.23	0.115	0.201	1.384	0.145	....	3.33	....	0.0176	0.67
D-2	Blue	0.13	1	261	0.0324	0.23	0.115	0.201	1.384	0.145	....	3.44	....	0.0172	0.69
D-3	Blue	0.13	1	261	0.0324	0.24	0.120	0.214	1.415	0.151	....	3.33	....	0.0180	0.71
D-4	Salt	15	5	262	0.0324	0.24	0.120	0.214	1.415	0.151	3.91	3.63	0.0160	0.0169	0.84
D-5	Salt	10	4	262	0.0324	0.235	0.118	0.207	1.399	0.148	3.68	3.44	0.0166	0.0174	0.76
D-6	Salt	10	4	262	0.0324	0.235	0.118	0.207	1.399	0.148	3.61	3.40	0.0168	0.0175	0.75
D-7	Salt	10	4	262	0.0324	0.21	0.105	0.176	1.320	0.133	3.38	3.19	0.0166	0.0173	0.59
D-8	Dichr	2.5	5	262	0.0324	0.21	0.105	0.176	1.320	0.133	3.56	3.36	0.0159	0.0166	0.63

[a] "Blue" denotes soluble Prussian blue dye (sodium ferri-ferrocyanide); "salt" denotes sodium chloride; "fluor" denotes fluorescein dye; and "dichr" denotes potassium dichromate. [b] Approximate weight. [c] Dissolving water required. [d] Slope of water surface, S, and mean depth of flow, d. [e] Mean values. [f] Col. 7 ÷ D (in which D = vertical diameter of the sewer). [g] V_c = mean value, centroid vertical; and V_m = mean time of appearance. [h] Based on the mean time of dye appearance for the dye tests, and on the centroid vertical for the salt and dichromate tests. [i] Titration of samples was used for these tests. The electrical-resistance method was used for other salt (and dichromate) tests.

two years prior to the time of making the velocity measurements, revealed that this part of the sewer was in good condition, with the exception of a 50-ft stretch where the arch was severely cracked. The gravel was loose along these cracks, and there were also quite a few transverse hair cracks and a few longitudinal hair cracks for short stretches (as in Fig. 5). The defective conditions would probably have little or no effect on the value of *n* under the conditions of the tests made. During this same inspection two short stretches of deposit, both in the vicinity of the 20-in. lateral, were discovered. One stretch extended 50 ft, averaging about 2 in. deep at the center of the sewer, and the other stretch extended 20 ft, averaging about 1 in. deep. Conditions at the time of the tests were substantially the same as when the inspection was made.

General Results.—Two salt-velocity tests were made in Sewer C by the titration method and two by the electrical method. The results of the electrical determinations are shown graphically in Fig. 6. The principal data and the results of all the tests made in this sewer are given in Table 2(*b*). As in Fig. 3, Points B, P, and E for Test No. C-5 denote, respectively: The beginning of increased chloride content; the peak of the salt content; and the end of the increased chloride content. The line of normal chloride content is marked BE, and M denotes the mean time of salt appearance.

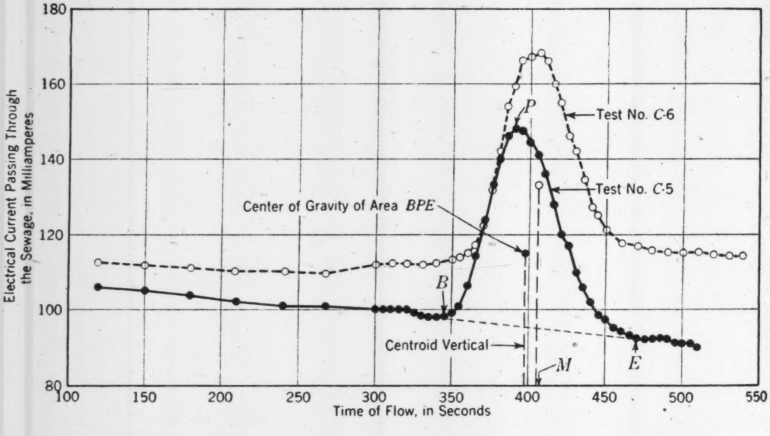


Fig. 6.—Electrical Determinations of Increased Chloride Content in Sewer C

Table 2(*b*) shows that there is a comparatively wide difference between the results of the first series of tests in this sewer, using dye and titration of sewage samples, and the results of the second series, using electrical methods with salt. No particular reason is known for this difference. The two series were made on different dates when there may have been a variation in the lateral sewer discharge or possibly in some other factor not apparent. The accuracy of the

FIG. 7.—A VIEW IN SEWER D, WITH FLOW CONDITIONS SIMILAR TO THOSE DURING VELOCITY MEASUREMENTS

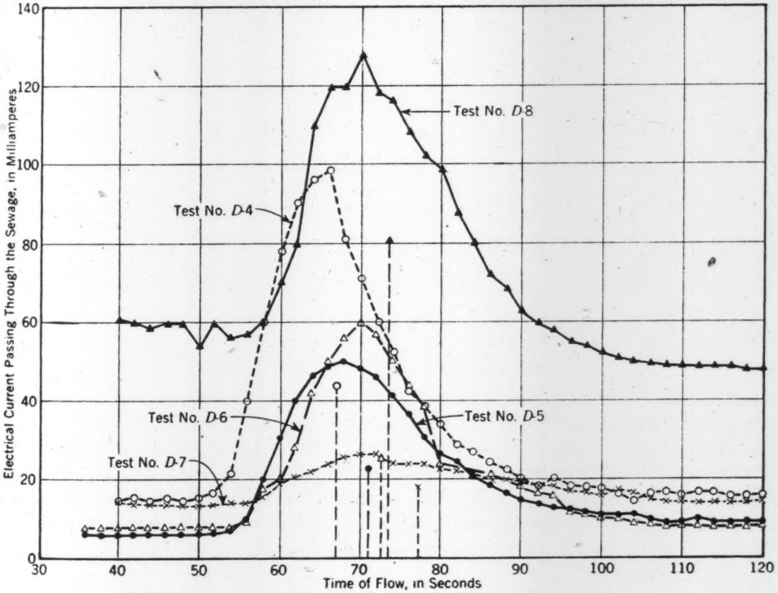


FIG. 8.—ELECTRICAL DETERMINATIONS OF INCREASED CHLORIDE CONTENT IN SEWER D

methods used during the second series certainly exceeded that of the first series; the crew was also considerably more experienced when making the second series. For these reasons the values of *n* determined from the later tests (Nos. C-5 and C-6) appeared the more reliable.

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer C, at the depths encountered, is 0.0139. However, the value accepted for use in the gaging work is 0.0135, giving more weight to the values determined from Tests Nos. C-5 and C-6—namely, 0.0131 and 0.0135. The value of *n* for this sewer when substantially filled is probably about 0.0130.

SEWER D

Description and Condition of Sewer.—This is the old brick sewer in Third Street, north of Main Street, constructed in 1873. Within the test reach the sewer is straight and on a uniform grade. There was little or no sewage being discharged into the sewer within this stretch while the tests were being made. At the time of the tests the condition of this sewer was good, considering its age (see Fig. 7). In general, the joints were well filled, and there were few pronounced irregularities in the brick surface. The walls were fairly straight—in fact, it was easy to see from manhole to manhole.

General Results.—Electrical methods were used exclusively for the salt tests in this sewer, since, in view of the extremely short time of flow (varying from 67 to 77 sec), it would have been impossible to take sewage samples sufficiently fast to obtain accurate results. As an experiment potassium dichromate was used, successfully, on the last test. The results of these tests are shown graphically in Fig. 8, and the principal data and the results of all the tests made in this sewer are given in Table 2(*c*). The results of the individual tests made in this sewer are fairly consistent in spite of the extremely short time of flow.

TABLE 3.—SUMMARY OF AVER

No.	Kutter's *n*, based on:	SEWER A		
		Dye	Salt	All
1	Peak, dye or salt content	0.0140	0.0137	0.0138
2	Centroid vertical	....	0.0140[a]	0.0140
3	Mean time, dye or salt appearance	0.0146	0.0145	0.0145
4	Discharge of flume	0.0151	0.0148	0.0149
5	Value adopted for gaging work		0.0140	

Value of n.—Based on the centroid vertical the average value of Kutter's *n* for Sewer D, at the depths encountered, is 0.0164. This value was accepted for use. The value of *n* for this sewer when substantially filled is probably about 0.0150.

SUMMARY OF AVERAGE VALUES OF n

The average values of Kutter's n for all the sewers are summarized in Table 3, which indicates that the average value based on the centroid vertical of the salt tests was accepted for the gaging work in every instance except one. The reason for the variation in this instance has been given.

COMPARISON OF KUTTER'S n WITH MANNING'S n

For comparison, the values of Kutter's n based on the centroid vertical of the salt-velocity tests are shown in Table 4 with the corresponding values of n in the Manning formula.[10]

In the case of Sewers B and C, the respective values of n are practically the same. These two sewers had the largest hydraulic radii of the four. There is a small difference in the two values for Sewer A, but the greatest difference is found in the case of Sewer D, for which the hydraulic radius was the least. In the light of similar comparisons previously made,[10] this relation between the values of n in the two formulas is not abnormal.

CONCLUSIONS

1. In general, higher values of n should be used for low flows in sewers than for flows of greater depths. The values given in Table 5, based largely on the tests described herein, are suggested for use in the absence of experimental data indicating other values. The values shown in the table are fairly consistent with such other tests on sewers as have come to the writer's attention, with a few exceptions. However, the values are higher than those found by others for conduits carrying clean water, probably because of the thin coating of slime and grease deposited on the sewer walls by sewage over a period of years.

2. The reasons for higher values of n for low flows are not clear. Friction between the free surface of the water and the air above has been suggested.[3]

AGE VALUES OF KUTTER'S n

SEWER B			SEWER C			SEWER D			No.
Dye	Salt	All	Dye	Salt	All	Dye	Salt	All	
....	....	....	....	....	....	....	....	....	1
....	0.0144[a]	0.0144	....	0.0139	0.0139	....	0.0164[a]	0.0164	2
0.0151	0.0150	0.0150	0.0146	0.0140	0.0142	0.0176	0.0171	0.0173	3
[a] This value was selected as the acceptable value for gaging work.									4
0.0144			0.0135			0.0164			5

---

[10] "Handbook of Hydraulics," by Horace W. King, 2d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1929, pp. 182, 283, and 318.

TABLE 4.—COMPARISON OF KUTTER'S *n* WITH MANNING'S *n*, BASED ON THE CENTROID VERTICAL OF THE SALT-VELOCITY TESTS

No.	Coefficient *n*	Test Numbers: 3	4	5	6	7	8	Average
				(*a*) Sewer A				
1	Kutter	0.0140	0.0141	0.0140	....	....	....	0.0140
2	Manning	0.0148	0.0145	0.0144	....	....	....	0.0146
				(*b*) Sewer B				
3	Kutter	0.0155	0.0144	0.0139	0.0139	....	....	0.0144
4	Manning	0.0160	0.0146	0.0140	0.0139	....	....	0.0146
				(*c*) Sewer C				
5	Kutter	0.0144	0.0145	0.0131	0.0135	....	....	0.0139
6	Manning	0.0143	0.0144	0.0130	0.0134	....	....	0.0138
				(*d*) Sewer D				
7	Kutter	....	0.0160	0.0166	0.0168	0.0166	0.0159	0.0164
8	Manning	....	0.0194	0.0203	0.0207	0.0206	0.0195	0.0201

The coating of slime and grease on the lower sections of the sewer walls may have some influence.

3. The dye-velocity and the salt-velocity methods can be applied conveniently for measuring velocities of flow in sewers partly filled. The salt-velocity method gives more satisfactory, and probably more accurate, results than the dye-velocity method, and the use of electrical resistance for determining relative salt content is decidedly much easier and quicker than titration of samples.

4. For dye-velocity tests the average time of passage of the dye is indicated best by the average time between the first and last appearance of the dye at the observation point. For salt-velocity tests the average time of passage of the salt is indicated best by the centroid vertical of the area of increased chloride content.

TABLE 5.—SUGGESTED VALUES OF *n* FOR SEWERS HAVING NO OBSTRUCTIONS

Depth of flow (1)	*n*: Minimum (2)	Average[a] (3)	Maximum (4)
	Monolithic-Concrete Sewers		
0 to ⅛ full..........	0.0140	0.0150	0.0160
⅛ to ¼ full..........	0.0135	0.0145	0.0155
¼ to ½ full..........	0.0130	0.0140	0.0150
½ full to full........	0.0125	0.0130	0.0135
	Old Brick Sewers		
0 to ¼ full..........	0.0160	0.0170	0.0180

[a] Recommended values for use where inside surfaces are fairly smooth and otherwise in good condition. Lesser or greater values should be used where conditions warrant.

5. For measuring velocities in sewers partly filled, the samples for titration should not be taken, nor should the electrodes be placed, at the center of the sewer. Additional field tests are desirable to indicate the best point in the cross section for taking the samples and for placing the electrodes.

Conclusion 4 indicates that the use of the average time of passage of the dye may give results approximating those obtained by use of the centroid vertical of the salt tests. This may appear illogical. Indeed, at first glance, the results of the measurements described herein (see Table 3) do not seem to substantiate the conclusion stated. However, recalling that the values of n based on these salt tests are thought to be possibly as much as 2% to 5% low, since only the fast water is timed, the results appear more consistent in this respect. Moreover, when using dye there is little or no alternative to accepting the average time between the first and last appearance of the dye for computing the velocity. There is no other time subject to satisfactory determination, and, indeed, this time is difficult to determine satisfactorily when using the dye in a sewer. However, for the salt tests there is ample evidence supporting the use of the centroid vertical of the area of increased chloride content.

(A possible reason why the average time of the passage of the dye should agree closely with the time indicated by the centroid vertical is advanced by C. M. Allen and E. A. Taylor.[6] This reason is that the eye cannot detect the entire curve of real color but sees only the portion of the curve formed when the dye is not too diluted; that is, portions of the color curve near each end are not detected. As a rule, the average time obtained by observing the visible part of the curve agrees fairly closely with the time obtained by use of the centroid vertical of the salt tests.)

ACKNOWLEDGMENTS

The work described herein was done by the engineers of the Commissioners of Sewerage of Louisville, Ky., under the general direction of the late Woolsey M. Caye, M. Am. Soc. C. E., at that time technical engineer for the commission. Aid and advice were given by the commission's consulting engineers, Metcalf and Eddy, of Boston, Mass. The work was under the immediate direction of the writer, then senior engineer for the commission. On some of the field tests assistance was rendered by Stuart E. Coburn, chemist for Metcalf and Eddy.

DISCUSSION

THOMAS R. CAMP,[11] M. AM. SOC. C. E.—The measured values of velocity and friction factor submitted by Mr. Johnson, for partly filled sewers, constitute a valuable contribution in a field where reliable measurements are very scarce. The data for Sewer A and Sewer D are of general interest because they apply to a circular cross section; and they are especially valuable because the depths are so small.

It has been known for some time (and the author has so stated) that the value of Kutter's n varies with the depth of flow in a sewer. Information on the manner of this variation has been so scarce, however, that it has been necessary in design to assume a constant value of n regardless of the depth of flow. This practice is illustrated in the shapes of the curves for the "hydraulic elements" appearing in most textbooks on sewer design; in which the velocity for a half-full circular pipe is shown to be the same as the velocity for the full pipe (see the broken-line curve in Fig. 9).

In measuring the friction factor in a sewer, the experimenter has no control over the discharge and must accept the flow at the time of the measurements. Therefore, the scarcity of measurements for any substantial variation in depth for the same sewer is quite understandable. In fact there was no substantial variation in depth during Mr. Johnson's measurements, and the conclusion that n varies with depth hardly can be drawn solely from his recorded observations.

It is to be regretted that the opinion is so widely held that measurements made in pipes carrying clean water are not applicable to the flow of sewage. Such is not the case for clean sewers. The amount of suspended matter normally present in sewage is so small that it has a negligible effect on the flow pattern in the pipe. The friction factor is a function only of the Reynolds number and the pipe roughness, and therefore measurements made with clean water are equally applicable to sewage flowing in the same pipe.

Measurements such as those made by Mr. Johnson are of limited value unless the results can be used to predict what the friction factor will be at other depths of flow. His estimates for the values of Kutter's n for the sewers flowing full are not substantiated by any experimental measurements, and they throw no light upon the value of n for other depths.

The best experimental measurements for sewers of circular cross section flowing partly full appear to be those of E. R. Wilcox[12] on 8-in. clay and concrete sewer pipe and S. M. Woodward, Hon. M. Am. Soc. C. E., and the late D. L. Yarnell, M. Am. Soc. C. E.[13] on open-butt-joint concrete and clay drain tile 4 in. to 12 in. in size. In Mr. Wilcox's work the pipes were clean and were laid carefully with good joints. The drain tile used by Messrs. Yarnell and Woodward was placed carefully in earth but with open-butt joints. The depth ranged from about $\frac{1}{4}$ full to full, in these experiments. The results showed conclusively that the friction factor decreases as the depth of flow increases, but the data were so erratic that reliable and accurate trends of friction factor with depth were not apparent.

---

[11] Associate Prof., San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[12] "A Comparative Test of the Flow of Water in 8-Inch Concrete and Vitrified Clay Sewer Pipe," by E. R. Wilcox, *Bulletin No. 27*, Univ. of Washington Eng. Experiment Station, March 1, 1924.

[13] "The Flow of Water in Drain Tile," by D. L. Yarnell and S. M. Woodward, *Bulletin No. 854*, U. S. D. A., 1920.

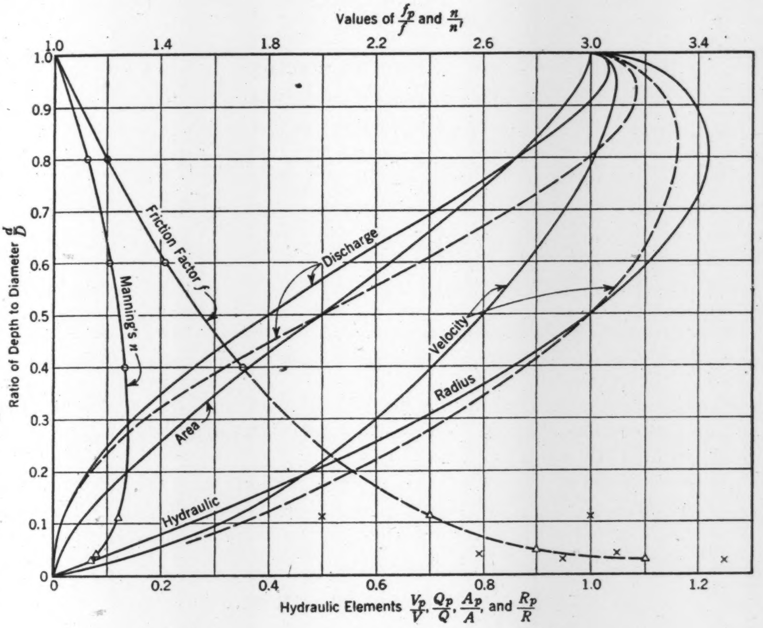


FIG. 9

The writer undertook a study of these data[12, 13] in an effort to find a trend of friction factor with depth.[14] The average value of the Weisbach-Darcy friction factor f in Mr. Wilcox's work was about 0.019 (Manning's $n = 0.0095$) for 8-in. vitrified sewer pipe flowing full and about 0.022 (Manning's $n = 0.0102$) for 8-in. concrete sewer pipe flowing full. In the experiments of Messrs. Yarnell and Woodward, the average values of f and Manning's n for the pipes flowing full were as shown in Table 6. These values are much less than those generally used in practice. (The subscript p herein denotes "partly full.")

A study of the ratios of the values of the friction factor for partly full f_p and full pipes f in these experiments indicates a trend as shown by the points marked with circles in Fig. 9. These points represent the average results of 824 experiments by Messrs. Yarnell and Woodward on well laid drain tile, both concrete and clay. The average values of $\frac{f_p}{f}$ for each pipe size for slopes varying from 0.0005 to 0.015 are shown in Table 6.

---

[14] "Final Report of the Committee to Study Limiting Velocities of Flow in Sewers, March 4, 1942," *Journal*, Boston Soc. of Civ. Engrs., October, 1942, p. 286.

The values of the ratios $\frac{f_p}{f}$ shown from the experiments of Messrs. Yarnell and Woodward as well as from those of Mr. Wilcox were quite scattered; but the averages for each ratio of depth to diameter, $\frac{d}{D}$, checked closely for both studies, the maximum deviation being about 5%. The experimental data seemed to show no consistent influence of the pipe material or of the variation in pipe size or slope. Mr. Wilcox used slopes of 0.005 to 0.04, so that the range of slopes for both sets of data was 80 to 1.

TABLE 6.—RATIOS OF FRICTION FACTORS $\frac{f_p}{f}$ FOR VARIOUS DEPTH-DIAMETER RATIOS

Pipe size (inches)	Friction factor, f	Manning's n	DEPTH RATIOS, d/D		
			0.8	0.6	0.4
4	0.033	0.0111	1.28	1.8	....
5	0.028	0.0106	1.15	1.34	1.9
6	0.027	0.0107	1.22	1.50	1.9
8	0.025	0.0108	1.18	1.38	1.7
10	0.023	0.0108	1.06	1.12	1.32
12	0.021	0.0107	1.21	1.40	1.70
Average	....	....	1.20	1.42	1.70

A study of the effective roughness projection k in these pipes was made from the von Kármán-Nikuradse[15] theory of turbulent flow in full pipes. The value of the friction factor f as given by Theodor von Kármán, M. Am. Soc. C. E., is, for smooth pipe—

$$\frac{1}{\sqrt{f}} = -0.8 + 2 \log (\mathbf{R} \sqrt{f}) \qquad (2a)$$

and, for rough pipe—

$$\frac{1}{\sqrt{f}} = 1.74 + 2 \log \frac{D}{2k} \qquad (2b)$$

in which $\mathbf{R}$ = the Reynolds number $= \frac{V D}{\nu}$ and k = the roughness projection on the pipe wall. The effective roughness projection on the walls of the pipe for the experiments of Messrs. Yarnell and Woodward is about 0.4 mm from Eq. 2b and is somewhat less for the pipe used by Mr. Wilcox.

A fairly good estimate can be made of the values of f for Mr. Johnson's Sewer D, and for Sewer A considered as a circular conduit, by guessing at the effective roughness projection k. From the description and from Fig. 2, a value of 0.5 mm seems reasonable for k, in Sewer A. From Eq. 2b, the corresponding value of f is 0.0117, assuming that turbulence is fully developed in the boundary film. For a smooth pipe, the value of f for Sewer A would be 0.0081 from Eq. 2a. An investigation of the thickness of the laminar film for the smooth pipe indicates that turbulence is not fully developed for k = 0.5 mm.

---

[15] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938.

Hence a somewhat smaller value of f, say 0.011 (Manning's $n = 0.0125$), is selected for Sewer A. From the photograph of Sewer D (Fig. 7), a value of $\frac{1}{4}$ in. seems reasonable for k; and, since turbulence is fully developed near the wall, the corresponding value of f is 0.038 (Manning's $n = 0.016$).

The writer has computed the values of f, from the author's measurements, and the corresponding ratios $\frac{f_p}{f}$ based on the foregoing values of f. For Sewer A, the average value of f_p for runs 1, 2, 4, and 5 is about 0.031 for $\frac{d}{D} = 0.042$, the corresponding value of $\frac{f_p}{f}$ being 2.8. For Sewer A, if $\frac{d}{D} = 0.028$, f_p is about 0.035, and $\frac{f_p}{f} = 3.2$. For Sewer D, if $\frac{d}{D} = 0.11$, the average value of f_p is about 0.09, and $\frac{f_p}{f} = 2.4$. The points corresponding to these values of $\frac{f_p}{f}$ are shown as triangles in Fig. 9, and the previously constructed curve has been projected through them.

For Sewer D, if the effective roughness projection k is $\frac{1}{2}$ in. or $\frac{1}{8}$ in., the corresponding values of $\frac{f_p}{f}$ are 2.0 and 3.0, respectively. For Sewer A, if f is taken at 0.010 or 0.012, the corresponding ratios $\frac{f_p}{f}$ are 3.1 and 2.6 for $\frac{d}{D} = 0.042$, and 3.5 and 2.9 for $\frac{d}{D} = 0.028$. These points are indicated as × in Fig. 9 for comparison with the selected values.

The effect of the variation of $\frac{f_p}{f}$ as shown by the curve upon the "hydraulic elements" is to displace the velocity and discharge curves from the positions indicated by the broken lines to the positions shown as solid lines.

The values of the ratio of Manning's coefficient,

$$\frac{n_p}{n} = \left(\frac{R_p}{R}\right)^{1/6} \sqrt{\frac{f_p}{f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

have been computed from the corresponding values of $\frac{f_p}{f}$ and are shown in Fig. 9. The positions of the curves $\frac{f_p}{f}$ and $\frac{n_p}{n}$ in this figure are based upon the best available experimental data, but these data are none too reliable for the purpose. Accurately controlled hydraulic laboratory work for a more precise evaluation of these functions is badly needed.

RICHARD G. COULTER,[16] JUN. AM. SOC. C. E.—In his work at Louisville, Ky., the author encountered a phenomenon reported by earlier investigators. In a given channel, Kutter's n tends to increase as the depth of flow decreases. In his conclusions, Mr. Johnson attributes the higher values of n at low flows to friction between the sewage surface and the air and the coating of slime and grease on the lower section of the sewer walls.

---

[16] Draftsman, C. C. Combs Organization, New York, N. Y.

The results of a very comprehensive series of experiments on drain tile, 4 in. to 12 in. in diameter, with slopes varying from 0.0005 to 0.015 and with depths of flow varying from about one fourth of the diameter to full,[13] were reported by S. M. Woodward, Hon. M. Am. Soc. C. E., and the late D. L. Yarnell, M. Am. Soc. C. E., in 1935. These were laboratory experiments conducted with great care; but the results with flow at part depth varied considerably. They showed, in general, that the value of n was greater when the drain was partly filled than when entirely filled, the excess at one fourth of the depth being roughly from 10% to 50%; but there were a few cases in which lower values of n for less depths of flow were found.

Theodore Horton, M. Am. Soc. C. E., published the result of gagings in the sewers of the North Metropolitan Sewerage System of Massachusetts.[17] In this work n both increases and decreases, with increases in the depth of flow (hydraulic radius). In a description of the sewers gaged it is stated that: Deposits of grease and organic matter appeared on both sides of the channels, greatest near the line of average flow; and there was practically no deposit on the bottom of the channels (probably due to the cleansing action of the sand being swept along).

Mr. Horton's results and description show that the channel condition influences n in operating sewers. The investigations of Messrs. Woodward and Yarnell, however, were on clean pipe in the laboratory. In their work, air friction may have some, but probably only slight, influence on the increasing value of n.

A more basic explanation of the variation in n with change in depth of flow may be obtained by recalling the laws of hydraulics used to derive the Chézy formula, Eq. 1. Fundamentally the resistance to flow is directly proportional to the frictional surface and variable with the velocity of flow in the relation, frictional resistance $= f\,V^y$. In addition, internal resistance in the fluid varying with the viscosity and diameter of the conduit must be taken into account. The effect of the diameter being inversely proportional to power, m, of the diameter.

Thus frictional resistance F, taking into account all the experimentally determined facts, becomes:

$$F = K \frac{V^y}{d^m} \qquad \text{(4)}$$

From this the elemental formula for difference in elevation of the hydraulic gradient (or surface of stream) is derived:

$$h = K \frac{l\,V^y}{d^x} \qquad \text{(5)}$$

in which K = a constant for wall friction; y = the coefficient of velocity relative to wall friction; and x = the coefficient of diameter or hydraulic radius relative to internal friction.

---

[17] "Flow in the Sewers of the North Metropolitan Sewerage System of Massachusetts," by Theodore Horton, *Transactions*, Am. Soc. C. E., Vol. XLVI (December), 1901, p. 78.

To change the form of Eq. 5: $V = \left(\frac{1}{K}\right)^{1/y} d^{x/y} \left(\frac{h}{l}\right)^{1/x}$; $V = K' \, d^{\alpha} \left(\frac{h}{l}\right)^{\beta}$;

or

$$V = C \, R^{\alpha} \, S^{\beta} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In Eq. 6 the coefficient C is constant for a channel of given material when α and β are assigned correct values. (Formulas giving varying exponents for different classes of channels have been introduced.[18]) By inspection, it will be seen that α and β are influenced by the velocity, friction, and diameter or hydraulic radius.

In the Chézy formula α and β are assumed constant at 0.5. Therefore, C is no longer a constant indicating channel friction; it becomes a variable chiefly influenced by roughness, modified by slope and cross section to compensate for errors in the exponents assigned to R and S.

The empirical Kutter formula, devised to obtain values for C, contains the variables n, R, and S, with n as a constant for roughness and R and S to correct for the conditions of flow. However, R and S have constant exponents when their inclusion is necessitated to correct for exponential variation. The result, when C is determined for various depths of flow in a given channel solving the Kutter formula for n, causes n not only to indicate roughness but also to reflect the changes in hydraulic radius.

C. E. RAMSER,[19] M. AM. SOC. C. E.—Since the paper appears to lack data on the effect of depth of flow on the value of n for tile pipes exceeding 12 in. in diameter, the writer offers the results of experiments on the flow of water in a 30-in. concrete tile pipe line under actual land drainage conditions in Murray County, Minnesota, during 1920.

For slope and current meter measurements, two manholes of 30-in. concrete tile were constructed 1,000 ft apart, the lower manhole being 234.5 ft above the outlet end of the tile line. Elevations of points at the manholes were established from which the elevation of the water surface could be determined by vertical measurements. From these elevations the slope of the water surface was computed, and current meter measurements were made with a rod-type current meter both in the lower manhole and in the ditch about 50 ft below the outlet of the tile. The manholes, consisting of 30-in. tile set up vertically, were built so as to cause practically no interference with the uniform flow of water in the tile. An 8-in. circular hole about in the center of the manhole was cut at a joint in the top of the tile. The manhole tiles were cemented around the bottom and at the joints to prevent leakage when the pipe was flowing under pressure.

The section of pipe where the experiments were conducted was straight and 1,000 ft in length. The tile pipe sections were laid with cemented butt joints along, and for some distance above, this section where a pressure head was expected.

---

[18] "Handbook of Hydraulics," by Horace W. King, 3d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1939, pp. 270–271.

[19] Chf., Hydrologic Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

Inspection at the two manholes indicated that the elevation of the abutting pipes at the joints varied from one half to three fourths of an inch. The tile was clean and free from any kind of growth.

Table 7 contains the results of the experiments. Measurements of the discharge and slope of the hydraulic gradient were made for depths of flow of from 0.67 ft to full with ratios of depth of flow to diameter of tile ranging from 0.27 to 1.00. The depth of flow was taken as the average of the depth at the upper and lower manholes. The area of flow used in the computations was taken as the average of the areas at the upper and lower manholes, which explains the fact that the average depths of flow given in Table 7 do not correspond to the average area of flow. The hydraulic radius was taken as the average of the hydraulic radii at the two manholes.

TABLE 7.—EXPERIMENTS ON THE FLOW OF WATER IN A 30-IN. CONCRETE TILE PIPE LINE

(D = Diameter of Tile; and A = Cross-sectional Area of Tile)

No.	Average depth of flow d (ft)	$\frac{d}{D}$	Average area of flow a (ft)	$\frac{a}{A}$	Average hydraulic radius R (ft)	Discharge Q (cu ft per sec)	Mean velocity V (ft per sec)	Slope S	Chézy coefficient C	Kutter coefficient n
					(*a*) UNDER PRESSURE					
1	2.50	1.00	4.91	1.00	0.625	29.96	6.10	0.00551	103.9	0.0133
					(*b*) PRESSURE AT LOWER MANHOLE					
2	2.48	0.99	4.90	0.99	0.638	23.00	4.69	0.00257	115.8	0.0122
3	2.48	0.99	4.90	0.99	0.638	21.21	4.33	0.00239	110.9	0.0126
4	2.45	0.98	4.87	0.99	0.666	20.16	4.14	0.00210	110.7	0.0126
					(*c*) FLOWING PARTLY FULL					
5	2.26	0.90	4.58	0.93	0.702	17.11	3.73	0.001345	121.3	0.01185
6	1.92	0.77	4.03	0.82	0.753	13.44	3.33	0.001126	115.0	0.0126
7	1.59	0.64	3.28	0.67	0.713	9.74	2.97	0.001017	110.3	0.0127
8	1.31	0.52	2.60	0.53	0.640	6.56	2.52	0.000966	101.3	0.0135
9	1.21	0.48	2.35	0.48	0.610	4.95	2.11	0.000966	86.9	0.0154
10	0.74	0.29	1.22	0.25	0.420	1.63	1.34	0.000976	66.1	0.0178
11	0.67	0.27	1.06	0.22	0.388	1.31	1.24	0.000966	60.1	0.0186

Measurement No. 1, Table 7(*a*), was made when the pipe was flowing under pressure for the entire length of the slope course, and measurements in Table 7(*b*) were made when the pipe was flowing under pressure at the upper end of the slope course as determined from observations at the manholes at both ends of the course. Measurements in Table 7(*c*) were made when the tile was flowing partly full under no pressure, the average depth of flow ranging from 0.67 ft to 2.26 ft, and the ratio of $\frac{d}{D}$ ranging from 0.27 to 0.90. Attention is called to the rather consistent decrease in value of n in Kutter's formula for an increase in depth of flow. The values obtained for the two smallest depths of flow are slightly higher than the values given by the author for the somewhat smaller depths of flow for the 24-in. brick circular sewer D.

C. Frank Johnson,[20] M. Am. Soc. C. E.—Discussers have added some information on the variation of Kutter's *n* for conduits partly filled. This information relates largely to conduits carrying clean water, rather than to sewers; and to laboratory experiments, rather than to field measurements. As previously stated by the writer, such information affords a valuable guide for use in sewerage computations, but cannot be used without modification.

Professor Camp expresses regret that "the opinion is so widely held that measurements made in pipes carrying clean water are not applicable to the flow of sewage. Such is not the case for clean sewers." The writer is of the opinion that there is no such thing as a "clean sewer," even though there are sewers which are free of deposits and sediment. The nature of sewage is such that the sewer walls are inevitably affected over a period of years after the sewer is put in use, the walls acquiring a surface different from that found in a conduit carrying clean water. This surface condition, rather than the suspended matter in the sewage, is what affects the flow in a sewer. The flow pattern in a sewer would probably be substantially the same for experiments made with clean water flowing in the sewer as for those made with sewage. Professor Camp's study of the ratios of friction factors for various depth-diameter ratios is of considerable interest.

Mr. Coulter mistakenly attributes to the writer the conclusion that higher values of *n* at low flows may result from friction between the sewage surface and the air. This conclusion was merely quoted from another published article. Mr. Coulter's explanation of the variation of *n* is of interest, and the writer has no quarrel with it. However, attention is again directed to the fact that the flow in sewers is affected to some degree by the peculiar condition of the walls, as well as by factors affecting other conduits.

The data contributed by Mr. Ramser are most valuable, ranging as they do from a condition of the conduit being about one fourth filled to completely filled.

In conclusion, the writer urges that additional field measurements be made in sewers partly filled, to supplement the meager information on this subject. Reliance on laboratory and other measurements made in conduits carrying clean water is not entirely satisfactory for sewerage work.

---

[20] Chf. Engr., Commrs. of Sewerage of Louisville, Louisville, Ky.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2210

ORGANIZING AND FINANCING SEWAGE TREATMENT PROJECTS

BY SAMUEL A. GREELEY,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. MILTON P. ADAMS, HAL F. SMITH, HERBERT MOORE, FRANCIS H. KINGSBURY, A. BRINTON CARSON, AND SAMUEL A. GREELEY.

SYNOPSIS

The creation of sewage treatment projects is limited, in this paper, to organizations established for the single purpose of sewage treatment and having this limited objective. According to administrative functions, organizations are considered under three classes:

1. Those which function as an arm of the state;
2. Those which function as an arm of a municipal corporation; and
3. Those which are created as new independent, overlapping municipal corporations.

According to geographical character, organizations are considered as local, metropolitan, or regional. The cause, creation, and administration of these special organizations are described with special reference to the Illinois Enabling Act of 1917.

Methods of financing sewage disposal projects are discussed with special reference to revenues from service charges and to loans secured by the revenue. The fundamental considerations in the computation of a charge for sewage disposal service relate to the three principal kinds of sewage for which the service is rendered—sanitary or domestic sewage, storm water, and trade waste. Methods of evaluating these services in terms of a rate of charge and some reference to court decisions and commission rulings are included.

GENERAL STATEMENT

The subject of this paper divides itself into two interrelated parts—organization and financing. These functions are governmental and political. Therefore, trends in the functions are influenced by motives and ideas other

---

NOTE.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] (Greeley & Hansen), Chicago, Ill.

than, and as well as, those of the engineering nature. However, the application of engineering yardsticks and experiences to the results of the organization of sewage treatment projects is useful and enlightening. A great variety of methods for acquiring and financing sewage treatment projects exist throughout the United States. Sanitary districts under the Illinois Enabling Act of 1917 have been successful generally. There has been a trend toward the use of revenue bonds which is likely to continue.

Meanings and Descriptions

The meaning of the word "organization," as used in this paper, requires some description. Citizens may consider that they are organized for the creation of sewage treatment projects when they have become incorporated into a city, town, or village whose charter provides the necessary procedure. Many sewage treatment projects have been undertaken, financed, and operated by this kind of organization. This paper, however, is limited to special organizations created for the one purpose of sewage treatment and having this limited objective. Such special organizations have been created under enabling acts in many of the United States. They differ by having different administrative powers, functions, and procedures, or by being different in geographical character.

In general, special organizations for sewage treatment projects fall into three classes of administrative functions and procedures. One class comprises organizations created to function as an arm of the state. This class remains under the general control of the state; its administrative officers are appointed by the state; its expenditures are so controlled; and it reports to the state. The Metropolitan District Commission of Massachusetts is an organization of this type. A second class is one created as an arm of the city, county, or other municipal corporation. This class functions under the control of an existing municipal corporation and, in general, its administrative officers are appointed by the mayor, or by the municipal governing bodies who also control the expenditures and to whom it reports. The Milwaukee Sewerage Commission is an organization of this type. A third class is one created as a new, independent, overlapping municipal corporation. This class, once it has been created, functions independently of any other local corporations or governing boards. Its administrative officers are either appointed, or elected, but it controls its own expenditures and reports only to the public. Special organizations of this type are The Sanitary District of Chicago, the Minneapolis-Saint Paul Sanitary District, and the Twenty-six Sanitary Districts in Illinois, which have been organized under the Enabling Act of 1917.

The classification of special organizations, in accordance with their geographical character, includes local, metropolitan, and regional districts. Local organizations are those having the same boundary and area as an existing municipal corporation, such as the Buffalo Sewer Authority, the Gary Sanitary District, and the Sanitary District of Indianapolis.

Metropolitan organizations are those comprising a large city, the adjoining municipalities, and, perhaps, some area of unincorporated land. Among these are The Sanitary District of Chicago, the Metropolitan Sewerage Commission of Milwaukee County, and the Minneapolis-Saint Paul Sanitary District.

Regional organizations are those comprising a large area, including a number of municipalities, not necessarily contiguous, and often including a considerable amount of county, township, or unincorporated area. There are relatively few regional organizations for sewage treatment projects. Examples are as follows: North Shore Sanitary District, Illinois, Washington Suburban Sanitary Districts, Maryland, Hampton Roads Sewerage Commission, Virginia, and Passaic Valley Sewerage Commission.

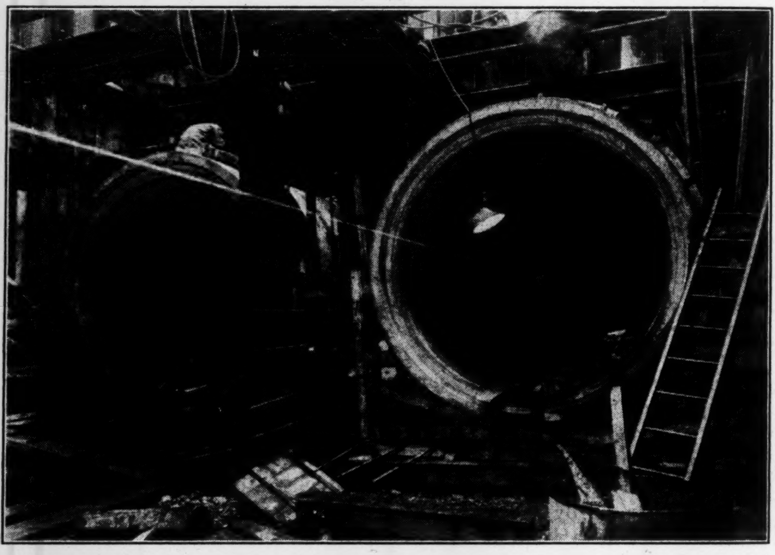


Fig. 1.—Metropolitan Organizations Are Those Comprising a Large City, the Adjoining Municipalities, and, Perhaps, Some Area of Unincorporated Land. The Buffalo (N. Y.) Sewer Authority Is a Metropolitan Organization of This Type. View Shows Construction of Harbor Crossing Which Connects to the Treatment Plant

The foregoing include only the principal kinds of special organizations. In California, for instance, organization is effected through contracts between adjacent municipal corporations. In some cases, special boards or commissions have been created by municipal governments to accomplish a specific sewage treatment project. In 1925, the River Conservancy District Act was passed in Illinois for the special organization of areas co-extensive with the drainage basins of rivers. In general, the trend in the organization of sewage treatment projects has been independent of other local governments.

A major reason for the creation of special organizations has been to secure taxing and financing powers additional to those permitted incorporated municipalities by constitutional restrictions. Thus, under the Illinois Enabling Act of 1917, the sanitary districts have separate powers to issue bonds and levy taxes under the constitutional limitation of the state, in addition to those which can be levied under the constitutional limit by the component cities, towns, and villages. In states with constitutional limitations, special enabling acts cannot give this additional power direct to an existing municipality. Thus, a strong motive behind the creation of special organizations arises from the fact that they permit the undertaking of difficult, highly specialized, and relatively expensive projects. The Illinois 1917 Act provides administration through the appointment, by the county judge, of three trustees who serve for the nominal salary of $300 per year. This permits the responsibility to be centralized on competent and trusted citizens who have no other duties. Thus, if special legislation is required in any event to provide for financing the project, a public desire to secure a special organization also, more or less outside the routine political field, may be fulfilled.

Special organizations have been called sanitary districts, sewerage commissions, authorities, and joint meetings. The term "sanitary district" is used in this paper. Most sanitary districts have power to finance, construct, and operate sewage disposal projects, including intercepting sewers, pumping stations, sewage treatment plants, and appurtenances.

STATUS IN ILLINOIS

Table 1 shows the dates of the enabling acts and of the creation of early sanitary districts. The first one (Item 1, Table 1) is the Boston Main Drainage, which was created by an Act of the Massachusetts General Court in 1876. Acting under authority of the city council, the Mayor of Boston petitioned the Legislature, and obtained the desired authority in an Act to "empower the City of Boston to lay and maintain a main sewer, discharging at Moon Island in Boston Harbor * * *." The second is The Sanitary District of Chicago, Ill., under an enabling act which became law in 1889, the creation of the district taking place the same year. The State of Illinois has had a long and complete experience in the organization of sewage treatment projects. The North Shore Sanitary District (Item 6, Table 1) was organized in 1913, following an Enabling Act of 1911. The first of the Illinois Sanitary Districts under the Enabling Act of 1917 was at Decatur. Thus, since 1889, Chicago and the State of Illinois have had actual experience with the special organization of projects for sewage disposal. Item 5, Table 1, refers to the Passaic (N. J.)

TABLE 1.—DATES OF EARLY SPECIAL ORGANIZATIONS FOR SEWAGE DISPOSAL PROJECTS

Item	Special organization	Enabling act	Creation of district
1	Boston (Mass.) Main Drainage	1876	1876
2	Sanitary District of Chicago, Ill.	1889	1889
3	North Metropolitan Sewerage District (Mass.)	1889	1889
4	South Metropolitan Sewerage District (Mass.)	1899	1899
5	Passaic (N. J.) Valley Sewerage Commission	1902 1907	1902 1910
6	North Shore Sanitary District (Ill.)	1911	1913
7	Decatur, Ill.	1917	1917

Valley Sewerage Commission. The original enabling act of 1902 was declared unconstitutional and the creation of the district was held in abeyance until another act was passed in 1907. The project was finally consummated in 1910. Table 2 lists eleven states in which sanitary districts have been organized. So far as the writer knows, no other states have organized sanitary districts. It should be noted, in the first place, that the record shows sixty sanitary districts in operation in the United States, with a total population of about fourteen million; 46% of the organized sanitary districts by number, and 36% by population, are in Illinois.

TABLE 2.—RECORD OF SPECIAL ORGANIZATIONS FOR SEWAGE DISPOSAL PROJECTS BY STATES

No.	State	Districts (No.)	Population[a]
1	California	1	260,000
2	Illinois	27	4,764,027
3	Indiana	4	623,512
4	Massachusetts	3	1,911,828
5	Maryland	4	288,125
6	Minnesota	1	805,031
7	New Jersey	8	1,594,366
8	New York	4	1,654,448
9	Ohio	1[b]	....
10	Wisconsin	4	901,693
11	Virginia	1	270,000[c]
12	Total	58	13,073,030

[a] Approximate population served in 1940. [b] Ten county sewer districts, but no sewer works. [c] 1933 population; the 1941 population was estimated at 450,000.

Table 3 is a brief summary of the progress and present status of sewage disposal in a number of states, in the United States as a whole, and in the State of Illinois. Table 3(*a*) shows the number of sewage treatment plants and the approximate population served in all of the eleven states having sewage treatment projects prior to 1900. There were no sewage treatment plants in any of the other states. In these eleven states there were forty-five sewage treatment plants with an approximate population of 546,000. As of 1940 (Table 3(*b*)), the total estimated population in the United States served by sewage treatment projects has increased to about forty million. The population served in the eleven states which had sewage projects prior to 1900 amounts to nearly

TABLE 3.—STATUS OF SEWAGE TREATMENT PRIOR TO 1900 AND IN 1940

No.	State	(*a*) PRIOR TO 1900[a]		(*b*) AS OF 1940[c]	
		Plants (No.)	Population[b]	Plants (No.)	Population[b]
1	Connecticut	3	41,000	54	840,000
2	Illinois	6	50,000	261	5,120,000
3	Iowa	1	1,500	281	795,000
4	Kansas	2	9,300	164	520,000
5	Massachusetts	15	147,000	43	781,000
6	Nebraska	1	7,200	103	280,000
7	New Jersey	7	43,000	156	2,600,000
8	New York	2	8,000+	269	5,400,000
9	Ohio	6	63,000	203	3,045,000
10	Pennsylvania	1	?	181	1,720,000
11	Rhode Island	1	176,000	11	400,000
12	Total	45	546,000	1,726	21,501,000
13	Entire United States	45	546,000		40,000,000

[a] From data compiled by Langdon Pearse, M. Am. Soc. C. E., and his staff, and from other sources. [b] Approximate population served. [c] From U. S. Public Health Service and from other records.

twenty-two million in 1940. Prior to 1900, the population served by sewage treatment plants in Illinois was about 10% of the total so served in the United States, whereas in 1940 it was about 24%.

PART I. ORGANIZATION

Special organizations for sewage treatment projects usually have been created because of the magnitude and difficulty of the project and the need for special financing. Civic leaders have been prompted either to initiate an enabling act, or to lead the community to avail itself of existing legislation. Therefore, sanitary districts are organized to obtain funds beyond the statutory limitations of the municipality or component municipalities, and, in many cases, to provide for a continuing qualified personnel to administer and accomplish the project.

Initiation and Creation

In a general way, sanitary districts are created by enabling acts of the state legislative bodies. Such enabling acts are either mandatory or permissive. Those which are mandatory bring the district into existence with the enabling act, or following the appointment of trustees or commissioners. All of the Illinois Acts are permissive. The area to be organized must petition for an election to determine the question of organizing a district. This step, of course, takes careful account of the rights and prerogatives of the small or local municipality and insures that the state does not usurp any of the local rights.

A recently created sanitation district, known as the Hampton Roads Sanitation Commission, in Virginia, is a regional organization. It was created by an enabling act which provided for an election to determine the question of establishing the Commission. Upon a favorable election, the Commission came into existence when the Governor appointed the five Commissioners provided by the Act. However, the local interests were still further protected by a provision in the enabling act which permits any individual municipality within the sanitary district to declare its intention of withdrawing within six months after the creation of the Commission. Such municipalities as declare their intention to withdraw must, within the next following six-month period, show to the satisfaction of the State Board of Health and the Commission that they have taken the necessary steps to finance, construct, and operate an acceptable sewage disposal project. During these two six-month periods, the Commission cannot take steps to issue bonds. Thus, in addition to the right to vote on the creation of a sanitary district, the local communities are afforded an opportunity to decide whether they will proceed by themselves, or jointly within the sanitary district.

The trend is to continue to safeguard the rights, powers, and interests of the local municipalities.

Area To Be Included

The boundary of the area to be organized is generally included in the originating petition and fixed by the court or other public body hearing and reviewing the petition. In some cases the boundary lines have been fixed by considerations of expediency, such as municipal corporation limits, property lines, and the like. Some enabling acts require a competent preliminary engineering investigation and report, the purpose of which is to assist the reviewing body in determining the area to be included. There is no such requirement in Illinois, but the trend seems to be to give more consideration to the preliminary aspects of organization. When the boundary of a sanitary district has been fixed and bonds issued and money used for permanent construction, it is difficult to diminish the area of the district. It is easier to annex than to disannex. Amendments have been made to existing Acts, providing for annexation or disannexation.

Administrative Board or Commission

In Illinois, the administrative board is elected in the case of The Sanitary District of Chicago, and is appointed by the county judge in all of the other districts. The principal question is the relative merit of election. The size of the district and of the project is likely to have some bearing, but the trend seems to be toward the appointment of the board. The Hampton Roads Sanitation Commission is composed of five members appointed by the governor. The Buffalo (N. Y.) Sewer Authority, one of the recent large special organizations, comprises five members appointed by the mayor.

Another question of interest relates to the compensation of members of the administrative board. In general, this compensation has been nominal. The Sanitary District of Chicago has nine Trustees, three of whom are elected every two years for a six-year term. This is a very large complex undertaking and requires considerable time by the Trustees. Therefore, they are compensated on a relatively liberal basis. In the other sanitary districts in Illinois, each Trustee now receives $300 per year. The members of the Buffalo Sewer Authority receive no compensation. The Chairman of the Hampton Roads Sanitation Commission, who is designated as a "full time member," may receive not more than $6,000 per year. The other members receive a small compensation for attendance at meetings, not to exceed $300 in any one year. The trend is toward a relatively small compensation.

Methods of Financing

The sanitary districts in Illinois are empowered to levy and collect taxes and to issue bonds upon favorable vote by the people. Amendments to the 1917 Act and others provide that the district may apportion and collect an additional charge for the disposal of industrial sewage and may also levy special assessments upon benefited property for the construction of collecting sewers. None of the districts in Illinois can establish and collect charges for sewage disposal service. The Buffalo Sewer Authority and the Hampton Roads Sanitation Commission are authorized to issue revenue bonds and, in fact, are limited to this method of raising money. The trend is to consider a sewage disposal project as a utility, to be operated on a revenue basis.

The Illinois sanitary districts are empowered to levy and collect taxes as soon as they are organized. Although considerable time may elapse before tax money comes in, it is possible for the board to issue tax anticipation warrants, or to borrow moderate sums for administrative expenses. In this way, much-needed engineering and legal services can be engaged promptly. The Hampton Roads Sanitation Commission is limited in this regard by the provision that the appropriation made by the state, except in the amount of $10,000, is not available to them before they are empowered, by an election, to issue bonds. The Buffalo Sewer Authority had no means of securing funds until they issued and sold revenue bonds, the first money being advanced by the Public Works Authority (PWA). Fortunately, however, they were able to use facilities provided by the city. Administrative boards should not be so limited in their early available funds as to handicap the progress of completing the project for which they were created.

Powers

The powers granted sanitary districts are not unusual and are more or less similar throughout the United States. The district is given power to engage engineers, lawyers, accountants, and clerks, to build and operate structures, to acquire right-of-way and property, to make contracts, and to formulate and enforce rules and regulations for the operation of the project. A special power, recently granted, is that of receiving and using grants from the federal government. Many enabling acts permit the district to contract with other districts or municipalities for sewage disposal.

Rules and Regulations

Enabling acts, in general, permit sanitary districts to adopt rules and regulations for the use of the facilities which they provide, but do not ordinarily give them police powers. This is understood to be true as regards The Sanitary District of Chicago. The police power rests with the state. The extent to which sanitary districts can regulate the use of their works has not been thoroughly tested, but, in general, the soundness of the rules and regulations has led to their acceptance by the public. The importance of such rules and regulations and their effect is obvious; for instance, it is important that sanitary districts have control over the construction of new sewers which, in turn, will require sewage disposal service.

General Trend

In recent years few sanitary districts have been organized for sewage disposal projects. In Illinois, the most recent important sanitary district was that at Danville in 1934. Since that time, only three small districts have been organized. Throughout the United States, there are relatively few new organizations, the most recent being the Hampton Roads Sanitation Commission, created in 1941, and, among large projects, the Buffalo Sewer Authority created in 1935. Two reasons are (1) the availability of federal grants and (2) the powers given to municipalities to issue revenue bonds. The experience in Illinois in the administration of sanitary districts under the 1917 Act has been so good that this method of procedure could well be extended to other parts of the United States.

Special Considerations

The creation and organization of sanitary districts in the various states and within each state vary in detail. A few illustrations are pertinent, as follows:

(*a*) The Passaic Valley Sewerage Commission is authorized to formulate projects, but cannot begin to create them until a prescribed number of municipalities within the district have approved the undertakings and obligated themselves to furnish the necessary funds.

(*b*) The East Bay Municipal Utilities District, in California, may purchase or construct only revenue-producing utilities. It cannot purchase or construct such utilities until their acquisition has been first approved by a majority of the voters of the district.

(*c*) The Washington (D. C.) Suburban Sanitary Commission may construct and maintain sewage disposal projects, as well as other utilities, and may extend them into adjacent territory outside of the district, wherever the property owners agree to such an extension. It may make and enforce plumbing regulations and build sewer connections directly to houses. It is empowered to borrow money and to levy a tax or to make special assessments and service charges. Appeals from assessments and service charges can be taken to the Maryland Public Service Commission.

An outline of the items included in the Illinois Enabling Act of 1917 is given in the Appendix.

PART II. FINANCING

The following methods of financing have been used by special organizations for sewage disposal projects:

(*a*) Taxes and borrowings through general obligation bonds;
(*b*) Funds contributed by constituent municipalities within the district;
(*c*) Special assessments on benefited property;
(*d*) Appropriations by the state;
(*e*) Voluntary contributions; and
(*f*) Revenues from service charges and loans secured by the revenues.

As the trend in financing has been toward service charges and revenue bonds, the discussion in this part of the paper will be limited to that method.

History of Revenue Bonds

The use of revenue bonds and charges for sewage disposal service is not new, having been resorted to as early as 1890. However, there are relatively few court decisions and rulings by regulatory commissions. From the viewpoint of general theory and the computation of rate structures, the application to sewage disposal projects is relatively undeveloped. There is need for a broad review of experience, and a development and presentation of procedures for review by courts and commissions. Among the early rate structures are those at Brockton, Mass., and Spokane, Wash., which were established between 1890 and 1900. The Brockton project is still financed by the collection of sewage disposal charges and is a good illustration of a sound rate structure. A survey of the United States made toward the end of 1938 indicated that more than 600 municipalities in thirty-five states were using revenue bonds and the income from sewage disposal charges for financing such projects. There is some confusion in reporting, classifying, and comparing these revenues. In some cases, the rates are sufficient to produce a revenue to meet operating expenses only, and in such cases the debt service is paid by taxes from the general fund. In other cases, the rates are sufficient to produce revenues sufficient for the entire annual cost of the project, including operation and debt service and, in some cases, a reserve or revolving fund for current extensions and replacements. Whenever rates are compared, the items covered by the annual revenue should be stated.

The power of municipalities to establish sewage disposal charges comes from special enabling acts or from sections in acts for establishing special organizations. The various acts grant different powers as to the amount and use of revenue. For instance, the Ohio law includes the following provision:

> The municipality may establish charges and equitable rates or charges of rents to be paid by those whose premises are sewered by a connection to the sewage disposal works, and these charges constitute a lien upon the property sewered by such a connection. The revenue shall be used for the payment of the cost of management, maintenance, operation and repair of the project, and any surplus in such fund may be used for enlarging or replacing parts of the project and for the creation of a sinking fund for the payment of any debt incurred for the construction; it shall not be used for the expansion of the system into unsewered areas.

Other acts provide that the revenues may be used not only for the expenses of operation and maintenance, but also to meet the debt service and to provide a reserve for improvements.

The use of revenue bonds received a considerable impetus from the grants donated to municipalities by PWA. During its active life, PWA aided in the construction of 1,524 sewage disposal projects, of which 230 were financed by revenue bonds. A brief summary of the sums involved follows:

Source of fund	
PWA loans, 22%	$101,500,000
Grants, 37%	171,500,000
Non-federal, 41%	189,000,000
Total	$462,000,000
Loans secured by revenue bonds	
PWA, 26%	$ 26,400,000
Non-federal, 14%	26,500,000
Total	$ 52,900,000

It is of interest to note that about 104 municipalities in Canada are reported as having established sewage disposal charges. It is reported that some 200 municipalities in Texas receive sewage disposal service from projects financed by revenue bonds, and there are a number in Massachusetts and New Jersey.

About 26 municipalities in Illinois have financed sewage disposal projects by revenue bonds and service charges.

DISCUSSION OF TERMS

The term "sewage disposal project" is used in a broad sense to include all degrees of sewage treatment, all of the structures included in the project, as it may have been financed, and, in some cases, the construction of sewers only. In short, it is a comprehensive term describing projects in the field of sewerage and sewage treatment which have been financed by revenue bonds. In some cases, the charges are referred to as sewer rentals[2] or sewer service charges. In the writer's opinion, the term "sewage disposal charge" is the better term.

FUNDAMENTAL CONSIDERATIONS

In general, there are three principal kinds of sewage for which service is rendered, as follows:

Sanitary or domestic sewage;
Storm water (usually comprising the first part of the runoff); and
Industrial sewage or trade waste.

The structures to provide the service will include combined sewers or separate sewers, and the service charge may be limited to a sewage disposal project for sanitary or domestic sewage only, or for combined sewage. This is an important fundamental consideration. In some cases, industrial sewage may be excluded, but, in general, much industrial sewage is discharged into the system. Thus, in considering revenues and rates, the kind of a sewage disposal project must be stated. On this basis, the elements of use or service are the following:

(*a*) Treatment and disposal of sanitary or domestic sewage;
(*b*) Handling and disposal of storm water; and
(*c*) Treatment and disposal of industrial sewage.

The collection of these kinds of sewage may or may not be part of the project, but a consideration and comparison of rate schedules should take the element of service and its cost into consideration. In addition to the foregoing, the municipality must make some provision for future growth and future population, and this is an element of service. Assuming that these are the proper elements of use or service, an effort should be made to measure them and to evaluate or price the quantity of the service. A quite general practice is to consider that sanitary or domestic sewage is reasonably measured by the quantity or use of water in the dwelling, exclusive, perhaps, of water used for lawn sprinkling, and discharged to the sewage disposal project.

Industrial sewages may affect the cost of providing sewage disposal service by their relatively greater strength, as compared with domestic sewage. They may have a relatively high oxygen demand or chlorine demand; they may contain large quantities of suspended solids and grease, difficult to handle;

---

[2] See "Sewer Rental Laws and Procedure," Third Progress Report of the Committee of the Sanitary Eng. Div., *Proceedings*, Am. Soc. C. E., November, 1942, p. 1585.

or they may be discharged in batches, and not uniformly, throughout the 24 hours. Some industrial sewages may contain toxic or corrosive substances. Unless these additional elements of service and their cost are covered in the rate structure, it is possible that a charge of discrimination could be supported on behalf of the domestic user. Fortunately, however, in many municipalities the amount of such annual cost and revenue is not a large proportion of the total, and adjustments can frequently be made by special rates or agreements with industries, in some cases providing pretreatment or equalization structures at the industry.

If the collecting sewers are combined, some (and occasionally much) of the storm water reaches the sewage disposal project. Capacity and treatment for this storm water must be provided and a part of the total annual cost is thus required for storm-water disposal. This service is more a function of the tributary area than of the use of water. Properties of greater area will shed a larger quantity of storm water to the sewers and require a greater sewer capacity, and to this extent a higher cost for disposal. Properties having more impervious surface than others will discharge relatively greater quantities of storm water; and properties of value, where interruptions to their use are costly, require a more extensive and expensive service. Thus, area is indicated as a possible yardstick for measuring this service. The evaluation of this yardstick, in terms of a sewage disposal charge or rate, so far has been computed from the assessed valuation, which gives some weight to the extent, area, and use value of the property. Based on present experience, a charge, on the basis of area and value, appears to be reasonable and expedient.

A fundamental consideration of service is to provide future capacity for the growth and development of the city. Such additional expenditures and annual costs are made for the benefit and use of presently vacant property and for an increased development and use of presently occupied property. Property, therefore, is served by the sewage disposal project. Its use and value are protected and stabilized. This element of service may also be related to area, perhaps through the assessed valuation.

Thus, for combined systems in which the financial structure covers both storm and sanitary sewers, a two-part rate is indicated. One part of the rate might be based on the use of water and the other part on the assessed valuation of property within the area served, or to be served, by the project. Such a two-part rate has been in use at Buffalo for more than three years and has recently been under review by the courts in the case of Philadelphia. Further, to accomplish a reasonable and equitable rate structure, the ordinance should provide for special charges, agreements, or considerations as regards industrial sewages.

COURT DECISIONS

The writer is not aware of many court decisions related specifically to sewage disposal projects. An early one is that of Carson versus Brockton Sewerage Commission, decided by the Supreme Judicial Court of Massachusetts on May 27, 1901. The principal contention of the petitioner was that, having paid an assessment for the installation of the sewers, he could not be forced to pay a special annual rental charge. The Court decided in favor of the City, on the basis that a sewer rental, computed on the quantity of water or the amount of the water bill, was not unreasonable. The Brockton revenue for sewage disposal service is in part a usage and in part a tax levy. The use charge brings in about 60% of the total annual revenue.

In 1942, there was a decision by the Court of Appeals of Richland County, Ohio, in connection with Gatton versus City of Mansfield. The appellant sought relief from the City for shutting off water furnished to her premises, because of her delinquency in payment of sewer rentals. The Court sustained the City. Recently, the Supreme Court of Pennsylvania has considered the case of a proposed sewer rental ordinance for Philadelphia, which establishes a two-part rate, comprising three mills for each dollar of the fair value of the property served, and an amount equal to one-quarter of the water rent. A very complete argument was prepared by the city solicitor, which included references and quotations from many pertinent court decisions, a few of which relate quite directly to sewage disposal service of one kind or another.

COMMISSION RULINGS

There are a number of sewage disposal utilities in the United States which are owned and operated privately, most of them being in New Jersey and in Texas. The New Jersey Board of Public Utility Commissioners has ruled on charges established by some of these private companies, as, for instance, the Lakewood Water Company. The rate approved by the Commissioners comprised a service charge related to the size of the water meter, a consumption charge related to the use of water, and a minimum charge. The service rendered is understood to have been mainly for domestic sewage and not for storm water.

TYPES OF RATE SCHEDULES

There are many types of rate schedules for sewage disposal service,[2] among which are the following:

(*a*) A charge related to the water bill, which may be a percentage of the water bill or may be a separate graduated or uniform scale, related to the quantity of water.

(*b*) A flat rate per connection, according to the class or kind of property.

(*c*) A charge based on the number and kind of fixtures connected to the project.

The kind of rate structure best suited to any community depends upon its size and complexity, whether the services are for sanitary or combined sewage, the number of water meters installed, the amount of revenue to be realized, and what it covers, and other local factors. The trend seems to be toward rates or charges related to the water consumption, with consideration being given to a two-part rate, where this is feasible and appropriate, under the Enabling Act, to local conditions.

EXPERIENCE WITH SERVICE CHARGES

A good indication of the feasibility of financing revenue bonds by sewage disposal charges is the relation between the cash collected and the total amount of the individual bills. Experience in the percentage of collections appears to have been favorable. Table 4(*a*) is taken from the annual reports of Lima, Ohio (population in 1940—44,711).

Experience at Battle Creek, Mich., indicates that current arrears do not, in general, exceed 5% of the bills. In 1936, the bills totaled $52,821, and the arrears $4,459, or 8.4%.

The experience at Buffalo has also been satisfactory. The sewage disposal charges were established to realize a total annual revenue of $1,500,000. The book record of revenue for the three fiscal years of operation is as shown in Table 4(*b*). The actual cash collections have been in excess of 93% of the current bills. Some proportion of the arrears has been in deferred payments by adjacent tributary municipalities, with whom contract adjustments were pending. The general experience with the administration of sewage disposal charges has been satisfactory. There seems also to have been a steady demand for revenue bonds so financed, at reasonable interest rates—generally below 4%.

TABLE 4.—EXPERIENCE AS TO THE PERCENTAGE OF SEWER RENTALS BILLED ACTUALLY COLLECTED

Year	Total billed (dollars)	TOTAL RECEIVED	
		Dollars	%
(*a*) LIMA, OHIO			
1932[a]	27,982.15	26,488.42	94.7
1933	49,054.50	46,244.71	94.3
1934	34,427.09	34,308.29	99.7
1935	33,974.39	33,901.54	99.8
1936	35,287.41	36,581.08	103.7
1937	36,044.23	36,511.83	101.3
1938	35,991.52	36,048.14	100.1
(*b*) BUFFALO, N. Y.			
1939[b]	1,505,004	1,410,254	93.8
1940[b]	1,456,449	1,354,586	91.1
1941[b]	1,485,932	1,395,132	94.0

[a] Nine-month period. [b] Year ending June 30.

GENERAL SUMMARY

There has been a marked trend toward the use of revenue bonds and charges for financing sewage disposal projects. Experience in selling revenue bonds and collecting revenue has been satisfactory. It seems likely that this method of financing will continue and expand. There is need for a further clarification of the structure of rate schedules for sewage disposal service and a classification of present experience in the light of fundamental considerations.

APPENDIX

ILLINOIS SANITARY DISTRICT LAW, ACT OF 1917
AS AMENDED 1919, 1923, 1927

The subjects of various sections, and brief comments on "An Act to Create Sanitary Districts and to Provide for sewage disposal," are as follows:

1. (As amended 1927.) Any area of contiguous territory containing one or more incorporated cities, towns, etc., which can construct and maintain treatment plants, may be incorporated as a sanitary district under this Act in the manner following:

(*a*) Petition to county judge by 100 legal voters to submit to legal voters question of whether territory shall be organized into sanitary district;
(*b*) County judge will then call two judges of the circuit court, and the three shall constitute a board of commissioners to consider boundaries;
(*c*) A 20-day notice of a meeting of Commissioners shall be published;
(*d*) After a meeting and hearing, the Commissioners fix boundaries; and
(*e*) Finally, the case is decided by an election within 60 days after step 1(*d*).

2. Courts must then take judicial notice of the existence of the sanitary district.

3. A board of trustees, consisting of three members, for the management, control, and government of the affairs and business of each sanitary district organized under this Act, shall be created in the following manner:

(*a*) The county judge will appoint trustees, who must be residents of the district. Not more than two shall be appointed from one incorporated town, village, etc., and for terms of 1, 2, and 3 years.
(*b*) (As added in 1919.) The county judge is given power to fill any vacancy.

4. (As amended in 1927.) The powers of the board are to manage and control all the affairs and the property of the district.

5. Ordinances.

6. Proof of ordinances.

7. (As amended in 1927.)

(*a*) The Board has power to provide for the collection and disposal of sewage and drainage, to maintain conduits, pipes, pumps, to contract with other sanitary districts, and to treat and purify sewage—"provided, nothing herein contained empowers the board to operate a system of water works for the purpose of furnishing or delivering water to any municipality or the inhabitants thereof." The Board is not authorized to flow sewage into Lake Michigan.
(*b*) Its duty is to provide sewers and treatment plant; a penalty is provided for violation.
(*c*) It may collect, from producers of industrial waste, the costs of fair additional construction, maintenance, and operating expense, over and above those covered by normal taxes.

8. It may acquire real or personal property by purchase, condemnation or otherwise, etc.

9. It may borrow, issue bonds, etc., but not to exceed 5% on the valuation of taxable property. The question must be submitted to a vote at an election.

10. It may impose a tax to pay indebtedness.

11. Contracts must be let to lowest responsible bidders.

12. (As amended in 1927.) The Board may levy additional taxes after an election has been held to authorize such levy; the tax can be terminated by a later election. The 1927 amendment provides for the certification and collection of taxes, the deposit of funds, etc.

13. Sewers, channels, etc., may cross public property, with the approval of the governor.

14. The Board may contract for sewage from a U. S. military post within its bounds.

15. Condemnation of private property.

16. Condemnation of property held for public use.

17. Power to permit outside territory to use its drains, etc.

18. (As amended in 1919.) Power to prevent pollution of water supply.

19. (As added in 1923.) Power to construct drains, sewers, pumping stations.

20. (As added in 1923.) Special assessments—annual installments.

21. (As added in 1923.) Bonds to anticipate collection of assessments.

22. (As added in 1923.) Cost of acquiring private property to be included in assessment, according to state local amendment act.

23. (As added in 1927.) Additional contiguous territory may be added to any sanitary district organized under this Act, as follows:

(*a*) Petition by 10% of the legal voters of the addition to submit to legal voters of addition such proposal, taxpayers thereof to assume proportionate share of bonded indebtedness, provided no territory disqualified in Section 1 of this Act is included.

(*b*) County judge to call all the county judges in the sanitary district and proposed addition, which body shall constitute a board of commissioners to have power and authority to consider boundaries.

(*c*) Notice of and hearing as provided in Section 1 of this Act.

(*d*) Trustees of district to accept by ordinance annexing the additional territory.

24. (As added in 1927.) Any contiguous territory within boundaries of district may become disconnected from such district in the following manner: Petition by 10% of legal voters of territory seeking disconnection to submit question to legal voters. County judges form commission. Such territory must have no bonded indebtedness incurred while it was a part of the district which has not been paid up, nor any special assessments. Notices and hearings are practically the same as in Section 1 of this Act.

DISCUSSION

Milton P. Adams,[3] M. Am. Soc. C. E.—In the "Synopsis" of this paper, Mr. Greeley limits the scope to "organizations established for the single purpose of sewage treatment and having this limited objective." The writer, in turn, since there are no such organizations in Michigan, must limit his discussion to a listing of the causes underlying the failure to achieve true metropolitan district financing and administration of the sewage treatment function in the area around Detroit. He will also mention briefly some of the problems that have been encountered elsewhere in the State of Michigan in order that the larger community may provide sewage collection and treatment to its smaller neighbor.

An amendment (Art. VIII, Sec. 31) to the State Constitution of Michigan, together with Act 312, Public Acts of 1929, as amended by Act 56, Public Acts of 1935, provides the legal foundation "for the incorporation of any two or more cities, villages or townships or parts of same" into metropolitan districts. Sewage treatment is but one of several objectives attainable through such organization. An intermediate but necessary step to the formation of such a district is the appointment and organization of a charter commission. Rules for selecting representatives on such commission are provided by the enabling statute. Instead of representation being based on outstanding local and nonpartisan qualification for such service, as under the Illinois Sanitary District law, unfortunately, in Michigan, one representative must come from each unit of government irrespective of size, whereas large cities are entitled to only one representative for each 200,000 population, over and above the first 200,000 of population, for which four representatives are allowed.

The charter subsequently must receive a favorable vote from each governmental unit. It is thereafter the magna charta of future administration, financing, and operation of the function sought.

Before sewage treatment was actually under way in Detroit as a city project, Wayne County administration of this service was seriously proposed and reported upon, both as a county drain commissioner's function and as that of the board of supervisors.

A regional committee on sewage collection and treatment, headed by the Hon. William A. Comstock, Governor of Michigan (1933–1934), was organized in 1933. Its objective was the solution of the problems not only of Wayne County's municipalities along the Detroit River but those of southern Oakland and Macomb counties as well. The latter counties border Wayne County on the north.

The following factors appear to account for the failure to achieve a separate and independent regional organization for sewage treatment in the metropolitan area:

1. The predominating size and political relation of Detroit with reference to some forty surrounding or neighboring municipalities of Wayne, Oakland and Macomb counties;

---

[3] Executive Secy.-Engr., Michigan Stream Control Comm., Lansing, Mich.

2. The statute provision, previously mentioned, in which one city with more than 85% of the tributary population and taxable property in the area would have been unable to exercise a controlling voice in the drawing of a charter necessary to the creation of such a district;

3. The situation created in 1935 when, with two applications before it—one by the City of Detroit, and one for Wayne County including Detroit—the PWA chose to deal with Detroit, leaving, for a subsequently submitted Wayne County application, the solution of the problems of the remaining municipalities not served by the Detroit project; and

4. The curb on metropolitan district powers of taxation and bonding inherent in the so-called 15-mill amendment to the State Constitution which became effective in 1933.

The nearest approach to regional administration of sewage treatment in Michigan is now operative under the provisions of Act 342, Public Acts of 1939, as amended by Act 353, Public Acts of 1941, in behalf of those communities in Wayne County not served by the Detroit project. During the spring and summer of 1942, under the stress of the war emergency and further conditioned upon favorable federal action as to the financing of certain necessary capital improvements, Wayne County has become the agent to serve the Grosse Pointe communities along Lake St. Clair as well as certain southern Macomb County municipalities. The treatment of all sewage so collected is by the City of Detroit acting under contract with Wayne County. A Sewage Disposal Committee of the Wayne County Board of Supervisors administers this function through its Board of County Road Commissioners.

Since October, 1939, Wayne County sewage treatment service has been available to its customer municipalities and townships at a uniform base rate independent of location. Until July 1, 1941, the basis of charge was $26 per million gallons of sewage, as measured by water passing the user's water meter. The current rate is $43 per million gallons, less discount, intended to be sufficient compensation to the customer governmental unit for collection of local service charges. The county's contract, therefore, is with the municipal subdivision served and not with the individual user.

The Detroit sewage treatment plant is now operated by the Detroit Water Board. The City Department of Public Works, on the other hand, is responsible for sewer maintenance and controls both city and out-of-city connections to the sewer system.

Only within the past year (1941) has the Common Council of Detroit adopted a so-called metropolitan rate for out-of-city and out-of-county sewage treatment service. To this rate, however, the City Department of Public Works seeks reimbursement as well for flowage or trunkage rights through that portion of its system not provided with PWA assistance. Distantly located municipalities find themselves at a financial disadvantage, therefore, as compared with those located more closely to the city's single sewage treatment plant.

During the summer of 1942, under authority of the county sewage disposal law, the City of Detroit and County of Oakland have reached an understanding whereby the sewage of several southern Oakland County municipalities will be received and treated by the City of Detroit. At the November 3, 1942, election, the municipalities of Ferndale, Royal Oak, Hazel Park, Huntington Woods, Berkley, Pleasant Ridge, and Clawson voted to so amend their respective charters as to enter contracts with the County of Oakland for treatment of their sewage on the county plan.

No privately owned sewage treatment utilities exist in Michigan, although Act 320, P. A. 1927, as amended, provides for their creation and control. Attempts were made to organize sanitary districts at Traverse City, Owosso, and Monroe during 1928–1930 without success. Failure of these efforts is charged to the 8% to 10% return then asked on the necessary capital outlay plus operating charges. Perhaps, reluctance on the part of Michigan communities to encourage additional types of privately owned, public utilities was also a factor.

Among the problems encountered under contract relation between governmental units for sewage treatment service, where well-conducted sanitary district organizations are lacking, are the following:

1. Reluctance to enter into the initial agreement because of uncertainty as to future year-to-year costs;
2. The governmental unit desiring accommodation may be asked to pay any amount in excess of the pro-rata service cost up to such amount as the "traffic will bear" at the will of the party providing the service;
3. The shifting legal responsibility for pollution where shut off or by-passing takes place, in an effort to enforce payment or obtain concessions as to charges; and
4. The Grand Rapids-East Grand Rapids problem of continuity of service to the latter municipality and compensation to the former was assured by a county circuit court hearing and signing by the judge of a contract mutually agreed upon in advance by representatives of the two cities. This contract has a "reopening clause" which has never been invoked since the original agreement was made early in the 1930's. The friendly suit or court hearing attested to by the county judge apparently has overcome the charter limitations of the municipalities which prevents their so contracting as to bind future legislative bodies of the two cities.

Hal F. Smith,[4] **Esq.**—A scholarly presentation of data concerning the development of organization and finance methods for sewage disposal projects is offered by Mr. Greeley. The paper includes a carefully considered description of the trend that the course of development is following. The writer's opinions, which are based entirely on personal experience in the field, are in accord with the author's findings.

The City of Detroit, Mich., made use of its regular municipal organization for construction and operation of its sewage disposal system, and assigned to its Department of Public Works, which includes the office of the city engineer, the task of constructing the system, which consists of intercepting sewers, sewage treatment plant, effluent conduit to the river, and its appurtenances. After completion, the operation of the system, including billing and collecting for service rendered, was transferred to the Department of Water Supply. The regular municipal organization was used because Detroit believes that, in general, economies in operation of municipal departments or projects lie in the path of consolidation of existing agencies rather than in the creation of new agencies.

---

[4] Senior Administrative Asst., Detroit Dept. of Water Supply, Detroit, Mich.

The City had little choice in the matter of financing its sewage disposal project. It had already reached its bonding limit and did not have sufficient cash resources to finance the project; so it decided to use the city funds that were available, accept the PWA grant, and issue revenue bonds to finance the remainder. The revenue bonds were issued by authority of a city ordinance, supported by a state enabling act, which also provided for raising funds to meet the system's debt charges and operating expense by imposing a sewage disposal charge upon the users of the system. It was decided to base the sewage disposal charge on metered water consumption because it appeared that water consumption was the most satisfactory and equitable measure of the service rendered by the sewage disposal system.

Its rate structure must be considered a two-part rate, wherein the part based on usage consists of a single step. It is a two-part rate because a substantial part of the capital costs of the project were paid from tax revenues; consequently, the "usage" part of the rate is required to produce only enough revenue to meet debt charges of the part that was financed through sale of revenue bonds and to meet operating expenses and reserve fund requirements.

A single-step for the "usage" part of the rate was adopted for the reason that the entire charge is for treatment and disposal of sewage that is picked up by the interceptors; consequently, the cost of treating and disposing of any given unit of sewage is identical to the cost of treating and disposing of any other unit, regardless of whether the unit was contributed by one large contributor or many small ones.

Consideration was then given to the problem of whether the charge should be billed on a separate bill form, or as a separate item on the water bill, or included in the water bill. For reasons of economy it was decided to include the sewage disposal charge in the water bill and to make a clear explanation of this combined billing on the bill form. This choice of procedure has made it possible for the department to take over the job of billing, collecting, and accounting for sewage disposal charges without any additional expense. In general, customers have understood the additional charge for sewage disposal and the combined form of billing, with the result that there has been no appreciable effect on customer relations. However, it did necessitate the preparation of a plan that could be depended upon to effect a complete and accurate segregation of water and sewage disposal receipts.

The question was then considered as to who should be held responsible for sewage disposal charges. Inasmuch as water charges in Detroit are a lien against the property served, the sewage disposal ordinance was drawn to make the sewage disposal charge also a lien and enforcible in exactly the same manner as is the lien for the water charge.

The next question considered was that of determining a satisfactory means of forcing payment of sewage disposal charges. Although the right of lien is a formidable weapon for use against delinquent customers, it is a most cumbersome and costly one because of the expense of foreclosure, which usually exceeds, to a considerable extent, the ordinary sewage disposal or water charge. Its principal value lies in its ability to fix responsibility and depends largely upon some other agency to force actual collection. The perfect combination is the right of lien to establish liability and the right to discontinue the sewer service or water service to the property for nonpayment of the sewage disposal service. The Detroit ordinance was drawn to provide for all of these measures.

Detroit makes a charge for sewage disposal service to all properties that are connected to the system, on the basis of water consumed, making adjustments under the following rule adopted October 20, 1941:

> The Sewage Disposal rate shall be applied uniformly to all water consumers, whose property is connected to the Sewage Disposal System, on the basis of the quantity of water used thereon or therein, regardless of the purpose for which the water is used, or the character or concentration of the sewage delivered from the property to the Sewage Disposal System, except that an additional charge may be made where the character of the sewage is such that it imposes an unreasonable burden upon the system, as provided for in the city ordinance; provided, however, that upon proper showing, exemption of the sewage disposal charge will be made on water delivered through a metered line where the entire amount of water delivered through said line is used for such a purpose, and in such a manner as to establish beyond reasonable doubt the fact that the water so taken does not enter the Sewage Disposal System.

Of particular interest is the fact that, since the sewage disposal charge has been in effect, less than thirty requests have been received for adjustment of the sewage disposal charge on the grounds that all or part of their water consumption did not reach the sewer. In general, the requests for adjustment were from railroads and large industrial plants.

An ordinance was passed by the Common Council requiring users of private wells, or any other private sources of water supply, to meter or measure such supply in a manner satisfactory to the Department of Water Supply, as a means of determining the proper charge for sewage disposal service. The regular sewage disposal charge is applied to water supplies from private sources.

Contrary to a rather widespread opinion that sewage disposal charges are difficult to collect and, that if added to water bills, would tend to slow up water collections, experience in Detroit indicates that inclusion of the sewage disposal charge in the water bill has had little if any effect on collections. As a matter of fact, the department's customer delinquent balance in October, 1941, hit a new low for the preceding thirteen-year period, in spite of the fact that the delinquent balance since June 19, 1940, carries sewage disposal charges as well as water service charges.

HERBERT MOORE,[5] ASSOC. M. AM. SOC. C. E.—The organization of large sanitary districts is the subject of this paper, such districts being required because of the "magnitude and difficulty" of projects for sewage treatment. Often, however, a need for relatively simple sewerage improvements can be met by the organization of a small sanitary district.

---

[5] Cons. Engr., Milwaukee, Wis.

In Wisconsin, enabling legislation makes this possible. Some forty sanitary districts have been organized under Wisconsin statutes 60.30 to 60.309, in unincorporated township areas. Districts have been formed for the most part to provide sewers only, in order to discharge sewage into the system of a neighboring incorporated community or metropolitan sanitary district. However, several small sanitary districts provide their own sewage treatment.

The first step in the organization of a sanitary district is a petition to the town board signed by at least 60% of the property owners of a given area. Following a public hearing, the town board establishes the boundary and appoints three commissioners at a nominal fee (generally about $25 each per year) to administer the district. The district may then construct sewerage improvements and finance the project by bond issue.

Thus, a sewerage and sewage disposal system serving a small suburban residential community in Metropolitan Milwaukee (Wis.) was constructed by the Westchester Sanitary District, Town of Brookfield, Wis., Waukesha County, and placed in operation in December, 1941. This project provides the main sewers, a sewage disposal tank, and sewer connections from house building lines. It is believed to be the first public sewerage and sewage disposal system to be constructed from homes through the disposal system complete. Furthermore no federal aid was received.

Located about one mile west of the limits of the area served by the Metropolitan Sewerage Commission of Milwaukee County, the Westchester Sanitary District was formed by the homeowners and residents of a housing community to correct nuisance conditions resulting from the use of septic tanks. Some forty houses had been constructed and septic tanks and gravel leaching beds were installed in accordance with best approved practice. However, soon after the houses were occupied, septic sewage effluent from the leaching beds emerged upon the surface of the ground, flowing into ditches and across neighboring property, thereby causing considerable odor and nuisance. The organization of the sanitary district resulted from the demand by property owners for immediate corrective measures.

Plans and specifications were prepared and advertised. The contract was awarded to the lowest of six bidders and the system, comprising some 8,000 ft of sewer lines and a concrete two-story-type tank, was constructed. Sewage effluent from the tank flows into a gravelly subsoil on the property of the District. Six months' time elapsed between the formation of the Sanitary District and the completion of construction of the project.

Work was financed by general obligation bonds in the amount of $4,200 and revenue bonds in the amount of $12,500, at interest rates of 2.4% and 4.5%, respectively. The general obligation bond issue was limited by statute to 5% of the assessed valuation at the time of the last previous assessment.

Francis H. Kingsbury,[6] M. Am. Soc. C. E.—An analysis of the various methods by which sewage treatment projects have been established, with particular reference to the organization and financing, is presented in this paper. Although three general types of organization are noted and consideration is given to various methods of financing, including reference to the trend toward service charges and revenue bonds, the writer believes that discussion of the various factors that can influence the selection of the type of borrowing may be appropriate.

---

[6] Associate San. Engr., State Dept. of Public Health, Boston, Mass.

The history of finance of the large investments for the Boston metropolitan sewerage districts and the change in the methods of public financing in Massachusetts since 1889 may be of interest. Until 1913 the general method of bonding in Massachusetts was that known as the "Sinking Fund Method" whereby the annual payments into a sinking fund plus the interest on the total bond issue remained at a constant total figure throughout the life of the bonds. A modification of this method of financing in the case of the North Metropolitan Sewerage District and the South Metropolitan Sewerage District, noted in Table 1, was adopted for the purpose of permitting a series of increasing annual payments for financing to parallel, as nearly as might be the estimated, future increases in population.

Chapter 439 of the Acts of 1889 and Chapter 424 of the Acts of 1899 of the Massachusetts General Court provide that:

> "* * * The treasurer or receiver shall, on issuing any of said scrip or certificates of debt, apportion thereto from year to year an amount sufficient with its accumulations to extinguish the debt at maturity; but any such apportionment or assessment shall be at the rate of one eightieth part of the whole amount in each of the first ten years, one sixtieth part in each of the second ten years, one thirtieth part in each of the third ten years, and the remainder equally divided in the next ten years * * *."

Since the population more than doubled during the life of the bonds, this method of financing was fortunate because it maintained a reasonably uniform per capita assessment.

Beginning in 1913 the Commonwealth adopted the serial bond method whereby bonds are issued for different periods of time to provide for payments on the principal each year in amounts as nearly equal as possible, with the further provision that no annual payment on the principal shall be less than any subsequent payment. The effect of this method of finance is to make the total requirements for interest and principal the greatest in the beginning, gradually decreasing to the last payment which includes its portion of principal and a relatively small charge for interest. Difficulties were experienced in meeting the first annual payments on new projects before revenue began to come in, and, in certain instances, special legislation permitted deferment of the annual payments on account of principal for a period of three to five years to delay the peak assessments.

The change from the sinking fund to the serial bond method was brought about by two general considerations:

1. The unfavorable experience with municipal sinking funds sometimes invested by financially inexperienced municipal officials;

2. The trend from private to public enterprise in many services which was having a tendency to pyramid annual expenses of municipal governments; and

the desire to accomplish the payment of a considerable portion of the principal of any borrowing before some new enterprise might be voted.

Fig. 2 shows a comparison of the annual payments for principal and interest by the sinking fund, modified sinking fund, and the serial bond methods of borrowing.

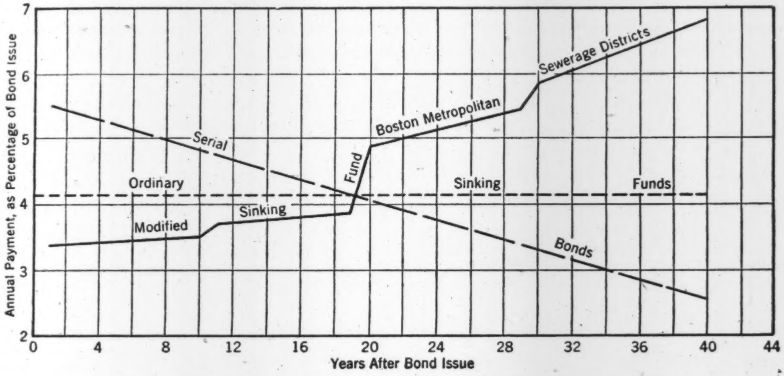


FIG. 2.—COMPARISON OF DEBT SERVICE FOR FINANCING BY VARIOUS METHODS OF BOND RETIREMENT (Interest Assumed at 3% Per Annum)

In financing a new project for sewage collection or sewage treatment, a careful analysis should be made of the probable future charges and their relation to probable population changes and to probable needs, not only for the project under consideration but for other municipal improvements. It might happen that a large initial assessment could defeat a project if it were financed by serial bonds requiring the largest payments in the beginning and that selection of the sinking fund method or a modification of the sinking fund method would be more appropriate. On the other hand, there are many projects for which the population is likely to be more or less stabilized, in which case the ordinary sinking fund or the serial bond method might be more appropriate.

A. BRINTON CARSON,[7] ASSOC. M. AM. SOC. C. E.—In his excellent paper Mr. Greeley states that the trend in the organization of sewage treatment projects has been independent of other local governments; but it appears, from his paper, that the trend proceeds at a snail's pace. It also appears that the existence of such a trend at all is due, not to the practical considerations of engineering, but to the legal exigencies of financing.

The evils of organizing sewage treatment projects along purely municipal or local governmental lines have always been apparent; so too have the engineering fallacies to which this practice frequently leads. Mr. Greeley has indicated, either consciously or unconsciously, the difficulties encountered in large municipalities. Those encountered in small communities are frequently defined more sharply.

The most important of these difficulties is the problem of operation. Because small plants frequently discharge their effluent into small streams, the required degree of treatment is generally higher than for large plants discharging into larger bodies of water. Yet it is precisely for small plants that the greatest difficulty is encountered in securing properly trained operators. The difficulty, of course, is basically financial. The proper operation of small plants tends to increase the per capita cost of operation, sometimes seriously. It is apparent, therefore, that larger plants, or groups of small ones operated as a unit, could operate more efficiently and at a much reduced per capita cost.

---

[7] Lt., CEC-V(S), U. S. N. R., Philadelphia, Pa.

There are other difficulties in small communities, however. Consider the case of C and D, two small adjoining towns on the same watershed. Because they were unable to agree on a joint plan, each built its own treatment plant. Both plants were finally constructed 200 yd apart on the same bank of one stream. Consider also the case of A, another small town. Agitation for sewerage came from a part of A built on low ground along the banks of a stream. Just outside the town limits of A, and also on low ground, was a thickly populated area, not incorporated as a town, a part of the adjoining township. Town A provided its citizens with sewerage and sewage treatment. It was willing to take the sewage from the adjoining area and treat it, but could do so legally only by agreement with the adjoining township. The township refused to enter into any agreement for the benefit of this small area, since the remainder of the township was sparsely settled, and it was afraid of setting up a precedent.

It is noted that even the Illinois Enabling Act permits territories the freedom of choice as to whether they shall annex or disannex themselves to or from a drainage district. No thought is given to the effect of this freedom of choice on the remainder of the sanitary district. Where such a territory lies in the center of a natural drainage area, the effect of its withdrawal might be disastrous on the entire remaining sanitary district.

It is true that the reason for many of the difficulties now being encountered lies in the fact that sanitary engineering, in terms of civilization, is relatively new. The education, for great masses of people, as to its necessity has scarcely begun. It has been necessary, practicable, and advisable to construct sewerage facilities as opportunity offered—almost, it might be said, as an educational guide.

It is probably true that the educational phase is not yet over even in the United States; but, coincidental with the decline of that phase, there should be inaugurated a new phase of broad-scale planning in the organization and financing of sewerage and sewage treatment plant construction. If it is begun early it will be well done in the end; if it is begun late it may never be done at all. As a basis for such planning, the writer offers the following suggestions for an ideal design, with the thought that existing conditions and practical considerations be permitted to modify it only by the purest compulsion.

Organization.—

1. What is (or should be) the definition of a sanitary district?

A sanitary district may be defined as a section of the earth's surface consisting of the whole or part of a natural drainage area or a series of such areas draining into a common basin.

2. How extensive should a sanitary district be?

A sanitary district should be large enough to warrant the construction of a sewage treatment plant with a capacity of at least 1,000,000 gal per day when the present density of the population in the area is 20% or more of the probable ultimate density. This arbitrary definition will be further modified by certain other factors such as:

(*a*) The availability of a plant site of sufficient size for present construction and future extension.
(*b*) The average slope of the basin. Where the average slope is relatively flat the district may be limited by the time required for sewage to reach the plant in a sufficiently fresh state for proper treatment.
(*c*) Undulating or special terrains, such as a community in a flat desert area; communities built along a shore and isolated by salt marshes from the mainland, etc.

3. To what extent should existing local governmental boundaries be used in defining a sanitary district?

Pre-existing governmental boundaries should be used only where they happen to coincide with the boundaries of a drainage area. It is a purpose of this procedure to ignore all local governmental boundaries, and to consider the question of sewage entirely from an engineering point of view. It is believed that such a procedure will reduce operational costs as well as overlapping and duplicating efforts.

4. When should such a sanitary district be organized?

When the density of population of any part of the district warrants the collection and treatment of sewage. Further definitions might be set up covering this phase, but it is likely that the pressure of local opinion may be more potent.

5. Who should determine the area of a sanitary district?

It should be first approximately defined by that agency of the state government customarily charged with reviewing and passing upon proposed sewerage facilities. After the district has been organized, by the procedure outlined herein, preliminary surveys should define the area more exactly. The preliminary definition of the district is delegated to the state as being a disinterested party. The state is also considered as a centralizing agency having knowledge of the proposed organization of all districts in the same drainage area, as well as in adjoining drainage areas.

6. When should a state agency delineate a sanitary district?

On a petition from 100 qualified voters in a continuous area. It is assumed that a town or community would be the continuous area referred to.

7. How should a sanitary district be activated?

After a petition of 100 qualified voters has been referred to the state agency controlling sewage, the agency would define the approximate limits of the district. This information would then be referred to the executive branch of the state government, which would appoint three commissioners to supervise an election in the district. The state would be empowered to advance the costs of this election, from funds set up for this purpose (see subsequent heading, "Financing," regarding preliminary financing). The question to be determined by this election, of course, is that of whether the district shall be organized and sewage facilities constructed.

It is possible that, under certain conditions, it might be advisable to include in this election the selection of permanent members of the sanitary district board or commission. In other cases, assuming that the vote of the electorate is affirmative, it may be preferred to select the district board by a separate election or by appointment by the executive arm of the state. However, this step of the procedure, being entirely politico-legal, will vary from state to state.

The next step, after establishing a sanitary district board or commission, would be to proceed with a comprehensive survey of the sewage handling requirements of the district. On completion, this would be presented to the state agency for review and approval. It is assumed that the area might vary from that originally proposed by the state, but it is believed that it will seldom vary to such an extent as to affect, materially, the population included and thus require a new election.

When the state agency and the district's engineers have agreed on a comprehensive plan of the facilities, area, etc., the district is then in a position to consider costs and methods of financing.

8. What jurisdiction should a sanitary district board have?

A sanitary district board should have full jurisdiction over the collection and treatment of all sanitary wastes within its area. This should include existing facilities as well as those proposed. Storm drainage, except where it is incidental to sanitary sewerage facilities, should remain a function of existing governmental bodies. The acquisition and operation of existing sanitary facilities will constitute a special problem in every case. Additional study, in general terms, should be given to this phase.

The treatment (and transportation) of prime industrial wastes should be limited to a percentage of the total volume of sewage to be treated. That percentage should be the quantity which a plant designed for sanitary sewage can absorb without impairing its operation. Where prime industrial wastes are treated at the source they become secondary industrial wastes. A larger percentage of secondary industrial wastes might be permitted.

In general, then, the jurisdiction of a sanitary district board should be complete over all sanitary sewerage and treatment or anything which might limit or interfere with, or affect, the cost of such treatment.

9. What powers should a sanitary district have, when fully constituted?

Its powers should include the following:

(*a*) To fix the extent of present and future requirements of sanitary sewage facilities for the area, and to engage the necessary professional guidance for so fixing.
(*b*) To own, construct, or acquire such facilities as may be required.
(*c*) To operate facilities so acquired.

(*d*) To finance the construction or acquisition of such facilities by the acceptance of grants, the issuance of bonds, or such other means as can be made available.

(*e*) To make such charges for the collection and treatment of sanitary sewage as shall be found necessary to provide for adequate operation and for the retirement of indebtedness.

10. How should these powers be vested in the sanitary district?

By appropriate state legislation.

Financing.—

11. How should the early stages in the establishment of a sanitary district be financed?

The legislative act providing for the creation of sanitary districts should provide for an appropriation from state revenues. This appropriation should be used as a revolving fund for the purpose of advancing funds for elections, comprehensive surveys, etc. (but not including the acquisition of facilities), to bridge the period until permanent financing has been obtained by the district. From its financing, the district would then repay to the state moneys so advanced. In cases where the district failed to complete its organization, the state would carry the funds so advanced as a charge against any such future organization.

12. How should the construction and acquisition of sewage treatment plants be financed?

The cost of the construction and acquisition of sewage treatment plants, pumping stations, main trunk connecting sewers, outfall sewers, etc., is essentially a function of area—first, because the system is an asset not merely to the part of the district being immediately sewered, but also to those parts having no immediate need for individual sewerage; second, because modern design, no matter how limited the proposed present construction, provides for considerable extension of use before additional facilities must be constructed. There is, therefore, no equitable basis for distributing these costs except on the basis of area.

Financing by taxation on assessed value is not equitable because the value of property does not necessarily reflect its requirements for sewage facilities. Assessments based on present usage requirements are likely to work to the disadvantage of those for whom facilities are provided at a later date, or vice versa.

The owner of a hundred acres presumably can use present facilities without new construction if he should develop the property. Therefore, it follows that the costs of sewage treatment plants, etc., should be assessed on an acreage basis for all area within the district. If later development within the district requires additional construction of this type, the additional cost should again be assessed on area.

13. How should the cost of branch sewers (that is, collection systems) be financed?

The cost of branch sewers is essentially a function of accessibility or possible use. Therefore, it is a function of frontage. It follows that the cost of branch sewers should be assessed on the frontage served. Assessments should be based on the cost of an average 8-in. or 10-in. sewer at average depth. Where trunk sewers are available for individual connection, only the cost of the standard branch sewer should be assessed against frontage, the remainder of the cost being carried under paragraph 12. Costs of connections to the line of the property should be assessed against frontage on the same standard basis, such as 1 per 50 ft of frontage or, for singly-owned properties, a fraction thereof.

14. How should the cost of operating and maintaining the sewerage and treatment facilities be financed?

The cost of operating and maintaining a sanitary sewage system is essentially a function of use. Therefore, its cost should be charged against those who use it. Data presented in the Third Progress Report of the Committee of the Sanitary Engineering Division on Sewer Rental Laws and Procedure[8] indicate that use rates are of three types: (*a*) A flat rate per connection, (*b*) a rate based on water consumption, and (*c*) a rate based on fixtures or type of connection. A flat rate is necessarily limited in application to relatively small communities with a uniform density of population. It is therefore unsuited for use in the proposed sanitary district.

Rates based on water consumption presuppose the accessibility of water records to the sewer authority. In the proposed sanitary district, this might frequently not be the case. Several different water suppliers, in some cases privately-owned water companies, might contribute to the sewage flow. As a standard method of rate fixing, therefore, water consumption is not universal enough in the possibility of its application. There is the additional factor that, with water consumption as a basis, the actual use and the presumptive use may vary widely. Extensive and unnecessary collection and treatment facilities may have to be provided which will not be used, due to the knowledge that, so long as they are not used, in terms of water, there will be no charge for these stand-by facilities.

It appears that the most equitable basis for sewage rates is that which examines and adheres the most closely to the probable use. The number and the type of fixtures, conditioned by their location as to residential or industrial use, appear to distribute the cost of operation and maintenance most satisfactorily and are the most available for universal application.

15. How should the collection of charges for sewage service be organized in a sanitary district?

A sanitary district is conceived, herein, as an administrative organization. To avoid, as fully as may be possible, the duplication of existing functions already organized, it is proposed not to authorize the sanitary district board to collect directly from the individual sewage contributor. The sanitary district would establish the assessments and rental charges, but the actual billing and collection would be in the hands of the several local governments or portions thereof that might be part of the sanitary district. They, in turn, would make periodic returns to the district. This would permit the inclusion of assessments on tax bills and, at least in certain cases, rental charges on water or other service bills issued periodically. A nominal fee for collection could then be deducted from returns to the district by the local governmental body.

---

[8] *Proceedings*, Am. Soc. C. E., November, 1942, p. 1585.

Summary.—It has been the intention, in the foregoing, to propose a broad-scale plan for the organizing and financing of sewage works programs. Mr. Greeley's paper has indicated the extent to which similar planning has progressed in Illinois and some of the difficulties encountered there. From Mr. Greeley's paper, it is evident that even the Illinois Enabling Act is a compromise of political and legal expediency and that the careful planning of the engineer has been only an afterthought. Any enabling act, of course, is better than none at all, but it is the writer's thought that the engineering profession should be prepared to propose its own enabling acts in the future.

Samuel A. Greeley,[9] M. Am. Soc. C. E.—The discussions of this paper have covered a considerable range. That submitted by Mr. Adams is a good record of a practical experience in the accomplishment of sewage treatment on a regional basis by separate organizations. Experiences of this kind are useful in developing better methods of organizing and financing such projects.

Mr. Smith describes the procedure followed in determining the method of financing the Detroit (Mich.) sewage disposal project. He calls attention to one matter of importance and that is the approximation of a two-part rate in which one part is designated as a charge for usage based on metered water consumption and a second part is based on the financing of general obligation bonds by a property charge or tax. Sometimes this division approaches an equitable distribution of the cost and should not be overlooked.

Mr. Moore describes the successful organization of a small sanitary district. There are undoubtedly localities where a small sanitary district meets the requirements; but it seems likely that, in general, the duplication of governmental units for small populations is not desirable.

Mr. Kingsbury gives a clear illustration of the effect of the type of bond and method of borrowing on the annual cost illustrating, for instance, the difference between an equal total annual cost and an equal per capita annual cost. Fixed charges of this kind should be related to the annual sums set up for depreciation and the method of allocating major repairs or replacements as between capital and operating accounts. Consideration should also be given to the lower total expenditure in dollars with serial bonds as compared to sinking fund methods. Presumably the serial bond maturities and interest rates can be adjusted to approximate equal annual total amounts. Frequently, however, general constitutional or charter provisions limit the choice, and the type of borrowing cannot be arranged to meet favorably different local conditions.

The discussion by Lieutenant Carson is of marked interest and describes a procedure for an ideal design in organizing and financing a sewage treatment project. The first seven sections under "Organization" relate primarily to the determination of the extent, area, and need of a sanitary district. In this connection, attention should be called to Chapter 112, Laws, Session of 1910 of the State of New Jersey. Paragraph 2 of this chapter authorizes governing bodies of any two or more municipalities in the state to unite and jointly cause to be made at their joint expense, by competent engineers, surveys, maps, plans, reports, and estimates of proposed works. This is a good method and should be used more widely. In the writer's opinion, the comments by Lieutenant Carson on methods of financing are somewhat limited, as full consideration is not given to the difference between systems handling sanitary sewage only and those with capacity for storm sewage as well. His description and outline of an ideal procedure, however, is very stimulating.

---

[9] (Greeley & Hansen), Chicago, Ill.

It would be highly desirable if experienced engineers were employed to advise in the preparation of enabling acts and in the formation of sanitary districts.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2211

PHYSICAL PROPERTIES THAT AFFECT THE BEHAVIOR OF STRUCTURAL MEMBERS

By Wilbur M. Wilson,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Jonathan Jones, A. B. Kinzel, Almon H. Fuller, C. F. Goodrich, R. L. Moore, Leon S. Moisseiff, Fred L. Plummer, A. A. Eremin, and Wilbur M. Wilson.

Synopsis

The physical properties of structural steels usually prescribed by specifications are the strength and the ductility. It is reasonable to infer that, if certain physical properties are prescribed, there must be some quite definite relation between these properties and the behavior of structural members. The purpose of this paper is to discuss this relation and to raise the question as to whether or not there are other characteristics of the material that should be considered.

Why Ductility Is Necessary

The two properties universally prescribed (and the only ones usually prescribed) for structural steels are strength and ductility. The value of high strength is self-evident. The value of ductility is so general that some discussion would seem to be in order. Engineers generally agree that ductility is necessary for the following reasons:

(1) To prevent fabrication and erection operations from breaking or weakening the members;

(2) To permit members that are slightly bent in handling to be straightened cold without injury;

(3) To enable plastic deformation to effect a more nearly uniform distribution in regions of concentrated stress due to inaccurate fabrication, thermal stresses due to welding, or other causes;

(4) To reduce greatly the extent to which moderate deformation stresses and load stresses are additive beyond the yield point; and

(5) To give the structure a shock-absorbing capacity that will enable it to withstand accidental shocks of considerable magnitude that are not quantitatively provided for in the design.

Note.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Research Prof., Structural Eng., Civ. Eng. Dept., Univ. of Illinois, Urbana, Ill.

All of these actions involve one or more occurrences in limited areas of deformation stress, exceeding slightly the yield point of the material.

(The term "deformation stress" is not in general use and an explanation of its meaning is in order: As the term is used in item 4, a deformation stress is a stress incident to a deformation, as distinguished from a load stress which is incident to a load. The vertical shear in the outstanding leg of the connection angles of a stringer is a load stress because it results from the load carried by the stringer. If the outstanding leg were to fracture because of the shear, the end of the stringer would fall. As distinguished from this action, the outstanding leg of the same angle is subjected to flexure because of the deflection of the stringer. This flexural deformation produces stress; but if

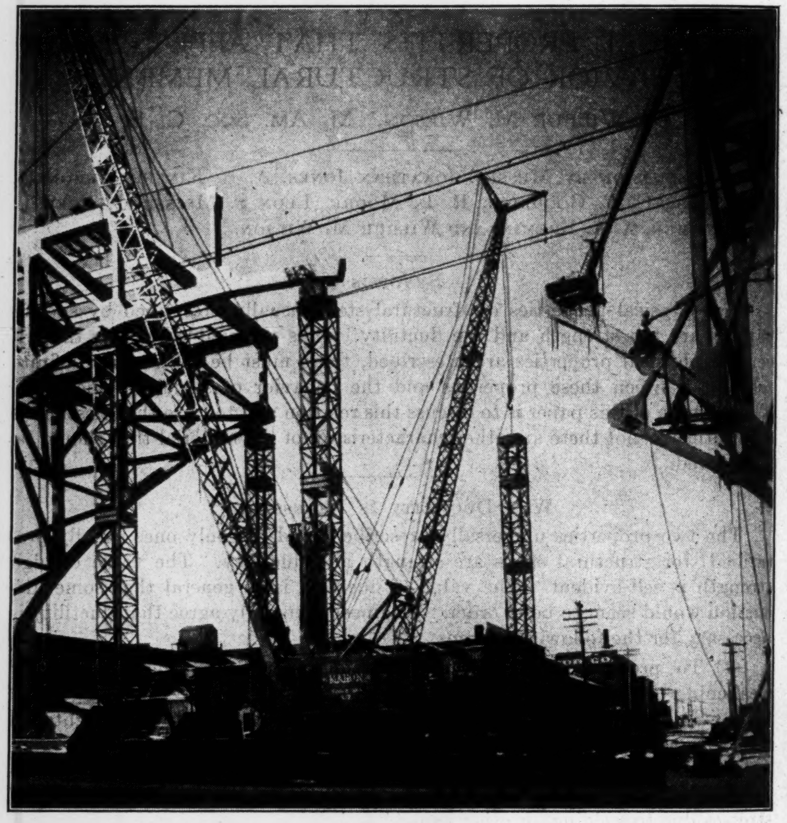


FIG. 1.—SUCCESSFUL CONSTRUCTION REQUIRES THE USE OF A STEEL TO WITHSTAND OVERSTRESS WITHOUT SERIOUS INJURY TO MEMBERS. CONSTRUCTION VIEW OF A HIGH-LEVEL VIADUCT OVER THE CUYAHOGA RIVER AT CLEVELAND, OHIO

the outstanding leg were equipped with hinges at both vertical edges, the necessary movement could take place without producing flexural stresses in the outstanding leg of the angle, and the connection angle would still hold the stringer in position. This latter case illustrates a primary characteristic of all deformation stresses, which can be stated as follows: If the movement which caused the deformation stress could take place without stress, the load-carrying capacity of the structure of which the member under consideration is a part would not be appreciably affected.)

Hazards of Fabrication and Erection (*Item* (*1*)).—Stresses slightly in excess of the yield point probably occur in most structures during fabrication and erection. Successful construction requires the use of a steel to withstand this overstress without serious injury to the members. For this reason, engineers, in general, consider the capacity of structural steel to elongate 1% or 2% at the yield point, without any appreciable increase in stress, to be one of its most valuable characteristics. This plastic deformation is twenty-five times the elastic deformation corresponding to usual design stresses, but standard A7 carbon steels[2] and a number of low-alloy structural steels can be subjected to this deformation without being stressed appreciably beyond the yield point. Moreover, if a steel originally capable of being elongated 25% is elongated locally 2% during fabrication, it still will retain more ductility than it is feasible to utilize. Furthermore, tests showed that an A7 steel, after being subjected to more than 2,000,000 repetitions of a cycle in which the stress (tension) varied from 25,000 to 50,000 lb per sq in., retained its original static strength, and practically all of its original ductility, although its yield point was raised. (The first application of this stress produced a plastic deformation of the order of 4% or 5%.[3]) There is a possibility, however, that age embrittlement may develop in some steels subsequent to plastic deformation. The possibility of this action taking place in structural steels has not been studied in a comprehensive manner.

For a half century, carbon structural steel similar to A7 has been successfully fabricated by riveting. Low-alloy structural steels have been fabricated, but to a lesser extent. Experience with both of these steels would seem to justify the conclusion that they are suitable for structural purposes after being subjected to the overstress incident to fabrication by riveting. Nevertheless, it would seem desirable, particularly with the low-alloy steels, to make comprehensive studies to determine the extent to which their ductility is reduced by age embrittlement subsequent to plastic deformation.

The foregoing comment pertains to the results of overstress incident to fabrication by riveting. During the past decade the structural engineer has been compelled to accept oxygen cutting and welding as fabrication processes because of their economy and convenience. An A7 carbon steel, with carbon and manganese content limited to 0.25% and 0.70%, respectively, can be welded without serious metallurgical damage, and specifications[4] have been adopted for the welding of highway and railway bridges.

---

[2] Specifications, American Society for Testing Materials, Designation, ASTM—A7.

[3] *Bulletin No. 327*, Univ. of Illinois Eng. Experiment Station, Urbana, 1941, p. 32.

[4] American Welding Society's 1941 "Specifications for Welded Highway and Railway Bridges."

The effect of oxygen cutting and welding upon structural steels may be summarized as follows:

(*a*) If carbon structural steels similar to A7 are oxygen-cut with a mechanically guided torch, the resulting edges will give tests equal to planed edges, but evidence is not available to justify a similar statement relative to low-alloy steels.[5]

(*b*) Although welding and oxygen cutting, when properly done, do no serious metallurgical damage to carbon structural steels with certain limitations of carbon and manganese content, they may result in serious injury to members subjected to repeated or reversed stresses. Surface irregularities or abrupt changes in section are serious "stress raisers" that greatly reduce the fatigue strength of a structural member. The indiscriminate use of oxygen cutting and welding may result in rough edges or abrupt changes in section, with an accompanying serious decrease in the fatigue strength of the member.

(*c*) Low-alloy steels can be oxygen-cut and welded, provided that the proper procedure is followed, but if the proper procedure is not followed the resulting member may be much more unsafe than a similar member fabricated of carbon steel; furthermore, the possibilities of serious damage are not limited to major operations. The welding of a small, incidental piece to a large member to facilitate erection (a process not supposed to affect the strength of the structure) may embrittle the member or actually cause a fracture before any external load is applied. Similarly, cutting a handhole in the web of a channel or I-beam may embrittle the member. These are the results of metallurgical damage. The damage is all the more dangerous because it results from a minor operation that may not be given serious consideration. No oxygen cutting or welding of any medium-carbon or high-carbon steels or any low-alloy steels should be done except in complete accord with the procedure developed for the particular steel in question, and then only under competent supervision.

Errors in Workmanship (*Item* (*3*)).—The plastic deformation at the yield point that prevents a member from being injured by the fabrication processes also effects a redistribution of uneven stresses and greatly reduces the extent to which load stresses and moderate deformation stresses are additive beyond the yield point. (This might not be true under a triaxial stress condition.) Errors of workmanship, structural members that must be forced into line, and thermal stresses due to welding are common sources of uneven stress distribution. The mechanism of this stress redistribution is apparent from the behavior of a column whose reinforcement is attached with longitudinal fillet welds. Measurements on columns with an $\frac{L}{r}$-ratio of 70, when reinforced in this manner, indicated that the thermal stresses due to welding were equal to the yield point of the material. Nevertheless, tests showed that the load-carrying capacity of the columns was increased in direct proportion to the increase in section.[6] This was possible because plastic deformation at the yield-point stress prevented the stress at any point from exceeding the yield point of the material by an appreciable amount. It also permitted low-stressed material adjacent to high-stressed material to limit the strain of the latter to a value only slightly greater than the elastic strain corresponding to a yield-point stress. Moreover, the high-stressed material was supported laterally by the adjacent low-stressed material. The action of a similarly reinforced tension member is much the same except that the load-carrying capacity of the member might not be increased quite as much as the area of the section because the stress-strain relation would not be the same in the heat-affected zone as in the unaffected zone. However, the difference would not be great for an A7 steel.

---

[5] *Proceedings*, Am. Soc. C. E., April, 1940, p. 627.

[6] *Bulletin No. 280*, Univ. of Illinois Eng. Experiment Station, Urbana, 1935, p. 25.

The effect of superimposing a load stress upon a deformation stress is similar to the effect of superimposing a load stress upon a thermal stress. In fact, a thermal stress is a deformation stress—a stress resulting from motion, or from the prevention of motion, as distinguished from a stress resulting from a load.

The action of a member reinforced without removing the dead load is similar to the action of a member to which reinforcement is attached with longitudinal fillet welds in that some portions of the section are stressed higher than other portions and in that a redistribution of stress takes place when the maximum stress reaches the yield point.

The stresses in the members used in the foregoing illustration exceeded the yield point of the steel over some portions of the section. The writer has taken the position that, in many situations, this high stress does not endanger the safety of the structure, for the following reasons:

(*A*) In each instance, the stress in excess of the usual design stress is due to a deformation, and this deformation is limited in magnitude to a value only slightly exceeding the elastic deformation corresponding to a yield-point stress by material other than the material that is overstressed; and

(*B*) The deformation at the yield point that equalizes local stress concentrations does not seriously injure carbon and some low-alloy structural steels.

Approval is given to the action described, subject to certain limitations, not with the idea of deliberately overstressing material in new construction, but with the idea of possibly retaining old structures in service which have deformation stresses due to foundation movements or which must be reinforced because of an increase in the loads. Even for this purpose, combined deformation stresses and load stresses equal to the yield point of the steel cannot be accepted for compression members for which the deformation will cause local wrinkling failure or for which eccentricity of the resultant stress will introduce objectionable flexural stresses in the column as a whole. Likewise, a high, combined, load-and-deformation stress cannot be accepted in a tension member that has a reduced section at a point of maximum stress. Such stress will cause the strain resulting from movement to be concentrated at the reduced section instead of being uniformly distributed over a considerable length, as it would be if the section were uniform.

Accidental Shock (*Item* (*5*)).—A fifth reason why ductility is an essential property of structural steels is that it gives the structure a shock-absorbing capacity that will enable it to withstand accidental shocks of considerable magnitude that are not quantitatively provided for in the design.

Some further explanation would seem necessary to convey a clear conception of this contemplated action. As ordinarily considered in the design of railway bridges, impact consists of the dynamic increment resulting from unbalanced wheels, "out-of-round" wheels, track irregularities, etc. These produce stresses in the members of a bridge, and a member is so designed that when impact stress is added to dead-load and live-load stresses the total does not exceed 18,000 lb per sq in., a value considerably less than the yield point of the material. In other words, the impact that is known to exist, and which is evaluated in the design, produces an elastic deformation of the material. This impact is to be distinguished from an accidental shock (such as might result from a derailment) that is unknown in amount and may be so great that it will bend or break a member. One occurrence of such a shock would cause failure in a tension member made of a brittle material and would cause permanent deformation in a member made of a ductile material. Failure of the brittle member might cause the collapse of the entire structure, whereas permanent deformation of the ductile member, in most instances, would require nothing more serious than its replacement.

In general, engineers recognize the value of a shock-absorbing capacity in a member that will prevent the collapse of the structure as a whole, due to a large accidental shock, but they are not willing to add material to the structure merely to provide safety against this unpredictable action; nor would this seem to be necessary. The proper solution of the problem would seem to be:

First, design the structure so that all loads and forces that are expected to come upon the structure, and which can be evaluated, will be resisted at stresses well below the yield point of the material (this is designing for strength); and

Second, use a material which, in providing the required strength, will also provide the greatest possible shock-absorbing capacity. Then, even though the magnitude of some possible accidental shock cannot be anticipated, provision will have been made to guard against the destructive effect of the shock with the greatest economy.

These two steps, consciously or unconsciously, enter into the design of all structures. The second step is introduced when the use of a ductile material is required. A part of the second step that is not always recognized is that the shock-absorbing capacity of a member depends as much upon the geometrical characteristics of the member as upon the ductility of the material.

The shock-absorbing capacity of an A7 carbon structural steel is approximately one thousand times as great in the plastic range as in the elastic range. This is true only if the entire volume of the member is equally stressed when failure occurs. The truth of this statement is apparent from the stress-strain diagram for such steels. The work done in a tension failure is represented by the area under the stress-strain diagram. This varies for various specimens but may have a value of the order of 18,000 in-lb per cu in. The work done in stretching a steel to 0.8 of the ultimate equals the area under the stress-strain diagram up to a point representing that stress. Since this point is on the ascending portion of the curve, the area under the portion of the curve to the left of that point will be relatively small—usually of the order of 10% or 15% of the area under the entire diagram.

An effective net section of 80% by test[7] is as great as is usually realized in fabricating by riveting. When a riveted member fails in tension, only a small part of its total volume—that adjacent to a transverse section through the rivet holes—is stressed to its ultimate capacity and undergoes the full plastic deformation of which the material is capable. This small volume absorbs energy that is equal, approximately, to 18,000 in-lb per cu in. Much the greater portion of the volume of the member is stressed to only 0.80 of its ultimate capacity and undergoes a correspondingly small plastic deformation. The energy that this greater volume has absorbed when the weakest section fails is of the order of 1,800 in-lb per cu in. The average value for the entire member depends upon the rivet pattern, but is not likely to exceed 3,000 to 4,000 in-lb per cu in. This is the basis for the statement that the shock-absorbing capacity of a member depends as much upon the geometrical characteristics of the member as upon the ductility of the material. Mechanical engineers, in their treatment of bolts that fail due to shock, have long recognized the principle involved. Such failures are largely eliminated by turning down the body of the bolt to a diameter less than the diameter at the root of the thread.

The fact that some types of members have a greater percentage of gross volume that is effective for absorbing shock than others is the justification for using a less ductile steel for some types of structural members than for others.

Cables of suspension bridges are fabricated of wires that may have a yield point of 150,000 lb per sq in., an ultimate strength of 220,000 lb per sq in., and an elongation in 10 in. as low as 4%. Because the wire is of uniform section, the entire volume will be stressed equally, and the energy required to break the wire is of the order of 7,000 in-lb per cu in., a value twice as great as the entire shock-absorbing capacity of many riveted tension members fabricated from an A7 steel.

The eyebar is another fabricated member that has a high shock-absorbing capacity because the section is uniform over the entire length except for enlargements at the ends. The cables of the Florianópolis (Brazil) suspension bridge were fabricated of medium-carbon, heat-treated eyebars.[8] The specifications for these eyebars prescribed a yield point and ultimate strength of 75,000 and 105,000 lb per sq in., respectively, and an elongation of 5% in 18 ft. The design stress was 50,000 lb per sq in. The energy required to break one of these bars is of the order of 4,300 in-lb per cu in., a value somewhat greater than the corresponding value for a riveted member of A7 steel.

The foregoing examples would seem to justify the use of high-strength, low-ductility steels for wires and eyebars, inasmuch as the uniformity of section more than offsets the low ductility of the material in their effect upon the shock-absorbing capacity of the member. This being true, it would seem that it might be profitable to consider whether or not it is possible to devise other methods of fabricating structural members so as to conserve a greater part of shock-absorbing capacity inherent in the material instead of discarding from 50% to 80% of this capacity, as is now usually done when fabricating by riveting. Any expedient that is used to stress the member uniformly high over the main part of its length must result in a member that has no section weaker than the body of the member, which is to be of constant section.

---

[7] "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1243.

[8] "The Eye-Bar Cable Suspension Bridge at Florianopolis, Brazil," by D. B. Steinman and William G. Grove, *ibid.*, Vol. 92 (1928), p. 280.

Effect of Speed of Loading upon Tensile Properties of Steel

Engineers have formed their conception of the tensile properties of steels from static tests. The term "shock" implies motion; that is, rapidity of application of the force. Some engineers have a vague feeling that the resulting stress, because of its suddenness of application, is more injurious than a static stress of the same intensity. Whether or not there is any basis in fact for this feeling depends upon whether or not the speed at which a steel is strained has any significant effect upon either its strength or its ductility. In the previous section of this paper, the comment relative to the shock-absorbing capacity of members is based upon the assumption that the ultimate strength and stress-strain relations are not significantly affected by the speed of loading.

H. C. Mann[9] reports tension tests on four alloy steels loaded at various speeds, to 28.5 ft per sec. His discussion of the tests contains the following paragraph:

> "It has been shown in the data presented that when similar specimens of the same material are tested at normal temperature, under both static and dynamic conditions, the total energy values obtained from each are the same, provided the velocity of impact is within a certain limiting amount, and that when this velocity is exceeded the impact values become considerably reduced. A possible explanation of this phenomena is obtained from a further consideration of the mechanism of the static test."

The tests cited showed that the shock-absorbing capacity of SAE-1035 steel[10] began to fall off slightly at a speed of 25 ft per sec. In considering this velocity in its relation to what might be experienced in a bridge, it should be noted that the significant velocity is that at which strain occurs rather than that at which a train crosses a bridge or at which one body strikes another.

Mr. Mann used short cylindrical specimens of constant diameter contained between heads of larger diameter, and they were subjected to a high-speed tension test. A few high-speed tension tests were made at the University of Illinois on specimens with geometrical characteristics of structural members.[11] The specimens included 3-in. by $\frac{3}{8}$-in. plates without joints; similar plates with open, drilled holes; similar plates connected with butt welds; and others connected with riveted joints. The force was derived from a 2,300-lb steel block which fell 10 ft and impinged on a nut on the lower end of a 2-in. steel rod suspended from the lower end of the specimen. The results indicated that the shock-absorbing capacity was as great under dynamic loading as under static loading.

---

[9] "The Relation Between the Tension, Static and Dynamic Tests," by H. C. Mann, *Proceedings*, A.S.T.M., Vol. 35, Pt. II, p. 323.

[10] Specifications, Society of Automotive Engineers, Designation, SAE-1035.

[11] "The Effect of Speed of Loading Upon the Ductility of Structural Steel," by Gordon L. Jeppesen, and "The Effect of Speed of Loading Upon the Ductility of Structural Steel," by Robert Zaborowski, theses presented to the Univ. of Illinois, Urbana, Ill., in 1938 and 1940, respectively, in partial fulfilment of the requirements for the degrees of Master of Science in Civil Engineering.

The results of these tests are highly significant to the designer of bridges and, although the work that has been done is only exploratory, it would seem that the discussion, "Why Ductility Is Needed," based on static tests, applies to high-speed loading, provided that the "limiting speed" in Mr. Mann's tests is not exceeded. It also would seem that an impact stress is no more destructive than an equal static-load stress, provided that the speed at which strain occurs does not exceed the limiting speed used by Mr. Mann. Additional high-speed loading tests are needed at both normal and low atmospheric temperatures.

Fatigue Failure

Structural members are designed for known loads, treated as though they were static, and are fabricated of a ductile steel to enable them to withstand shock. It is now known that members subjected to repeated or reversed loads may fail by progressive fracture (unfortunately designated as a "fatigue failure") and at a relatively low average stress. Moreover, it is known that the fatigue strength of small machined-and-polished specimens is not a dependable criterion of the fatigue strength of a structural member fabricated of the same material, and that the fatigue strength of a structural member does not necessarily increase with the static strength of the steel from which it is fabricated.[3,12]

The ordinary formulas of mechanics of materials have been developed mainly for the design of structural members subjected to static loads, and they almost entirely neglect to consider localized stresses due to nonhomogeneity of material and sudden changes of cross section in structural parts. There are many localized regions in structural parts where the stress is higher, sometimes several times higher, than that computed by the ordinary formulas for strength of materials.

Any cause which produces a localized area of high stress is called a stress raiser. Abrupt changes in section, notches, reentrant angles, grooves, screw threads, surface irregularities, and discontinuities such as cracks, holes, or inclusions are stress raisers. The effect of a round hole in a plate, and of other simple stress raisers, has been determined by mathematical analysis and studied by means of transparent models viewed with polarized light; but, because the effect of a stress raiser upon the fatigue strength of a specimen is a function of the size of the specimen and of the physical properties of the material, as well as of the stress raiser, these studies are of limited value in predicting the fatigue strength of a particular specimen. They are of real value, however, in determining types of geometrical stress raisers that are to be avoided.

The cumulative experiences of half a century indicate that there is little reason to fear fatigue failures in riveted carbon-steel chords of trusses, or other members subjected to relatively few repetitions of the maximum design stress, provided that the members contain no unusually severe stress raisers. There has not been sufficient experience to justify a similar statement relative to

---

[12] *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, Urbana, 1938.

welded members in carbon steel or relative to either welded or riveted members of the low-alloy steels. Experience has demonstrated that riveted carbon-steel members subjected to a large number of repetitions of the design stress do fail in fatigue. Fatigue failures of similar details made of low-alloy steels may be expected.

A limited number of fatigue tests have been made on steel structural members.[3,12,13,14] More fatigue tests of both welded and riveted connections are needed, and they should be made on specimens as large and as nearly like the connections used in structures as it is possible to test. Tests are also needed to determine the impact-fatigue strength of structural steels, and these, likewise, should be made on specimens with the geometrical characteristics of structural members.

Shear-Slip Relation for Riveted Joints

The extent to which the shear on a riveted joint is resisted by the friction between the plates depends upon the tension in the rivets. Laboratory tests show that carbon-steel rivets with a grip of 3 in. consistently have a tension of 20,000 to 30,000 lb per sq in.[12] The shear required to produce slip between plates clamped together by such rivets is of the order of 20,000 to 25,000 lb per sq in. of rivet section, a value considerably in excess of the usual working stress for carbon-steel rivets. The tension in some low-alloy steel rivets with a 3-in. grip varied considerably with individual rivets and was as low as 5,000 to 10,000 lb per sq in. for some.[15] The shear required to produce slip between the plates was correspondingly erratic and low. In general, the tension in the rivet increased with the grip in both magnitude and uniformity, with a maximum value somewhat less than the yield point of the steel.

A short, properly driven rivet will fill the rivet hole but probably will have a low tension. Unless the end of a long rivet is cooled before driving, or unless it is tapered, it probably will not fill the hole, but will have a high tension. A rivet that either fills the hole or has high tension will be satisfactory in service if the rivet retains its tension.

The limited number of fatigue tests that are available on this subject indicate that the rivets or bolts of a joint will have a much higher fatigue strength if the shear is resisted principally by the friction between the plates than if it is resisted principally by shear on the rivets or bolts.[16]

Summary and Conclusions

This paper contains ideas, relative to the behavior of structural members, which have resulted from many years of experience in testing engineering materials and structural members. The writer has assumed that the reader is familiar with the literature on the subject but, in a few instances, reference has been made to tests that bore directly upon the particular question being studied. The ideas of the writer, discussed in the body of the paper, may be summarized as follows:

---

[13] *Bulletin No. 310*, Univ. of Illinois Eng. Experiment Station, Urbana, 1939.

[14] *Bulletin No. 317*, Univ. of Illinois Eng. Experiment Station, 1939.

[15] *Bulletin No. 337*, Univ. of Illinois Eng. Experiment Station, 1942.

[16] *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, 1938, pp. 58, 63, and 64.

(1) Structural members should be designed for known forces and then checked against fatigue failure. They should be fabricated of a steel whose ductility, considered in conjunction with the geometrical characteristics of the members, will enable them to resist large, unpredictable shocks.

(2) The plastic deformation that occurs at the yield point without an appreciable increase in stress enables the steel to be fabricated and erected without serious injury, permits redistribution of uneven localized stresses, and reduces the extent to which load stresses and moderate deformation stresses are additive beyond the yield point. Although stresses somewhat greater than the yield point may result at some portions of some sections of the members, they will not reduce the load-carrying capacity of the member appreciably unless the member is made of a steel subject to age embrittlement subsequent to plastic deformation, or unless the conditions are such that the high stresses will cause buckling or local wrinkling of a compression member or excessive elongation at reduced sections of tension members. Additional information is needed relative to the susceptibility of structural steels to age embrittlement subsequent to plastic deformation.

(3) The shock-absorbing capacity of a member depends upon the geometrical characteristics of the member as well as upon the ductility of the steel.

(4) Impact stresses are no more destructive than static stresses of the same magnitude unless the rate of strain exceeds the "limiting speed" of the material, except for the possible effect of impact fatigue, about which practically nothing is known. Additional information is needed relative to the "limiting speed" of structural steel at various temperatures and when fabricated into specimens with various geometrical characteristics.

(5) Oxygen cutting and welding may be used in the fabrication of steel structures, but only under the most careful and enlightened supervision.

(6) Fatigue failures are not likely to occur in riveted carbon-steel chords of trusses, or other members subjected to relatively few repetitions of the maximum design stress, provided that the members contain no unusually severe stress raisers. There has not been sufficient experience to justify a similar statement relative to welded members of carbon steel or relative to either welded or riveted members of low-alloy steel. Carbon-steel members subjected to a relatively large number of repetitions of the design stress have failed in fatigue. Fatigue failures of similar details made of low-alloy steels may be expected. Additional fatigue tests of riveted and welded structural members of both carbon and low-alloy structural steels are needed. Impact-fatigue tests are particularly needed.

(7) A short, properly driven rivet will fill the rivet hole but probably will have a low tension. A long rivet, unless its end is cooled before driving, or unless it is tapered, will not fill the hole but will have a high tension. A rivet that either fills the hole or has high tension will be satisfactory in service if the rivet retains its tension.

ACKNOWLEDGMENT

Fig. 1 was supplied through the courtesy of Link-Belt Company.

DISCUSSION

Jonathan Jones,[17] M. Am. Soc. C. E.—The following discussion of a problem in structural design may be considered a corollary to the author's exposition. In the design of long riveted plate girders, it is frequently economical to construct the flanges of structural silicon steel[18] which, although more expensive per pound, ordinarily permits an increase in unit stress from 18,000 to 24,000 lb per sq in. with a proportionate reduction in section and weight. If, however, a proportionate increase be made in the shearing unit stress for the web plate, it frequently results in a plate undesirably thin as regards either future corrosion or the possibility of buckling. If the shearing unit stress is not increased over that for carbon steel and if a silicon-steel plate is used, an "extra" of some $20 per ton is paid for capacity in shear which cannot be utilized. If carbon steel is retained for the web, it is cheaper, is safe in shear, and is of adequate thickness; but, because of its interaction with the silicon-steel flange, its outermost fibers are inevitably stressed to almost 24,000 lb per sq in., or one third higher than specifications permit.

However, engineers now seem to be in general agreement on the propriety of the evidently economical construction, carbon-web and silicon flanges. As the author has stated, the justification is that not only the unit stress, but also the possibilities of deformation, must be taken into account. In an all-carbon girder, if underdesigning, under future loading, led to an increase of flange unit stress to 33,000 or 36,000 lb per sq in., both web and flange would enter the phase of plastic flow, and the girder would be in danger. In a girder with carbon-web and silicon flanges, however, any future overloading that might bring the flange unit stress to 33,000 or 36,000 lb per sq in. could not cause the web to flow, because the silicon flanges would still be 9,000 to 12,000 lb per sq in. below their yield-point stress of 45,000 lb per sq in., and would still be elongating under practically their original modulus. The carbon web is locked in a high-yield-point flange that will not let it flow, and without flowing it cannot fail, being a material of high ductility.

The author refers to the eyebar (evidently the forged-head bar of several generations' standing) as an advantageous form from the standpoint of energy absorption, the advantage arising from the lack of any spots or localities of reduced area. Attention might here be called to a description[19] of a different type of tension-resisting bar, in which reinforcing-pin plates were welded on to each end of a long flat plate. In the case described,[19] this bar proved more economical of manufacture than a forged-head bar, and it obviously possesses the characteristic praised by the author of having no point of reduced cross section from end to end.

The last two sentences of the "Synopsis" open up a wider field than that of which the author disposes. The vital question of correlation between uses to which a material is to be put, and tests specified for its acceptance, is suggested by the "Synopsis" but not conclusively treated in the paper. Tests for strength and ductility are shown to be appropriate in specifications for structural steel, but does the author answer his final question of "whether or not there are other characteristics of the material that should be considered"?

---

[17] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[18] Specifications, American Society for Testing Materials, Designation, A.S.T.M.-A94.

[19] "Test of Model of the Tacoma Narrows Anchorage Bar," by Bruce Johnston and H. J. Godfrey, *Welding Journal*, Welding Research Supplement, August, 1939, p. 253s.

For structural carbon steel, it may be stated safely that there are no other characteristics, unless and until new types of application must be considered; if, for instance, welding is contemplated, a test that would check directly on "weldability" might be wanted. In general, however, considering riveted bridges, buildings, and similar structures, years of experience and millions of tests have convinced the engineer that if structural carbon steel passes the routine tests for strength and ductility, it may be considered also to possess all of that dependability under adverse conditions which the engineer has learned to expect of it.

The danger lies in carrying the same reasoning, by analogy, over into the specifying of tests for harder steels and for other materials, where no such background of experience may yet exist. Carbon steel, with 60,000 lb per sq in. tensile strength and, say, 26% elongation in 8 in., can safely be subjected to 18,000 lb per sq in. average stress combined with some form of coped or otherwise suddenly altered cross section. Can some harder steel, with 80,000 lb per sq in. and, say, 18% elongation in 8 in., be used safely in the same notched form at some higher unit stress? Often not. Often, it would not safely carry the same unit stress as the carbon steel. A new requirement has been introduced by the sudden change of cross section—accommodation to point concentrations of stress. Will the routine tension and bend tests establish that such a power of accommodation exists? They will not.

Supplementary requirements are not always overlooked. Sometimes for a harder structural steel, a "drifting" test has been imposed, at least on occasional samples, if not in a routine way. In many instances, however, the end use and the acceptance test seem to be further apart than is logical. (In a "drifting" test, punched holes in a narrow bar with sheared edges are considerably enlarged by driving tapered pins, the measure of acceptability being the degree of enlargement before fracture at edge of hole or edge of bar.)

The end use of bridge cable wire is to withstand tension combined with a highly overstraining cold bend around a small-radius sheave at the anchorage, and also to withstand tension combined with heavy cross loading in cable bands and saddles. That the customary tensile and bend tests are not prima facie evidence of this characteristic was proved by the serious failure, in service, of wire that had passed such tests with ease, but that, by an innovation in the process of manufacture, had been rendered vulnerable to the critical conditions just named.

The end use of a bronze bushing or bearing plate is to withstand rubbing on steel under heavy pressure without excessive wear, and to withstand this without seizing or scoring despite considerable fault in the lubrication; but the specified acceptance tests cover sometimes compressive deformation and sometimes tensile strength and elongation.

The end use of arc-welding filler metal is to make a sound, strong, and ductile joint with the material to be welded, under a wide range of variables such as position and temperature; but the specified physical tests are made upon the filler metal as laid in and upon itself, under the most uniform and ideal conditions of deposition.

None of the situations just described need necessarily to be dangerous, but the frame of mind they imply is so. Danger is absent if and when a sufficient body of experience has been built up, to prove that, if the acceptance tests are passed, none of the truly important properties, even though the tests do not challenge them, can be absent. Prudence dictates a greater effort than is always in evidence, to make the acceptance tests fit the critical aspects of the end use; and the fact should be squarely faced that in many instances an increased cost of testing will have to be absorbed by the user.

A. B. Kinzel,[20] Esq.—The scope of this paper is very broad, and Professor Wilson is to be congratulated both for the conciseness of his treatment and for the closeness of his reasoning. In treating so broad a subject, general terminology is necessary. In fact, the terms used are so all-inclusive that they frequently are not defined with sufficient completeness. An unconscious change in breadth of a term may lead to unwarranted conclusions.

Because of this method of approach, there is a strong tendency in the paper to reach conclusions that are ultraconservative and give rise to implications which are even more so. Various engineering codes have proved to be effective. These codes have been prepared by conservative engineers, but do not engender the feeling, present in Professor Wilson's paper, that welding, cutting, riveting (under some conditions), and low-alloy steels are to be applied only with fear and trepidation. Certainly there is sufficient experience to indicate that, as in most engineering problems, there need be no such feeling after application of intelligent judgment. As the paper is reasoned very concisely, discussion of it requires considerable detail and close following of the text.

One of Professor Wilson's first pleas is for more information on strain-aging embrittlement of carbon and low-alloy steels. It has been well demonstrated that fully killed steels—either the silicon or aluminum types—are not subject to this phenomenon. This fact removes a very large part of steel from consideration. Most of the low-alloy steel is fully killed. As to those steels which are not fully killed and may or may not exhibit strain aging, it should be stated that forming, punching, drilling, and similar processes used in general construction, and particularly in preparation for riveting, produce more than sufficient strain to cause the phenomenon, and such strain is set up at points of high stress concentration. Nevertheless, as Professor Wilson well demonstrates, experience justifies the conclusion that such steels are suitable for structural purposes after fabrication by riveting. Although one must be in accord with any plea for more information on any subject, it would seem that there is no need for the implication that further information on strain-age characteristics is needed vitally for immediate engineering use.

The author then discusses welding and oxygen cutting and very rightly stresses the importance of surface irregularities or other stress raisers. Here again the matter of degree must be considered carefully. An ordinary sheared edge very frequently contains surface irregularities in the form of notches, and even minute cracks, which may be far more deleterious to fatigue strength than the broader, less sharp irregularities resulting from cutting or welding. In the discussion of the application of these processes to low-alloy steels, it is stated (see heading, "Why Ductility Is Necessary: Hazards of Fabrication and Erection (Item (1)) (*c*)") that, if the proper procedure is not followed, the resulting member may be "much more unsafe." The use of this phrase is typical of the kind of implication to which many engineers may object. It is necessary to consider low-alloy steels. One category of such steels, broadly characterized by having less than 0.14% carbon and a tensile strength of some 75,000 lb per sq in., shows ductility in the heat-affected zone not dissimilar to that of plain very low-carbon steels and very much greater than that of plain, medium-carbon steels. Professor Wilson's statement obviously is not intended to apply to that group. The other group, with tensile strength approaching 90,000 lb per sq in. or more, generally contains more carbon and does exhibit reduction in ductility caused by the heat cycle of normal cutting or welding. By following the proper procedure, particularly with the addition of pre-heating or post-heating by torch or otherwise, the reduction in ductility becomes a minor matter. The same applies to the incidental welding, but here still another factor enters the situation. Damage due to such incidental welding of a small piece to a large member or cutting a handhole in the web of a channel may produce very serious conditions, primarily because of the internal stress resulting from the operation, and this is of the same order of magnitude even when low-carbon steels are considered. Moreover, local stress due to riveting a bracket in place likewise may reach high levels. Thus, although the writer is in accord with the author's word of caution, it would not appear that the entire subject is one for major alarm nor that the need for proper procedure and competent supervision in such operations is restricted to welding or cutting.

---

[20] Chf. Metallurgist, Union Carbide and Carbon Research Laboratories, New York, N. Y.

Regarding the matter of uneven stress distribution produced by fabrication, it is regrettable that Professor Wilson has not included riveting as a source of uneven stress distribution. Again, with respect to the measurements of internal stress due to producing a structural column by welding, it should be emphasized that similar measurements made on rolled members show that internal stress in rolled members likewise reaches the yield point. Stress distribution well may be different, and, for that reason, welded members must be considered very carefully from the standpoint of locked-up stresses; but, to reach such a conclusion, requires much more knowledge than the simple fact that internal stresses of yield-point magnitude do exist. The effect of the heat-affected zone is mentioned in this connection together with the conclusion that the difference between zones affected and not affected by heat would not be great for A7 steel. Neither would the difference be great for any other structural steel delivered for use in the as-rolled or normalized condition.

Regarding deformation at the yield point to equalize local stress concentration, the statement (see heading, "Why Ductility Is Necessary: Errors in Workmanship (Item (3)) (*B*)") is made that it does not seriously injure "carbon and some low-alloy" steels and the writer is at a loss to know just why "all" low-alloy steels were not included with "all" low-carbon steels or "some" low-carbon steels not included with "some" low-alloy steels. This wording again illustrates the matter of unfortunate implication. Actually from the standpoint of the structure, when deformation does occur, stresses are relieved, and the structure is in a highly desirable condition. The danger appears when deformation does not occur at all because of three-dimensional stress conditions. Also, damage is imminent when, because of stress concentration, total deformation must take place over an extremely limited zone and the percentage elongation not only exceeds the yield point deformation but goes on to approach deformation corresponding to the ultimate strength of the material.

In setting up the general solution of the problem, the author cites design first and material second and asks for material which will provide the very greatest possible shock-absorbing capacity. Particularly apt is his emphasis on the fact that shock-absorbing capacity of a member depends as much upon the geometrical characteristics of the member as upon the ductility of the material. This matter of geometry deserves to be emphasized, as it is the essence of the attack on limitation of ductility by three-dimensional stress which probably has been the most potent factor in such few service failures as have occurred. In this connection, suspension-bridge wires and eyebars are examples of uniform stress distribution. In these cases there is almost pure tension —that is, almost complete absence of the three-dimensional effect—and these members pointedly emphasize the need for geometry because of three-dimensional stress effects as well as stress uniformity.

Referring to the section on fatigue failure, although it is true that the fatigue strength of a structural member does not increase necessarily with the static strength of the steel from which it is fabricated, it is likewise true that, given reasonable surface conditions and geometry, fatigue strength does increase with the static strength in most instances. It is stated that there is little reason to fear fatigue failure in riveted trusses or other members subjected to relatively few repetitions of the maximum design stress; and it also is stated that there is not enough experience to make the same statement regarding welded members in carbon steel or any members in low-alloy steel. Just how much experience would be necessary for such a statement is a matter of opinion, but it is highly significant that, among the few bridge failures which have taken place in the hundreds of welded bridges, particularly abroad, no one of them has been reported as a fatigue failure. A statement is made regarding a large number of repetitions of design stress which would produce fatigue failures in riveted carbon-steel members. The author also states that were these details made of low-alloy steels, "similar fatigue failures might be expected"; but the probabilities would be definitely less as the basic fatigue strength of the higher tensile low-alloy steels is higher than that of the structural carbon steels.

The summary and conclusions of the paper are naturally open to the same type of arguments as the foregoing. For example, in conclusion (2), it is stated that deformation stresses above the yield point will not reduce the load-carrying capacity of the member appreciably unless the member is made of a steel subject to age embrittlement. There is no evidence to indicate that, even if the member is subject to age embrittlement, its load-carrying capacity will be reduced by local yielding. In the case of some designs that well may be so. In the case of others, it is not true. It should be remembered that strain-aging embrittlement does not reduce pure tensile impact resistance unless the velocities reach the order of magnitude involved in ballistics. The fact that a specimen of steel which shows strain-aging embrittlement under notched impact tests will not show loss of ductility in a straight tension impact test should be emphasized and again gives weight to the author's plea for good geometry.

As for conclusion (5), it is unfortunate that, in view of all the arguments presented by Professor Wilson, riveting has not been included with oxygen cutting and welding among those processes which should be used only under the most careful and enlightened supervision. The statement as given applies to any fabrication operation. The need for supervision and control in welding operations is stated well in the various codes, and it certainly seems unnecessary to single these operations out from all the others. This is particularly true of cutting where the condition of the surface in general is much better than that generally obtained by shearing and where, in addition, such effects as may be produced are so close to the surface that a stress of appreciable magnitude in the third dimension cannot exist. It also seems to be unnecessary in view of the really vast experience with oxygen-cut members.

Welding and cutting are not as old in the art as riveting but rapidly are approaching a mature age. They may be misapplied just as any process may be misapplied, but the fact that there are no hidden menaces involved already has been well established by experience. Low-alloy steels are even newer in this picture, but they have been used commercially on an appreciable scale for well over a decade. Thousands of freight cars bear witness to their adequacy. Good engineering judgment should be applied to their selection and use. Structures built in accordance with such engineering judgment and existing codes are performing satisfactorily, and the engineers responsible for such structures need not be too disturbed by the possible existence of phenomena which are understood fairly well, even though, in some instances, these phenomena have not been measured quantitatively.

ALMON H. FULLER,[21] M. AM. SOC. C. E.—Matters of much interest to the designing engineer are dealt with in this paper although the full implications of them may be outside of his experience. It raises some questions, frequently overlooked, which need to be considered in revising specifications and in the designing of unusual structures.

The author states, in "Summary and Conclusions" (2), that "* * * Additional information is needed relative to the susceptibility of structural steels to age embrittlement subsequent to plastic deformation." The writer would expect age embrittlement to follow oft-repeated plastic deformations but not to follow the occasional overstress which results from fabrication and erection procedures, secondary stresses, and deformation stresses in general. As the effect of such overstresses is to produce a slight change in the material and to raise the elastic limit, it seems likely that, in many a case, a repetition of the condition which produced the overstress might keep the unit stresses within the raised elastic limit. When elastic stability was not established after the first overstress, it might be established in such a small number of overstresses as to restore elastic behavior without danger of embrittlement. Pending further researches on the subject it may be possible to make pertinent observations of existing conditions.

---

[21] Prof., Civ. Eng., Iowa State College, Ames, Iowa.

In the paper on "Theory of Limit Design,"[22] by J. A. Van den Broek, M. Am. Soc. C. E., as well as in the one under consideration, questions are raised and helpfully discussed which demand thoughtful consideration for those situations for which close designing is essential. The effect of both papers should be to allay and not to create any fear that material will not adjust itself to apparently unfavorable conditions wherever the design has been made intelligently.

Can it be possible that, for structural steel, some of the adjustment (perhaps plastic flow) really takes place within the range of stress called yield point, elastic limit, and even proportional limit? Concrete is accepted as imperfectly elastic, and therefore flow is recognized well within accepted working stresses. Steel is usually regarded as perfectly elastic although the physicists are doubtless correct when they assert that it is not. Although the plastic flow (or creep) at normal temperatures is too slight to be measured within the precision of the usual deformation apparatus within the time of laboratory readings, two publications[23,24] record deformations over several weeks' time which are clearly greater than would occur within an hour or so for the same unit stresses. In his "Synopsis," the author expresses one purpose of the paper "* * * to raise the question as to whether or not there are other characteristics of the material [steel] that should be considered." Not even the flow of concrete is great enough to make undesirable changes in the dimensions of a structure; but quite likely such flow does relieve local stress concentrations and thereby does contribute to satisfactory functioning of the structure as a whole. Is it not likely that the very slight plastic yielding of steel, within working stresses, also contributes to the best functioning of a steel structure?

C. F. Goodrich,[25] M. Am. Soc. C. E.—The author justifies the practice of reinforcing structural columns by attaching the reinforcement with longitudinal welds, even though the thermal stresses due to the welding are equal to the yield point of the material. He is careful, however, to give approval only to the idea of retaining such old structures in service as must be reinforced to carry safely increased loads.

The simplicity of welding reinforcing material on old structures, as compared to riveting, makes welding the ideal solution of the problem of strengthening overloaded bridges. Tests have shown that the high thermal stresses resulting from such practice do not necessarily impair the load-carrying capacity of the reinforced structure.

---

[22] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 638.

[23] *Bulletin No. 72*, Eng. Experiment Station, Iowa State College, Ames, 1924.

[24] *Bulletin No. 40*, Eng. Experiment Station, Ohio State Univ., Columbus, 1928.

[25] Chf. Engr., Am. Bridge Co., Pittsburgh, Pa.

A somewhat different problem is presented when two different steels with different physical properties are used for one new member. Long riveted plate girders have been built with a carbon-steel web plate and silicon-steel flanges. When designing the web plate to resist buckling, it is found that the shearing capacity of a silicon plate is not fully utilized and a carbon-steel plate is often more economical. Many engineers have questioned this practice because the carbon-steel web will necessarily be stressed the same amount as the silicon-steel flange, in which a working stress of 24,000 lb per sq in. is usually specified. Thus the determination of the capacity rating of such a girder presents a nice problem. A capacity stress for carbon steel of 0.8 times the 33,000 lb per sq in. yield point is 26,400 lb per sq in., which is only 10% more than working stress of the silicon flange; whereas a capacity stress for silicon steel of 0.8 times the 45,000 lb per sq in. yield point is 36,000 lb per sq in., which is above the yield point of the carbon web.

In the accepted theory of girder design, the web is assumed to take its proportion of the bending moment and will doubtless do so in a well-designed, well-built girder of carbon web and silicon flange under static loads up to the yield point of the carbon web. The expectation that the web will continue to take its proportionate part of the bending moment beyond this point or under repeated loads somewhat below this point would seem to be in the realm of wishful thinking.

The part of the web between the compression flange angles probably is locked there so that it cannot yield in compression because it will have no place to go. This hardly seems true, however, of the part of the web between the tension flange angles, or just outside of them, where necking down (which accompanies plastic yielding in tension) is not restrained by the clamping action of the flange angles, especially under repeated loads and the possible loosening of rivets. It seems more than probable that the carbon web, under these conditions, no longer could be relied upon to take its proportionate part of such overload. Thus, the silicon flange would have to take all of it. It is probably true that the silicon flange will continue to act elastically up to its yield point, provided rivets have not become loose and the girder is otherwise integral.

The question might well be asked: Would responsible engineers evaluate a girder with silicon flanges and carbon web to carry future overloading and to resist impact and fatigue as highly as they would a girder of the same make-up but with silicon web? The answer is probably negative. The entire matter then resolves itself into whether the engineer is willing to accept this loss of load-carrying capacity in order to effect the undoubted first cost economy of using the carbon web.

R. L. MOORE,[26] ASSOC. M. AM. SOC. C. E.—An excellent general discussion of the need for ductility in structural steels is presented in this paper. The author's ideas also are applicable to other materials than steel. Although the functions of ductility generally are recognized, it is difficult to place a quantitative value on the importance of this property or to establish limits within which it may be varied without sacrificing structural safety. The author's statement in the "Synopsis" implying a definite relation between prescribed physical properties, including ductility, and the behavior of structural members, seems to the writer to require some elaboration.

---

[26] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

As long as the structural engineer was concerned principally with the use of carbon steel, the question of ductility was not important. The present trend toward the use of less ductile alloy steels and nonferrous metals for special uses, however, particularly where weight saving is an important consideration, makes it necessary to examine ductility specifications more critically. The question of greatest interest is not, "How is ductility useful?" but rather, "How much ductility is needed?"

Ductility generally is expressed in terms of percentage elongation and reduction in area as determined from standard tensile tests. Ratios of tensile yield to ultimate strength also give a measure of ductility. Specifications for certain alloy steels limit this ratio to a maximum of 0.70,[27] whereas for carbon steels it usually ranges from 0.50 to 0.60. It should be emphasized that all materials do not exhibit the same characteristics of ductility. Among the heat-treated aluminum alloys, for example, alloy 61S–T, with a ratio of tensile yield to ultimate strength of 0.85 and an elongation in 2 in. of 14%, may show better ductility as far as forming operations are concerned than alloy 17S–T, with a ratio of tensile yield to ultimate strength of 0.65 and an elongation of 20%. If reduction in area is used as the measure of ductility, however, rather than the ratio of tensile yield to ultimate strength, or elongation, a more consistent indication of the relative forming characteristics of these particular aluminum alloys is obtained.

Carbon steel apparently has such a margin of ductility beyond that required for most structural uses that there has been little need to attempt to establish a definite relation between this property and the behavior of structural members. Observations on the behavior of actual structures are of little help in this respect since the latter are not designed to function in a stress range where ductility is a significant factor. Few structural tests appear to have been made in which the effect of ductility was one of the principal variables considered. It has been shown by static tests on riveted joints of steel and aluminum alloys[28] that differences in elongations ranging from a minimum of 11% to a maximum of 39% (measured over gage lengths of four diameters) had no apparent effect upon structural behavior. The joint strengths were within a few per cent of predicted values, indicating that the needs for ductility to level off stress concentrations and to effect a uniform distribution of load between rivets were amply met.

The author points out that the welding of structural steels may produce stress concentrations that are detrimental to the fatigue strength of a member. No mention is made of the effect of welding upon ductility. Tests by Professor Wilson, with Messrs. Bruckner and McCrackin,[15] as reported in 1942, have shown, however, that elongation values for butt welded tensile specimens of low-alloy steels may be as low as 5% in 8 in., when fractures occur in the welds. The effect of this "local brittleness" upon shock resistance and fatigue strength would appear to be an important consideration, although the increasing use of welded steel construction might be interpreted to mean that this reduction in ductility is not a critical factor.

---

[27] "Specifications for Steel Railway Bridges," A.R.E.A., Pt. 3, Alloy Steels, 1938.

[28] "Riveted and Pin-Connected Joints of Steel and Aluminum Alloys," by Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1359.

Experience is needed, of course, to indicate to what extent ductility may be utilized in a structural material. The civil engineer, accustomed to thinking in terms of the ductility of carbon steel, will naturally have a different conception of this property, and of minimum structural requirements, than the aeronautical engineer, whose principal structural materials are alloy steels and nonferrous metals. The successful structural use, in aircraft, of metals having elongations of 10% to 14% in 2 in. (which is appreciably less than those generally prescribed for other types of structures) emphasizes the need for a better understanding of the functions of ductility with respect to structural behavior.

LEON S. MOISSEIFF,[29] M. AM. SOC. C. E.—The experience that engineers and metallurgists have had with structural members has brought to their minds in recent years an increasingly important problem. Professor Wilson deals with the information essential to this problem. He asks what physical properties of metals are desirable and may be required to assure the best behavior of structural members. The problem is not an outcome of theoretical considerations; it is a purely practical one. Based on his wide experience, Professor Wilson poses the problem on observations, engineering practice, and tests.

In the "Synopsis" of the paper the statement is made that "It is reasonable to infer that, if certain physical properties are prescribed, there must be some quite definite relation between these properties and the behavior of structural members." This suggests that the author doubts the correctness of the inference—and so do many engineers with him. It has been found in many cases that the physical properties prescribed in widely adopted specifications are insufficient to assure the desired behavior of structural members. As stated by Professor Wilson, many years of experience in fabricating and using structural carbon steel have proved that these specifications and the prescribed tests for the acceptance of the material have served well the requirements of the structures in operation. In fact, it should be emphasized that the specified physical properties of structural carbon steel and the acceptance tests for them have been developed by engineers during these many years of experience by closely following the behavior of the material, constituent members, and structures in use. Also, it should be kept in mind that commercial acceptance tests are limited by practical considerations. From the practical side it is important that such tests should be relatively simple and should consume little time. Acceptance tests have been standardized for fair comparisons and definite results and also to attain speed in testing.

Engineers early recognized that the strength of the material is the first requisite. The behavior of the material under action of load or strain also soon attracted the attention of the practical master mechanic—some kind of a bend test was used for metals long before scientific testing. In developing the standard tension test for strength, the phenomenon of ductility attracted attention and was made part of the tension test, largely because it was concomitant with it. (This is available with the tension test and consumes little additional time.) The observation and recording of the reduction of area also fall in line with the tension test. Should additional physical properties be specified, they will require separate tests which may be more time consuming.

---

[29] Cons. Engr., New York, N. Y. (Mr. Moisseiff died on September 3, 1943.)

It was very fortunate that, in the transition from wrought iron to steel, the structural carbon steel of the present time was developed. Its wide use for structures is largely due to its dependability and workability. Most of the latter is due to its high ductility. The high percentage of elongation of structural carbon steel is a result of its manufacture; it was not aimed for especially. Low-alloy and high-alloy steels, as well as lightweight metals, however, do not show such high elongation as structural carbon steel. This has brought up one of the questions of the new problem: How high an elongation does a metal require to meet the requirements of modern fabrication and erection of structures? The author discusses at length the several factors which demand from the metal an elongation through ductility that surpasses the elastic elongation. There are indications of a quantitative evaluation of these factors in the paper. It is highly desirable, however, to establish experimentally the percentage of elongation required in a given metal to meet, safely, the possible plastic deformations that may occur during fabrication, erection, and operation.

To study the effect of higher and lower ductility of metals and the effect of higher and lower ratios of yield strength to ultimate strength and the effect of the moduli of elasticity on distribution of stress, a series of tests of riveted and pin-connected joints was made.[28] Five metals—two steels and three aluminum alloys—having a fairly wide range of ductilities and ratios of yield strength to ultimate strength were included. All the joints tested behaved in a manner consistent with the basic assumptions of design and with the properties of the materials. Furthermore, the tests showed that the plastic yielding necessary to effect a uniform distribution of load among the rivets must be relatively small and that differences in ratios of yield strength to ultimate strength, elongation values, and moduli of elasticity were not significant factors as far as static ultimate strengths were concerned.

In these tests it was shown that the plastic yielding necessary to attain a uniform distribution of load in the riveted joints was relatively small. Tests on various structural parts in which stress-raising is expected may furnish information on the percentage of ductility required for practical purposes.

In discussing physical properties that affect the behavior of structural members, the writer thinks it is appropriate to mention some desirable physical properties which metallurgists may strive for.

One such property is the physical durability of the metal, its resistance to corrosion. In recent years a fraction of 1% of copper has been introduced to the chemical constituents in structural steels as protection against rust. There probably are other corrosion retarding agents which could be utilized economically. The assurance of a very low rusting quality would allow engineers to improve the design of structural members.

Another property of metals which is all important in the behavior of structures is their modulus of elasticity. It is amazing how the numerical value of the modulus remains practically unchanged for a given metal. It appears to be independent of the other physical properties and of the process of manufacture. Thus, rolled structural steel and high-strength stainless steel will show practically the same modulus as the cold-drawn bridge wire, which is nearly four times as strong as structural steel. Various grades of aluminum and duralumin have the same aluminum modulus; and other metals also have a specific value. This phenomenon is so striking that it arouses the curiosity of engineers. The satisfaction of this curiosity and raising the modulus of some metals would result in tremendous advantages of structural and economical value. These facts call for research of the highest kind. Whether the modulus of elasticity results from the forms of the crystals and their pattern in the metal and whether these could be modified by controlled processes affecting their formation is a problem of transcendental interest. Structural engineers need not be told that successful research in these problems will result in substantial gains.

FRED L. PLUMMER,[30] M. AM. SOC. C. E.—The importance of the ductility of structural steels is emphasized in this paper as necessary in order to avoid possible failures due to: Fabrication and erection operations; stress concentrations caused by inaccurate fabrication, irregularities, welding heat or forced deformations; or excessive stresses caused by shock loads. J. A. Van den Broek and Hardy Cross, Members, Am. Soc. C. E., and others also have discussed the capacity of ductile metals to undergo relatively large deformations when stressed to the yield point, thereby modifying the distribution of stresses and possibly increasing the safe load-carrying capacity of the structure.

The ductility, to which these authors refer, is measured usually in connection with tension tests in which uniaxial stress is applied at room temperature and in small increments separated by definite time intervals or at a relatively slow but uniform rate. Thus, the reported values do not reflect the possible effect of temperature, rate of propagation of stress, or biaxial or triaxial stress distributions. Service failures of steel structures almost invariably show none of the deformations which are characteristic of failures in the tension tests. This indicates that ductility has failed to perform the functions suggested by the author, at least in these specific cases.

Professor Wilson refers to the structural steel covered by specification A7.[2] Many structural shapes are rolled from steel meeting this specification. Steel plates used for structural work often are selected to meet specifications A10 or A70. Such steel plates, ordered from different manufacturers and meeting all requirements of these specifications, may consist of "rimmed steel," "killed steel," or "semi-killed steel." However, metallurgists state that the resistance of these steels at low temperatures to impact forces and their susceptibility to age embrittlement following plastic deformation may be quite different.

---

[30] Chf. Research Engr., Hammond Iron Works, Warren, Pa.

Steel plate structures frequently consist of plates that have been bent or dished into curved shapes by rolling or forming. Thus such plates are subjected to more "cold working" than most of the structural shapes used in bridges and buildings although these members sometimes receive similar treatment. In steel plate structures the principal elements of the structure are subjected frequently to biaxial stresses of approximately equal intensities whereas the principal members of bridge and building structures normally resist only uniaxial stresses.

The writer would appreciate a statement from the author indicating whether he considers safe the use of steel plates meeting the requirements of specification A70 in a code-designed structure: (1) For which the plates will be formed cold to a dished shape (age embrittlement following plastic deformation); (2) in which biaxial stress conditions will exist throughout most of the structure (partial loss of effective ductility); (3) which will be subjected to atmospheric temperature variations in an area when for short periods the temperature will fall below 0° F (lower resistance to impact forces); (4) of which the various plates will be assembled by carefully controlled welding processes (stress concentrations due to welding heat); and (5) which, during such a period of low temperature, might be subjected to an impact force while resisting the normal design loads (speed-of-stress propagation). If the author does not consider the specifications adequate, in what way would he suggest that they be modified so as to insure the production of a steel that would meet the foregoing requirements adequately?

Structural engineers are widening continuously the scope of their activities. This applies to the types of structures which they design and to the kinds of metals to be used in such structures. Not many years ago the typical structural steel designer was concerned primarily with the use of one material, one method of fabrication, and the proper proportioning of members subjected to bending forces or to a direct thrust or pull, together with relatively simple "connections." Now he may choose from a wide variety of alloy steels and nonferrous metals in proportioning the parts of complicated continuous structures, including parts such as "skin structures" whose strength depends upon their elastic stability. He may connect the parts of his structure with rivets or by welding. It is only natural, but perhaps fatal, to try to apply old standards, tried procedures, and tested ideas to new materials used in new types of structures. Professor Wilson serves all engineers in outlining some of the problems and indicating a few about which additional information is needed.

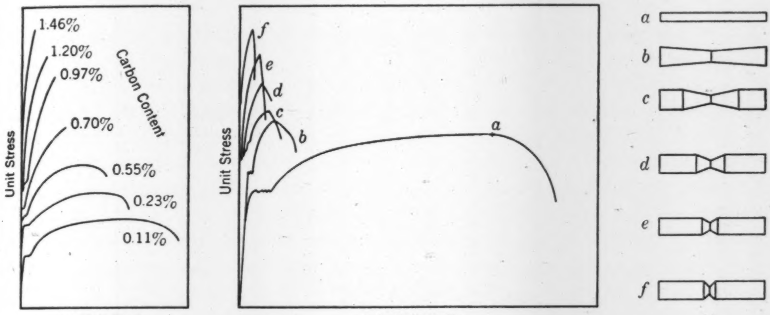
A. A. Eremin,[31] Assoc. M. Am. Soc. C. E.—The physical properties of steels affecting the design of structures may be determined from diagrams expressing stress-strain relations for tests on steel. Sufficient data are not yet available to draw the diagrams for various types and shapes. However, considering the similarity of some characteristic features of steels, the diagrams available at present are sufficient to determine the approximate influence of the physical properties of steels on the behavior of structures. Interesting diagrams of this stress-strain relation have been presented by S. Timoshenko.[32]

---

[31] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[32] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., Van Nostrand, p. 553.

In Fig. 2(*a*) stress-strain diagrams for steels with various carbon content are shown. The ductility is decreased with an increase of the carbon content. The alloy steels are characterized by a carbon content varying from 0.25% to 0.50%. Therefore, Fig. 2(*a*) makes it possible to determine an approximate relation between the plastic flow of alloy steels and A7 steel. These curves prove the author's statement that, where plastic flow is desirable, the low-alloy steels or A7 steel should be used. Fig. 2(*b*)[33] shows relative stress-strain diagrams for steel members with reduced mid-sections such as those in Fig. 2(*c*). Members in which notches are cut, or members with variable cross section, lose their ductility although their ultimate strengths are increased.



(*a*) Unit Strain (*b*) Unit Strain (*c*)

Fig. 2.—Relative Stress-Strain Diagrams for Steel

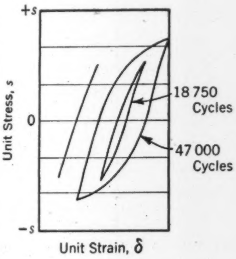


Fig. 3

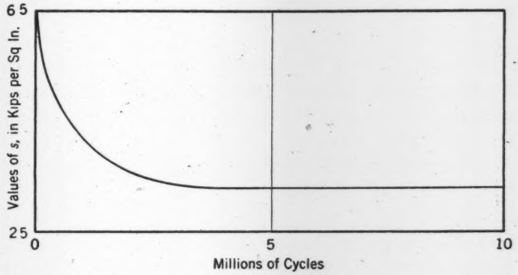


Fig. 4

The capability of steel to harden under repeated loading is shown in Fig. 3. It is obvious that the alternate loading in a member may raise its yield point and produce a stiffening effect.

In conclusion (1), the author states that structural members should be designed for known forces and then checked against fatigue failure. From Fig. 4 it is obvious that the fatigue effect on stress in the members may be estimated and considered as a special type of loading. Therefore, it is possible to design the structure for a combined loading including the fatigue effect.

---

[33] *Der Stahlbau*, June 19, 1936, p. 109.

In conclusion (2), Professor Wilson states that the plastic deformation permits the redistribution of uneven localized stresses. From Fig. 2 it is evident that the plastic deformation at the reduced sections, where the localized stresses occur, is considerably reduced. Therefore, its effect on the redistribution of stresses is negligible.

Wilbur M. Wilson,[34] M. Am. Soc. C. E.—The action of a composite plate girder with silicon-steel flanges and a carbon-steel web has been presented in the discussion. The problem, which is within the scope of the paper, is one of considerable importance and would seem to justify a more detailed consideration than has been given. The following discussion is based upon the premise that a member with given geometrical characteristics buckles at a certain average critical strain rather than at a definite average critical stress. Actually, the buckling strength depends upon both stress and strain, but, for a stress range in which Hooke's law does not apply, the average strain at which buckling begins is much more nearly constant than the average stress. The discussion is based upon the further premise that the carbon-steel web of the composite girder will have the same thickness as the silicon-steel web of the all-silicon girder which otherwise might have been used.

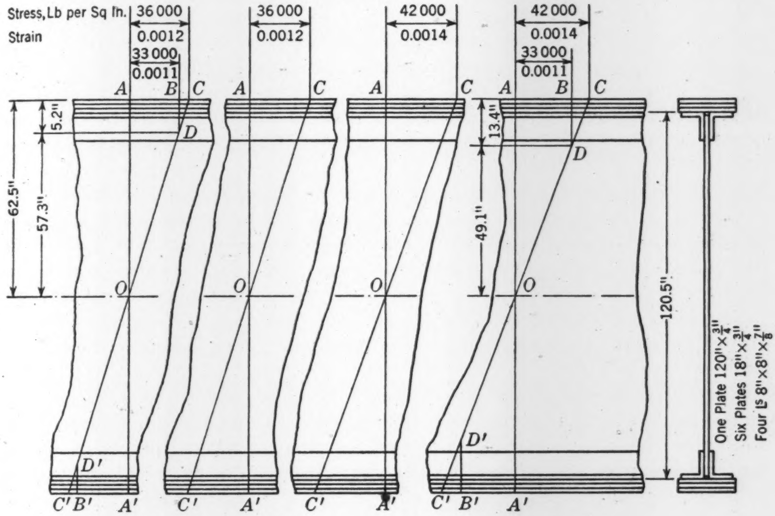
The distribution of the stress and strain on a transverse section of a plate girder is shown by the diagrams of Fig. 5. To simplify the problem, the computations have been made on the basis of the gross section so that, for a girder symmetrical about the center line, the tensile and compressive stresses are equal. The girder is composed of a 120-in. by $\frac{3}{4}$-in. web, four 8-in. by 8-in. by $\frac{7}{8}$-in. angles, and six 18-in. by $\frac{3}{4}$-in. cover plates. Figs. 5(*a*) and 5(*b*) are for a section at which the maximum stress is 36,000 lb per sq in.; this is eight tenths of the yield point of silicon steel and corresponds to a capacity rating.

The area OACOC′A′O in Fig. 5(*b*) represents both the stress and the strain on a section of an all-silicon girder. The yield point has not been exceeded, and the stress is proportional to the strain over the entire section. The resisting moment corresponding to a maximum stress of 36,000 lb per sq in., based upon the gross section and computed by the moment-of-inertia method, is 28,373,400 ft-lb.

The area OACOC′A′O in Fig. 5(*a*) represents the strain on a section of a composite girder with a carbon-steel web and silicon-steel flanges. The same area also represents the stress on the silicon part of the girder, but the stress on the carbon web is represented by the area OABDOD′B′A′O. The resisting moment corresponding to a maximum stress of 36,000 lb per sq in., based upon the gross section and computed by the moment-of-inertia method, is 28,357,800 ft-lb, which is 99.94% of the resisting moment of the all-silicon girder. Although a small part of the carbon web between the angles is stressed beyond the yield point, the maximum strain in the web due to a moment of 28,373,400 ft-lb (the moment-resisting capacity of the all-silicon girder) is only 0.06% greater for the composite girder than it is for the all-silicon girder. Therefore, the composite girder with carbon web and silicon flanges would appear to have as great a moment-resisting capacity as the all-silicon girder when the maximum stress is 36,000 lb per sq in. Furthermore, there would seem to be no more danger of buckling and no more tendency for the rivets to loosen for the composite girder than for the all-silicon girder since both tendencies to fail are functions of the strain and there is no significant difference between the maximum strains for the two types of girders.

---

[34] Research Prof., Structural Eng., Civ. Eng. Dept., Univ. of Illinois, Urbana, Ill.



(a) SILICON FLANGE CARBON WEB (b) ALL SILICON GIRDER (c) ALL SILICON GIRDER (d) SILICON FLANGE CARBON WEB

FIG. 5.—STRESS-STRAIN RELATIONS FOR VARIOUS GIRDERS

Consider now the conditions when the girder is subjected to an accidental maximum flexural stress of 42,000 lb per sq in.—a stress greater than any that would be considered in either the design or the rating of a bridge. The resulting stress and strain are shown in Fig. 5(*c*) for the all-silicon girder and in Fig. 5(*d*) for the composite girder. Since no part of the all-silicon girder is stressed beyond the yield point, the strain and stress are both represented by OACOC′A′O; for the composite girder, the strain is represented by OACOC′A′O, but the stress in the web is represented by OABDOD′B′A′O. The resisting moment corresponding to a maximum stress of 42,000 lb per sq in. is 33,102,300 ft-lb for the all-silicon girder and 32,881,300 ft-lb for the composite girder, the latter being 99.15% of the former. Although this extreme increase in stress (to 42,000 lb per sq in.) has increased the part of the carbon web that is stressed to the yield point, the maximum strain in the web due to a moment of 33,102,300 ft-lb (the moment-resisting capacity of the all-silicon girder) is only 0.85% greater for the composite girder than it is for the all-silicon girder. Inasmuch as the tendency to buckle and the tendency to loosen rivets are both functions of the strain, these tendencies to fail are not significantly different for the two kinds of girders.

The statement has been made in the discussions that the writer did not answer the question which he raised in the last sentence of the "Synopsis." This question restated is: "Are there characteristics of the material that should be considered which are not now determined in acceptance tests?" The writer listed or implied in the "Summary and Conclusions" some characteristics of structural steels which are not now generally familiar to structural engineers but which he believes should become known before the steels are subjected to the services and fabrication methods that are becoming more common. This is particularly true for the newer steels, as stated by Mr. Jones.

Mr. Kinzel expresses the belief that the writer has been unduly conservative in his attitude toward low-alloy structural steels with special reference to the possible injurious effects of welding and flame-cutting. Mr. Kinzel is a metallurgist and is accustomed to addressing metallurgists. The writer is a structural engineer with a limited knowledge of metallurgy and is accustomed to addressing other structural engineers, many of whom likewise have a limited knowledge of metallurgy. Warnings to an expert in any field may be uncalled for whereas warnings to a novice in the same field may be entirely in order. Many structural engineers are novices in the field of metallurgy.

A conservative engineer is naturally reluctant to use steels that can be embrittled during fabrication by deviations from prescribed procedure. His problem is to be sure that, in the confusion of shop fabrication and field erection, the proper procedure is always followed. What is most to be feared is incidental welding and flame-cutting not called for on the drawing and done without the knowledge of the engineer and by other than approved procedure. The writer agrees with Mr. Kinzel that low-alloy steels can be flame-cut and welded satisfactorily when the proper procedure is followed, but he also believes that a word of caution intended to insure that the proper procedure is followed is in order. The warnings in the paper were to emphasize the need for caution and not to discourage the use of welding and flame-cutting or the use of low-alloy steels.

Mr. Kinzel's reference to the undesirable features of riveted structural members is quite in order. There are locked-up stresses in structures fabricated by riveting. Punching and riveting do injure both carbon and low-alloy structural steels. Riveted members are particularly susceptible to fatigue failure as the writer's own tests indicate.[35]

With regard to embrittlement subsequent to plastic deformation, Mr. Kinzel states in the third paragraph of his discussion:

> "* * * It has been well demonstrated that fully killed steels—either the silicon or aluminum types—are not subject to this phenomenon. This fact removes a very large part of steel from consideration. Most of the low-alloy steel is fully killed. As to those steels which are not fully killed and may or may not exhibit strain aging, it should be stated that forming, punching, drilling, and similar processes used in general construction, and particularly in preparation for riveting, produce more than sufficient strain to cause the phenomenon, and such strain is set up at points of high stress concentration. Nevertheless, as Professor Wilson well demonstrates, experience justifies the conclusion that such steels are suitable for structural

---

[35] *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, Urbana, 1938, p. 103.

> purposes after fabrication by riveting. Although one must be in accord with any plea for more information on any subject, it would seem that there is no need for the implication that further information on strain-age characteristics is needed vitally for immediate engineering use."

Mr. Kinzel does not include any mention of rimmed steels. Some rimmed steels are susceptible to embrittlement subsequent to plastic deformation. Structural specifications do not prohibit the use of rimmed steel. Unquestionably rimmed steel is used. One reason this mysterious (to the structural engineer) phenomenon should be investigated is that there have been occasional brittle failures of structural members in service for which there has been no acceptable explanation. Some years ago the writer tested fifteen wide plates without joints[36] (20 in. to 40 in. wide) to compare the unit strength of the wide plates with the unit strength of control specimens cut from the same parent plates. Of three supposedly identical specimens 20 in. wide, the elongation in 8 in. was 35.1% for one and 23.5% for another. The third broke with a brittle fracture with practically no elongation. The structural engineer rightly wants to know why these brittle fractures occasionally occur.

Mr. Kinzel asks how much experience would be necessary to determine whether or not any particular method of fabrication was susceptible to fatigue. Fatigue failures may occur after a few thousands of cycles or after a few millions of cycles. Except for temporary bridges, designs should be based upon a long life. Fatigue failures have developed in carbon-steel members with bad stress raisers after many years of service. If a railroad bridge is designed for 50 years of service, the fact that the members were intact after 30 years of service would not be proof that it was properly designed against fatigue.

In discussing the relative fatigue strength of low-alloy and carbon-steel structural members, Mr. Kinzel states:

> "* * * it is likewise true that, given reasonable surface conditions and geometry, fatigue strength does increase with the static strength in most instances."

Unfortunately, riveted structural members and welded structural members, except members with butt welds with the reinforcement ground flush, do not have "reasonable surface conditions" as judged by their ability to resist fatigue.

As a result of comparative fatigue tests of riveted joints in carbon, silicon, and nickel steels,[35] the Committee on Iron and Steel Structures of the American Railway Engineering Association revised its specifications for railway bridges, reducing the allowable stress in silicon-steel and nickel-steel members subjected to fatigue.[37] The present requirements may be somewhat more drastic than necessary but have been adopted until such time as more information is available.

Mr. Kinzel takes exception to the many cautions relative to low-alloy steels, flame-cutting, and welding. Not all of these have been answered in particular because, as stated previously, the differences between Mr. Kinzel and the writer seem to be in degree only and are due to differences in background and viewpoint. Although the writer believes that his cautions to structural engineers are in order, he believes that Mr. Kinzel's viewpoint is necessary to complete the picture and welcomes the discussion in which it has been presented.

---

[36] *Bulletin No. 239*, Univ. of Illinois Eng. Experiment Station, Urbana, p. 11.

[37] "Specifications for Steel Railway Bridges," A.R.E.A., 1942, Pt. 3, Alloy Steels, paragraph 9 of the Foreword.

The writer was quite gratified to note that Messrs. Moisseiff and Moore recognized the implication in the statement in the "Synopsis"—

> "It is reasonable to infer that, if certain physical properties are prescribed, there must be some quite definite relation between these properties and the behavior of structural members."

This statement does contain the implication that all of the physical properties prescribed may not be needed, at least not for all structural members. Specifications of a material are usually a compromise between what the engineer thinks he wants and what the manufacturer can produce at a reasonable cost. The structural engineer has been spoiled by the high ductility combined with a fair strength that is obtained in carbon structural steel, one of the cheapest structural materials available. When he began to seek a material of higher strength or of lighter weight, he was reluctant to accept a lower ductility. Conservativeness and inertia probably are at least partly responsible for the demand for high ductility in the high-strength steels.

The objective is shock-absorbing capacity of the structural member rather than ductility of the material; and, as stated in the paper, shock-absorbing capacity depends upon the geometrical characteristics of the member as well as upon the physical properties of the material. Also, as cited in the paper, there are structural members fabricated of steels of low ductility that have a greater shock-absorbing capacity than the usual type of member fabricated of A.S.T.M.-A7 steel. Yet geometrical characteristics of the member have few restrictions in the specifications, and, in so far as the literature indicates, have been given little consideration by the profession. Usually where the final objective is a function of two factors, as is true for the shock-absorbing capacity of a structural member, the characteristics of both factors are considered. Investigations to determine the interrelations of the ductility of the material and the geometrical characteristics of the member are greatly needed. A few exploratory tests have been reported.[38] Inasmuch as ductility does not affect the shock-absorbing capacity except as yield-point stresses occur, and the latter generally occur only under accidental loads, high-speed tension tests which simulate accidental shocks are needed. Also, because the geometrical characteristics are equally as important as ductility, tests on small specimens of constant cross section and smooth surface would be of little value in evaluating the behavior of a structural member. What would seem to be needed are tests of specimens with the geometrical characteristics of structural members and as large as it is possible to test.

Mr. Plummer asks the writer whether or not he considered it safe to use A.S.T.M.-A70 steel under certain conditions which he prescribed involving: Age embrittlement subsequent to plastic flow, multiaxial stress conditions, low temperatures, welding, and shock. What a collection of unsolved problems! If the writer had authoritative answers to all of the problems involved in Mr.

---

[38] *Journal*, Am. Welding Soc., July, 1943, p. 317-s.

Plummer's question, he would not have written this paper. As stated by Mr. Plummer, the changes that are taking place in structural engineering practice are so great that it is not safe to assume that old practices necessarily apply to new conditions. Moreover, the impatience of modern times makes it necessary to learn in the laboratory in a few years what, in the past, was learned to a considerable extent by experience over a long period of years.

The last paragraph of Mr. Eremin's discussion reads as follows:

> "In conclusion (2), Professor Wilson states that the plastic deformation permits the redistribution of uneven localized stresses. From Fig. 2, it is evident that the plastic deformation at the reduced sections, where the localized stresses occur, is considerably reduced. Therefore, its effect on the redistribution of stresses is negligible."

This is the action which the writer had in mind in the discussion of combined deformation stresses and load stress in which he stated (see heading, "Why Is Ductility Necessary: Errors in Workmanship (Item (3))"),

> "Likewise, a high, combined, load-and-deformation stress cannot be accepted in a tension member that has a reduced section at a point of maximum stress."

In discussing fatigue, Mr. Eremin states,

> "From Fig. 4 it is obvious that the fatigue effect on stress in the members may be estimated and considered as a special type of loading. Therefore, it is possible to design the structure for a combined loading including the fatigue effect."

It is the opinion of the writer that there are two distinct steps in the design of structural members subjected to fatigue. The allowable static stress has one value, which is constant for a given material. The allowable fatigue stress usually differs from the allowable static stress. Moreover, it is different for different types of members fabricated of the same steel and is different for different members in the same bridge due to the fact that some members are subjected to more stress cycles than other members of the same bridge. (Hip verticals are more susceptible to fatigue than end posts.) Likewise, the maximum stress for which the member is designed would be different in designing for static load than it would be in designing for fatigue. In designing a member for static load, the maximum possible stress that could be produced by the design load should be used. In designing for fatigue, only the stress that is likely to occur frequently should be considered. To illustrate, consider a two-truss double-track bridge. There is a possibility that the two tracks will be loaded simultaneously so, in designing for static loads, both tracks should be considered loaded; but, because for most bridges the simultaneous loading of both tracks would occur at very infrequent intervals, only one track should be considered loaded in checking for fatigue. Of course, the empirical methods of designing for reversed stresses now in use do not take into account these possibilities, but it should be borne in mind that these empirical rules were written when there was relatively little known about the fatigue strength of structural members, as distinguished from the fatigue strength of small machined specimens. Tests have recently been made to determine the fatigue strength of structural members,[39] which probably will result in some revision of the paragraph in the specifications which controls the design of structural members subjected to fatigue. Curves like the one shown in Fig. 4 are needed for the types of members fabricated of various materials. The data for some of the curves are available, but a large amount of testing remains to be done.

The best method of designing structural members subjected to reversals and repetitions of stress has yet to be determined. The number of structural members seriously affected by fatigue is relatively small. The design load is not the same for fatigue as for static stresses. The ratio of the allowable fatigue to allowable static stress is not the same for low alloys as for carbon steel; it is not the same for all types of members; and it is not the same for all members of the same structure.[40] It would seem to the writer, therefore, that the best way to design is to proportion the members for the static loads (dead load, live load, and impact), and then check those members subjected to repeated or reversed loads for the possibility of a fatigue failure.

The writer believes that the paper, together with the discussions which it has evoked, has served a useful purpose in that it has:

1. Emphasized the manner in which the ductility of the material redistributes the stress when static stresses reach the yield point in regions of stress concentration;
2. Pointed out conditions under which yield-point stresses are and are not dangerous;
3. Called attention to the fact that the final aim sought, large shock-absorbing capacity of a member, depends upon the geometrical characteristics of the member as well as upon the physical properties of the material;
4. Indicated the possibilities of developing new types of structural members with geometrical characteristics which will result in a large shock-absorbing capacity even though the material has less ductility than has generally been considered necessary; and
5. Enumerated research projects which might lessen the risk that would otherwise accompany a too rapid adoption of new materials and new fabricating processes.

---

[39] *Bulletins Nos. 302, 310, 317, 327,* and *344,* Univ. of Illinois Eng. Experiment Station, Urbana.

[40] *Bulletin,* A.R.E.A., Vol. 45, No. 440, p. 25.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2212

PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING

BY MERRILL BERNARD,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. EDGAR DOW GILMAN, CLARENCE S. JARVIS, IVAN E. HOUK, W. G. HOYT, L. R. BEARD, ROBERT E. KENNEDY, L. K. SHERMAN, C. H. EIFFERT, GAIL A. HATHAWAY, L. STANDISH HALL, AND MERRILL BERNARD.

SYNOPSIS

This paper undertakes to report on certain progressive developments in applied hydrology, now a recognized field of engineering, and the science of meteorology, heretofore primarily associated with the prediction of weather and with the generalized influences of weather on the climatic régimes of a region.

Inadequacy in the quality, extent, and length of precipitation records has given incentive and justification to the development of more rational design techniques so that the flood to be accommodated by a structure may bear a direct relation to type and magnitude of the storm that will ultimately produce it.

Precipitation data for approximately 350 of the nearly 1,300 storms of record have been reviewed, the analysis beginning with the rainfall observers' notes. Of this number about 150 are presented in the paper, the tabulations including representative time-area-depth values.

The principal meteorological elements are defined. The distribution of moisture in the atmosphere and the dynamic action between air masses are discussed in sufficient detail to give basic understanding of the mechanism of storms.

Modern methods of water control and use have demanded a means for quantitative expression of the physical upper limits of storm rainfall. Efforts to meet this need are described, and although the studies which are the basis for this paper have been successful in achieving this end, they have also disclosed weaknesses in long-established techniques, inadequacies in quality and the extent of hydrometeorological data, and the need to readjust common concepts of flood-storm relationships.

It is the purpose of this paper to report on these relative successes and failures.

NOTE.—Published in January, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was published in *Proceedings*.

[1] Hydrologic Director, U. S. Weather Bureau, Washington, D. C.

THE HYDROLOGIC CYCLE FUNCTIONALIZED

The march of events that marks the progress of an elemental particle of water from the sea surface into the atmosphere, to the land, and back to the sea is known as the hydrologic cycle. In its passage two major influences are exerted—heat and the gravitational pull of the earth.

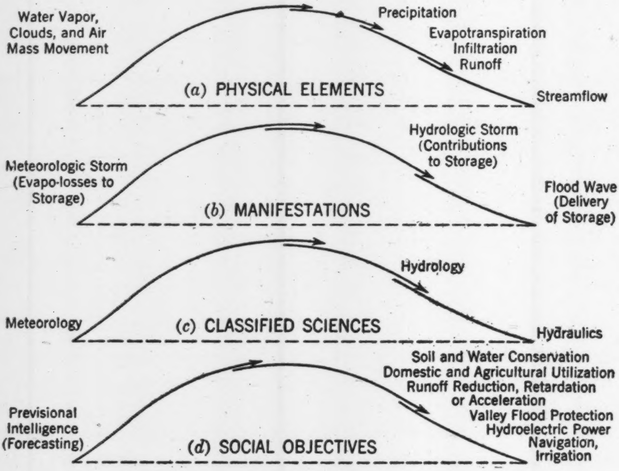

Water Vapor, Clouds, and Air Mass Movement
Precipitation
Evapotranspiration
Infiltration
Runoff
(a) PHYSICAL ELEMENTS
Streamflow
Meteorologic Storm (Evapo-losses to Storage)
Hydrologic Storm (Contributions to Storage)
(b) MANIFESTATIONS
Flood Wave (Delivery of Storage)
Meteorology
Hydrology
(c) CLASSIFIED SCIENCES
Hydraulics
Previsional Intelligence (Forecasting)
Soil and Water Conservation
Domestic and Agricultural Utilization
Runoff Reduction, Retardation or Acceleration
Valley Flood Protection
Hydroelectric Power
Navigation, Irrigation
(d) SOCIAL OBJECTIVES

FIG. 1.—THE HYDROLOGIC CYCLE FUNCTIONALIZED

Heat emanating from the sun is absorbed by the earth's surface and radiated back into the masses of free air moving over this surface. Heat then becomes the basic cause of turbulence in the air layers coming into contact with ground, vegetal, and water surfaces. Thus, through convectional displacement, repeated opportunity is provided for the absorption and removal of evaporated moisture.

The hydrologic cycle is not completely represented in the usually accepted sequence—from sea to land to sea. A particle of water evaporated from an ocean surface may be deposited very quickly at sea as part of a raindrop and so may never reach the land. Another particle may be taken into an air mass that moves to and over an extensive land area and out to sea again before meeting the influences necessary to convert it from vapor to liquid. Still another representative element in the train of moving water vapor may be deposited as rain or snow on the land, retaken into the atmosphere by evaporation, and transported back to the sea by air.

Much of the transitional water of the earth follows the more typical cycle that deposits the raindrop on the land surface, where it joins the runoff in overland flow, and enters a stream that ultimately flows into the sea. A modified cycle is that in which the raindrop enters the ground and reaches the stream later, through orifices below the flow line of its channel, whereas still another drop may continue downward under the action of gravity to some deep ground-water reservoir from which it may ultimately reach a stream or percolate into the sea.

If all the moisture in the atmosphere were suddenly precipitated, there would be only enough to produce an average depth of about 1 in. of water over the total surface of the earth. What, then, accounts for the fact that rainfalls of more than 20 in. in 24 hr are experienced in many parts of the world? The answer is given by the meteorologist who observes and analyzes the factors known to bring about the convergence of vast air masses heavily burdened with moisture.

Fig. 1 has been prepared to show how the meteorologic, hydrologic, and hydraulic phases of the water cycle are related functionally as well as physically. Inescapable overlapping of interest is apparent and the need for coordinated effort becomes obvious.

It is believed that meteorology must play an increasingly important rôle in engineering planning. Although the science is not truly "exact," the meteorologist, nevertheless, deals with a medium for which well-established physical laws are known. His greatest limitation lies in the impossibility of sampling the vast extent of the atmosphere in a manner which even approaches statistical exactness.

INADEQUACY OF FLOOD DISCHARGE RECORDS

It would have been difficult before May, 1935, to convince the residents of "Wakefield, Kans., that the discharge in the Republican River could have greatly exceeded 70,000 cu ft per sec. The maximum flood flow of record had increased from 37,500 to 47,500 cu ft per sec in 1903, and to 70,000 cu ft per sec in 1915, but for the next twenty years the river failed to reach or exceed the 1915 crest discharge. In May, 1935, the Republic River Basin was visited by a storm which produced a flood peak estimated at 180,000 cu ft per sec. The prevailing maximum-flood discharge, as it progressed throughout

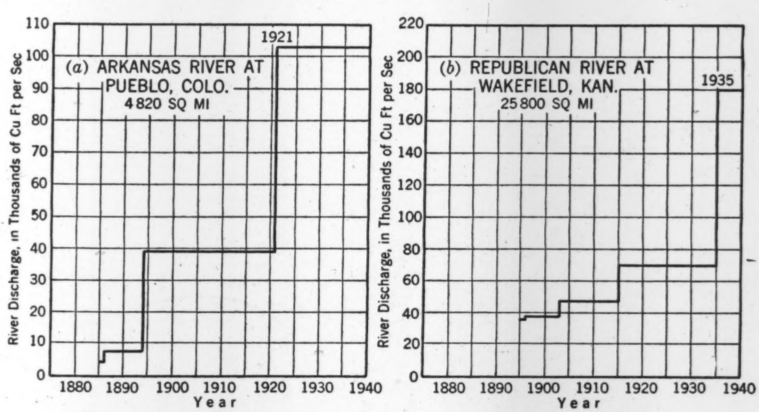
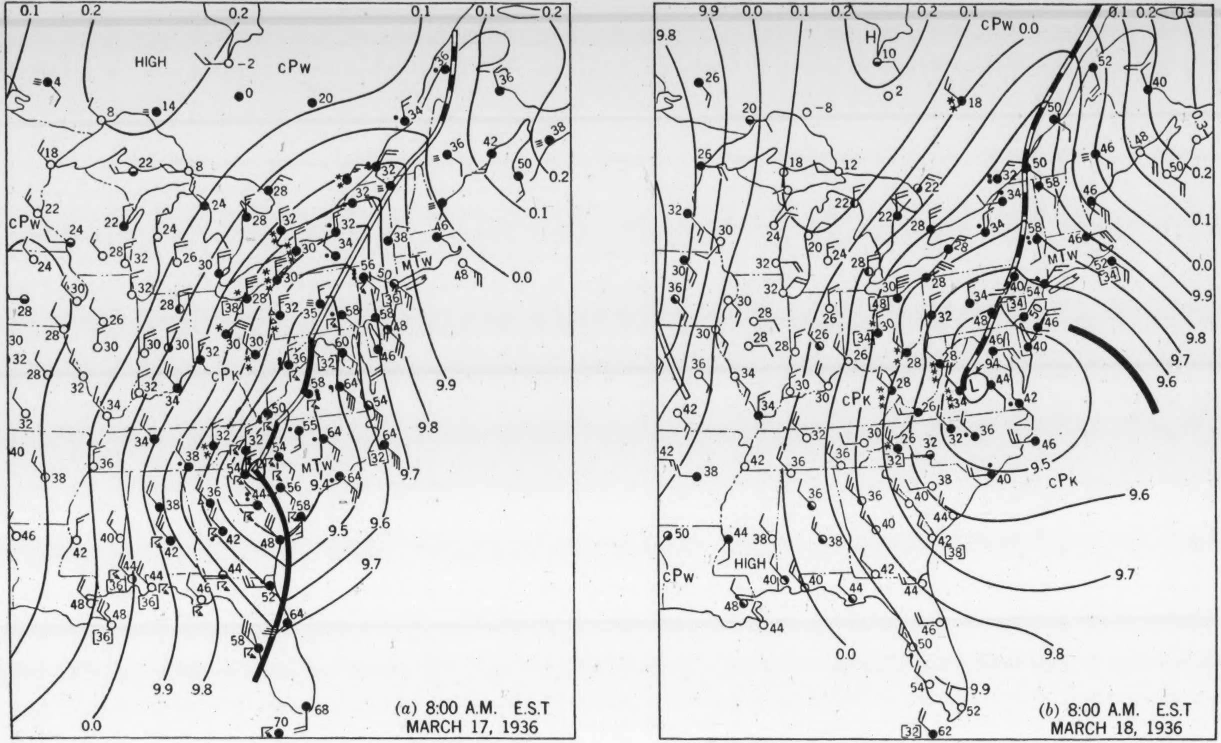


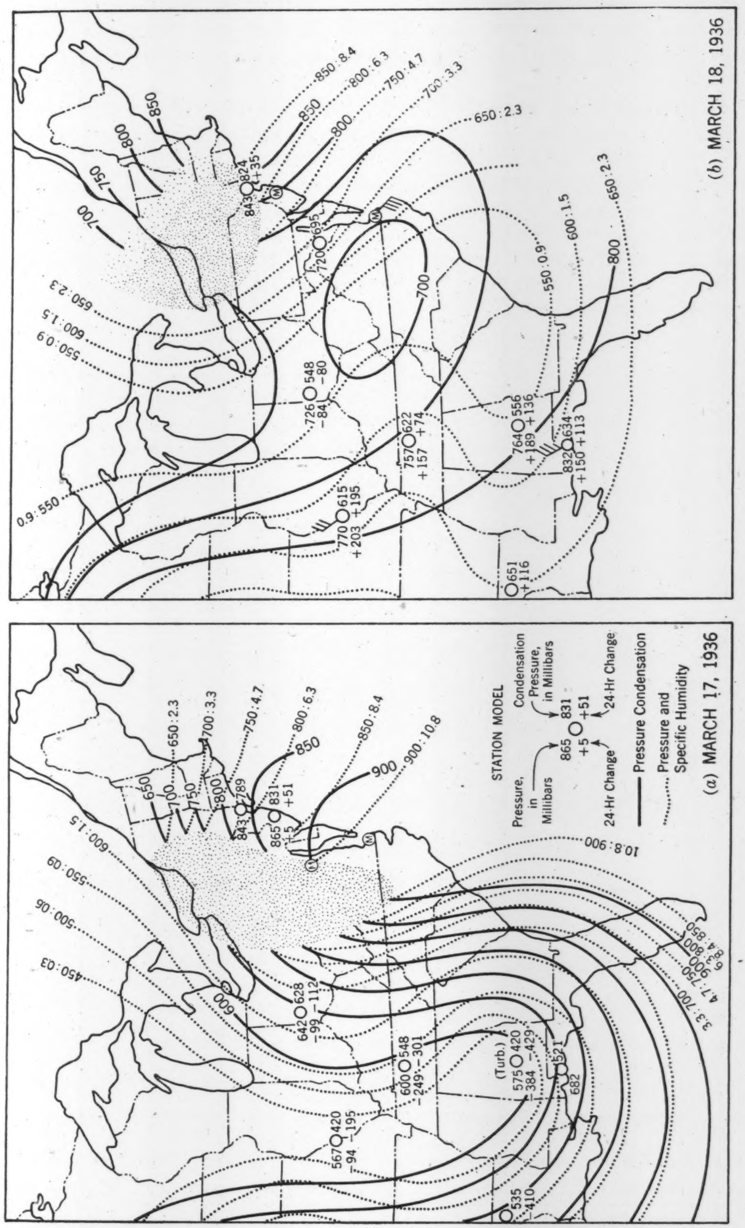
FIG. 2.—MAXIMUM FLOOD OF RECORD, TO AND INCLUDING ANY YEAR


HIGH
cPw
cPk
MTw
9.4
9.5
9.6
9.7
9.8
9.9
0.0
0.1
0.2
H
L
HIGH
cPw
cPk
MTw
0.3

(*a*) 8:00 A.M. E.S.T MARCH 17, 1936

(*b*) 8:00 A.M. E.S.T MARCH 18, 1936

Fig. 3.—Synoptic Surface Weather Maps for a 24-Hr Period


STATION MODEL
Pressure, in Millibars
Condensation Pressure, in Millibars
865
831
+5
+51
24-Hr Change
24-Hr Change
Pressure Condensation
Pressure and Specific Humidity
650:2.3
700:3.3
750:4.7
800:6.3
850:8.4
900:10.8
600:1.5
550:0.9
500:0.6
450:0.3
850
900
600
(Turb.)
850:8.4
800:6.3
750:4.7
700:3.3
650:2.3
700
750
800
550:0.9
600:1.5
0.9:550

(a) MARCH 17, 1936

(b) MARCH 18, 1936

Fig. 4.—Isentropic Charts for a 24-Hr Period; 8:00 A.M. (E.S.T.)

the period of record at Wakefield, and a similar relation between increasing record length and maximum-flood discharge at Pueblo, Colo., on the Arkansas River, is shown in Fig. 2.

The City of Wakefield will have established 100 years of record of stream flow in the year 1995. As this record progresses the prevailing maximum flood will approach the asymptotic limit represented by a flood which will result from the most critical combination of maximum-storm rainfall, snowmelt contribution, and minimum losses to storm runoff. The insecurity in estimating maximum-flood discharge by extrapolating the curves in Fig. 2 accounts largely for the rationalized design methods developed by the Engineer Department of the Corps of Engineers, and others bearing a responsibility for flood-control design.

INADEQUACY OF PRECIPITATION RECORDS

A number of the larger cities in the United States have records covering approximately 100 years. These records are valuable in statistical studies relating rainfall intensity to duration, but are of limited value in problems involving the areal distribution of flood-producing rainfall over a basin. Only a few of the historical storms, that of August 5, 1843, in New Jersey and eastern Pennsylvania being an example, are sufficiently represented by data to permit the plotting of isohyetal maps. In the main, it has been necessary to make best use of the record period beginning about 1892, a conclusion also reached by the Miami Conservancy District in its classic work, "Storm Rainfall of Eastern United States" (1).[2]

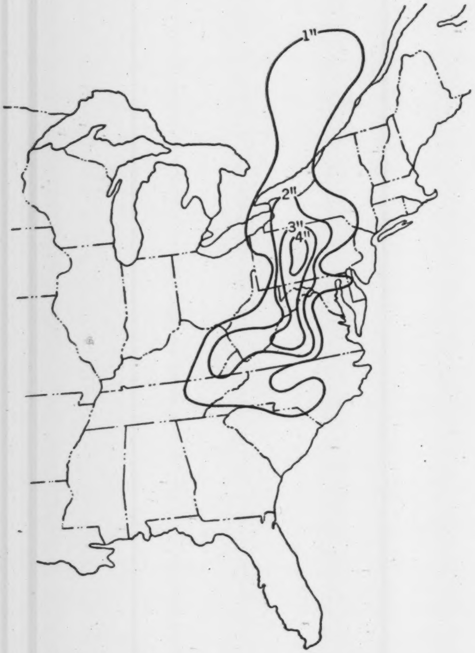


FIG. 5.—ISOHYETAL MAP FOR THE 24-HR PERIOD ENDING 8:00 A.M. (E.S.T.), MARCH 18, 1936

This paper will not undertake to review the development of flood-control planning. It is sufficient to state that early attempts in utilizing recorded storm rainfall as a guide to the measure of future storms failed in answering the questions: "Could this storm, having occurred here as recorded, been greater in intensity, duration and extent?" or, "Could this storm, having occurred here, be considered as representative of one which will ultimately center over a particular basin located outside its limits?" The meteorologist, whose field embraces the causes of precipitation, has been called upon to consider these questions.

---

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix 1).

THE STORM AS THE DESIGN UNIT

Storm rainfall can be converted into flood flow in a rational and realistic manner. All of these rational methods require that rainfall be expressed in terms of average depth occurring within a fixed period of time, distributed over an area of known dimensions and pattern. Only rainfall recorded concurrently can become the basis for such analysis, a requirement seldom fulfilled by the older records which often fail to disclose the actual period during which the rain fell. It can be seen that if stations "A" and "B" are sufficiently close they will produce practically identical records of rainfall. As they are separated by increasingly greater distances the record at "B" becomes less and less dependent upon that at "A" until the records are no longer related and, therefore, cannot be combined even when statistically identical (4).

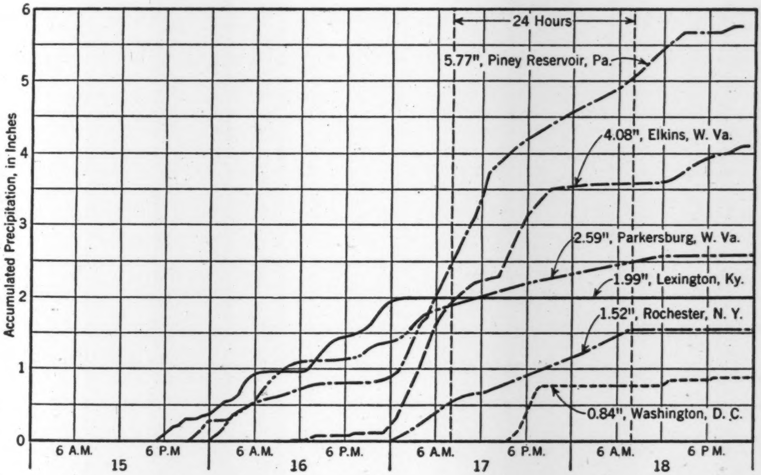


FIG. 6.—MASS CURVES OF RAINFALL FOR MARCH, 1936

Thus far the writer has treated only the "hydrologic storm," shown graphically as an isohyetal map and usually limited in total extent to areas less than 200,000 sq miles. The meteorologist, quite naturally, is interested in the hydrologic storm as it expresses the results of his analysis. However, he has been more directly interested in what will be referred to in this paper as the "meteorologic storm," a three-dimensional phenomenon of considerably greater extent than the hydrologic, or "rain" storm. His medium is the earth's atmospheric envelope—the field of action being the greater part of the northern

TABLE 1.—Area-Duration-Depth

Region	(a) North											
Locality	Pa., N. J., N. Y., and Conn.				Northeastern States				Eastern Pa. and N. J.			
Center	Smith's Corner, Pa.				Jewell, Md.				Camden, N. J.			
Date	September 18 to 20, 1894				July 26 to 29, 1897				August 3 to 5, 1898			
Area (sq miles)	100	1,000	10,000	40,190	100	500	2,500	26,241	100	500	5,000	30,600
Average Depth (In.):												
12 hours	4.9	4.0	2.6	1.6	12.9	8.9	4.1	1.9	4.9	4.5	3.2	2.4
24 hours	6.9	5.5	3.5	2.2	13.0	9.0	4.4	2.2	5.0	4.6	3.4	2.6
36 hours	7.7	6.3	4.4	2.9	13.8	9.6	4.6	2.5	6.2	6.1	4.6	3.6
48 hours	8.1	6.8	4.9	3.4	....	....	....	....	6.2	6.1	5.0	4.0
72 hours	....	....	....	....	....	....	....	....	....	....	....	....
Total Storm:												
Area	40,190 sq miles				26,241 sq miles				30,600 sq miles			
Duration	48 hours				96 hours				54 hours			
Depth	3.4 in. (average)				3.3 in. (average)				4.0 in. (average)			

TABLE 1.—Area-Duration-Depth

Region	(a) North Atlantic											
Locality	New England			New England			No. Atlantic States			New England		
Center	Somerset, Vt.			York, Pa.			Scituate and Westerly, R. I.			Provincetown, Mass.		
Date	November 2 to 7, 1927			August 20 to 24, 1933			September 16 to 17, 1932			September 14 to 18, 1933		
Area (sq miles)	2,500	25,000	100,000	100	500	2,000	500	2,500	5,000	500	2,500	10,000
Average Depth (In.):												
12 hours	5.8	2.6	1.4	11.3	9.4	7.8	....	....	....	2.4	2.1	2.0
24 hours	7.4	4.2	2.6	13.2	11.1	9.3	7.9	6.4	6.0	3.6	3.3	2.9
36 hours	8.4	5.1	3.3	13.2	11.1	9.3	....	....	....	5.6	4.9	3.9
48 hours	....	5.8	3.5	....	11.0	9.2	9.3	7.8	7.3	6.6	6.0	4.9
72 hours	....	....	....	....	....	....	10.8	8.7	8.1	9.7	8.8	7.3
Total Storm:												
Area	100,000 sq miles			5,000 sq miles			10,000 sq miles			10,000 sq miles		
Duration	48 hours			120 hours			48 hours			96 hours		
Depth	3.5 in. (average)			10.1 in. (average)			7.5 in. (average)			8.9 in. (average)		

TABLE 1.—Area-Duration-Depth

Region	(a) North Atlantic Region (*Continued*)						(b) South							
Locality	Pa., N. J., and Del.			New Jersey			Virginia and West Virginia				Central Maryland			
Center	New Jersey			Ewan, N. J.			Bloomery, W. Va.				Baltimore, Md.			
Date	August 19, 1939			September 1, 1940			September 28 to 30, 1896				July 12, 1903			
Area (sq miles)	100	1,000	10,000	100	500	2,500	500	2,500	10,000	54,467	100	500	2,500	4,280
Average Depth (In.):														
12 hours	15.8	11.3	4.6	18.4	13.1	7.4	5.5	5.1	4.4	2.6	...	...	....	....
24 hours	...	...	....	...	...	....	6.9	6.6	5.8	3.5	...	...	....	....
36 hours	...	...	....	...	...	....	...	...	....	3.7	...	...	....	....
48 hours	...	...	....	...	...	....	...	...	....	....	...	...	....	....
72 hours	...	...	....	...	...	....	...	...	....	....	...	...	....	....
Total Storm:														
Area	10,000 sq miles			2,500 sq miles			54,467 sq miles				4,280 sq miles			
Duration	18 hours			12 hours			120 hours				8 hours			
Depth	5.4 in. (average)			7.4 in. (average)			3.8 in. (average)				2.2 in. (average)			

Values for Selected Storms

Atlantic Region

Md., Del., Pa., N.J., and N.Y. Colora, Md. September 24 to 27, 1902				Pa., N. J., and N. Y. Dover, N. J. June 7 to 15, 1903			New Jersey Paterson, N. J. October 7 to 11, 1903				Atlantic States New Jersey July 18 to 23, 1919			
500	2,500	10,000	37,980	500	2,500	13,663	100	500	2,500	5,000	500	2,000	10,000	25,000
5.0	3.3	1.9	1.0	4.0	3.4	2.3	8.7	7.2	5.8	5.2	2.5	2.0	1.4	1.0
6.2	4.7	3.2	1.8	6.2	5.2	3.4	14.6	12.2	10.1	8.7	3.7	2.8	1.9	1.4
7.0	5.6	4.0	2.6	6.3	5.2	3.5	15.2	13.2	10.9	9.7	3.7	3.1	2.5	2.1
7.6	6.3	4.8	3.3	6.4	5.3	3.5	15.3	13.2	....	....	4.4	4.3	3.0	2.4
7.6	6.3	5.1	3.6	6.6	5.6	3.7	....	....	....	....	6.1	5.8	4.1	3.3
37,980 sq miles 72 hours 3.6 in. (average)				13,663 sq miles 216 hours 5.8 in. (average)			2,000 to 6,000 sq miles 48 hours				25,000 sq miles 120 hours 5.6 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)

Southern New York Hector, N. Y. July 6 to 10, 1935			New Hampshire Pinkham Notch, N. H. March 9 to 15, 1936			New England Pinkham Notch, N. H. March 17 to 19, 1936			New England Providence, R. I. December 19 to 21, 1936			Northeastern States Barre, Mass. September 17 to 21, 1938		
100	500	5,000	100	500	2,000	100	500	2,500	500	2,500	25,000	500	2,500	5,000
9.2	7.5	5.0	5.0	4.0	3.0	3.2	2.7	2.0	2.6	2.3	1.7	6.8	5.0	....
11.1	9.4	6.4	7.1	5.8	4.4	6.7	5.1	3.5	2.7	2.4	2.0	9.6	7.2	6.2
11.2	9.7	7.0	8.5	6.9	5.4	8.9	6.8	4.7	....	....	....	10.9	9.0	8.3
....	....	....	9.0	7.2	5.5	10.8	8.0	5.2	....	....	....	11.7	10.0	9.1
....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
5,000 sq miles 36 hours 7.0 in. (average)			2,000 sq miles 48 hours 5.5 in. (average)			2,500 sq miles 48 hours 5.2 in. (average)			50,000 sq miles 24 hours 1.8 in. (average)			40,000 sq miles 108 hours 8.3 in. (average)		

Values for Selected Storms (*Continued*)

Atlantic Region

S. E. No. Carolina New Bern, N. C. July 29 to 31, 1908			Eastern Pennsylvania Gordon, Pa. August 21 to 22, 1915				Florida Peninsula St. Petersburg, Fla. August 1 to 3, 1915			The Carolinas Altapass, N. C. July 13 to 17, 1916			
100	500	2,500	100	500	2,500	5,310	100	500	2,500	500	2,000	10,000	3,7000
3.7	3.6	3.4	4.7	4.5	3.8	3.1	13.6	10.6	6.8	10.5	9.1	5.5	2.2
6.0	5.7	5.1	....	....	....	....	14.3	11.8	9.1	16.6	13.5	8.6	3.8
7.2	6.8	6.2	....	....	....	....	14.5	12.5	10.0	18.8	15.5	9.9	5.6
7.9	7.6	7.1	....	....	....	....	15.2	13.2	10.5	19.5	16.3	10.6	7.0
....	....	....	....	....	....	....	....	....	....	20.1	16.8	11.0	7.8
15,000 sq miles 54 hours 5.9 in. (average)			5,310 sq miles 15 hours 3.2 in. (average)				10,000 sq miles 66 hours 8.3 in. (average)			37,000 sq miles 108 hours 8.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(b) SOUTH ATLANTIC											
Locality	S. E. No. Carolina			Md., Pa., and Va.				Florida				
Center	Hatteras, N. C.			Baltimore, Md.				New Smyrna, Fla.				
Date	September 14 to 16, 1917			October 9 to 10, 1922				October 4 to 12, 1924				
Area (sq miles)	500	2,500	10,000	100	500	2,500	12,350	100	500	2,500	5,000	8,579
Average Depth (In.):												
12 hours	5.0	4.4	3.2	4.4	3.6	2.5	1.7	18.3	14.8	8.6	5.6	3.2
24 hours	6.0	5.3	4.6	4.6	3.8	3.0	2.1	22.0	18.1	11.3	8.0	5.3
36 hours	7.0	6.4	5.6	....	....	....	....	23.4	19.4	12.7	9.5	7.1
48 hours	....	....	....	....	....	....	....	24.1	20.2	13.6	10.6	8.2
72 hours	....	....	....	....	....	....	....	24.7	21.1	14.6	11.7	9.3
Total Storm:												
Area	10,000 sq miles			12,350 sq miles				8,579 sq miles				
Duration	42 hours			27 hours				216 hours				
Depth	5.9 in. (average)			2.2 in. (average)				13.1 in. (average)				

TABLE 1.—AREA-DURATION-DEPTH

Region	(c) OHIO RIVER													
Locality	N. Ind. and Ohio, W. Pa., and Northern W. Va.				S. Ill., Ind., and Kentucky			Arkansas to Maine			Central West Virginia			
Center	Johnstown, Pa.				Fairfield, Ill.			Richmond, Ind.			Holcomb, W. Va.			
Date	June 14 to 18, 1912				June 14 to 18, 1912			March 23 to 27, 1913			March 10 to 15, 1918			
Area (sq miles)	500	2,500	25,000	49,330	500	2,500	50,000	500	2,000	25,000	500	2,500	10,000	17,495
Average Depth (In.):														
12 hours	3.1	2.5	1.4	1.0	3.0	2.5	0.8	4.7	4.1	2.8	2.9	2.6	2.1	1.8
24 hours	4.0	3.6	2.3	1.9	4.1	3.5	1.6	6.7	6.2	3.9	4.3	3.8	2.9	2.5
36 hours	4.2	3.8	2.6	2.2	4.2	3.6	1.9	7.6	7.0	4.7	4.7	4.2	3.2	2.7
48 hours	4.8	4.2	3.0	2.6	4.5	4.1	2.4	8.6	7.9	6.0	5.2	4.5	3.4	3.0
72 hours	...	....	....	2.8	4.8	4.3	2.7	9.8	9.0	7.1	5.8[a]	5.0[a]	3.8[a]	3.3[a]
Total Storm:											[a] Total storm duration, 66 hours			
Area	49,330 sq miles				62,775 sq miles			25,000 sq miles			17,495 sq miles			
Duration	120 hours				108 hours			96 hours			66 hours			
Depth	2.9 in. (average)				3.1 in. (average)			7.9 in. (average)			3.3 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(c) OHIO RIVER												
Locality	Ind., Ky., N. Y., Ohio, Pa., and W. Va.				Miss., Ala., Tenn., Ky., W.Va., and Ga.			Ky., Ill., Mo., Ind., Ohio, and Tenn.			Near East Central Ohio		
Center	Various centers				Rock Island, Tenn.			Earlington, Ky.			Near Keene, Ohio		
Date	September 1 to 4, 1922				March 21 to 24, 1929			June 21 to 22, 1935			August 6 to 7, 1935		
Area (sq miles)	500	2,500	10,000	22,972	500	2,500	25,000	500	2,500	25,000	500	2,500	19,297
Average Depth (In.):													
12 hours	4.8	3.4	2.2	1.4	8.1	7.0	4.1	7.6	6.0	2.3	8.7	6.4	2.9
24 hours	4.8	3.6	2.3	1.6	9.3	8.2	5.3	9.2	7.5	3.6	9.1	6.8	3.4
36 hours	4.8	3.7	2.9	2.4	9.6	8.6	5.6	....	....	....	....	....	3.4
48 hours	4.8	3.7	3.1	2.8	9.8	8.8	5.9	....	....	....	9.1	6.8	3.4
72 hours	4.9	3.9	3.3	2.9	....	....	....	....	....	....	....	....	....
Total Storm:													
Area	22,972 sq miles				122,070 sq miles			41,000 sq miles			19,297 sq miles		
Duration	84 hours				54 hours			42 hours			48 hours		
Depth	2.9 in. (average)				3.9 in. (average)			3.5 in. (average)			....		

Values for Selected Storms (*Continued*)

Region (*Continued*)											(*c*) **Ohio River Region**			
Alabama and Georgia Glenville, Ga. September 23 to 28, 1929				Fla., Ga., N. C., and S. C. Vernon, Fla. September 29 to October 3, 1929			South Atlantic States Keysville, Va., and Swansboro, N. C. August 10 to 17, 1940				Arkansas to Ohio Golconda, Ill. October 5 to 7, 1910			
500	2,500	50,000	100,000	500	5,000	50,000	500	2,500	23,536		500	2,500	25,000	75,000
11.6	9.6	1.9	1.0	7.4	5.7	3.1	7.8	6.2	2.7		5.7	4.4	3.4	2.6
13.9	11.6	3.8	2.4	9.7	8.0	5.3	10.4	9.0	4.6		8.2	6.8	6.0	4.8
15.8	13.6	5.2	3.1	13.0	10.2	7.2	10.6	9.7	6.1		10.1	8.5	6.7	5.5
17.3	14.9	6.4	3.9	....	10.5	7.9	14.7	12.5	7.4		12.7	11.0	8.5	6.2
18.5	16.2	7.0	4.4	....	....	....	....	....	....		14.5	13.2	10.4	7.8
100,000 sq miles 120 hours 5.4 in. (average)				70,000 sq miles 60 hours 7.7 in. (average)			170,000 sq miles 186 hours 6.4 in. (average)				75,000 sq miles 90 hours 7.8 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)										
Louisiana to Illinois Perryville, Tenn. March 14 to 17, 1919				Texas to Ohio Rolla, Mo. October 26 to 30, 1919			Western Virginia and North Carolina Marion, N. C. October 29 to November 2, 1921			
500	2,500	25,000	100,000	500	2,500	50,000	500	2,500	10,000	24,274
....	....	....	....	....	....	....	4.4	3.1	1.9	1.1
8.1	6.0	....	....	5.8	4.9	....	5.0	3.6	2.7	2.0
....	....	....	....	....	....	....	5.1	4.0	3.4	2.5
10.0	8.9	6.4	4.5	9.5	8.9	5.2	6.1	4.8	4.3	3.5
10.6	9.6	6.7	4.8	10.4	9.6	6.0	6.4	5.1	4.7	3.9
100,000 sq miles 96 hours 4.9 in. (average)				100,000 sq miles 120 hours 5.2 in. (average)			24,274 sq miles 96 hours 4.0 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)												
Ohio Newcomerstown, Ohio August 6 to 7, 1935			Texas to Pennsylvania Taylorsville, Ky. January 20 to 25, 1937				Arkansas to Ohio Fords Ferry, Ky. March 28 to 31, 1938			Kentucky Simpson Post Office, Ky. July 4 to 5, 1939		
100	500	2,000	500	2,500	25,000	100,000	500	2,500	10,000	100	500	5,000
10.4	8.7	6.4	3.6	3.2	2.4	2.0	4.4	3.4	2.4	14.4	10.5	5.2
....	....	....	5.9	5.3	3.9	3.3	6.7	5.4	3.6	....	....	....
10.9	9.1	6.7	8.0	7.1	5.6	4.8	6.7	....	....	....	....	....
....	....	....	9.2	8.6	6.9	5.6	6.7	5.6	4.2	....	....	....
....	....	....	11.4	10.5	8.7	7.3	8.0	7.2	5.9	....	....	....
2,000 sq miles 36 hours 6.7 in. (average)			100,000 sq miles 144 hours 9.7 in. (average)				25,000 sq miles 72 hours 4.5 in. (average)			5,000 sq miles 12 hours 5.2 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(d) GREAT LAKES										
Locality Center Date	N. Central Wisconsin Butternut, Wis. July 25 to 27, 1897			Northwestern Wisconsin Medford, Wis., and Hart, Mich. June 3 to 8, 1905				Wisconsin and Michigan Merrill, Wis. July 20 to 24, 1912			
Area (sq miles)	500	2,500	10,000	500	2,500	25,000	100,000	500	2,500	25,000	100,000
Average Depth (In.):											
12 hours	5.8	5.1	3.7	7.0	5.3	2.8	1.3	6.2	5.1	2.2	0.6
24 hours	9.0	7.8	5.3	7.6	6.2	3.9	1.8	10.2	7.7	3.7	1.2
36 hours	....	....	....	8.6	7.2	4.8	2.3	....	7.9	4.0	1.4
48 hours	....	....	....	9.3	7.8	5.0	2.5	....	7.9	4.0	1.4
72 hours	....	....	....	9.9	8.4	5.8	3.1	....	7.9	4.0	1.4
Total Storm:											
Area	15,000 sq miles			100,000 sq miles				100,000 sq miles			
Duration	66 hours			120 hours				120 hours			
Depth	4.7 in. (average)			3.4 in. (average)				3.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(d) GREAT LAKES REGION										
Locality	Central New York				New York and New England				New York		
Center Date	Syracuse, N. Y. June 9 to 12, 1922				Severance, N. Y. September 11 to 16, 1925				Philadelphia, N. Y. October 17 to 20, 1930		
Area (sq miles)	500	2,500	10,000	20,371	500	2.500	10,000	13,825	100	500	1,000
Average Depth (In.):											
12 hours	4.2	3.5	2.1	1.7	2.1	1.9	1.6	1.4	4.2	3.1	2.4
24 hours	4.3	3.8	2.7	2.3	2.6	2.3	1.8	1.6	5.4	4.3	3.3
36 hours	4.9	4.0	2.8	2.5	4.2	3.4	2.4	2.0	5.7	4.5	3.6
48 hours	5.0	4.2	3.1	2.7	4.2	3.5	2.7	2.5	5.7	4.7	3.9
72 hours	5.2	4.4	3.4	3.0	....	....	....	2.7	....	....	....
Total Storm:											
Area	20,371 sq miles				13,825 sq miles				2,560 sq miles		
Duration	84 hours				126 hours				48 hours		
Depth	3.0 in. (average)				4.1 in. (average)				2.6 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(e) UPPER MISSISSIPPI								
Locality	Minnesota and Wisconsin				N. W. Minnesota and Eastern North Dakota				E. Kansas and S. E. Nebraska
Center Date	Pine River Dam, Minn. June 2 to 6, 1898				New Folden, Minn. July 1 to 6, 1901				Salina, Kans. May 28 to 31, 1903
Area (sq miles)	100	1,000	5,000	28,877	500	2,500	25,000	51,170	2,000
Average Depth (In.):									
12 hours	5.4	4.7	3.2	1.4	5.4	4.3	1.5	1.2	....
24 hours	6.7	5.7	3.7	2.0	6.7	5.0	2.2	1.7	5.4
36 hours	7.3	6.3	4.4	2.3	7.0	5.2	2.4	2.0	....
48 hours	8.0	6.9	5.1	2.9	7.0	5.2	2.6	2.1	....
72 hours	8.7	7.5	5.5	3.2	8.9	6.4	3.6	3.1	....
Total Storm:									
Area	28,877 sq miles				51,170 sq miles				2,000 sq miles
Duration	102 hours				108 hours				24 hours
Depth	3.6 in. (average)				3.1 in. (average)				5.4 in. (average)

VALUES FOR SELECTED STORMS (*Continued*)

REGION

Near Central New York Greene, N. Y. June 30 to July 5, 1915				New York and New England Mount McGregor, N. Y. July 7 to 9, 1915				Near Western New York York, N. Y. May 14 to 20, 1916			
100	500	2,500	18,021	500	2,500	10,000	17,178	500	2,500	10,000	21,351
2.6	2.3	2.0	1.1	3.0	2.8	2.2	1.8	3.5	2.7	1.9	1.5
3.6	3.5	3.1	1.9	3.7	3.5	3.1	2.8	4.1	3.3	2.5	2.1
....	....	....	2.2	....	....	....	....	4.4	3.8	2.9	2.4
....	....	....	2.2	....	....	....	....	4.8	4.1	3.1	2.6
4.5	4.3	3.8	2.7	....	....	....	....	4.9	4.2	3.5	2.9
18,021 sq miles 150 hours 3.4 in. (average)				17,178 sq miles 66 hours 2.8 in. (average)				21,351 sq miles 162 hours 3.2 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

(*Continued*)						(*e*) UPPER MISSISSIPPI VALLEY REGION							
Near Eastern New York Elks Park, N. Y. October 4 to 7, 1932			Western New York Buffalo, N. Y. June 17 to 22, 1937			S. Minnesota and Wisconsin Minneapolis, Minn. July 24 to 28, 1892				N. W. Minn. and Eastern North Dakota Lamberton, Minn. July 18 to 22, 1897			
500	2,500	28,948	100	500	9,890	500	2,500	5,000	24,360	500	2,500	25,000	100,000
5.8	4.0	2.6	4.6	3.4	1.3	6.4	4.9	3.8	1.6	4.4	3.8	2.8	1.5
7.6	5.9	4.1	5.1	3.8	1.6	6.9	5.4	4.4	2.2	6.0	5.2	3.6	2.1
8.5	6.7	4.7	5.1	3.8	1.6	7.0	5.5	4.5	2.3	....	....	....	....
9.1	7.4	5.3	5.1	3.8	1.8	7.0	5.7	4.8	2.9	6.5	5.8	4.1	2.7
9.1	7.4	5.3	5.1	3.8	1.8	7.3	5.9	5.0	3.1	7.6	6.8	4.8	3.2
28,948 sq miles 98 hours 5.3 in. (average)			9,890 sq miles 120 hours 3.4 in. (average)			24,360 sq miles 108 hours 3.4 in. (average)				100,000 sq miles 102 hours 3.4 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

VALLEY REGION (*Continued*)

Minnesota, Wisconsin, and Michigan Ironwood, Mich. July 20 to 22, 1909			Northern Minnesota and Wisconsin Beaulieu, Minn. July 18 to 23, 1909				Southern Minnesota New Ulm, Minn. July 13 to 17, 1916				West Central Illinois Galva, Ill. August 18 to 20, 1924		
500	2,500	25,000	500	2,000	20,000	60,000	100	1,000	2,500	23,700	500	2,500	25,000
5.3	4.4	2.4	10.1	8.4	2.1	1.5	7.0	5.7	4.4	0.9	5.7	4.3	1.8
7.1	5.7	3.0	10.6	8.7	2.8	1.9	7.0	5.8	4.7	1.5	7.8	6.1	2.6
10.0	7.9	4.2	11.1	9.4	4.0	2.8	7.1	5.9	4.8	1.7	....	6.3	3.2
10.5	8.4	4.6	11.1	9.5	4.7	3.1	7.1	5.9	4.9	2.0	....	....	3.7
11.5	9.0	6.0	11.4	9.8	6.4	4.0	7.2	6.0	5.1	2.2	....	....	....
60,000 sq miles 108 hours 4.4 in. (average)			60,000 sq miles 96 hours 4.4 in. (average)				23,700 sq miles 96 hours 2.4 in. (average)				28,629 sq miles 54 hours 3.6 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(*e*) UPPER MISSISSIPPI VALLEY REGION (*Continued*)			(*f*) MISSOUR			
Locality	Eastern Missouri and Western Illinois			Iowa			
Center	Crystal City, Mo.			Woodburn, Iowa			
Date	June 9 to 11, 1938			August 26 to 28, 1903			
Area (sq miles)	500	2,000	10,000	500	5,000	20,000	60,000
Average Depth (In.):							
12 hours	6.8	5.4	3.7	9.3	6.4	4.0	2.1
24 hours	7.2	5.9	4.2	11.2	7.8	5.2	2.8
36 hours	....	....	....	12.2	8.7	5.9	3.4
48 hours	....	....	....	....	....	....	....
72 hours	....	....	....	12.6[b]	8.8[b]	6.2[b]	3.8[b]
Total Storm:				[b] Total storm duration, 54 hours			
Area	10,000 sq miles			60,000 sq miles			
Duration	24 hours			96 hours			
Depth	4.2. in (average)			4.3 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(*g*) LOWER MISSISSIPPI							
Locality	Texas to New England					Southern States		
Center	Thomasville, Ala.					Elba, Ala.		
Date	June 1 to 5, 1928					March 12 to 15, 1929		
Area (sq miles)	100	500	2,500	25,000	100,000	500	2,500	50,000
Average Depth (In.):								
12 hours	9.0	8.1	7.0	4.5	2.1	13.9	9.9	2.7
24 hours	12.2	11.7	10.8	6.7	3.2	18.1	13.8	5.4
36 hours	13.9	13.4	12.4	7.8	3.8	20.5	16.8	7.2
48 hours	14.9	13.7	13.6	8.9	4.7	23.5	18.6	8.1
72 hours	15.0	14.7	13.9	10.0	5.5	25.5	21.0	....
Total Storm:								
Area	100,000 sq miles					135,000 sq miles		
Duration	108 hours					60 hours		
Depth	6.6 in. (average)					7.3 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(*h*) SOUTHWESTERN											
Locality	Texas					Oklahoma and Texas			Texas and Oklahoma			
Center	Hearne, Tex.					Chattanooga, Okla.			San Marcos, Tex.			
Date	June 26 to July 2, 1899					May 22 to 26, 1908			December 1 to 5, 1913			
Area (sq miles)	100	500	2,500	10,000	25,000	2,500	10,000	50,000	500	2,500	25,000	50,000
Average Depth (In.):												
12 hours	12.0	10.7	7.7	4.9	3.3	4.4	4.1	3.2	....	....	....	....
24 hours	22.3	18.0	12.8	8.7	5.5	6.4	5.8	4.2	8.5	6.5	4.0	....
36 hours	27.2	24.0	18.2	12.1	8.8	8.3	7.8	5.8	....	....	....	....
48 hours	28.4	25.1	20.8	14.6	10.2	....	....	....	11.2	9.2	6.3	4.8
72 hours	30.6	29.4	23.3	17.1	12.0	....	....	....	13.3	11.8	....	....
Total Storm:												
Area	40,000 sq miles					50,000 sq miles			50,000 sq miles			
Duration	72 hours					96 hours			96 hours			
Depth	9.5 in. (average)					6.7 in. (average)			7.4 in. (average)			

Values for Selected Storms (*Continued*)

River Region							(*g*) Lower Mississippi Valley Region						
Iowa, Illinois, and Indiana Bonaparte, Iowa June 9 to 10, 1905				Kansas, Missouri, Iowa, and Illinois Bethany, Mo. July 5 to 7, 1909			Louisiana Alexandria, La. June 13 to 17, 1886			Southeastern Louisiana Near Saint Francisville, La. March 20 to 22, 1926			
500	2,500	10,000	25,000	2,500	25,000	50,000	250	1,000	5,000	500	2,500	10,000	28,210
10.9	8.8	5.5	4.0	....	....	....	17.7	15.8	9.9	6.1	4.7	3.2	2.2
....	....	....	....	5.7	....	....	22.5	20.1	12.6	7.1	5.8	4.3	3.2
....	....	....	....	....	....	....	26.5	23.7	14.8	7.1	5.8	4.3	3.2
....	....	....	....	6.4	4.8	3.9	28.6	25.6	16.0	7.6	6.2	4.7	3.4
....	....	....	....	9.0	6.4	5.4	29.0	26.0	16.2	....	....	....	....
25,000 sq miles 12 hours 4.0 in. (average)				60,000 sq miles 108 hours 4.4 in. (average)			6,900 sq miles 100 hours 14.7 in. (average)			28,210 sq miles 66 hours 4.2 in. (average)			

Values for Selected Storms (*Continued*)

Valley Region (*Continued*)													
Arkansas, Louisiana, and Texas Logansport, La. July 22 to 27, 1933					Tex. to Ill. and Ky. Waldron, Ark. June 12 to 18, 1935			La., Miss., and Ark. Park, La. October 1 to 4, 1937			S. W. Louisiana Near Jennings, La. August 12 to 15, 1938		
100	500	2,000	20,000	100,000	500	2,500	50,000	500	2,500	25,000	500	2,500	25,000
11.4	10.5	8.5	4.1	0.8	6.3	5.2	1.8	8.9	7.2	2.7	10.7	8.3	2.8
16.9	15.7	13.0	6.5	1.6	7.2	6.0	2.6	11.9	10.0	4.4	13.1	10.2	3.9
18.7	17.4	15.5	8.6	2.6	7.7	6.5	3.1	13.2	11.3	5.1	....	....	....
19.4	18.5	16.9	10.9	3.4	7.8	6.9	3.7	14.4	12.0	5.3	13.7	10.8	4.7
20.6	19.7	18.4	12.4	5.6	8.4	7.6	4.9	....	....	....	....	....	....
100,000 sq miles 126 hours 7.3 in. (average)					102,450 sq miles 144 hours 5.0 in. (average)			25,000 sq miles 114 hours 5.4 in. (average)			34,210 sq miles 90 hours 4.3 in. (average)		

Values for Selected Storms (*Continued*)

Region													
Texas and Arkansas Houston, Tex. August 16 to 21, 1915				Tex., La., Ark., and Mo. Houston, Tex. August 17 to 20, 1915			Arkansas and Tennessee Jessieville, Ark. April 20 to 21, 1927			Southwestern Texas Near Kerrville, Tex. June 30 to July 2, 1932			
500	5,000	50,000	100,000	1,000	10,000	50,000	500	2,500	5,000	Point	100	2,500	10,000
6.2	4.8	2.6	1.8	6.2	4.5	....	....	....	....	21.6	18.0	10.7	6.8
9.2	7.3	4.0	3.0	8.0	6.8	4.2	10.8	8.8	7.5	35.0	30.4	19.6	11.5
13.5	10.1	5.7	4.3	....	....	....	....	....	....	....	....	....	....
16.8	13.1	7.3	5.6	15.0	10.8	7.0	....	....	....	35.6	30.8	19.6	12.1
18.6	17.1	10.1	7.8	19.5	16.2	9.8	....	....	....	....	....	....	....
100,000 sq miles 120 hours 9.1 in. (average)				225,000 sq miles 96 hours 6.4 in. (average)			5,000 sq miles 24 hours 7.5 in. (average)			10,000 sq miles 48 hours 12.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(h) SOUTHWESTERN								
Locality Center Date	North Central Oklahoma Enid, Okla. August 5 to 17, 1932			Oklahoma Near Cheyenne, Okla. April 2 to 4, 1934			Southern Texas D'Hanis, Tex. May 31, 1935		
Area (sq miles)	500	2,500	9,500	100	500	5,000	100	500	2,500
Average Depth (In.):									
12 hours	8.1	5.9	3.4	16.7	12.2	5.1	16.6[a]	11.0[a]	6.1[a]
24 hours	9.5	7.0	4.5	....	....	....	....	....	....
36 hours	9.7	8.2	4.9	....	....	....	....	....	....
48 hours	10.1	8.6	5.2	....	....	....	....	....	....
72 hours	....	....	....	....	....	....	....	....	....
Total Storm:							[a] Total storm duration, 4 hours		
Area	9,500 sq miles			5,000 sq miles			5,000 sq miles		
Duration	48 hours			12 hours			4 hours		
Depth	5.2 in. (average)			5.1 in. (average)			4.6 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(i) NORTH PACIFIC REGION (*Continued*)						(j) SOUTHERN			
Locality Center Date	N. Idaho and N. W. Montana Near Roland, Idaho December 17 to 19, 1933			Washington and Oregon Cougar, Wash. December 28 to 30, 1937			Southern California Opids Camp, Calif. December 17 to 27, 1921			
Area (sq miles)	500	2,500	21,964	100	500	1,600	100	500	2,000	10,000
Average Depth (In.):										
12 hours	2.2	1.8	1.0	....	....	....	8.3	6.4	4.4	2.0
24 hours	2.6	2.1	1.3	....	....	....	14.0	10.6	7.4	3.4
36 hours	3.0	2.6	1.6	....	....	....	16.6	12.7	8.9	4.4
48 hours	3.7	3.2	1.9	10.5	8.8	7.2	19.3	15.9	11.6	5.9
72 hours	....	....	....	....	....	....	23.8	19.7	14.6	8.0
Total Storm:										
Area	21,964 sq miles			1,600 sq miles			10,000 sq miles			
Duration	60 hours			48 hours			234 hours			
Depth	2.1 in. (average)			7.2 in. (average)			15.0 in. (average)			

hemisphere. He has classified the interacting air masses on the basis of their heat and moisture properties, and has determined the manner in which one will oppose or yield to another. This generalized concept of cause and effect is illustrated in Figs. 3 to 6, in which the meteorologic storm represented by synoptic surface weather maps (Fig. 3) and isentropic charts (Fig. 4) is compared with the hydrologic storm expressed as an isohyetal map (Fig. 5) and mass curves of rainfall at selected points throughout the storm area (Fig. 6).

Beginning, then, with a general atmospheric disturbance of nation-wide proportions, the engineer-hydrologist moves progressively through the extensive area of general precipitation to the concentrations of rainfall of basin-wide extent. Here he assumes the responsibility for routing flood water over the ground as surface runoff and through the channel system as the flood wave.

Values for Selected Storms (*Continued*)

Region (*Continued*)							(*i*) North Pacific Region						
Southern Texas Rocksprings, Tex. June 12 to 15, 1935			Texas San Angelo and Carlsbad, Tex. September 13 to 17, 1936				Western Washington Wind River, Wash. December 16 to 19, 1917				Central Washington Elk Park, Wash. February 23 to 28, 1932		
500	2,500	10,000	100	500	2,000	5,000	500	2,500	10,000	15,675	100	1,000	10,000
8.1	6.7	4.4	16.5	13.3	8.8	6.8	3.2	2.6	1.9	1.7	4.5	3.3	2.3
8.6	7.4	5.4	22.6	17.5	10.7	7.7	4.8	4.0	3.1	2.7	8.8	6.7	3.8
8.8	7.6	5.5	22.8	18.5	12.1	9.0	6.3	5.3	4.1	3.5	12.0	9.2	4.6
9.8	8.7	6.5	22.8	18.6	12.2	9.3	8.1	7.1	5.6	4.9	13.1	10.4	5.2
....	....	....	....	....	....	....	10.3	8.7	6.7	5.9	16.2	13.2	6.4
10,000 sq miles 48 hours 6.5 in. (average)			25,000 sq miles 108 hours 10.1 in. (average)				15,675 sq miles 96 hours 6.5 in. (average)				10,000 sq miles 90 hours 7.5 in. (average)		

Values for Selected Storms (*Continued*)

Pacific Region								
Southern California Hoegees' Camp, Calif. December 29, 1933, to January 1, 1934			Southern California Big Bear Lake, Calif. February 11 to 16, 1937			California, Nevada, and Oregon Sierra Slope December 9 to 12, 1937		
100	1,000	10,000	500	2,500	10,000	500	5,000	90,000
8.6	6.5	2.9	4.1	3.0	2.0	7.3	5.2	1.8
12.6	10.5	5.0	5.7	4.2	3.0	11.5	8.1	2.9
14.3	11.6	5.7	....	....	....	15.6	11.5	4.4
15.2	12.2	6.2	6.2	4.7	3.4	17.6	13.7	5.5
15.9	12.7	6.6	....	....	....	19.6	15.3	6.3
10,000 sq miles 72 hours 6.6 in. (average)			10,000 sq miles 48 hours 3.4 in. (average)			90,000 sq miles 72 hours 6.3 in. (average)		

HISTORICAL BACKGROUND OF STORM RECORDS

"Storm Rainfall of Eastern United States" (1) by the Miami Conservancy District, continues to be outstanding as an example of organized storm data. This excellent compendium, however, does not provide the details essential to the use of more recently developed techniques for storm analysis.

In 1937, when cooperative studies were begun by the Corps of Engineers, the U. S. Department of Agriculture, and by the Weather Bureau in the U. S. Department of Commerce, it was necessary to organize the precipitation data within individual storms. Since the greater number of records were based on observations made with the nonrecording rain gage, it became a major problem to identify the period within which the rain actually fell. The observers' notes in the manuscript records would occasionally indicate the time of beginning and ending of rainfall, but the records of the comparatively few recording gages supplemented by a meteorological analysis of the synoptic weather maps for the period provided the means for making the most accurate distribution of cumulative rainfall depth throughout the duration of the storm.

Of the approximately 1,300 storms of record, about 350 have been analyzed in varying degree by the field offices of the Corps of Engineers and the Hydrometeorological Section of the Weather Bureau. Of this number 150 have been listed in Table 1. In addition to the generalized information giving the date and location of the storm, area-duration-depth values are included.

General Classification of the Principal Storm Types

The following classifications describe the basic actions which produce storm rainfall and, either singly or in combination, can be used to explain heavy rainfall in almost any part of the world. The primary requisites for intense rainfall are cyclonic convergent flow and a potential source of warm moist air. The addition of a lifting mechanism, such as a mountain barrier, or a wedge of cold air, enhances the opportunity for intense rainfall.

Type 1. Quasi-Stationary Front with Active Minor Waves.—A cold front that becomes quasi-stationary with sharp temperature and moisture contrasts between the air masses preceding and following the front, thus resulting in the development of active minor waves along the front.

Type 2. Rapidly Developing Waves Along a Cold Front.—Waves that develop rapidly into major cyclonic systems under favorable meteorological conditions.

Type 3. Major Occluded Cyclonic Systems.—Cyclones which, under favorable temperature and moisture conditions, develop into deep intense systems and cause cold polar air to sweep entirely around the system, thus cutting off the source of warm moist air and occluding the warm sector.

Type 4. Tropical Cyclones.—Storms of tropical origin which move inland, bringing with them unusual quantities of moisture to be precipitated as the storm slowly dissipates.

Type 5. Local or Frontal Thunderstorms.—

Local Thunderstorms.—Storms within an air mass resulting from convective overturning due to convergence, heating, or orographic lifting of moist, unstable air, but not associated with frontal activity.

Frontal Thunderstorms.—Storms produced by releasing the convective energy in moist, unstable, tropical air through frontal lifting.

Type 6. Moist Air Flow Up Mountain Slopes.—Storms resulting from steep pressure gradients which cause moist, unstable air to flow up mountain slopes.

A glossary of meteorological terms is presented as Appendix 2.

Regions of Meteorological Homogeneity

The mean annual isohyetal map for the United States shows the rainfall progressively decreasing from the seacoasts inland, indicating that its distance and direction from the source of moisture are important influences on the amount and duration of rainfall for a locality. Parts of the country having marked departures in annual rainfall are usually regions of rugged topography—surface irregularities exercising an almost independent influence on rainfall. Differences in annual rainfall between regions of similar physiographic characteristics and moisture accessibility confirm the opinion that still other factors help to determine a rainfall régime.

Since there are no distinct boundaries of regions with records of consistent rainfall experience, it must be concluded that any combination of all factors causing rainfall is transitional, and regions of meteorological homogeneity can be outlined only after certain tolerances have been accepted. It is obvious that such regions, under any definition, must vary widely in extent. Great areas can be embraced within prescribed limits in the central third of the United States, whereas it may be impossible to assign boundaries of homogeneity to areas of appreciable extent in the quite rugged humid southeastern quarter of the country.

A region of meteorological homogeneity is defined as one in which all points will have the same, or reasonably similar, records of rainfall intensity and duration at the end of a significant period; or, it may be stated that a storm entering such a region can center over any point with equal probability.

Studies to date have not been sufficiently extensive to allow the exact delineation of all meteorologically homogeneous regions throughout the United States. A tentative subdivision, however, can be made in the 21 areas defined below:

Area	Region
1	Western side of the Pacific Coast Ranges
2	The lee side of the Coast Ranges
3	The Great Central Valley, California
4	The western side of the Cascades
5	The eastern half of Washington and Oregon
6	The plateau region of Utah, with emphasis on the local topographic features
7	Southern Arizona and California
8	The western slope of the Rockies
9	The eastern slope of the Rockies
10	Northeast Arizona and northwest New Mexico
11	Eastern New Mexico and extreme west Texas
12	Western Oklahoma, Kansas, and Nebraska
13	Dakotas and eastern Montana
14	Minnesota and northern Wisconsin
15	Central Mississippi Valley
16	Upper Ohio Valley
17	Western slope of the Appalachians
18	Northeastern States
19	Middle Atlantic Coast
20	Eastern Gulf States
21	Western Gulf States

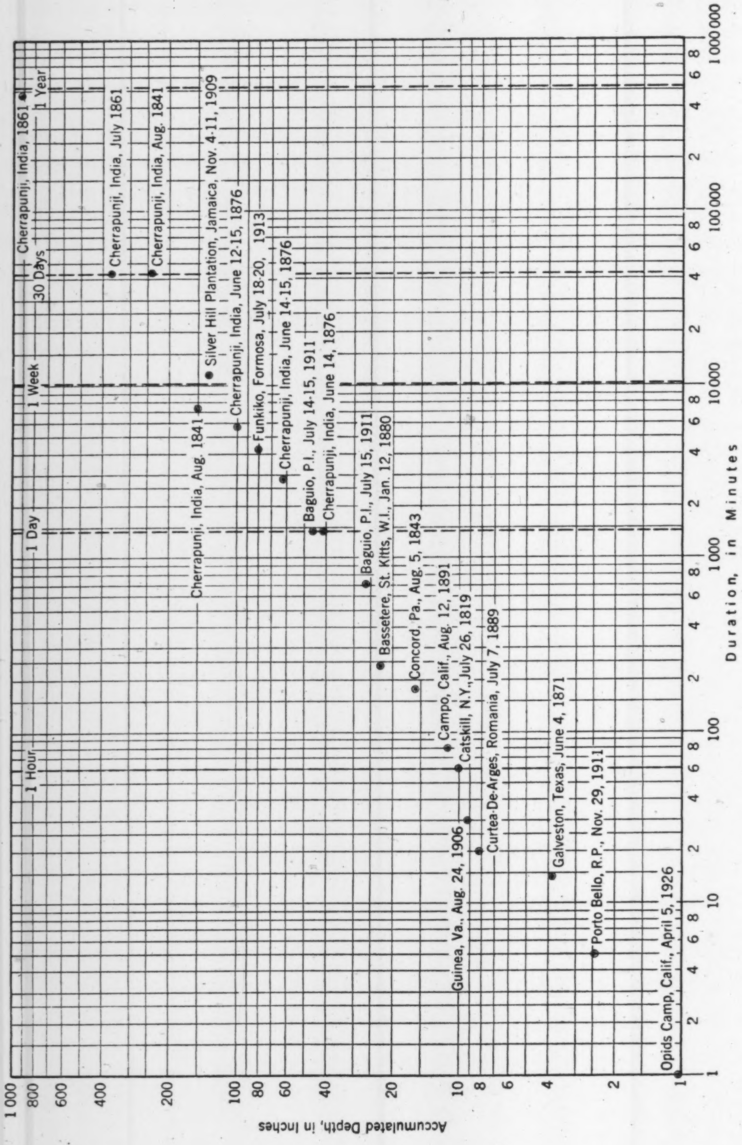


FIG. 7.—DURATION-DEPTH VALUES FOR POINT RAINFALL (WORLD'S RECORD)

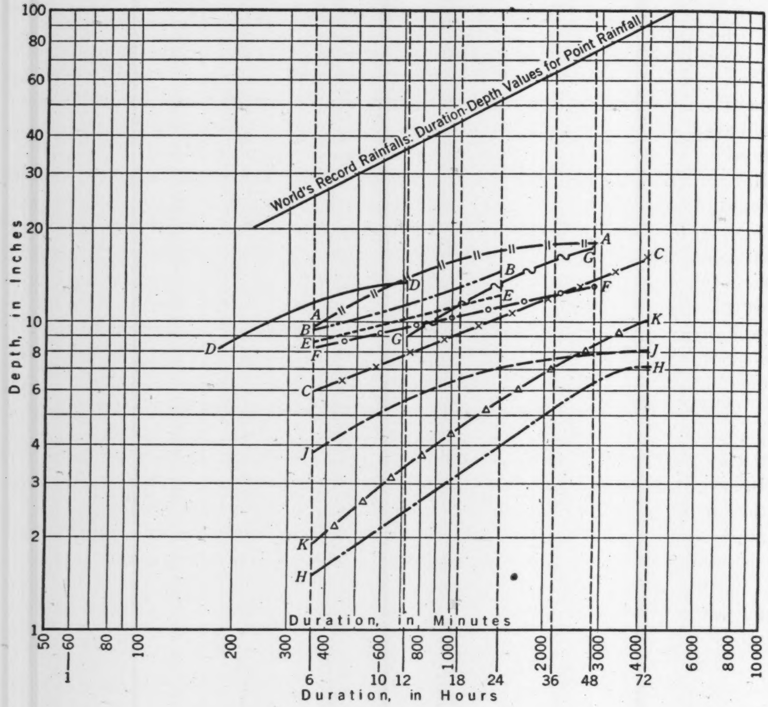
THE MAXIMUM POSSIBLE OR ULTIMATE FLOOD

An immediate distinction to be made is that between the maximum possible flood and the maximum possible storm. Factors other than rainfall, such as melting snow, infiltration rates, and basin characteristics, enter into the determination of the maximum flood stage. The storm of September 11–13, 1878, produced more rain over smaller areas than did that of March 24–26, 1913. However, the former was the result of a decadent tropical hurricane which could not have occurred under conditions permitting a snow mantle over the upper Ohio Basin, whereas rainfall from a storm of the type and proportions of the latter could combine with snow melt under optimum cover and melting conditions to produce a flood of greater proportions. The maximum possible storm, then, is limited to that type which can produce the greatest amount of rainfall over a selected basin within a critical period fixed primarily by the concentration time for the basin and under conditions which are optimum for maximum snow melt and minimum infiltration and retardation.

Although rainfall amounts may be expected to increase as the record for a point or a small basin continues, it is a sound statistical inference that, within the average record period of about 50 years, every extensive region in the United States has experienced a storm which resulted from a critical combination of major factors and at least two of these have approached their physical upper limits. It is reasonable to assume also that at the end of an indeterminate period every basin in the region will have experienced the visitation of such a storm. The statistical evidence of these conclusions is supported by computations of maximum possible rainfall which are based on conservative theoretical assumptions.

Phenomenal precipitation records have been reported from various points over the world throughout the past 100 years. Their news interest has been greater than their value as records of flood-producing rains because no indication is given of the extent of the storm, of which they represent only a dimensionless point. Fig. 7 shows point rainfall depths accumulating throughout periods of varying durations. There is a well-defined upper limit. It should be noted that the lower range of the curve is well represented by values observed in the United States. Although a curve enveloping these points could be modified to represent average depth over specified area, its use in design would be extravagant in many cases, disregarding as it must the physiographic influences particular to the region in which the basin is located.

Limiting depth-duration curves for a number of basins in the United States are compared with that of world-wide point rainfall in Fig. 8. Differences in depth-range and curve-shape reflect differences in basin area and restraining regional influences which make it impossible for all basins throughout the country to have the same maximum rainfall experience. The primary objectives of the hydrometeorological studies have been to make it possible for the designer to consider economy by taking advantage of circumstances favoring a particular basin, while aiding him to gain security by disclosing the most critical storm-flood situation which his structure will ever be called upon to meet.



Curve	Basin	State	Area
A	Ompompanoosuc River above Union Village	Vermont	124
B	Mill Creek	Ohio	154
C	White River	Washington	400
D	Cherry Creek	Colorado	416
E	Youghiogheny River	Pennsylvania	430
F	St. Francis River above Wappapello	Missouri	1,310
G	Wolf Creek above Fort Supply	Oklahoma	1,660
H	Arkansas River above Caddoa	Colorado	19,000
J	Ohio River tributary above Pittsburgh	Pennsylvania	19,117
K	Red River above Denison	Texas-Oklahoma	38,400

FIG. 8.—LIMITING DURATION-DEPTH CURVES

PRINCIPLES OF STORM TRANSPOSITION

Storm transposition is not a new idea. It is a comparatively simple procedure to place an isohyetal map in critical position over that of a basin and convert the rainfall into a flood wave at a selected point. In so doing the designer has merely stated, "Had this storm occurred over the basin under the assumed conditions of runoff, it would have produced a flood of these pro-

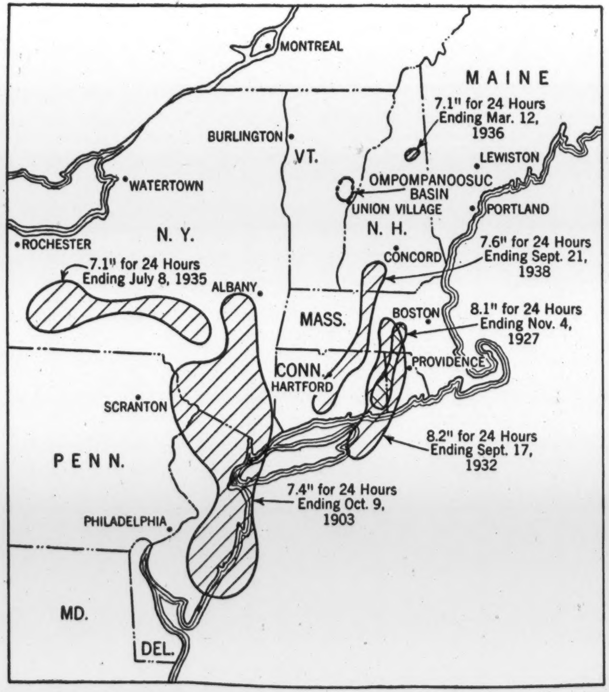

MONTREAL
MAINE
7.1" for 24 Hours Ending Mar. 12, 1936
BURLINGTON
VT.
LEWISTON
WATERTOWN
OMPOMPANOOSUC BASIN
UNION VILLAGE
PORTLAND
N. H.
ROCHESTER
N. Y.
CONCORD
7.6" for 24 Hours Ending Sept. 21, 1938
7.1" for 24 Hours Ending July 8, 1935
ALBANY
MASS.
BOSTON
8.1" for 24 Hours Ending Nov. 4, 1927
CONN.
HARTFORD
PROVIDENCE
SCRANTON
8.2" for 24 Hours Ending Sept. 17, 1932
PENN.
7.4" for 24 Hours Ending Oct. 9, 1903
PHILADELPHIA
MD.
DEL.

FIG. 9.—OMPOMPANOOSUC BASIN UPSTREAM FROM UNION VILLAGE, VT. Approximately Fifty Storms Were Reviewed or Analyzed, Seven of Which Became Control Storms

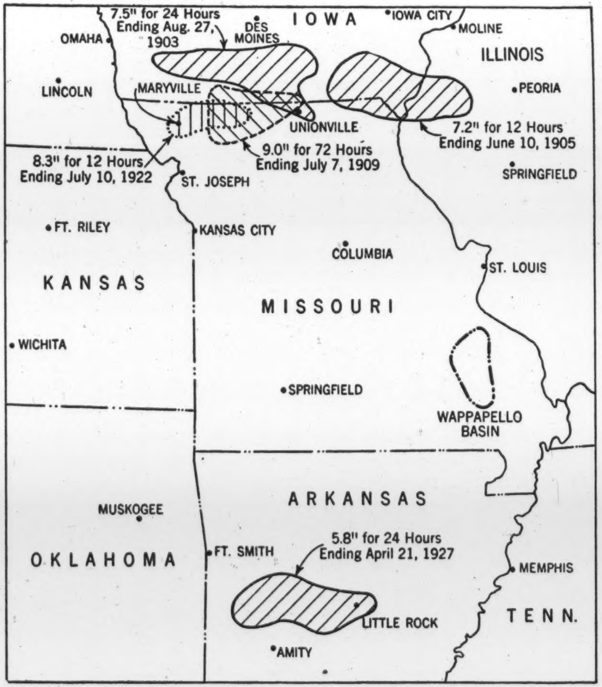

7.5" for 24 Hours Ending Aug. 27, 1903
DES MOINES
IOWA
IOWA CITY
MOLINE
OMAHA
ILLINOIS
LINCOLN
MARYVILLE
PEORIA
UNIONVILLE
7.2" for 12 Hours Ending June 10, 1905
9.0" for 72 Hours Ending July 7, 1909
8.3" for 12 Hours Ending July 10, 1922
ST. JOSEPH
SPRINGFIELD
FT. RILEY
KANSAS CITY
COLUMBIA
ST. LOUIS
KANSAS
MISSOURI
WICHITA
SPRINGFIELD
WAPPAPELLO BASIN
ARKANSAS
MUSKOGEE
5.8" for 24 Hours Ending April 21, 1927
FT. SMITH
OKLAHOMA
MEMPHIS
LITTLE ROCK
TENN.
AMITY

FIG. 10.—ST. FRANCIS RIVER UPSTREAM FROM WAPPAPELLO, MO. Note the Absence of Significant Storms in the Immediate Vicinity of the Basin; Fifty-Five Storms Were Investigated, of Which Eight Were Considered Critical

portions." He is not yet in position to know that the storm could have occurred within the season implied by the assumed conditions of runoff; nor that it could occur such a distance and direction from its actual location. Nor can he, if interested in maximum values, be sure that the storm could not have been much greater in magnitude.

To proceed beyond this point in reaching the objectives previously set forth, it is necessary to develop a philosophy which recognizes the limitations of basic data and scientific knowledge, strikes an intelligent balance between the empirical and the rational, resorts to theory as a guide in expressing qualified judgment, and in a common-sense fashion proposes to make the best use of whatever data are available—about 50 years of fairly dependable precipitation records, 40 years of acceptable synoptic weather maps, and a scanty 8 years of reasonably adequate upper-air data.

It is more the exception than the rule to find adequate record of hydrologic data on a basin designated for flood study. More often, it is necessary to anticipate the basin's future rainfall experience from an extensive study of the region in which the basin is located. It is known, to begin with, that the basin will ultimately (at the end of *X* years) produce a record of storm precipitation which will have in it one storm experience at or near the upper limit fixed by the characteristics of the region of which the basin is a part. If the basin approaches the region in size it will have this experience in a much shorter period than if it is small and storms entering the region have opportunity to center elsewhere.

As basins depending upon reservoir storage for flood control are usually much smaller than the region in which they are located, the procedure is usually to transpose to them those storms of record which, for a particular duration, have produced rainfall under conditions which are known to be supercritical. It would be possible, of course, to superpose all storms over the basin, plot average depth for the basin area for selected durations and obtain an enveloping curve, the increments of the curve becoming the synthetic design storm of typical pattern. The results of flood-storm analysis are given greater utility if area-duration-depth curves are prepared for the region which, under the assumptions of meteorological homogeneity, can be applied to any basin within it. In some instances it is necessary to transpose from one extreme to the other within a region, or between adjacent regions, in which case the transposed storm is adjusted to its physical upper limit for both the observed and transposed locations.

In Figs. 9 to 16 the assigned basin is shown in its relation to those storms contributing to or supporting maximum depth-duration values for the region. The depths indicated for each area are the average of actual rainfalls within a selected isohyet of major storms.

THEORETICAL DETERMINATION OF STORM RAINFALL

To determine theoretically the maximum possible precipitation over an area, it is necessary to develop an equation of flow. This equation expresses the obvious principle that the amount of precipitation is the difference between moisture inflow and moisture outflow.

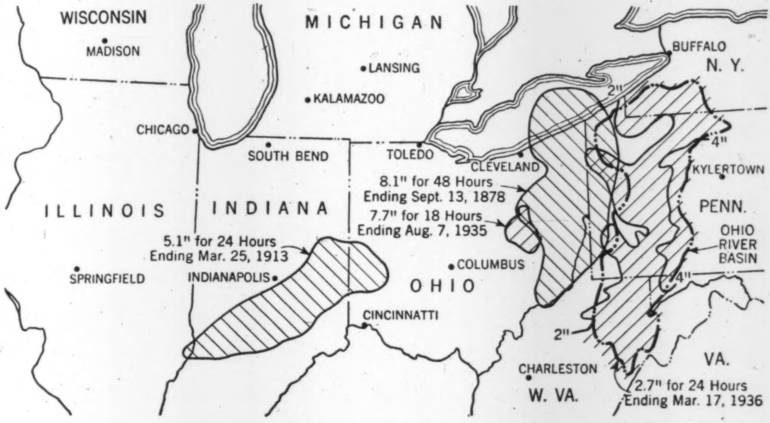

WISCONSIN
MADISON
MICHIGAN
LANSING
KALAMAZOO
BUFFALO
N. Y.
2"
4"
CHICAGO
SOUTH BEND
TOLEDO
CLEVELAND
KYLERTOWN
8.1" for 48 Hours
Ending Sept. 13, 1878
ILLINOIS
INDIANA
7.7" for 18 Hours
Ending Aug. 7, 1935
PENN.
OHIO
RIVER
BASIN
5.1" for 24 Hours
Ending Mar. 25, 1913
COLUMBUS
SPRINGFIELD
INDIANAPOLIS
OHIO
4"
CINCINNATTI
2"
VA.
CHARLESTON
W. VA.
2.7" for 24 Hours
Ending Mar. 17, 1936

FIG. 11.—OHIO RIVER UPSTREAM FROM PITTSBURGH, PA.

Two Hundred and Sixty-Five Storms Were Listed and Reviewed; of which Forty-Three Were Classified and Seven Were Adopted as Representative of Upper Limit Conditions within Their Respective Types

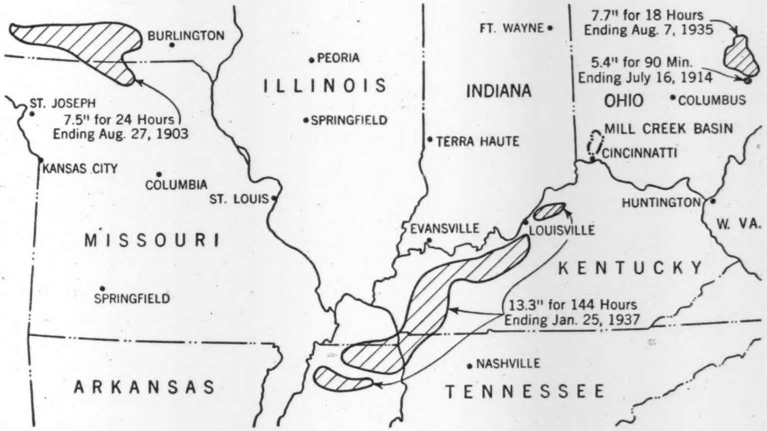

7.7" for 18 Hours
Ending Aug. 7, 1935
FT. WAYNE
BURLINGTON
PEORIA
5.4" for 90 Min.
Ending July 16, 1914
ILLINOIS
INDIANA
OHIO
COLUMBUS
ST. JOSEPH
7.5" for 24 Hours
Ending Aug. 27, 1903
SPRINGFIELD
MILL CREEK BASIN
TERRA HAUTE
CINCINNATTI
KANSAS CITY
COLUMBIA
ST. LOUIS
HUNTINGTON
EVANSVILLE
LOUISVILLE
W. VA.
MISSOURI
KENTUCKY
SPRINGFIELD
13.3" for 144 Hours
Ending Jan. 25, 1937
NASHVILLE
ARKANSAS
TENNESSEE

FIG. 12.—MILL CREEK BASIN, OHIO

One of the Early Studies Drawing Upon Storm Data Organized for Use in Other Reports

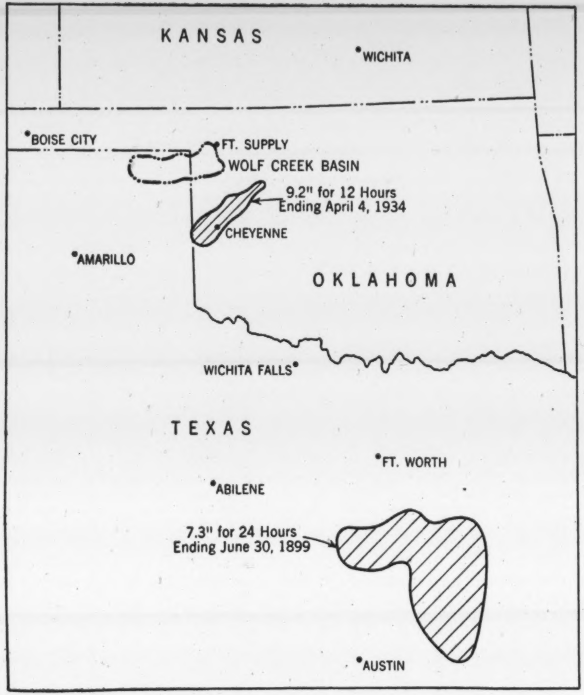

KANSAS
WICHITA
BOISE CITY
FT. SUPPLY
WOLF CREEK BASIN
9.2" for 12 Hours
Ending April 4, 1934
CHEYENNE
AMARILLO
OKLAHOMA
WICHITA FALLS
TEXAS
FT. WORTH
ABILENE
7.3" for 24 Hours
Ending June 30, 1899
AUSTIN

FIG. 13.—WOLF CREEK BASIN UPSTREAM FROM FORT SUPPLY, OKLA.
A Very Limited Number of Storms Had Been Analyzed at the Time This Report Was Prepared; Detailed Analyses Were Made of Only the Three or Four Most Significant

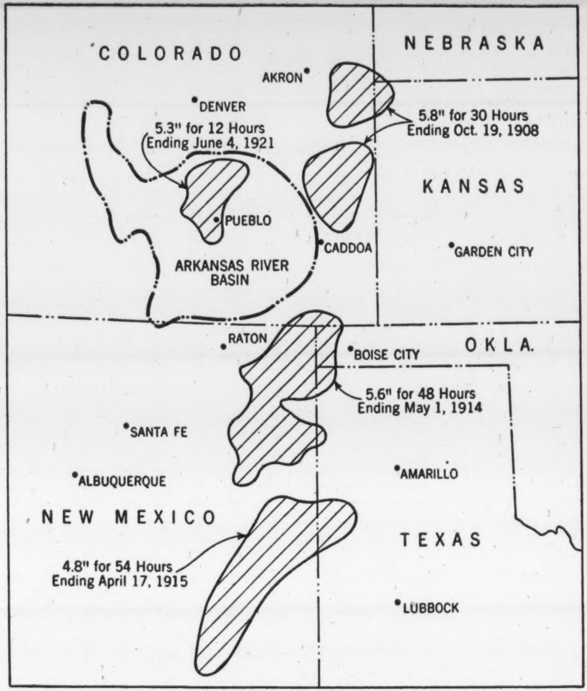

COLORADO
NEBRASKA
AKRON
DENVER
5.3" for 12 Hours
Ending June 4, 1921
5.8" for 30 Hours
Ending Oct. 19, 1908
KANSAS
PUEBLO
CADDOA
GARDEN CITY
ARKANSAS RIVER
BASIN
RATON
BOISE CITY
OKLA.
5.6" for 48 Hours
Ending May 1, 1914
SANTA FE
AMARILLO
ALBUQUERQUE
NEW MEXICO
TEXAS
4.8" for 54 Hours
Ending April 17, 1915
LUBBOCK

FIG. 14.—ARKANSAS RIVER BASIN ABOVE CADDOA, COLO.
One Hundred and Thirty-Three Storms Were Considered, of Which Six Contributed to the Maximum Possible Storm

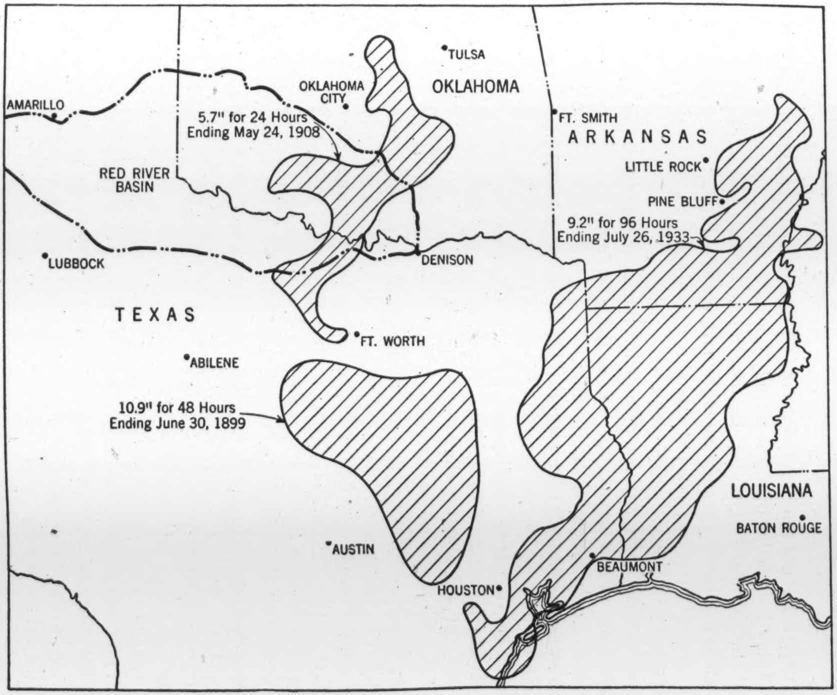

TULSA
OKLAHOMA
OKLAHOMA CITY
AMARILLO
5.7" for 24 Hours Ending May 24, 1908
RED RIVER BASIN
LUBBOCK
DENISON
TEXAS
FT. WORTH
ABILENE
10.9" for 48 Hours Ending June 30, 1899
AUSTIN
HOUSTON
FT. SMITH
ARKANSAS
LITTLE ROCK
PINE BLUFF
9.2" for 96 Hours Ending July 26, 1933
BEAUMONT
LOUISIANA
BATON ROUGE

FIG. 15.—RED RIVER UPSTREAM FROM DENISON, TEX.
By Elimination, Nine Storms Were Found to be Significant, of Which the Three Shown Controlled the Limiting Depth-Duration Curve Developed in This Study

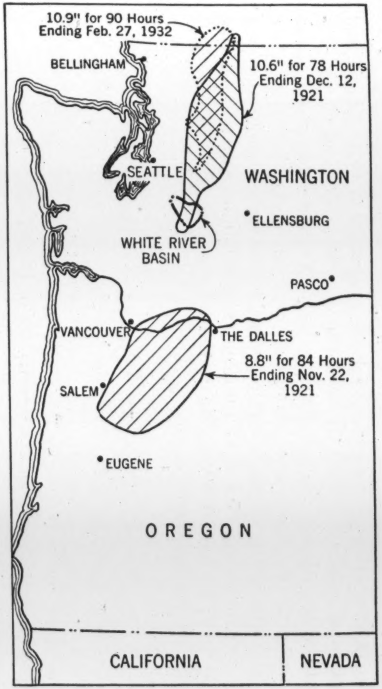

10.9" for 90 Hours Ending Feb. 27, 1932
BELLINGHAM
10.6" for 78 Hours Ending Dec. 12, 1921
SEATTLE
WASHINGTON
ELLENSBURG
WHITE RIVER BASIN
PASCO
VANCOUVER
THE DALLES
8.8" for 84 Hours Ending Nov. 22, 1921
SALEM
EUGENE
OREGON
CALIFORNIA
NEVADA

FIG. 16.—WHITE RIVER BASIN UPSTREAM FROM MUD MOUNTAIN DAM SITE, WASHINGTON
It was Necessary to Analyze Twenty-Six Storms in Order to Identify the Two Representing Maximum Storm Conditions for This Basin

It is evident that moisture inflow will be proportional to the horizontal component wind velocity. The wind velocity in the free air, of course, is a function of the density and the pressure gradient. From a study of thousands of upper-air soundings, it has been found that, at moderately high levels, there exists on the average a direct linear relationship between temperature and pressure. For example, at 5 km, at a pressure of 560 millibars the temperature is found to be 0° C, and at 495 millibars the temperature is − 50° C. This relationship, by the equation

$$\frac{p}{T} = R\rho \quad \text{(1)}$$

also fixes the density, ρ. (In Eq. 1: p = atmospheric pressure, in millibars; T = temperature, in degrees absolute; and R = the gas constant for dry air.) In the foregoing example, the density for 0° C and 560 millibars is 713 g per cu m; whereas that for − 50° C or 495 millibars is 773 g per cu m. Thus the extreme wind will depend, in general, on the extreme temperature gradients aloft. Without delving too deeply into the subject, it may be stated that with the present-day knowledge of the thermodynamics of the atmosphere the limits of extreme temperature aloft can be determined with reasonable accuracy. These limits are found to agree reasonably with observed data. There are also a vast number of pilot-balloon and anemometer observations that have been used in determining the maximum values.

To produce heavy rain two conditions involving moisture must be fulfilled: The air must be saturated; and the temperature-lapse rate must be such that rising air would be in unstable equilibrium.

These requirements definitely establish the upper limit of moisture content of a column of air as a function of the dew-point temperature at a fixed level. The moisture content of an atmospheric column, expressed in terms of water, is called precipitable water (W_P), which is the total amount of vapor in the column, and not the amount that can be removed. The problem now is to determine that which can be removed.

Nature has various ways of producing rainfall, the convective process being considered the most efficient. From a detailed study of the seasonal variation of all the phenomena associated with convective overturning, the functional relationships shown in Fig. 17 between the size of the convective cell and moisture charge have been derived. Surface dew point can then be taken as the measure of the amount of precipitable water in the atmosphere under the two conditions previously stated. The W_P-curve shows that with a dew point of 55° F at the surface there is 1 in. of precipitable water in the air column above, whereas a surface dew point of about 71° F accounts for 2 in. of water in the column.

The idealized model of a convective cell shows the existence of three layers—the inflow layer, the layer of maximum lift, and the outflow layer. The model to the left has a height of 18,000 ft conforming to a surface dew point of 50° F, whereas that to the right shows the cell size fixed by a dew point of 80° F. Intermediate values are represented by the curves.

Knowing the total precipitable water in the column and the total lift experienced, the amount removed as precipitation, and called effective precipitable water (W_E), can be computed. These values are shown by the curve in Fig. 17. For example, in the case of a dew point of 55° F the total water in the column is 1 in., but only 0.55 in. can be removed.

The flow equation, then, is:

$$R = \int K\, v\, W_E\, dt \tag{2}$$

in which: K = a coefficient depending upon the basin characteristics; v = wind velocity in miles per hour; W_E = effective precipitable water in inches; and t = time in hours.

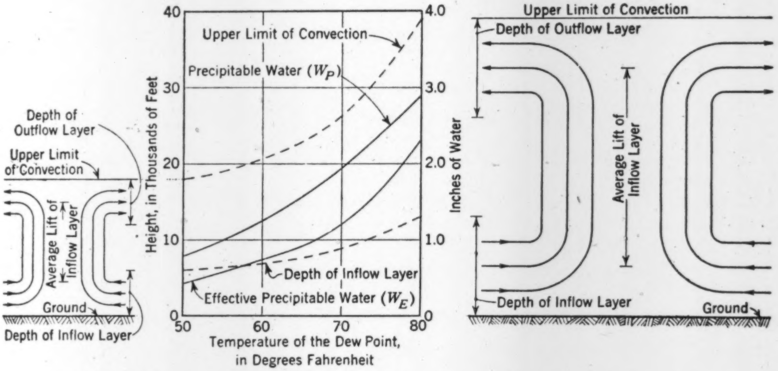


FIG. 17.—EFFECT OF MOISTURE CHARGE ON THE STRUCTURE OF A CONVECTIVE CELL

This means that the maximum precipitation will equal the product of the number of air columns moving into the basin and the maximum amount of moisture that can be squeezed out of each column by the most efficient natural process.

The minimum basin length for which the assumption of linear inflow is justifiable can be estimated from a comparison of the relative magnitudes of the vertical and horizontal components of the wind. If the width is approximately equal to the length, the minimum area is then about 1,000 sq miles for short durations, and about 10,000 sq miles for longer durations. A further restriction on potential rainfall is imposed by the maximum width of moist tongues which limits the moisture inflow column to a maximum of approximately 250 miles.

CONTRIBUTION OF SNOW MELT TO STORM RUNOFF

Precipitation in the form of snow acquires significance as a flood hazard when it accumulates over a basin beyond normal depth and water content. As a source of flood flow, snow can be considered a hydrologic element, but in

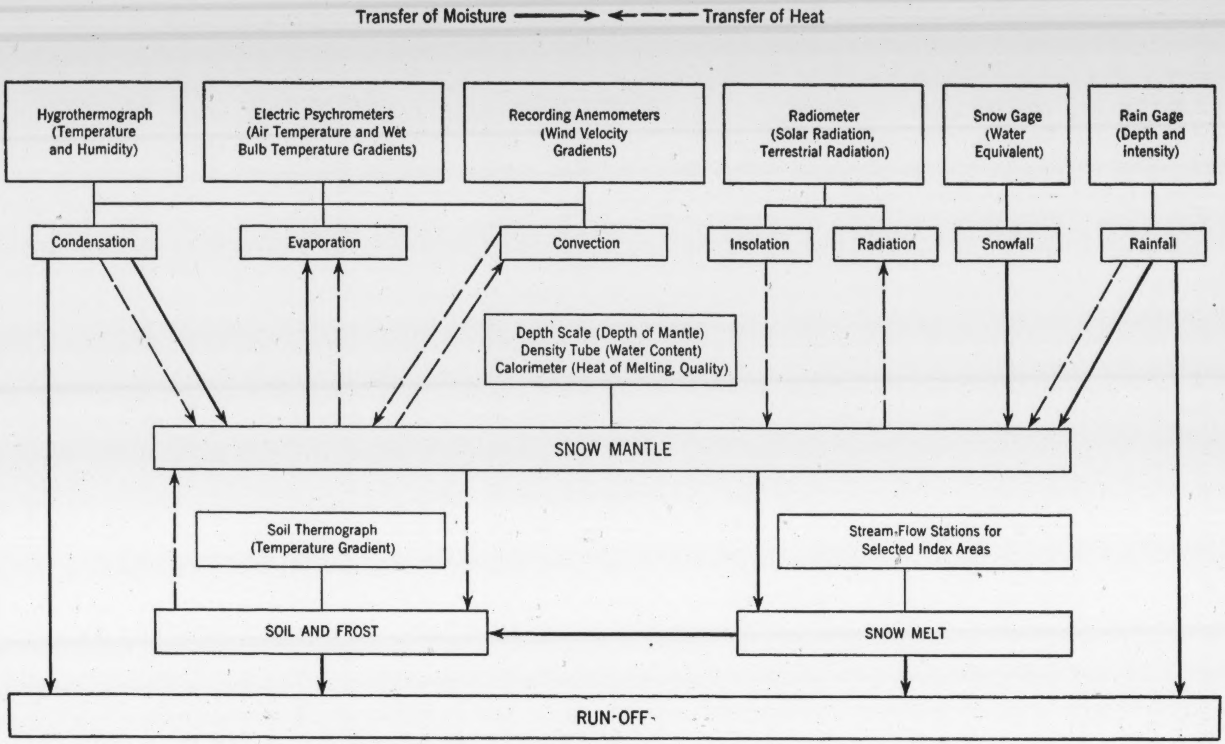

Transfer of Moisture ⟶ ⇠ Transfer of Heat
Hygrothermograph (Temperature and Humidity)
Electric Psychrometers (Air Temperature and Wet Bulb Temperature Gradients)
Recording Anemometers (Wind Velocity Gradients)
Radiometer (Solar Radiation, Terrestrial Radiation)
Snow Gage (Water Equivalent)
Rain Gage (Depth and intensity)
Condensation
Evaporation
Convection
Insolation
Radiation
Snowfall
Rainfall
Depth Scale (Depth of Mantle)
Density Tube (Water Content)
Calorimeter (Heat of Melting, Quality)
SNOW MANTLE
Soil Thermograph (Temperature Gradient)
Stream-Flow Stations for Selected Index Areas
SOIL AND FROST
SNOW MELT
RUN-OFF

FIG. 18.—SNOW-HEAT-RUNOFF BALANCE; FUNCTIONAL RELATIONS AND INSTRUMENTATION

so far as the melting process is wholly involved with the factors, heat, moisture, and air movement, the problem more normally falls in the field of meteorology. The interrelationships between the atmosphere, the snow mantle, and runoff are shown in Fig. 18.

Engineers have made practical use of the fact that the temperature of the air is the most significant single factor in the melting process expressing this index in terms of degree-days above freezing. However, other sources of heat must be considered in any rational approach to snow-melt determination. These are—conduction from underlying soil, warm rain, radiation, condensation, and latent heat. Heat is transferred from the air to the snow surface through convection or turbulent exchange, a complex phenomenon which still defies exact analysis.

It is necessary, if the most critical snow-melt and runoff conditions are to exist:

1. That the snow is in a ripened state (that is, previous heat transfer has raised the temperature of the entire snow mass to the melting point; any further heating, therefore, produces melting);
2. That sufficient depth of frost exists in the underlying soil to check infiltration during the melting period; and
3. That rain occurs simultaneously with melting.

Under conditions (1) and (2) no temperature discontinuity exists at the soil-snow interface. This eliminates soil conduction as a factor in either melting or absorbing heat from the snow.

In ordinary circumstances, there is an appreciable heat exchange due to radiation. Snow absorbs a varying amount of solar energy or insolation depending on the character of its surface and angle of the sun. At the same time it radiates back to space as a "black body." Both the incoming and outgoing radiation are affected by the cloud cover which acts as a blanket to suppress radiational loss or gain. Continuous rain predicates an overcast sky and hence reduces the possible radiative heat transfer to a negligible amount.

Warm moist air transfers heat to the snow in two ways: By sensible heat exchange due to the temperature difference between the air and snow, and by the release of heat through condensation on the snow surface. This transfer of heat results in melting, or the conversion of the solid portion of the snow into liquid form at the rate of 1 g per 80 calories, the latent heat of fusion of ice. Since the heat of fusion of ice is only 80 calories and the heat of vaporization of water is 600 calories per g, the moisture condensed on the snow surface melts 7½ times its own weight of snow.

In moving air the vertical flow of heat and moisture depends on the degree of turbulence of the air. According to the theory of atmospheric turbulence, the temperature, humidity, and strong wind movement measured near the ground are approximately logarithmic functions of height. Within the higher wind-velocity range, therefore, melting rates can be reasonably defined in terms of velocity, dry-bulb temperature, and dew point.

For maximum melting rates accompanying rainfall, the air is close to saturation and, accordingly, the dew point approaches the dry-bulb temperature. Hence, the wind velocity and either dew-point or dry-bulb temperature characterize the air layer next to the snow and can be used in the determination of melting rates throughout the course of a critical storm. As an example, a limiting value of 55° F has been found to be representative of air temperature over snow cover for the upper Ohio Basin, while the warm sector of the storm remains over the basin. This value is to be reduced as the larger basins are considered, due to the influence of the extensive snow cover on the temperature of the lower layer of the atmosphere.

It has been generally conceded, heretofore, that flood threat from snow melt is proportional to the average depth of snow or its water equivalent over a basin. However, it is believed that a more accurate criterion for maximum contribution from snow melt is the existence of only a sufficient amount of snow in storage to react to the melting influences throughout the entire passage of a storm over the basin—the storm to be characterized by extreme temperatures and humidities. The presence of more snow than can be melted under these critical conditions may serve as a deterrence to runoff in the early stages of the storm; or at least it may exercise a regulatory influence on runoff. As melting continues, successive areas of the basin become depleted of snow, causing a progressive decrease in the contribution of melt to runoff. The sequence of melting takes a characteristic pattern beginning with a relatively sharp rise as temperature increases from freezing to some maximum value which is followed by a gradual decrease as snow disappears over the basin. Thus it is evident that the storm and not the depth and distribution of snow determines the amount of snow-melt contribution to the maximum possible flood.

Procedure in Developing the Maximum Possible Storm for an Individual Basin

Four years of intensive study have disclosed no method for determining maximum possible storm rainfall which will apply consistently to all basins. The Hydrometeorological Section of the Weather Bureau, in complying with its assignments, has had to begin with the unproductive end of the problem; that is, it has had to deal progressively with (1) an area sufficiently great to insure the inclusion of the region of meteorological homogeneity embracing the basin; (2) the major drainage area containing the assigned watershed in which orographic influences are to be evaluated; and (3) the assigned tributary basins themselves, where the problem reaches its most difficult phase in that both area and time are reduced to units smaller than those inherent in the available storm and rainfall data.

To present the quantitative results of storm transposition in suitable form for predicting the maximum possible flood, use is made of a procedure, familiar in engineering practice, involving the use of a duration-depth curve. Values of maximum rainfall for arbitrary periods of the storm sequence are plotted against corresponding values of duration which then define an enveloping duration-depth curve of the particular storm. Successive increments of rainfall for convenient units of time may be read off this curve and then rearranged, subject to possible meteorological restrictions, according to a pattern that would produce the maximum flood peak for a basin. This particular time distribution of rainfall is referred to as the critical arrangement of a storm.

The prevailing techniques are best illustrated from a recent study and report on the maximum possible precipitation over the Ohio River tributary basins above Pittsburgh, Pa. In this case, the storm of March 22–27, 1913, became the key storm which, after modification and adjustment made necessary by transposition, contributed the controlling points of the limiting depth-area-duration curves. The storm of March 16–19, 1936, which caused the greatest flood of record at Pittsburgh, and the storm of January 20–25, 1937, similar in type to that of March 19, 1913, were analyzed in their actual and transposed positions.

A typical storm analysis leading to the determination of the maximum possible precipitation over a particular river basin is presented in that of March, 1913. This storm began with the rapid northeastward movement of a low pressure area and a relatively slow eastward movement of the cold front which finally became stationary over the northern tributaries of the Ohio River.

No upper air data are available for March, 1913, but by comparison with storms of recent years it can be inferred reasonably that there must have been an intense cyclonic circulation of cold air aloft over the Central Plains and upper Mississippi Valley region, and a strong west-southwest to east-northeast flow of moist tropical air over the Ohio Valley. The sharp contrast of heat properties between these two air masses made available large quantities of potential energy which could be utilized in releasing excessive amounts of precipitation over a narrow band. The boundary between the two air masses was obviously unstable, thus tending to be broken intermittently by intense wave developments of small magnitude. These caused sharply alternating fluctuations of cold dry air and warm moist air to move along the frontal zone. The large-scale, up-glide motion of the warm air over the cold air tended to prolong the rainfall, but it was the sudden lifting and intensely localized convergence associated with the minor waves which caused intermittent occurrences of intense rainfall.

Such minor waves appear to move in a general eastward direction, but it is almost impossible to predict at what point they will reach the maximum of their intensity. Also, in the extreme case a number of these waves may successively concentrate their activity at approximately the same point, thus accumulating a small but intense center of rainfall. Fortunately, this concentration of rainfall on small areas usually occurs at the expense of rainfall over the surrounding areas, and for basins as large as 20,000 sq miles it is reasonable to assume that in a period of 24 hr or less the concentration observed in March, 1913, was near the maximum.

In considering the transposition of the March, 1913, storm to the Ohio tributaries above Pittsburgh, there were two questions to be answered: (1) Could the storm have been greater in its original position? (2) How much increase or decrease should be allowed in transposing it?

The following steps were taken to answer these questions:

(1) Analysis of the weather charts on the dates of heaviest rainfall showed that a dew point of 64° F was representative of the moisture charge of the warm sector in the storm area. This corresponds to $W_e = 0.88$ in.

(2) The analysis of maximum possible dew points from the Gulf of Mexico to Pittsburgh for various seasons revealed that in late March in the vicinity of the 1913 storm, a dew point of 69° F ($W_e = 1.13$) was possible. In other words, the March, 1913, storm rainfall could have been 28% greater or 128% of the observed average depths.

Maximum possible dew points at Pittsburgh indicate a dew point of 63° F ($W_e = 0.84$) for late March. Therefore, the maximum possible storm of type I near Cincinnati, Ohio, would have to be reduced in the ratio of 1.13 to 0.84 in transposing it to the Pittsburgh area. This represents a 26% reduction in rainfall amounts.

It is realized that no statistical approach can provide a valid adjustment factor for type I storms occurring near Cincinnati and transposed to the Pittsburgh area. However, the comparative data in Table 2 are in general agreement with the fact that the maximum possible storm values will be greater near Cincinnati than near Pittsburgh.

TABLE 2.—COMPARISON OF AVERAGE RAINFALL (IN INCHES) AT CINCINNATI, OHIO, AND PITTSBURGH, PA.

No.	Description (1)	Cincinnati (2)	Pittsburgh (3)	Ratio, $\frac{\text{Col. (3)}}{\text{Col. (2)}}$ (4)
1	Annual precipitation	38.41	35.81	0.92
2	March precipitation	3.82	3.10	0.81
	Rainfall, Type I Storms:			
3	Greatest 35 storms	1.44	1.05	0.73
4	Greatest 10 Pittsburgh storms	2.73	2.14	0.78
5	Greatest 35, 12-hr storms	0.42	0.31	0.74

The value of 26%, computed by the dew-point technique, was accepted and used in the following adjustment: Considering the observed depths in the March, 1913, storm equal to 100%, the maximum possible type I storm in the same locality would be 28% greater or equal to 128%. However, this maximum possible storm must be decreased 26% when transposing it over the Ohio River above Pittsburgh. The maximum possible transposed storm therefore becomes 95% of the observed.

The same result can be reached quite simply by adjusting the observed 1913 depths to the maximum possible dew point at Pittsburgh. The effective precipitable water for the respective areas was 0.88 in. in the actual storm and 0.84 in. (the maximum possible) at Pittsburgh. This is a 5% decrease. Therefore, in transposing the 1913 storm to the Ohio tributaries above Pittsburgh a decrease of 5% was applied to all depths for all durations, on the assumption that the storm could have been greater in its original locality, but could not have been so great if the storm had centered over the Ohio tributaries above Pittsburgh.

In regard to the orientation of isohyetal patterns resulting from type I storms, it must be emphasized that such storms occur with upper-air currents from the west-southwest or southwest, and rainfall must necessarily take place along a band which is at right angles to the major axis of the Pittsburgh basin. Since this particular direction of flow is necessary to bring moisture, unaffected by topography, into the basin, there can be no justification for changing the orientation of the original storm pattern. For the foregoing reasons a mechanical transposition without re-orientation was considered most representative of the March, 1913, storm over the Ohio tributaries above Pittsburgh.

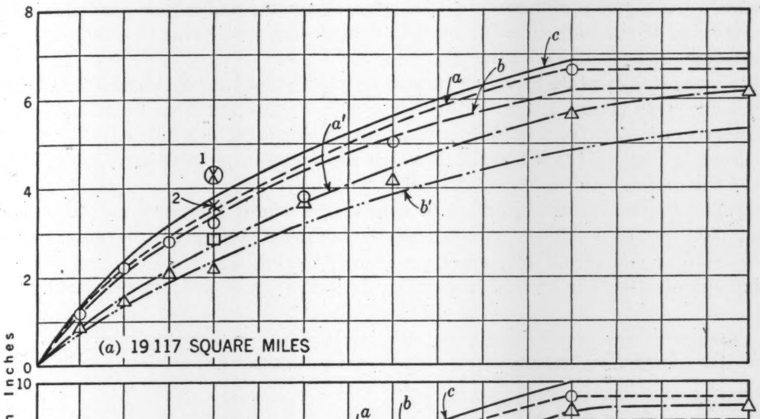


FIG. 19.—DURATION-DEPTH CURVES OF MAXIMUM POSSIBLE RAINFALL; STORM TYPES I AND II

Two typical groups of duration-depth curves are shown in Fig. 19. Each block in the diagram contains a family of five curves applicable to the area of a particular subdivision of the basin. Two curves, one for each of the major storms classified as types I and II, represent duration curves of rainfall as it actually occurred over the upper Ohio Basin (curves a and b, Fig. 19). Two other curves give corresponding values of rainfall obtained by transposing these storms to the most critical position over the basin (curves a′ and b′, Fig. 19). The enveloping curve is the duration-depth curve of maximum possible rainfall, representing as it does an upper limit fixed by actual values which have been increased with meteorological justification and modified according to the influences involved in transposing the storms to their assumed locations (curves c, Fig. 19).

Two additional 24-hr values have been plotted for the total basin (19,117 sq miles). One (Point 1, Fig. 19(*a*)) is the value computed from theoretical considerations involving moisture charge and velocity of air inflow. The other (Point 2, Fig. 19(*a*)) is the value which recognizes a possible error of 20% in the theoretical computation.

The maximum contribution to runoff from snow melt will result from a combination of maximum melting rates, maximum storage of free water in the snow and a uniformly deep snow cover over the basin. A series of snow-melt increments (see Fig. 20(*a*)) have been derived from principles previously described, assuming the distribution of snow cover which will respond most actively to the series of meteorological events characterizing the maximum possible storm.

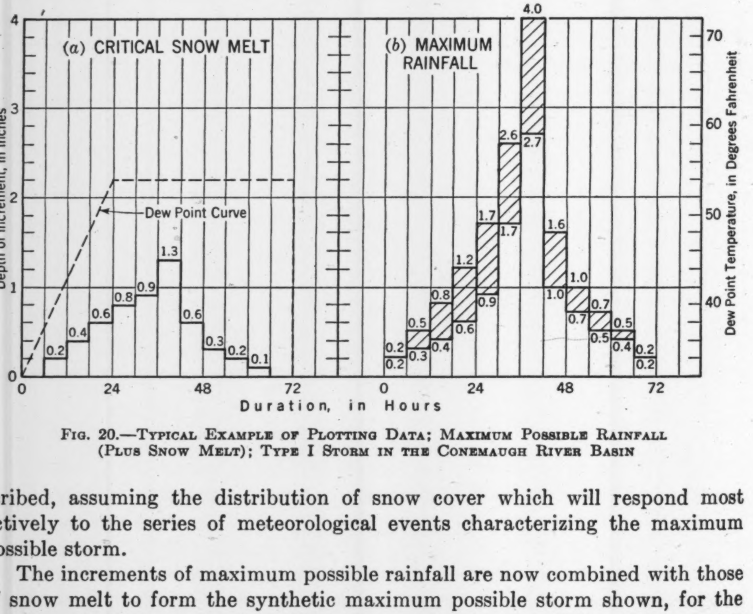


FIG. 20.—TYPICAL EXAMPLE OF PLOTTING DATA; MAXIMUM POSSIBLE RAINFALL (PLUS SNOW MELT); TYPE I STORM IN THE CONEMAUGH RIVER BASIN

The increments of maximum possible rainfall are now combined with those of snow melt to form the synthetic maximum possible storm shown, for the Conemaugh River Basin (as a typical example), in Fig. 20(*b*). The rainfall blocks have been arranged in an order which associates the highest rate of snow melting with the highest rate of rainfall, both being compatible with the meteorological features of storms of this type. Since the first 24 hr of the storm sequence is fixed by the meteorological situation and the accompanying snow melt, the rainfall-snow-melt increments of this period should not be interchanged among themselves or with those of the last 48 hr of the storm. Increments within the last 48 hr of the storm, however, may be interchanged as desired. The individual snow melt and rainfall increments must be considered inseparable, the necessity for which is readily seen from a consideration of the snow-melt computation procedure. These restrictions apply to all sizes of drainage basins.

The Bibliography in Appendix 1 will serve as a guide to selected reading for those whose interest in meteorology has carried them beyond the scope of this paper.

The writer should like to close by expressing his conviction that an engineer is not prepared to deal adequately with precipitation, the principal hydrologic factor, until he has gained a working knowledge of the processes which take from the sea, transport, and precipitate over the land the myriad of rain drops constituting a great storm. Thus equipped he will find his concepts broadened, his techniques made flexible, and his judgment strengthened.

ACKNOWLEDGMENTS

The hydrometeorological studies described herein are conducted for, and in cooperation with, the Engineer Department, Corps of Engineers, U. S. Army. The writer wishes to acknowledge with thanks the valuable assistance of Albert K. Showalter, Samuel B. Solot, Phillip Light, Alexander L. Shands, and other members of the Hydrometeorological Section, Weather Bureau.

APPENDIX 1

BIBLIOGRAPHY

(1) **"Storm Rainfall of Eastern United States,"** Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed., Dayton, Ohio.

(2) **A Report on the Maximum Possible Precipitation over the Ompompanoosuc Basin above Union Village, Vermont,** Hydrometeorological Section, U. S. Weather Bureau, in cooperation with the U. S. Engr. Dept., Corps of Engrs., March, 1940.

(3) **A Report on the Maximum Possible Precipitation over the Ohio River Tributary Basins above Pittsburgh, Pa.,** Hydrometeorological Section, U. S. Weather Bureau, in cooperation with the U. S. Engr. Dept., Corps of Engrs., June, 1941.

(4) *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 660.

(5) **"Hydrometeorology—A Coordination of Meteorology and Hydrology,"** by Merrill Bernard, *Transactions*, Am. Geophysical Union, Vol. 19, 1938, Pt. II, pp. 598–602.

(6) **"Flood Forecasting. Climate and Man,"** by Merrill Bernard, *Yearbook of Agriculture*, U. S. D. A., 1941, pp. 565–576.

(7) **"Physical and Dynamical Meteorology,"** by David Brunt, Cambridge Univ. Press, 1939.

(8) **"The Meteorological Phase of Flood-Forecasting,"** by H. R. Byers, *Transactions*, Am. Geophysical Union, Vol. 20, 1939, Pt. II, pp. 205–207.

(9) **"Snow Melt. Climate and Man,"** by C. L. Forsling, *Yearbook of Agriculture*, U. S. D. A., 1941, pp. 557–560.

(10) **"Dynamic Meteorology,"** by B. Haurwitz, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

(11) **"Meteorology for Pilots,"** by B. C. Haynes, *Bulletin No. 25*, C. A. A., 1940.

(12) **"Storms and Floods. Climate and Man,"** by B. Holzman and A. Showalter, *Yearbook of Agriculture*, U. S. D. A., 1941, pp. 551–557.

(13) **"Physics of the Air,"** by W. J. Humphreys, McGraw-Hill Book Co., Inc., New York, N. Y., 1940.

(14) **"Analysis of High Rates of Snow-Melting,"** by Phillip Light, *Transactions*, Am. Geophysical Union, Vol. 22, 1941, Pt. I-B, pp. 195–205.

(15) **"Synoptic Analysis of Some Excessive Rainfalls in the Mississippi Basin Due to Squeezing of Tropical Air between Anticyclones,"** by E. J. Minser and G. A. Hathaway, *Bulletin*, Am. Meteorological Soc., Vol. 19, January, 1938, pp. 34–42.

(16) **"Introduction to Meteorology,"** by S. Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

(17) **"Weather Analysis and Forecasting,"** by S. Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1940. (*a*) p. 124, Fig. 68.

(18) **"The Scientific Basis of Modern Meteorology. Climate and Man,"** by C. G. Rossby, *Yearbook of Agriculture*, U. S. D. A., 1941, pp. 599–655.

(19) **"Manual of Meteorology,"** by Sir Napier Shaw, Cambridge Univ. Press, 1930.

(20) **"Computation of the Maximum Possible Storm,"** by A. K. Showalter and S. B. Solot, *Transactions*, Am. Geophysical Union, 1942, Pt. II, pp. 258–274. (*a*) Fig. 2, p. 261. (*b*) p. 259.

(21) **"Storm,"** by George R. Stewart, Random House, 1941.

(22) **"An Outline of the Thermodynamics of Snow-Melt,"** by W. T. Wilson, *Transactions*, Am. Geophysical Union, Vol. 22, 1941, Pt. I-B, p. 182.

(23) **"Flood Flow Characteristics,"** by C. S. Jarvis, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 985–1032.

(24) **"Rainfall Characteristics and Their Relation to Soils and Runoff,"** by C. S. Jarvis, *ibid.*, Vol. 95 (1931), pp. 379–423. (*a*) p. 391.

(25) **"Weather Forecasting,"** by W. Napier Shaw, Constable & Co., Ltd., London, England, 1941, p. 220.

(26) **Dictionary of Applied Physical Science,** Vol. III, p. 61.

(27) **"The Unit Hydrograph and Its Application,"** by L. K. Sherman, *Journal*, Associated State Eng. Societies, April, 1941, p. 4.

(28) **"Flood Flow on Missouri Streams,"** *Bulletin No. 30*, Univ. of Missouri Eng. Series, 1942.

(29) **"Unit Hydrograph Method and Storm Transposition in Flood Problems Relating to Great Storms in Eastern and Central United States,"** by Merrill Bernard, *Water Supply Paper No. 772*, U. S. Geological Survey, 1936.

(30) **"Meteorological Data,"** *Proceedings*, Am. Soc. C. E., January, 1933, pp. 153–182.

(31) **"Handbook of Engineering Fundamentals,"** by Ovid W. Eshbach, John Wiley & Sons, Inc., New York, N. Y., 1936, p. 2–127.

(32) **"Computation of Depth of Precipitable Water in a Column of Air,"** by S. B. Solot, *Monthly Weather Review*, April, 1939, pp. 100–103.

(33) **"Maximum Possible Precipitation Over the Red River Basin Above Denison, Tex.,"** by Merrill Bernard and others, U. S. Weather Bureau, October 26, 1939.

(34) **"Storm Rainfall of Eastern United States,"** *Technical Reports of the Miami Conservancy District*, Pt. V, Revised Ed., p. 335 (1917 Ed., p. 273).

(35) **"Floods in the Sacramento Valley, Calif., Dec. 1937,"** by E. H. Fletcher, *Monthly Weather Review*, January, 1938, pp. 441–444.

(36) **"Floods of March 1928, in Sacramento Valley, Calif.,"** by N. R. Taylor, *ibid.*, March 1928, pp. 100–102.

(37) **Sacramento Flood-Control Project,** California State Reclamation Board, revised plans of 1925, pp. 109–115.

(38) **"Maximum Possible Precipitation Over the Sacramento River Basin of California,"** *Hydrometeorological Report No. 3*, Hydrometeorological Section, U. S. Weather Bureau, in cooperation with the U. S. Engr. Dept., Corps of Engrs., May, 1942, p. 113.

(39) **"Advantages in the Non-Uniform Hour of Observation in the Interpretation of Published Precipitation Data,"** by J. R. Miles, *Monthly Weather Review*, April, 1940, p. 99.

APPENDIX 2

GLOSSARY OF METEOROLOGICAL TERMS

Synoptic Chart.—A chart, such as the ordinary weather map, which shows the distribution of meteorological conditions over an area at a given moment.

Anticyclone.—An area of high barometric pressure and its attendant system of winds (clockwise in the northern hemisphere).

Cyclone.—An area of low barometric pressure with its attendant system of winds (counterclockwise in the northern hemisphere).

Air Mass.—An extensive portion of the earth's atmosphere approximating horizontal homogeneity.

Front.—The boundary between two different air masses.

Polar Front.—The surface of discontinuity separating an air mass of polar origin from one of tropical origin.

Cold Front.—Front at which relatively cold air displaces warmer air.

Warm Front.—Front at which relatively warm air displaces colder air.

Quasi-Stationary Front.—Front whose movement is such that there is no appreciable displacement of warm air by cold air or vice versa.

Cyclogenesis.—Process which creates or develops a new cyclone, or which produces an intensification of an old one.

Frontogenesis.—Process which creates a front or intensifies an existing front.

Frontolysis.—Process which tends to weaken or destroy an existing front.

Occlusion.—Process whereby the air in the warm sector of a cyclone is forced from the surface to higher levels.

Occluded Front.—The type of front resulting when a cold front overtakes a warm front.

Cold Front Type Occlusion.—Occlusion in which cold air in rear of the cold front is colder than and underruns the cold air in advance of the warm front.

Warm Front Type Occlusion.—Occlusion where cold air in rear of cold front is warmer than cold air in advance of warm front.

Isentropic Chart.—Synoptic chart plotted on a surface of constant potential temperature, and is so called since a surface of constant potential temperature, if unsaturated, is also a surface of constant entropy.

Potential Temperature.—Temperature which an air parcel would have if expanded or compressed dry adiabatically to a standard pressure of 1,000 millibars.

Equivalent Potential Temperature.—The potential temperature of an air parcel after all the latent heat of condensation of the contained water vapor is realized.

Dew-Point Temperature.—The temperature at which, under ordinary conditions, without change of pressure, condensation begins in a cooling mass of air.

Wet-Bulb Temperature.—Lowest temperature to which air may be cooled by evaporating water into the air at the same barometric pressure.

Specific Humidity.—Ratio of the weight of water vapor to that of air in a given sample, in grams per kilogram.

Lapse Rate.—Rate of decrease of temperature in the atmosphere with height.

Dry Adiabatic Lapse Rate.—Rate at which an ascending body of unsaturated air will cool due to adiabatic expansion, equal to approximately 1° C per 100 m.

Pseudo-Adiabatic Lapse Rate.—Lapse rate equal to the rate at which an ascending body of saturated air will cool during adiabatic expansion. Its value is not the same under all conditions, but is never greater than the dry adiabatic rate.

Precipitable Water.—Total water vapor contained in an atmospheric column of unit cross-section area expressed in terms of a column of water of the same cross-section area.

Effective Precipitable Water.—The greatest amount of precipitable water which can be removed from an atmospheric column by convective action.

Tongue.—A projecting stream of air which may be dry or moist, revealed by the isentropic chart or atmospheric cross-section charts.

Trough.—An elongated area of low barometric pressure.

Stability.—Thermodynamic state in which vertical displacements are resisted.

Instability.—Thermodynamic state in which vertical displacements are favored.

Convective Process.—Process, mechanical or thermal, causing the upward or downward movement of a limited portion of the atmosphere. Cumulus type clouds are indicative of the occurrence of convective processes.

Level of Free Convection.—Level in the atmosphere at which, when reached, an air parcel is warmer than its surroundings and will continue to be displaced upward.

Lifting Condensation Level.—Level in the atmosphere at which, when reached, an air parcel will cool at the pseudo-adiabatic rate.

Conditional Instability.—Thermodynamic state which is unstable for saturated, but stable for dry or unsaturated, air.

Convective Instability.—Thermodynamic state of a layer which will become unstable after sufficient lift.

Up-Slope Winds or Up-Glide Winds.—Winds occurring when there is a net flow of air moving up a slope. When the slope is a front, the warm air moves up with respect to the colder air beneath the front.

Down-Slope or Down-Glide Winds.—Winds occurring when there is a net flow of air moving down a slope.

Orographic Rainfall.—Rainfall caused by the interference of rising land in the path of moisture-laden wind. A horizontal air current striking a mountain slope is deflected upward and the consequent dynamical cooling associated with the expansion of the air produces rain if the air contains sufficient aqueous vapor.

Convergence.—Condition that exists when distribution of winds within a given area is such that there is a net horizontal inflow of air into the area.

Gradient Wind.—The wind resulting from the balance (neglecting friction) between the force due to the pressure gradient, the apparent deflecting force of the earth's rotation, and the centrifugal force due to the curvature of path.

Geostrophic Wind.—The wind resulting from the balance of the force due to the pressure gradient and the apparent deflecting force of the earth's rotation, neglecting curvature of path and friction.

DISCUSSION

Edgar Dow Gilman,[3] M. Am. Soc. C. E.—Much valuable data are contained in this paper, including theories and conclusions on storms and precipitation. Precipitation is undoubtedly one, but only one, of the major factors in the production of floods and is not in itself a controlling factor. Although the subject of precipitation is handled interestingly in this paper, the title, "Flood Estimating," appears to be broader than the material actually covered. For example, the heading of one section is "The Maximum Possible or Ultimate Flood," in which the author sets forth the maximum possible or ultimate precipitation, which does not seem to the writer to be synonymous with floods.

From a study of Ohio River flood records and flood observations in more recent years, there appear to be two factors in floods, regarding which more data and study would be enlightening. The first is the variation in the longitudinal profiles of flood flow and crests. The second is the effect on such profiles of the "timing" of the tributary crests at their outlets relative to the moving crest in the river.

Different floods have quite different longitudinal profiles. When these profiles are plotted as of any instant, at wherever the crest is at that instant, with miles as the horizontal axis and feet elevation as the vertical axis, it is found that the slopes in front of, and behind, crests are quite dissimilar in different floods—some will be gradual and some will be sharp. The lengths of the higher elevations or crests will also vary; some will be short and some will be long. How high a flood is produced in a big river with many tributaries, when there has been a given precipitation over the watershed, depends upon how that volume of water is stored in the moving flood. A given volume in a long, flat crest with gradual slopes will not result in as high a flood as if the same volume were contained in a short, sharp crest with steep slopes. This introduces the factor of "timing"—that is, whether crests in tributaries reach their mouths at such a time as to add to the height or length of the crest in the river or some of each.

In 1913 the Miami River (which enters the Ohio River about 17 miles below Cincinnati) reached its crest on March 26, producing the great Dayton flood. On that date the Ohio River was 4 ft under the flood stage of 52.0 (gage height) at Cincinnati. This caused a pool in the river, and, for a considerable period, water was actually flowing upstream in the Ohio River. In the meantime, a crest in that river was on its way down, but did not reach Cincinnati until April 1, when it crested with a gage height of 69.9, the second greatest recorded flood stage at Cincinnati up to that time (exceeded in 1884 and 1937). The gage height of the Miami River at Hamilton, Ohio, was 34.0 ft at its crest on March 26; and, on April 1, when the Ohio River crested at the mouth of the Miami River, the gage height of the latter was only 4.0 ft. In these six days the Miami River had dropped 30 ft. Presumably the greater portion of the volume of flood discharge from the Miami River added to the length of the flood crest of the Ohio River, rather than to its height, which was probably increased somewhat because of the additional water ahead of the crest. However, the problem is what would have been the effect on this Ohio River crest if the greatest discharge of the Miami River had reached its outlet on March 31, April 1, or April 2, instead of on March 26?

---

[3] Director, Dept. of Public Utilities, Cincinnati, Ohio.

Time obscures details and, in the minds of most people, the 1913 flood in the Miami River and the 1913 flood in the Ohio River at Cincinnati are considered one and the same flood; but the six-day interval between them, with due regard to the flashy character (rapid rise and fall) of floods in this smaller river, probably was a vitally important timing.

That all tributaries to a river would release their maximum discharge at the exact time to produce the greatest increase in height of the crest of the major river as it passed the mouth of each tributary might be equivalent to assuming an improbable, if not an impossible, series of cumulative coincidences. With the maximum precipitation over the watershed of each tributary and such an assumption, a theoretical maximum flood could be shown that at least would be interesting. Although such a drastic eventuality will never be realized, it seems true that floods in major rivers, such as the Ohio, vary in height as more or fewer of the total number of tributaries approach such timings in their discharge; and that, because of variations in the number of such coincidences, it is a practical possibility to have higher floods with other than the maximum precipitations. In other words, unless the precipitation is of very long duration over the entire watershed of a big river, it cannot be stated that the greatest precipitation will necessarily produce the greatest floods.

The writer would suggest again that all river gage heights be read in sea-level elevations. Past records from existing gages could be referenced to the new base and not be lost. Such procedure would add materially to the information to be gained from reading a table of gage heights throughout the length of the Ohio River. It seems unnecessarily confusing to the writer to refer to a gage height of 30 ft in the Miami River at a time when the gage height in the Ohio River is 48 ft, and, at the same time, to realize that water was flowing from the Miami River into the Ohio River.

CLARENCE S. JARVIS,[4] M. AM. SOC. C. E.—This timely, concise report on "relative successes and failures" has a special appeal because of its clarity of concept, its modesty of claims, and its wealth of significant data in their proper relationships. The final paragraph preceding "Acknowledgments" deserves particular emphasis, which may be afforded by an illustration from the writer's personal experience.

In connection with a paper on the characteristics of flood flow (23), the insertion of a few brief paragraphs on rainfall phenomena, particularly in their relation to floods, was found desirable; and several months of study under the local Weather Bureau officials were required before that portion of the text attained its final form. Incidentally, that course of intensive study opened the way to the writing of a companion paper (24), for which the nucleus was prepared and diverted from the earlier paper. Fully five years were needed to encompass the first phase of that project, namely, the assembly and summarization of representative precipitation data from stations throughout the world. Basic principles and interesting relationships were set forth as a preliminary to solving some of the outstanding problems involving rainfall and resulting runoff. The data proved to be so abundant and varied, and the field for investigation so broad, that it seemed best to publish the assembled records with such brief discussion of fundamentals as had been prepared and to invite cooperation from all hydrologists, engineers, and interested scientists or research workers in the interpretation of facts brought forth and in the extension of the hydrologic field.

---

[4] Hydr. Engr., With Office, Chf. of Engrs., Washington, D. C.

In the course of writing chapters for recent volumes, all dealing with some phase of hydrology, the writer has observed that the progress is so rapid that extensive study is required for one to avoid becoming out of date.

After reviewing the most notable recent advances in practical hydrology and meteorology now largely associated with the demands, as well as the facilities, afforded by aeronautics for obtaining three-dimensional weather data, one is impressed with the insignificance of any new ideas he has brought forth, in comparison with the important problems awaiting and inviting solution. Frankly, within the writer's experience, some of the methodology and findings that rated highest according to his judgment were promptly relegated to the rôles of "incomplete forward passes." Probably the surest way to have new procedures adopted for general use is to have them devised and announced by responsible, thoroughly organized agencies. This leads inevitably to the conclusion that such official representatives should continue to set the pace and to direct the vanguard in notable advances. Unofficial utterances possibly may attain the importance of "aerial reconnaissance," or even desultory "bombing" of a hostile, unknown area. Complete mastery of the situation will continue to depend on the organized efforts of "ground troops" and their occupation in force.

Although accepting the authoritative data and their analysis as presented in this paper, the writer submits herewith some qualifying notes and queries, which seem to invite further study:

1. Solar radiation may contribute directly to the heating of the atmosphere through which it passes—probably somewhat less than 10% for clear, dry air when the sun is near the zenith, but correspondingly more as the rays become more inclined. Examples of equivalent transparent masses are 34 ft of clear, placid water or 14 ft of the best grade of glass. With the formation or the gathering of cloud masses, the reflecting and radiation surface is elevated high above the ground and applied to the heating of a lesser mass of atmosphere, in a rarefied condition, resulting in pronounced thermal changes and gravitational instability therein.

Is it not quite probable that much of the turbulence and convective movement results directly from the local effects of solar energy applied to the upper surface of the cloud masses? Such heating should dispel the upper fringe of cloud forms, and the induced buoyancy would give an upward component to the air column's movement.

2. The author and other writers assume that contained in the atmospheric column covering the globe is a single inch of water, on an average, with appropriate excess in torrid regions and deficiency near the poles. If this is correct, it seems inconsistent to assume a depth of only 0.88 in. for the precipitable moisture on the date of heaviest rainfall, during the storm which most nearly represents the maximum possible. It is of record that more than 1 in. fell at Cincinnati, Ohio, in the hour extending somewhat beyond midnight of March 25–26, 1913; and considerable areas recorded as high as 6 in. in 24 hr. It would have required more horizontal wind movement than has been recorded to account for the seven complete displacements and renewals of air columns over those extensive areas of high rainfall; and those same areas would have been flanked by regions of deficient rainfall, if the air masses had been "wrung dry" in the convective cells.

On the contrary, the 1,100 sq miles of area with precipitation depth 6 in. or greater were flanked by areas of moderate rainfall depth; 12,000 sq miles with 4 in. or more, and some 20,000 sq miles with 3 in. or more, and much greater areas for the 2-in. and 1-in. isohyetals. Assuming that the width of incoming tropical airstream was 200 miles, then the movement required for extracting the average depth of 4.7 in. within the 4-in. isohyetal is expressed by:

TABLE 3.—STORM OF MARCH 24 TO 27, 1913, AS RECORDED AT U. S. WEATHER BUREAU STATIONS IN CINCINNATI (C) AND DAYTON (D), OHIO

March	TEMPERATURE, IN DEGREES FAHRENHEIT						WIND VELOCITY, IN MILES PER HOUR				PRECIPITATION (INCHES OF DEPTH)	
	Maximum		Mean		Minimum		Maximum		Mean			
	C	*D*	*C*	*D*	*C*	*D*	*C*	*D*	*C*	*D*	*C*	*D*
24	70	64	62	58	55	51	18 W	50 S	....	18	2.21	2.95
25	70	59	63	54	56	48	26 SW	46 S	....	....	4.15	2.27
26	57	56	47	44	37	32	17 NW	....	....	....	1.11	....
27	37	32	32	28	27	23	22 W	....	....	....	trace	1.90 (estimated)

Horizontal wind velocity in miles per hour, $V = \dfrac{4.7 \times 60}{0.88 \times 24} = 13.3.$

To supply the larger areas with lesser depths included in this particular storm pattern, much greater wind velocities would have been required. Assumption of greater precipitable depths of moisture would reduce the corresponding wind movement, at least to stay within the maximum velocities observed at the Cincinnati station, as recorded in Table 3.

Quoting from a previous paper (24*a*):

> "The condensation of 1 in. of rain releases the equivalent of 2 h.p.-hours of energy per sq.ft. of surface, or nearly 87 000 h.p.-hours per acre. If applied exclusively toward heating the first mile in height of air column above sea level, the resultant increase of temperature would be 30° Fahr., or more, depending on the density and humidity; and this thermal change would cause an expansion ranging from 6 to more than 10 per cent. The resulting buoyancy would be of the same order as that possessed by ice or logs floating in water."

There is no more assurance of confining the released energy to the heating of the lower mile of the atmospheric column than of applying similar heat units solely to the melting of snow cover as the vapor condenses upon its surface. A considerable time factor is required to complete the transfer of the heat units—some portion to the condensed moisture, some to the snow, and the remainder to the surrounding atmosphere. Furthermore, the natural surface runoff on perceptible slopes may limit the time of contact for the condensed moisture and the snow surface, thus preventing complete transfer of the released energy. During a cold spell following a heavy snowstorm in Washington, D. C., attempts were made to remove some of the cluttering snow heaps at intersections by the application of blowtorch flames to tunnel through the compact mass. The slight trickles of water reaching the gutters and the slow progress in snow removal plainly demonstrated that heat was being dissipated into the surrounding atmosphere instead of into the packed snow.

Inasmuch as the first volume of daily *Climatological Data* began in January, 1914, before much progress had been made in regular observations aloft, it may be well to present some of the available records, pertaining to the storm of March 24 to 27, 1913, in Ohio (see Table 3).

In the absence of direct observations as to upper air conditions, only estimates and assumptions are available. Nevertheless, condensation and precipitation were proceeding vigorously at the prevailing temperatures as observed at U. S. Weather Bureau stations; therefore, the actual dew point for some of the air masses involved in the turbulence might have been somewhat higher than the station temperatures. At Kew, near London, England, it was observed that the normal variation of dew point under foggy conditions was nearly half the diurnal variation in station temperature, thus amounting to some 13° C or 23.4° F (17*a*). W. Napier Shaw has presented tabular data (25) showing the water-vapor capacity of saturated air up to 30,000 ft above sea level as more than 0.73 in. in depth when the temperature at the base of the column is 50° F. Field observations at St. Louis, Mo., with a maximum observed temperature of 77° F, gave the atmospheric moisture capacity at saturation as 2.28 in., of which 1.67 in. was subject to precipitation; and other stations recorded moisture capacities of 2.60 and 2.92 in., respectively, for temperatures of 77° and 73°, with 1.67 and 1.41 in. subject to precipitation (26).

3. Whether the temperature gradient was subjected to inversions, increasing the moisture capacity, or to other irregularities resulting from the general turbulence, the heavy precipitation and the corresponding release of energy from the condensing vapor was not recorded, as upper air data were not observed. No one can say authoritatively and convincingly, therefore, that the total moisture content of the air column was only 1.40 in. nor that the precipitable moisture at the beginning of the storm was only 0.88 in. or double that amount. Of vastly more importance are the foresight, determination, and courage displayed both in the establishment and the maintenance of the Hydrometeorological Division and in its guidance toward the solution of vital problems affecting the national security and economy.

After all, the rainfall-runoff relationship is a simple one, despite the complicated, elusive, and partly indeterminate factors involved. One of the important keys to its solution, the "unit hydrograph" (27), written following the

November, 1927, floods in New England, seems destined to stand as a monument to organized research. This clear concept, by L. K. Sherman, M. Am. Soc. C. E., was developed further by Mr. Bernard's keen perception and adroit adaptations—together with the determined efforts of the U. S. Geological Survey; the State of Pennsylvania; the Corps of Engineers, U. S. Army; the U. S. Weather Bureau; the Soil Conservation Service; and other federal or local agencies—to provide a safe and logical basis for analysis, forecasts, and design related to floods and plans for their control. Naturally, important contributions have come from the vanguard among consultants, such as W. W. Horner, M. Am. Soc. C. E., Robert E. Horton, M. Am. Soc. C. E., and Horace W. Wood, Jr., M. Am. Soc. C. E. Mr. Wood's work, "Flood Flow on Missouri Streams" (28), gives an unusually clear résumé of methods now in vogue, including empirical and other formulas together with the "unit hydrograph" in their appropriate settings.

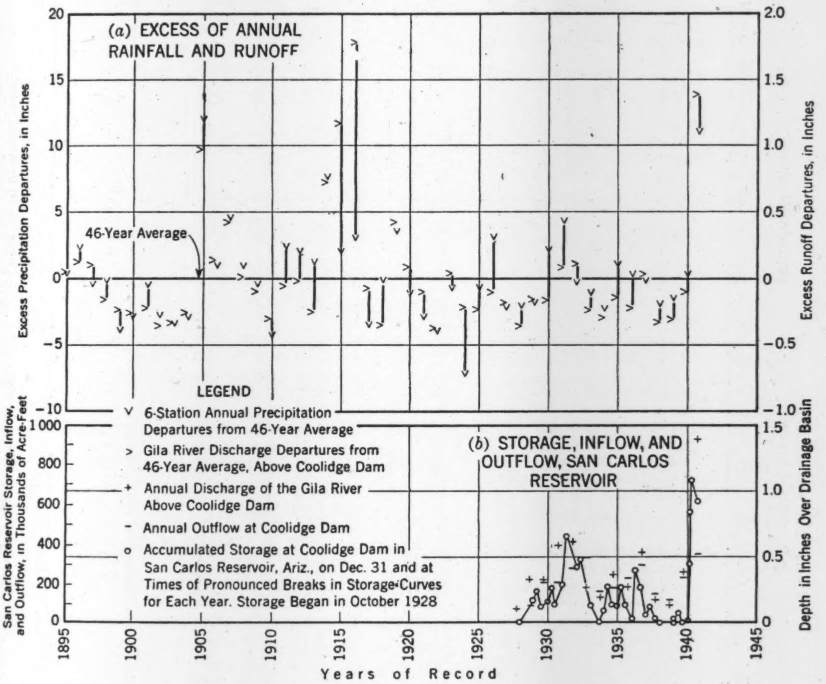


FIG. 21.—GILA RIVER BASIN, AS OBSERVED AT COOLIDGE DAM, ARIZONA; ANNUAL PRECIPITATION AND RUNOFF EXCESS COMPARED WITH 46-YR AVERAGES

In spite of the varied topography and hydrologic as well as meteorological characteristics of the semiarid Southwest, the writer has found some very consistent relationships between rainfall and runoff excess or deficiency, year by year, from 1895 to 1941, inclusive.

The 46-yr average values are as follows:

Description	Gila River basin	Rio Grande basin
Precipitation, in inches	11.80	12.83
Runoff:		
Inches of depth	0.492	0.840
Cubic feet per second	968	1,500
Thousands of acre-feet per year	339	1,084
Area:		
Square miles	12,880	24,200
Thousand acres	8,243	15,488

The foregoing yearly means of rainfall depths at six stations on the Gila River watershed above Coolidge Dam, Arizona (Fig. 21), and of the nine stations on the Rio Grande watershed above Elephant Butte, N. Mex. (Fig. 22), were subtracted from the respective 46-yr means for those same station groups, to give the excess or deficiency for each year. Corresponding departures from annual mean runoff depths per year throughout the same record periods, plotted on a scale 10 times as great, show fair agreement generally,

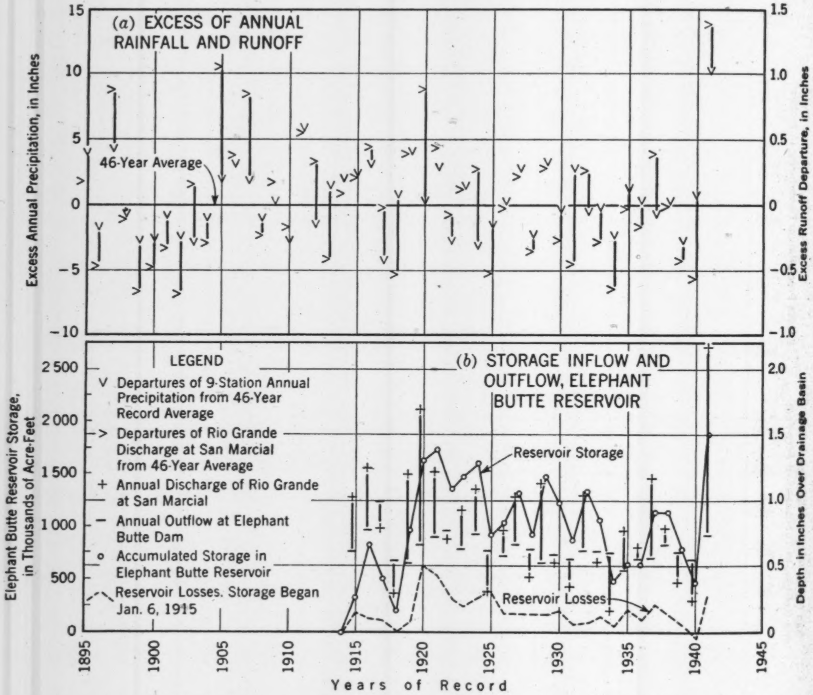


FIG. 22.—RIO GRANDE BASIN, AS OBSERVED AT SAN MARCIAL DAM, NEW MEXICO; ANNUAL PRECIPITATION AND RUNOFF EXCESS COMPARED WITH 46-YR AVERAGES

with several of the years recording nearly coincident plottings. Likewise, the cumulative effect of a series of relatively wet years, such as 1918 to 1920 on the Rio Grande and 1905 to 1907 or 1911 to 1916 on the Gila River, and the subnormal rainfall generally from 1932 to 1940 on both watersheds, culminated in great departures from normal runoff and associated storage volumes, either in underground or in major surface reservoirs.

In 1941, with rainfall nearly double the normal, runoff was recorded representing nearly 4 times the mean for the Gila River, and more nearly 2.7 times the mean for the Rio Grande at Elephant Butte (where the excess was 1.7 times the mean).

Sufficient data were available to determine the annual losses in Elephant Butte reservoir, with a mean of about 200,000 acre-ft and a maximum of 648,000 acre-ft in 1920 (during the initial filling of the higher contours which required heavy contributions to bank storage). During 1918, the reservoir losses were nearly balanced by return flow from bank storage; and, during the critically dry season of 1940, the return flow not only equaled the storage losses, but also provided a dividend of 48,000 acre-ft for use in the lower valley.

A similar approach would disclose interrelationships between either monthly or daily rainfall and runoff, provided all of the principal factors, such as antecedent rainfall and temperature, and their effect upon soil moisture, are given due consideration. Similar relationships for the time, distance, and volumetric quantities, which affect both peak and total discharge at successive stations along the river channel, would be found.

Neither the microscopic approach, through rainfall intensities per hour or minor fraction thereof, nor the telescopic approach, through record means and departures therefrom is capable of answering all the vital questions pertaining to water supply, flood forecasting, and best schedule for release of reservoir storage. Both of those approaches, and others occupying intermediate ground, should be regarded as independent witnesses contributing evidence on the problems involved. The author has done much toward integrating the available records as well as the logically derived implications, and thus has completed one of the "forward passes" previously attempted by the writer (23)(24) before such a wealth of data, methodology, and coordinated teamwork was at hand.

IVAN E. HOUK,[5] M. AM. SOC. C. E.—Estimates of maximum possible flood flows must often be made for use in designing spillways and other hydraulic structures, as well as for use in planning major flood control projects. The author's comprehensive discussion of meteorological problems involved in such estimates shows that real progress is now being made on this important phase of hydrologic work. The accompanying data regarding maximum possible storms and their relation to maximum possible floods should prove valuable to practicing hydraulic engineers, to flood control specialists, and to all scientists actively engaged in meteorological and hydrological researches.

Naturally, the occurrence of the maximum possible storm on a given drainage area does not necessarily mean the occurrence of the maximum possible flood, even though the duration of the storm rainfall may equal the time of concentration for the given area. This is especially true for relatively large drainage basins in temperate zones, where conditions of snow cover and surface soil moisture content constitute important influences on rates of flood runoff. It gradually becomes less certain as the size of the basin decreases. In the case of a comparatively small area subject to intense rainfall of the thunderstorm or cloudburst type, the maximum possible flood probably is caused by the maximum possible storm, since the rate of infiltration (even under optimum conditions) usually is only a small proportion of the rate of cloudburst precipitation.

---

[5] Senior Engr., Technical Investigations, Bureau of Reclamation, Denver, Colo.

The world's records of most intense point rainfall shown in Fig. 7 suggest the question as to whether or not they represent maximum possible rates of rainfall. The question is primarily of academic interest, since it probably never would be economically advisable to design bridges, culverts, sewers, spillways, or other hydraulic structures for the intense rates of runoff which such precipitations would produce. Probably the points shown on the diagram represent rates somewhere between 90% and 100% of maximum possible precipitation rates.

The discussion of the theoretical determination of storm rainfall constitutes an interesting introduction to a complicated meteorological problem. Although some items involved in the suggested method of calculation are not at present susceptible of accurate evaluation, the general method of attack seems logical and the continuance of such studies should lead eventually to a better general understanding of the problem. The application of Eq. 2 to some actual storm records and the publication of the results, including values of K, v, and W_E, would constitute a valuable addition to the paper.

W. G. HOYT,[6] M. AM. SOC. C. E.—In summarizing the progress made in one phase of hydrometeorology during the ten years since his earlier study in 1934 (29), Mr. Bernard has performed an excellent service to the engineering profession. In 1934 no attempt was made to answer the question as to whether or not the transposed storm was a "limiting" storm. The principal question raised at that time by the advising committee of hydrologic experts was whether or not, with reasonable likelihood, the great storms could have shifted slightly within a region, to center in a critical position on the particular basin of interest. As a result, however, of the research studies and observations described by Mr. Bernard, the hydraulic engineer of 1943 not only can transpose storms with reasonable surety but can determine, at least theoretically, with the meteorologic information now at his disposal, the potential limits of storm precipitation.

Credit certainly should accrue to the personnel of the various government agencies who have worked on the interrelated hydraulic and meteorologic problems and to the Corps of Engineers who in many instances made funds available for such studies. Not to be forgotten is the engineering staff of the Miami Conservancy District who early organized storm data for flood studies.

On June 15, 1932, a special Society Committee on Meteorologic Data submitted a progress report (30) in which they concluded among other things that, as of 1931, "Research in meteorology by the [Weather] Bureau, particularly along the lines which are used by the Engineering Profession, has not kept pace with the growing importance or utilization of this type of data, nor with research in other lines of science, either pure or applied." One reasonably might conclude from the evidence of Mr. Bernard's paper that, as of 1943, the research in meteorology—especially as it relates to the engineer in flood problems—is outstanding.

---

[6] Hydr. Engr. (Prin.), Water Resources Branch, U. S. Geological Survey, Washington, D. C.

L. R. BEARD,[7] JUN. AM. SOC. C. E.—Considerable progress has been made during the past several years in developing the science of hydrology. Perhaps the most difficult and most interesting development concerns the relation of rainfall to flood flow. This newer feature promises to rival that development in both difficulty and interest.

Because of the value of resolving a variable into its more fundamental components, it is advantageous to deal with meteorological factors rather than with observed precipitation. A disadvantage of dealing with meteorological factors is that, as Mr. Bernard states (see heading: "Principles of Storm Transposition"), there is available "a scanty 8 years of reasonably adequate upper-air data." These data are of utmost importance in the quantitative determination of precipitation.

In replacing a general function such as flood flow by its several component functions—infiltration, absorption, evaporation, return flow, transpiration, snow melt, detention, siltation, air moisture, wind velocity, frontal activity, convective activity, cyclonic activity, etc.—it is primarily necessary to relate the various component functions properly. However, if advantage is to be gained by this replacement, the probable error of the result obtained by using the component functions should be less than the probable error of the result obtained by using the general function. With the probable error of the result of each component function known, the probable error of the combined result can be computed by the equation (31):

$$R = \left[\sum_{j=1}^{j=m} \left(\frac{\partial Z}{\partial z_j} \right)^2 (r_j)^2 \right]^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which there are m component functions, and in which: R = probable error of the combined result; Z = the combined result expressed in terms of the component results; z = a component result; and r = probable error of a component result. The relation of runoff to the component factors is not often a simple differentiable function, and much simplification would be necessary in order to apply Eq. 3 mathematically. However, it should be noted that the probable error of the sum or difference of m quantities is equal to

$$r_m = \left[\sum_{j=1}^{j=m} (r_j)^2 \right]^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

and the probable error of the product of two numbers is equal to

$$r = [(z_1)^2 (r_2)^2 + (z_2)^2 (r_1)^2]^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

---

[7] Asst. Engr., U. S. Engrs., War Dept., Los Angeles, Calif.

The important point to be made is that the possibility of evaluating one factor very accurately should not be interpreted to mean that the combined result is necessarily evaluated accurately. If the maximum possible rainfall could be evaluated with great accuracy, then the probable error of the maximum possible flood derived therefrom would be slightly greater, on a percentage basis, than the probable error of the maximum possible over-all runoff coefficient. In general, the function "runoff coefficient" expressed in terms of infiltration, snow melt, detention, etc., is less reliable than the "runoff function," since it is derived from the latter. However, the runoff-coefficient function may converge more rapidly than does the runoff function, for which reason it is questionable whether the maximum possible runoff can be determined more accurately from rainfall than from runoff. The method by which maximum accuracy is obtained must be selected independently for each case.

In the determination of a maximum possible value of a function such as wind velocity, the procedure is not entirely mathematical. In general, such a value is merely considerably greater than any observed value (this is substantiated by the fact that the value is referred to as "maximum probable" by many). It is true that, in most cases, one scarcely can imagine a larger value. However, one hardly could imagine the occurrence of the Republican River flood of 1935 prior to that time. When the determination of the many factors (component functions) is made by "engineering judgment," a suggestion that the accuracy of the combination be investigated may be looked upon as a bit farfetched. However, the accuracy may be evaluated in the same manner as the quantity with the same justification.

There is one other point to be considered which is somewhat related to the accuracy discussion. Spillway and other designs sometimes are based on the maximum possible or maximum probable flood. In these cases, maximum possible values necessarily are evaluated as low as judgment allows (in order that construction costs will not be prohibitive). Remote or hazy possibilities are discounted. As a result, since the value cannot be strictly designated as maximum possible, it often is called maximum probable. The best that can be said for such a quantity is that it is perhaps a very large quantity and that it is the result of a decision making a design possible. Such a decision might not have been made if a more definable quantity were demanded. In other words, it "gets the job done." The worst that might be said for such a quantity is that its use as a maximum possible value would cause the following:

(*a*) The engineer would use the value as a basis for his calculations and his judgment and, consequently, for his decisions, and therefore would not have a sound basis for such decisions; and

(*b*) The economic calculations would be in error because the possibility of a disaster resulting from an occurrence in excess of the maximum probable occurrence is neglected or overlooked.

One example of a remote possibility which is ordinarily neglected is the extreme condition of snow melt. Partly melted snow could exist as slush in such a state that a small amount of heat would release tremendous quantities of water. Under these conditions, the maximum possible runoff volume virtually would be unlimited.

In designating a region as being meteorologically homogeneous, certain exceptions must be admitted. The "western side of the Pacific Coast Ranges," for example, can be considered meteorologically homogeneous for air-mass qualities, upper wind velocities, and many other qualities, with certain toleration. However, one rightfully cannot transpose the rainfall data shown in Table 1(*j*) to any part of the region. This is because the areas in which these excessive rainfall amounts occurred are areas of normally high rainfall and cannot be considered to be on a par with other areas in the region. Opids Camp, Calif., has a normal seasonal rainfall of almost three times that of Los Angeles, Calif., which is about 15 miles distant. Almost invariably the rainfall at Opids Camp is two to three times that at Los Angeles. In the mountains north and east of Los Angeles, the area which could be said to have reasonably constant rainfall characteristics would be limited to a few square miles in extent. There are undoubtedly many other areas in the United States where simple storm transposition would be impracticable and where the condition of meteorological homogeneity is restricted.

The convective cell of Fig. 17 is the type of phenomenon which produces cloudburst or thunderstorm precipitation. The height of such a cell normally is limited by the depth of an instability layer and always is limited by the height of the tropopause. Experiments with fluids which are not exactly analogous to air have shown that the horizontal diameter of such a cell is on the order of three to four times the depth of the cell. As Fig. 17 shows only the rising half of the cell's cross section, the drawing of the descending (outside) part of the cell would complete a cell whose diameter, in this case, is about two times the depth. Condensation would occur in the part of the cell shown (ascending air), and rainfall therefore would occur below and near this part. Only very light rainfall, if any, would occur below the descending part of the cell.

The type of rainfall which is steady and almost uniformly distributed over large areas generally is associated with a cyclone and, often, a frontal system. The cyclone, which may extend far into the stratosphere, is a result of dynamic instability and cannot be classed as a convective cell, which is a result of thermal instability. Often, however, thermal instability is present in the vicinity of a cyclone, in which case both types of rainfall are superimposed.

Numerous other complicated factors should be considered before a meteorological analysis can be attempted for any area. Some admirable work has been done in this line by J. Bjerknes, consulting meteorologist, and by the Pacific Division Office of the U. S. Engineer Department at San Francisco, Calif. This work should be of considerable value for the formulation of a procedure of analysis.

ROBERT E. KENNEDY,[8] M. AM. SOC. C. E.—This paper should be read in connection with a similar paper (20) on the same subject—a bit more technical perhaps—by A. K. Showalter and S. B. Solot, who were mentioned by Mr. Bernard under "Acknowledgments." The writer is interested in the meteorological conditions described by these men as limits beyond which nature cannot go but which nature reaches quite frequently. If that is so, students of this subject are approaching the ultimate sources.

---

[8] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

The background of approach to the problem in both these papers is to divide it into three parts: First, there is the maximum amount of moisture available for precipitation that can be brought to the region under consideration; second, there is the maximum wind velocity in the region that will feed the moisture into the storm vortex; and, finally, there is the maximum time limit of duration of the disturbance.

The first step is comparatively simple. In 1939 Mr. Solot outlined a method (32) of ascertaining the water in a column 1 sq in. in area and some miles high. From this point it is a logical step to ascertain the precipitable water in a column of saturated air and to tie it to the dew point at the surface. This was done by Messrs. Showalter and Solot (20*a*).

The moisture is brought in by some maritime air mass and is spread out like a blanket over an area extending from its source region over the ocean. Studies of air mass movements account for the disturbances that develop in this blanket. The trigger action that sets off the storm is described in both papers; but, whatever starts it, the action itself results in a maelstrom of updraft turbulence fed by the energy of latent heat released when the moisture is precipitated. The updraft provides temporary storage for large quantities of the neighboring blanket of moisture fed in at high wind velocity.

Both papers represent this action by the same figure. This is Fig. 17 in Mr. Bernard's paper, which represents the typical maximum storm as extending vertically to near the stratosphere with about one third of its height indraft, one third updraft, and the top one third outdraft of that part that gets away. All the precipitable moisture does not finally fall out as rain.

Fig. 17 appears to be entirely empirical based largely on visual appearance, mainly as seen in the miniature model of a summer thunderstorm. It seems to be a concept that has been generally accepted for a long time.

The curve, W_E, of effective precipitable water as related to surface dew point is another empirical relation, justified no doubt because it works within the limits of the data from which it was derived.

The limits of the surface dew points are described by Messrs. Showalter and Solot. The upper limit is about 80° F, which is about the maximum possible over the ocean from which the moisture blanket originates. These investigators find that it will not become less than 50° no matter how far the moisture is carried inland.

Just how the maximum probable dew point for the season is derived for a particular region is not indicated, but the authors appear confident they have approached the ultimate. The reader is referred by Messrs. Showalter and Solot to the Sacramento report for discussion of that feature.

The second step in the procedure is to estimate the velocity of inflow. Local anemometer records yield information as to wind velocities near the ground. Velocities in the upper part of the lower one third of the cell representing inflow are derived from the records of temperature and pressure at the 5-km level in the rear of major storms. A complete survey of the extreme minimum temperatures and pressures at that elevation has been made and is being kept up to date, according to Messrs. Showalter and Solot, who state that the maximum rate of upper air inflow is assumed to be reached in most of the major outstanding storms (20*b*)—another ultimate. These men have outlined briefly the procedure involved to estimate the geostrophic wind velocity at that elevation.

The third and final step is to estimate how long the storm will last. Apparently, Messrs. Showalter and Solot failed to find meteorological relations that can be evaluated readily. Resort is made largely to the statistical procedure of the hydrologists in which depth of precipitation, in inches, is plotted against duration in time. Both papers show such curves.

The reasons given for making these studies are twofold. The maximum storm as developed statistically by the hydrologists may be larger than actually can fall from the skies. Messrs. Showalter and Solot do not cite any such instance but it would be interesting if they had. Second, these studies give some measure of control in storm transposition. At least one is less likely to transpose a storm entirely out of its meteorological region. Storm transposition does not seem to be encouraged by these men. Mr. Bernard uses it cautiously. It is being supplanted by area-duration-depth data.

L. K. Sherman,[9] M. Am. Soc. C. E.—One of the tests, as to whether the occurrence of the ultimate maximum storms presented by the author are within the realm of reasonable probability, is furnished by computing the runoff that such storms would produce. Such computed runoff per square mile of basin may be compared by analogy with the large range of recorded runoffs. In any event, it is the amount of the possible flood that is wanted. The storm is simply a means to estimate the flood magnitude.

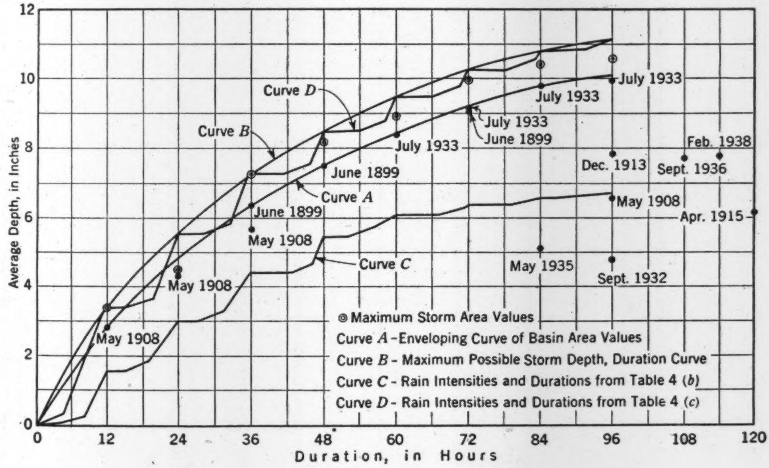


Fig. 23.—Revised Depth-Duration Curves, Red River Basin Above Denison, Tex.

---

[9] Cons. Engr., Chicago, Ill.

TABLE 4.—RECORD OF STORM ON THE RED RIVER BASIN, ABOVE DENISON, TEX., MAY, 1908; DRAINAGE AREA, 38,400 SQ MILES

Item No.	Description	ORDER OF MAGNITUDE								Total
		1	2	3	4	5	6	7	8	
	(a) OBSERVED DATA									
1	Order of events	2	3	4	1	5	6	7	8	....
	Rainfall, Wet Area:									
2	In inches	1.51	1.48	1.39	1.06	0.62	0.27	0.18	0.11	6.62
3	Percentage of basin	100.0	97.4	83.1	40.4	67.0	45.0	28.1	9.2	....
4	Rainfall, weighted, on the basin, in inches	1.51	1.44	1.15	0.43	0.42	0.12	0.05	0.01	5.13
5	Infiltration capacity, f, in inches per hour	0.30	0.23	0.23	0.30	0.23	0.20	0.20	0.20	....
	Duration, in Hours:									
6	Noneffective	3.9	4.0	3.9	3.7	4.1	3.9	5.0	7.0	35.5
7	Effective	3.9	5.1	4.9	1.9	3.8	1.3	0.0	2.0	22.9
8	Depth of noneffective rainfall, in inches	0.26	0.26	0.31	0.20	0.24	0.18	0.15	0.15	1.75
9	Average effective-rain intensity, i, in inches per hour	0.37	0.25	0.26	0.45	....	....	....	....	....
10	Basin runoff, in inches	0.528	0.312	0.250	0.094	0.121	0.034	0.001	0.000	1.33
	(b) RAINFALL IN ITEM 2 IS ASSUMED TO COVER THE ENTIRE BASIN; AND f IN ITEM 5 IS TAKEN AS $f_e = 0.20$									
	Rainfall, Wet Area:									
11	In inches	1.51	1.48	1.39	1.06	0.62	0.27	0.18	0.11	6.62
12	Percentage of basin	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
13	Rainfall, weighted, on the basin, in inches	1.51	1.48	1.39	1.06	0.62	0.27	0.18	0.11	6.62
14	Infiltration capacity, f_e, in inches per hour	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
	Duration, in Hours:									
15	Noneffective	3.9	4.0	3.9	3.7	4.1	3.9	5.0	7.0	35.5
16	Effective	3.9	5.1	4.9	1.9	3.8	1.3	0.0	2.0	22.9
17	Depth of noneffective rainfall, in inches	0.26	0.26	0.31	0.20	0.24	0.18	0.15	0.15	1.75
18	Average effective-rain intensity, i, in inches per hour	0.49	0.29	0.30	0.49	0.36	....	....	....	....
19	Basin runoff, in inches	0.76	0.41	0.42	0.47	0.17	0.074	0.0053	0.0007	2.31
	(c) MAXIMUM STORM ON THE RED RIVER BASIN (WRITER'S DERIVATION OF INTENSITY AND DURATIONS BASED ON CURVE B, FIG. 23)									
	Rainfall, Wet Area:									
20	In inches	....	....	....	....	....	....	....	....	11.1
21	Percentage of basin	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
22	Rainfall, weighted, on the basin, in inches	3.4	2.2	1.7	1.2	1.0	0.7	0.6	0.3	11.1
23	Infiltration capacity, f_e, in inches per hour	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
	Duration, in Hours:									
24	Noneffective	4.45	4.45	4.45	4.45	4.45	4.45	4.45	4.45	35.5
25	Effective	7.1	4.6	3.5	2.5	2.1	1.4	1.2	0.6	23.0
26	Depth of noneffective rainfall, in inches	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	2.16
27	Average effective-rain intensity, i, in inches per hour	0.45	0.42	0.41	0.37	0.35	0.31	0.28	....	....
28	Basin runoff, in inches	1.72	1.02	0.73	0.44	0.32	0.13	0.08	0.01	4.45

When he was called upon to estimate the maximum possible flood from the Red River basin above Denison, Tex., the writer utilized the author's maximum design storm shown as curve K in Fig. 8. This depth-duration curve, drawn to a natural scale, with more detail, is shown in Fig. 23, taken from a report of the U. S. Weather Bureau (33). Curves C and D were plotted in Fig. 23 by the writer.

The derivation of runoff from this rain depth-duration curve B presents some problems not encountered in deriving runoff from an observed storm. The storm patterns, intensities, and durations of rainfalls are unknown in this case.

As a guide in deriving these unknown quantities, the writer utilized the data from the largest storm of record which fell on the Red River basin above Denison—the storm of May 21–25, 1908. The data derived from the records of this storm are presented in Table 4(*a*). The 4-day storm period was divided into eight periods of 12 hr. Each 12-hr period was computed as if it were a separate storm. In this way, cognizance was taken of the continuous changes in the position of storm intensities, areas wetted, and durations of effective, noneffective, and no rain.

In this 1908 storm, the point of maximum rain intensity started at Childress, Tex.; then proceeded to Hobart and Pauls Valley, Okla; thence south and west along the Red River back to Childress; and finally moved to Harrington, Okla., and Denison. The movement was always in a clockwise direction. The four highest spot rainfalls in the 12-hr periods were 4.29 in., 4.42 in., 4.27 in., and 4.39 in. There were forty-four rain recording stations in and about the basin, with four first-class stations outside. Mass curves of rainfall were made for most of the rain stations by the U. S. Weather Bureau. With a template set to the angle of f (Item 5, Table 4(*a*)), the writer derived the time durations of rains given in Items 6 and 7, Table 4(*a*). The weighted averages used in deriving runoff were obtained with Theissen's method. For moving storms the writer prefers this method to isohyetal maps. The averaged 12-hr amounts and durations in Table 4(*a*) actually represent a large number of downpours and lulls, varying from zero to very high intensities throughout the 12 hr. They are grouped into two classes: Effective and noneffective rain.

Table 4(*b*) assumes that the storm of May, 1908, covered the entire basin of 38,400 sq miles to the average depths covered on the wetted fractional areas found in Table 4(*a*). It further assumes that the ground has been wetted by antecedent rainfall so that its infiltration capacity has been reduced to f_c, the ultimate minimum capacity of the basin for the early growing season, in which the major floods have occurred. Durations of rain are kept as per the actual storm on the basin. The resulting rain amounts have been plotted as curve C in Fig. 23. The assumptions here made are conservative, as evidenced by the fact that the amounts of rainfall are under the 12-hr amounts given by the U. S. Weather Bureau, marked "May, 1908." These latter points represent amounts from the May, 1908, storm when transposed to its most effective position on the basin.

Table 4(*c*) is a computation based on the maximum rainfall depth-duration curve B of the U. S. Weather Bureau. This rainfall totals 11.1 in. average depth over the basin. This is more than twice the amount of the largest storm of record.

Tables 4(*a*) and 4(*b*) indicate that depths of noneffective rain, in the largest observed storm on the basin, followed fairly uniform averaged amounts in each of the 12-hr periods. The observed average intensity of all the noneffective rains at each of the stations on the basin was about 0.05 in. per hr or 0.06 in. per hr. The total period of noneffective rain in the maximum storm probably will be longer than that of 1908. How much longer is unknown. It is not reasonable to suppose that the maximum storm will have a total duration less than that of 1908. Any assumption of increase in the duration of noneffective rain will reduce the volume of runoff. Therefore, the writer has assumed that the average intensity of noneffective rain will be 0.06 in. per hr and that the duration will be 35.5 hr, the same as in 1908. This gives 35.5/8 = 4.44 hr for each period. At 0.06 in. per hr the depth of noneffective rain is 0.27 in. This appears in Item 26, Table 4(*c*).

Following the same line of reasoning, the writer has adhered to the total duration of effective rains (23 hr) as it existed in the largest storm of record. However, this value is subject to comparison with the resulting average intensities. The average effective-rain duration for a 12-hr period is 22.9/8 = 2.86 hr. Item 7, Table 4(*a*), and Item 16, Table 4(*b*), indicate that the effective rain durations are longer than the average for the larger rains and shorter than average for the lesser rains. Accordingly, the writer has prorated the average duration in the proportions of each given 12 hr of effective rain depth to the average depth. The average effective rain depth is 11.1/8 = 1.39 in. Prorating to derive Item 25, Table 4(*c*):

$$\begin{aligned} 2.86 \text{ hr} \times 3.4/1.39 &= 7.1 \text{ hr} \\ 2.86 \text{ hr} \times 2.2/1.39 &= 4.7 \text{ hr} \\ 2.86 \text{ hr} \times 1.7/1.39 &= 3.5 \text{ hr, etc.} \end{aligned}$$

The derivations of infiltration capacities f for the basin were made from a number of observed rainfall-runoff records including the 1908 storm.

Values of average effective-rain intensity for 12 hr over the basin were derived thus (using Table 4(*a*) as an example):

$$\frac{\text{Item 4} - \text{Item 8}}{\text{Item 5}} = i \text{ or Item 9}$$

Runoff from the basin in Tables 4(*a*) and 4(*b*) was derived by taking the sum of the rain excess on each Theissen subarea multiplied by its percentage of the entire area. Runoff in Table 4(*c*) was found thus:

$$\text{Item 22} - \text{Item 26} - \text{Item 25} \times \text{Item 23} = \text{Runoff or Item 28}$$

Runoffs, in cubic feet per second, were derived by the unit-hydrograph method; 12-hr distribution graphs were used against the eight derived depths of runoff in the sequence of their occurrence. In the cases where f_c was a constant, the sequence or order is immaterial.

The Red River basin above Denison includes two major streams which unite just above Denison. One is the Red River proper with a drainage area of 30,600 sq miles, and the other is the Washita with 7,700 sq miles. Hydrographs from several storms showed that peak from the Washita sometimes followed and sometimes preceded the Red River peak, thereby showing that synchronization of the two-peak flow was possible. Accordingly, separate unit hydrographs were derived for the Red and Washita rivers and the two combined to derive the ultimate possible peak flow.

The peak of the flood hydrograph, from the design storm (curve B, Fig. 23) at Denison, will be 1,500,000 cu ft per sec or 39 cu ft per sec per sq mile. This is based on the possible synchronization of the Red and Washita peaks.

The more probable peak flow, without synchronization, will still range from 1,100,000 to 1,300,000 cu ft per sec. The peak flow over a spillway, of course, will be reduced in accordance with available flood storage back of the dam.

The peak runoff of 1,500,000 cu ft per sec does not mean that the total volume of the maximum flood is increased. That volume remains the same whether the peaks synchronize or not.

In combining unit hydrographs from different areas, the distribution graph must be expressed in inches and not in percentages.

The writer is of the opinion that: (*a*) The author's "design storm" is what it purports to be—"the maximum possible storm"; (*b*) it should be used in the design of all structures which, in the event of failure, would involve large loss of life and property damage; (*c*) the values for runoff thus derived call for no additional factor of safety, since that factor has been cared for by the concatination of possible events; and (*d*) the author's "design storm" furnishes far more reliable limits than any data derived by statistical analysis. The latter, based on a long-time record, does furnish a possible check on the engineer's computations and may aid him in deciding whether a particular structure warrants the elimination of certain remote hazards.

C. H. EIFFERT,[10] M. AM. SOC. C. E.—Much new information has been collected in this field during the past few years, perhaps largely because of the increased interest in aviation. The U. S. Weather Bureau and Mr. Bernard are to be commended for the progress in this direction.

When the Miami Conservancy District investigations were begun about 1913, little information was available outside of the Weather Bureau records of precipitation, temperature, and wind movement. Therefore it is quite gratifying to note that what Mr. Bernard calculates to be the maximum possible storm that could occur, in the area where the 1913 flood actually did occur, agrees quite closely with what was decided on as the maximum possible when the Miami Conservancy District studies were made.

In connection with his calculations Mr. Bernard reaches the following conclusion (see heading, "Procedure in Developing the Maximum Possible Storm for an Individual Basin"): "* * * the March, 1913, storm rainfall could have been 28% greater or 128% of the observed depths."

The following paragraph is quoted from "Storm Rainfall of Eastern United States," Part V, *Technical Reports of the Miami Conservancy District* (34), under the heading, "Reasons for Choosing as Basis of Design a Flood 40 Per Cent Greater than that of March 1913":

---

[10] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

> "The extensive investigation of storms in the eastern United States leads to the belief that the March, 1913, flood was one of the great floods of centuries in the Miami Valley. In the course of three or four hundred years, however, a flood 15 or 20 per cent greater may occur. It is not believed that a flood will ever occur which is more than 20 or 25 per cent in excess of that of March 1913. There is a factor of ignorance, however, which should be provided against, and the only way this could be done was arbitrarily to increase the size of maximum flood provided for. If longer records were available a closer estimate could be made, but in planning works on which the protection of the Miami Valley depends, it was necessary to go beyond human judgment. This was done on all the other phases of the design, and it was believed that it would not be good engineering practice to stop at individual judgment on this phase. The planners of the project felt that they must be able to say that the engineering works are safe in every respect. For this reason provision was made for a flood nearly 40 per cent greater than that of March 1913. This is 15 or 20 per cent in excess of what is believed to be the greatest possible flood that will ever occur."

In this discussion the writer cannot refrain from referring again to the possibility of Weather Bureau records being kept in such a way as to be more useful and less difficult to interpret by engineers making hydrological studies. The principal Weather Bureau stations record their precipitation from midnight to midnight; many observers take evening readings; many others make their observations in the morning; and some are recorded from noon to noon. When storm rainfall maps and studies involving such records are prepared, it is often very difficult and sometimes impossible to determine on which day a considerable part of the rainfall actually occurred. This is particularly true in connection with storms of short duration. Why would it not be possible to change this system so that all readings are taken at approximately the same time of day? Most observers could make either morning or evening observations, and those who could not read at the required time could be replaced gradually. Obviously it is not practical for the ordinary observer to make his observations at midnight. However, the midnight stations have recording gages so that their records are adaptable to any time and could be used in connection with either morning or evening records. Originally, the Weather Bureau rainfall observations were not intended for this purpose, being thought of mainly in connection with agriculture. The extensive use of these records in recent years by engineers and hydrologists would seem to be of sufficient importance to justify such a change.

On the first page of his paper, the author refers to time-area-depth values for about 150 storms. Throughout the remainder of the paper these values are referred to as area-duration-depth values. On the Miami Conservancy District work the term time-area-depth was used. Although the word "duration" is slightly more specific in its meaning, either expression is readily understandable. "Time-area-depth" would seem to this writer to be preferable for the sake of its euphony and greater ease of oral expression.

GAIL A. HATHAWAY,[11] M. AM. SOC. C. E.—Excellent progress is being made in this extremely important phase of flood control as well as in connection with all problems involved in the development and utilization of water resources in the United States. The paper is a comprehensive report of this progress. The investigation of the meteorological and hydrological aspects of major storms by the Hydrometeorological Section of the U. S. Weather Bureau has furnished the engineer with a much clearer picture of the probable limiting rates of storm rainfall.

---

[11] Hydr. Engr., War Dept., Office, Chf. of Engrs., Washington, D. C.

In discussing the general subject of storm studies it may be of interest to review briefly some of the events leading to the present work of the Hydrometeorological Section of the Weather Bureau which has proved so valuable in the flood control work of the Corps of Engineers. Shortly after the great storm of May 30–31, 1935, in the headwaters of the Republican River basin, the writer was engaged on a study to determine a spillway design flood for the Kingsley Dam (previously the Keystone Dam) on the North Platte River near North Platte, Nebr. The question immediately arose as to whether it would be reasonable to expect a storm similar to that of May, 1935, to occur over the drainage area of the North Platte River above Kingsley Dam, which is about 150 miles north of the Republican River headwaters. At that time E. J. Minser (chief meteorologist, Transcontinental and Western Air, Inc.) and the writer made a meteorological analysis of the storm to ascertain the feasibility of transposing it to the North Platte River drainage area. After careful study the conclusion was reached that there was no good reason, from the meteorological or physiographical viewpoint, why the storm could not have occurred over the North Platte basin above Kingsley Dam. Similar meteorological studies were made by the writer in 1935 and 1936, while he was employed by the U. S. Engineer Department, for the Sardis Dam on the Little Tallahatchie River in Mississippi, the Bluestone Dam on the New River in West Virginia, and the Possum Kingdom Dam on the Brazos River in Texas.

During the summer of 1936, arrangements were made by the Corps of Engineers for the assignment to the Office, Chief of Engineers, of the late Wilson Reed, Jr., to conduct meteorological studies of selected river basins. However, the demand from the field offices of the Engineer Department for meteorological studies was so great that, later in 1937, a group was set up in the Weather Bureau to make the studies. As stated by the author, this group had been working for, and in cooperation with, the Engineer Department in the review of the meteorological aspects of storms and the determination of maximum storm rates for specific drainage basins.

The first line of attack in any hydrological problem involving runoff is a thorough analysis of rainfall data. The author has reviewed the historical background of storm records and the early investigation and analysis of storm rainfall data. The early studies of storm rainfall are extremely valuable, but the present-day hydrologic technique requires that rainfall intensities be broken down into periods of less than 24 hr. Analysis of a large number of major storms in the United States shows that the maximum rainfall intensities usually occur in periods of 12 hr or less, interspersed with periods of minor rainfall rates. It is considered essential, therefore, that maximum rainfall quantities be determined for periods of 6 hr and in some cases for shorter time intervals. Because of the importance of a thorough analysis of storm rainfall the Engineer

Department, in 1937, organized a program for the study of several hundred major storms covering the entire continental area of the United States. The program also includes the analysis of current storms of major proportions. These recent storm studies are extremely valuable because of the existing network of recording rainfall stations, and the large number of upper-air observations made by means of pilot balloons and "radiosondes."

The study of major storms by the division and district offices of the Engineer Department has been divided into Part I and Part II. Part I of the study consists of the compilation of basic precipitation data, preparation of mass rainfall curves, and isohyetal maps. In addition to published records of the Weather Bureau, Part I includes all available precipitation data and miscellaneous information on storms obtainable from the manuscripts of original records, files of municipal agencies, newspapers, testimony of witnesses, and similar sources. In the event of a major storm, representatives of district offices conduct a field survey in the areas of excessive rainfall to obtain a record of rainfall measurements secured by local residents. The location and method of obtaining these "bucket" or "washtub" records are analyzed carefully on the ground and every effort is made to classify them as to their reliability. Part I of the study, assembled and organized by the various district offices, is reviewed by the Hydrometeorological Section of the Weather Bureau and the mass rainfall curves are correlated with the meteorological analysis of the storm.

Upon completion of the review by the Hydrometeorological Section, Part I is returned to the originating district office for completion of Part II of the study. The preparation of Part II involves the following steps:

(*a*) Preparation of the final total-storm isohyetal map based on the selected storm period, or delineation of zones on the isohyetal map and grouping of rainfall stations by zones;

(*b*) Tabulation of contemporaneous rainfall quantities as scaled from mass rainfall curves, for periods usually increasing by 6-hr increments;

(*c*) Tabulation of absolute maxima rainfall quantities for durations of 6, 12, 18, and 24 hr for stations within the zones of excessive rainfall intensity;

(*d*) The computation of mass rainfall curves representing the average depth of rainfall over selected areas of the storm; and

(*e*) The computation of maxima duration-depth-area data for various combinations of zones.

The final isohyetal map showing the zone subdivisions and the duration-depth-area curves for a typical major storm are shown in Figs. 24 and 25. In this case, mass rainfall curves were prepared for stations underscored with a single line; and recording-gage records were available for stations underscored with a double line. In most cases, rainfall values of less than 3 in. were interpolated from published records without constructing mass curves. Hence they are approximate. For the purpose of clarity, the odd numbered isohyets above 4 in. have been deleted from the isohyetal map in Fig. 24. All of the isohyets (that is, 1 to 21 in.) were used in computation of the depth-area data in Fig. 25. Pertinent data sheets similar to that shown in Fig. 26 are prepared

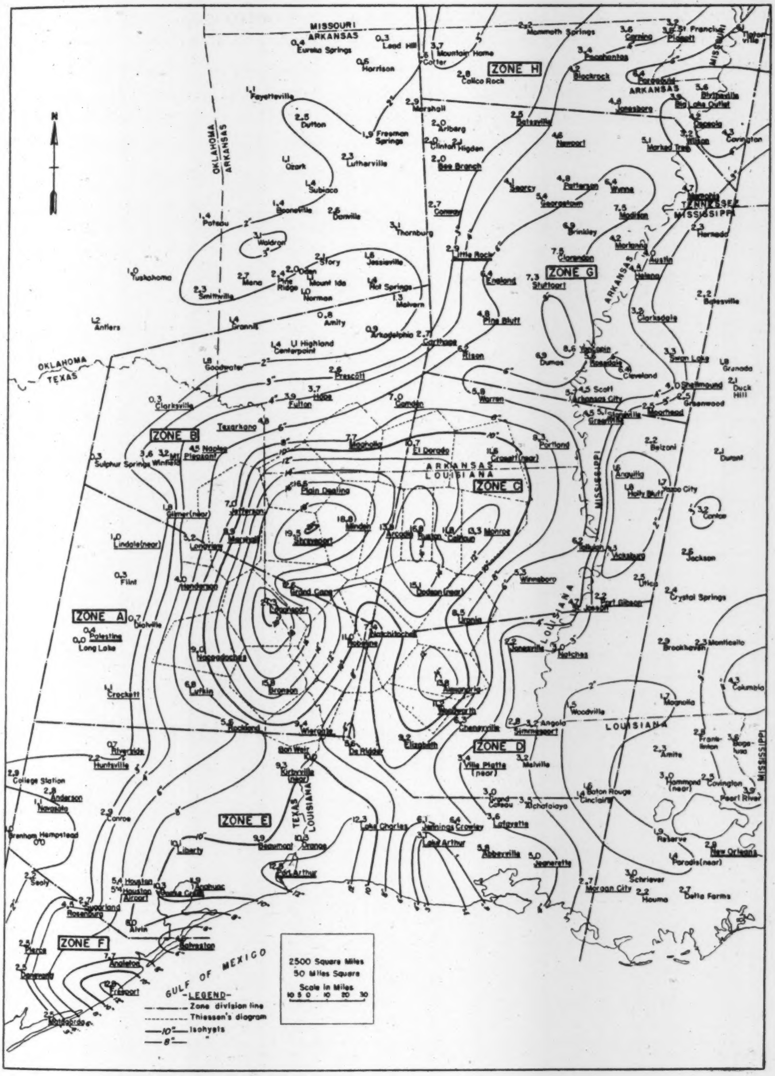
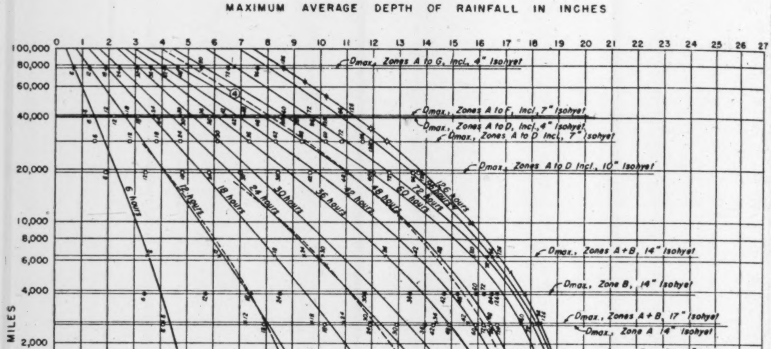

MISSOURI
ARKANSAS
OKLAHOMA
ARKANSAS
OKLAHOMA
TEXAS
ARKANSAS
LOUISIANA
TENNESSEE
MISSISSIPPI
TEXAS
LOUISIANA
LOUISIANA
MISSISSIPPI
GULF OF MEXICO
N
ZONE A
ZONE B
ZONE C
ZONE D
ZONE E
ZONE F
ZONE G
ZONE H
0.4 Eureka Springs
0.3 Lead Hill
0.6 Harrison
3.7 Mountain Home
1.6 Cotter
2.8 Calico Rock
2.2 Mammoth Springs
3.6 Corning
3.8 Piggott
St Francis
Tiptonville
3.4 Pocahontas
4.2 Blackrock
6.4 Paragould
5.6 Blytheville
3.9 Big Lake Outlet
4.6 Jonesboro
4.2 Osceola
3.2 Wilson
4.3 Covington
5.1 Marked Tree
1.1 Fayetteville
2.5 Dutton
1.9 Freeman Springs
2.9 Marshall
2.0 Ariberg
2.0 Clinton
2.1 Higden
2.0 Bee Branch
2.5 Batesville
4.6 Newport
1.1 Ozark
2.3 Lutherville
1.4 Subiaco
1.4 Booneville
2.6 Danville
1.4 Poteau
3.1 Waldron
2.7 Conway
3.1 Thornburg
4.1 Searcy
4.8 Patterson
6.4 Wynne
5.4 Georgetown
7.5 Madison
4.7 Memphis
2.3 Hernando
6.9 Brinkley
4.2 Marianna
4.0 Austin
4.5 Helena
2.9 Little Rock
7.5 Clarendon
6.4 England
7.3 Stuttgart
2.1 Story
1.6 Jessieville
2.0 Oden
1.1 Mount Ida
1.0 Tuskahoma
2.7 Mena
2.4 Pine Ridge
1.0 Norman
1.6 Hot Springs
1.3 Malvern
2.3 Smithville
0.8 Amity
1.2 Antlers
1.4 Grannis
0.9 Arkadelphia
2.7 Carthage
4.8 Pine Bluff
2.2 Batesville
3.2 Clarksdale
1.4 Centerpoint
U Highland
8.6 Yancopin
5.5 Rosedale
6.4 Cleveland
6.5 Rison
6.9 Dumas
3.3 Swan Lake
1.8 Granada
1.8 Goodwater
2.6 Prescott
2.1 Duck Hill
5.8 Warren
4.5 Scott
5.7 Arkansas City
4.0 Shellmound
2.5 Greenwood
0.3 Clarksville
3.9 Fulton
3.7 Hope
7.0 Camden
4.5 Greenville
5.1 Stoneville
2.5 Moorhead
4.8 Texarkana
7.7 Magnolia
10.7 El Dorado
8.3 Portland
2.2 Belzoni
2.1 Durant
0.3 Sulphur Springs
3.6 Winfield
3.2 Mt Pleasant
4.5 Naples
11.6 Crossett (near)
1.6 Anguilla
1.8 Holly Bluff
1.7 Yazoo City
16.6 Plain Dealing
3.2 Canton
1.8 Gilmer (near)
7.0 Jefferson
18.8 Minden
13.8 Arcadia
16.8 Ruston
11.8 Calhoun
13.3 Monroe
19.5 Shreveport
8.5 Marshall
3.2 Longview
1.0 Lindale (near)
6.2 Tallulah
4.3 Vicksburg
2.6 Jackson
0.3 Flint
4.0 Henderson
12.6 Grand Cane
15.1 Dodson (near)
5.3 Winnsboro
2.5 Utica
2.4 Crystal Springs
Logansport
0.7 Diolville
0.4 Palestine
0.0 Long Lake
4.7 St Joseph
2.2 Port Gibson
7.4 Natchitoches
11.0 Robeline
8.5 Urania
9.0 Nacogdoches
2.2 Jonesville
3.0 Natchez
2.9 Brookhaven
2.3 Monticello
15.9 Bronson
6.8 Lufkin
Alexandria
4.3 Columbia
1.1 Crockett
11.2 Woodworth
1.5 Woodville
1.7 Magnolia
5.6 Rockland
9.4 Wiergate
6.3 Cheneyville
2.8 Simmesport
3.2 Angola
0.7 Riverside
Bon Weir
5.6 De Ridder
9.2 Elizabeth
2.8 Franklinton
3.6 Bogalusa
2.2 Huntsville
10.0
2.3 Amite
2.9 College Station
9.3 Kirbyville (near)
3.4 Ville Platte (near)
3.2 Melville
2.8 Anderson
1.1 Navasota
3.0 Hammond (near)
2.5 Covington
3.0 Grand Coteau
3.5 Atchafalaya
1.6 Baton Rouge
1.4 Cinclaire
3.9 Pearl River
2.9 Conroe
12.3 Lake Charles
6.1 Jennings
6.4 Crowley
3.6 Lafayette
1.0 Brenham
0.0 Hempstead
10.1 Liberty
9.9 Beaumont
10.3 Orange
3.7 Lake Arthur
1.9 Reserve
5.8 Abbeville
5.0 Jeanerette
2.9 New Orleans
1.4 Paradis (near)
12.8 Port Arthur
2.2 Sealy
5.4 Houston
5.4 Houston Airport
10.3 Goose Creek
4.9 Anahuac
3.0 Schriever
2.7 Morgan City
2.2 Houma
2.7 Delta Farms
2.7 Sugarland
4.5 Rosenburg
8.0 Alvin
Galveston
2.5 Pierce
7.7 Angleton
2.5 Danevang
12.8 Freeport
2.5 Matagorda
2"
3"
4"
5"
6"
8"
10"
12"
14"
16"
18"
2500 Square Miles
50 Miles Square
Scale in Miles
10 5 0 10 20 30
-LEGEND-
Zone division line
Thiessen's diagram
10" Isohyets
8" "

FIG. 24.—ISOHYETAL MAP; STORM OF JULY 22 (1:00 A.M.) TO JULY 27 (7:00 A.M.), 1933

for each storm study for use of the various district offices in determining whether the completed storm studies have a direct bearing on problems in their respective offices. The sheets also serve as an index to the data assembled in each study.

At the beginning of the storm study program, 1,350 storms were assigned to district offices of the Engineer Department. However, it is not expected



Explanation

"D_{max}, Zones A + B, 14" isohyet" refers to maximum depth-duration values computed for the area within zones A + B and the 14 in. isohyet.

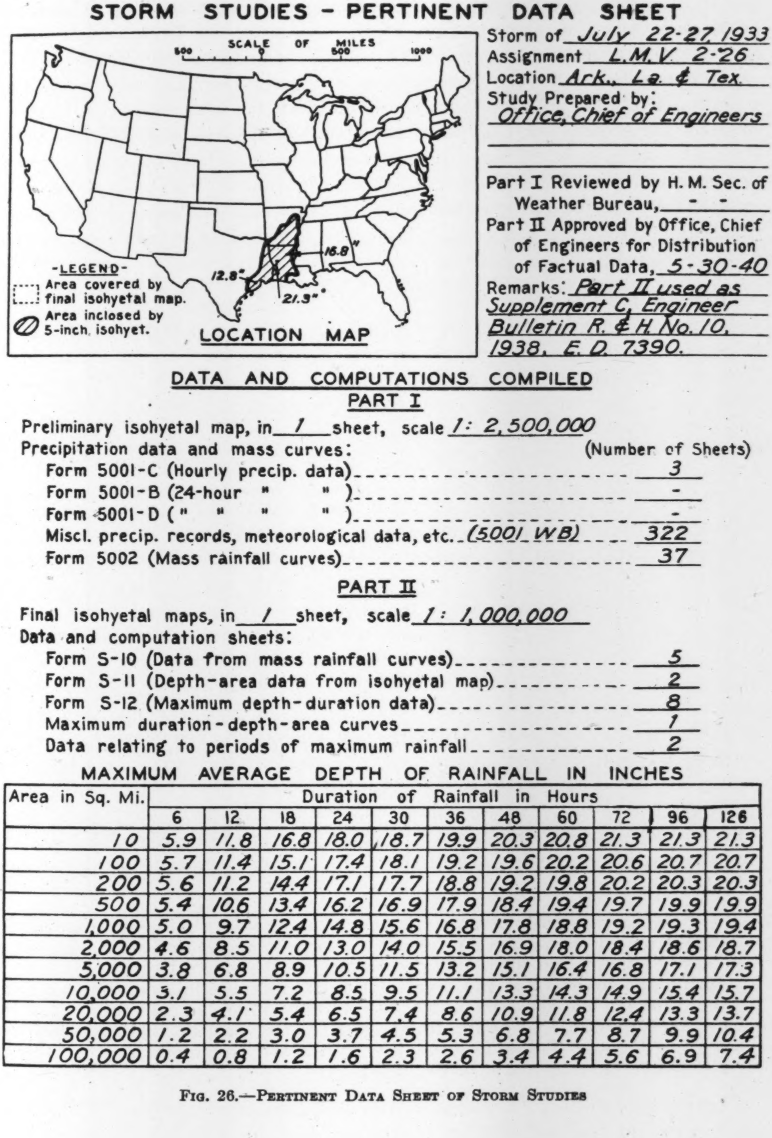
Curves shown by solid lines represent final maximum depth-area curves.

Depth-area curves shown in broken line (- - -) were computed (for comparison only) by planimetering isohyetal maps for following periods. (1) 12 hr ending 1 A 24, (2) 24 hr ending 7 A 24, (3) 48 hr ending 7 P 24, (4) 48 hr ending 1 P 25.

Plotted points represent maximum average depth of rainfall, over the area designated, within the number of hours indicated by figures beside the points.

Identical legends were used for all depth-duration curves within the same zone or combination of zones.

FIG. 25.—MAXIMUM DEPTH-AREA CURVES; STORM OF JULY 22 TO 27, 1933


SCALE OF MILES
500
0
500
1000
16.8"
12.8"
21.3"

STORM STUDIES - PERTINENT DATA SHEET

-LEGEND-
Area covered by final isohyetal map.
Area inclosed by 5-inch isohyet.

LOCATION MAP

Storm of July 22-27, 1933
Assignment L.M.V. 2-26
Location Ark., La. & Tex.
Study Prepared by: Office, Chief of Engineers

Part I Reviewed by H. M. Sec. of Weather Bureau, - -
Part II Approved by Office, Chief of Engineers for Distribution of Factual Data, 5-30-40
Remarks: Part II used as Supplement C, Engineer Bulletin R. & H. No. 10, 1938, E. D. 7390.

DATA AND COMPUTATIONS COMPILED

PART I

Preliminary isohyetal map, in 1 sheet, scale 1: 2,500,000

Precipitation data and mass curves: (Number of Sheets)

Form 5001-C (Hourly precip. data)	3
Form 5001-B (24-hour " ")	-
Form 5001-D (" " " ")	-
Miscl. precip. records, meteorological data, etc. (5001 WB)	322
Form 5002 (Mass rainfall curves)	37

PART II

Final isohyetal maps, in 1 sheet, scale 1 : 1,000,000

Data and computation sheets:

Form S-10 (Data from mass rainfall curves)	5
Form S-11 (Depth-area data from isohyetal map)	2
Form S-12 (Maximum depth-duration data)	8
Maximum duration-depth-area curves	1
Data relating to periods of maximum rainfall	2

MAXIMUM AVERAGE DEPTH OF RAINFALL IN INCHES

Area in Sq. Mi.	Duration of Rainfall in Hours										
	6	12	18	24	30	36	48	60	72	96	126
10	5.9	11.8	16.8	18.0	18.7	19.9	20.3	20.8	21.3	21.3	21.3
100	5.7	11.4	15.1	17.4	18.1	19.2	19.6	20.2	20.6	20.7	20.7
200	5.6	11.2	14.4	17.1	17.7	18.8	19.2	19.8	20.2	20.3	20.3
500	5.4	10.6	13.4	16.2	16.9	17.9	18.4	19.4	19.7	19.9	19.9
1,000	5.0	9.7	12.4	14.8	15.6	16.8	17.8	18.8	19.2	19.3	19.4
2,000	4.6	8.5	11.0	13.0	14.0	15.5	16.9	18.0	18.4	18.6	18.7
5,000	3.8	6.8	8.9	10.5	11.5	13.2	15.1	16.4	16.8	17.1	17.3
10,000	3.1	5.5	7.2	8.5	9.5	11.1	13.3	14.3	14.9	15.4	15.7
20,000	2.3	4.1	5.4	6.5	7.4	8.6	10.9	11.8	12.4	13.3	13.7
50,000	1.2	2.2	3.0	3.7	4.5	5.3	6.8	7.7	8.7	9.9	10.4
100,000	0.4	0.8	1.2	1.6	2.3	2.6	3.4	4.4	5.6	6.9	7.4

FIG. 26.—PERTINENT DATA SHEET OF STORM STUDIES

that all of the storms under study will be completed to the point of computing 6-hr rainfall quantities and preparing duration-depth-area data for short-time intervals. Only the major storms in each section of the United States with critical rates for different intervals of time will be analyzed completely.

In connection with the presentation of duration-depth-area data by the author in Table 1 it appears desirable to point out that the depth-area relationships for the August 6–7, 1935, storm in Ohio, and the July 4–5, 1939, storm in Kentucky were derived largely from unofficial or so-called "bucket" observations; but no footnote has been added to indicate that the results were obtained from other than official observations. A brief footnote in such instances, concerning the source and relative accuracy of the data, would seem appropriate.

L. STANDISH HALL,[12] M. AM. SOC. C. E.—A valuable summary of modern developments in meteorology for application to flood flow estimating is contained in this paper. In general, the rôle of meteorology as applied to flood flows is in the determination of maximum rates of precipitation. The analysis must include time-area-depth studies to cover drainage areas of various sizes, and for this purpose Table 1 contains valuable data. Having made similar studies for specific storms the writer can appreciate the time and effort necessary in the compiling of such a table.

Fig. 7 is of great interest in developing an upper limit for maximum rainfall rates. As pointed out by the author, before one can arrive at the maximum rate for a specific area the physiographic influences particular to the region in which the basin is located must be considered. As an example, on the western side of the Coast Range in the vicinity of San Francisco, Calif., a rainfall rate of 1 in. per hr can be expected once in 25 years. Whereas a rainfall rate of 2 in. per hr would occur less frequently than 100-yr intervals. This rate is materially below the world's record of between 10 and 15 in. in 1 hr, and the use of a rate in excess of 2 in. per hr for the San Francisco area would be extravagant. On many structures the use of an even smaller "design" flood would be justified when the damage caused by the rare flood would not justify the added expense of the larger structure.

Storm transposition must be used with caution in regions where orographic influences are a predominant factor. The author has not stressed this feature sufficiently. The report of the Miami Conservancy District focuses attention on this point (1*a*):

> "In beginning this investigation it was uncertain what geographic limits should be set to obtain the largest amount of useful and pertinent data. These limits at first were assumed to include the entire United States, but during the progress of the investigation it became apparent that rainfall conditions in the Western part of the country differ radically from those in the Eastern part, that by limiting the area to that east of the 103d meridian, running through Texas, Colorado, Nebraska and North and South Dakota, no applicable data would be excluded."

Under the heading, "Regions of Meteorological Homogeneity," the author has recognized the variation in rainfall intensity in California by including portions of the state within four areas (Nos. 1, 2, 3, and 7). Even with this subdivision two important areas are omitted, namely the west side of the Sierra Nevada and the east side of the Sierra Nevada. Marked variation of rainfall intensity created by the rugged topography could be cited within the boundaries of these six areas.

---

[12] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

With reference to the contribution of snow melt to storm runoff, the author's statement, that the flood hazard increases when the snow accumulates beyond normal depth and water content, must be modified with respect to the Sierra Nevada in California. For those unacquainted with this area it should be noted that the depth of annual snowfall is not exceeded in any other part of the United States, and during the period from January to March the normal depth of snow on the ground above El. 6,000 is from 6 ft to 10 ft. Heavy snow cover over the high Sierra acts as an influence in the suppression of flood conditions.

Rainfall over the mountain areas of the Sierra is not the only element increasing floods of large magnitude in the Central Valley of California, but air temperature during the storm and the amount of snow on the ground prior to the storm are contributing factors. If cold storms have placed snow at low altitudes which is followed by a warm rain, the danger of a large flood is greatly increased. The greatest flood danger occurs when there is less than 4 ft of snow over all or a large part of the drainage area. After the snow has accumulated to a greater depth, warm rain serves only to compact the snow but does not yield much runoff. When the depth of snow is less than 4 ft, and particularly when it is less than 2 ft, a warm rain and the accompanying warm air currents can frequently melt all of the snow. Observations have been made at an elevation of approximately 4,000 ft near Calaveras Big Trees, Calaveras County, California, of a depth of 4 ft of snow being melted entirely between weekly visits during a period of high temperature and warm rains. The large flood of December, 1937, was the result of a warm rain at a time when there was less than 2 ft of snow over a large portion of the Sierra (35). The same conditions occurred in 1928 during the large flood of that year (36). Prior to the warm rain in the latter part of March, a maximum of only 4 ft of snow had been accumulated on the higher altitudes of the watersheds. This rain melted most of the snow, with the result that very large floods occurred. Prior to the flood of 1928, there was a general belief among engineers that a major flood could not occur in a year of subnormal runoff. The reports available indicate that, in the record floods of 1861–1862, there was snow at very low elevations in the foothills prior to the storm. There were really three major floods in that year; one in December, 1861, and two in January, 1862, when warm rains melting snow augmented the runoff (37). To be sure, rains of unprecedented volume occurred at the same time. Without citing further examples, it can be stated as a result of observations of the snow-pack and meteorological conditions accompanying large floods in the Sierra that, whenever the snow is at low altitudes in the foothills and a warm rain of great intensity occurs, these are the conditions that produce the maximum flood runoff.

The procedure outlined by the author in developing the maximum possible storm for an individual basin offers a new approach to this problem of applied hydrology. With proper modifications based on the characteristics of individual basins it should yield dependable estimates of maximum flood flow. In addition to precipitation and depth of snow on the ground there are many other factors that have a very marked influence on volume of flood flow to be expected from a specific rate of precipitation. These factors for the most part are outside of the scope of the paper.

MERRILL BERNARD,[13] M. AM. SOC. C. E.—Mr. Gilman raises the natural doubt that the maximum storm necessarily produces the maximum flood. The writer's definition of the ultimate or maximum storm will be found to qualify magnitude in considering the principal exterior influences on floods—the snow-melt contribution to runoff; the loss to flow volume, infiltration; and the loss to peak flow, retardation. Perhaps it would be better to modify still further the definition by referring to the "effective" maximum storm. It is more important to note the differences in net effect between storms producing equal volumes of rainfall, in which case differences in distribution through time and over area result in flood hydrographs of varying shape and peak discharge. The apparent inability to sustain excessive intensities over any but relatively small areas would indicate the impossibility of experiencing concurrent maximum precipitation over the watersheds of every tributary of a basin except in the case of a very small one. The critical synchronization of tributary flow, like the most critical storm rainfall pattern, will depend largely upon the characteristics of the particular basin. On the larger basins, synchronization is involved in the direction of storm movement. This is not to be confused with the so-called "storm track" comprised of the trace of movement of an area of low barometric pressure. In certain localities physiographic controls will establish a fairly stable régime in the direction of movement of the "rain front" of "rain pattern." More often, the direction is fortuitous, depending usually on the orientation of an extended quasi-stationary front and the timing of energy releasing "waves" along it. When physiographic influences do not predominate and the region of homogeneity is sufficiently extensive to permit the transposition and superposition of storm maps to and upon basin maps, all, or the greater number, of the steps given in Table 5 are necessary to the synthesis of the flood hydrograph.

Mr. Jarvis' particularly comprehensive discussion warrants far more consideration than can be given to it in these closing paragraphs. His questions in regard to the apparent inadequate values of W_E, effective precipitable water, are answered in references (2), (3), and (38).

Mr. Hoyt has referred to the progress report (30) of the special Society Committee on Meteorological Data. This piece of constructive criticism, to the writer's knowledge, has contributed much in guiding the Weather Bureau to its present important place as a source of engineering data. However, in retrospect, a principle in democratic evolution seems to have revealed itself; namely, the need must be demonstrated before fulfilment is made possible through financial support. Engineers have always wanted more precipitation data. The Weather Bureau (on appropriations of less than 5¢ per capita) was trying to provide them; but it took the demonstrated needs of flood control and soil erosion prevention to bring about the approval of Congress for the establishment of a network of nearly 3,000 recording (intensity) rainfall stations, several hundred badly needed stream discharge stations, and a score of field and indoor laboratories—all now playing important rôles in engineering programs.

---

[13] Hydrologic Director, U. S. Weather Bureau, Washington, D. C.

TABLE 5.—STEPS IN FLOOD HYDROGRAPH SYNTHESIS

No.	Step	Means of determination
1	Maximum storm volume	Total isohyetal map; area-volume relations
	Variation of Rainfall Depth—	
2	With time	Mass curves of rainfall
3	With duration	Depth-duration relations
4	With area	Depth-area relations
5	Distribution of rainfall depth over area	Isohyetal pattern, actual or synthetic
6	Critical time increment	Equal to or multiple of basin's concentration period
7	Critical time pattern	Arrangement of increment depths in critical time order
8	Maximum runoff volume	Superposition of total storm to give greatest volume of runoff under minimum loss conditions, to include the snow-melt increment when appropriate
9	Critical concentration of runoff	Superposition of storm increments to give greatest concentration of runoff in critical period
10	Critical synchronization of tributary flow	Progressive routing of tributary flow to disclose critical storm pattern and position
11	Maximum flood peak	Most effective combination of conditions for critical concentration of runoff and critical synchronization of tributary flow

Mr. Houk's comments on the distinction to be made between the maximum storm and the maximum flood emphasize those of Mr. Gilman. In general, the magnitude of storm and flood is of comparable order as the upper limits are approached. On small watersheds, rainfall must be expressed in terms of intensity for short periods. On large basins the volume of rainfall produced by an extensive storm will determine the magnitude of the resulting flood. In the former case, there will be a considerable fluctuation in ground surface effects, thereby disrupting the frequency relations between rainfall and the flood. In the latter case, initial rains will have reduced infiltration rates and will have provided sufficient water to occupy pondage and channel storage fully. Thus, the resulting flood will bear a far more consistent relation to storm rainfall than the rainfall of a brief storm over a small area.

The difficulties in determining storm rainfall over a prescribed basin through the solution of an equation of flow are partly revealed through an analogy in which the problem is to determine the deposition of suspended material in a silt laden stream throughout a reach in which the slope has been abruptly flattened. The flow is confined within an unyielding channel of fixed dimensions, and all the principal factor values are determinable, including the volume of deposited material and the residual of silt which leaves the reach at its outlet. In the equation of moist air flow, adequate means for measuring and giving quantitative expression to the comparable variables are largely lacking. The imaginary boundaries of the flow channel beyond the assumed inflow section have no assurance of reality. The residual moisture leaving the basin is not subject to measurement. Improved instrumentation and a more dense upper-air observation network can be expected to overcome most of these deficiencies.

It is fairly simple to determine the prevailing direction of inflow and, therefore, the basin constant K. Gradient winds can likewise be estimated with fair accuracy. The value of R can be found by planimetering the isohyetal map. It is not always possible, however, to disassociate v and W_E in actual storms. This does not invalidate the basic method of computing the maximum rainfall amounts since, for the physical upper limit, it would be necessary to assume that these two factors combined at their respective maximum effectiveness. It is not altogether logical to combine them at their physical upper limits because they are not completely independent.

In order to develop and maintain maximum velocities (which increase with height and, therefore, must be integrated throughout the depth of the inflow layer to provide maximum rates of inflow), there must be steep gradients of temperature as well as of pressure. At the same time the maximum moisture charge must necessarily be contained in a broad tongue of moist air characterized by the maximum possible saturation temperature at all levels. Also, the moisture charge must be horizontally isothermal throughout the full width of the tongue. Thus, the introduction of a temperature gradient automatically places limitation on the moisture charge.

Mr. Houk's suggestion that values for K, v, and W_E for the outstanding storms of record be published strikes a note of accord among all engineers interested in hydrometeorology. An illustration (38) is given in the case of the Sacramento River Basin where the principal lifting agencies providing the mechanism for producing rainfall are the Coastal Ranges and the Sierra Nevada. Since these barriers remain stationary, the problem of determining rainfall amounts through computation is somewhat simplified. The Point Reyes (Calif.) station was found to be representative for inflow wind velocities, and the dew points at San Francisco (Calif.) were used as an index to the moisture charge of the inflowing air mass. It was possible, therefore, to complete the computations of rates of rainfall by use of the basic formula, $R = \int K\, v\, W_E\, dt$ (see Eq. 2). Fig. 27 compares the theoretically computed average depths of rainfall over the basin with the observed depths obtained by planimetering isohyetal maps. Fig. 28 shows a comparison of the computed and observed time-intensity patterns for the storm of February 25–29, 1940.

The writer can agree with Mr. Beard in his principles of relative accuracy, the validity of which must await the time when observations of the variables he has enumerated have covered a period sufficiently long to have statistical significance. Engineers are now merely groping toward a state of knowledge that will ultimately introduce reasonably precise procedures into hydrologic design. In the meantime, efforts to refine the knowledge on any one of the factors in the sequence of events from the "meteorological storm" through the "hydrologic storm" to the culminating flood would seem warranted.

The designation of "maximum possible" to the storm values discussed in the writer's paper has the intended meaning that such values can be approached but not exceeded, and therefore can be considered only as the asymptotic limit of the magnitude-frequency curve. As Mr. Sherman has stated, such values should not be the basis for design in any except the case where social and psychological considerations dictate security beyond economic justification. In any case, it would seem as much an advantage to the flood control designer to know the upper limit of magnitude of his major factor, the storm, as it is to the structural designer to know the ultimate strength of his materials.

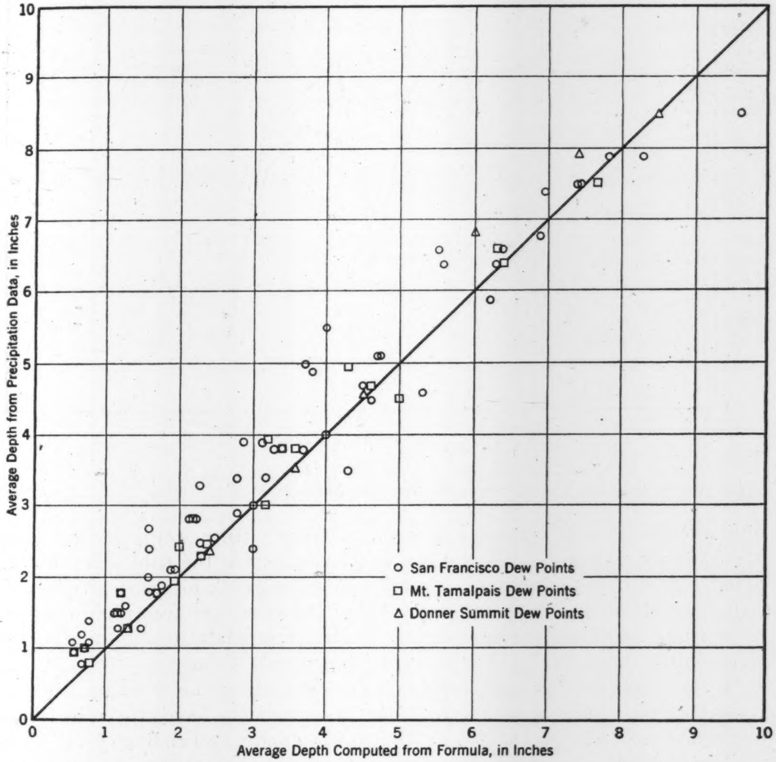


FIG. 27.—AVERAGE PRECIPITATION DEPTHS OVER TOTAL BASIN

The criticism of listed regions of meteorological homogeneity, particularly as the basis for justifying storm transposition, is readily admitted. Apparently, there is a compensative circumstance in certain mountainous areas where orographic controls so stabilize the rainfall régime that the seasonal pattern can be used as a guide to the distribution of storm rainfall over a large basin. The opinions of Messrs. Beard, Kennedy, and Hall have not overemphasized the cautions to be observed in storm transposition.

The writer is indebted to Mr. Kennedy for his review of the work of Messrs. Showalter and Solot (20). The theoretical determination of storm rainfall could be given only a limited place in the original paper.

It is hardly correct to say that storm transposition is being supplanted by area-duration-depth data. The designer will always resort to the simple expedient of superimposing storms over basins wherever homogeneity of rainfall régime exists. Values taken from area-depth curves are meaningless until they have been expressed as areal patterns—either actual or synthetic. Mr. Eiffert's reference to the apparent inconsistency in the title of Table 1 is noted. Area-depth curves constructed on a single graph for successive cumulative periods of time from the beginning of the storm are accurately referred to as time-area-depth curves, whereas those for separate durations, with no common beginning time, are more appropriately called duration curves.

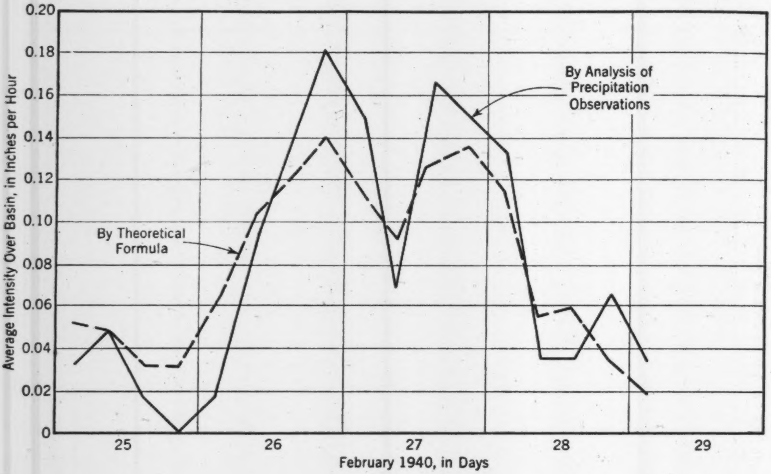


FIG. 28.—COMPUTED PRECIPITATION INTENSITY AVERAGE OVER TOTAL BASIN; STORM OF FEBRUARY 25–29, 1940

In the work of the Hydrometeorological Section of the Weather Bureau, it has been found that the staggered hours of observation have proved blessings in disguise, disclosing as they do the times of beginning and ending of rain (39).

Mr. Hathaway has made a particularly valuable contribution to the paper in illustrating the steps toward design objectives beyond those involving meteorology. That pertinent storm data will ultimately be made available in the concise terms characteristic of the Corps of Engineers should be appreciated by the civil engineering profession.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2213

APPLICATION OF SOIL MECHANICS IN DESIGNING BUILDING FOUNDATIONS

By A. Casagrande,[1] Assoc. M. Am. Soc. C. E., and
R. E. Fadum,[2] Jun. Am. Soc. C. E.

With Discussion by Messrs. Jacob Feld, Leonard Zeevaert, Gregory P. Tschebotarioff, Guthlac Wilson, W. H. McAlpine, Karl Terzaghi, D. P. Krynine, O. G. Julian, A. A. Eremin, D. M. Burmister, Abraham Woolf, Lazarus White, George L. Freeman, Soil Mechanics Committee of the Los Angeles Section—Trent R. Dames, Chairman, and Frederick J. Converse, Sterling S. Green, and Paul Baumann—and A. Casagrande and R. E. Fadum.

Synopsis

The first part of this paper reviews the causes of building settlements, and summarizes what soil mechanics has to offer as a basis for the design of building foundations. In the second part are described the foundations of two large office buildings in Boston, Mass., which were designed specifically for the purpose of reducing total and differential settlements to a tolerable amount. The settlements of these buildings are compared with those of other large buildings designed without the use of soil mechanics. The methods of reducing existing differential settlements are not discussed. The success of any method such as underpinning, the application of additional load to the lighter sections of a structure, the provision of continuous jacking arrangements, or the method of "bleeding" the underground by means of borings adjacent to and under those parts of a structure that have not settled are dependent on accurate observations of the behavior of the structure which is to be remedied.

Introduction

Although problems dealing with building settlements may be relatively simple as compared with other problems in soil mechanics, knowledge of soil properties is not yet sufficiently developed to permit an accurate solution even of this problem. It must be kept in mind that, in general, even the most

---

Note.—Published in November, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Associate Prof., Civ. Eng., Harvard Univ., Cambridge, Mass.

[2] Instr. in Civ. Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass.

elaborate underground explorations do not disclose a complete picture of the soil conditions. Therefore, the results of a settlement analysis, as well as of most problems in applied soil mechanics, should be expressed by a range which includes the possible limits of all factors entering the computations. A clear discussion of the possible variations of each factor should be presented to facilitate a final decision by those who use the information for design purposes. It cannot be emphasized too strongly that the correct application of the results of such an investigation to the design of earthworks or foundations will, in addition to a thorough knowledge of soil mechanics, always require experience and judgment.

Differences Between Soils and Structural Materials.—The deformation of soils, even under small loads, is much larger than that of steel or concrete. Furthermore, the deformation of many types of soils does not occur simultaneously with the application of a load; it develops in the course of time. Therefore, one may not be aware of a dangerous situation until a building begins to crack many years after it has been built.

Another difference between structural materials and soils resides in the fact that in the former deformation is chiefly a result of a change in shape. In soils, however, deformation is the result of both a change in shape and a change in volume. In some problems, and in particular problems dealing with the settlement of buildings resting on clay, the deformation due to change in volume of the underlying soil layers is usually much more important than the deformation due to change in shape.

Much confusion and harm have resulted in the past from the belief that a satisfactory allowable load for a soil can be obtained by reducing the failure load, derived from surface-loading tests or pile-loading tests, by a liberal factor of safety. The resulting stresses will not produce rupture of the soil. There is no assurance, however, that these "safe" stresses in the soil will not produce objectionable or dangerous settlements.

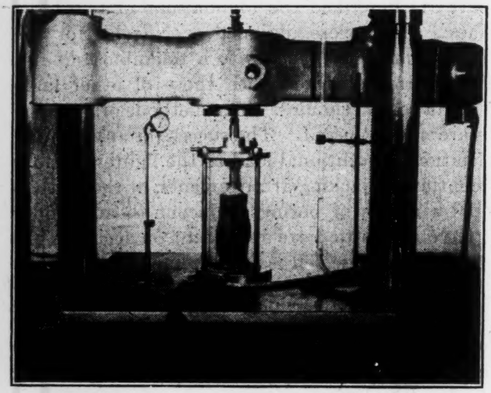


FIG. 1.—TRIAXIAL COMPRESSION APPARATUS.

SOIL PROPERTIES RESPONSIBLE FOR SETTLEMENTS

General. — Since, for practical purposes, soils possess no tensile strength, only the deformation characteristics resulting from compression forces are of interest. Therefore, the principal testing methods are designed to subject soil samples to compression. Theoretically, it should be possible to determine all the stress-deformation characteristics from so-called "triaxial compression tests." In this type of test the principal stresses can be varied at liberty, thereby producing any desired combination of normal and shearing stresses within a sample. Actually, the only practical type of triaxial compression test apparatus thus far developed is one in which two of the principal stresses are kept equal. These stresses are produced by the pressure of a liquid surrounding a cylindrical specimen (Fig. 1). If such a compression test is made without this liquid pressure, the test is called an "unconfined compression test." This type of test is similar to cylindrical compression tests used in concrete testing. In a third type of compression test, which is of particular importance in the testing of clays, the specimen is confined laterally in a metal ring and is compressed between two porous plates. In this "confined compression test" (which is also called a "consolidation test"), the lateral displacements are prevented entirely, and one measures only the relation between stress, volume, and time.

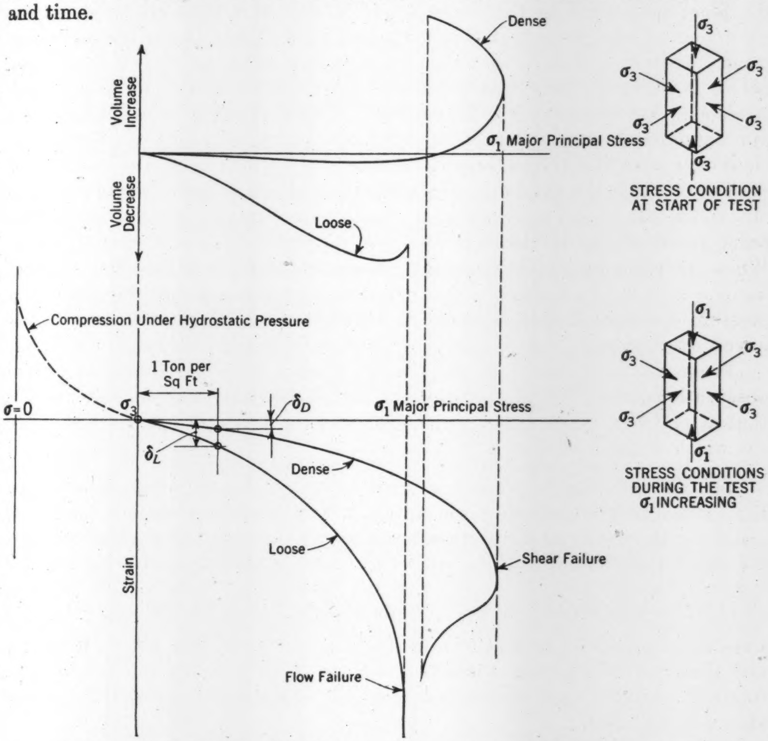


FIG. 2.—TYPICAL STRESS-STRAIN AND STRESS-VOLUME CHANGE CURVES FOR SAND

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

Stress-Deformation Characteristics of Cohesionless Soils.—When a sample of sand is tested in triaxial compression, starting, for example, from a hydrostatic stress condition, and then one of the principal stresses is increased until failure occurs, stress-strain and stress-volume change curves of the type shown in Fig. 2 are obtained. A loose sand decreases in volume, whereas a dense sand expands during loading. Although this behavior is of great importance in connection with the question of stability of dams consisting chiefly of fine-grained or medium-grained sands, it is of limited interest in problems attending building construction. Buildings resting on loose fine sands or inorganic silts are likely to undergo sudden settlements if the underlying soil is disturbed—for example, by heavy pile-driving operations in adjacent building areas. Most sand deposits, however, are sufficiently dense that no such dangers exist.

The chief concern is with ordinary deformation under stress and its effect upon the distribution of soil reactions. It can be noted from Fig. 2 that an increase in stress of (for example) 1.0 ton per sq ft produces a larger strain in the loose sand than in the dense sand. If the two tests are repeated on specimens of the same material, subjected to different intensities of minor principal stress, one finds that the strain corresponding to $\sigma_1 - \sigma_3 = 1.0$ ton per sq ft changes approximately in inverse proportion to the minor principal stress σ_3. Furthermore, it is found that the shear strength of sands increases approximately in direct proportion with the minor principal stress. The coefficient expressing this proportionality is called the coefficient of internal friction.

The fact that the resistance to deformation, as well as the ultimate strength, of a cohesionless solid increases approximately in direct proportion to the confining pressures can be demonstrated easily by placing sand in a rubber bag. When the air is evacuated from the interior of such a soft bag, the contents become as hard as a brick. The evacuation of air subjects the sand to a pressure corresponding in intensity to that produced by approximately 20 ft of soil overburden.

From consideration of these physical characteristics of sands, and from observations, the following conclusions, applicable only to noncohesive soils that extend to a depth below the loaded area for a distance at least equal to the width of the loaded area, may be drawn:

(*a*) The intensity of pressure exerted on a rigid, loaded area supported on the surface of a cohesionless soil varies in such a manner that a maximum occurs at the center and no pressure occurs at the edge. For small footings for which the product of the width of the footing and the unit weight of soil ($b\,\gamma$) is small in relation to the average unit load on the footing $\left(p = \dfrac{P}{A} \right)$, the pressure diagram is parabolic in shape, as shown in Fig. 3(*a*). When $b\,\gamma$ is of the same order of magnitude as $p = P/A$ (which is usually the case for large footings, or for a rigid foundation mat), the diagram of soil reactions is as shown in Fig. 3(*b*).

(*b*) As the depth of a rigid, loaded area beneath adjacent ground surface is increased, the soil reactions change until the intensity of pressure is nearly uniform, as shown in Fig. 3(*c*). This is frequently the case in caissons and pier foundations.

(*c*) The ultimate bearing capacity of a cohesionless soil increases approximately in direct proportion to the width of a loaded area.

(*d*) The ultimate bearing capacity of a cohesionless soil increases approximately as the square of the depth of the loaded area beneath adjacent ground surface.[2a]

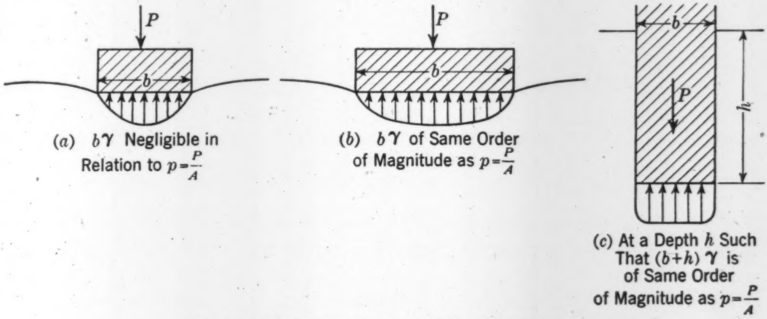

P
b
(a) $b\gamma$ Negligible in Relation to $p=\frac{P}{A}$
P
b
(b) $b\gamma$ of Same Order of Magnitude as $p=\frac{P}{A}$
b
P
h
(c) At a Depth h Such That $(b+h)\gamma$ is of Same Order of Magnitude as $p=\frac{P}{A}$

FIG. 3.—RIGID FOOTING ON THE SURFACE OF SAND

(*e*) Specifications for the allowable loads for sands, as generally given in building codes, do not take sufficiently into consideration the favorable influence of depth. Such "bearing values" are far too conservative for the case of footings founded at considerable depth, whereas, for narrow, interior, wall footings that are founded at the surface or at a shallow depth beneath immediately adjacent ground surface, the values are not sufficiently conservative.

(*f*) When fairly heavy loads are supported at shallow depths on narrow footings resting on loose sand, it is well to test the bearing capacity by loading a test area of similar width. However, for large areas on dense sand, or for areas which are to be founded at a considerable depth below ground surface on dense sand, even the heaviest loading which building codes permit is exceedingly conservative, so far as strength of the soil is concerned, and requires no verification by load tests.

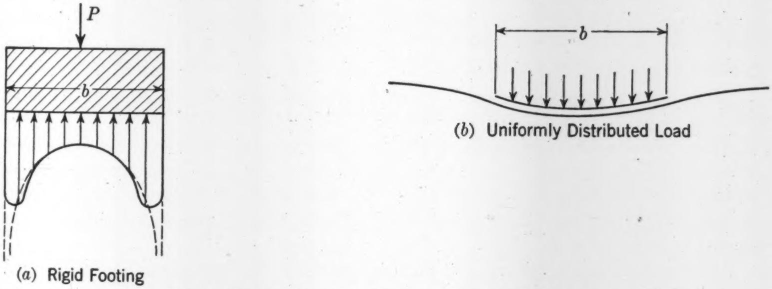
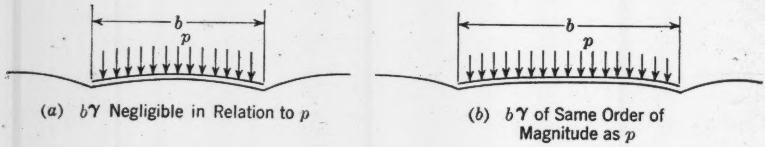

P
b
(a) Rigid Footing
b
(b) Uniformly Distributed Load

FIG. 4.—LOADS ON THE SURFACE OF ELASTIC MATERIAL

(*g*) From the theory of elasticity it follows that, for a perfectly elastic soil, the reactions on rigid areas and the settlements of uniformly loaded areas on the ground surface should be as shown in Fig. 4. The variations in pressure intensities and the settlement patterns thus obtained are just the opposite from those obtained in corresponding cases for sand, as shown in Figs. 3(*a*) and 5(*a*), respectively. Thus, in the case of sand, less harm would result from just guessing intelligently at the deflections or the distribution of soil reactions than from making elaborate computations on the basis of hypothetical theories which definitely do not apply. Furthermore, the fact that Hooke's law does not apply to cohesionless soils also invalidates the law of superposition. (Modifications of Boussinesq's formula,

---

[2a] See corrected statement in the closing discussion.



(*a*) $b\gamma$ Negligible in Relation to p (*b*) $b\gamma$ of Same Order of Magnitude as p

FIG. 5.—UNIFORMLY DISTRIBUTED LOAD ON THE SURFACE OF SAND

$$\sigma_z = \frac{3\,P}{2\,\pi}\,\frac{z^3}{R^5} \qquad (1a)$$

by introducing a "concentration factor," α, in the form,

$$\sigma_z = \frac{\alpha\,P}{2\,\pi}\,\frac{z^\alpha}{R^{\alpha+2}} \qquad (1b)$$

yield, when integrated over finite areas, essentially the same type of distribution as shown in Fig. 4. Therefore, this hypothetical solution, utilizing a concentration factor, is no improvement over the theory of elasticity. Both are equally unsuited for computing soil reactions and deflections on sand.)

(*h*) The settlement of a uniformly loaded area is a minimum in the center and a maximum along the edge. The deflection curve is as shown either in Fig. 5(*a*) or 5(*b*), depending on the relative magnitude of $b\,\gamma$ and p.

(*i*) The average settlement corresponding to a given unit load increases for small areas with increasing width of the footing, whereas for large areas it is practically independent of the width.

(*j*) The settlement for a given unit load decreases rapidly as the depth of the loaded area beneath adjacent ground surface increases.[2b]

(*k*) Sand layers provide an excellent medium for spreading load and for reducing differential settlements, due to the small strain which corresponds to ordinary loads and to the fact that the resistance of sand to deformation increases in proportion to the applied pressure. The presence of sand strata should be utilized as much as possible for this purpose.

(*l*) Conventional design of building foundations by means of individual column footings and wall footings, with or without piles, is in general entirely adequate for foundations on medium-grained and coarse-grained cohesionless soils, provided that borings to a depth at least equal to the width of the building have verified the fact that the sand is not underlain by a compressible stratum.

[2b] See modification in the closing discussion.

(*m*) When founding on very fine loose sand, settlements may become excessive. Very fine loose sands, rock flour, etc., may settle seriously at some later time, due to a lowering of the ground-water table, the driving of piles, or other disturbances caused by near-by construction operations.

Stress-Deformation Characteristics of Clays and Other Cohesive Soils.—Clays are very fine-grained soils having grain sizes which run well into the colloidal range. Such soils, when dried, become hard and appear like a soft rock. When remolded at consistencies between certain limiting water contents, clays exhibit the property of plasticity. With relatively little experience, clays can be identified in the field from an examination of these properties.

Since the plasticity, as well as the compressibility and permeability of a clay, is related to its content of colloidal, scale-like particles, the Atterberg liquid and plastic limit tests are useful for general classification purposes. Such classification tests, however, are far from adequate in quantitatively describing the pertinent properties of an undisturbed clay. With the same raw material, nature has made clays possessing widely different structural characteristics. These are dependent on many factors, as, for example, whether sedimentation took place in fresh or salt water and the amount of load under which consolidation proceeded.

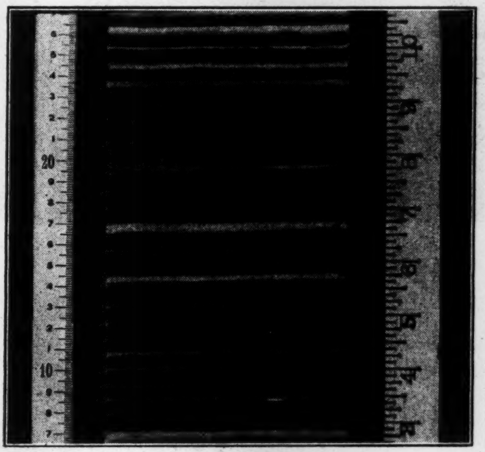


FIG. 6.—TYPICAL VARVED CLAY

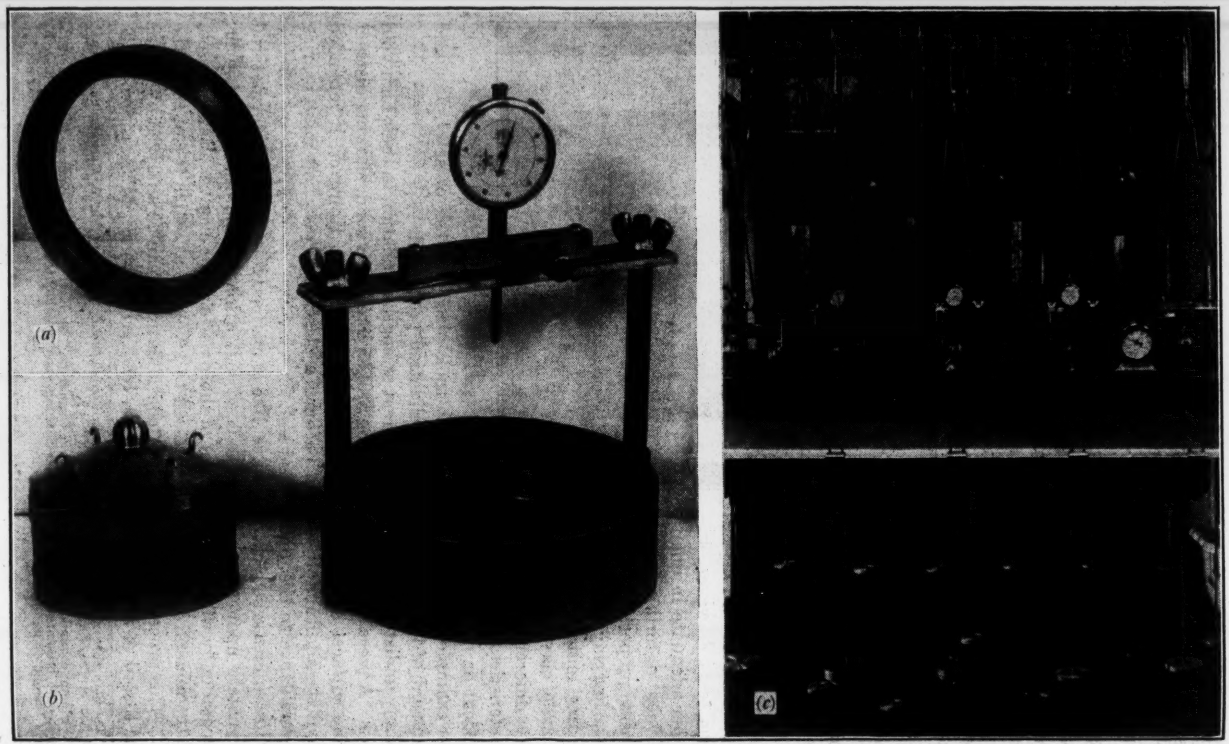
Many undisturbed clays possess great strength in relation to their high void ratios.[3] A piece of such a clay can be broken like a soft solid, showing distinct rupture surfaces similar to those of brittle materials. However, if remolded without the addition of water, it becomes a plastic and very soft mass bearing no resemblance to the undisturbed soil.

Most undisturbed clays behave like a perfectly incompressible material immediately upon application of a load. If a load is maintained, however, water gradually will be squeezed out of those parts of the clay which are stressed, until the mineral skeleton is compressed to a volume at which it can carry the load. This process, which is called "consolidation," can be analyzed by means of the theory of consolidation[4] set forth by Karl Terzaghi, M. Am. Soc. C. E.

---

[3] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

[4] "Erdbaumechanik auf bodenphysikalischer Grundlage," by Karl Terzaghi, Leipzig, Deuticke, 1925. For up-to-date presentation, see "Theorie der Setzung von Tonschichten," by Karl Terzaghi and O. K. Fröhlich, Vienna, 1936.



(a) and (b) Detail Views (c) Setup in a Loading Frame

FIG. 7.—CONSOLIDATION APPARATUS

Deformation of Clay Without Volume Change.—Stress-strain curves obtained from unconfined compression tests or surface-loading tests, on most undisturbed clays, are almost straight lines within the load range of working stresses. Hence, it should be possible to determine that part of the settlement of a structure which occurs during the application of load, and which is due to lateral deformation without volume change, by simply determining the modulus of elasticity and then applying appropriate formulas from the theory of elasticity. This approach has been tried by many investigators, and almost invariably it has been found that the actual settlements are only a fraction of the computed settlements. Some of this discrepancy is frequently due to the fact that the so-called "undisturbed samples" have suffered some disturbance during the sampling operations. In many cases this discrepancy is due to anisotropic characteristics of natural clay deposits.[5] Most clays are stratified, even though the stratification may not be visible. The most pronounced stratification is found in the so-called "varved" clays, in which thin layers of rock flour are closely laminated with clay layers of varying degrees of plasticity and frequently of varying colors. An excellent example of such a clay is shown in Fig. 6.

Whenever any type of stratification is present, the soft layers are confined laterally, to a certain extent, by the stronger sand and silt layers or partings. Hence, under the load of a building, that part of the settlement which is due to the lateral deformation of the underlying clay strata can develop only to a limited extent, as compared with the settlement which would take place if the clay were homogeneous and isotropic.

Consolidation of Clays.—There are many types of consolidation apparatus in use, all of which incorporate the principle, introduced by Professor Terzaghi, of compressing a thin cylindrical sample confined in a ring between two porous stones. The consolidation apparatus shown in detail in Fig. 7(*a*) and set up in a dead-weight loading frame in Fig. 7(*c*) is a simple and efficient example.[6] The test is performed by applying the load in increments. Each increment is allowed to remain on the sample a sufficient length of time for consolidation to take place, the extensometer being read at appropriate time intervals. The data of such a test are plotted in the form of a "pressure-void ratio curve" (Fig. 8) and a series of "consolidation curves" (Fig. 9).

Without presenting Professor Terzaghi's theory of consolidation of fine-grained soils in detail, the following points will be mentioned: The time required to reach a given degree of consolidation is directly proportional to the square of the height of a layer of clay, drained on both sides; it is inversely proportional to the coefficient of permeability; and it is directly proportional to the slope of the pressure-void ratio curve.

It is a simple matter to convert consolidation curves obtained in the laboratory to a curve showing the probable progress of settlement with time for a structure supported on the material from which the samples were obtained, provided the thickness and drainage characteristics of the loaded clay layers are known accurately. Unfortunately, this is seldom the case. When the soil conditions are complex, a tolerably accurate knowledge of the drainage characteristics can be obtained only by means of detailed underground explorations in which continuous samples are obtained. Even such an extensive study will provide information sufficient only to establish certain limits within which the actual conditions may lie. The range thus established may be quite wide.

---

[5] "Influence of Anisotropic Properties of Clay Deposits on Settlements," by Austin B. Mason, a thesis presented to the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in June, 1939, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering.

[6] "Notes on Soil Testing for Engineering Purposes," by A. Casagrande and R. E. Fadum, *Soil Mechanics Series No. 8*, Graduate School of Eng., Harvard Univ., Cambridge, Mass., January, 1940, p. 42.

Preconsolidation Load.—Correct interpretation of the results of consolidation test data is an important prerequisite for a comprehensive settlement analysis. The relation between pressure and void ratio, which is obtained from a consolidation test, may differ widely from the true relationship which a clay will follow in its undisturbed condition in the ground. It is necessary, therefore, to reconstruct this true relationship from the test data.

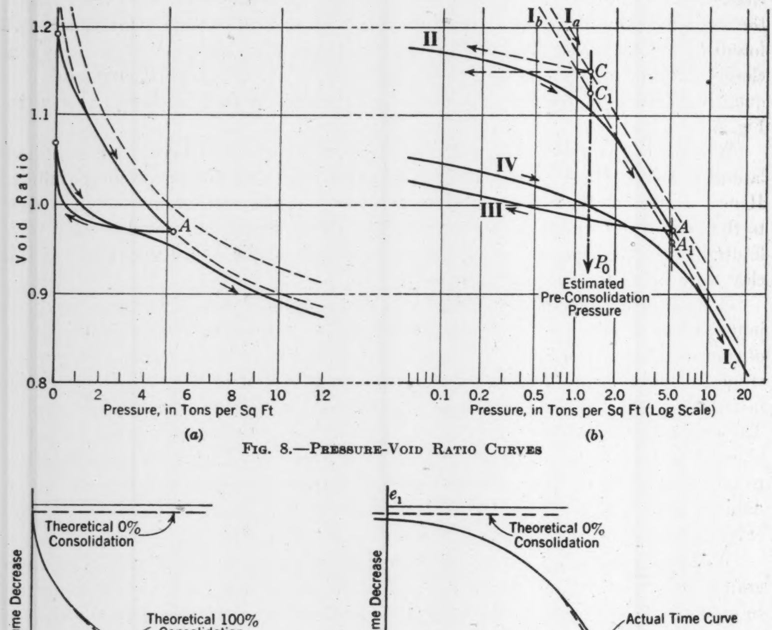


FIG. 8.—PRESSURE-VOID RATIO CURVES

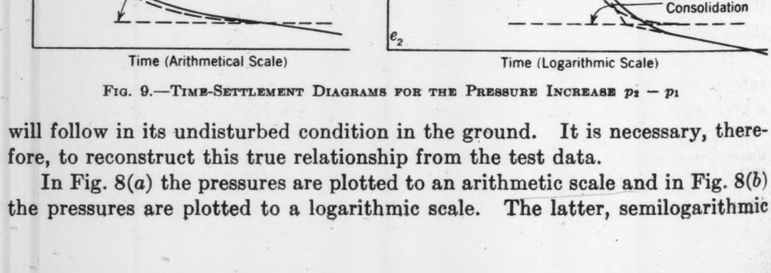


FIG. 9.—TIME-SETTLEMENT DIAGRAMS FOR THE PRESSURE INCREASE $p_2 - p_1$

In Fig. 8(a) the pressures are plotted to an arithmetic scale and in Fig. 8(b) the pressures are plotted to a logarithmic scale. The latter, semilogarithmic plot lends itself readily to an analysis of the loading history of a sample. The first part (II) of the compression curve is in reality a recompression curve which meets the virgin branch I_b and then continues along that branch as a straight line. At an arbitrary load, corresponding to point A, the load is reduced by a series of decrements to zero, and a rebound curve III is obtained. When load is applied again as before, the recompression curve IV is obtained. This latter curve meets the virgin compression curve I_c at a point corresponding to a higher pressure than that corresponding to point A. This diagram is typical of all very fine-grained soils. The magnitude of the drop in the position of a virgin branch after each rebound cycle depends chiefly on the structural characteristics of a soil.

The similarity in the shape of branch II, the relative positions of branches II and I_b with the shape of the recompression curve IV, and the relative positions of curves IV and I_c suggest that it should be possible to estimate the so-called preconsolidation pressure p_0, under which the soil was consolidated in the ground, from the data of a properly conducted consolidation test.

From a large number of tests on different types of soils, it has been found that the preconsolidation pressure for most clays can be determined with a satisfactory degree of accuracy by means of the empirical method shown in Fig. 10 and explained as follows: First, the position of the virgin compression line (I) is established by extending the straight-line part of the pressure-void ratio curve. Then, on the branch marked II, a tangent (t) and a horizontal line (h) are drawn at the point T which corresponds to the point having the smallest radius of curvature. The angle between (t) and (h) is then bisected, and the abscissa of the point of intersection C, of the bisector (c) with the virgin line I, determines the magnitude of the preconsolidation pressure (P_0).

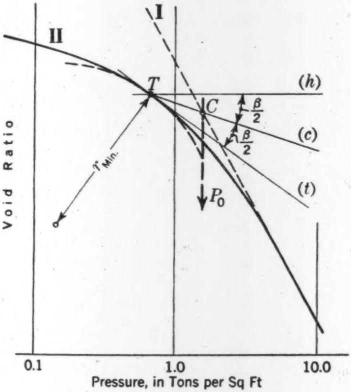


FIG. 10

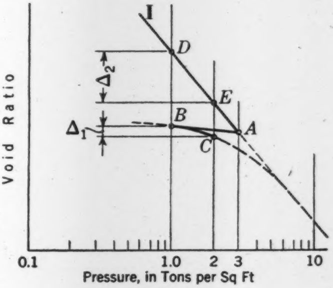


FIG. 11

The significance of this pressure can be demonstrated as follows: The slope of the virgin compression branch on a semilogarithmic plot is not noticeably affected by swelling and minor deformations of a sample. Hence, if a soil is completely consolidated under a pressure p_1 produced by an existing overburden, the compression due to any additional load Δp is determined easily by multiplying the thickness of the compressible stratum by the ratio of the change in void ratio $(e_1 - e_2)$ corresponding to a change in stress $(p_2 - p_1)$, divided by one plus the initial void ratio $(1 + e_1)$.

If the largest overburden to which a soil has been consolidated during its geologic history later has been partly removed, the compression due to the

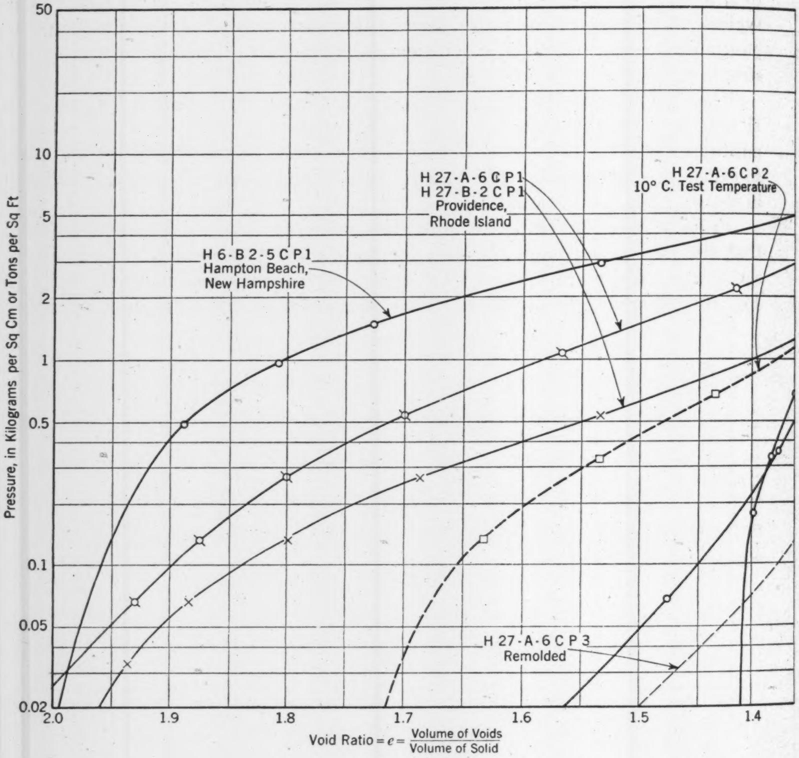
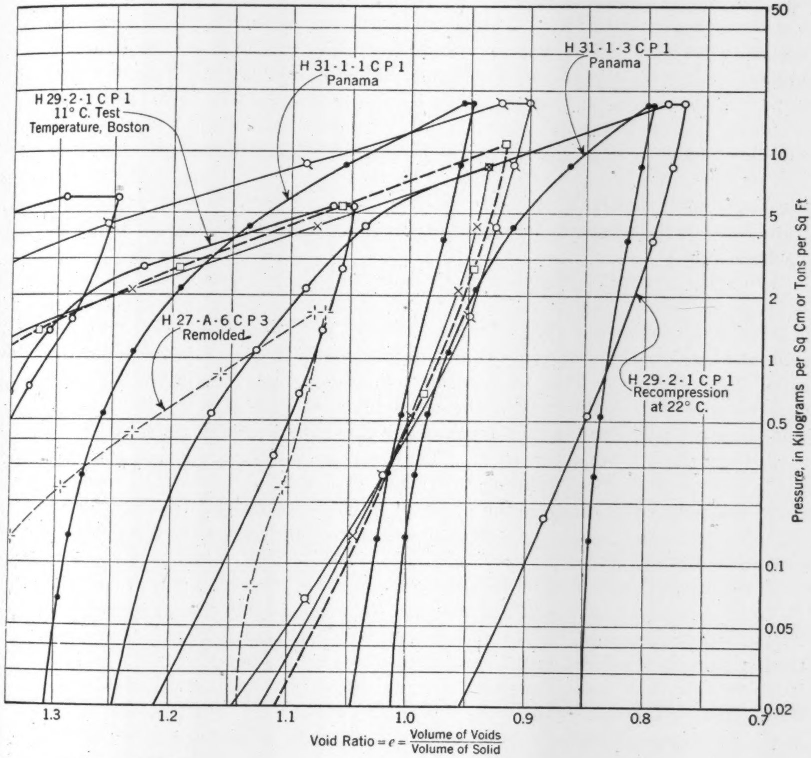


FIG. 12.—COMPRESSION

same pressure increase Δp will be much smaller, in spite of the fact that the virgin compression curve remains unchanged. For example (Fig. 11), if a clay layer has been compressed by an overburden pressure of 3 tons per sq ft and this pressure is later reduced by erosion to 1 ton per sq ft and then again increased to 2 tons per sq ft by the weight of a building, the consolidation due to the building load will correspond to a change, Δ_1, in void ratio. If, however, the clay has never been consolidated by a pressure in excess of the existing overburden pressure of 1 ton per sq ft, the additional stress due to the weight of the building would have produced a change, Δ_2, in void ratio, which is approximately five times as great as Δ_1. This example should suffice to illustrate the fundamental importance of a careful study of the preconsolidation pressure.

Many attempts have been made to arrive at a reasonable estimate of the compressibility of clays from tests which are more simple than the time-consuming consolidation test and which preferably could be performed on small, inexpensive samples. Utilizing a discovery by P. C. Rutledge,[7] Assoc. M. Am. Soc. C. E., that the pressure-void ratio curves for clays of the same general character follow a "pattern," it is possible in many cases to determine the compressibility approximately from the natural water content of the soil. This method will be explained by means of an example.



DIAGRAMS OF ORGANIC CLAYS

Example of a Settlement Analysis Based on Simple Soil Tests.—A power station had suffered serious settlement in spite of the "fact" that borings showed only sand to a depth of 35 ft and that piles had been driven to a depth of 25 ft to 30 ft. The writers were requested to make a preliminary investigation of the foundation conditions for a proposed extension of the plant. Four exploratory borings were made which revealed a stratum of slightly organic clay extending from approximately 39 ft to 51 ft beneath the ground surface. The clay stratum was underlain by a thick stratum of very compact sand. Almost continuous cores were obtained from the clay stratum by means of 4-ft sections of thin-walled steel tubing, 2 in. in diameter. The entire stratum was found to be fairly homogeneous, rather brittle in the undisturbed condition, and very soft and sticky when remolded. The natural water content ranged between 37% and 58%, the liquid limit between 37% and 65%, and the plastic limit between 20% and 27%. (Whenever the natural water content of a clay, which is consolidated by at least 40 ft of overburden, is almost as high as the liquid limit, it is an indication that the clay has the type of complicated structure which probably results from sedimentation in sea water. Numerous shells which were found in the clay were identified as being of marine origin.)

---

[7] "Compression Characteristics of Clays and Application to Settlement Analysis," by P. C. Rutledge, a thesis presented to the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in June, 1939, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering.

From the general conditions at the site and the appearance of the clay stratum, it seemed likely that at no previous time did the clay layer carry a heavier overburden than that which existed at the time of inspection. It was evident also that this relatively thin stratum was fully consolidated under the existing overburden. The preconsolidation pressure was computed from the unit weights and thicknesses of the overlying materials at a depth of 45 ft, which corresponded to the mid-depth of the clay stratum, and was found to be approximately 1.5 tons per sq ft. The average water content of the clay stratum was 48%, corresponding to a void ratio of $e = 1.30$. The problem thus was reduced to finding the slope of a virgin compression curve for an organic clay which would compress under a load of 1.5 tons per sq ft to a void ratio of 1.30.

Pressure-void ratio curves for a number of different organic clays and silt-clays are assembled in Fig. 12 (rotated 90° to accommodate the page size). It happens that the curve for the soil used in test No. H27-B-2CPl passes practically through the point corresponding to $p = 1.5$ tons per sq ft and $e = 1.30$. If this were not the case, however, it would be a simple matter to draw a line through this point having the same slope as the other virgin compression curves in the immediate vicinity. Assuming that the average increase in stress due to the contemplated structure is 1.0 ton per sq ft, this increase in stress, according to Fig. 12, will reduce the void ratio of such a clay by about $\Delta e = 0.11$. Therefore, the total estimated compression of the clay stratum under the proposed load will be equal to $\frac{\Delta e}{1 + e}$ times the thickness of the layer—that is, $\frac{0.11}{2.3} \times 12 = 0.57$ ft.

Strength of Clay and Safety Against Rupture.—The question of safety against actual failure of the soil arises in connection with the question of the allowable pressure which a spread footing, pier, or caisson may exert on a soil. It cannot be overemphasized that a safe load, as far as rupture of the soil itself is concerned, may be anything but safe as far as the structure as a whole is concerned.

In a preceding section it was stated briefly that, with the exception of the case of a heavy load on narrow footings supported on or slightly below the ground surface, the possibility of stressing a cohesionless material to the point of rupture practically does not exist. In the case of clays, however, the situation is quite different.

The simplest approach to the problem of determining the ultimate bearing capacity of a clay is one which was originally suggested by Professor Terzaghi.[4] He assumed a wall footing of width 2 b resting directly on the ground surface, and assumed that the soil was divided into three square prisms as shown in Fig. 13(a). If the application of a load produces failure of the soil, the underlying prism will deform laterally. The resistance offered to this deformation will be dependent on the resistance to failure of the adjacent prisms.

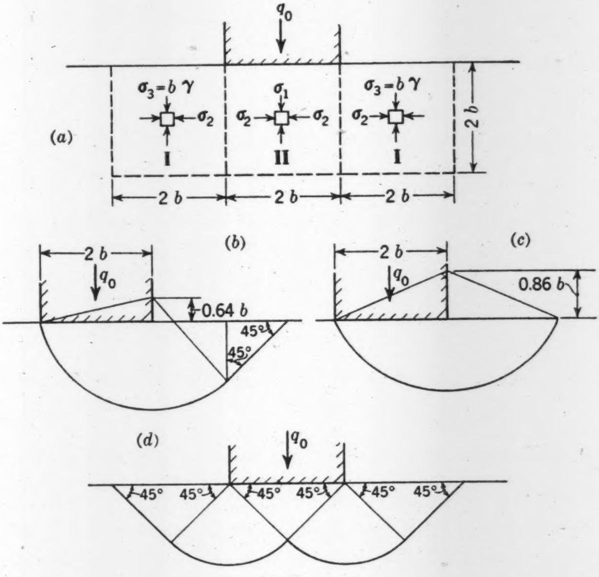


FIG. 13

If a load p per unit of area is applied quickly enough so that a clay will have no time to consolidate, the shear strength will be equal to a constant c; the Mohr rupture curve is represented by a straight line parallel to the pressure axis as shown in Fig. 14(a). Hence, if a clay is subjected to a quick loading, it may be considered to act as a purely cohesive material. This constant shear strength is frequently called "the cohesion of a clay." (The term "cohesion" has been used loosely in soil mechanics and has led to much confusion. In most cases it is used to designate the shear strength under zero normal pressure. This strength can vary between wide limits for the same clay.)

Referring to Fig. 14(*a*), circle I represents Mohr's circle of stress for the rupture condition of a small element in the center of prism I. The major principal stress, $\sigma_z = b\,\gamma + 2\,c$, which will produce failure, must be equal to the minor principal stress acting on a small element in the center of prism II. For the latter element, a major principal stress of $(b\,\gamma + 4\,c)$ will produce failure. Since the weight of the overlying soil produces a pressure $b\,\gamma$, it follows that the pressure q_o, which the footing must exert in order to produce failure according to this hypothesis, is equal to $4\,c$.

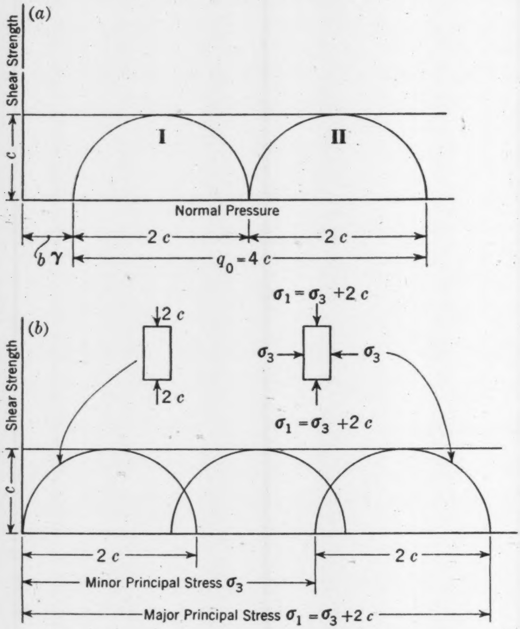


FIG. 14

Utilizing the principle of the Swedish method of cylindrical sliding surfaces, H. D. Krey[8] found that a combination of a circular sliding surface and a wedge, as shown in Fig. 13(*b*), will give the smallest resistance and a corresponding value of $q_o = 6\,c$. The senior writer found that, by the use of a circular sliding surface only, without a wedge, a value of $q_o = 5.5\,c$ is obtained. Guthlac Wilson,[9] M. Am. Soc. C. E., determined analytically that the exact position of the center of this circle is as shown in Fig. 13(*c*) and the exact value of $q_o = 5.4\,c$.

---

[8] "Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes," by H. D. Krey, 4th Ed., Berlin, Ernst, 1932.

[9] "The Calculation of the Bearing Capacity of Footings on Clay," by Guthlac Wilson, *Journal*, Institution of Civ. Engrs., November, 1941.

A satisfactory approach to the problem of bearing capacities of clay from an analytical point of view is by means of the theory of plasticity formulated by L. Prandtl.[10] He found that the shearing surfaces for an ideally plastic material are as shown in Fig. 13(*d*) and that, for a shear strength c, the failure unit load is $q_o = 5.14\ c$.

If clay were an ideally plastic material, the data from unconfined and triaxial compression tests on undisturbed samples would furnish a series of stress circles as shown in Fig. 14(*b*). In reality, the envelope obtained from a series of such tests is slightly inclined to the horizontal for homogeneous clays and is considerably inclined and curved for fissured clays and remolded, compacted clays. Unconfined compression tests on fissured clays have a tendency to produce rupture at a much smaller value than that indicated by the results of a series of triaxial tests. This is due to a premature disintegration of the specimens. If the confining pressure of a minor principal stress is acting, such as is the case in a surface-loading test or a triaxial compression test, this disintegration does not take place. For this reason, triaxial compression tests conducted at small lateral pressures are more satisfactory than unconfined compression tests for the determination of the c-value, which value one should use when translating results of compression tests on undisturbed clay specimens into ultimate bearing capacity for surface loading.

Compression tests on undisturbed soil cylinders are usually much less expensive than loading tests performed on sufficiently large areas. Therefore, the determination of the ultimate bearing capacity by means of cylinder compression tests is recommended. Since the allowable soil pressures commonly specified in building codes are conservative from the standpoint of safety against rupture of the clay, this indirect check of the ultimate bearing capacity is entirely satisfactory.

The problem becomes more difficult if the loaded areas are at a considerable depth below adjacent ground surface as, for example, in the case of caisson foundations. In such a case, the hydrostatic uplift of the soil may be added to the foregoing value of $5\ c$ and used as the ultimate bearing capacity. Actually, this value in such cases will be in excess of $(5\ c + \gamma\ h)$. There are no satisfactory theoretical solutions available, however.

EXAMPLES OF THE APPLICATION OF SOIL MECHANICS TO THE DESIGN OF BUILDING FOUNDATIONS

General.—Since it is difficult at present to set up general rules for the design of foundations on soft ground, the application of principles of soil mechanics can be demonstrated best by presenting the soil conditions in a large city, describing the types of foundations there used, and comparing the settlement records of five large buildings.

Foundation Conditions in Boston.—Large areas in Greater Boston are underlain by a thick stratum of soft glacial clay. A typical soil profile is shown in Fig. 15(*a*). The organic silt is so compressible that only very light buildings can be erected directly on the fill without suffering serious distress as a result of settlement. Therefore, it has been general practice to transfer building loads to the layer of hard or stiff clay below this stratum. Load transfer is accomplished either by means of spread footings, piles, or concrete piers with enlarged bottoms. In some areas a layer of sand instead of the hard clay is found overlying the soft clay, and in these areas piles usually have been used.

---

[10] "Über die Eindringungsfestigkeit plastischer Baustoffe und die Festigkeit der Schneiden," by L. Prandtl, *Zeitschrift für angewandte Mathematik und Mechanik*, February, 1927.

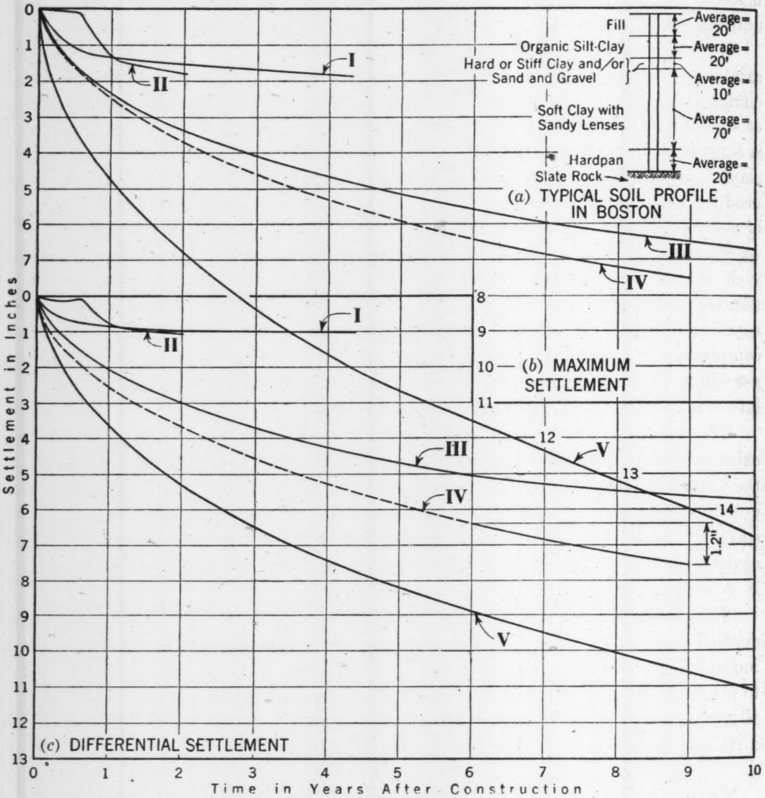


FIG. 15.—TIME-SETTLEMENT RECORDS, BOSTON, MASS.

Settlements of Buildings in Boston.—The weight of soil removed to provide basement space will compensate for the weight of an average building three or four stories in height. The presence of the thick, soft, clay substratum in Boston has had little effect on buildings of this type. Larger buildings, however, have suffered from progressive and differential settlements. Such settlements have been most pronounced in cases where a building, covering an extensive area, has produced different intensities of load on various sections of the building area. If no provision is made to equalize the net increase in pressure throughout the building area by increasing the depth of basement or sub-basement under the more heavily loaded sections, or to stiffen the structure, serious differential movements may occur.

Typical examples are shown by curves III, IV, and V in Figs. 15(*b*) and 15(*c*). The curves in Fig. 15(*b*) show the progress of movement of the point of maximum settlement in each of three different structures located within the confines of Greater Boston. The curves in Fig. 15(*c*) show the progress of differential settlement in these buildings.

The settlement represented by curves III occurred in a building supported on wood piles. These piles were driven into and, in some areas, through a stratum of sand underlain by soft clay. The thickness of soft clays varies from 55 ft to 85 ft throughout the area occupied by the building. In the most heavily loaded area, under a tower section, the average building load is equal to 1.2 tons per sq ft, and under a lighter wing of the building to 0.9 ton per sq ft. The average depth of soil removed under both sections was 10 ft. Hence, the net increase in pressure on the underlying soft clay varies between 0.6 ton per sq ft and 0.3 ton per sq ft. Furthermore, the thickness of the soft clay substratum is greatest under the more heavily loaded section. It is not surprising, therefore, that in the 10-yr period following construction the differential movement that developed between these two sections was equal to approximately 85% of the maximum settlement which occurred in the more heavily loaded section.

Curves IV correspond to another large building, for which settlement records were available only for a 3-yr period starting six years after construction. From an analysis of these data,[11] movements that had occurred prior to this period were estimated to be as shown by the dotted portions of these curves. The loads of this building are transmitted by spread footings to the top of a sand and gravel stratum, 12 ft thick. This stratum is underlain by 65 ft of soft clay. The soil conditions are comparatively uniform throughout the building area. The intensity of building load, however, varies from 2.1 tons per sq ft under a tower section to 1.2 tons per sq ft under an adjacent wing. The net increase in pressure, after deducting the weight of 15 ft of soil removed to provide basement space, is 1.4 tons per sq ft and 0.5 ton per sq ft, respectively. A contour map of the differential settlements which occurred in the 3-yr period starting six years after construction is shown in Fig. 16. (Contours represent points of equal settlement in reference to point A. The observation period equals three years, beginning six years after the erection of the building.) Curve IV in Fig. 15(*b*) shows the progress of settlement of point C in Fig. 16. Since a part of this building has not settled, this curve also represents the progress of differential movement. The rate at which this differential movement is progressing, approximately nine years after construction of the building, is 0.40 in. per yr.

---

[11] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering (pp. 209–217).

A final example is shown by curves V in Figs. 15(*b*) and 15(*c*). The settlement represented by these curves occurred in a building supported by spread footings and concrete piers with enlarged bottoms bearing directly on a stratum of sand and gravel underlain by 90 ft of soft clay. The building loads in this case varied from 2.40 tons per sq ft under a tower section to 0.80 ton per sq ft under an adjacent wing. The net loads, after deducting the weight of soil removed from these areas, were 2.0 tons per sq ft and 0.7 ton per sq ft, respectively. The settlement of the tower section is shown by curve V in Fig. 15(*b*), and the differential settlement between the tower and this adjacent wing is shown by curve V in Fig. 15(*c*).

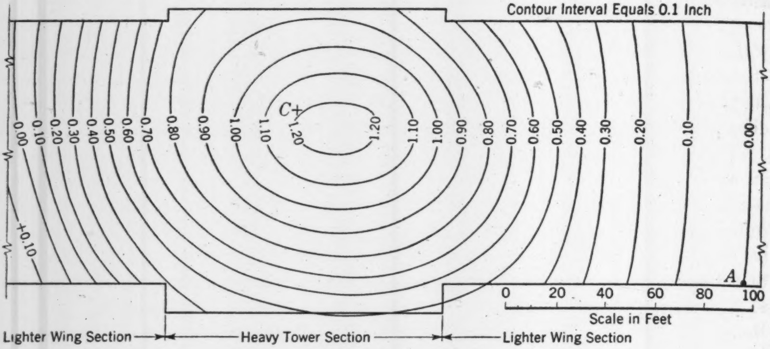


FIG. 16.—DIFFERENTIAL SETTLEMENT CONTOURS

Many other striking examples of serious settlements could be cited from other cities, and, were it not for a feeling of false shame that induces engineers and architects to withold permission for publication of the records of such settlements, it would be possible to add still more. Probably the largest settlements on record are those which have occurred in Mexico City, Mexico, where settlements are recorded in units of feet rather than inches. It is not surprising, therefore, that the principle of a "floating foundation" has been incorporated there without compromise in the design of a modern building.[12]

The experience gained from a study of observed settlements indicates forcefully that, for buildings covering large areas and producing considerable differences in loading between various sections, differential settlements can be reduced to a tolerable amount by excavating a quantity of soil approximately equal in weight to the load of the superimposed structure, so that there will be practically no net increase in stress in underlying soft clay strata. Even the fulfilment of this requirement will not eliminate settlements entirely, for the following reasons:

(*a*) The removal of a large mass of soil will produce an elastic deformation of the underlying soil resulting in a slight upheaval of the bottom of the excavated pit. Upon replacing the weight of soil removed by the weight of the building, corresponding settlement will take place. Because of the confining effect of the edges and the corners of an excavated pit, this upheaval and subsequent settlement theoretically should be largest in the center of the area, approximately one half of this value along the edges and one quarter at the corners. The actual magnitude of the upheaval is dependent chiefly on the lateral reinforcement of the soft soil by sand layers and lenses. Although it is known that this lateral deformation can assume large proportions for more homogeneous deposits of very soft clay, such as those which exist in Mexico City, no serious movements have ever been observed in Boston. This upheaval was measured, as will be described subsequently, during construction of a large building in Boston.

---

[12] "The Floating Foundation of the New Building for the National Lottery of Mexico," by Jose A. Cuevas, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., June, 1936.

(*b*) Since building loads are usually transferred on to a bearing stratum by means of individual column footings, wall footings, caissons, or pile clusters which cover only part of an entire building area, loads concentrated on this stratum will cause a slight deformation and consolidation of the soil immediately below the footings. Again the amount of settlement due to this cause will be reduced along the edges and particularly at the corners of a building area, due to the increased lateral confinement of the soil at these places.

(*c*) If the bearing stratum is very thin, there will be a net increase in stress in limited volumes of the softer soil underlying the bearing stratum. The soil will adjust itself to these increased stresses chiefly by consolidation and to a small degree also by lateral deformation.

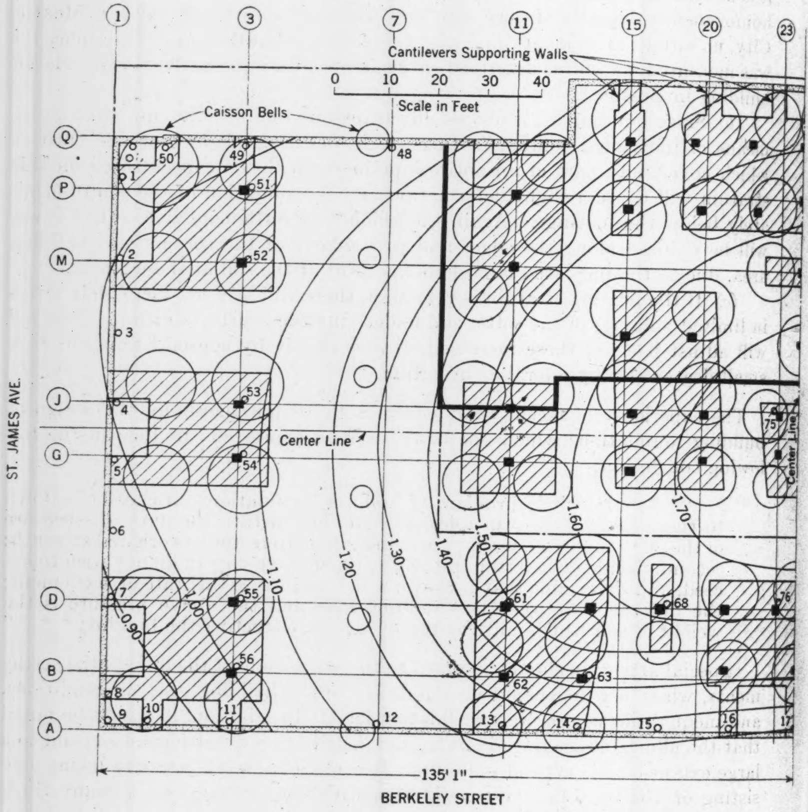
The experience of recent years confirms the requirements for the design of building foundations which one of the writers expressed[3] in 1932 in the following conclusion:

> "* * * the whole problem of building foundations on clay boils down to these two simple principles: first, do not disturb the natural structure of the clay; if you do, no human being is able to restore its original strength; second, decide on a certain rate of settlement which you do not wish to exceed, and determine that pressure which will cause this rate of settlement; the difference between the building load and the above pressure is the weight of soil which must be removed before erecting the building * * *."

Special attention should be given to the magnitude of the differential settlements, which are dependent on the design of the building, the soil conditions, and inequalities in the preconsolidation load. In addition, it should be noted that the allowable magnitude, rate, and distribution of settlement depend to a large extent on the type of building. It is obvious that a masonry facing consisting of 4-ft by 6-ft limestone blocks, with very thin joints, is many times more sensitive to differential settlements than, for example, a brick building.

The Liberty Mutual Insurance Company Building in Boston.—An example of the application of soil mechanics to the design of the foundations for a typical office building is furnished by the Liberty Mutual Insurance Company Building which was built in 1936–1937. This building has a plan area 270 ft by 116 ft, and consists of a heavy tower section and two lighter wings. It is faced with large, thin, limestone blocks which are very sensitive to differential movements. The basement was so designed that the weight of the soil excavated was slightly less than the building load on all sections of the plan area occupied by the building.

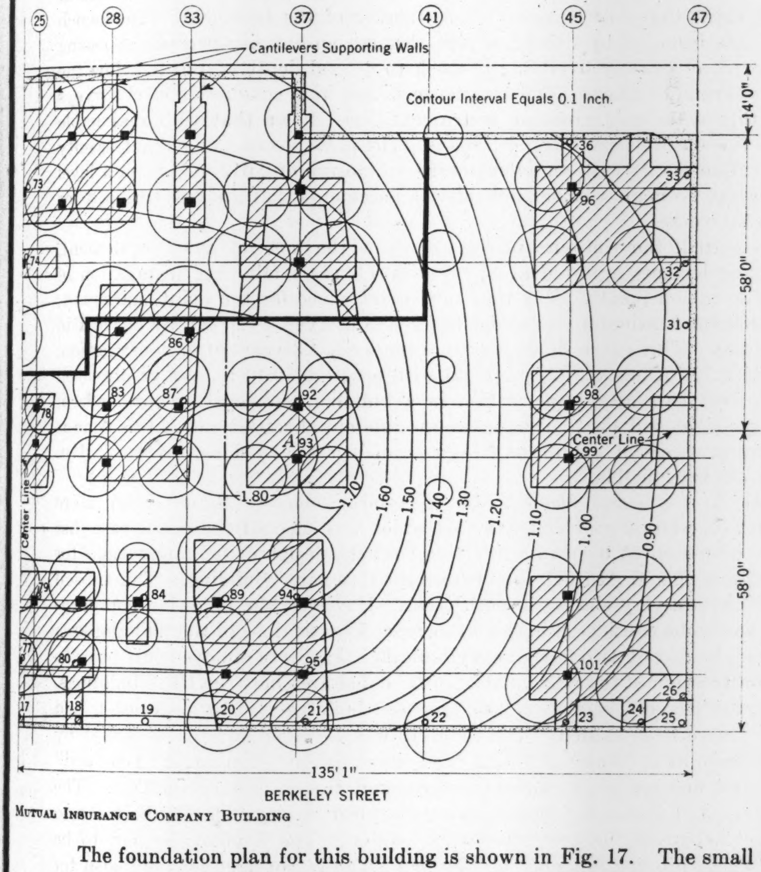
The building load is transferred to the basement by a great number of columns, which in turn are transferred by concrete piers on to the surface of a hard clay stratum at a depth of about 40 ft below ground surface. The bottoms of the piers are increased in area by means of bells, the diameters of which are governed by the column loads as well as by the thickness of the underlying hard clay stratum. The allowable load on the hard clay was determined from compression tests on undisturbed samples. The hard clay, which averages 10 ft in thickness, is underlain by 60 ft of soft clay.



1
3
7
11
15
20
23
Cantilevers Supporting Walls
0
10
20
30
40
Scale in Feet
Caisson Bells
Q
P
M
J
G
D
B
A
ST. JAMES AVE.
Center Line
Center Line
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
0.90
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
48
49
50
51
52
53
54
55
56
61
62
63
68
75
76
135' 1"
BERKELEY STREET

FIG. 17.—FOUNDATION PLAN, LIBERTY

To reduce the differential settlement of the exterior foundation walls to a minimum, they were designed as heavily reinforced girders. To reduce the differential settlement between the exterior foundation walls and the interior footings, these walls are supported by cantilever beams which transfer the wall loads to piers carrying interior columns. This arrangement increases the absolute settlement of the circumferential wall, but in so doing it decreases the differential movement in reference to the interior of the building. Only at those points where it was not possible to arrange a cantilever system were piers placed directly beneath the walls.


25
28
33
37
41
45
47
Cantilevers Supporting Walls
Contour Interval Equals 0.1 Inch.
14'0"
58'0"
58'0"
STUART STREET
Center Line
Center Line
36
33
96
32
31
86
83
87
92
98
78
A
93
99
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
79
84
89
94
76
77
80
95
101
26
17
18
19
20
21
22
23
24
25
73
74
135' 1"
BERKELEY STREET

Mutual Insurance Company Building

The foundation plan for this building is shown in Fig. 17. The small dark squares represent interior column locations, the shaded areas represent mats, and the circles represent the diameters of pier bells bearing on the stratum of hard clay. The contours represent points of equal settlement in reference to a permanent bench mark for a period of 4.3 years.

The building load varies from 1.3 tons per sq ft under the central tower section to 0.8 ton per sq ft under the adjacent wings. To equalize the net increase of pressure throughout the building area, a sub-basement was installed under the tower section. Thus, the net increase in pressure was made equal to 0.2 ton per sq ft in all areas.

The small numbered circles in Fig. 17 designate the locations of permanent settlement observation points which were established during the construction of the substructure. The elevations of these points were determined periodically by means of precision water-level surveys in reference to a permanent bench mark that was installed in one corner of the basement. The bench mark was installed by driving a pipe through an exterior permanent casing into hardpan directly overlying bedrock to a total depth of 120 ft below the original ground surface. The experience gained from conducting level surveys with this water-level precision instrument[13] has shown that with reasonable care a line of levels can be made around a closed loop containing about twenty observation points with an average error of closure of 0.012 in. Such a level survey can be made without difficulty in basements that are fully utilized for storage purposes.

As stated, the contours in Fig. 17 show the lines of equal settlement. These settlements occurred in a period of 4.3 years following installation of the observation points. The time-settlement curve for the point marked A, at which the maximum settlement has been observed, is shown by curve I in Fig. 15(*b*). This curve shows a rate of increase, 4.3 years after construction, of only 0.12 in. per yr, as compared with a rate of 0.40 in. per yr observed some nine years after construction in a building, previously mentioned, of similar weight and size. The maximum differential settlement at this time is slightly in excess of 1.0 in., and the rate of increase as shown by curve I in Fig. 15(*c*) is negligible.

The New England Mutual Life Insurance Company Building.—A more difficult foundation problem than that which was encountered in the case just described presented itself in connection with the design of the foundations for the New England Mutual Life Insurance Company Building. The subsoil conditions are very similar in both cases. However, the following complications had to be considered in the latter case: Two sections of the building area were pre-loaded by two fairly heavy buildings that were removed just prior to the construction of the new building. An equal area between these buildings, where the heaviest section of the proposed building was to be located, had never before been loaded. The building was to cover an area of 340 ft by 200 ft and was to consist of a heavy ten-story section occupying a "T-shaped" area, and one one-story, two two-story, and two four-story sections. The one-story and four-story sections, at some future time, might be built up to the full height of the heavy ten-story section. The foundations had to be designed to provide not only for the first stage of construction, but also for all enlargements contemplated for the future.

Extensive underground explorations were undertaken. Nine exploratory borings were made from which almost continuous thin-walled tube samples were obtained from the hard and soft clay strata. Two large-diameter borings were then made for the purpose of obtaining undisturbed samples for laboratory investigations of the compressibility characteristics of the material below the elevation at which the building was to be founded.

---

[13] The type of instrument used in these surveys is described in "Practical Application of Soil Mechanics: Settlement of Structures in Europe and Methods of Observations," by Karl Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 1432–1448, particularly pp. 1433–1436.

Settlement investigations for various preliminary designs of the building indicated that, with the aforementioned complications, serious differential settlements could be avoided only by using the principle of a floating foundation, thereby excavating as much soil as the building, including the anticipated future enlargements, would weigh. This estimated weight was 130,000 tons as compared with 40,000 tons for the Liberty Mutual Building.

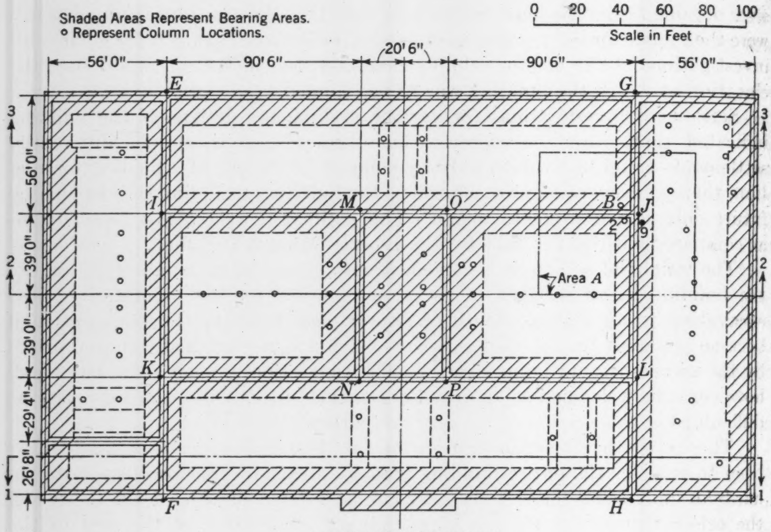
The weight of soil to be removed, necessary to counterbalance the load of the building in all sections of the area occupied by the building, required excavating to an average depth of approximately 35 ft. For this reason it became apparent that a sub-basement, which was not originally contemplated by the owners, would be necessary. The suggestion of such a sub-basement, however, was welcomed as an area for garaging automobiles, since the parking difficulties in this section of the city are particularly acute.

The only other feasible solution of this foundation problem would have been to transfer the entire building load by means of piles or piers on to the hardpan underlying the soft clay stratum at a depth of at least 125 ft beneath the original ground surface. A preliminary comparison of the cost of this type of foundation with a floating type of foundation showed that the transfer of the load to the hardpan would be considerably more expensive.

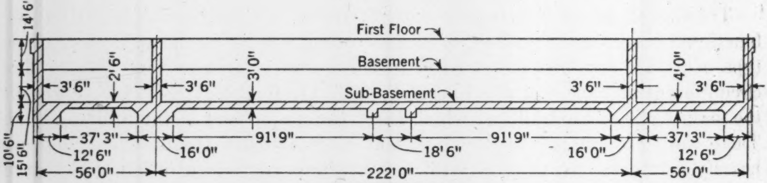
The design of the foundation was influenced vitally by the fact that the functional requirements called for clear working spaces, free of columns. This type of load distribution lent itself to the design shown in Fig. 18(*a*). The circumferential lines and the lines EF, GH, IJ, KL, MN, and OP represent the main walls of the building which carry the major part of the building load. These walls are extended through the sub-basement and the basement and are heavily reinforced to provide sufficient stiffness to reduce differential settlements that might result from irregularities in thickness and consistency of the hard clay stratum. It was determined, from load tests and unconfined and triaxial compression tests, that the allowable load on the hard clay should not exceed 4 tons per sq ft. The widths of the wall footings as shown by the dotted lines in the foundation plan, Fig. 18(*a*) are such that this unit load is not exceeded. The bearing pressure under the circumferential walls was increased in relation to the unit load for the interior wall footings to reduce the differential settlements between the outside and the inside walls. An additional increase in the design unit load on the hard clay was provided at the corners of the building by decreasing the footing widths at these locations.

Typical cross sections, showing the sections of the walls, wall footings, and basement slabs, are given in Figs. 18(*b*), 18(*c*), and 18(*d*).

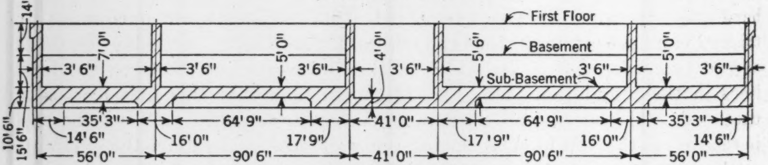
The uplift produced by a head of 22 ft of water controlled the design of the sub-basement slab in most instances. Exceptions to this are found in those areas where columns and heavy equipment, such as boilers, rest directly on the slab. Thicker slabs were necessary to provide for these loads. The column loads are not carried directly down to the hard clay, but are transferred


Shaded Areas Represent Bearing Areas.
○ Represent Column Locations.
0
20
40
60
80
100
Scale in Feet
56' 0"
90' 6"
20' 6"
90' 6"
56' 0"
E
G
3
3
56' 0"
I
M
O
B
J
2
9
39' 0"
2
2
Area A
39' 0"
K
N
P
L
29' 4"
26' 8"
1
1
F
H

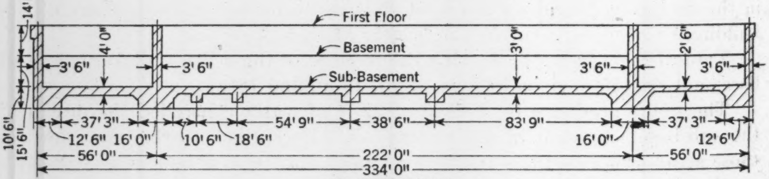
(a) FOUNDATION PLAN


14' 6"
First Floor
Basement
Sub-Basement
2' 6"
3' 0"
4' 0"
3' 6"
3' 6"
3' 6"
3' 6"
37' 3"
91' 9"
91' 9"
37' 3"
12' 6"
16' 0"
18' 6"
16' 0"
12' 6"
10' 6"
15' 6"
56' 0"
222' 0"
56' 0"

(b) SECTION 3-3


14' 6"
First Floor
Basement
Sub-Basement
7' 0"
5' 0"
4' 0"
5' 6"
5' 0"
3' 6"
3' 6"
3' 6"
3' 6"
3' 6"
3' 6"
35' 3"
64' 9"
41' 0"
64' 9"
35' 3"
14' 6"
16' 0"
17' 9"
17' 9"
16' 0"
14' 6"
10' 6"
15' 6"
56' 0"
90' 6"
41' 0"
90' 6"
56' 0"

(c) SECTION 2-2


14' 6"
First Floor
Basement
Sub-Basement
4' 0"
3' 0"
2' 6"
3' 6"
3' 6"
3' 6"
3' 6"
37' 3"
54' 9"
38' 6"
83' 9"
37' 3"
12' 6"
16' 0"
10' 6"
18' 6"
16' 0"
12' 6"
10' 6"
15' 6"
56' 0"
222' 0"
56' 0"
334' 0"

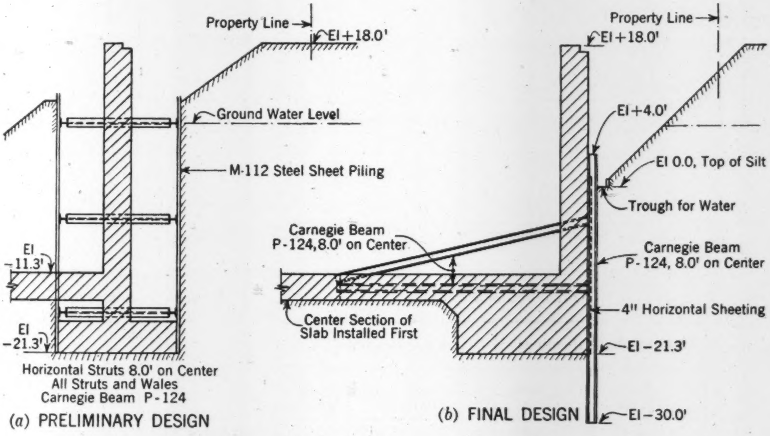
(d) SECTION 1-1

Fig. 18.—New England Mutual Life Insurance Company Building

to the main walls for the purpose of eliminating serious differential movements between the individual columns and the main walls. The objection might be raised that a more economical transfer of load might have been arranged by the use of individual girders beneath the sub-basement floor. Unfortunately, the depth of girders required would have been such that the girders would have borne on the hard clay and thereby defeated the purpose of the load transfer. With the exception of the small rectangular area bounded by MNOP in Fig. 18(*a*), all of the sub-basement floors are supported on the main walls and are separated from the hard clay by at least 3 ft of the highly compressible, organic silt-clay. It was specified that, if it were discovered, during the progress of construction, that the hard clay in some areas approached the bottom of the sub-basement slabs, it would be necessary to excavate to a sufficient depth and replace the hard clay by soft organic silt-clay.

The unsymmetrical footings of the circumferential walls, shown in Fig. 19(*b*), evolved from adapting considerations of soil mechanics and requirements for simplicity and economy of construction to the conventional type of footing shown in Fig. 19(*a*). By cutting off the outside of the footing and extending the inside, and by filling the space between the footing and slab, the following advantages were gained:

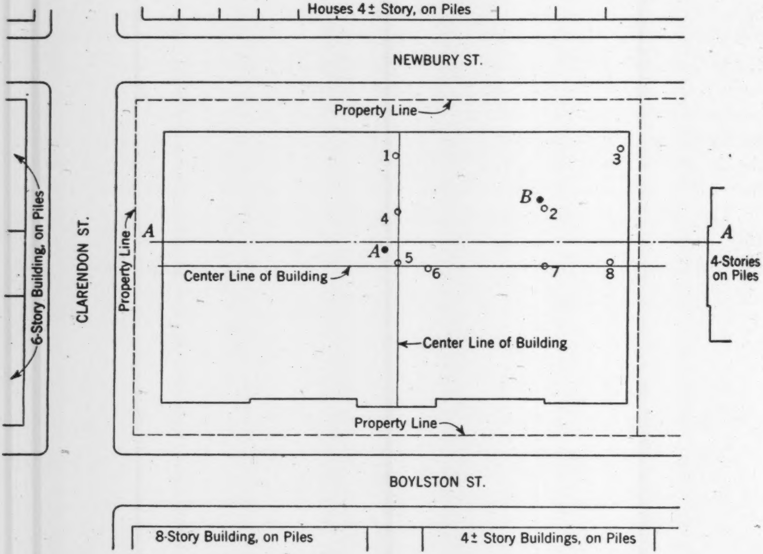
(1) Increased settlement of the circumferential walls, and thereby a decrease in differential settlement;
(2) Material reduction in the span of the sub-basement slabs; and
(3) A simple and economical method of construction.


Property Line
El +18.0'
Ground Water Level
M-112 Steel Sheet Piling
El -11.3'
El -21.3'
Horizontal Struts 8.0' on Center
All Struts and Wales
Carnegie Beam P-124
Property Line
El +18.0'
El +4.0'
El 0.0, Top of Silt
Trough for Water
Carnegie Beam P-124, 8.0' on Center
Carnegie Beam P-124, 8.0' on Center
4" Horizontal Sheeting
Center Section of Slab Installed First
El -21.3'
El -30.0'

(*a*) PRELIMINARY DESIGN (*b*) FINAL DESIGN

FIG. 19.—UNSYMMETRICAL FOOTINGS OF THE CIRCUMFERENTIAL WALLS

The general excavation was carried first to a depth of about 20 ft, to the top of the organic silt-clay, and from there vertical H-section piles, as shown in Fig. 19(*b*), were driven. As the excavating proceeded, horizontal and inclined H-section braces were installed. These bore against sections of the slab that had been installed prior to excavating the final berm. The horizontal braces were later concreted into the slab, and the inclined braces were cut off.


Houses 4± Story, on Piles
NEWBURY ST.
Property Line
1
3
B
2
4
A
A
A
5
6
7
8
Center Line of Building
Center Line of Building
Property Line
Property Line
6-Story Building, on Piles
CLARENDON ST.
4-Stories on Piles
BOYLSTON ST.
8-Story Building, on Piles
4± Story Buildings, on Piles

Point	Overburden Removed				Pouring Dates (1939)	
	Intermediate Readings		Final Readings		Slab	Wall
	Date (1939)	Rise (ft)	Date (1939)	Total rise (ft)		
1	November 26	0.11	....	....	November 29	December 11
2	September 5	0.14	September 11	0.20	September 19	December 6
3	October 24	0.16	October 30	0.19	November 2	December 1
4	October 12	0.20	October 23	0.22	October 27	December 26
5	October 12	0.24	....	....	October 27	December 26
6	October 19	0.20	October 23	0.23	October 27	December 26
7	September 5	0.16	September 11	0.21	September 22	November 27
8	October 24	0.15	October 30	0.19	November 2	December 1

Points 1 to 8 were established in the ground prior to the removal of the overburden (June, 1939).
Overburden (wet) = 2.24 tons per sq ft.

FIG. 20.—FOUNDATION MOVEMENTS, NEW ENGLAND MUTUAL LIFE INSURANCE COMPANY BUILDING

The vertical piles and horizontal wooden sheeting remained in place and were used as the outside form work. The development of this ingenious solution is essentially due to L. S. Homer, M. Am. Soc. C. E., superintendent for the Turner Construction Company.

Observations of Movements.—It was stated that, if a foundation is designed in such a manner as to produce a zero net increase in stress in underlying compressible soil after excavating and building operations are completed, the major part of the subsequent settlement of a structure is due to the elastic deformation and the consolidation of the soil immediately below the footings. Were it possible to distribute the building load uniformly over the building area, and were it possible to excavate and apply the building load simultane-

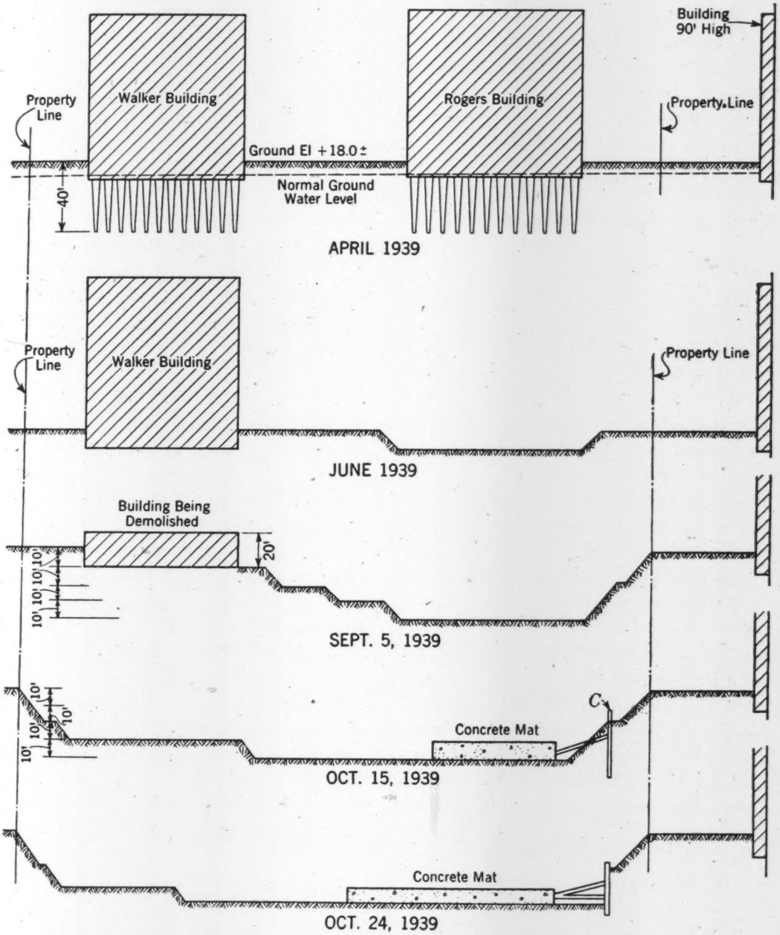

Building 90' High
Property Line
Walker Building
Rogers Building
Property Line
Ground El +18.0±
Normal Ground Water Level
40'
APRIL 1939
Property Line
Walker Building
Property Line
JUNE 1939
Building Being Demolished
20'
10'
10'
10'
10'
10'
SEPT. 5, 1939
10'
10'
10'
10'
C
Concrete Mat
OCT. 15, 1939
Concrete Mat
OCT. 24, 1939

Fig. 21.—Successive Measurements of Section A–A, Fig. 20

ously, no settlement would result. However, it is impracticable to design and construct a building in such a manner that these requirements are fulfilled, and it is necessary, therefore, to investigate the consequences that might result if these two conditions are not fulfilled.

For this purpose an effort was made to obtain a record of the movement of the bearing stratum during and subsequent to all construction operations. To obtain this record, it was necessary to measure the rise in elevation of this stratum, resulting from the removal of an average of 35 ft of overburden, and then to determine the downward movement which took place when the weight of the foundations and superstructure was added.

Prior to excavating, underground reference points were established at eight locations in one quadrant of the building area, as shown in Figs. 20 and 21. These reference points consisted of 2-in. tubes, each 6 ft in length, which were driven into the hard clay. This was accomplished at each location by advancing the casing to the top of the hard clay. After the casing had been cleaned out, a steel tube was lowered and driven 6 ft. The elevation of the top of the tube was then established. For the purpose of marking the location of each tube, colored wooden sticks, which were weighted to prevent flotation, were lowered as the casing was withdrawn.

The movements recorded for each point are given in the table in Fig. 20. The maximum upward movement observed when the eight points were first uncovered was 2.9 in., at point 5 in the center of the area, and the minimum was 1.3 in., at point 1, which is located on the shorter axis near the periphery of the building area. The second set of observations, which was made on six of the points one week after they were first uncovered, and just prior to installing the sub-basement slab, showed that in this period they had continued to rise so that the ultimate total rise for each point was approximately the same.

Settlements resulting from the placing of wall footings 10 ft thick and adjacent sub-basement slabs were determined at the location marked "B" in Fig. 20 by observing the movement of the top of a 1½-in. steel rod which had been driven 3 ft into the hard clay. This rod, which projected up through a piece of pipe encased in the concrete, settled approximately 0.5 in. during the placing of the concrete. Subsequent observations which were made at periodic intervals in reference to a permanent bench mark (of the type installed in the Liberty Mutual Insurance Company Building) showed that this point moved downward as follows:

Time of observation	Approximate load (tons per sq ft)	Total downward movement (inches)
As concrete was placed	0.75	0.5
After six days	0.75	0.75
After one month	1.00	1.00
After two and one-half months	1.25	1.00
After three months	1.75	1.25

—a total of 1¼ in. during the three-month period following the installation of the concrete slab and footings at this location. At the end of this three-month period, during the installation of the basement walls, permanent observation points were established at seventy-three locations throughout the sub-basement area. The elevations of these points were observed periodically in reference to the permanent bench mark by means of precision water-level surveys. Point 9 (see Fig. 18(*a*)) was located within 10 ft of the locations of underground observation points 2 and B, thus affording a means of continuing observations of the movement of the bearing stratum at this location.

At point A, Fig. 20, a 4-in. boring had been made in April, 1938. When the overburden was removed in October, 1939, this hole was found to have been reduced to 2.5 in.

The top of a pile driven at point C (see Fig. 21) was observed in October, 1939, to have moved inward ¾ in. and to have been raised ½ in. In December, 1939, after the exterior footings and walls had been poured, this pile had moved downward ¾ in.

After the excavation and shoring were completed, the street curbs moved in about 1¼ in. and down about 1¼ in. No horizontal or vertical movement was observed in adjacent buildings.

A continuous record of the movements that occurred during the construction period, following the demolition of buildings which formerly occupied the site, together with diagrams which show the change in net unit load at this location, are shown in Fig. 22. The shaded diagram represents the net unit load on the bearing stratum at the location of the intersection of two interior cross-wall footings (see Fig. 18(*a*)) where points 2, B, and 9 are located. The net unit load at point B is 1.41 tons per sq ft. The dotted diagram shows the average net unit load (= − 0.39 ton per sq ft) on the area marked "A" in Fig. 18(*a*). This area includes one half of the unloaded area adjacent to these wall footings. The ordinates of both diagrams show the net change in unit load resulting from all construction operations—that is, the algebraic sum of the unit load

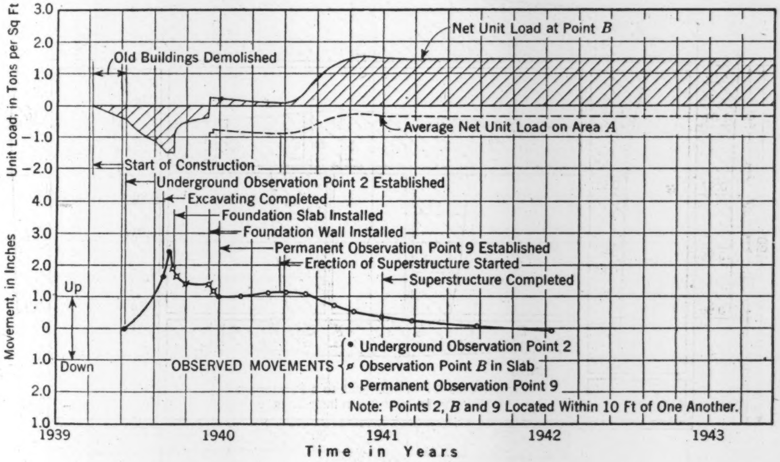
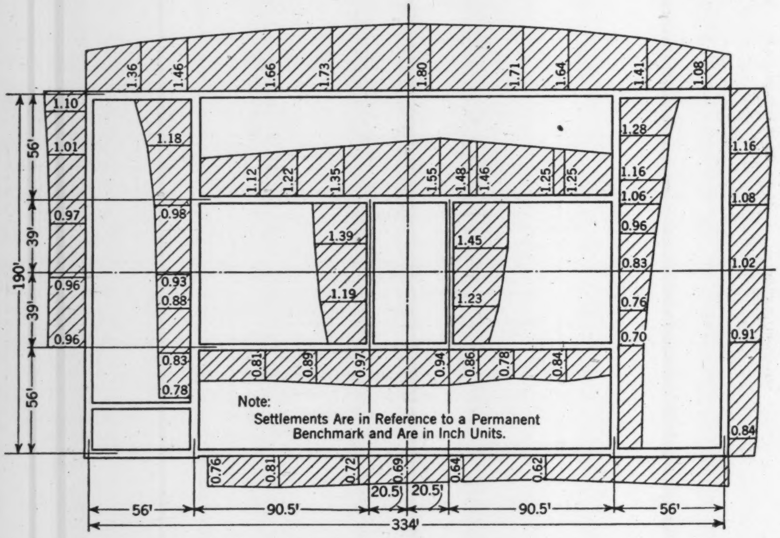


FIG. 22.—TIME-LOAD DIAGRAMS AND OBSERVED MOVEMENTS

removed by the demolition of old buildings, the unit load removed by excavating an average of 35 ft of soil, the temporary unit load added by the lowering of the ground-water table during construction, and the unit load of the foundations and superstructure of the new building.

The shaded diagram shows an ultimate local increase in pressure immediately below the wall footings. The dotted diagram, which represents the average change in unit load for a typical section of the building area, including unloaded parts, shows an ultimate net decrease at the conclusion of all construction operations. Therefore, the subsoil is subject to an increase in pressure only to a limited depth below the footings, and for this reason only a comparatively small volume of soil will be compressed.

The loading diagrams assist in an interpretation of the curve showing the foundation movements. This latter curve shows that during the excavating period the bearing stratum rose 2.40 in. After the foundations had been installed, it settled back 1.40 in. In the six-month period that followed, during which time hydrostatic pressure was increasing under the slab with very little building weight being added, a slight rise in elevation took place. Then, as the building load increased rather rapidly during the erection of the superstructure, the stratum began to settle. As of January 15, 1942, which was approximately one year after the superstructure was substantially completed, the net movement was 0.06 in. downward. It will be noted that, of the total downward movement (2.46 in.), 1.40 in. occurred during construction of the foundations prior to erection of the superstructure.


1.36
1.46
1.66
1.73
1.80
1.71
1.64
1.41
1.08
1.10
1.01
0.97
0.96
0.96
1.18
0.98
0.93
0.88
0.83
0.78
1.12
1.22
1.35
1.55
1.48
1.46
1.25
1.25
1.39
1.19
1.45
1.23
0.81
0.89
0.97
0.94
0.86
0.78
0.84
1.28
1.16
1.06
0.96
0.83
0.76
0.70
1.16
1.08
1.02
0.91
0.84
0.76
0.81
0.72
0.69
0.64
0.62
56'
39'
39'
56'
190'
56'
90.5'
20.5'
20.5'
90.5'
56'
334'

Note:
Settlements Are in Reference to a Permanent Benchmark and Are in Inch Units.

FIG. 23.—SETTLEMENT PROFILES, NEW ENGLAND MUTUAL LIFE INSURANCE COMPANY BUILDING

The movements that have occurred at other points throughout the building area, as determined from periodic precision water-level surveys of the permanent observation points, are shown by the settlement profiles in Fig. 23. The settlements that are represented by these profiles have occurred in the 2-yr period following the installation of the foundation walls. The settlements thus represented include the movements which took place during erection of the superstructure. These profiles show little differential movement.

The progress of settlement of the point at which the maximum settlement has occurred is shown by curve II in Fig. 15(*b*). The maximum differential settlement determined from the seventy-three points under observation is shown by curve II in Fig. 15(*c*). The trend of these curves is sufficiently well established to indicate that future settlements will be of a small order of magnitude.

CONCLUSIONS

(1) The analysis of observed settlements of buildings in Boston shows clearly that settlements can be reduced to a tolerable magnitude by removing a quantity of soil which, in weight, is approximately equal to that of the superimposed structure.

(2) This method of "floating" a building is particularly adaptable to locations where a suitable bearing stratum is located from 20 to 40 ft below the ground surface and where soil and ground-water conditions permit the necessary excavating.

(3) A structure that is underlain by soft soil, and which obtains support from such soil, will undergo some settlement even though the weight of the structure does not exceed the weight of soil removed. The removal of a mass of soil will cause an upheaval of the bottom of an excavated pit, due in part to a deformation and in part to a swelling of the underlying soil. Upon reestablishing the original stress conditions, a settlement at least equal to the magnitude of the upheaval will take place.

(4) The settlements due to deformation and recompression following swelling take place essentially during the construction period. Progressive settlements will occur subsequent to this period for many years at a decreasing rate if the weight of the structure is greater than the weight removed from the building area prior to its construction.

(5) Differential settlements are best controlled by designing the foundation and the superstructure as an integral unit, with special attention to stiffness. Where this is not acceptable, design for stiffness must be confined to the foundation, which increases the cost and decreases its effectiveness.

(6) Differential settlements can be controlled to some extent by increasing the unit loading on those areas where the smallest progressive settlements are anticipated and decreasing the unit loading where the largest progressive settlements are anticipated.

ACKNOWLEDGMENTS

The writers wish to acknowledge gratefully the cooperation of Chester Lindsay Churchill, architect for the Liberty Mutual Insurance Company

Building; C. N. Godfrey of the architectural firm of Cram and Ferguson, which designed the New England Mutual Life Insurance Company Building; the Turner Construction Company, and in particular L. S. Homer, superintendent of construction for both the Liberty Mutual and the New England Mutual Insurance Company buildings; H. A. Mohr, M. Am. Soc. C. E., of the Raymond Concrete Pile Company; and the City of Boston Building Department. Messrs. P. C. Rutledge, F. M. Baron, Assoc. M. Am. Soc. C. E., and E. M. Fucik and K. S. Lane, Juniors, Am. Soc. C. E., former members of the staff of the Graduate School of Engineering, Harvard University, Cambridge, Mass., assisted the writers in the design of the foundations of these buildings. J. A. Delaney, a present member of the staff, assisted in conducting level surveys. M. J. Hvorslev, Assoc. M. Am. Soc. C. E., supplied copy for Fig. 6; and reproduction of the information in Figs. 20 and 21 was made possible through the courtesy of the Turner Construction Company.

APPENDIX

NOTATION

The following letter symbols, used in this paper, conform essentially to "Soil Mechanics Nomenclature"[14] presented in 1941 by the Committee of the Soil Mechanics and Foundations Division on Glossary of Terms and Definitions and on Soil Classification:

A = area covered by a footing;
b = width of a footing;
c = a constant shear strength;
e = void ratio;
h = depth of a footing;
P = concentrated, or equivalent concentrated, load on a footing;
p = average unit load on a footing;
q_o = average unit pressure exerted by a footing;
$R = \sqrt{r^2 + z^2}$;
r = radius or radial distances from an axis;
z = distance parallel to the z-axis (depths), measured from the base of the footing;
α = a concentration factor (Eq. 1*b*);
β = an angle in Fig. 10;
γ = weight of soil per unit volume;
Δ = increment of increase; thus: Δp = any additional unit pressure; Δe = change in void ratio, etc ;
δ = unit strain; and
σ = principal stress.

---

[14] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

DISCUSSION

Jacob Feld,[15] M. Am. Soc. C. E.—The use and the limitations of soil tests and soil theories that should be considered in building foundation design are summarized in this paper. Definite rules are set forth for cohesionless soils, and the general advice is given that the "floating method" be used in cohesive soils.

The thirteen conclusions and recommendations listed for cohesionless soils (see heading: "Introduction—Stress-Deformation Characteristics of Cohesionless Soils") are open to question since, in some instances, they do not agree with either experiment or experiences. For example:

(*a*) There is no justification for the assumption that the intensity of pressure at the edge of a rigid area is zero. The experimental works of W. S. Housel,[16] M. Am. Soc. C. E., of R. B. Fehr and C. R. Thomas,[17] and of Frederick J. Converse,[18] Assoc. M. Am. Soc. C. E., definitely show the existence of some pressure along the edges of footings laid in sand.

(*b*) The uniformity of base pressure in pier foundations is the result of the rigidity of a deep footing rather than of the depth of the footing. Similar uniformity of base pressure should be expected under a gravity type of abutment built in a shallow excavation.

(*c*) A rewording of this rule might clarify its meaning, to the effect that the unit bearing is independent of the width of the loaded area. However, this is not consistent with conclusion (*i*). The effect of footing shape should be considered, as well as the corresponding effect of edge resistance in sustaining loads.

(*d*) This rule is not substantiated by tests of bearing values on deep caissons. The more likely rule should read:

"The increase in ultimate bearing capacity of a cohesionless soil with depth is approximately as the square of the depth of the loaded area."

(*e*) and (*f*) The author suggests correctly that, taking depth into account, different allowable bearing values should be assumed for interior and exterior wall footings.

(*g*) The elimination of the Boussinesq formula as a tool to indicate the range of expected values is not wise until a more accurate method is devised. Although the Boussinesq formula is based on the assumption of isotropic elastic materials (which assumption does not include soils at failure), it should be emphasized that the theory of elasticity of materials is similarly limited. Since, under small loadings, the strains in soils are substantially homogeneous and the lack of tensile strength does not come into play until the tensile components of internal shear strains exceed the initial compression, there is a range of loading for which the Boussinesq formula, in the original or modified form, does serve a useful purpose.

---

[15] Cons. Engr., New York, N. Y.

[16] *Research Bulletin No. 13*, Univ. of Michigan, Ann Arbor, Mich., 1929.

[17] *Bulletins*, Pennsylvania State College, State College, Pa., 1912–1915.

[18] *Civil Engineering*, April, 1933, p. 207.

In the list of recommendations for noncohesive soils, at least a warning of the effect of unwatering on settlements also should be included.

The statement that the action of foundations in clays can be properly predicted only if the loads are placed in excavations deep enough to "float" the building, is wise. Mention might be made of the practice in clay localities (Texas, Connecticut, etc.) of not removing the last foot or so of the excavation until just before placing the concrete so as to reduce the settlement from recompression of the clay swelling at subgrade.

Much can be learned of foundation design by the analysis of corrective measures for buildings showing differential settlements. The writer described a few cases in a paper in 1930.[19] A list of references on the subject of bearing capacity published before 1913, in readily available literature as determined for the design of structures now in use, is as follows:

(1) **"Preliminary Foundation Tests for the St. Paul Building,"** *Engineering Record*, Vol. 33, May 2, 1896, p. 388.
(2) **"Safe Load on Soil at New Orleans, La.,"** *Engineering News*, Vol. 41, May 11, 1898, p. 330 (correction, p. 333).
(3) **"Foundation Construction for the New Capitol for South Dakota,"** by Samuel H. Lea, *Engineering Record*, Vol. 57, April 4, 1908, p. 437.
(4) **"Bearing Tests for Heavy Foundation Loads,"** *Engineering Record*, Vol. 60, July 10, 1909, p. 55.
(5) **"Testing Bearing Power of Hard-Pan. Extension, Whitehall Building, New York City,"** *Engineering Record*, Vol. 61, June 18, 1910, p. 792.
(6) **"Tests and Costs of Making a Test of the Bearing Power of Soil for a Building,"** *Engineering-Contracting*, Vol. 34, July 13, 1910, p. 31.
(7) **"Test of Bearing Capacity of Sand under Municipal Building, New York City,"** *Engineering Record*, Vol. 62, July 9, 1910, p. 46; July 16, 1910, p. 57; and *Engineering-News*, Vol. 63, January 6, 1910, p. 24; Vol. 64, November 17, 1910, p. 525.
(8) **"Device for Making Sub-Surface Tests of the Bearing Power of Soils with Some Examples of Operation,"** *Engineering-Contracting*, Vol. 34, August 3, 1910, p. 94.
(9) **"Testing Soil Below the Surface for Foundation Loads,"** *Engineering Record*, Vol. 62, July 16, 1910, p. 71; Vol. 63, May 6, 1911, p. 512.
(10) **"Testing Foundations at the Municipal Building, New York,"** *Engineering Record*, Vol. 63, February 18, 1911, p. 196.
(11) **"Standard Test of Soil,"** by Rudolph P. Miller, *Engineering Record*, Vol. 66, July 27, 1912, p. 112.
(12) **"Soil-Bearing Tests,"** *Engineering Record*, Vol. 66, September 14, 1912, p. 304.
(13) **"Hard-Pan and Other Soil Tests,"** by J. Norman Jensen, *Engineering News*, Vol. 69, March 6, 1913, p. 460 (also editorial, p. 463).
(14) **"Building Foundations,"** by J. A. Smith, *Journal*, Associated Eng. Soc., Vol. 36, April, 1906, p. 155.

---

[19] "Several Foundation Repair Jobs," by Jacob Feld, *Proceedings*, Brooklyn Engrs. Club, April, 1930.

(15) **"Results of Tests on Chicago Hard-Pan at a Depth of 57 Feet Below Lake Level,"** by Frank A. Randall, *Engineering-Contracting*, Vol. 37, April 17, 1912, p. 436.
(16) **"Supporting Power of Soils,"** by Randall Hunt, *Journal*, Associated Eng. Soc., Vol. 7, June, 1888, p. 189; and *Engineering News*, Vol. 19, June 16, 1888, p. 484.
(17) **"Construction of the Buildings, Bridges, Piers and Docks at Jackson Park,"** *Engineering Record*, Vol. 28, August 26, 1893, p. 199.
(18) **"Allowable Pressure on Deep Foundations,"** by Elmer L. Corthell, *Engineering News*, Vol. 56, December 20, 1906, p. 657; and *Engineering Record*, Vol. 54, December 15, 1906, p. 647 (editorial).
(19) **"Foundation Pressure on Hard-Pan; Proposed Rule,"** by Rudolph P. Miller, *Engineering News*, Vol. 74, December 29, 1910, p. 727; and *Engineering Record*, Vol. 62, December 31, 1910, p. 783.
(20) **"Sand Foundations for High Buildings,"** *Engineering Record*, Vol. 66, September 21, 1912, p. 310.
(21) **"Report on Unit Pressure Allowable on Road-Beds of Different Materials,"** *Proceedings*, A.R.E.A., Vol. 13, 1912, p. 388.
(22) **"Failure of the Transcona Grain Elevator,"** *Engineering News*, Vol. 70, November 6, 1913, p. 944.

LEONARD ZEEVAERT,[20] JUN. AM. SOC. C. E.—The material contained in the paper is of great value and very encouraging for it shows the ready application of soil mechanics in the design of building foundations. The writer has had the opportunity to design the foundations for three buildings constructed in Mexico City, Mexico, in which principles similar to the ones stated in the paper have been used.

In each of these buildings the basement was excavated to variable depths to give a more uniform and favorable distribution of loading and thus to decrease the dishing effect. Early indications show greatly decreased dishing as compared to similar adjacent buildings and to other buildings in the city which, according to usual practice, have equal depth of excavation throughout the foundation area. Decreased average settlements are obtained because of the more uniform stresses within the clay strata. The writer found that the idea of excavating to variable depths in order to create more uniform stresses in the compressible strata, and therefore to reduce differential settlements, helps considerably and is readily applicable to very compressible soils, such as Mexico City clays. He has called this type of foundation a "compensated foundation."

To compute the load distribution on the foundation slab when a rigid foundation is to be used in connection with underlying clay, "the edge load problem" was devised. The compensated excavation and an edge load applied along the boundary of the foundation are the means used to obtain an essentially uniform stress in the clay stratum below the structure. The load thus applied at the surface to hold these uniform stresses within the clay stratum is the load used to compute the foundation structure of the building in question.

---

[20] Prof., Soil Mechanics, School of Civ. Eng., Univ. of Mexico; Cons. Engr., Mexico City, Mexico.

One example of the aforementioned items is the Wong Office Building (165 by 56 ft and 105 ft high) under which there is a stiff fill underlain by a very soft clay stratum extending about 90 ft to the underlying sand stratum. The building is of concrete with an average net load of 0.9 ton per sq ft. In this case, the ratio of the load of the building to the weight of the soil excavated was about 9 to 5. In the analysis it was recognized that adjacent buildings transmitted considerable stress to the clay beneath the Wong Building and these stresses were taken into account in the settlement analysis.

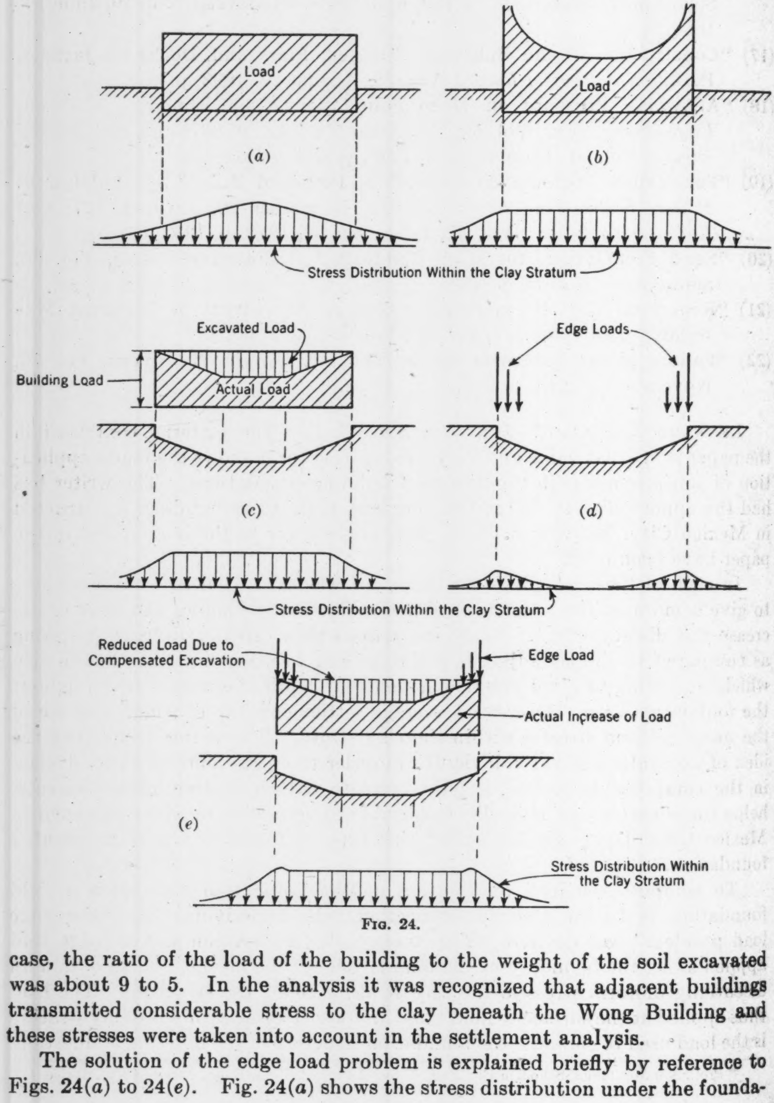

Load
(a)
Load
(b)
Stress Distribution Within the Clay Stratum
Excavated Load
Building Load
Actual Load
(c)
Edge Loads
(d)
Stress Distribution Within the Clay Stratum
Reduced Load Due to Compensated Excavation
Edge Load
Actual Increase of Load
(e)
Stress Distribution Within the Clay Stratum

FIG. 24.

The solution of the edge load problem is explained briefly by reference to Figs. 24(*a*) to 24(*e*). Fig. 24(*a*) shows the stress distribution under the foundation and strata due to a flexible foundation on clay, and Fig. 24(*b*) shows the same for a rigid foundation. Therefore, in a foundation that may be considered rigid, the stress distribution in the clay stratum and the load distribution on the surface must be approached as demonstrated in Fig. 24(*b*). By "compensating" the excavation, the first step to this approach is obtained. Fig. 24(*c*) shows the improved distribution of stress in the clay strata. However, there will be a drop in the stress distribution within the clay strata (Fig. 24(*c*)) at the ends corresponding to the boundary of the excavation. Therefore, unless the foundation of the building is sufficiently rigid, dishing the foundation structure is possible. By applying edge loads as shown in Fig. 24(*d*), a fairly uniform stress to the very edge of the foundation and, at the same time, a load distribution on the surface that approaches the distribution for a rigid foundation (Fig. 24(*e*)) may be obtained. This load can be supplied by designing the foundation sufficiently rigid.

This building indicated an average settlement of 1.2 in. in nine months after the excavation load was reached, and, three months later, a total settlement of 1.6 in. after application of practically all load. The differential settlements were of the order of 0.08 in. (maximum) at points of about 50 ft apart. The settlement analysis indicated 2 in. at the end of the construction period and about 16 in. per decade after application of the load. Early in the construction period the settlements were 0.8 in. greater at one end, because this end during the construction period was more heavily loaded than the remainder of the building, but the tilting of the foundation did not exceed 0.8 in. at the completion of the loading period. Differential settlements remained under 0.08 in., and no dishing effect has been noticed. The excavation depths were not appreciably greater than those commonly used in Mexico City. A regular concrete foundation with beams and 10-in. thick slabs was used, and no cracks have developed. This is of particular significance as, in Mexico City, most buildings with fewer stories have shown visible cracks after the construction period. The writer has used his theory of foundation computations in two other buildings with similar results.

The Boussinesq formula derived from the theory of elasticity was used for stress estimates. Predicted settlements have been of the order of 30% greater than observed settlements. The void ratio of clays encountered at these sites is of the order of 9, and compressibility coefficients at the loads usually encountered are of the order of 1 cm^2 per kg. Samples were taken from the top, bottom, and middle of the clay stratum and tested in the writer's private laboratory. The coefficients used were the average weighted values of the data obtained from the three depths.

Concerning the heaving of the foundation, the writer[21] has found that when the excavation extended through the stiff fill underlying the clay, heaving became very noticeable but ceased after a short period. Therefore, the opinion of the writer is that when excavations occur in very soft clay the heaving is mainly in the nature of lateral plastic flow into the excavation rather than an expansion of the underlying clay.

---

[21] "Aplicación de la Mecánica de los Suelos al Estudio de las Excavaciones en Arcillas Saturadas," by L. Zeevaert, *Ingeniería y Arquitectura*, Agosto, 1942.

Gregory P. Tschebotarioff,[22] M. Am. Soc. C. E.—Two successful foundation designs, based on the principles of soil mechanics, are brought to the attention of the engineering profession by the authors. The time-settlement records of the two buildings, designed by the authors, as compared with the records of three other older buildings founded on similar soil strata at Boston, clearly indicate the advantages of the "floating" type of foundation. Both the maximum and the differential settlements of these two new buildings are much smaller than the ones for buildings with foundations designed in the conventional manner. For a long time Professor Casagrande has been an ardent advocate of the "floating" type of foundations on clays. The behavior of these two buildings, therefore, should give him justifiable cause for satisfaction.

The design of the latter of the two appeals more to the writer. The use of basement walls as stiffening foundation girders provides for a much greater rigidity of the entire structure as compared with the cantilever system used in the first building. This should be of considerable importance in other localities where new designs might have to be made for soil deposits about which fewer data are available than in Boston. The greater general rigidity and the equalizing action of the deep-wall girders should prove very helpful in all cases where the compressibility of a deposit is not entirely uniform. In both buildings, an attempt is made to further equalize the "dishing" effect of deep-seated settlements by increasing the loading at the periphery, and especially at the corners. In the second building, this is done in a very simple manner by decreasing the width of the footings at the periphery and at the corners. The resulting likely variation in the shallow-seated settlements should help in compensating the usual tendency for a greater settlement of the center of a foundation supported by a deep clay stratum. This arrangement, in combination with the use of basement girder walls for load distribution purposes, appears to the writer as being much more economical and effective than the cantilever loading system adopted in the first building with a similar aim. This second arrangement also should be more helpful in counteracting possible harmful effects of any deeper future excavations (for instance, subways) in the vicinity of structures of this type.

The writer heartily agrees with the general Conclusions (1), (2), (3), (5), and (6). However, insufficient evidence is yet available to justify Conclusion (4) that "The settlements due to deformation and recompression following swelling take place essentially during the construction period. * * *" This statement appears to be true under the loading and soil conditions described by the authors, but it does not follow necessarily that it will hold under different conditions. An examination of Fig. 22 shows that the heaving of the bottom of the excavation pit continued without interruption, and even at an increased rate, after the excavation was completed. Only the installation of the foundation slab stopped and decreased the heaving. In the data presented, there is no evidence to show that the heaving would not have continued for an appreciable time had the excavation pit been left open for a longer period. It is reasonable to assume that the heaving of the bottom of the excavation pit is due to two causes: (1) A deformation involving the squeezing of soil into the pit without change of density; and (2) a swelling of the clay as a result of the removal of the overburden. The first cause would predominate in very soft clays, and the greater part of the action should be of relatively short duration. The second cause would be of greater importance in stiffer materials and might last longer. It would appear that, in the case of the Boston buildings, no appreciable swelling could have occurred because of the relatively short period of excavation.

---

[22] Asst. Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

The time element in this kind of work is likely to prove of considerable importance. Gradual absorption of water causes the clay to swell. If this were permitted to occur through a stratum of considerable depth, it is only natural to assume that the settlements following recompression might last for a correspondingly long period and would be much larger. It is not customary to leave excavation pits open for a long time, but it has happened. The writer believes, therefore, that the likely importance of the time element should be emphasized.

Practically no information is contained in the paper concerning the physical characteristics of the clay underlying the two buildings. It is to be hoped that, in their closing discussion, the authors will give this supplementary information. Such descriptions as "hard" or "soft" clay generally are recognized as insufficient to describe a soil properly and in a manner permitting other engineers to visualize its nature.[23] In the paper, the only data concerning the type of clay supporting the buildings described, is contained in Fig. 15, with the clay defined only as "hard" or "stiff" and "soft." No water contents, either in the form of average or limit values, nor any other numerical values of properties important for the visualization and classification of the soil are given. The stratified nature of the clay was mentioned. Therefore, both average values and values related to the coarser and finer thin strata would be of interest. Without such data, the paper will be incomplete, as it will not permit engineers in other localities to relate the observations on their soils to those reported by the authors. It is to be hoped, therefore, that these data will be given in the closing discussion.

The general discussion of the basic soil mechanics principles underlying the design of foundations is presented in a clear and concise manner. There is one point of secondary importance, however, with which the writer does not agree entirely, and which should be stressed since considerable misconceptions have existed on the matter in the past. The authors (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils (*g*)") state that the "concentration factor" is no improvement over the theory of elasticity. The reason offered is that when an integration over a finite area is made (using the concentration factor) then essentially the same type of stress distribution results as that shown in Fig. 4.

The writer fully agrees with the conclusion that the concentration factor, as used so far, does not present any actual improvement. However, the very attempt to integrate the concentration factor over a finite area indicates a misconception concerning the physical conditions responsible for the variation of the soil reactions under foundations underlain by sand, as compared with those underlain by clay. In this connection, the writer wishes to point out that the deflection curve of a large foundation resting upon sand, as shown in Fig. 5, does not appear to be correct. Much more likely is the form of the deflection curve shown in Fig. 25(*b*), the idea suggested by the authors being as shown in Fig. 25(*a*).

---

[23] "Exploration of Soil Conditions and Sampling Operations," by H. A. Mohr, *Soil Mechanics Series No. 9*, Harvard Univ., February, 1940, 2d Ed., p. 5.

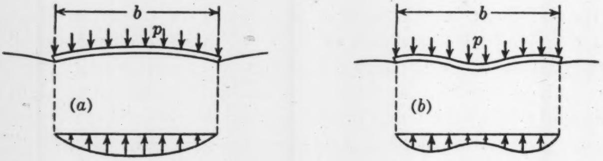


Fig. 25.—Nature of Deflection (*a*) of a Large Uniformly Loaded Flexible Foundation Mat Resting on Sand, and (*b*) the Distribution of Soil Reactions

A zero value of the soil reaction at the edge of a footing underlain by sand is largely due to the fact that the sand can have a shearing resistance only in the presence of a normal stress mobilizing the friction between grains. Such a stress is present under the central part of a footing, but is nonexistent at the edge. In other words, this decrease in the value of the soil reaction is essentially a boundary condition. An attempt to express this boundary condition by the concentration factor can lead to results in agreement with known facts only if the concentration factor is assumed as varying with the distance from the edge of a footing. An integration with a constant value of a concentration factor over the entire area naturally leads to unreasonable results, since the basic assumption underlying such action has no foundation in physical facts.

A large area is likely to deflect in the manner shown by Fig. 25(*b*), since at a certain depth the pressures on a horizontal layer of any soil will be greater beneath the center of a foundation than at its edges. In other words, the effect of the boundary surface condition will decrease with depth, and, for a large loaded area, will be noticeable only at the edges; whereas, beneath the central part of the foundation, the stress distribution in sand should not vary greatly from that of other materials. The point is only of theoretical interest since settlements on sand of normal density are very small in any case.

In conclusion, the writer wishes to emphasize that his criticisms are only of secondary importance, and that the authors are to be complimented on their successful designs.

Guthlac Wilson,[24] M. Am. Soc. C. E.—All engineers concerned with the design of building foundations will find much valuable material in this paper. The writer is particularly interested in the excellent descriptions of two floating or ship-like foundations, a type of foundation that was first brought to his attention by S. E. Faber, M. Am. Soc. C. E., in the course of discussions in Shanghai, China, in the 1930's. It is most interesting to note that this type of foundation is economically superior to the piled foundation even where, as in Boston, a stratum of hardpan is within easy reach of piles.

---

[24] Director, Constructional Design, Ministry of Works, London, England.

However, although in a strict sense the authors are correct, they seem to have been somewhat too sweeping in their statement (see heading, "Examples of the Application of Soil Mechanics to the Design of Building Foundations: The New England Mutual Life Insurance Company Building—Observation of Movements"):

> "* * * Were it possible to distribute the building load uniformly over the building area, and were it possible to excavate and apply the building load simultaneously, no settlement would result. However, it is impracticable to design and construct a building in such a manner that these requirements are fulfilled, * * *."

It appears to the writer that these requirements could be more nearly satisfied, and would be, were it justifiable economically. Assume that a building similar to those described were to be built in another location and that there were no layer of hard or stiff clay overlying the soft clay, as in Boston. The engineer's object would then be to change an overburden of 2 tons per sq ft of organic silt clay and fill at the foundation depth of 40 ft into a uniform foundation pressure of 2 tons per sq ft with the minimum of disturbance. This change could be best accomplished by an adaptation of "The Chicago Method." The outer basement walls and the interior main walls, each with a sufficient area of footing, could be constructed in trenches; the ground floor and a suitable amount of the superstructure could then be built upon these; the basement volume would later be excavated by sections between interior main walls —perhaps each section would be excavated in a series of parallel slices; and the

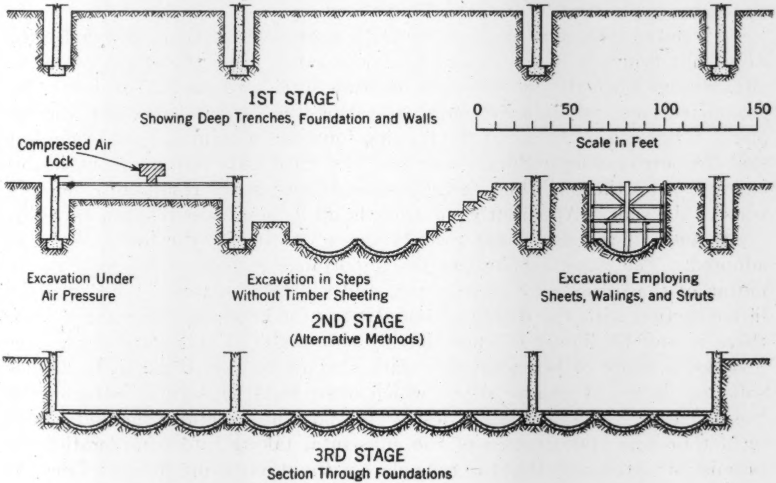

1ST STAGE
Showing Deep Trenches, Foundation and Walls
0
50
100
150
Scale in Feet
Compressed Air Lock
Excavation Under Air Pressure
Excavation in Steps Without Timber Sheeting
Excavation Employing Sheets, Walings, and Struts
2ND STAGE
(Alternative Methods)
3RD STAGE
Section Through Foundations

FIG. 26.—SUGGESTED METHOD OF CONSTRUCTING FLOATING FOUNDATION

first basement floor and the main foundation floor could be constructed. As only a small area of the foundation would be opened up at one time, the vertical swelling, or "negative settlement" would be minimized. In extreme cases, it might be worth while to resort to compressed air. The main basement floor could probably be built economically as a series of parallel and continuous barrel-vault arches, designed on the lines of the well-known Zeiss-Dewidag roof. Such an arch of 60-ft span by 30-ft width would bear approximately 2 tons per sq ft if it were 1.5 ft thick. A suggested foundation along these lines is illustrated by Fig. 26.

There is so much information packed into the paper that the writer rather hesitates to ask for more. However, he hopes that the authors will enrich their paper by the addition of information stating what live loads were considered in the foundation computations, what reduction factors, if any, were used, and whether these reduction factors were the same as those used in the design of the columns. Also, they should add diagrams showing the story heights of various parts of the New England Mutual and Liberty Mutual buildings.

W. H. McAlpine,[25] M. Am. Soc. C. E.—The analysis of the design of foundation conditions for the New England Mutual Life Insurance Company Building and the results obtained in the completed structure are of special interest to the writer, perhaps in part because the new structure replaces old structures reminiscent of his school days long ago.

It is noted that, as a result of the settlement investigations, it was decided to use the principle of a floating foundation rather than to carry the load on piles driven through the soft clay stratum into hardpan 125 ft below the ground surface, preliminary plans indicating the former to be the cheaper plan. The decision to adopt the floating foundation plan, in spite of the fact that the surrounding buildings were on piles, must have required considerable courage and a thorough knowledge and confidence in the results of laboratory tests of the soils. Apparently the work in all its steps was planned carefully.

A study of the settlement records appears to justify the foundation plan adopted. The paper emphasizes the importance of both a thorough examination of the subsoils by deep borings and soil tests of the sampled material in connection with the design of foundations for important structures where there is any likelihood of encountering soft material. Too frequently, the mistake is made of being satisfied with shallow borings that fail to disclose soft clay layers at greater depth which often result in serious settlement of the structure. The foundation design of important structures should be based on a thorough investigation of the substrata, taking into consideration the permissible settlement that can take place without impairing the structure. In some cases, considerable inequalities in settlement can be tolerated, whereas, in a gun block for large coastal batteries, any inequality in settlement would be serious. In some instances, taking relative economy into consideration, it may be a close decision as to whether to use piles to carry the load to firm material or to adopt the floating type of foundation.

---

[25] Chf. Engr., Office, Chf. of Engrs., U. S. Army, Washington, D. C.

Referring to Fig. 23, an explanation as to the reason for the greater settlement on the Newbury Street side than on the Boylston Street side would be desirable. Also, a statement as to whether any of the adjacent streets or buildings were adversely affected during or after the construction period would be of interest.

The "Conclusions" of the authors appear sound on the basis of the facts presented.

Karl Terzaghi,[26] **M. Am. Soc. C. E.**—A description of the foundations of two large buildings in Boston, Mass., forms the basis of this stimulating paper. The success of these foundations is due primarily to the fact that the weight of the structures is roughly equal to the weight of the soil removed before the application of the weight of the buildings.

The principle of this simple method for avoiding important settlement of buildings on soft ground has been known for more than a century. It is described in a classical handbook by G. Hagen whose third edition appeared in 1870.[27] After outlining the general procedure, Hagen states that the total weight of the structure should not be greater than the weight of the mass of mud which previously occupied the site of the building. As an example of the successful application of the method he mentions the foundation of the Albion mills in London, England, by Rennie (1761–1821).

In the design of the foundations described in the paper advantage has been taken of the stiffness of the top crust of the Boston blue clay. This procedure was also recommended by Hagen, who quotes as an example the foundation of a ropery in Rochefort in France, designed by Blondel, early in the nineteenth century. The structure rests on a 12-ft layer of stiff mud which constitutes the top layer of a very deep stratum of soft mud.

These examples demonstrate that the most important method at the disposal of present-day engineers, for reducing or eliminating the settlement of structures, is very old. Therefore, many readers of the paper may feel tempted to inquire exactly what contribution to soil mechanics is included in it. To find an answer to this question one may turn to the more recent handbooks—for instance, the German handbook on foundations,[28] published in 1906, which constitutes the successor to the older writings. This more recent and more elaborate treatise does not even mention the method for eliminating the settlement which was well known to the older generation of engineers. The reason is quite obvious: The designs of Rennie and of Blondel originated in flashes of engineering intuition, and the capacity for intuition is not hereditary. Methods of design based on intuition cannot be, and are not, practiced by the average engineer until they become an integral part of a logical system which demonstrates their usefulness on the basis of theoretical principles. Soil mechanics has established such a system.

Still more important is the fact that soil mechanics succeeded in establishing methods for estimating the settlement of structures in advance of construction. Some of the theories on which the computations are based originated late in the nineteenth century. Nevertheless in none of the handbooks on foundations published before 1920 did settlement receive more than casual attention. The consequence of this prevalent ignorance concerning settlement is stored away in the records of countless lawsuits, and they are demonstrated by the curves III to V in Fig. 15. The method for preventing important differential settlements, such as those shown by the curves III to V, was already known prior to 1870, but the designers had no inducement to use it, because they were unable to estimate in advance the settlement of the foundation which they designed. None of the important and spectacular settlements that have come to the writer's attention have been anticipated. They came as "Acts of God." This fact makes the issue clear enough. Soil mechanics did not grow out of the need for novel methods of foundations. It grew out of the urgent need for reliable rules for using the existing ones with less risk. Once the knowledge of the factors that determine the settlement of foundations becomes commonplace, the frequency of inadequate foundations and of surprise settlements will diminish automatically. In other words, in the future, the average engineer will be capable of accomplishing that which, in the past, has been the prerogative of those very few who are gifted with engineering intuition. The average person does not see except that which he has been taught to look for and soil mechanics serves the function of a mentor.

---

[26] Winchester, Mass.

[27] "Handbuch der Wasserbaukunst," by G. Hagen, Berlin, Ernst & Korn, 1st Pt., Vol. 2, 3d Ed., 1870.

[28] "Handbuch der Ingenieurwissenschaften," III. Band, Der Grundbau, 4th Ed., 1906, W. Engelmann, Leipzig.

In the paper the description of the two Boston foundations is preceded by a set of general statements concerning the factors that determine the settlement of structures. The writer believes that many of these statements require some qualification.

Statements (*a*) to (*m*), under the heading "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils," deal with cohesionless soils. According to the first sentence of statement (*a*), the contact pressure on a rigid area resting on the surface of a cohesionless soil increases from zero at the edge to a maximum at the center. This statement is confirmed by both theory and experiment. However, in practice, the base of every footing is at a certain depth h below the surface. If γ is the unit weight of the sand and ϕ the angle of internal friction, the unit pressure at the edge of the footing can be as great as

$$q_o = \gamma\, h \tan^4 \left(45° + \frac{\phi}{2}\right) \ldots . (2)$$

Therefore the pressure at the edge of a footing is by no means necessarily equal to zero. As a matter of fact, the writer has never seen a record obtained from pressure-cell observations on actual footings which would justify the assumption that the contact pressure along the edge of a footing is equal to zero.

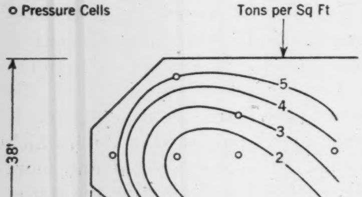


FIG. 27.—DISTRIBUTION OF CONTACT PRESSURE ON THE BASE OF A CONCRETE BRIDGE PIER ON SAND, AT A DEPTH OF 34 FT BELOW THE RIVER BOTTOM

Statement (*b*), illustrated by Fig. 3(*c*), is at variance with the only one set of observations concerning the distribution of the contact pressure on the base of a pier that has come to the writer's attention. Fig. 27 shows the base of the pier, at a depth of about 34 ft below the bottom of the Rhine River. It rests on a stratum of dense, coarse sand with fine gravel. The pier was constructed by the compressed air method. Using the notation indicated in Fig. 3, the value $p = \frac{P}{A}$ was about 4 tons per sq ft and the value $(b + h)\,\gamma$ about 1.8 tons per sq ft. The contact pressure was measured by means of eight pressure cells of the modified Goldbeck type.[29] The lines of equal pressure indicate that the contact pressure increased very considerably from the central part of the base toward the edges. If Fig. 3(*c*) is based on sets of field measurements unknown to the writer, a presentation of the factual data would help in clarifying the issue.

According to statement (*c*) the ultimate bearing capacity of a cohesionless soil increases approximately in direct proportion to the width of the loaded area. This statement is valid only if the coefficient of internal friction (ϕ) of the sand is almost independent of the pressure. Since the ultimate bearing capacity increases approximately in direct proportion to the fourth power of $\tan\left(45° + \frac{\phi}{2}\right)$, a small decrease of ϕ causes an important deviation from the rule which is based on the assumption that ϕ is constant. Therefore, the rule does not apply to sands whose angle of internal friction decreases appreciably with increasing pressure.

The validity of the rule expressed by statement (*c*) has been contested repeatedly, the last time by B. K. Hough, Jr.,[30] Assoc. M. Am. Soc. C. E. According to Mr. Hough, the rule is not in accordance with the results of large-scale field tests. The preceding paragraph leaves no doubt that, in many instances, properly conducted large-scale field tests would disclose important departures from the rule expressed by statement (*c*); yet the writer has not yet seen any reliable record that would either confirm or contradict the statement. The reason is consistently the same: The small footings are loaded to failure whereas the test on the larger ones is discontinued at an earlier stage. The elastic properties of sand are such that the soil support of a footing on sand does not really fail until the settlement exceeds a certain percentage, $n\%$ of the width of the loaded area, regardless of what this width may be. For dense sand the value $n\%$ is about five and for loose sand ten or fifteen. Extrapolation from the load corresponding to a smaller settlement to the failure load cannot be relied upon. In this connection it should be noted that the unit load required to produce a given settlement or a conspicuous increase of the slope of the load-settlement curve has nothing in common with the ultimate bearing capacity referred to in statement (*c*).

Statement (*d*) implies that the ultimate bearing capacity of a loaded area increases with the square of the depth. This statement seems to be open to objections. In the 1920's the writer developed an approximate theory for computing the ultimate bearing capacity of shallow foundations such as circular footings.[4] According to this theory the ultimate bearing capacity q_h of a circular footing with a radius R which rests at a depth h below the surface on a mass of dry sand with an angle of internal friction 36° is roughly equal to

---

[29] "Der Bau der neuen Rheinbrücke bei Ludwigshafen-Mannheim," by R. Burger, *Die Bautechnik*, Vol. 10, 1932.

[30] Discussion by B. K. Hough, Jr., of the paper, "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), pp. 1184–1186.

$$q_h = q_o \left[1 + \frac{h}{R} + 0.146 \left(\frac{h}{R}\right)^2\right] \qquad (3)$$

in which q_o is the ultimate bearing capacity of the same footing for $h = 0$. Fig. 28(a) is a vertical section through the footing. The vertical pressure that acts on the sand within zone I causes the sand to spread in radial, horizontal

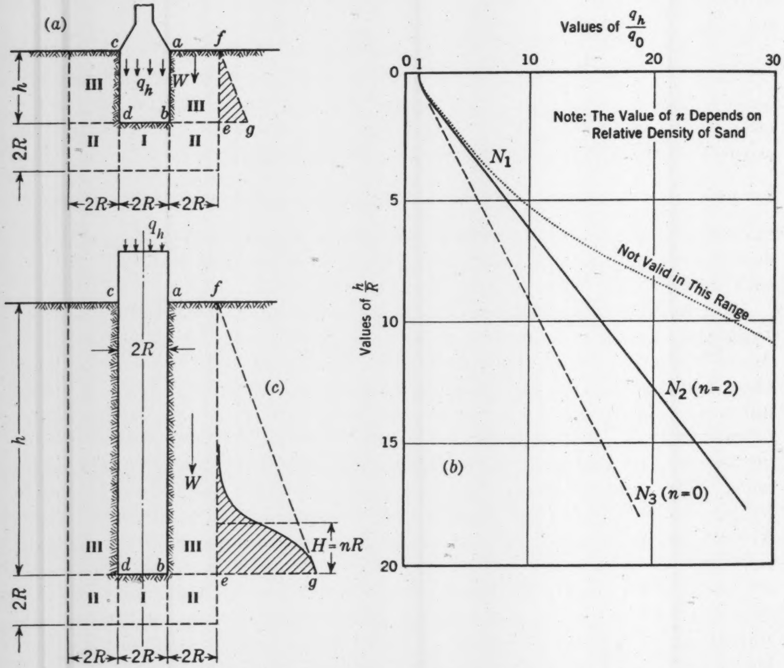

(a)
c
a
f
h
2R
III
q_h
W
III
d
b
e
g
II
I
II
2R
2R
2R
q_h
c
a
f
2R
(c)
h
W
III
III
$H = nR$
d
b
e
g
2R
II
I
II
2R
2R
2R
Values of $\frac{q_h}{q_0}$
0
1
10
20
30
0
5
10
15
20
Values of $\frac{h}{R}$
Note: The Value of n Depends on Relative Density of Sand
N_1
Not Valid in This Range
N_2 (n=2)
N_3 (n=0)
(b)

FIG. 28.—RELATION BETWEEN BEARING CAPACITY AND DEPTH OF FOUNDATION FOR CIRCULAR FOOTINGS (a) AND CYLINDRICAL PIERS (c).

directions. The sand within zone II tends to rise, because it is laterally confined. The rise is resisted by the weight W of the sand within zone III, by the skin friction along the vertical surface of the footing and by the full shearing resistance of the sand along the outer, cylindrical boundary ef of zone III. This shearing resistance is represented by the shaded area efg in Fig. 28(a).

In Fig. 28(*b*), Eq. 3 is represented by the curve N_1. For values of $\frac{h}{R}$ between 1 and 4 the curve is almost straight and it can be represented almost exactly by the simple equation

$$q_h = 1.8 \left(\frac{h}{R}\right) q_o \qquad \left(4 > \frac{h}{R} > 1\right) \ldots\ldots\ldots\ldots\ldots (4)$$

Eq. 4 covers the range of $\frac{h}{R}$ for circular footings. On the other hand, if $\frac{h}{R}$ is greater than 4, Eq. 3 is not even approximately valid. This statement is illustrated by Fig. 28(*c*) which represents a vertical section through a cylindrical pier. Since the height of zone III is great compared to its width, the shearing stresses along ef must necessarily become equal to zero at a considerable depth below the surface, as indicated by the shaded area in the diagram. The average height, $H = n\,R$, of the shaded area depends on the relative density of the sand. In the writer's judgment it may range between zero for loose sand and about $2\,R$ for dense sand. On this assumption the writer obtained the lines N_2 and N_3 for the relation between the depth and the ultimate bearing capacity of the pier. This result suggests that the ultimate bearing capacity increases approximately with depth and not with the square of the depth. As a matter of fact, the writer is unaware of any acceptable theory, or of any record of field observations that would sustain statement (*d*).

According to statement (*j*) the settlement due to a given load decreases rapidly as the depth of the loaded area beneath the adjacent ground surface increases. In order to apply a load on an area at a considerable depth below the surface it is necessary to dig a shaft. While the shaft is being excavated a stress relaxation develops within a zone whose width is several times the diameter of the shaft.[31] In the vicinity of the bottom of the shaft which later constitutes the loaded area and to a considerable horizontal distance from the bottom, the state of stress in the sand is practically independent of the depth of the shaft, provided the depth of the shaft exceeds several times the diameter of the shaft. The settlement of a loaded area depends to a large extent on the initial state of stress in the sand within the zone of potential plastic equilibrium. Since the state of stress in the sand below the bottom of the shaft is practically independent of depth, one would expect that the settlement of a pier at a given soil pressure at the base of the pier should also be practically independent of depth. This tentative theoretical conclusion is illustrated by Fig. 29. According to this figure the ultimate bearing capacity of the pier increases with depth, but the settlement for a given unit load is practically independent of the depth to the loaded area provided the load does not exceed the customary bearing value for the base of piers. The writer's experience in this matter is limited to a large-scale loading test which he made in 1928 on the bottom of a shaft on Houston Street, in New York, N. Y., at a depth of 55 ft below street surface. The bearing block covered the entire bottom of the shaft, from wall to wall. The results of this, and of all the other tests and observations which he made in the shaft, corroborated the opinion illustrated by Fig. 29. Hence they are incompatible with statement (*j*). However, a single set of observations does not justify a final conclusion. Therefore, the writer would appreciate information concerning the field data on which Professor Casagrande's statement (*j*) is based.

---

[31] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943.

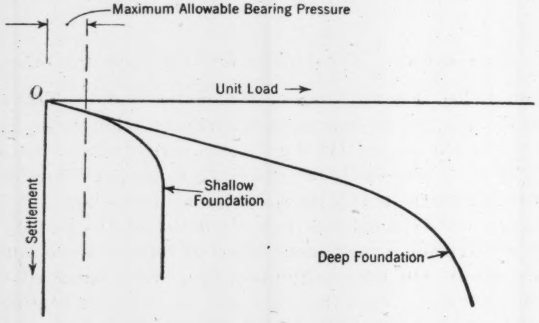


Fig. 29.—Writer's Conception of the Influence of Depth of the Foundation on the Relation Between Unit Load and Settlement of Cylindrical Piers

Under the heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Clays and Other Cohesive Soils," the method for determining the preconsolidation load on clays receives considerable attention. This method was devised by Professor Casagrande in 1936.[32] The text of the original paper conveys the impression that the method is based on the following experimental investigation: Undisturbed samples of different clays have been consolidated in the laboratory under an arbitrary load. Then the load was removed and applied again. The curve which represents the renewed application of a load in the pressure, void-ratio diagram is known as recompression curve. After many recompression curves were obtained a graphical procedure was devised which made it possible to determine the laboratory preconsolidation load, if the corresponding laboratory recompression curve is known. To this point, the procedure was strictly empirical.

The next step consisted in using the procedure without any modification for the determination of a preconsolidation load that was applied by nature at an extremely low rate, thousands of years ago. Although this step constitutes a daring extrapolation, its implications have nowhere been discussed. In order to decide whether it is justified, it is necessary to compare the real value of the preconsolidation pressure produced by nature with those determined by means of the graphical method.

The real value of the preconsolidation load for overconsolidated clays can only be evaluated on the basis of geological evidence but, as a rule, geological evidence regarding the thickness of vanished strata leaves a wide margin for interpretation.

---

[32] "The Determination of the Pre-Consolidation Load and Its Practical Significance," by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics, Vol. III, 1936, pp. 60–64.

On account of the uncertainty of geological estimates the degree of accuracy of the graphical method can be evaluated only on the basis of records pertaining to clays for which the preconsolidation load is identical with the present overburden pressure. This condition is satisfied only for normally consolidated clays. Since these clays have never carried any load other than their own weight, the preconsolidation pressure increases approximately in simple proportion to depth.

In 1936 Professor Casagrande investigated a normally consolidated deposit of glacial clay about 100 ft thick and found that the graphically determined preconsolidation load below a depth of 50 ft is practically independent of the depth, whereas the true load increased in simple proportion to depth from about 1½ tons per sq ft to about 3 tons per sq ft.[32]

Since he assumed that his graphical method is fairly reliable he suggested that the fissured rock beneath the clay may contain artesian water under an exceedingly high hydrostatic pressure. In 1941, D. M. Burmister, Assoc. M. Am. Soc. C. E., published the results of soil investigations of a soft clay deposit at the site of Flushing Meadow Park.[33] Between the depths of about 20 ft and 65 ft below the surface the actual overburden pressure on the clay increased from about 0.6 to 1.4 tons per sq ft. Within the same range of depth the overburden pressure determined by means of the graphical method decreased from 0.6 ton per sq ft at a depth of 20 ft to about 0.26 ton per sq ft at a depth of 65 ft; yet there was no evidence of the presence of artesian water in the sand beneath the clay.

To the writer's knowledge, no other set of data pertaining to drill-hole samples from normally consolidated clays has ever been published. Since the values obtained by means of the graphical method failed to agree with the known values of the preconsolidation pressure for the clays covered by the two published records, it seems unlikely that the agreement should be better in those instances in which the actual preconsolidation pressure is unknown.

On various occasions it has been stated that the graphical method has been applied successfully to the interpretation of the results obtained by tests on shaft and tunnel samples of normally consolidated clays. To eliminate the objection that the favorable results obtained on individual samples may be accidental, it would be necessary to make such tests on a set of samples which have been taken within the same stratum at very different depths; yet, thus far, no data of this type have been published.

The practical implications of the uncertainties concerning the evaluation of the preconsolidation load have been discussed elsewhere by the writer.[34] A systematic analysis of the striking discrepancies between reality and test results may increase, materially, the knowledge of physical properties of natural clays, but this promising source of information has not yet been tapped.

In discussing Fig. 10 the authors state:

> "From a large number of tests on different types of soils, it has been found that the preconsolidation pressure for most clays can be determined with

---

[33] "Laboratory Investigations of Soils at Flushing Meadow Park," by Donald M. Burmister, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 187.

[34] Discussion by Karl Terzaghi of the paper, "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *ibid.*, Vol. 109 (1944), p. 1196.

> a satisfactory degree of accuracy by means of the empirical method shown in Fig. 10 * * *."

Considering the records quoted, the term "satisfactory degree of accuracy" requires qualification. The writer has used the graphical method repeatedly to determine whether or not a clay has been heavily overconsolidated; but he has ceased to give any weight to the numerical results because both the published records and his own experience indicate that the difference between the computed and the real value for samples from the same drill hole may range between zero and several hundred per cent of the computed value. If the maximum preconsolidation load p_o exceeded the present overburden pressure p_1 by less than a few tons per square foot, the graphical method does not indicate whether p_o exceeded p_1 at all. In such instances, the writer relies entirely on geological evidence. During the seven years since the graphical method was devised, a wealth of data has been accumulated. Therefore the time seems to be ripe for a revision of the method on a broader empirical basis.

Whatever the merits of the present graphical method may be, the research which centered about the preconsolidation load served a useful purpose. It gave an additional impetus to the development of the present methods of sampling and it broadened present knowledge of the physical properties of clays. Many of the refined procedures which grew out of these efforts will become obsolete because it will be found that the required expenditure of time and money is out of proportion to the practical value of what can be gained. Nevertheless the efforts were not wasted because they constitute part of the general exploration of the field and each one of them constitutes one more step toward the ultimate goal. The goal is the development of simple rules and practicable procedures which will reduce the hazards associated with the purely empirical methods of dealing with earthwork and foundation problems.

Under the heading, "Examples of the Application of Soil Mechanics to the Design of Building Foundations," the foundations of two new buildings in Boston are described. In the following discussion the new buildings will be briefly referred to as I (Liberty Mutual Insurance Company Building) and II (New England Mutual Life Insurance Company Building). Building I is supported by cylindrical piers whose enlarged base rests on a stiff clay crust with an average thickness of 10 ft. Fig. 30 shows a vertical section through the base of four piers. The actual thickness of stiff clay crusts is likely to change from place to place and the customary test borings are not numerous enough

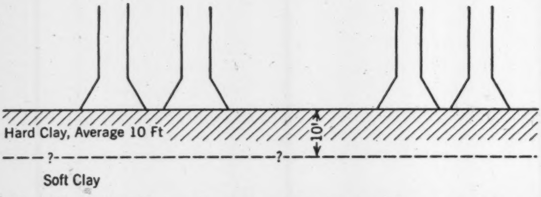


Fig. 30.—Hard Clay Crust Supporting Foundation Piers for Building

for detecting weak spots. The writer would appreciate information concerning the method of surveying the stiff crust or for bridging weak spots.

The foundation for building II was excavated to an average depth of 35 ft. Prior to excavating, eight underground reference points were established in the stiff crust. During the process of excavation the points rose by distances up to 2.9 in. The observed settlement, to date, is roughly equal to the preceding rise of the stiff crust. In 1927 the writer designed a similar foundation. The weight of the building was roughly equal to the weight of the soil and water removed. The excavation penetrated through fine sand to a depth of 30 ft below the surface.[35] At a depth of about 14 ft below the bottom of the excavation the sand rested on the surface of a very thick stratum of soft glacial clay whose water content was close to the liquid limit. Before excavation was begun an underground reference point was established in the sand, a short distance above the soft clay. To the surprise of every one, the rise of the point was too small to be measured with an engineer's level. As a consequence, the settlement of the finished structure was also too small to be measured. However, the writer still believes that the heave of the bottom of the pit was only delayed. If the pit had been open for a couple of months, the heave might have started. Such a possibility is suggested by the well-known delay in the beginning of the swelling of homogeneous clays in tunnels and by the delay of the beginning of the settlement due to the consolidation of clay strata which has occasionally been observed.[36] There are no reliable means for predicting the amount of heave on the basis of test results. If no observations are made, even an important heave may pass unnoticed until it is too late to modify the design or the construction procedure in accordance with the reaction of the subsoil on the change of loading produced by excavation. Therefore the excavation for a deep, floating foundation should not be started without previously establishing a set of underground reference points.

Fig. 18 shows several vertical sections through the basements in building II. It would clarify the concept of the unsymmetrical continuous footings (Figs. 18 and 19(*b*)) if the authors would furnish some data concerning the assumptions on which the computation of the earth pressure on the outside of the basement walls was based.

The description of the two foundations constitutes a valuable contribution to current knowledge of the behavior of foundations on soft soil and it demonstrates again the intrinsic merits of the method of floating foundations.

D. P. Krynine,[37] M. Am. Soc. C. E.—Soil mechanics data concerning sands and clays are summarized in the first part of this interesting paper. These data refer to pressures and settlements and also to the methods of testing. It is regrettable that, in the second part of the paper, which deals exclusively with foundations on clay, this valuable information on sands did not find any application. The discussion hereafter follows the order of presentation of the material in the paper.

---

[35] "Wellpoint Method for Handling Excavation of Foundation Pit at New Sewage Pumping Station, Lynn, Mass.," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., Vol. 14, 1927, pp. 389-397.

[36] "The Actual Factor of Safety in Foundations," by Karl Terzaghi, *Structural Engineer*, Vol. 13, 1935.

[37] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

Concentration Factor.—The authors are to be commended for stating very clearly that the principle of superposition is valid only if Hooke's law holds. The so-called "concentration factor" (or "concentration index"), as expressed in Eq. 1*b*, may be applied if the loaded area is very small, as in the highway subgrade. The writer admits that the integration of Eq. 1*b* over large finite areas is decidedly erroneous. A good substitute for the concentration factor is the rule proposed by A. E. Cummings,[38] M. Am. Soc. C. E. If there is a rigid underlying boundary at a depth, z, from the earth surface, the vertical pressure on that boundary may be computed according to the Boussinesq formula using $0.75\ z$ instead of z.

Varved Clays.—Fig. 6 represents a cross section of typical varved clay. It should be added that there are varieties of varved clays in which not two, but three, different materials are combined to form a clay mass. Geologists have studied the origin of varved clays,[39] particularly in New England. Engineering properties of varved clays in New England have been investigated by the U. S. Army Engineers in the Boston (Mass.) District and by the Connecticut State Highway Department, but nothing has been published to date (1943).

Time-Settlement Diagrams.—It is regrettable that Fig. 9 is not discussed by the authors. To the writer's knowledge, time-settlement diagrams, involving the fitting of the curves according to Professor Casagrande's method, have not been treated sufficiently well in engineering literature, and further clarification of Fig. 9 would be important. On the other hand, Figs. 8, 10, and 11 have been discussed adequately elsewhere.[32]

Example of a Settlement Analysis Based on Simple Soil Tests.—A simple method of determining the expected settlement without performing a consolidation test on a given sample is presented in the paper, providing only that results of tests on other clays are available. It is obvious, however, that, in addition to the tests enumerated in the paper, the specific gravity of grains must be determined or assumed.

Caissons.—To the knowledge of the writer, there has been no case of failure of deep-caisson foundations by shear (except, perhaps, during construction). Friction of a deep caisson against the earth mass considerably decreases the pressure on the base and increases the factor of safety. Uniform distribution of pressure at the base of a caisson as in Fig. 3(*c*) is rather a theoretical probability. In reality, because the chamber is filled with concrete at the end of the work, probably there is a slight overloading at the edges of a caisson. An analogous situation is created when tunneling by the liner-plate method. In this case, there is an overloading under the legs of the frame and relief of pressure under the invert.

Differential Settlement Contours.—A uniformly thick clay layer decreases in thickness in proportion to the vertical pressure at a given point. The vertical pressure in the case of a nonrigid rectangular foundation is at a maximum at the middle of that rectangle. Hence, the maximum settlement is to be expected at the middle of a rectangular nonrigid building, especially if the central part of that building is overloaded as in Figs. 16 and 17. There is no way to avoid the settlement completely; but it is possible to decrease its absolute value by making excavations of variable depths and to shift the excess of vertical pressure from one part of the building to another and thus to decrease the differential settlement. The quotation from Professor Casagrande's previous work (under the heading, "Settlements of Buildings in Boston") will stand repeating. It is very true and should be kept in mind by the designers of foundations on clay deposits. In this connection, it is valuable to recall a statement, credited to Glennon Gilboy, M. Am. Soc. C. E., that driving piles into stiff clay is analogous to driving nails into a mirror to reinforce it.

---

[38] "Foundation Stresses in an Elastic Solid with a Rigid Underlying Boundary," by A. E. Cummings, *Civil Engineering*, November, 1941, p. 665.

[39] "Conditions of Formation of the Varved Glacial Clay," by E. Antevs, *Bulletin*, Geological Soc. of America, Vol. 36, 1925, pp. 171–172 (abstract); and other works of the same author and other geologists.

The central contour lines of the differential settlement in Figs. 16 and 17 are elliptical; and the further a contour line is from the center of disturbance (overloaded central part), the less it is elongated and the more it approaches the shape of a circle. This is in accordance with Saint Venant's principle as applied in a horizontal plane.

The Liberty Mutual Insurance Company Building in Boston.—This building has a nonrigid foundation since the load is applied to the clay stratum by numerous caisson bells supporting piers, mats, and columns. Each bell is practically an independent load. For some reason point A, where the maximum settlement has been observed, is located somewhat eccentrically, closer to Stuart Street. This may be due: (*a*) To overloading or (*b*) to a change either in properties or in thickness, or both, of the underlying clay layer. It is very regrettable that in Fig. 17 observed contour lines only are shown, and no information with respect to the forecast settlement is given. This circumstance makes it impossible to judge how close the anticipated values of settlement were to the actual ones and to examine the causes of discrepancy, if any.

The New England Mutual Life Insurance Company Building.—This building differs from the Liberty Mutual Insurance Company in its weight and in the rigidity of the foundation, which, together with the structure itself, represents a rigid box. The building in question furnishes an excellent opportunity to show what a designer and builder of foundations must know about soil mechanics and how this knowledge should be used.

Prior to the final design of the building in question, the following was done:

(*a*) Extensive underground explorations were undertaken and the samples obtained in borings were tested in a laboratory (mostly for consolidation); and

(*b*) On the basis of this information a soil profile was prepared and analyzed. It was found that only two kinds of foundations are to be considered—a "floating" foundation or long piles reaching to hardpan. For economical reasons the former was preferred.

In general, a designer does not need to master the technique of borings and that of laboratory tests; but he must be able: (*a*) To order borings to be made at proper places and to see that they are taken properly; and (*b*) to see that the laboratory makes the proper tests and makes these tests properly. True knowledge of soil mechanics is needed in analyzing the soil profile. However, mere acquaintance with soil mechanics theories, not combined with extensive practice, and especially with what is called "sound judgment," is dead knowledge. In analyzing the soil profile, the designer must be prepared to answer the question: "What might happen to my structure if I build it this way?" Obviously, to answer such a question the Fourier series or the equation of the logarithmic spiral are not helpful. Similar details are required in studying soil mechanics for the first time, but, in applying this study to the design of structures, a thorough understanding of fundamental principles is required; and this can be seen very clearly from the examples of the second part of the paper.

A convincing proof of the necessity of combining theories with considerable practice is the ingenious detail proposed by Mr. Homer, as shown in Fig. 19(*b*). This detail simply could not have been worked out by a theorist without considerable building practice.

If soil mechanics were an exact science (which it is not), no measurement of settlements during and after construction would be necessary. In fact actual settlements would be exactly the same as the settlements exactly computed on the basis of exact soil and load information. In reality, settlements are carefully watched and measured at any project of importance. This is done to check computations made and theories underlying them and also in order to be able to take urgent measures during construction in the case of some abnormality. Particularly the authors are to be commended for the instructive nature of Fig. 22 which, to the knowledge of the writer, is the first in the literature to show both the elastic rebound due to the removal of an old building and the subsequent downward travel of the structure to reach, in the final analysis, the original ground surface.

In conclusion, the writer wishes to congratulate the authors for bringing to the attention of the profession the interesting and important results of their studies on foundations.

O. G. JULIAN,[40] M. AM. SOC. C. E.—A cogent and synoptic thesis on a subject which appears at long last to be emerging into the realm of rationality has been presented by the authors. The differences between soils and what are commonly known as "structural materials" are stated. However, since ultimately all structures are supported on soils (or rock), there appears to be no good reason for not considering the latter as structural materials. Also, it would appear evident that architects and structural engineers who are responsible for the safe and efficient design of buildings should be as well informed regarding the properties of soils as they are regarding other materials such as steel, concrete, brick, wood, etc., on which the stability of their structures depends.

As emphasized by the authors (see heading, "Introduction: Differences Between Soils and Structural Materials"), the statement that a given soil is "safe for" a certain unit load means very little. In addition to the maximum principal stresses, at least the orthogonal stresses and the relationship between stress, strain, moisture content, and time must be considered intelligently; otherwise, the structure may be subject not only to excessive settlement as a whole but to differential settlements which may possibly, in the course of time, wreck the superstructure. The authors correctly emphasize the point that the supporting soil, foundation, and superstructure should be designed as an integral whole. In fact, these components ordinarily form a continuous system which is statically indeterminate to a high degree—the intelligent design of such a system requires knowledge of the properties of the soil and other materials, judgment, and experience to a greater degree than the ability to carry out precise mathematical analyses.

---

[40] Chf. Structural Engr., Jackson & Moreland, Boston, Mass.

It is interesting to note from Fig. 2 the reversal from positive to negative of the change of slope of the strain-stress curve for sand at the point where the major principal stress exceeds the minor principal stress, that beyond this point the slope of the curve for "dense sand" is nearly constant up to the point where the major principal stress is twice the minor, and that dense sand increases in volume with increase in the major principal stress when the latter is three or more times the minor. This latter effect is due to the fact that in order to deform substantially the grains must slip over each other and in so doing cause expansion. The increase in volume indicates that the point of failure is approaching. As appears natural, the "elastic properties" and "shearing strength" of a given sand mass appear to be dependent almost entirely upon the ratio of the principal stresses—that is, on the degree of confinement. A loose liquid sand mass if confined may make excellent foundation material; however, emphasis should be placed on the importance of confining the entire mass. As stated by the authors (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils," conclusion (*m*)), serious settlements may result from lowering the ground-water table. Quantitative values pertaining to the curves given in Fig. 2 and definitions of "dense" and "loose" would be most instructive and interesting. It is hoped the authors will publish these data in their closing discussion.

With the exceptions of conclusions (*d*), (*g*), and (*i*), the thirteen conclusions pertaining to noncohesive soils (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils") appear to be intuitively evident, at least qualitatively.

Conclusion (*d*) is not evident; it would appear that at least in some cases the ultimate bearing capacity would increase as the first power rather than as the square of the depth. It would be interesting to know whether this conclusion is based on deductive reasoning or was arrived at empirically.

Conclusion (*g*) raises the question as to whether or not the theory of elasticity is valid in any degree when considering the stress or deformation condition of a mass of cohesionless soil. A persual of recent literature on the subject shows considerable divergence of opinion and indicates that the question is at least debatable. Since the theory of elasticity is based on continuity, Hooke's law, and its corollary, the principle of superposition, it would appear a priori that this theory cannot possibly be valid in the case under discussion. On the other hand, it is conceivable that, if subjected to considerable pressure, a confined mass of cohesionless particles may acquire, to some degree, continuity of action and elastic properties so that formulas based on the principle of superposition may be rough approximations and, in the absence of better methods, lead to conclusions that are of considerable value.

According to the theory of elasticity the deformation of an isotropic mass of semi-infinite extent, subjected to a uniformly distributed load over part of the free surface, depends on (1) the shape of the loaded area, (2) the area of the loaded area, (3) the intensity of pressure, (4) the elastic properties of the loaded material, and (5) the rigidity of the structure by means of which the load is applied.

For either a round or a rectangularly loaded area, provided the structure is flexible so that the intensity of loading is uniformly distributed, the average settlement, w_{aver}, may be obtained from Schleicher's formula:[41]

$$w_{\text{aver}} = m \sqrt{A}\, p \frac{1 - \nu^2}{E} \qquad \text{(5)}$$

in which ν = Poisson's ratio; E = Young's modulus; and m = the "shape factor." The latter is a function of the ratio of length, a, to the breadth, b, of the loaded area. The value of m may be determined from the following approximate formulas:[41]

$$m = 0.37 + 0.61\, \epsilon^{-0.052\, a/b} \qquad \text{(6)}$$

provided a/b is greater than unity and less than 100. In Eq. 6, ϵ = base of Napierian logarithms. Approximately, provided a/b is greater than unity and less than 10:

$$m = 0.98 - 0.027 \left(\frac{a}{b}\right) \qquad \text{(7)}$$

For a circle $\frac{a}{b}$ may be taken as unity without substantial error. It may be noted that Eqs. 5, 6, and 7 confirm the authors' conclusion (*i*) as applicable to small areas. Also, as might be surmised from consideration of the resistance due to shearing rigidity, the settlement decreases as the ratio of the perimeter to the area increases.

If the structure is absolutely rigid and the loaded area is circular, Eq. 5 is applicable; but the value of the shape factor[42] becomes 0.89, making the settlement approximately 6% less than the average settlement of a flexible structure of the same shape and weight. From this it may be inferred that the rigidity of the structure has little influence on the average settlement.

Still assuming that the theory of elasticity applies, and further that the structure is absolutely rigid and that the bearing area is round (radius a), the intensity of pressure, q, immediately beneath the structure may be found from the well-known Boussinesq formula,

$$q = \frac{p}{2\sqrt{1 - \left(\frac{r}{a}\right)^2}} \qquad \text{(8)}$$

According to Eq. 8, the intensity of pressure increases from a minimum of one half the average intensity at the center to a maximum at the perimeter; it approaches infinity as r approaches a. This latter intensity, of course, is not realized because the material yields and the pressure is redistributed. By Saint Venant's principle, this redistribution is accomplished without materially affecting the distribution at points some distance from the perimeter. This is well illustrated by Fig. 4(*a*). It may be noted that the distribution of pressure according to the theory of elasticity is the reverse of that which is intuitively evident for a footing on the surface of a sand mass and indicated by Figs. 3(*a*) and 3(*b*). However, comparison of these figures with Fig. 3(*c*) indicates that the slope of the distribution curve decreases at a rapid rate as $(b + h)\,\gamma$ increases and is practically zero for the case where $(b + h)\,\gamma$ equals p. It appears reasonable that further increase of $(b + h)$ would cause reversal of the slope of the distribution curve and that the intensities would approach the values, given by Eq. 8 and represented graphically by Fig. 4(*a*), as the breadth and depth increased indefinitely. Attempts by various investigators to express the variation in intensity, as the properties of the material and degree of confinement of the soil change, have led to the "concentration-factor" (α) concept. In addition to the Froehlick-Griffith formula (Eq. 1*b*),[43, 44] the following empirical formula[45] proposed by D. P. Krynine, M. Am. Soc. C. E., for the case of an absolutely rigid round disk appears of special interest:

---

[41] "Theory of Elasticity," by S. Timoshenko, *Engineering Societies Monograph*, p. 338, Eq. 207; also references on p. 335.

[42] *Ibid.*, compare Eqs. 207 and 209.

$$q = p\left(\frac{\alpha}{2} - 1\right)\left[1 - \left(\frac{r}{a}\right)^2\right]^{0.5\alpha - 2} \qquad (9)$$

in which q, r, and a have the same meaning as in the corresponding elastic Eq. 8 to which Eq. 9 reduces if α is taken as 3. If α is taken as 4, Eq. 9 states that the intensity of pressure is uniformly distributed, as illustrated by Fig. 3(*c*). As α assumes higher integral values, 5 and 6, the distribution curve becomes parabolic as in Fig. 3(*b*) and Fig. 3(*a*), respectively. When $\alpha = 8$, the distribution curve assumes a shape similar to that of a frequency curve. In addition to the aforementioned objections, it appears that the concentration factor, α, itself should be a variable increasing with r and decreasing with z. For example, in the case of a large foundation, the concentration factor for a given depth might be 3 near the center and about 8 as the perimeter is approached. Also, with r held constant, the concentration factor might be 8 near the surface and approach 3 as the depth increases indefinitely. This difficulty, however, might be overcome by a statistical approach—a soil mass of given properties and depth and loaded with a given intensity over a defined area being assigned an over-all constant concentration factor. Not only are careful settlement observations on a multitude of full-sized structures with known underground conditions required, but such observations should be supplemented by pressure-intensity measurements at various depths and horizontal locations. Since the distribution of pressure may change with time, these studies should be extended over a protracted period. F. E. Giesecke, M. Am. Soc. C. E., cites a case[46]

---

[43] "The Pressure Under Substructures," by J. H. Griffith, *Engineering and Contracting*, March, 1929, p. 113.

[44] "Distribution of Stresses Under a Foundation," by A. E. Cummings, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1072 *et seq.*

[45] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *ibid.*, Vol. 103 (1938), p. 827 et seq.

[46] "The Distribution of Soil Pressure Beneath a Footing," by F. E. Giesecke and others, *Bulletin No. 43*, Texas Eng. Experiment Station, 1933.

in which, at the end of the construction period, values of σ_z under the corner and center of the rigid part of a footing bearing on clay were each approximately equal to 2,000 lb per sq ft; but, after the expiration of seven months, the pressure at the corner had increased to 2,240 lb per sq ft and that at the center had decreased to 1,760 lb per sq ft approximately. As stated by the proponents,[47] until sufficient field observations regarding the pressure distribution beneath foundations are made available, formulas such as Eqs. 1*b* and 9 must be considered rather hypothetical. Nevertheless, such formulas appear to be a step in the right direction and may give some idea of the distribution of pressure. Also, it would appear possible to derive corrective parameters which might be applied to elastic settlement formulas such as Eq. 5. However, as has been stated elsewhere by Professor Casagrande[48] in discussing clay masses, the problem is complicated by the introduction of layers of various materials in the underground. The introduction of comparatively stiff diaphragms or slabs, of course, will spread the load and materially reduce the settlement.

Sharp distinction should be made between such materials as sands, which are noncohesive, and clays, which, on account of the "molecular attraction" between the finely divided soil particles and the surface tension of the included moisture, are cohesive and therefore have intrinsic shearing strength even when not subjected to pressure. The clays possess definite elastic properties for loads of short duration; and, therefore, for such loads the distribution of stress and settlements may be calculated approximately from elastic formulas. However, since the major part of the settlement is due to the slow forcing out of the entrapped moisture over a period of many years and, since this process is proportional to the stress to which the soil is subjected, there is a tendency for the distribution of stress beneath a foundation bearing on clay to approach uniformity as time elapses. Either a rigid or a flexible structure supported on clay tends toward convexity downward. In the case of a rigid structure, this may produce stresses of considerable magnitude which should be considered in the design of the foundation and superstructure.

In addition to the elastic settlement which occurs upon application of the load, there may be additional settlement due to plastic and viscous flow.[49, 50] If the working pressure is not excessive, the plastic flow will be negligible; the viscous flow may continue at a diminishing rate indefinitely. Last, but by no means least, there will be settlement due to the entrapped water being forced out of the clay. This effect, due to the highly impermeable nature of the material, will continue at a diminishing rate for many years or even centuries. A method of determining the difference in void ratio corresponding to the difference in pressure caused by the removed overburden and the imposed load, the initial void ratio, and the ultimate magnitude and the rate of the settlement have been described clearly and in detail by the authors.

The foundation design for the New England Mutual Life Insurance Building appears to be almost ideal. Since the design is based on the most certain principle in foundation engineering—that is, buoyancy—it should remove many uncertainties as to probable settlement and magnitude of stresses.

---

[47] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 846, conclusion 12.

[48] *Ibid.*, Vol. 101 (1936), p. 1119 *et seq.*

[49] "Theory of Heat," by James Clerk Maxwell, Longmans Green, London, 1899, p. 295.

[50] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 316.

The basic requirement, stated by Professor Casagrande in 1932 and quoted in the paper (see heading, "Examples of the Application of Soil Mechanics to the Design of Building Foundations: General," paragraph (c)), is perfectly sound. To it might be added:

> "Design the foundation and superstructure so that they together may tip slightly but will not bend appreciably under the application of reasonable variations in the intensity of soil pressures."

These principles have been applied by the writer to the design of structures for the support and housing of units of machinery which are sensitive to differential settlement and which, if bent, would be rendered inoperative. In certain cases it may be found advisable to introduce steel trusses in foundation walls. Such foundations are comparable to boats inasmuch as they are, to some degree, buoyant. The New England Mutual Life Insurance Building is comparable to a boat floating on still water.

Without discussing the question of pile foundations in detail, it may not be out of place to mention that the action of a group of friction piles, or piles bearing on an intermediate hard layer under which the soil may yield, is subject to the principles stated in the paper. The soil is subjected to a distributed load of varying and somewhat speculative intensity comparable to that which would be introduced by a foundation mat. The bearing capacity of such a group of piles cannot be evaluated rationally as the capacity of a single pile times the number of piles. Also, since their driving disturbs the clay, a group of friction piles in clay may aggravate rather than minimize settlement.

The authors, in this paper, have made a definite contribution to foundation and structural engineering. It is believed that the value of the paper would be enhanced considerably if, in their closing discussion, they would include a tabulation of range in, and typical values for, void ratios, shearing strength, compressive strength on unconfined specimens, Young's modulus, and Poisson's ratio for clay and other soils.

A. A. EREMIN,[51] ASSOC. M. AM. SOC. C. E.—In this paper the authors have opened an interesting discussion of soil mechanics as applied to foundation design. It is true that correct application of soil mechanics requires experience and judgment. Therefore, a discussion of cases encountered in practice is highly desirable.

Two groups of problems in soil mechanics are generally involved. The first is the distribution of stresses produced by the superimposed loading, and the second is the computation of the resulting deformations in soil, or the settlement of foundations.

Various stress distributions in soils have been classified, the most important being those based on the Boussinesq formula cited in Eq. 1*a*. D. P. Krynine,[52]

---

[51] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[52] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 1938), p. 827.

M. Am. Soc. C. E., has made a valuable contribution by simplifying the graphical application of the Boussinesq formula, and F. Kögler and A. Sheidig[53] have developed interesting simple methods of computing stresses in soil, the final results of which are in close agreement with that obtained with the Boussinesq theory.

In computing the critical failure stresses in soil, shown in Figs. 13(*a*) and 14(*a*), the authors have assumed that the shear stresses are determined by the horizontal tangent to the circle of stresses. A more exact expression of shear stresses in a homogeneous material is determined from the curved line, or from the sloping tangent shown by S. Timoshenko.[54]

For several reasons, the computation of deformations and settlement yields indefinite results in soil mechanics. The computed deformations can be verified only by measuring the settlements of completed structures, which measurements generally are not readily obtainable. Furthermore, the settlement of a foundation varies not only with the physical properties of the soil, but also with the methods of constructing the foundation. For instance, it is well known that the effect of upheaval in a deep excavation may be reduced by driving the cofferdam sheathing to a greater depth than that generally required for the lateral stability of the cofferdam. Likewise, the effect of upheaval may be reduced by digging the last layer immediately before building the foundation; or it may be reduced by pre-stressing the soil with a temporary loading.

Computation of settlement by plotting graphs of the ratio of voids in soil, as shown in Fig. 12, may produce results that vary within a wide range. In applying this method, the authors did not take into consideration the facts that the stresses in soil vary with a variation in depth of foundation, and also that at various horizontal planes under a foundation the stresses are not uniform.

The authors' conclusion that the settlement of a foundation can be reduced to a tolerable magnitude by removing a quantity of soil which, in weight, is approximately equal to that of the superimposed structure should be considered true only in the case of a limited depth of foundation.

It is true, as the authors state, that the settlement of building foundations is greater in the middle of the structure than that at the walls of the structure. However, the reverse is quite possible. For illustration, reference may be made to experience with the government building in Cairo, Egypt, where the foundations settled less in the middle than at the walls. The Cairo building was designed in 1930, careful consideration being given to the principles of soil mechanics.[55]

D. M. Burmister,[56] Assoc. M. Am. Soc. C. E.—The achievement possible in designing and constructing satisfactory, as well as economical, building foundations by the proper application of the principles of soil mechanics is demonstrated in this paper. The practical value of reliable methods of reducing the total magnitude, and particularly differential settlement, is obvious. No less interesting and important are the actual design features and methods of construction for the two structures analyzed in the paper.

---

[53] *Die Bautechnik*, 1929, p. 268.

[54] "Strength of Materials," by S. Timoshenko, D. Van Nostrand, New York, N. Y., Vol. 2, 1941, p. 712.

[55] *Die Bautechnik*, 1935, p. 93.

[56] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

In such situations, where the methods described are applicable, the uncertainties involved in settlement investigations may become somewhat less important, and the designer can proceed with greater assurance of satisfactory results on the basis of the available information on subsurface conditions, such as usually can be obtained economically by present methods of soil explorations. For other situations, where these methods are not directly applicable, the uncertainties involved in the settlement investigation may become a major concern in design. It was emphasized by the authors that subsurface conditions cannot be learned in sufficient detail, even by the most elaborate explorations, to serve as an entirely adequate basis for the selection, design, and installation of a satisfactory and economical type of foundation. Consequently, as stated, the results of a settlement investigation should be expressed by a range of values, which include estimates of the possible limits of all factors involved.

The writer considers the following as being the more important sources of uncertainty and error in settlement investigations:

(1) The possible variation in thickness of the compressible stratum between borings from an assumed stratification;
(2) The natural state of stress in the clay deposit or overburden pressure, p_n;
(3) The consolidation properties of the clay;
(4) The foundation stresses in the clay stratum, p_f; and
(5) The settlement characteristics of stratified soil deposits.

Information on the first three items is obtained from subsurface explorations and laboratory soil tests. The sources of uncertainty are due primarily to the necessity of selecting average values on the basis of limited information, and to unknown variations in subsurface conditions between borings which must be assumed, guided by experience in the given area and by judgment. Fairly reasonable values usually can be assigned without much difficulty to the possible limits of variation of these quantities. Their influence, expressed as a range of settlement values in terms of percentage variations from the average, can be estimated by taking partial derivatives of the basic settlement equations with respect to each variable quantity.

In addition to the notation of the paper, let: S = settlement; H = thickness of clay layer; C_c = compression index; C_v = coefficient of consolidation; e_n = void ratio of material in its natural state; p_n = overburden pressure; p_f = foundation stress in the clay stratum; U = consolidation, expressed as a percentage of the total; t = time; T_v = time factor corresponding to values of U; and Δ = variation in any quantity.

For total settlement:

$$S = H\left(\frac{C_c}{1 + e_n}\right)\log_e\left(\frac{p_n + p_f}{p_n}\right) \ldots\ldots\ldots\ldots\ldots (10)$$

and, for time rate of settlement:

$$\text{Settlement (in percentages)} = S\,U \qquad (11a)$$

and

$$t = \frac{T_v\,H^2}{C_v\,(1 + e_n)^2} \qquad (11b)$$

The influence of variations of quantities on settlement is treated as follows:

(1) For percentage variations in thickness ($\pm\,\Delta H$)—

$$\text{Settlement} = \frac{\partial S/\partial H}{S}\,100\%, \quad \text{or} \quad \pm\,\frac{\Delta H}{H}\,100\% \qquad (12a)$$

and

$$\text{Time} = \frac{\partial t/\partial H}{t}\,100\%, \quad \text{or} \quad \pm\,\frac{2\,\Delta H}{H}\,100\% \qquad (12b)$$

(2) For percentage variations in overburden pressure ($\pm\,p_n$, due to possible variations in unit weights or in ground-water elevations)—

$$\text{Settlement} = \pm\,\frac{\dfrac{p_f\,\Delta p_n}{p_n\,(p_n + p_f)}}{\log_e \dfrac{p_n + p_f}{p_n}}\,100\% \qquad (13)$$

(3) For percentage variations in consolidation properties $\left(\pm\,\Delta\,\dfrac{C_c}{1 + e_n}\right)$—

Settlement, variation in reduced compression index

$$= \pm\,\frac{\Delta\left(\dfrac{C_c}{1 + e_n}\right)}{\dfrac{C_c}{1 + e_n}}\,100\% \qquad (14)$$

and

Time, variation in the coefficient of consolidation

$$(\pm\,\Delta C_v) = \mp\,\frac{\Delta C_v}{C_v}\,100\% \qquad (15)$$

These factors actually may combine to produce favorable or adverse combinations, and the actual settlements may be decreased or increased accordingly with respect to the predicted values. Although statistical methods do not help much in arriving at a reasonable probable combination, it is suggested that the square root of the sum of the squares of the percentage variations expressed by Eqs. 10 to 15 will give a reasonable probable range of values, due to a combination of these factors, which may amount to ± 25% to ± 50%, depending on conditions. Variations in conditions at any point influence conditions at all neighboring points because the structure and the supporting soil behave as a continuum. Experience has indicated that the average soil properties in many stratified deposits do not vary as much as one might expect in a horizontal direction within each stratum, although they may vary in a vertical direction in a more or less consistent fashion within the given stratum, due to changing conditions during deposition.

The sources of uncertainty and error under items (4) and (5), which are inherent in every settlement analysis, may be of much greater consequence, because they affect primarily the estimates of differential settlement and the rate of settlement in the early stages, which are needed for appraising the situation as to what should and can be done. These uncertainties arise from the fact that the foundation stresses in natural soil deposits and the settlement characteristics may be quite different from those assumed in the simplified theoretical approach used in the solution of settlement problems. It is difficult and even impossible at present to assign numerical values to the influences of these uncertainties.

The writer is in agreement with the basic soil mechanics principles underlying the design of foundations presented by the authors, but wishes to add a few remarks regarding some of them. Under item (4), the stratification of a deposit, the degree of rigidity of the structure, and the depth of the foundation below the surface of the ground have important influences on the intensity and distribution of stresses in a compressible clay layer. It must be realized that at best the Boussinesq stress equation, as stated by the authors in item (*g*) (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils"), can give only a rough approximation for the stress conditions and that the degree to which the law of superposition of stress and strain is valid in a sand stratum overlying clay is open to question. Nevertheless, both represent most important tools in settlement investigations; otherwise, a solution would be impossible. Therefore an assumption of their validity must be made, even though accepted with reservations. It is necessary to distinguish between near surface phenomena in connection with relatively small bearing areas on sand, where the theory of elasticity does not apply, and the intensity and distribution of stresses transmitted through a sand stratum into a clay layer, where the agreement may be much better, provided the solution for a two-layer soil system were known.

Under item (5), the settlement characteristics of stratified soil deposits, both as to total magnitude and rate of settlement, are affected markedly by conditions, because the structure and the supporting soil behave as a continuum. They will tend to adjust themselves in accordance with a kind of "principle of least work," so that the weaker spots tend to relieve themselves of stress and transfer stress to adjacent stronger areas with corresponding adjustments in settlement. The degree of rigidity in the structure tends to reduce differential settlement materially, because the foundations of the actual structure and the supporting soil cannot "follow down" exactly in accordance with the theoretical estimates based on the assumption of individual foundation loads, which are free to move vertically without restraint. The question is: What is a satisfactory measure of the rigidity of a structure, and how much reduction in differential settlement can be achieved economically?

The authors stated in item (*k*) that sand layers provide an excellent medium for spreading load and for reducing differential settlements. The restraints and resistance to deformation of a fairly thick sand stratum overlying a compressible clay stratum also may reduce the total settlement very materially. It is believed that the solution to this problem is best approached empirically from the settlement rather than the stress "angle" by comparing actual settlements with predicted settlements, after making proper allowances for the other factors discussed herein. When a number of structures have been observed for which the subsurface conditions are fairly well known, an empirical coefficient or multiplying factor may be determined by plotting the ratio of the actual to the predicted settlements against the ratio of the width of the structure to the thickness of the reinforcing sand layer. The decrease in settlement with depth below the adjacent ground surface, as stated in item (*j*), may be approached in a similar manner.

In addition to the foregoing factors, which tend to decrease settlement materially, two other factors must be mentioned. Two-dimensional consolidation (see Biot's solution[57]) in contrast to the simplified approach in the one-dimensional theory and the lateral subsurface displacements (lateral bulging effects), which always accompany and contribute to the total magnitude of settlement, may tend to increase settlement somewhat at the walls and decrease differential settlements in the early stages of consolidation.

In order that engineers in other localities may compare their situations with those reported, the authors should include in their closing discussion information concerning the physical properties of the soils in the deposit. It would be of immense value to the engineering profession if the authors would include what they have judged to be the possible variations and the limits for the different factors involved in the settlement investigations and what they have learned from the actual settlement observations of the two structures as a full-scale check on their assumptions as to foundation stresses and settlement characteristics of the deposit.

In conclusion the writer wishes to congratulate the authors on their valuable contribution to applied soil mechanics.

ABRAHAM WOOLF,[58] ASSOC. M. AM. SOC. C. E.—Together with the architects and owners, the authors should be complimented, not only for their excellent study and report, but for their courage in the design and actual construction of a type of foundation radically different from the usual Boston type of foundation. From the settlement results shown in Fig. 15, it is evident that the floating type of deep foundation is the solution to the problem as applied to the conditions in the City of Boston. However, the conclusion that this type of foundation is the general solution in every locality, where at least similar conditions exist, is not justified reasonably. The authors were in a position to assume responsibility for the floating type of design, with the certainty of the resulting settlements as shown in Fig. 15, only after extensive study and observation of the action of several similar structures in Boston. It is to their credit that they correlated their observations and experiences in the field, together with laboratory practice, and prepared a definite procedure for the design of a floating type of foundation. The experience of the authors was the prerequisite to the design. This fact alone should call attention to how important it is for municipalities, universities, and engineers to keep accurate records of the settlements of important structures and also to make laboratory soil tests so that any design which departs from the usual type would be justified in a particular locality.

---

[57] "Consolidation Settlement Under a Rectangular Load Distribution," by M. A. Biot, *Journal of Applied Physics*, Vol. 12, No. 5, May, 1941, pp. 426–430.

[58] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

In underconsolidated clays, such as exist in some parts of Detroit, Mich., there is a possibility that the installation of a floating type of foundation might cause settlements beyond those expected. The character of clays, preconsolidated under loads less than the overburden, presents a problem of sensitive nature, and the floating type of foundation needs careful study if adapted to such clays. The installation of a deep basement, no doubt, reduces the load on the substrata below and at the same time provides sufficient depth to allow for the design of a rigid foundation; but the installation disturbs the firm upper strata and also provides additional drainage immediately below the concrete foundation. The improved drainage increases the rate and amount of consolidation by relieving the excess hydrostatic pressure in the underconsolidated clay. The result, of course, would be increased settlements. However, in normal clays, which are preconsolidated under heavier loads than the existing overburden, no such excess hydrostatic pressures exist, as in the underconsolidated clays, and, even though the deep basement does provide a better drainage under the foundation, there can be no evidence of any additional excessive settlements because the stress condition in the substrata is in balance due to the fact that the excavated loads are approximately equal to the building loads.

Conclusion (5) reads,

> "Differential settlements are best controlled by designing the foundation and the superstructure as an integral unit, with special attention to stiffness. Where this is not acceptable, design for stiffness must be confined to the foundation, which increases the cost and decreases its effectiveness."

This statement becomes of interest to the designing structural engineer because it is his design which controls the internal differential settlements. The writer wishes to classify the differential settlements into two types—the external and internal. External differential settlements are measured at relative points on the periphery of the building, the internal are measured at points within the structure. The purpose of segregating the differential settlement into two types is to help analyze the condition of equilibrium of the foundation, itself, as a structure. The variation in stiffness of a rigid substructure has no material effect on the external differential settlements; but it definitely controls the internal condition. Fig. 23 shows a maximum external differential settlement of 1.16 in. between the two long sides of the building on Boylston Street and Newberry Street, whereas the maximum internal differential settlement on the walls, running crosswise, does not amount to more than 0.5 in. It may be that the Boylston Street subway acts to reduce the settlement rate on the Boylston Street side, thereby increasing the differential on the two long sides. The almost balanced condition of the two shorter sides indicates equilibrium.

The internal differential settlements and resulting distortions are a measure of the shears and moments of component parts of the rigid foundation. In Fig. 23 three of the cross-walls show tension on the top, and the fourth shows tension on the bottom. The writer has spot-checked the wall showing tension on the bottom and found that the moment developed is approximately 635,000,000 in-lb based on a differential deflection of 0.2 in., requiring about 60 sq in. of reinforcing steel at 24,000 lb per sq in. in the bottom of the wall. Since the actual amount of steel installed was not known, it was felt that it would be an interesting check on the original design.

At present, there appears to be no definite answer to the question, "How stiff to make the foundations."[59] A rational approach to the problem would be to design the integral members of the foundation so that structural stability is insured and then to determine what elastic distortions these members will develop under this stability. These distortions can be checked with the soils engineer, and, of course, it would be his responsibility to advise whether the structure is too stiff or too flexible or whether the original assumption of soil reaction distribution needs revision. Also he must decide what adjustment should be made to satisfy the elastic distortion in the soils. The coordination of the instantaneous elastic yield of steel and concrete with the time lag in the elastic yield in clays is perhaps somewhat beyond the present-day developments. Attempts have been made to assume upper and lower limits in the soil and then to interpolate the results obtained by means of reasoning at least to set the direction of the stress.[60] The theory involves the so-called K-values (moduli of subgrade reaction which are given in pounds per square inch per inch), but these values do not introduce the time effect, which is quite important in the analysis of both the structural stability of the foundations and the soil substrata beneath.

In an investigation of recent date regarding the structural stability of a building which had undergone considerable total and differential settlements and still appeared to be sound and not under any serious condition of distress, the following was deduced:

(1) In a distorted, rigid, steel structure, in which the distortions are due to the displacements of column reactions because of settlement, if the calculated stresses in the structure, due to this distortion are at, or beyond, the yield point of the steel, the structure, itself, will undergo elastic relief and these calculated stresses will not exist. The actual stresses in the structure are reduced, by this relief, to the yield-point values or below; otherwise, the entire structure would be in a state of complete collapse, as indicated by the high calculated stresses.[61]

(2) In a concrete structure a similar condition prevails, but the relief is developed by rupture of the concrete in tension, made evident by the appearance of cracks at those points where the reinforcing steel is omitted. The

---

[59] "Economics of Rigid Frames for Building Foundations," by K. R. Kimball, *Engineering News*, September 26, 1935, p. 427.

[60] "Stresses in Concrete Runways of Airports," by H. M. Westergaard, presented to Highway Research Board, Washington, D. C., December, 1939; published by Portland Cement Assn.

[61] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 638.

elastic range between the yield and rupture points for concrete in tension is so close that there is no opportunity for relief to develop without rupture.

These observations are mentioned merely to indicate that, in the design of a rigid concrete substructure, the possibility of a tension stress in the concrete should be eliminated and all concrete members subjected to such possible conditions of stress should be reinforced both top and bottom. The observations also indicate that moderate settlements are not too serious when applied to an analysis of the stability of a structure.

Conclusion (*m*) does not imply that lowering the ground water decreases the bearing capacity of the loose sand. Rather, it refers to the disturbances in the sand structure due to the lowering of the ground water. The writer has made observations where the raising of the ground water has decreased the ultimate bearing capacity of dense sands.[62] The observations indicate ratios for different kinds of sands of $\frac{\text{bearing capacity, dry}}{\text{bearing capacity, wet}}$ as follows:

Item	Grade of sand	Ratio
(*a*)	Nonuniform, well graded	2.83
(*b*)	Uniform, medium grain size	1.98
(*c*)	Uniform, fine grain size	1.52

In Fig. 15(*b*), curve IV is established in its position relative to curves III and V without mentioning the fact that the total settlement of the particular point was known six years after the building was finished and the extrapolation was established for curve IV with those data. In several settlement studies settlement readings were not taken from the construction completion date to the point where the differential distortions become troublesome; and it was impossible to establish the total settlement definitely, particularly where there have been no permanent bench marks and the site immediately surrounding the structure had undergone subsidence. In this connection, it would be of value for any building to establish at least one level of the completed structure definitely tied in with some permanent city bench mark, instead of with a curb, manhole, or any other flexible point. The authors might clarify this point with regard to curve IV.

In discussing Fig. 18(*a*), the authors remark that,

> "It was specified that, if it were discovered, during the progress of construction, that the hard clay in some areas approached the sub-basement slabs, it would be necessary to excavate to a sufficient depth and replace the hard clay by soft organic silt-clay."

The radical deviation from the usual construction practice makes this clause difficult to accept, particularly with respect to the method of backfilling a soft organic silt-clay.

Fig. 19 shows the unsymmetrical footings of the outside walls of the New England Mutual Life Insurance Company Building. The final design indicates a procedure better suited to deep foundations which require a minimum disturbance than to areas outside the building or structure. This type of wall footing is usually standard practice in underpass and open-cut tunnel design. The method of thickening the slab under the wall without indicating structural continuity at the joint between the slab and the wall is not clear. It would be interesting to know the distribution of the soil reaction under this type of footing in order to establish design procedure for the structural stability of the slab and wall.

---

[62] "An Experimental Investigation of the Relation Between the Bearing Capacity of Dry and Wet Sands," by J. P. Luby and A. Woolf, thesis presented to Massachusetts Institute of Technology, Cambridge, in 1928, in partial fulfilment of the requirements for the degree of Bachelor of Science.

LAZARUS WHITE,[63] M. AM. SOC. C. E.—A large part of this paper is devoted to theoretical soil mechanics, particularly as applying to unconsolidated clays. Considerable valuable information regarding the nature of building settlements and actual records of settlements of buildings in the Boston district are given.

The writer does not wish to discuss theoretical soil mechanics, although he regrets that the use of photoelastic methods as applied to soil mechanics is not mentioned. Also, he regrets that the plottings of the lines of equal vertical pressure beneath the building foundations, described in detail, are not given. For some reason, soil mechanics, as exemplified by the paper, seems to have relegated Boussinesq to the rear and eschewed the graphical representations of pressures so prominent in soil mechanics of only a few years ago. In fact, by a circuitous route, the authors arrive at a conclusion so simple that one wonders whether all the elaborate mathematics of present-day soil mechanics is necessary. All one needs to do, according to the authors, is to excavate from the cellar and subcellar of the proposed buildings a weight of material equal to the weight of the proposed building. The structure can then be erected on mats of reinforced concrete to gain a foundation free of undue settlements, with additional cellar and basement space, all for the cost of a few thousand yards of additional excavation—surely a small price to pay for the benefits realized.

However, in justice to the authors, the writer wishes to call attention to their statement (see first paragraph of "Introduction"),

> "It cannot be emphasized too strongly that the correct application of the results of such an investigation to the design of earthworks or foundations will, in addition to a thorough knowledge of soil mechanics, always require experience and judgment."

This conforms to an old test of any engineering design. Does it conform to good practice?

The writer hesitates to take issue with the authors as to the theories of soil mechanics and confesses that he has been unable to follow the elaborate mathematics of this paper. Nevertheless, there are some basic principles by which to test the conclusions; also, there is considerable experience as to what is good practice. The writer begs to differ with the conclusions of the paper, particularly Conclusions (1), (2), and (6).

---

[63] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

There is an old recognized principle that, in a structure, the loads tend to go to the hard points. As applied to a building with numerous columns extending to cellar grade, the loads can best be picked up by the foundations directly beneath the column bases. The best foundation is based upon this principle. For instance, in the first Chicago (Ill.) "skyscrapers," mat foundations were used on the Chicago clay with unsatisfactory results; then the Chicago caissons were introduced, which carried the loads directly from the columns to hardpan or rock. Later, various types of steel piling and caissons were used successfully. Conclusions (1) and (2) would set the profession back forty years.

When an excavation is made, various disturbances take place. The groundwater level usually is lowered in an irregular way, and, as the authors state, the interior bulges, which may or may not be serious. The placing of sheeting around the exterior of the excavation creates disturbances.

Usually it takes several weeks before the foundation can be completed, during which time the interior is exposed to the weather. It dries out if exposed to the sun, or is saturated if exposed to the rain.

Now if, instead of designing a foundation to carry the loads directly to a secure stratum and by-passing the weak upper stratum, the engineer resorts to a "raft" or "mat" foundation and attempts to avoid differential settlements by an elaborate system of reinforced concrete girders, what risks are run. In the first place, uniformity of pressure is impossible. As amply demonstrated by photoelastic experiments and the application of the Boussinesq formulas, differential pressures bring about differential settlements which may be very damaging. This is covered quite fully in the Progress Report of the Special Committee on Earths and Foundations.[64]

It happens that the Boston clay has been studied intensely over quite a long period, and its qualities are well known; it is geologically a simple formation founded upon hardpan or impervious rock. Calculations of settlements of buildings on Boston clay, computed by the use of the Boussinesq equations and the consolidation test and methods devised by Professor Terzaghi, have agreed fairly well with observed settlements, but nevertheless in other and more complex areas the engineer should be on his guard; consolidation is a drainage problem and the condition must be simple and capable of analysis. However, there are wet areas with difficult conditions, such as that adjacent to the Great Lakes where an oxidized yellow clay prevents upward drainage and an underlying limestone may keep the intermediate soft blue glacial clay under an artesian pressure built up to high values—in some cases by natural gas.

Deposits of clay are often associated with sand pockets under pressure. Such pockets may complicate the drainage of the clay so as to defy computation. Again, an underlying bed of porous limestone may keep the clay under such hydrostatic pressure as to prevent consolidation.

At one location in the Hudson Valley a heavy glacial clay deposit of varved clay is still very soft after a period dating back to the glacial age. This is because the wet blue clay is confined between two layers—an upper, oxidized, very tight yellow clay, and a lower porous limestone under pressure from ground water in a near-by mountain. In the vicinity of Detroit, Mich., a similar condition prevails—an upper crust of yellow clay is underlain by a thick deposit of soft blue clay, also under pressure from a porous layer above the rock floor. For these conditions, the writer knows of no formula which could give the correct rate of consolidation for the blue clay. In both cases, structures placed on the upper yellow clay came to grief—in one case, the blue clay beneath a heavy building squeezed into an adjacent excavation, and in the other case a structure on a heavy mat, in turn resting upon an oxidized layer, settled and upset because of plastic flow in the underlying soft blue clay. The subsoil in any city is subject to changes; it is usually being drained, with lowering of ground water, or a broken water main may cause a rise in the ground-water level.

---

[64] *Proceedings*, Am. Soc. C. E., May, 1933, p. 777.

To the writer, it seems wise to design any important structure with its foundations extended to rock or some deep, sound stratum. There are several methods available. The cost may be a little higher than for a spread footing or mat foundation, but it may prove to be cheap insurance.

The writer has every respect for the authors but does not believe that soil mechanics has advanced far enough to furnish the basis for the design of mat foundations upon unconsolidated strata such as wet clays or silts. Engineers cannot predict confidently what settlements will occur. Such mat foundations are subject to many dangers—from neighboring structures, from neighboring excavations and pile-driving operations, and from the shifting of floor loads.

Methods of installing foundations, particularly to great depths, have been so improved that in most instances the old-fashioned shallow foundations have been superseded. However, in some districts there are thick strata of unconsolidated clays and silts, and the risk of uncertain and irregular settlements of shallow footings must be taken. Soil mechanics may be invoked in such cases to diminish the risk.

Soil mechanics alone is not sufficient for choosing a site or designing a foundation. It is a great aid to the engineer, but not his master. For instance, a building may be "plumped" in the middle of an extinct pond in which, at depths affected by the structure, are layers of unconsolidated peat. Such localities may be detected by the practiced eye of a trained geologist.

The shape of a building is a great consideration in the design of a foundation —whether it is of uniform height or of a tower design, and whether or not it contains courts and projecting wings. Such dangers may be revealed by a plotting of the iso-pressure lines computed from the Boussinesq formula.

GEORGE L. FREEMAN,[65] M. AM. SOC. C. E.—Foundation engineers will find this paper particularly valuable because it follows through, from a statement of the basic principles of soil mechanics affecting the designs, to a description of the "finished products," with records of the settlement phenomena accompanying and following the construction periods. The description of the foundation of the New England Mutual Life Insurance Company Building is of interest, with its record of subgrade heaving and surrounding soil movement, and the adaptation of the construction methods to job conditions. It is obvious that close control of construction methods is essential on such work; otherwise, subsoils and bearing strata may be disturbed and much of the care and skill expended in the design may be nullified.

---

[65] Cons. Engr. (Moran, Proctor, Freeman & Mueser), New York, N. Y.

It may be assumed that the two substructures created by the authors reflect to full extent the benefits of thorough soil studies and their application in the designs. From the performance records, it is evident that both structures have proved satisfactory in service, and the designers are to be congratulated on their success.

The settlements of the belled piers under the Liberty Mutual Insurance Company Building are partly dependent on the thickness and continuity of the hard crust which overlies the deep, soft clay bed. It would be interesting to know what steps were taken to prove the integrity of the crust under each bearing area. The value of the 10-ft hard bearing stratum in distributing load concentrations to the underlying soft clay is evidenced by the regularity and symmetry of the settlement contours in Fig. 17. In both this design and that of the foundation of the New England Mutual Life Insurance Company Building, the designers have taken full advantage of the fortunate existence of the hard crust 40 ft below the ground surface, without which both designs would have been materially different from those shown.

The writer prefers the type of design used for the foundations of the New England Mutual Life Insurance Company Building, as its wall arrangement, if strongly reinforced, can lend great stiffness to the substructure, assisting to smooth out local deflections. Had there been no bearing crust at the 40-ft depth, a full mat design would have been indicated, with the walls modified to some extent to provide proper stiffness.

The provision for future increases in heights of certain sections of the New England Mutual Life Insurance Company Building greatly complicated the problem of designing a floating foundation. With soil excavated, as stated, in an amount to equal the weight of the building, including its future extensions, it would appear that a greater differential of settlement than shown might be expected between the present lightly and heavily loaded areas, unless some compensating factors were introduced into the design. Footing widths shown in Fig. 18(*a*) were apparently designed for ultimate building heights. Would the authors please clarify this point, and indicate the locations of the building sections of varying heights mentioned in the paper?

The tabulation of vertical movements of the hard clay stratum shows that point 5 was up 2.9 in. on October 12, and that other points continued to rise until October 30. It seems from the record that the upward movement must have continued at all points until checked by the weight of the concrete, which was placed in the vicinity of point 5 on October 27. Does not this mean that the area around point 5 might have risen considerably more than the 2.9 in. shown, before the rise was arrested by the weight of the concrete?

The amount and rapidity of the upward movement, prior to the placing of the concrete, leave the writer with a suspicion that a definite soil movement occurred due to lateral plastic flow induced by the weight of the surrounding unexcavated areas, causing uplift of the hard clay blanket. This thought is supported by the fact (see discussion of Figs. 21 and 22) that "the street curbs moved in about 1¼ in. and down about 1¼ in.," with a certainty that the subsidence did not stop at the curbs but extended for some distance over the adjoining street areas. Moreover, it is noted that the total weight imposed on the soft clay at the underside of the hard clay blanket, outside the sheeted area, due to about 50 ft of overburden, was about 5,000 lb per sq ft, whereas the excavation within the sheeted area left a weight of only about 1,600 lb per sq ft to counteract uplifting pressure.

It would be of interest to know how the designers of the two Boston foundations applied live loads in their load-settlement computations. It is the practice in the writer's firm to determine as closely as possible, from the expected occupancy of a building, the live loads that will actually exist, and to use these live loads, plus the actual dead loads, in load-settlement computations. As an example, in the design of the floating foundation for the Albany (N. Y.) Main Telephone Building,[66,67] the total live load assumed in settlement studies amounted to but one third of the total of the designed live loads for the several stories; this was believed to approximate the actual live load contributing to settlement.

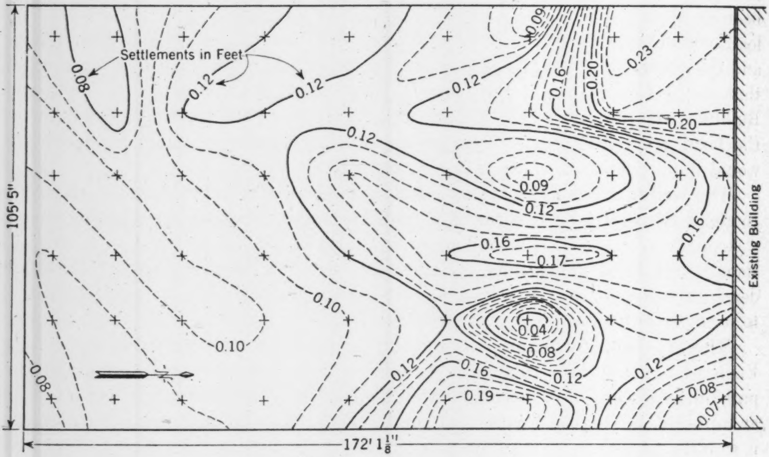


FIG. 31.—SETTLEMENT OF THE FOUNDATION OF THE ALBANY (N. Y.) MAIN TELEPHONE BUILDING AS OF DECEMBER 11, 1930

It is unfortunate that so few settlement records are available, and that those available are so incomplete. The settlement of the foundations of the Albany Main Telephone Building, to December 11, 1930, is shown in Fig. 31. At that time practically all the live and dead loads were on the foundations, the dead load having been in place for practically a year. The south half of the mat

---

[66] "Rigid Rectangular Frame Foundation for Albany Telephone Building," by G. W. Glick, *Engineering News-Record*, November 27, 1930, pp. 836–838.

[67] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 1, p. 278.

settled normally, and about as expected, with corners high and a tendency to dish downward at the center. The north half settled much more, and erratically, due to a cofferdam bracing failure at the north end of the site when the excavation was near subgrade level, more than 25 ft below the ground surface. This illustrates strikingly how the exigencies of construction may upset the calculations of the soil mechanic.

Fig. 32 shows settlement contours, as of February 9, 1927, for the 5-ft thick mat supporting the main office building of the Ohio Bell Telephone Company in Cleveland, Ohio. (The numbers in circles are the settlements, in sixteenths of an inch, extrapolated from records to December 3, 1926, or January 6, 1927.)

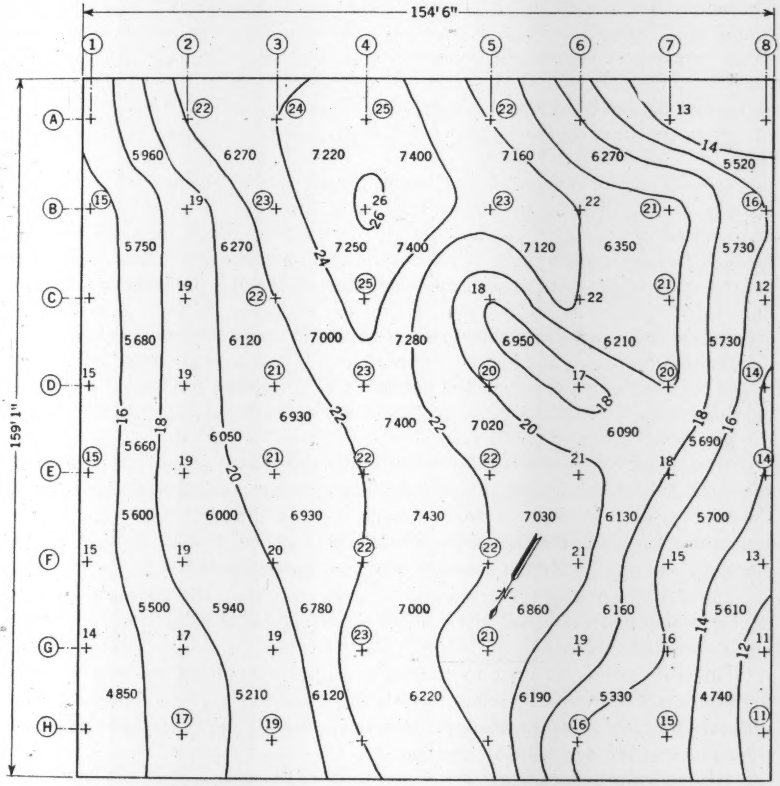


FIG. 32.—SETTLEMENT OF THE FOUNDATION, OHIO BELL TELEPHONE COMPANY BUILDING, AS OF FEBRUARY 9, 1927

This mat is nearly square, and is supported on a great depth of "stiff gray clay." The area had previously been loaded by a theater and a hospital. The bearing level is about 52 ft below the original ground surface, and the mat was designed for varying unit loads, the pressures on the soil being about 5,600 lb per sq ft under the easterly and westerly side panels, and increasing to about 7,400 lb per sq ft in the north and south center panels. (To these loads should be added the uniformly distributed weight of a 5-ft thick concrete mat.) The average intensities of pressure on the soil, exclusive of the weight of the mat, are shown in each panel in Fig. 32, and it will be noted that the settlements followed rather consistently the variation in loading, resulting in a barrel-shaped deflection. The mat was completed in December, 1925, and all foundation work, including three levels of basement space, by March, 1926. Practically all of the dead weight was on the structure at the time of the settlements shown in Fig. 32.

With respect to statements (*a*) to (*m*) (see heading, "Stress-Deformation Characteristics of Cohesionless Soils"), concerning foundations supported on cohesionless soils, it would appear that the general principles outlined are based largely on theoretical considerations, and may or may not apply in practice. Foundations of structures of any magnitude or importance cannot be supported safely at the surface of a cohesionless soil, and are necessarily supported on bearing levels below the ground surface. Under actual conditions, rule (*a*) would not apply.

Rule (*b*) may or may not be applicable for deep caisson foundations. In the absence of data directly bearing on this point, the writer believes that uniform distribution of load over the bearing area is the only assumption that can be made. In the actual sinking of pneumatic and open-dredged caissons to great depths, so many factors upset any calculation of bearing pressures that much uncertainty exists as to the actual pressure distribution in the soil. The excavation in the area of a caisson shaft results in stress release over that area, and, to some extent, outside the excavated area. At the same time, the weight of the caisson, which is supported partly by skin friction and partly by direct bearing under the cutting edges, tends to set up high load intensities around the perimeter of the excavated area.

Subsequent filling and loading of the caissons results in some adjustment of supporting values of skin friction, cutting edge pressure, and direct bearing over the shaft area. In many cases, caissons sunk in granular soils may derive substantial support through side-wall friction, although such support is not generally allowed in the determination of safe bearing areas.

The writer questions the validity of rule (*d*)—that the ultimate bearing capacity of cohesionless soil increases as the square of the depth of the loaded area below the surface.

This rule seems to lead to fantastic results, if applied to very shallow foundations. The writer prefers, from present knowledge, to consider that the ultimate bearing value varies approximately with the depth, rather than with the square of the depth below the surface.

Although the general statements in (*e*) and (*f*) appear sound, the last statement in (*f*) may not be an infallible guide; certainly judgment and experience are required in its application. Settlements do follow the loading of cohesionless materials, and vary with the intensity of loading and density of the supporting soil. Over a large building area the density of the soil will usually vary considerably and differential settlements are to be expected. Load tests will occasionally be advisable to check settlements, especially where high soil bearing intensities are used in the foundation design.

Rule (*j*) may not apply for deep caisson foundations where settlements may be due more to effect of construction operations than to soil characteristics. The writer's experience would indicate that settlement of a deep caisson foundation supported at a safe unit load on cohesionless soil is practically independent of the depth of the bearing level below the surface, and is largely dependent on soil disturbance and other factors connected with the construction procedure.

SOIL MECHANICS COMMITTEE OF THE LOS ANGELES SECTION—TRENT R. DAMES, CHAIRMAN,[68] FREDERICK J. CONVERSE,[69] AND STERLING S. GREEN,[70] ASSOC. MEMBERS, AM. SOC. E., AND PAUL BAUMANN,[71] M. AM. SOC. C. E.—The Soil Mechanics Committee of the Los Angeles (Calif.) Section first read the authors' paper individually and, surprisingly enough, the reaction of each individual was the same. Each felt that the authors had gone too far in publishing a series of brief generalizations regarding the behavior of sand and clay as the publication of generalizations of this nature, unsupported in the text by the data on which they were based and with the exceptions and limitations not stated, may mislead the uninitiated and lead to a loss of confidence on the part of, and the accusation of impracticability by, the construction industry.

When the committee met for detailed discussion of the paper, a new difficulty developed. No two members of the committee were in agreement concerning what the authors intended by some of their statements. For example, under the heading, "Soil Properties Responsible for Settlements," mention is made repeatedly of sand in the loose and dense states without qualification or definition. Sand usually is defined as consisting of more than 80% of particles ranging from 1/20 mm to 2 mm in grain size and the behavior of sands of different composition may be quite different. The division line between loose sand and dense sand presumably is the critical density; the latter, however, is not a soil constant but varies with the moisture content and loading condition and has been defined differently by different authorities. Another example of rather indefinite language is the reference to the "ultimate bearing capacity of a cohesionless soil." It is not clear whether the authors intend the ultimate bearing capacity to be the greatest load which it is permissible to place on a spread foundation or the load that will cause a footing to penetrate continuously through the soil in the manner of a pile being driven. There are several instances in which the authors state that one quantity varies in proportion to another quantity. In some of these cases it is not clear whether the authors are stating that the relationship is one of linear proportion or of some unstated curvilinear proportion.

---

[68] (Dames & Moore), Los Angeles, Calif.

[69] Associate Prof., Civ. Eng., California Inst. of Tech., Pasadena, Calif.

[70] Materials Engr., Bureau of Water Works and Supply, Los Angeles, Calif.

[71] Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

Apparently the authors intend the statement (*c*) (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils")—

> "The ultimate bearing capacity of a cohesionless soil increases approximately in direct proportion to the width of a loaded area"

—to be the basis for selecting design loads for different sizes of footings. This rule is valid only for a foundation on the ground surface. For footings founded at any depth, this rule does not apply even for the case of final failure. Furthermore, the yield point bearing capacity, which should be used for design purposes, will vary in proportion to the shearing strength of the supporting soil at the depth of the maximum concentration of stress. Statement (*d*)—

> "The ultimate bearing capacity of a cohesionless soil increases approximately as the square of the depth of the loaded area beneath adjacent ground surface"

—is not borne out by the observations of the writers of this discussion. Certain very recent river-deposited sands in the Los Angeles area are capable of supporting, at their yield point bearing value, small spread foundations loaded to 5,000 lb per sq ft at a depth of 10 ft below the surface. Based on the authors' theory, the same deposits at a depth of 50 ft would support 125,000 lb per sq ft, and at 100 ft, 500,000 lb per sq ft. On the contrary, it would be expected that the yield point bearing capacity would vary as a straight-line function of the depth as the support offered a loaded area is a function of the resistance of the supporting material to shearing displacements by lateral or plastic flow. For a uniform soil condition such as the authors evidently are considering, the shearing strength increases approximately in direct proportion to the depth below the ground surface, and therefore it follows that the bearing value would increase approximately as a straight-line function of the depth.[72]

In paragraph (*f*) the statement is made that—

> "* * * for large areas on dense sand, or for areas which are to be founded at a considerable depth below ground surface on dense sand, even the heaviest loading which building codes permit is exceedingly conservative, so far as strength of the soil is concerned, and requires no verification by load tests."

The writers agree that, in general, building codes have restricted allowable bearing loads on dense sand to too low values. However, some advanced codes, such as the new Los Angeles City Building Code, permit deviation from arbitrary allowable foundation pressures after the performance of special foundation investigations. The only requirement of the Los Angeles Code is that "Stresses and deformations within the foundation shall be determined by the general principles of soil mechanics." If building codes impose costly and unnecessary restrictions on construction methods, the codes should be altered.

Statement (*g*)—

> "* * * the fact that Hooke's law does not apply to cohesionless soils also invalidates the law of superposition"

---

[72] "Practical Shear Tests for Foundation Design," by Trent R. Dames, *Civil Engineering*, December, 1940, pp. 784 and 785.

—should be expanded to say "Hooke's law does not apply to any soil." In all soils, the resistance to compression increases approximately as an exponential function of the load, similar to the behavior of metals under compressive stresses beyond the proportional limit.[73] However, the various equations for stress distribution, although certainly not precise, are valuable as an indication of the relative behavior of different sizes and types of foundations.

In paragraph (*l*) it is stated that, for buildings founded on cohesionless soils, borings should be driven—

> "* * * to a depth at least equal to the width of the building * * *."

Here again confusion and uncertainty result from the authors' failure to qualify the meaning of "building." For example, a typical airplane manufacturing building might be from 300 to 400 ft wide by 800 ft or more long with clear spans of 150 ft or more between rows of columns. It is unreasonable in this instance to require that borings extend to 300 or 400 ft in depth.

The authors devote several pages to a consideration of the characteristics of the consolidation diagrams of clay and particularly emphasize the determination of the preconsolidation loads without restricting the statements made to specific clays, such as the glacial clays of New England, or explaining the practical significance and use of the values obtained. The writers have not been able to apply the conclusions and rules cited to the residual and alluvial clays and bay muds of the Pacific Southwest as the consolidation curves of the clays of this region practically never possess straight-line parts on semilogarithmic graphs. The consolidation curves obtained for the clays of this region in general approximate straight lines on double-logarithmic graphs.

In connection with Fig. 13, discussing "Strength of Clay and Safety Against Rupture," the authors state "* * * the possibility of stressing a cohesionless material to the point of rupture practically does not exist." Apparently the authors are referring to a dense, cohesionless soil far below the ground surface, although the limitations intended are not specified. Experience in the design, construction, and behavior of a variety of structures in the Pacific Southwest indicates that the possibility of overstressing a cohesionless material to the point of progressive failure is often the principal consideration. Within the capacity of the soil to resist lateral displacement by plastic flow, the deflection due to consolidation and compression of cohesionless soils (and also dense residual clays) is usually negligible.

In presenting Fig. 14, the authors make the statement that the shear strength of clay "* * * will be equal to a constant c * * *," regardless of the confining surcharge (normal pressure). If the clay is not completely saturated, increased granular pressures will be effective immediately upon application of load, and the shearing strength therefore will increase with the confinement and be expressed by the equation,

$$s = c + n \tan \phi \qquad (16)$$

---

[73] "Recent Work in the Field of High Pressures," by P. W. Bridgman, *American Scientist*, January, 1943, Fig. 15.

in which s is the total shearing strength; c, the apparent cohesion; n, the confining surcharge; and ϕ, the apparent angle of internal friction of the soil. For clays of this region, while the apparent angle of internal friction, ϕ, may be as low as 3° or 4°, it may also be as high as 20°.

In further discussing Fig. 14(a), the statement is made "* * * that the pressure q_o, which the footing must exert in order to produce failure * * * is equal to $4\,c$." By the use of another theory quoted, q_o was equal to $5.5\,c$ and again,[10] $5.14\,c$. Apparently, the authors are considering the conditions of final failure which is reached at considerably higher loads than those at which initial yielding occurs. The load which would produce this final failure would certainly not be a desirable basis for determining an allowable design load. A better appreciation of the behavior of loaded soils may be obtained through a study of the stresses required to produce yielding and plastic flow.[74,75] The yield point of a semi-infinite body loaded by a circular area on its surface is equal to π times the shearing strength. The ratios of unit load to shearing strength for other shapes of loaded areas do not vary appreciably from π. Practical confirmation of this relationship has been secured over a period of years.[72]

In the last few paragraphs under "Soil Properties Responsible for Settlements," the authors recommend the performance of cylindrical compression tests to determine the bearing capacity of clays. It is not clear whether the authors are referring to the use of triaxial compression tests or compression tests on unsupported soil cylinders. The authors, of course, realize that tests on unsupported soil cylinders are frequently impossible because of the character of the material. The direct shear test may be used effectively for all kinds of materials, without exception, and offers a means of determining bearing capacity with sufficient exactness for practical purposes at a cost which is comparable to the free cylinder test and which is much less than that of the time-consuming triaxial test.

The statement in the very last paragraph under "Soil Properties Responsible for Settlements" that, in the case of caisson foundations, "the hydrostatic uplift of the soil may be added to the foregoing value of $5\,c$ and used as the ultimate bearing capacity" is very dangerous and subject to a long series of limitations most important of which is that the bearing capacity given by this rule is actually the capacity at ultimate failure and not a basis for allowable design loads.

The value of the second part of the paper, "Examples of the Application of Soil Mechanics to the Design of Building Foundations," would have been much enhanced if the authors had included information on the physical characteristics of the soils upon which the buildings in question are founded as this would enable the reader to make a comparison with similar soils, the behavior of which had been studied and experienced. The only such data, so far as the writers are aware, is that shown in Fig. 15 (a), which appears to be reminiscent of the usual well-driller's log except for the fact that it gives no record of ground water. Above all, it is the clear and specific identification of the foundation

---

[74] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

[75] "Plasticity," by A. Nádai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

soils which the writers (and probably many other readers interested and somewhat familiar with soil mechanics) hope to find in papers of this nature. Under the circumstances, this part of the paper, although definitely intriguing, is not as enlightening as it could be.

In closing, the writers hope that their discussion will be accepted as constructive criticism. They are well aware of the difficult, even impossible, task of setting down theories and principles of soil mechanics in the form of generalities, which have many exceptions, without the time or space for adequate discussion of the exceptions or clear statements regarding the limitations of the generalities.

A. CASAGRANDE,[76] M. AM. SOC. C. E., AND R. E. FADUM,[77] JUN. AM. SOC. C. E.—The discussions, which represent much thought and labor on the part of the contributing engineers, are very stimulating to constructive thinking. Careful study of their contents is recommended to all who are seriously interested in this subject. In addition to presenting a great deal of new material, the discussions contain a divergence of views which should encourage the development of independent thought so important in this field in which design is still largely dependent on judgment.

The discussions have broadened the scope of the paper to such an extent that the authors are not able to comment in this closure on all the new material presented, nor to answer all of the many requests for additional information. In the following, emphasis is placed on material which would not otherwise be available to the readers.

PROPERTIES OF CLAYS AFFECTING SETTLEMENT ANALYSES

The numerous requests in the discussions, for detailed information on the properties of the clays underlying the buildings described in the paper, prompted condensed use of material originally intended for a separate, comprehensive publication on this subject. In this form it also serves as an answer to comments on methods developed by the senior author for analyzing the preconsolidation pressure of clay deposits.

General Description of the Boston Clay Deposits.—The clays underlying Greater Boston are probably among the best explored and most thoroughly tested clays for foundation purposes. They have served as a proving ground for much research in soil mechanics and also for the development of soil sampling methods. In particular, the explorations for the two buildings described in the paper have assisted greatly in improving soil sampling tools and methods. This development culminated in the best sampling of this clay so far accomplished—namely, the samples taken during the explorations for the Mystic Power Station of the Boston Edison Company during 1941–1942. Because considerable trouble in undisturbed sampling was still experienced at the sites of the two buildings referred to in the paper, for which the explorations were conducted from 1936 to 1938, the results of explorations at all three sites will be presented. For the sake of brevity the sites will be referred to as follows:

---

[76] Associate Prof., Civ. Eng., Graduate School of Civ. Eng., Harvard Univ., Cambridge, Mass.

[77] Asst. Prof., Soil Mechanics, School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

Site	Building
I	Liberty Mutual Insurance Company Building
II	New England Mutual Life Insurance Company Building
M	Mystic Power Station

At all three sites the soil profiles are similar. Directly overlying rock is glacial till, sand and gravel, deposited and pre-loaded by glaciers. Overlying this material is a thick stratum of inorganic clay, water deposited during the retreat of the last ice sheet. This clay is a fairly continuous formation, but contains many lenses of sand and local pockets of gravel and boulders. Except for the upper hard crust, which at one time was exposed to surface conditions (drying and oxidation), the color of the clay varies between darker and lighter shades of olive green and gray. The term "blue clay" should not be taken literally. This simplified color designation has become part of the name of this clay.

The effect of the drying of the surface has produced the so-called "hard yellow clay" which again is an exaggerated color designation. Beneath this hard clay there is a gradual transition into stiff and medium and finally into soft blue clay.

Because of irregular erosion, the thickness of the hard yellow and stiff blue clay is variable. Over limited areas these materials are entirely absent, and in some places sand has been deposited in their stead.

After this period of drying and erosion, the land subsided again during the past five thousand years or so, bringing the surface to an average El. −20 mean low water (M.L.W.). On top of the clay a deposit of organic silt was formed, averaging 20 ft thick, and this was finally covered by artificial fill as thick as 20 ft. The thickness of the soft clay averages between 50 ft and 100 ft. This clay deposit is chiefly responsible for building settlements in Greater Boston.

The physical characteristics of good undisturbed samples of the blue clay vary between a very brittle material which upon remolding becomes extremely soft and sticky and a material that shows no brittleness, upon remolding softens much less, and does not become sticky. To distinguish between these extreme types, they will be referred to as "brittle" and "plastic" types, respectively.

The brittle type is easily sampled with thin-wall samplers,[78] the clay practically "melting" under the pressure of the cutting edge. Such samples show no bending of the bedding planes, although improper sampling may result in shear planes. Even rather stiff soils of this type can be sampled with little pressure and can be easily pushed out of the tubing. This type usually has a slightly more greenish color than the plastic type.

The plastic type offers much more resistance to sampling and to the removal of the samples from the tubes, although the samples may be only medium soft. Even with the best sampling procedure in bore holes some bending of the planes of stratification cannot be prevented. This type of clay often contains individual larger grains of sand and here and there a pebble. It is usually less regularly stratified, and sometimes mottled, as if at one time the deposit had been badly disturbed. Also, it is often found adjacent to boulder nests or lenses of sand or gravel.

---

[78] "Exploration of Soil Conditions and Sampling Operations," by H. A. Mohr, *Soil Mechanics Series No. 21*, Harvard Univ., Cambridge, Mass., 3d Ed., November, 1943.

In addition to these extreme types there are intermediate types which exhibit some brittleness. Since the grain-size distribution and the liquid and plastic limits of all types vary within the same general range, no differentiation is possible by means of such classification tests. The raw material seems to be the same. Differences in intensity of flocculation at the time of deposition (in other words, differences in the structure of the clay) are assumed responsible for the wide variations in physical characteristics. An attempt was made to determine from the contents of Foraminifera whether individual samples are marine or fresh-water deposited. H. C. Stetson, of the Department of Paleontology at Harvard University, Cambridge, Mass., found Foraminifera in all types of the Boston blue clay, but very sparse (only three or four individuals to about 40 g of dry clay), indicating that the deposit might have been laid down in an estuary at some distance from the sea, under brackish conditions.

The distribution of plastic and brittle type clays over Greater Boston shows no regularity. For example, in the Lynn area the highly brittle type, with a natural water content at about the liquid limit, is present at El. −40, whereas at the same elevation in other areas the less brittle types are usually found. (Elevations refer to Boston city base unless they are marked M.L.W. Boston city base is 6 in. below M.L.W.) In the deep borings at site M the plastic and brittle types alternate. At one location in Cambridge the hard yellow clay was distinctly of the brittle type. Yet, in spite of its hardness, it was considerably above the plastic limit and became medium soft upon remolding. From these and other observations it was concluded that, at the same time and with the same raw material, brittle types of clay would appear to have been deposited in some areas and plastic types in others.

At several sites the brittle type predominated near the bottom of the soft clay stratum. This order of occurrence may be responsible for earlier conclusions that the Boston clay at greater depths is extremely soft and not yet fully consolidated. Actually, good undisturbed samples of such brittle clay, from depths of about 100 ft, have been found to have compressive strengths up to 2 tons per sq ft; but the use of thick-wall samplers, and the practice of unwatering the bore holes, creating great unbalanced stresses in the clay to be sampled, resulted in such serious disturbance to the samples that, upon recovery, they had to be classified as very soft. This condition, together with the fact that sampling operations become unwieldy and time consuming with great depths, has generally resulted in samples being increasingly more disturbed with greater depth. The use of thin-wall samplers of 2-in. and 3-in. diameter, operating in water-filled, cased bore holes, has eliminated most of these difficulties. Excellent undisturbed samples of brittle clay were obtained at site M from depths as great as 120 ft.

The practice adopted by boring contractors of classifying the consistency of the clay, according to the driving resistance of the sampling tube, as hard, stiff, medium, soft, and very soft, does not reflect the type of clay. The designation "very soft" may actually refer to a medium stiff, very brittle type of clay which "melts" under the concentrated pressure of the cutting edge.

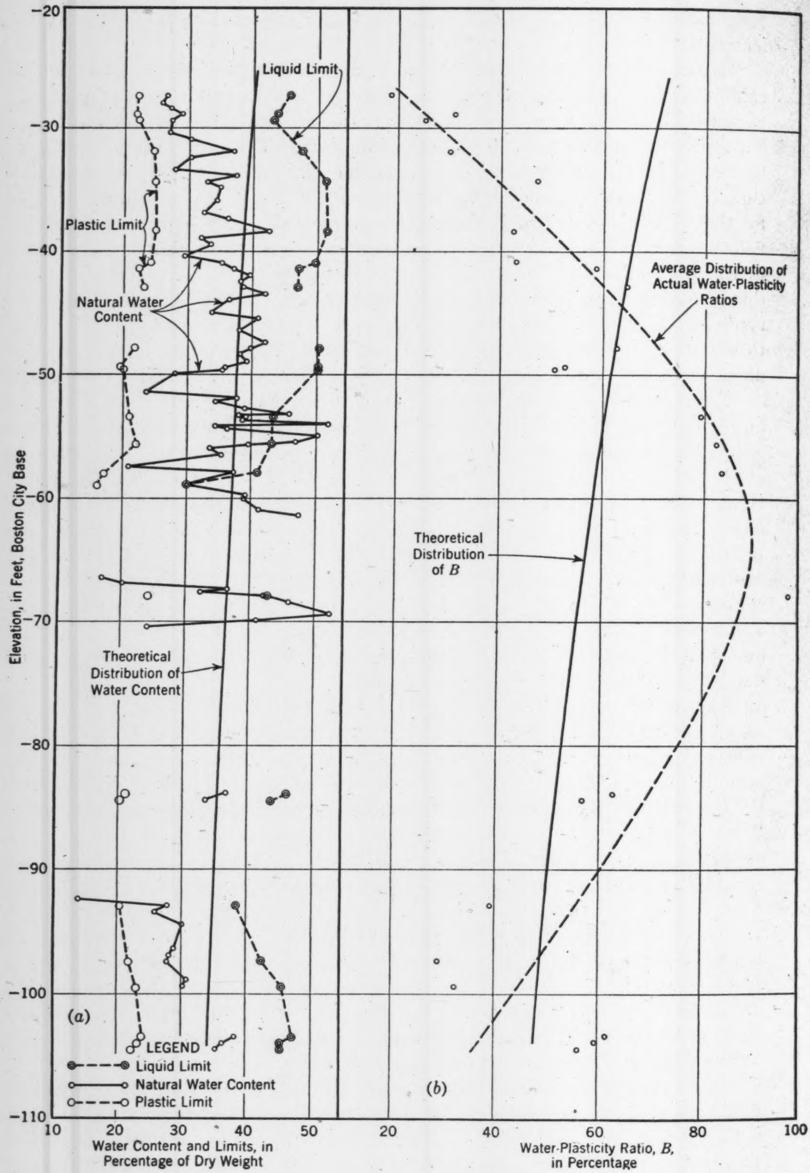
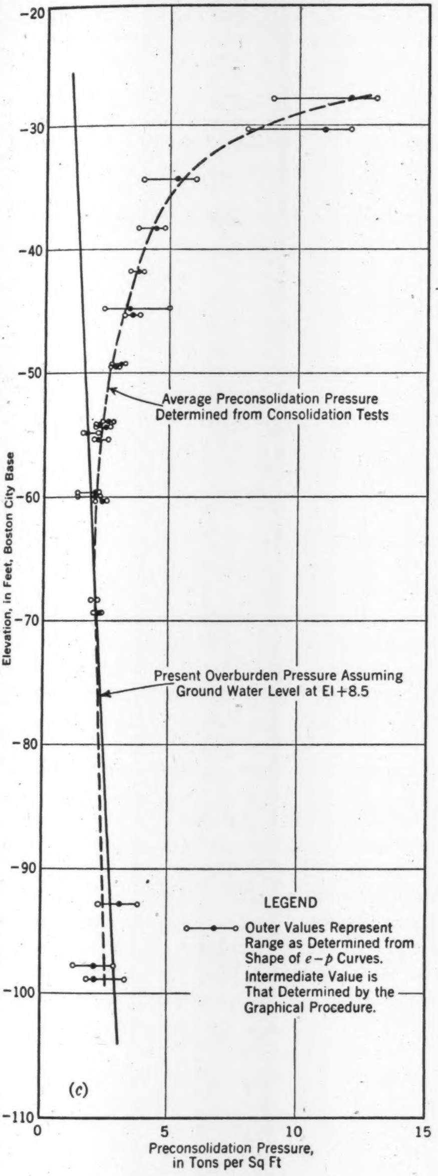
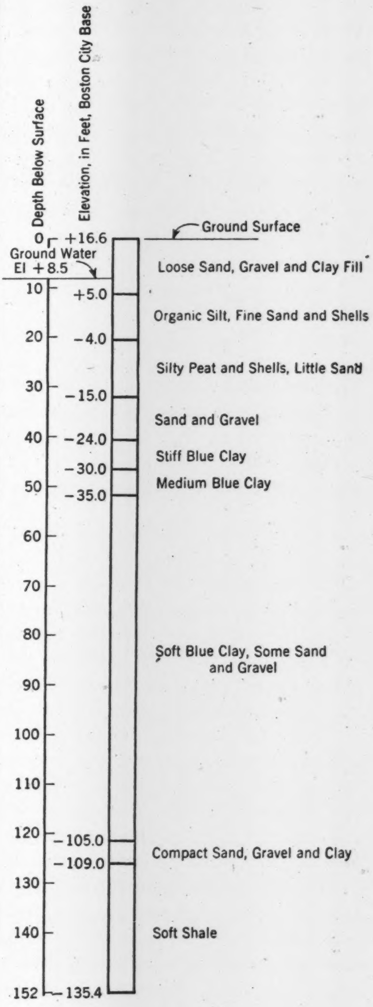


FIG. 33.—PROFILE OF w_n, P_w, L_w, B, AND



p_p FOR BORING NO. 3, MYSTIC SITE


Depth Below Surface
Elevation, in Feet, Boston City Base
Ground Surface
Ground Water El +8.5
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
152
+16.6
+5.0
−4.0
−15.0
−24.0
−30.0
−35.0
−105.0
−109.0
−135.4
Loose Sand, Gravel and Clay Fill
Organic Silt, Fine Sand and Shells
Silty Peat and Shells, Little Sand
Sand and Gravel
Stiff Blue Clay
Medium Blue Clay
Soft Blue Clay, Some Sand and Gravel
Compact Sand, Gravel and Clay
Soft Shale

(*d*) RECORD OF BORING NO. 3
MYSTIC SITE

Variations in Water Content and Plasticity in Boston Clay.—The great variations in natural water content can be analyzed by relating water content to the liquid and plastic limits.

From hundreds of limit determinations of samples obtained at many sites in the Greater Boston area the total range of variation for the liquid limit was found to be 30 to 76; and for the plastic limit, 16 to 33. Omitting the very exceptional values, an average range of 40 to 55 is found for the liquid limit; and one of 20 to 25, for the plastic limit.

For sites I, II, and M, for which very detailed soil investigations were made, the average arithmetic means are as follows:

Site	Liquid limit, L_w	Plastic limit, P_w
I	48.3	23.8
II	48.0	22.6
M	45.6	22.3

The over-all average for the liquid limit is 47; and for the plastic limit, 23.

The most complete record yet available in the Boston area for a deep boring is illustrated in Fig. 33, from site M. Fig. 33(*a*) shows the natural water content w_n, and limit values L_w and P_w, and Fig. 33(*b*) gives the so-called water-plasticity ratio (percentage),

$$B = \frac{w_n - P_w}{L_w - P_w} 100 \qquad (17)$$

which gives the relation of the natural water content to the limits. For $w_n = P_w$ this ratio is 0%, and for $w_n = L_w$ the ratio is 100%. Values greater than 100% indicate that the natural water content is above the liquid limit. In Figs. 33(*a*) and 33(*b*), the solid line denotes the theoretical distribution of water content w_n and B, respectively, computed from the average laboratory virgin compression curve, shown subsequently as curve T in Fig. 36.

For sites I and II the B-values are plotted in Fig. 34. In this diagram observations also cover a range in elevations for which no values are available in Fig. 33 because of loss of samples.

The spread in the B-values in Figs. 33 and 34 is not entirely due to variations in the type of clay but, to some extent, is the result of inaccuracies in correlating the water content and limit values.

Fig. 35 shows the results of the most recent tests of this type, determined from samples obtained from a boring in Boston Harbor. Special care was taken to make sure that water content and limits were determined from identical material, and also that lean and fat clay layers were not mixed. To check the latter, the different color shades were used as a basis for differentiation. When the tests for Figs. 33 and 34 were made, the importance of such extreme care was not fully realized.

Fig. 35 does not represent the full range of variations in water content, since every determination is the average for a height of several inches. By slicing the clay into thin layers, sometimes a value as low as 25% is obtained for a thin layer of light colored very silty material; and a value as high as 60%, for a dark olive greenish clay layer.

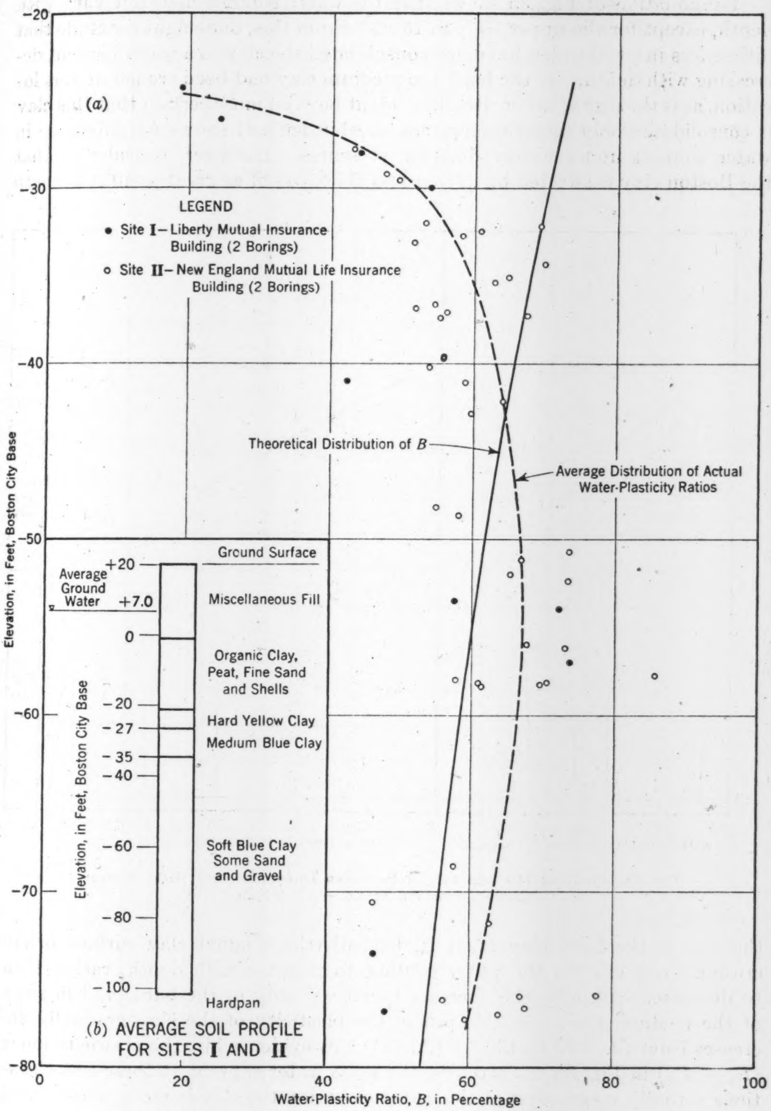


FIG. 34.—WATER-PLASTICITY VALUES FOR BUILDING SITES I AND II

Examination of Fig. 35 shows that the water content does not vary with depth, except for the upper 10 ft to 15 ft. From this, one might conclude that differences in overburden have not consolidated the clay to a water content decreasing with depth. If the hard and medium clay had been eroded at this location, as is the case at other sites, one might be even more certain that this clay is consolidated only under the present overburden and shows no difference in water content under widely differing pressures. However, considering that the Boston clay is affected by drying to a thickness of as great as 40 ft beneath the top of the hard clay (that is, beneath the original clay surface before erosion), one expects the water content to increase with depth, rather than to decrease. Actually this increase is noticed only in the hard, and in some of the medium clay. At this profile the plasticity of the clay gradually decreases from El. −23 to El. −40 M.L.W., as evidenced by the plasticity index ($L_w - P_w$) in Fig. 35. Therefore, the same water content at these two elevations actually means that at the upper elevation the clay is more consolidated than at the lower elevation. In this case, only a study of the relation of the water content to the limits—that is, of the water-plasticity ratio—can assure correct interpretation of the meaning of the constancy of the water content

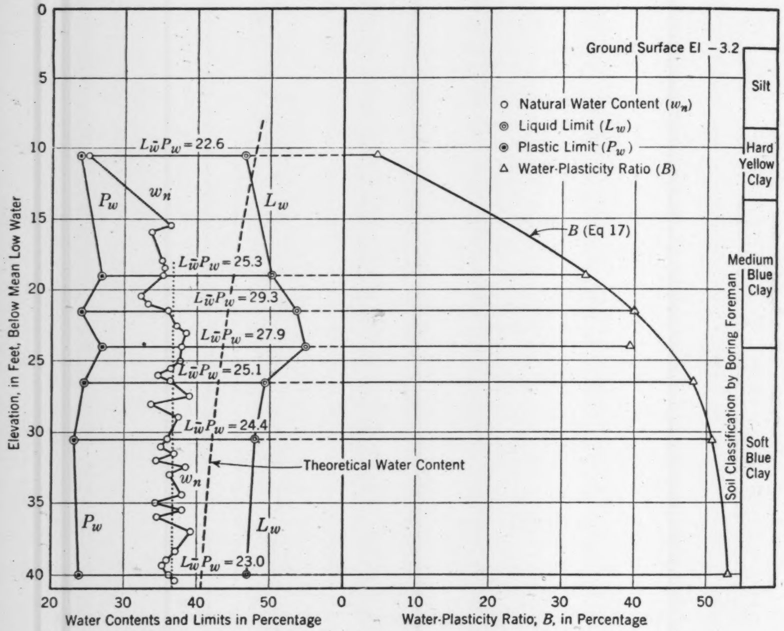


Fig. 35.—Borings 14A and 14B in Proposed Dredging Area, East Boston (Ground Surface at El. − 3.2 M.L.W.)

with depth. A study of the water content alone would lead to erroneous conclusions.

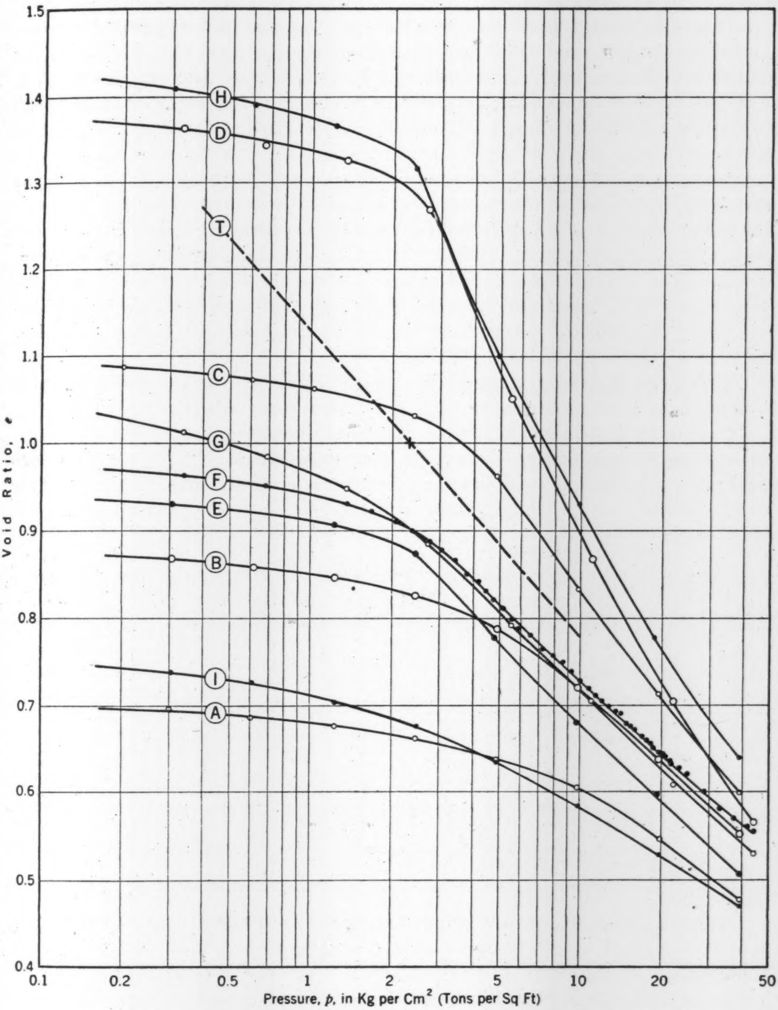


FIG. 36.—TYPICAL PRESSURE VERSUS VOID RATIO CURVES OF BOSTON BLUE CLAY FROM UNDISTURBED SAMPLE BORING NO. 3, MYSTIC SITE

Water Content of Hard Yellow Clay.—During remolding of an undisturbed sample the water content of the hard yellow clay may appear to be slightly below the plastic limit, because the clay crumbles before it can be molded into a thread. However, the plastic limit of the same sample, after it has been remolded with water for the determination of both limits, is actually slightly below the natural water content, with the corresponding B-value between 0 and 20%.

This discrepancy between the "feel" of the material remolded from an undisturbed state, and the "feel" of that which has been kneaded with the addition of water, has been noticed also for other clays near the plastic limit. Apparently, remolding of hard clay with the addition of water effects a thorough breakdown of the structure, which mere remolding at the natural water content will not accomplish.

Consolidation Tests and Preconsolidation Pressure for Boston Clays.—Fig. 36 shows typical pressure versus void ratio curves for undisturbed samples from the same boring for which other test results are assembled in Fig. 33. The preconsolidation pressure p_p for each curve, determined by the graphical procedure, as well as the probable range of p determined from the shape of the transition curve, is given in Table 1. Generally, the more brittle a sample, the sharper is the transition curve between the recompression and virgin branches, and the narrower is the estimated range for p_p.

The virgin curves are generally straight lines on a semilogarithmic plot, except those for samples with initial void ratios considerably greater than 1.0.

Curves D, E, F, and G, Fig. 36, were obtained from specimens only a few inches apart in elevation. Curve D corresponds to a specimen with a high natural water content, whereas the other three curves represent another type of clay with a water content approximately equal to the over-all average of this boring.

TABLE 1.—PRECONSOLIDATION PRESSURE IN KILOGRAMS PER SQUARE CENTIMETER (TONS PER SQUARE FOOT)

Curve	Laboratory No.	Depth (ft)	Elevation (city base)	Graphical method	PROBABLE RANGE	
					From:	To:
A	H75-3-4-4A	47.0	−30.4	11	8	12
B	H75-3-5-4A	51.0	−34.4	5.3	4.0	6.5
C	H75-3-7-3B	58.5	−41.9	3.8	3.5	4.0
D	H75-3-10-5A	70.6	−54.0	2.7	2.5	2.9
E	H75-3-10-5B	70.8	−54.2	2.4	2.2	2.5
F	H75-3-10-6A	71.0	−54.4	2.6	2.2	2.8
G	H75-3-11-6B	71.5	−54.9	1.9	1.7	2.3
H	H75-3-12-4B	87.0	−70.4	2.4	2.2	2.5
I	H75-3-14-4	109.5	−92.9	3.3	2.5	4.0
T	Average (e–p)-curve for the computation of the theoretical distribution of w_n and B (see Figs. 33, 34, and 35)					

Curve E, Fig. 36, was obtained from a standard test that lasted one week, whereas curve F was obtained from a long-time test that lasted 85 days. Comparison of the shape of the (e-p)-curves and of the preconsolidation pressures, obtained from them, indicates essentially the same results. These two tests, and similar comparisons on other clays, permit the conclusion that consolidation tests lasting several months give practically the same results as the standard tests performed in a few days.

Curve G shows a rather steep recompression branch and a flat transition curve to the virgin branch. This condition is usually caused by excessively distorted samples. For comparison the results are included, showing that the preconsolidation pressures obtained from this test are about 20% to 25% lower than those from the other three tests performed on specimens from the same elevation.

The preconsolidation values obtained from the twenty-four tests on undisturbed samples from this boring are plotted in Fig. 33(*c*), and for comparison the actual stresses under the present overburden load are shown. From the surface of the clay stratum to about El. −65 the clay appears to be consolidated by drying. Below this elevation there is a good agreement between the p_p-values obtained from the tests and the actual overburden pressure, indicating that the soft clay below about El. −65 may be considered consolidated under the present overburden load. In this case there is fairly clear geological evidence that overburden was never higher than at the present time.

The importance of great care in the performance of consolidation tests, if they are to be used for analyzing the preconsolidation pressure, cannot be overemphasized. Such tests should be supervised and checked carefully, since irregularities in loading, differences in the time allowed for each increment if not adjusted to the same period, etc., easily cause changes in the shape of the $(e–p)$-curves which may result in large errors in the derived p_p-values. Tests in which some or all increments were not allowed to stand long enough to develop the slope of the secondary compression curve clearly, as indicated by semilogarithmic plots of the time curves (see Fig. 37), should be discarded.

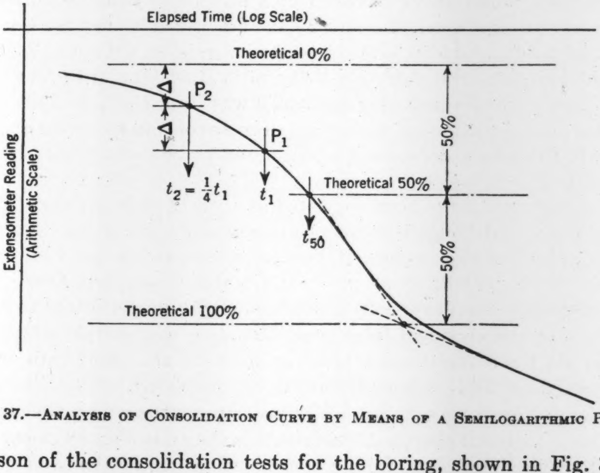


FIG. 37.—ANALYSIS OF CONSOLIDATION CURVE BY MEANS OF A SEMILOGARITHMIC PLOT

Comparison of the consolidation tests for the boring, shown in Fig. 33, with older tests on samples of Boston blue clay obtained with thick-walled samplers indicates that the general magnitude of the preconsolidation pressures, determined by the shape of the $(e–p)$-curves on less perfect samples, is still of the correct order of magnitude,[79] but that the error in the graphical procedure may increase to 50%.

---

[79] "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1155.

Comparison of Actual and Theoretical w_n-Values and B-Values for Boston Blue Clay.—Fig. 33 also shows the theoretical decrease of water content w_n and water-plasticity ratio B with depth, computed on the basis of an average laboratory compression curve, shown in Fig. 36 as the broken line T ($C_c = 0.355$, through $e = 1.0$, or $w_n = 36.3\%$, and $p = 2.3$ kg per cm²).

For the zone below the influence of drying, the theoretical decrease in water content from a depth of 80 ft (El. −63.4) to a depth of 120 ft (El. −103.4) is from 36.8% to 34.4%. This decrease is so small that it is completely overshadowed by the actual variations in water content. For the same depths the theoretical B-value (assuming $L_w = 47$ and $P_w = 23$) is reduced from 58% to 48%. The actual decrease in B-value is much greater than the theoretical decrease.

Investigations of Other Clays.—The Laurentian clays permit study of a large range of preconsolidation pressures. The hard Laurentian clays, investigated by Irving B. Crosby, Affiliate Am. Soc. C. E., and the senior author in 1930,[80] are preconsolidated under pressures of from 7 to 15 tons per sq ft, whereas the soft Laurentian clays investigated in recent years by the U. S. Army Engineers are consolidated under pressures of only 1.0 to 1.5 tons per sq ft. Both types of clay are of marine origin and are extremely brittle in the undisturbed state. When remolded, the hard clay becomes very soft and sticky, and the soft clay changes practically into a fluid. Grain-size curves and the Atterberg limits fall in the same ranges. The liquid limits for the hard clay were found to lie between 29.8 and 53.4; and the corresponding plasticity indexes, between 12.5 and 29.8. For the soft clay the liquid limits range between 36.5 and 55.0; and the plasticity indexes, between 12.3 and 28.4.

For the hard clay the water-plasticity ratio B was found to range between 40% and 100%; for the soft clay the range was from 150% to 240%.

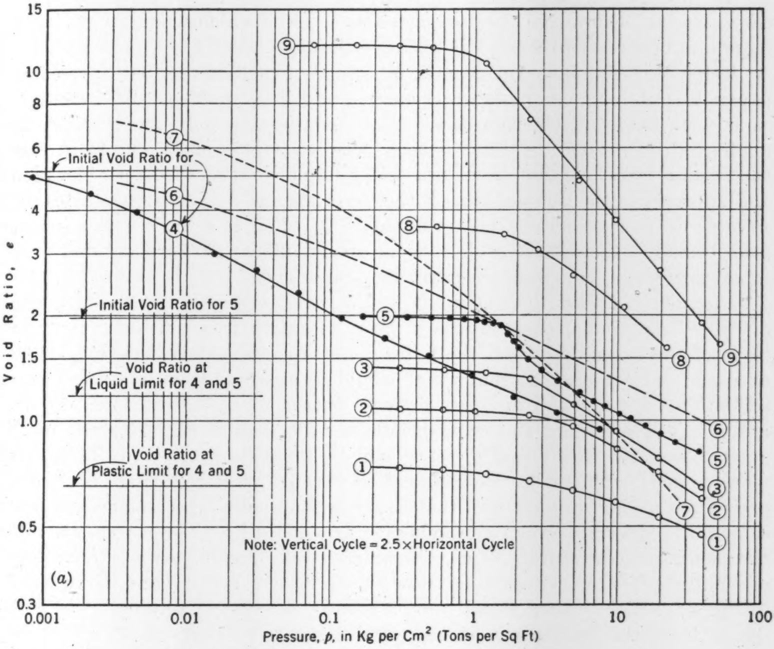
Comparing the theoretical and actual decrease of water content and B-value with depth, the actual decrease is again found to be considerably larger than the theoretical decrease.

Such a clay must have been deposited at a water content very much above the liquid limit. Although it is not possible to reproduce in the laboratory the very slow growth of such a deposit, one can arrive at least at a lower limiting void ratio for the condition at which the material was deposited. For this purpose dispersed samples of soft Laurentian clay were permitted to flocculate. Irrespective of the chemical treatment, including one sample which was not treated at all, the deposits were found to have the same void ratio of about 5 after flocculation and sedimentation were completed. Consolidation tests were performed on these samples. The pressure versus void ratio curve of one of these tests (without chemical treatment) is shown in Fig. 38 as curve No. 4. The relationship determined from a consolidation test on the same clay in the undisturbed state is shown by curve No. 5. Thus, even with a void ratio of about 5.0 at the beginning of the test, the flocculated sample yielded an (e–p)-curve considerably below the curve for the undisturbed material. Probably the starting void ratio under the extremely slow natural growth of such a deposit is considerably greater than that obtained by rapid flocculation in the laboratory.

---

[80] Report on an Extensive Investigation of Slides in Eastern Canada, made by A. Casagrande to Irving B. Crosby, Cons. Geologist, January, 1931.

Curves Nos. 6 and 7 are speculations on the possible true virgin $(e\text{–}p)$-curve according to which this clay might have been consolidated by nature. There is some doubt about the meaning of the sharp initial curvature for curve No.5 after the preconsolidation pressure has been exceeded. Because of the imperfect fit of the sample in the consolidation ring, a lateral strain could be introduced which might result in a partial breakdown of the clay structure. However, three different tests on the same material gave almost identical results. One test was slow (70 days). If the shape of the curve beyond this



(b) IDENTITY OF CURVES IN (a)

No.	Laboratory No.	Description of material
1	H75-3-14-4	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve I)
2	H75-3-7-3B	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve C)
3	H75-3-12-4B	Undisturbed, soft Boston blue clay from site M (see Fig. 36, curve H)
4	H32-12-F2	Flocculated Laurentian clay
5	H32-12-C2	Undisturbed, soft Laurentian clay; Massena, N. Y.
6	....	Hypothetical virgin compression curve for soft Laurentian clay
7	....	Hypothetical virgin compression curve for soft Laurentian clay
8	H29-2-2CP1	Undisturbed, organic silt-clay, Boston
9	H28-1B-CP1	Undisturbed, volcanic clay, Mexico City

FIG. 38.—COMPARISON OF PRESSURE VERSUS VOID RATIO CURVES BY LOGARITHMIC PLOTTING

initial "disturbance" should continue about parallel to the natural virgin curve, one obtains curve No. 6, which could be assumed to begin at a void ratio of about 5. Taking into consideration the much greater drop of w_n-values and B-values with depth than is derived from the laboratory virgin (e–p)-branch, the natural virgin curve must be much steeper than either curve No. 5 or curve No. 6. Curve No. 7 is suggested as a curve which would explain this more rapid decrease of w_n and B with overburden pressure.

Ralph B. Peck, Jun. Am. Soc. C. E., is one of the few who have presented papers [81] containing complete water content and limit profiles. His investigations were made on Chicago clay. When considering only the clay that is unaffected by drying, his data[82] reveal, between depths of 25 and 45 ft, that the natural water content starts nearer the liquid limit at the upper elevation and approaches the plastic limit at the lower elevation. In both profiles the B-value drops from about 70% to about 30%, which is again a much greater drop than the theoretical drop of from about 70% to 60%, computed from laboratory virgin compression curves for similar clays.

Discussion of Variations of w_n, B, and p_p with Depth.—From the foregoing evidence, and similar observations at other localities, it is concluded that, for clays of the types described, the natural water content decreases with depth at a rate at least equal to that which would be expected from laboratory consolidation tests on undisturbed samples. In the foregoing examples the decrease in water content and water-plasticity ratio with depth is actually much greater than the theoretical decrease, which differs from Professor Terzaghi's views on this subject.[83,84] No explanation for this greater decrease is offered, except the tentative suggestion that the magnitude of the secondary compression, which develops over thousands of years, might increase in proportion to the overburden pressure. Thus, differences would result between laboratory and natural (e–p)-curves such as those shown by curve No. 5 and curve No. 7 in Fig. 38, which in turn would cause a greater decrease of water content with depth than the laboratory curve would indicate. However, it should be emphasized that very slow laboratory consolidation tests have so far shown rates of secondary compression along the virgin curve which are independent of pressure.

It is unlikely that the settlements of buildings are governed by (e–p)-curves as steep as curve No. 7 along which nature may have consolidated the clay over long periods. Rather, the (e–p)-curves, obtained from laboratory tests on good samples, seem to form a reliable basis for settlement analyses, provided that the preconsolidation pressure p_p can be determined with sufficient accuracy from laboratory tests, or geological evidence, or both. Without any knowledge of p_p one is forced to assume that the clay is consolidated only under the present overburden. The settlements so computed may be far too large.

The necessity of obtaining as accurate information as possible on the preconsolidation characteristics of clay deposits as a prerequisite for settlement analyses cannot be overemphasized, and the question of how to determine the true p_p is of major importance. Referring to the procedure for determining p_p from consolidation tests, Professor Terzaghi states that he has

---

[81] "Earth Pressure and Shearing Resistance of Plastic Clay," A Symposium, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1008.

[82] *Ibid.*, Figs. 19 and 20, pp. 1009 and 1110.

[83] "Undisturbed Clay Samples and Undisturbed Clays," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., July, 1941.

[84] Discussion by Karl Terzaghi of the paper, "Relation of Undisturbed Sampling to Laboratory Testing," by P. C. Rutledge, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1198.

> "* * * ceased to give any weight to the numerical results because both the published records and his own experience indicate that the difference between the computed and the real value for samples from the same drill hole may range between zero and several hundred per cent of the computed value. If the maximum preconsolidation load p_o exceeded the present overburden pressure p_1 by less than a few tons per square foot, the graphical method does not indicate whether p_o exceeded p_1 at all. In such instances, the writer relies entirely on geological evidence."

In the experience of the present writers it is just those cases, in which the maximum preconsolidation load exceeds the present load by a few tons per square foot, that require greater accuracy in analysis than is generally provided by geological evidence. The writers believe that this greater accuracy can be furnished by properly conducted consolidation tests on properly taken samples. As to geological evidence, Professor Terzaghi himself states: "* * * but, as a rule, geological evidence regarding the thickness of vanished strata leaves a wide margin for interpretation."

Furthermore, the thickness of vanished strata alone does not determine the preconsolidation pressure of clays. Except for very recent deposits, most clays have experienced one or more periods of drying which geological evidence will usually not reveal. Many apparent disagreements between p_p as determined from reliable tests and from geological evidence are due to preconsolidation of the clay by drying. Such differences can easily be "several hundred per cent of the computed value." In any event geological evidence would not inform the investigator of the degree and depth to which the clay was preconsolidated by drying. Where the telltale, hard, discolored crust has been eroded, only test results such as those shown in Fig. 33, and those obtained by any other testing methods that may be developed for this purpose, will reveal the existence and extent of consolidation due to drying.

If the large deviations between the true and computed p_p-values are caused by poor samples, or by poorly conducted consolidation tests, the procedure for determining p_p from the test results should not be blamed. Unfortunately, much unreliable information has been derived from tests made on badly distorted samples. Therefore, it is rather treacherous to rely on published data which are several years old, unless one has had an opportunity to check, scrupulously, all the many important details of sampling and testing which may cause erroneous results. It is regrettable that so much data obtained from soil testing of the past, including extensive material in the writers' files, must be eliminated as a source of reliable information because of the use of excessively distorted samples.

Further improvements or development of new methods for analyzing the pre-loading of clays from their stress-deformation characteristics are desirable. The loading history of a clay is a latent picture, somewhat like an undeveloped negative. The success of a settlement analysis depends largely on the ability to develop the picture.

Consolidation of clay by drying affects the magnitude of settlements of buildings in two ways: (1) Reduction in settlements compared to those of a clay of similar thickness that does not contain zones of dried clay; and (2) reinforcement of the clay stratum in the horizontal direction which assists in reducing settlements due to lateral deformation. The latter effect is similar to the reinforcement by layers of coarse-grained soil. Thick flood-plain deposits with their alternating layers of clay, sand, and silt, and with many of their clay layers well precompressed by drying, may show relatively small settlements as compared with the total thickness of clay in such deposits.

The idea of hydrostatic excess pressures in rock underlying clay was advanced by the senior author in 1936 as a tentative explanation for discrepancies in water-content distribution and consistency of so-called undisturbed samples of Boston clay. Observations have indicated that the water pressure in the rock underlying the clay corresponds to elevations above high tide.

It is difficult, if not impossible, to determine the intensity of the true water pressure in the rock underlying clay by water-level measurements in ordinary bore holes. Only by the use of special installations such as those devised for pore-water pressure measurements in clays can the pressure intensity be measured with sufficient accuracy.

Even these measurements must be viewed very critically because of the many technical difficulties involved in eliminating large errors. Not until perfectly reliable methods are developed, will it be possible to prove or disprove for a given site the existence of excess hydrostatic pressures in rock underlying clay.

Beneath extensive impervious clay deposits considerable excess hydrostatic pressure may be a natural and frequent phenomenon that may seriously affect the condition of consolidation of such clay deposits. Any relief in the excess hydrostatic pressure (as produced, for example, by wells) will result in consolidation of the clay under its own weight. Large changes in water pressure beneath clay deposits may result in spectacular settlements, such as those observed in the Santa Clara Valley in California.[85]

In this connection, the writers note with interest Mr. White's remarks, revealing a rather emphatic belief in the existence of such pressures.

Logarithmic Versus Semilogarithmic Plots of Pressure Versus Void Ratio Curves.—The remarks in the discussion by the Soil Mechanics Committee of the Los Angeles Section on the logarithmic plot of pressure versus void ratio curves touch a question which has been debated for a number of years.

In 1931 the senior author started plotting the (*e*–*p*)-curves of consolidation tests on undisturbed clay samples on semilogarithmic graph paper, chiefly because he found that it facilitated the estimation of preconsolidation pressures. As the results of tests on undisturbed samples on different clays accumulated, he noticed that the virgin compression branch plotted as a straight line for most of these clays. Only for clays with a natural void ratio considerably greater than 1.0, and with tests carried to large pressures, did he find that the virgin branch was more or less curved.

---

[85] "Ground-Water, Salt-Water Infiltrations, and Ground Surface Recession in Santa Clara Valley, California," by C. F. Tolman and J. F. Poland, *Transactions*, Am. Geophysical Union, 1940, Pt. I, pp. 23–35.

Shortly thereafter, investigators in Europe started independently to use the semilogarithmic plot as well as the logarithmic plot for presentation of $(e\text{–}p)$-curves.

Within the narrow working range of pressures which apply to the settlement analysis of a given structure, the virgin compression curves are practically straight lines, both on semilogarithmic and logarithmic plots. Then the choice may depend on whether the mathematical expression of a logarithmic function, or of a power function, is easier to use in a particular problem. The writers consider the logarithmic equation more easily handled in settlement analyses.[7]

The logarithmic plot has the advantage over the semilogarithmic plot in that it facilitates assembling on one diagram the results of soils with widely different compressibilities, as illustrated in Fig. 38. A number of sedimentary soils, which are curved on this plot, show straight-line virgin curves on a semilogarithmic plot. On the other hand, the clay from Mexico City, Mexico, which is not an ordinary clay but the result of chemical decomposition of volcanic ash, is a straight line on the logarithmic plot.

A minor disadvantage of the logarithmic plot is the necessity of using a much larger cycle size for the void-ratio scale than for the pressure scale to enable sufficiently accurate plotting. When plotting curves on symmetrical log paper, the $(e\text{–}p)$-curves of ordinary clays are so extremely flat that they would always give the impression of being straight lines.

Analysis of Time Curves from Consolidation Tests by Means of Semilogarithmic Plots.—Professor Krynine requested further information on this method. It consists of three parts: (1) Determination of the "theoretical zero," (2) determination of the boundary between primary and secondary compression, and (3) computation of the coefficient of permeability.

The necessity for finding a "theoretical zero" was realized by the senior author in 1928, after he discovered that the air content in soils invalidates the original procedure developed by Professor Terzaghi for analyzing time curves.[86] The method for finding the theoretical zero is based on the assumption that the upper half of the time curve is approximately a parabola, and consists of the following steps (on arithmetic and semilogarithmic plots): Choose an arbitrary point P_1 on the time curve, within the estimated first 50% consolidation. Divide the time, t_1, of this point by 4 and find the corresponding point P_2 on the curve. The difference in ordinates, Δ, of these two points is transferred above P_2, as illustrated in Fig. 37. This procedure is repeated for several points and the horizontal line passing through the average of the resulting locations is assumed to be the "theoretical zero coordinate."

With this modification, the senior author continued using the Terzaghi procedure, but in 1930 discovered very large discrepancies between the coefficient of permeability computed from the time curves and the directly measured values for undisturbed clays. After considerable experimentation with various types of plots, plotting the time on a log scale was found to permit visual differentiation between the primary and secondary compression. Also the intersection of the tangent to the inflection point and of the straight-line continuation of the secondary compression branch was found to correspond approximately to a theoretical consolidation of 100%.

---

[86] "Principles of Final Soil Classification," by K. Terzaghi, *Public Roads*, Vol. 8, 1927, p. 46.

Since both the theoretical zero point and the theoretical 100% point are subject to errors, the theoretical 50% point is considered least affected by these errors and was chosen instead of the 90% point in Professor Terzaghi's original procedure. Then for the time, t_{50}, corresponding to this theoretical 50% point, the coefficient of permeability can be computed from the following formula, based on the Terzaghi theory of consolidation:[87]

$$k = \frac{a_v \gamma_w H^2}{20 (1 + e_m) t_{50}} \qquad \text{(18)}$$

in which $a_v = \frac{p_2 - p_1}{e_1 - e_2}$ = coefficient of compressibility obtained from $(e\text{-}p)$-curve for increment; γ_w = unit weight of water; H = thickness of layer drained on both sides; $e_m = \frac{e_1 + e_2}{2}$ = average void ratio for the increment; and t_{50} = time for theoretical 50% consolidation, from Fig. 37.

Using the semilogarithmic plot for time curves, the writers have always been able to check, with satisfactory accuracy, the results of direct permeability tests with the k-values computed from the time curves, provided that the slope of the secondary compression branch was not so steep as to obliterate the bend in the curve indicating the transition from primary to secondary compression. This obliteration happens frequently in the case of peaty soils, but in inorganic clays only when very small load increments are used. The latter reaction is due to the fact that the slope of the secondary compression branch on a semilogarithmic plot is independent of the magnitude of the load increment; in other words, the greater the load increment, the sharper the line of demarcation between primary and secondary compression.

The use of the square root scale for the time curve, as introduced by D. W. Taylor, Assoc. M. Am. Soc. C. E., assists in determining the theoretical zero coordinate, but does not eliminate the difficulty of distinguishing between primary and secondary compression. The large discrepancies between the k-values computed from arithmetic or square root plots and the directly measured values are due to this difficulty.

In addition to more accurate computation of the coefficient of permeability, the semilogarithmic plot has an advantage over other plots in that the shape of the theoretical curve remains unchanged regardless of changes in the variables in Eq. 18. Once one is acquainted with the characteristic shape of the theoretical curve, a glance at an observed curve on a semilogarithmic plot will reveal numerous important details regarding the primary and secondary compression that cannot be seen in the other plots. For example, it will show whether the time allowed for consolidation has been sufficient, will furnish an estimate of the degree of accuracy of the computed k-values, or will show whether the secondary compression obliterates the primary compression to such an extent that it is not possible to compute k.

When performing consolidation tests one should always plot the time curves on semilogarithmic paper as consolidation progresses, to enable proper control of the test. Without such a precaution many tests are ruined by allowing insufficient time for consolidation.

---

[87] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., 1943, p. 274.

STRESS DISTRIBUTION IN CLAY AND HOOKE'S LAW

The omission of stress diagrams showing the results of computations of stress distribution in the clay beneath the buildings was not intended to convey the impression, gained by several discussers, that the writers consider this subject of no importance. In the settlement analyses of the two buildings described, extensive computations were made to determine the stress distribution in the clay for different designs.

A number of papers in *Proceedings* in the past few years have described methods for making these computations. Since the results from all of these methods are sufficiently accurate for practical purposes, the choice of method is a matter of getting results with the minimum effort. For the analysis of building II, the junior author developed a set of influence charts, both for isotropic and anisotropic soils, which proved useful.[88, 89]

The distribution of stress in clay is probably more affected by anisotropic properties of such deposits than by deviations from Hooke's law. In the design of building II, various assumptions of anisotropy were made.[90]

Mr. Julian has presented a clear and scholarly review of those aspects of the theory of elasticity involved in the computation of stresses and settlements. His discussion reveals several interesting suggestions that should be of assistance to those desiring more information on this subject. Poisson's ratio of a given clay is not a constant quantity. For the purpose of computing the initial stress distribution clay is usually assumed to be a perfectly incompressible material.

The discussion of the Soil Mechanics Committee of the Los Angeles Section contains two conflicting statements on the question of the validity of Hooke's law. On the one hand, the committee proposes to expand the writers' statements on the limitations of the validity of Hooke's law to various types of soils, as follows: "Hooke's law does not apply to any soil." On the other hand, it proposes that shear stresses under rigid footings should be computed by means of the theory of elasticity, which presupposes the validity of Hooke's law. In addition, the shear stresses as computed for a uniform loading on a perfectly flexible footing are assumed to hold also for rigid footings. According to the theory of elasticity, infinite concentrations of shear stresses will occur under a rigid footing and a statement that the maximum shear stress will be equal to p/π is an unwarranted assumption. Either the material is almost perfectly elastic, in which case the maximum shear stress will be much greater than p/π; or the material is capable of making plastic readjustments where stresses are high, and then the maximum shear stress may be smaller than p/π. Fortunately, most soils have the property of distributing stresses from such parts where the shear strength is approached, to adjacent parts of the soil where shear stresses are moderate. This action can be compared with the manner in which steel adjusts stress concentrations which occur in almost every steel

---

[88] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering (pp. 265–294).

[89] One of these influence charts is reproduced in "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 485.

[90] "A Problem of Elasticity Suggested by a Problem in Soil Mechanics: Soft Material Reinforced by Numerous Strong Horizontal Sheets," by H. M. Westergaard, *Contributions to the Mechanics of Solids*, Stephen Timoshenko 60th Anniversary Volume, 1938.

structure. Just as the structural engineer relies on this property of steel, so must the foundation engineer rely on the capability of soils to release high stress concentrations.

PROPERTIES OF COHESIONLESS SOILS AFFECTING SETTLEMENTS AND BEARING CAPACITY

Have Simplified Rules a Place in Foundation Engineering?—In his contacts with the profession, the senior author has received frequent requests for a simplified presentation of the results of soil mechanics to serve as a guide to engineers who have neither the time nor the opportunity for a detailed study of soil mechanics. It cannot be denied that even today many architects and engineers are guided by the misleading concept that each soil has a constant bearing capacity, independent of the size and depth of the loaded area, and of the type of structure (see most handbooks and building codes). The designers are not entirely to blame if many building foundations are still designed by such completely inadequate, although age-old, rules.

Simplified formulations of the findings of soil mechanics have an important place in the design of building foundations and are practicable for coarse-grained soils. These formulations should convey at least qualitative information on the influence of possible variations in the design—for example, width, rigidity, and depth of loaded areas for the pertinent range in which such quantities ordinarily vary in building foundations. However, settlement analysis of buildings on soft soil does not as yet seem ready for such simplification.

In the first part of the paper a number of simplified formulations are presented concerning the behavior of loaded areas on cohesionless soils. These statements were included in an attempt to provide information which might be useful chiefly as a qualitative guide to the designer. Valuable comments were contributed on several of these statements.

(*a*) Discussers object to assuming zero pressure along the edge for rigid, loaded areas on the surface of cohesionless soil because no measurements by pressure cells have ever indicated zero pressure. It would require extremely small pressure cells to measure the pressure along the edge; but extrapolation from measurements made in the interior of such loaded areas has indicated practically zero pressures along the edge. In addition, from the strength characteristics of a cohesionless soil it follows logically that for the zero confining pressure at the edge the other stresses at this point must also disappear.

It was also objected that building footings are always extended to a certain depth below adjacent ground surface. However, there is a definite trend toward placing foundations for smaller buildings directly on the surface of a sand or gravel fill. This is also true of foundations for tanks. Therefore, a designer should know that zero edge pressure is a possible extreme limiting condition which he may have to consider.

In support of his claims that "There is no justification for the assumption that the intensity of pressure at the edge of a rigid area is zero," Mr. Feld refers to tests by Professor Converse.[18] These tests were not performed on the surface but at a depth of 4 ft. Furthermore, the soil was not cohesionless, but is described as "* * * a clay-loam, according to the classification of the

U. S. Bureau of Public Roads, and was made up of 29 per cent clay, 30 per cent silt, and 41 per cent sand."[91]

(*b*) In regard to pressure distribution on a rigid area supported at great depth, Mr. Feld states: "The uniformity of base pressure in pier foundations is the result of the rigidity of a deep footing rather than of the depth of the footing." Rigidity only insures uniform settlement. In the case of cohesionless materials, there is adequate proof that deformation does not occur in accordance with Hooke's law; therefore a uniform settlement does not insure a uniform base pressure at any depth.

The comments of Messrs. Terzaghi, Krynine, and Freeman on statement (*b*) are pertinent. Construction procedure, disturbance of the soil, and flexibility of the contact surface will result in considerable deviation from a uniform contact pressure. In most cases it is not possible to evaluate the influence of all these factors, and the assumption of a uniform contact pressure usually will be found to be on the safe side.

(*c*) Mr. Feld suggests rewording the statement on increase of ultimate bearing capacity with width of footing "* * * to the effect that the unit bearing is independent of the width of the loaded area." This is just the opposite of the writers' statement.

Professor Terzaghi states that the increase of ultimate bearing capacity approximately in direct proportion to the width of the loaded area "* * * does not apply to sands whose angle of internal friction decreases appreciably with increasing pressure." In the writers' experience, the angle of internal friction for a given sand can be assumed substantially constant in the range of pressure intensities encountered in normal building loads.

(*d*), (*e*), and (*f*) The ultimate bearing capacity, q_o, on the surface of a mass of cohesionless soil can be determined by load tests, keeping in mind that it increases approximately in direct proportion to the width of the area. By placing the same loaded area at increasing depths, h, the ultimate bearing capacity, q_h, will increase rapidly as compared with the value of q_o. This increase is due to a greater confinement of the soil underlying the area and to side friction. Thus,

$$q_h = q_o \left[1 + f(h)\right] \qquad \text{(19)}$$

in which the function of depth, $f(h)$, determines the increase with depth. The senior author has generally assumed a linear increase with depth, and in 1934 he was instrumental in introducing a modest linear increase into a proposal for a new building code for Boston.[92] More recently he has been led by observations in practice to consider a linear increase too conservative for shallow foundations.

For deep foundations the linear increase always applies, but in the writers' opinion the ultimate bearing capacity becomes so large that other factors than the ultimate bearing capacity control the design. For example, if a load test on an area 2 ft by 2 ft on the surface of a sand shows an ultimate bearing capacity of 4 tons per sq ft, the ultimate bearing capacity at a depth of only 4 ft would be more than 20 tons per sq ft; and, at a depth of 25 ft, more than 100 tons per sq ft, assuming a linear increase according to the conservative Eq. 22 which follows. Therefore, any simplified statement on the increase of ultimate bearing capacity with depth should apply primarily to comparatively shallow depths.

---

[91] *Civil Engineering*, April, 1933.

[92] "Proposed New Boston Building Code," by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., 1936, Vol. II, p. 311, paragraph (*d*).

The senior author considers Eq. 3 in its general form the best available expression for the pertinent shallow range. Professor Terzaghi considers it applicable for the range from zero depth to a depth equal to twice the width of a footing. He suggests that the linear formula, Eq. 4, represents the part of Eq. 3 within the range $1 < \frac{h}{R} < 4$. A better agreement with Eq. 3, including validity for the important range from the surface to a depth $h = R$, is afforded by the following equation in which the increase with depth is a parabolic function:

$$q_h = q_o \left[1 + \left(\frac{h}{R} \right)^{1.35} \right] \ldots\ldots (20)$$

In general form this equation may be written

$$q_h = q_o \left[1 + \left(\frac{h}{R} \right)^{m} \right] \ldots\ldots (21)$$

The exponent m depends largely on the soil properties and for shallow foundations varies from values larger than 1.0 to almost 2.0.

Most of the discussers expressed their belief in a linear increase of ultimate bearing capacity with depth, which is undoubtedly the simplest rational formulation. The question arises as to what form of linear increase would be suitable for safe application with a minimum of soil investigations. The following linear equation—

$$q_h = q_o \left(1 + \frac{h}{R} \right) \ldots\ldots (22)$$

—has the advantage that it is reliable for very shallow depths and that it can be safely used for any depth, since side friction is entirely neglected. For q_o one may use the results of load tests, or the following well-known formula,

$$q_o = R \, \gamma \tan^4 \left(45 + \frac{\phi}{2} \right) \ldots\ldots (23)$$

in which γ is the unit weight of the soil, buoyant below the ground-water surface. The angle of internal friction ϕ of a cohesionless soil can often be estimated from its general character and density.

When greater accuracy is desired for the usual working range from the surface to depths equal to several times the width of a footing, detailed soil investigations will be required. In this case, substitution of the increase along a parabolic curve by a straight line is not justified, and a more accurate formula, such as Professor Terzaghi's Eq. 3, in its general form, should be used.

For the purpose of a simplified statement the writers change formulation (*d*) in the paper to:

> The ultimate bearing capacity of a cohesionless soil increases with the depth of the loaded area beneath adjacent ground surface, approximately according to linear Eq. 22.

Mr. Freeman and the Soil Mechanics Committee of the Los Angeles Section disagree with the writers' statement that in the case of deep foundations on dense sand even the heaviest loading allowed by building codes is exceedingly conservative as far as the strength of the soil is concerned. They refer to the possibility of considerable differential settlement, which is not the issue in this statement. A footing may undergo objectionable settlement from a load which is still entirely safe as far as the strength of the soil is concerned. The occurrence of objectionable differential settlements does not imply that a safe load, as far as the strength of the soil is concerned, was exceeded.

(*g*) and (*h*) In discussing the validity of Hooke's law for cohesionless soils, Mr. Feld is apparently of the opinion that the Boussinesq formula does give satisfactory results for cohesionless soils—at least for small stresses. Mr. Julian states that there is a possibility that a cohesionless mass may have elastic properties when confined under high pressures. The writers agree with Mr. Julian, but believe that for most practical problems the assumption of Hooke's law for cohesionless materials leads to a distribution of soil reactions which are grossly in error.

(*i*) and (*j*) Messrs. Terzaghi and Freeman state that the decrease of settlements with depth may not apply to statement (*j*), depending largely on the effect of construction operations on the soil underlying the area to be loaded.

The writers agree fully with these qualifications. For example, when in an open excavation upward flow of water is allowed to develop, even the most compact and highly consolidated sand may change into a quick condition and become loose and highly compressible.

Within the usual working range of building footings (say, from depths of zero to 10 ft) the decrease of settlements for sand does apply and should be considered when designing such foundations.

In regard to the objections raised in the discussions, the writers propose the following modification of (*j*):

> With increasing depth of a loaded area, the settlement due to a given unit load will decrease appreciably only if the soil under the loaded area remains undisturbed. When disturbance is inevitable, the settlement may be assumed to be independent of depth.

The Liberty Mutual and New England Mutual Life Insurance Buildings

Building I, a foundation plan of which is shown in Fig. 17, is a ten-story tower-shaped office building having two setbacks. The plan area of the first two floors and basement is approximately 32,000 sq ft; the area of the third through sixth floors, approximately 20,000 sq ft; and the area of the seventh through tenth floors, approximately 10,000 sq ft. The dead load of this building, as computed from data furnished by the Turner Construction Company and based on actual quantities used in construction, is 39,500 tons. This dead load was approximately 5% less than the design dead load. The actual live load, as determined from an inspection of the building during which representative items of live load were weighed and unit intensities computed, is 1,500 tons. The effective weight of the soil removed to form a general basement, and in addition a sub-basement under the more heavily loaded ten-story section, is 35,000 tons. The net increase in load for the entire area is 6,000 tons.

Building II, a foundation plan of which is shown in Fig. 18(*a*), covers a plan area of approximately 66,000 sq ft. The section to the left of line EF and that to the right of GH are four-story sections. The area bounded by EGJI is occupied by a ten-story section; areas IMNK and OJLP, by two-story sections; and area FKLH, by a one-story section, with the exception of the center part, approximately 60 ft in length, which carries two stories. In the center of the building is an ornamental tower section approximately 40 ft square which rises 125 ft above the ten-story height. The dead load of this building, as determined from data furnished by the Turner Construction Company, and again based on actual quantities used in construction, is 110,000 tons. In this case 104,000 tons was the design dead load. The estimated actual live load is 6,000 tons. The effective weight of soil removed from this area is 127,000 tons. The estimated weight of two buildings which formerly occupied this site was 16,700 tons. The total weight removed from this area exceeds the weight applied in the form of the completed structure. This additional weight was removed to provide for dead-load increases and live-load increases in additions contemplated for the future and for which the foundations were designed.

The widths of all wall footings for building II were designed to provide for additional loads contemplated for the future. In many respects, however, the existing load distribution is more critical, and it is anticipated that, when the more lightly loaded sections are increased in story height, the differential settlements will diminish.

Both foundations were designed for dead loads plus actual live loads which were estimated as accurately as possible.

The thickness of the crust of hard and stiff clay was thoroughly explored during the construction of both buildings. For building I, the thickness was ascertained by making one hand-auger boring in the bottom of every caisson. The diameters of the caisson bells were proportioned in relation to the actual thickness of the crust. For building II numerous auger borings were made during the construction period to determine the variations in thickness of the crust. As a result of these explorations, it was decided during construction to raise the elevation of the base of the Berkeley Street exterior wall so as not to cut into the hard crust and unduly reduce its thickness. At this location the elevation of the base was slightly higher than at other places in the building area.

The design of the outside foundation walls for building II for horizontal forces was largely determined by the hydrostatic pressure. To this was added earth pressure at rest, buoyant below ground-water level. The temporary sheeting and bracing was designed on the basis of triaxial tests on undisturbed samples of the organic silt, making due allowance for a modest yield. Later the earth pressure was measured directly and found to check well with the design values.

Mr. Woolf takes exception to the specification for removing hard clay from under the sub-basement slabs to prevent them from bearing directly on hard material. This was necessary to assure that all loads would be transferred through the wall footings. Otherwise it would not have been possible to determine the soil reactions for which the slabs should be designed. Even if these pressures had been known, it would have been uneconomical to permit the slabs to carry the building load. If the crust of hard clay were not available, then a continuous mat would have been necessary, as emphasized by Mr. Freeman.

The main reasons for designing eccentric footings for the outside walls were: (1) Simplification of construction, and (2) more uniform distribution of the contact pressures on the base, considering water and earth pressure against the walls. The Building Department did not permit transmitting the earth pressure horizontally through the floor systems, thus necessitating transmitting these forces through the wall footings.

From the observed deflections, because he desired a comparison with the actual design, Mr. Woolf spot-checked the design of an interior wall for building II and computed the amount of reinforcing steel required to be 60 sq in. These walls were provided with 100 sq in. of steel in both the top and bottom.

Several discussers expressed a preference for the design of the foundation of building II over that of building I. When comparing these designs, the difference in the design of the superstructure should be kept in mind. The type of foundation designed for building II was made possible by the "columnless type" of building design.

Settlements.—The most recent settlement observations of buildings I and II were conducted after 6.5 and 3.8 years had elapsed, respectively, since installation of the observation points. The progress of the settlements is consistent with those reported in the paper. Table 2 summarizes the essential values.

TABLE 2.—SUPPLEMENTARY REPORT OF SETTLEMENT

Observation	Years elapsed	
(*a*) BUILDING I, LIBERTY MUTUAL INSURANCE COMPANY BUILDING		
Elapsed period, in years	4.3	6.5
Maximum settlement for point A93, Fig. 17 (curve I, Fig. 15(*b*))	1.85	2.04
Maximum differential settlement (curve I, Fig. 15(*c*))	1.02	1.18
(*b*) BUILDING II, NEW ENGLAND MUTUAL LIFE INSURANCE COMPANY BUILDING		
Elapsed period, in years	2.1	3.8
Maximum settlement (curve II, Fig. 15(*b*) and Fig. 23)	1.80	1.98
Settlement of point 9, Fig. 22	1.06	1.17
Maximum differential settlement (curve II, Fig. 15(*c*))	1.18	1.29

Referring to Fig. 20, Mr. Freeman concludes from the available observations that point 5, building II, might have risen considerably more than 2.9 in. before the rise was arrested by the weight of the concrete. The additional rise of point 5 which occurred in the period from October 12 to October 23, 1939, is estimated by the writers from a study of movements of points 4 and 6, Fig. 20, to be between 0.3 and 0.4 in. In this connection it should be emphasized that point 9 rose slightly after the foundation slabs and walls were installed at this location, due to an increase in hydrostatic pressure on the slab.

Mr. McAlpine inquires whether any of the adjacent buildings were adversely affected during or after construction. Great care was exercised in frequently observing elevations of observation points established on all structures in close proximity to the site of operations. No movements could be detected in any building surrounding the area. In this respect the soil conditions in Boston are more favorable than in some other cities where such deep excavations would invariably result in lateral flow of the soil and settlements of adjacent buildings.

The lack of symmetry in the settlement contours of the central area of building I, Fig. 17, is due to a nonuniform depth of excavation. The area enclosed by the heavy black line defines the sub-basement area. A depth of soil varying from the basement level to the sub-basement level was excavated from the area enclosed by the dot-dashed line on the Berkeley Street side of the sub-basement section. Had it been possible to excavate more soil at location A, the settlement pattern would probably have been symmetrical.

In reply to the requests by Messrs. Krynine and Burmister, for a comparison of computed and actual settlements, the following is quoted from a comprehensive investigation by the junior author:[93]

> "The analysis of data for six buildings located within the confines of Boston has suggested the following conclusions:
>
> "(1) Settlements due to consolidation, as computed from information disclosed by boring records, numerical data obtained from carefully performed consolidation tests of representative undisturbed samples, and stresses computed on the basis of the elastic theory, varied between values equal to and up to twice the magnitude of the observed settlements.
>
> "(2) Conservative estimates of the predicted differential settlements are obtained if, in computing the stresses by the elastic theory, the following simplifying assumptions are made: (*a*) The soil above the plane of loading and adjacent to the loaded area does not influence the distribution of the load below the plane of loading. (*b*) A structure is relatively flexible and does not redistribute the load as settlement occurs. It was generally observed that the ratio of the observed settlement to the computed settlement was low for points of large settlement and high for points of small settlement."

Recent investigations indicate that the excess of predicted settlements over actual settlements can be further decreased by considering more carefully the effect of preconsolidation of a substantial thickness of the clay stratum by drying.

CONTROL OF SETTLEMENTS BY COMBINING FLOTATION AND STIFFNESS

The interesting historical remarks by Professor Terzaghi on early applications of the principle of flotation to the design of foundations, and his discussion of the time lag between the inception of a new idea and its general acceptance, explain why the profession has so overlooked this method for almost one hundred years that it had to be rediscovered. However, he still leaves unanswered the question: Why is not more use made of this method in the design of modern buildings? The writers believe that the principle of flotation by itself is not sufficient to design satisfactory floating foundations for large buildings on soft ground. Just as ships require stiffness to withstand fluctuating support and variable loading, so must buildings floating on soft ground be designed to withstand nonuniform support and loading. In this connection, conclusion (5) of the paper deserves to be underscored. The problem of the design of such foundations is by no means solved in a completely satisfactory manner. Cooperation between architect and engineer should lead to novel and more practical designs in which full use is made of the superstructure in obtaining adequate stiffness.

---

[93] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering (pp. 237 and 238).

If those structures are excepted in which functional requirements demand excavation at least equal to the weight of the structure, the method of flotation would probably be worth while only for very heavy structures, or structures with nonuniform load distribution.

Professor Zeevaert describes the application of an interesting and novel idea for compensating for the dishing effect by decreasing the depth of the excavation toward the edges of the building, thus assuring a more uniform stress distribution in the underlying soil. This method is theoretically sound and it would be worth while to have the details of this design made available to the profession. Professor Zeevaert expresses the opinion that heaving due to excavating in very soft clay is mainly caused by lateral plastic flow rather than by vertical expansion of the underlying clay. This is probably correct for Mexico City, although at the site of building II it was proved that the ground lost because of minor subsidence of the immediately adjacent street surface was of the same order of magnitude as the volume lost because of outward movement of the sheeting. The volume of the heave over the entire building area was of a much larger magnitude.

The writers concur with Professor Tschebotarioff's statement that, under different loading and soil conditions than those at building II, recompression following swelling may not be completed during the construction period.

Mr. Wilson suggests an interesting foundation design, which is intended more nearly to satisfy the requirement that the building load should be removed and applied simultaneously.

FLOATING FOUNDATIONS VERSUS DEEP FOUNDATIONS

The writers agree fully with Mr. White that deep foundations, which transfer load through soft clay on to hard underlying material, are preferable to floating foundations, when the cost of the former type is only a little higher than that of the latter. However, it was not the purpose of the paper to discuss foundations in which settlements are eliminated by carrying the loads on practically incompressible materials, but to present some applications of soil mechanics to the design of foundations which transmit load on to soft soil. The question as to when deep foundations should be selected was also beyond the scope of the paper.

For both buildings described, a pile or caisson foundation to rock would have been much more expensive than the floating foundation. For building II comparative cost studies were made for steel piles to rock, for caissons to rock, and for the floating foundation. These studies left no doubt that a deep foundation would be much more expensive than the floating foundation. A cost analysis of building II, after its completion, showed a unit cost for the substructure of $5.50 per ton load. This value may be compared with the actual cost of considerably more than twice this amount for a foundation to rock, consisting of open-end steel pipe piles, which was installed at a site in Greater Boston where similar subsoil conditions prevailed. When considering the value of the additional basement space provided by the floating foundation, the comparison is still more in favor of the floating foundation.

It was intimated in the discussions that only on the basis of detailed soil investigations, as conducted for these two buildings, would it be possible to design satisfactory floating foundations. However, most of these investigations served to analyze the settlements for foundations of a conventional type. These studies showed that such foundations would result in excessive differential settlements. Could it have been possible to decide beforehand on a floating type of foundation, most of these soil investigations would not have been necessary.

CONCLUDING REMARKS

In retrospect, several conflicting statements appear rather prominently in the discussions. On close scrutiny one finds that many of these conflicts are caused by a scarcity of reliable observations.

The designer is often confronted with the dilemma of whether to put more trust in observations whose accuracy he may have reason to doubt or in a theory which at best is a crude approximation. A theory can be modified to yield such a sufficiently wide range of results as may reasonably be expected to include the actual conditions. An observation is an empirical fact with which one cannot argue, unless one is thoroughly familiar with all the details.

Many observations from field and laboratory are subject to errors which render them valueless or actually misleading. For example, only a small percentage of measurements conducted with pressure cells can be considered reliable. The measurement of pore-water pressure in clays is assuming increasing importance, but it remains to be proved whether any of the existing measuring devices really give accurate results. Even the simplest field measurement (namely, the observation of settlements) has occasionally been subject to serious errors. Undisturbed samples of the same clay have been sent to different laboratories for shear strength tests. The results were so far apart that an objective observer must greatly wonder about the confidence with which such data were used in "accurate" stability computations. Therefore, let the frequently heard call for more observations and research on the behavior of earthworks and foundations be tempered by emphasis on quality rather than on quantity.

Acknowledgment.—The data from explorations of the clay deposits at the site of the Mystic Power Station are used with the permission of the Boston Edison Company and Jackson and Moreland, Engineers, for whom the writers made these investigations. The writers wish to thank R. P. Burden, of the Staff of the Harvard Graduate School of Engineering, for his assistance in the preparation of material and review of the text for this closure; and J. A. Delaney, of the staff of the Harvard Soil Mechanics Laboratory, for the performance of tests.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2214

STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS

By Silas H. Woodard,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. M. Goodrich, Maurice P. van Buren, Joseph A. Wise, David B. Hall, Roberto Contini, I. Nelidov, Bernard L. Weiner, Richard W. Albrecht, Duff A. Abrams, C. A. P. Turner, Mark W. Huggins, A. A. Eremin, and Silas H. Woodard.

Synopsis

Formulas for strains and stresses on planes oblique to direction of principal stresses are developed in this paper. The thesis is offered that stresses are limited to tensile and compressive stresses; that shearing stress and shearing strength are misnomers; that shear is resisted by simple diagonal tensile and compressive stresses; that ruptures or failures occur when material is overstressed in tension; and that all failures in the field of engineering structures (except where plastic flow is classed as a failure) are tensile failures.

Fundamental Problems of Stress in Terms of Strain

Let Fig. 1 be the side elevation of a prism $A_1B_1C_1D_1$ of length H, and 1 unit by 1 unit in cross section, acted upon by force, p, parallel with the axis—that is, with the dimension H. During the application of the force, p, the prism changes from $A_1B_1C_1D_1$ to $A_2B_2C_2D_2$ or vice versa depending on whether p is compression or tension; the length of $\frac{H}{2}$ changes to the length $\left(\frac{H}{2} - h\right)$; the dimensions normal to the axis change to the length $(1 - 2\,b)$; the length of $a = \overline{OB_1}$ changes to the length $(d = \overline{OB_2})$.

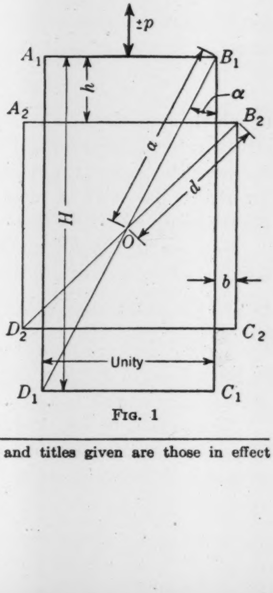


Fig. 1

Note.—Published in February, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., New York, N. Y.

Let δ = strain in direction of $p = 2\frac{h}{H}$; δ_2 = (strain normal to direction of p) $= 2\frac{b}{1} = 2b$; δ_3 = (strain in direction of $\overline{\mathrm{OB}_1}$) $= \frac{a-d}{a} = 1 - \frac{d}{a}$; δ_n = (strain normal to direction of $\overline{\mathrm{OB}_1}$); $\mu = \frac{\delta_2}{\delta}$; and α = the angle between the direction of p and $\overline{\mathrm{OB}_1}$. Furthermore,

$$\sin\alpha = \frac{1}{2a} \qquad \text{(1a)}$$

$$\cot\alpha = \frac{H}{1} = H \qquad \text{(1b)}$$

$$a = \frac{1}{2\sin\alpha} \qquad \text{(1c)}$$

$$h = \frac{\delta\cot\alpha}{2} \qquad \text{(1d)}$$

and

$$d = \sqrt{\left(\frac{H}{2} - h\right)^2 + (\tfrac{1}{2} + b)^2} \qquad \text{(2a)}$$

Substituting values of H, h, and b, in Eq. 2a,

$$d = \tfrac{1}{2}\sqrt{(1-\delta)^2\cot^2\alpha + (1+\mu\,\delta)^2} \qquad \text{(2b)}$$

and

$$\delta_3 = 1 - \frac{d}{a} = 1 - \sin\alpha\sqrt{(1-\delta)^2\cot^2\alpha + (1+\mu\,\delta)^2} \qquad \text{(3)}$$

Simplifying Eq. 3,

$$\delta_3 = 1 - \sqrt{(1+\mu\,\delta)^2 + [(1-\delta)^2 - (1+\mu\,\delta)^2]\cos^2\alpha} \qquad \text{(4a)}$$

Usually the designer is interested in the stresses and strains normal to a given plane. From Eq. 4a it follows that

$$\delta_n = 1 - \sqrt{(1+\mu\,\delta)^2 + [(1-\delta)^2 - (1+\mu\,\delta)^2]\sin^2\alpha} \qquad \text{(4b)}$$

in which δ_n is the strain normal to any plane cutting a prism subject to axial stress in terms of the axial strain, Poisson's ratio μ and the sine of the angle α between the axis of the prism and the cutting plane. Eq. 4b is rather awkward for ready use, but it may be plotted into curves that are both useful and illuminating.

For practically all materials of construction the elastic limit is less than 0.004 E (E = modulus of elasticity) and working stresses are less than 0.001 E. It can be shown that the ratio, $\frac{\delta_n}{\delta}$, if computed by Eq. 4b, will be the same to the second decimal place for all values of δ less than 0.005. The uncertainties in the values of Poisson's ratio and the modulus of elasticity would vitiate any greater refinement.

The curves of Fig. 2 show, for various values of Poisson's ratio and for all angles, the strain normal to a plane that cuts a prism which is stressed axially, in terms of the strain in the direction of the axis as computed by Eq. 4b. The curves are for the following values of Poisson's ratio:

μ	Material
0.1	An approximate value for neat cement mortar
0.15	An approximate value for 1 : 3 mortar
0.2	An approximate value for ordinary concrete
0.25	An approximate value for stone
0.275	Value frequently used for steel
0.3	Another reported determination for special steel
0.33	Value generally used for copper
0.40	Value reported for lead

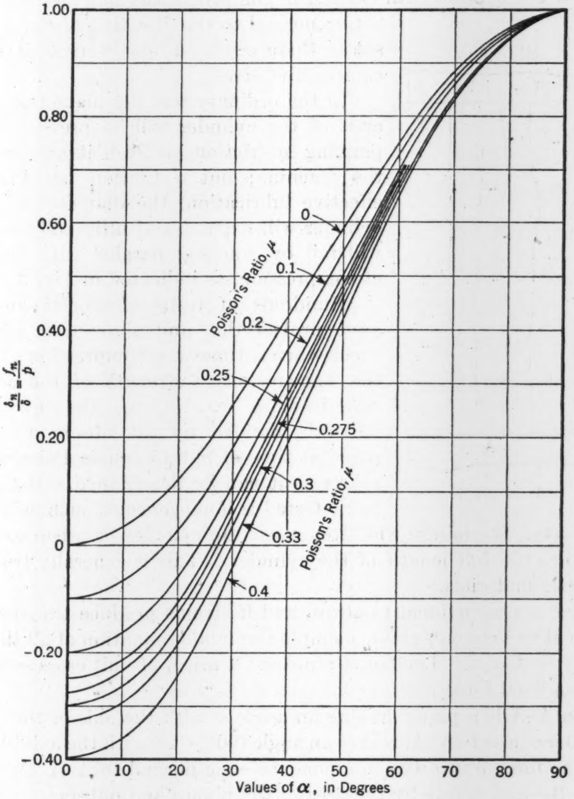


FIG. 2.—STRESS NORMAL TO A PLANE THAT CUTS A PRISM WHICH IS STRESSED AXIALLY

Interpolation may be made for other values of Poisson's ratio by simple proportion. If stress is proportional to strain, the curves of Fig. 2 can be used to read not only the strains normal to any plane cutting a prism in terms of axial strain of the prism but also the stresses, in terms of the principal axial stress, p.

In Fig. 2, the ordinates are marked to read both $\frac{\delta_n}{\delta}$ and $\frac{f_n}{p}$, in which f_n is the stress normal to the inclined plane. Of course, if p is tension, the ordinates above the x-axis will indicate tension and those below, compression. If p is compression, the ordinates above will indicate compression and those below will indicate tension.

Consider the case of a test specimen of a concrete cylinder or prism being subjected to axial pressure in a compression testing machine. By theory the diameter of the cylinder will expand in the proportion of Poisson's ratio and, acting normal to the direction of the compressive stress, there will be a tensile stress of μ times the compressive stress.

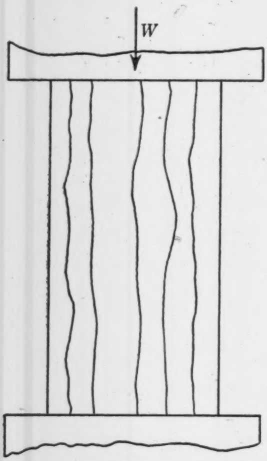


Fig. 3

In the ordinary test the diameters of the two ends of the cylinder will be prevented from expanding by friction between it and the heads of the machine; but if friction be eliminated by effective lubrication, the diameter of the entire cylinder will expand and ultimately the material will fail by cracking parallel with the direction of compression, as indicated in Fig. 3.

Hundreds of tests[2] of mortar and concrete have proved this and show that such failures occur when μ times the compression stress equals the known tensile strength of the concrete; or stated more exactly, f_t, the known tensile strength, divided by the observed compressive stress at time of failure equals Poisson's ratio as nearly as it can be determined. If the material being tested is homogeneous, such as 28-day neat cement mortar, the failure will often cause the cylinder or prism to break into small needles the full length of the cylinder. This is generally true of amorphous brittle materials.

If stress is proportional to strain and if strains produce transverse strains proportional to Poisson's ratio, a simple formula or equation of all the curves of Fig. 2 may be found. Let Fig. 4 represent a prism of unit cross sections acted upon by an axial force p.

Section A–A is a plane making an angle α with the axis of the prism; and plane B–B, normal to A–A, makes an angle $(90° - \alpha)$ with the axis of the prism. Resolve the force p into two components—one normal to A–A, the other normal to B–B—and divide by the area of the planes and obtain:

The normal force on unit area of plane A–A,

$$p_a = p \sin^2 \alpha \qquad (5a)$$

---

[2] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1092.

and the normal force on unit area of plane B–B,

$$p_b = p \cos^2 \alpha \ldots\ldots (5b)$$

Then by hypothesis the force p_a produces a strain normal to A–A, of

$$\delta_a = \frac{p_a}{E} = \frac{p \sin^2 \alpha}{E} \ldots\ldots (6a)$$

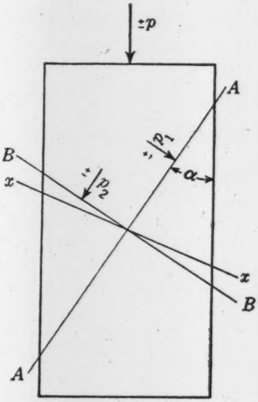


FIG. 4

but, force p_b produces a strain of opposite sign normal to A–A, of

$$\delta_b = -\frac{\mu\, p_b}{E} = -\frac{\mu\, p \cos^2 \alpha}{E} \ldots\ldots (6b)$$

The net strain normal to plane A–A is $\delta_a + \delta_b = \delta_n$. Then

$$\delta_n = \frac{p}{E}(\sin^2 \alpha - \mu \cos^2 \alpha)$$

$$= \frac{p}{E}[(1 + \mu) \sin^2 \alpha - \mu] \ldots (7)$$

and

$$f_n = E\, \delta_n = p\,[(1 + \mu) \sin^2 \alpha - \mu] \ldots (8a)$$

Another form for Eq. 8*a* is,

$$f_n = p\,[1 - (1 + \mu) \cos^2 \alpha] \ldots\ldots (8b)$$

In Eqs. 8*a* and 8*b*, p is the axial stress in a prism and f_n is the stress normal to a plane making an angle α with that axis. These formulas plot curves identical with those of Fig. 2. However, they can be derived directly from Eq. 4*b*. For example, write Eq. 4*b* in the form

$$\delta_n - 1 = -\sqrt{(1 + \mu\,\delta)^2 + [(1 - \delta)^2 - (1 + \mu\,\delta)^2] \sin^2 \alpha} \ldots\ldots (9a)$$

Squaring both sides of Eq. 9*a* and expanding the binomials:

$$\delta^2_n - 2\,\delta_n + 1 = 1 + 2\,\mu\,\delta + \mu^2\,\delta^2$$
$$+ (1 - 2\delta + \delta^2 - 1 - 2\,\mu\,\delta - \mu^2\,\delta^2) \sin^2 \alpha \ldots\ldots (9b)$$

But, δ^2_n, $\mu^2\,\delta^2$, and δ^2 are extremely small fractions, of the order of less than one millionth. Also they tend to cancel each other by opposite signs. If these can be dropped, Eq. 9*b* becomes

$$\frac{\delta_n}{\delta} = 1 - (1 + \mu) \cos^2 \alpha = \frac{f_n}{p} \ldots\ldots (10)$$

which is the same as Eq. 8*b*. The evidence indicates that Eq. 10 is applicable for all materials of construction up to the strains at which rupture occurs.

STRESS AND INTENSITY OF FORCE

It is common to conceive, and sometimes to define, stress as the normal component of the force known to be transferred across a unit area of cross section of a material. Referring to Fig. 4, the normal components of the force,

p, upon unit area of the planes A–A and B–B are

$$p_a = p \sin^2 \alpha \qquad (11a)$$

$$p_b = p \cos^2 \alpha \qquad (11b)$$

$$p_a + p_b = p \qquad (11c)$$

The normal stress f_a on plane A–A is given by Eq. 8*b*.

The difference between the intensity of normal force, p, and normal stress, f_a, on the plane is

$$p_a - f_a = \mu\, p \cos^2 \alpha \qquad (12)$$

It becomes important to distinguish between stress and force intensity. As used in this paper, stress f is defined as the magnitude of internal force normal to unit area which is accompanied by, and proportional to, strain in the same direction; or conversely, if the distance between two particles in homogeneous matter be changed, the proportional change is a measure of the stress on a unit area normal to the straight line between the particles. Force intensity p is defined as the sum of the normal components of all forces acting on unit area.

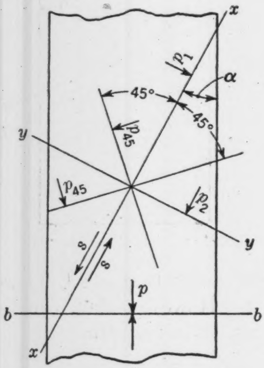


Fig. 5

Stress f may be greater than, equal to, or less than, force intensity p. This study indicates that f may be greater or less than p by a percentage not greater than Poisson's ratio, the variation being determined by the angles between the direction of principal stresses and the plane on which the stress or force intensity is acting.

Force intensity, although not a measure of stress, is a necessary and useful unit in problems of mechanics, and will be used in all appropriate equations of applied mechanics. Stress and force intensity are numerically the same when the stress is a single, maximum, principal stress, as, for example, p in Figs. 1 and 5. Furthermore, stress and force intensity are always numerically the same in any and every direction in a material for which Poisson's ratio is zero—but there is probably no such material.

The writer hereby enlists with the partisans of the Maximum Stress and Maximum Strain Theories of failure as opposed to the Maximum Shear Theory, the Maximum Distortion Energy Theory, and the Maximum Strain-Energy Theory. Although they are inseparable, the Maximum Strain Theory and Maximum Stress Theory are sometimes referred to as two independent theories.

SHEAR AND STRESS

It is common to use the terms shearing stress and shearing strength, but two logical deductions in the following study are that, (*a*) in materials strained by exterior forces including shearing forces, only two kinds of stresses are set up—compressive stresses and tensile stresses—and (*b*) shear is accompanied by and resisted by internal forces of tension and compression. In other words, there is no such thing as shearing stress or shearing strength as distinguished from compressive and tensile stresses and compressive and tensile strength.

However, the same deductions develop a rigid relation between shear and tensile and compressive stress so that shear and shearing intensities may be used to compute stresses.

Consider the simple case illustrated by Fig. 5 in which (x–x) is a plane making an angle α with the direction of a principal stress p (tension or compression). This stress is assumed to be the result of a single exterior force acting in the same direction. On plane (b–b) normal to the direction of the exterior force there is no shear and the stress is a maximum and is the same as the force intensity. Suppose the cross section (b–b) to have unit area, then the exterior force will also be p. The force intensity p_a normal to (x–x) will be as expressed by Eq. 5*a*, and the stress f_a will be as expressed by Eq. 8*b*. On plane (y–y) normal to (x–x), the force intensity p_b will be as expressed by Eq. 5*b*, and the stress f will be

$$f_b = p\,[1 - (1 + \mu)\sin^2] \qquad \text{(13a)}$$

The unit shear on either (x–x) or (y–y) will be

$$s = p \sin\alpha \cos\alpha \qquad \text{(13b)}$$

The force intensity normal to one of the planes making an angle of 45° with (x–x) and (y–y) will be (see Eq. 5*a*):

$$p_{45} = p \sin^2(\alpha + 45) = p\,(\tfrac{1}{2} + \sin\alpha\cos\alpha) \qquad \text{(14a)}$$

and normal to the other it will be

$$p_{-45} = p \sin^2(\alpha - 45) = p\,(\tfrac{1}{2} - \sin\alpha\cos\alpha) \qquad \text{(14b)}$$

Eqs. 14*a* and 14*b* represent two forces acting diagonally across the shear planes. The net effect of the components of p_{45} and p_{-45} parallel to the shear planes is their algebraic difference. These components on unit area of the shear planes are found by multiplying Eqs. 14*a* and 14*b* by $\sin^2 45°$, with the result that the net effect is found to be, $p \sin\alpha \cos\alpha$, which is the value of unit shear as given by Eq. 13*b*. In other words, the force intensities p_{45} and p_{-45} of Eqs. 14*a* and 14*b* are real forces and constitute a sufficient answer to the question of how shear is resisted without calling upon an imaginary and inexplicable thing called shearing stress.

If the shear per unit area on any pair of shear planes and the force intensity on each of the planes is known, then the direction and intensity of the maximum and minimum stresses can be computed (keeping in mind that the maximum and minimum stresses usually mean the maximum tensile and compressive stress or the minimum tensile and compressive stress and that the engineer is principally interested in finding the maximum stress to compare it with the known breaking stress). Let p_a = the force intensity on one shear plane; p_b = the force intensity on the other shear plane; s = the shear per unit area of the shear planes; μ = Poisson's ratio for the stressed material; and β and

$(90° - \beta)$ = the angles between the shear planes and any other plane such as (x–x), Fig. 4.

The stress $f_{(x-x)}$ on plane (x–x), then, is equal to

$$f_{(x-x)} = p_a [1 - (1 + \mu) \sin^2 \beta]$$
$$+ p_b [1 - (1 + \mu) \cos^2 \beta] \pm 2 s (1 + \mu) \sin \beta \cos \beta \ldots\ldots\ldots (15)$$

Eq. 15 follows if it be assumed that there are four principal stresses acting separately in the material under consideration, p_a and p_b acting normal to the shear planes and $+ p_c$ and $- p_c$ (numerically equal to s) acting at 45° to the shear planes. Then by determining the stress on the plane (x–x) making an angle β and $(\beta - 90°)$ with the shear-plane equation, by use of the preceding equations, Eq. 15 is obtained. To find the value of β which corresponds with the maximum value of f, place the first differential of f with respect to $\beta = 0$.

$$\frac{\delta f}{\delta \beta} = - 2 (1 + \mu) p_a \sin \beta \cos \beta + 2 (1 + \mu) p_b \sin \beta \cos \beta$$
$$- 2 (1 + \mu) s \sin^2 \beta + 2 (1 + \mu) s \cos^2 \beta = 0 \ldots\ldots\ldots\ldots (16)$$

from which

$$\sin^2 \beta = \tfrac{1}{2} \pm \tfrac{1}{2} \sqrt{\frac{(p_b - p_a)^2}{(p_b - p_a)^2 + 4 s^2}} \ldots\ldots\ldots\ldots\ldots (17)$$

Having found the values of β from Eq. 17, they may be substituted in Eq. 15 and the maximum and minimum stress, f, determined. As stress is numerically the same as force intensity p, if Poisson's ratio, μ, is zero, then

$$p = p_a \cos^2 \beta + p_b \sin^2 \beta \pm 2 s \sin \beta \cos \beta \ldots\ldots\ldots\ldots (18)$$

The two values of p (maximum and minimum force intensities) obtained from Eq. 18 are frequently called the principal stresses, but, according to the definition of stress in this paper, they are not equal to stress except in special cases; however, they are important and usable values. In the special case where $s^2 = p_a \times p_b$ (see Fig. 4), the maximum values of p and f are the same but not the minimum values.

When using Eqs. 15 and 17 it is usually more convenient to express p_a and p_b in terms of s. If $p_a = k_1 s$ and $p_b = k_2 s$,

$$\sin^2 \beta = \tfrac{1}{2} \pm \tfrac{1}{2} \sqrt{\frac{(k_2 - k_1)^2}{(k_2 - k_1)^2 + 4}} \ldots\ldots\ldots\ldots\ldots (19)$$

Consider some special examples of pairs of shear planes subject to unit shears equal to s and normal force intensities of p_a and p_b, respectively.

Example 1.—The case of pure shear on a given plane, such as at the neutral axis of a simple beam or on the cross section of a shaft subjected to torsion only. Here $p_a = 0$ and $p_b = 0$, and Eq. 17 becomes $\sin^2 \beta = \frac{1}{2}$, or $\beta = 45°$; $f = \pm (1 + \mu) s$, and $p = \pm s$. That is, there will be a unit stress of $+ (1 + \mu) s$ on one of the planes making an angle of 45° with the shear plane and a unit stress of $- (1 + \mu) s$ on the other; and on the same planes there will be a force intensity, $p = \pm s$.

Example 2.—In the case of a column or simple tension member under uniform axial stress equal to p; then $p_a = k\,s$; $p_b = \frac{s}{k}$; or $p_a\,p_b = s^2$. This will be true for every shear-plane section, and k will be equal to $\tan \alpha$, in which α is the angle between the shear plane and the axis of the member. Then, if β is the angle between a shear plane and the plane of maximum stress:

$$\sin^2 \beta = \tfrac{1}{2} \pm \tfrac{1}{2} \sqrt{\frac{k^2 - 1}{k^2 + 1}} = \cos^2 \alpha \ldots\ldots\ldots\ldots\ldots (20)$$

Eq. 20 is sometimes useful, and it furnishes confirmation of Eqs. 8*b*, 15, and 18.

Example 3.—The case of a combination of shear with one simple stress, such as a shaft under torsional unit shear s, and an axial tension or compression of, say, $p_b = k\,s$, then

$$\sin^2 \beta = \tfrac{1}{2} \pm \tfrac{1}{2} \sqrt{\frac{k^2}{k^2 + 4}} \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

TABLE 1.—MAXIMUM AND MINIMUM STRESSES (IN POUNDS PER SQUARE INCH) COMPUTED BY EQS. 15 AND 17
(Proof Tensile Strength 23,400 Lb per Sq In.)

Test No.	Applied shear	Applied tension	$\sin^2 \beta$	β	COMPUTED STRESS	
					Tensile	Compressive
3SA3	10,300	15,700	0.8031	63°–39′	22,500	10,900
3SA24	10,300	14,900	0.7927	62°–53′	21,910	11,110
3SA15	11,060	13,200	0.7510	60°– 0′	21,520	12,680
3SA18	11,060	13,800	0.7643	60°–57′	21,980	12,500
3SA21	11,060	14,300	0.7715	61°–26′	22,250	12,300
3SA4	12,860	11,700	0.7087	67°–26′	22,710	15,550
3SA20	12,860	14,100	0.7399	59°–21′	24,230	15,010
3SA10	12,720	10,300	0.6894	56°– 6′	21,750	15,700
3SA17	12,720	9,950	0.6823	55°–41′	21,500	15,750
3SA7	13,100	0	0.5000	45°– 0′	17,500	17,500

Table 1 shows the maximum and minimum stresses by Eqs. 15 and 17 for nine tests of combined torsion and tension on samples of aluminum, reported by Joseph Marin, Assoc. M. Am. Soc. C. E., and R. L. Stanley[3] in 1940.

The value of Poisson's ratio μ for the material tested is assumed to be 0.33. The proof tensile strength, 23,400 lb per sq in., was the mean of three tests which varied by 4.4%.

It will be seen that the computed maximum tensile stresses agree with the proof tensile strength as closely as the test results in each group agree with each other. Unfortunately, there were no compression tests to determine the proof compressive strength. The single pure shear test (torsion only) yielded at a unit shear of 13,100 lb per sq in., for which the stress computed by Eqs. 15 and 17 would be 17,500 lb per sq in. compression on one diagonal and 17,500 lb per sq in. tension on the other. It may also be noted that the stress-strain curves in the determination of the "proof strength" indicate the yield point to be about 17,500 lb per sq in. Possibly the compressive proof strength would be approximately 17,500 lb per sq in.

---

[3] "Failure of Aluminum Subjected to Combined Stresses," by Joseph Marin and R. L. Stanley, *Supplement, The Welding Journal*, February, 1940, Vol. 19, pp. 74s–80s.

Conclusions

It is obvious that—

(*a*) Poisson's ratio must be used in the more accurate computations of stresses; and

(*b*) Shear intensity is not a stress but is always accompanied and resisted by diagonal tensile or compressive stresses or both.

Other conclusions may be deduced from the formulas developed herein, therefore, such as,

(*c*) If shear exists on a plane section at a given point, then the maximum or principal stress at that point is not normal to the plane. This fact is frequently overlooked;

(*d*) If the principal stress is known at any point, the stresses in all other directions may be found;

(*e*) Conversely, if the shearing and force intensities on a pair of shear planes at any point be known, the directions and magnitude of the maximum or principal stresses may be found;

(*f*) The stresses in all other directions may be computed either from the principal stresses or direct from the shear and force intensities.

Both from the formulas herein and from practical observation, it is concluded that ruptures of material under compression are tensile failures; also, that tests to determine compressive strength as usually made, especially those on mortars and concrete, give incorrect results and indicate too high strength because of the resisting effect of friction on the ends. If this is true, the testing methods should be revised to eliminate friction which can be done easily, or a correction factor should be determined to apply to the results of tests as now made.

DISCUSSION

C. M. Goodrich,[4] M. Am. Soc. C. E.—The newer theories of Sandel and Schleicher[5] are in better agreement with tests than were the older theories. Mr. Sandel considers the change of length in the long diagonal in a cubic element to be the measure of allowable distortion and equates this to the distortion under a single axial load to obtain his criterion of safety. Mr. Schleicher[6] follows the same argument, "Gestaltänderung" (or "Change of Shape"). Their results differ (but not greatly; see Table 2) possibly because of a difference in the value of Poisson's ratio.

TABLE 2.—Comparison of Stress Theories[a]

For:	Sandel	Schleicher
σ_x, σ_y, τ	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - 0.865\,\sigma_x\,\sigma_y + 2.865\,\tau^2}$	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - \sigma_x\,\sigma_y + 3\,\tau^2}$
σ_x, σ_y	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - 0.865\,\sigma_x\,\sigma_y}$	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - \sigma_x\,\sigma_y}$
σ_x, τ	$\sigma = \sqrt{\sigma^2_x + 2.865\,\tau^2}$	$\sigma = \sqrt{\sigma^2_x + 3\,\tau^2}$
τ	$\sigma = 1.69\,\tau$	$\sigma = \sqrt{3}\,\tau$

[a] σ_x is the unit stress in one direction, σ_y is the unit stress at right angles to σ_x, and τ is the shear.

To illustrate the procedure, consider the case of pure shear in comparison with tension. Under a single tensile load the distortion $\epsilon_1 = \frac{\sigma}{E}$ and $\epsilon_2 = \epsilon_3 = -\frac{\mu\,\sigma}{E}$; and for the diagonal, $\epsilon = \sqrt{\epsilon^2_1 + \epsilon^2_2 + \epsilon^2_3} = \frac{1}{E}\sqrt{\sigma^2 + 2\,\mu^2\,\sigma^2}$. If $\mu = 0.3$,

$$\epsilon = 1.086\,\frac{\sigma}{E} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

In a case of shear only (see Fig. 6), since the cube of investigation is infinitely small, a cube of the same size at 45° with the old cube is its equivalent. In this new cube each unit axial stress equals the old unit shear stress. The change of length of the diagonal in the square is $\epsilon = \sqrt{\epsilon^2_1 + \epsilon^2_2}$. Now $\epsilon_1 = \frac{\tau}{E}(1 + \mu)$, and $\epsilon_2 = -\frac{\tau}{E}(1 + \mu)$, whereas $\epsilon_3 = 0$.

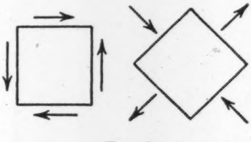


Fig. 6

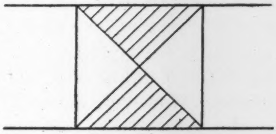


Fig. 7

---

[4] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[5] *Stahlbaukalender*, 1937, p. 24.

[6] *Bauingenieur*, 1928, p. 253.

Accordingly, the distortion of the long diagonal of the square (which, since $\epsilon_3 = 0$, is also the distortion of the long diagonal of the cube) is

$$\epsilon = \frac{\tau}{E} \sqrt{2\,(1 + 2\,\mu + \mu^2)}; \text{ or, if } \mu = 0.3:$$

$$\epsilon = 1.8386 \frac{\tau}{E} \qquad (23)$$

Equating formulas 22 and 23

$$\sigma = 1.69\,\tau \qquad (24)$$

When the bearing surfaces testing a stone cube are well greased,[7] the characteristic pyramids shown in Fig. 7 no longer appear, and the pressure required to break the cube drops to one half or less of what it is where friction helps the specimen.

The stresses indicated in Table 2 are to be taken as equivalent stresses. Consider a steel cube under compression in the vertical direction; its sides bulge and then flow when the elastic limit is exceeded. The writer was taught, and still believes, that the sum of all forces in any direction should equal zero.

MAURICE P. VAN BUREN,[8] ASSOC. M. AM. SOC. C. E.—Interesting adventures in mathematics can be developed from false premises. In referring to a concrete cylinder subjected to axial pressure in a compression testing machine (see heading, "Fundamental Problems of Stress in Terms of Strain"), the author states

> "* * * By theory the diameter of the cylinder will expand in the proportion of Poisson's ratio and, acting normal to the direction of the compressive stress, there will be a tensile stress of μ times the compressive stress."

From this postulate, various unusual conclusions are derived. One of these is Eq. 12 which expresses a difference between the intensity of normal force and normal stress on a plane.

In the generally accepted theory, the effect of an axial force on the dimensions of a body resembles temperature change in that, for given conditions, there is a natural size and shape which the body tends to assume. Lateral expansion or contraction from such causes produces stress only if prevented, in which case the stress will be of the same kind as the axial force. This is obvious from Fig. 1, where, by the principles of equilibrium, there can be no resultant lateral force unless external compression forces are applied to the sides of the piece. However, in short thick cylinders, the inner layers may be restrained by the resistance of less heavily loaded outer fibers, causing a balance of equal and opposite internal circumferential forces. The amount of these forces will bear a definite relation to Poisson's ratio and may explain lateral tension failures under compressive tests of certain brittle materials already subject to high initial cohesive stresses.

If derived from correct concepts, formulas expressing the stresses and strains involved in lateral expansion and contraction will follow accepted theory and can be found in any standard handbook of mechanics. Such deformations naturally affect the behavior of all materials, to a greater or less extent, depending on particular characteristics, shape, loading, and initial stress. In any event, in themselves, the deformations prove nothing regarding the fundamental causes of failure in general.

---

[7] "Vorlesungen über Technische Mechanik," by August Föppl, 9th Ed., III 68; see also "Zwang und Drang."

[8] Cons. Engr. (J. Di Stasio & Co.), New York, N. Y.

JOSEPH A. WISE,[9] M. AM. SOC. C. E.—Some interesting ideas are presented in this paper, but unfortunately some fundamental errors are involved that vitiate most of the conclusions. The first of these errors is the tacit assumption that in an elastic solid there must be a stress proportionate to, and in the direction of, every strain. Consider the illustration, used in the paper, of a concrete cylinder with a uniformly distributed compressive force applied axially. There is a lateral strain of μ times the axial strain (μ being Poisson's ratio), but it does not follow that there is any lateral stress. Incidentally the paper limits its discussion to co-planar states of stress, although, for a thorough presentation, a possible triaxial state of stress must be considered.

Fig. 8 shows a concrete cylinder with a compressive load applied in the direction of the z-axis, so that the unit force or unit stress is σ_z. Assume complete freedom of lateral displacement on the end surfaces in order to avoid unnecessary complexities introduced by lateral restraint. The state of stress will be assumed homogeneous; that is, all elements will have the same stress, σ_z. Take an element at the outer surface, as shown. On inspection $\sigma_x = 0$, since there is no external force applied in the x-direction. Similarly, σ_y must be zero, otherwise one could not satisfy the equation of static equilibrium, $\sum y = 0$. From tests it is known, however, that transverse strains occur; and therefore strains occur, but no stresses, in the x-direction and y-direction. It would do no good to assume any hypothetical arrangement of tensions on the outer surface and compressions in the core, as, when the conditions for static equilibrium have been met, the net total strain in the x-direction and in the y-direction would be zero (due to any such set of forces) and would not account for lateral strain. The same error renders Eqs. 8*a* and 8*b* of the paper incorrect. Then in Eq. 8*a*, f_n is not equal to $E \delta_n$, and likewise Eq. 8*b* is incorrect since the stress is not necessarily equal to E times the strain. This error is implicit in the complex definition of stress used in the paper and invalidates most of the subsequent work. In defining stress as normal force per unit area which is accompanied by, and is proportional to,

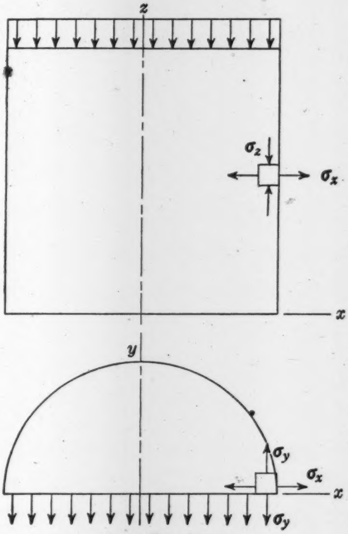


FIG. 8

---

[9] Lt. Comdr. (CEC), U. S. N. R., Public Works Office, 3d Naval Dist., New York, N. Y.

strain in the same direction, an unnecessary and unusual limitation is placed upon the concept of "stress" and, by implication, this leads to the erroneous concept that every strain must have a stress proportional to it and in the same direction.

Definitions are limitations imposed on words so that speech and writing will convey more accurate concepts. In setting up his new definitions of stress, the author does not convey more accurate concepts thereby and no longer is using the language that is basic to the various theories of failure enumerated. The customary definition of stress, heretofore used, is that stress is force per unit area within a solid. This is amplified by defining normal stress (tension or compression) as force per unit area acting perpendicularly to the area in question, whereas shearing stress is force per unit area acting parallel to and on the surface of this area. Since most of the references to the subject under discussion used these definitions, the writer will use them also. It may be pointed out that some of the errors inherent in the paper arise from the definitions which implicitly contain false assumptions. This is the reason for rejecting the proposed definitions and adhering to the customary ones. Based on these generally accepted definitions, the value of the principal stresses for Table 1 of the paper have been recomputed in Table 3.

In stating that the Maximum Stress Theory and the Maximum Strain Theory are inseparable, the author fosters a misconception that stems from the proposed definitions. The Maximum Stress Theory is the naive one which states that materials rupture when a definite maximum normal stress exists. This has been shown to be untenable, and Table 3 shows how far from true it is. The tensile strength of the aluminum was 23,400 lb per sq in.; yet all the specimens failed at lower tensile stresses. In test 3SA7, although the tensile stress was only 56% of the ultimate tensile strength, rupture occurred.

The Maximum Strain Theory, which states that materials fail when a maximum elongational strain occurs, is the theory usually considered the explanation for the failure of brittle materials such as concrete. The interpretation that the strain must be due to a stress in the same direction is incorrect, as has been shown. The proper interpretation of tests on concrete mentioned by the author, according to this theory, is that in pure tension specimens the elongational strain is equal to $\frac{\sigma}{E}$, of course, and failure is ascribed to excessive

TABLE 3.—RECOMPUTATION OF MAXIMUM AND MINIMUM STRESSES IN TABLE 1

Test No.	Applied shear	Applied tension	Maximum tension	Maximum compression	Maximum shear
3SA3	10,300	15,700	20,800	5,100	12,950
3SA24	10,300	14,900	20,160	5,262	12,712
3SA15	11,060	13,200	19,433	6,233	12,833
3SA18	11,060	13,800	19,890	6,090	12,990
3SA21	11,060	14,300	20,275	5,975	13,125
3SA4	12,860	11,700	19,978	8,278	14,128
3SA20	12,860	14,100	21,716	7,616	14,666
3SA10	12,720	10,300	18,873	8,573	13,723
3SA17	12,720	9,950	18,632	8,682	13,657
3SA7	13,100	0	13,100	13,100	13,100

strain, not excessive stress. In a compression specimen, when the lateral strain reaches this value, failure occurs although there is no lateral stress present.

The Maximum Shear Theory states that materials fail when the shearing stresses reach the maximum shearing strength of the material. This is in much better agreement with test results on ductile materials than the two preceding theories, as may be seen from Table 3. The shearing strength should be 50% of the ultimate tensile strength, or 11,700 lb per sq in. This theory has been modified by Mohr[10] to make the shearing strength a function of the maximum tensile stress and is in somewhat better agreement with some test results than the unmodified Maximum Shear Theory. Mohr's theory can be best illustrated by means of Mohr's circle of stress as in Fig. 9.

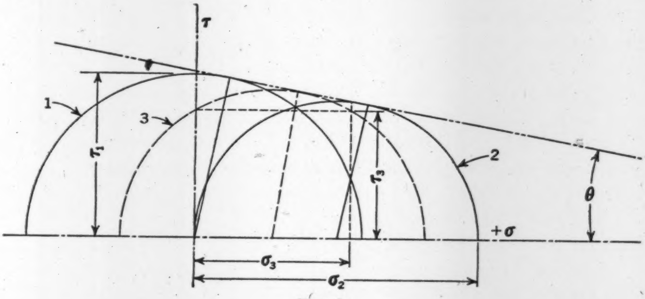


FIG. 9

Here τ_1 in circle 1 is the maximum shearing strength in pure shear, σ_2 in circle 2 is the maximum tensile strength in pure tension, and σ_3, τ_3 are the tensile and shearing stresses in any combined shear and tension. Mohr's theory states that these circles all touch a common envelope, which approximates a straight line making an angle θ (approximately 15°) with the horizontal. Circles of stress and circles of strain[11] are useful graphical aids in presenting the relationships between stresses and strains.

The Maximum Strain Energy Theory states that materials rupture when the strain energy introduced exceeds a maximum value. The Maximum Distortion Energy Theory states that materials rupture when the distortion energy exceeds a limiting value. Distortion energy is defined as total strain energy minus volume-change energy. This last theory was shown to be in better agreement with test results than the others for the tests referred to in the paper[12] from which Table 1 was taken.

The failure or disruption of materials is a subject that will probably require much further research before engineers can select the best theory of failure, and several theories may be valid, each for its own realm and for certain types of materials. Pending such research, present theories should be regarded as purely tentative and expedient approximations—using the Maximum Strain Theory for brittle materials and either the Mohr Maximum Shear Theory or the Maximum Distortion Energy Theory for ductile materials.

---

[10] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 480–482.

[11] "Circles of Strain," by Joseph A. Wise, *Journal of the Aeronautical Sciences*, August, 1940, Vol. 7, No. 10, pp. 438–440.

[12] "Failure of Aluminum Subjected to Combined Stresses," by Joseph Marin and R. L. Stanley, *Supplement, The Welding Journal*, February, 1940, Vol. 19, pp. 74s–80s.

David B. Hall,[13] Assoc. M. Am. Soc. C. E.—Before discussing any of the conclusions of this paper, the writer finds it necessary to take exception to the terminology used. Throughout the literature of elasticity, plasticity, and strength of materials, the term "stress" is used universally to designate the forces that must act across any plane in a body to maintain equilibrium. Since matter is not continuous but consists of thinly dispersed particles, it is understood, of course, that stress in the foregoing sense is a statistical, or average quantity, much the same as gas pressure, which, although actually consisting of a bombardment by gas molecules, can be, and is, measured in pounds per square inch. There is no reason why the author should abandon the term "stress" and substitute "intensity of force." The interatomic phenomena in a stressed body are no doubt very complicated. The author has elected to reserve the term "stress" for some phenomenon, apparently of this kind, although he has offered no explanation of its nature further than that stress is something that is proportional to strain. As so defined, stress serves no purpose which could not be served equally well by attributing any phenomenon related to strain directly to strain itself. That the author's transposition of accepted terms is confusing is well illustrated by his own statement in his discussion of failure theories: "Although they are inseparable, the Maximum Strain Theory and Maximum Stress Theory are sometimes referred to as two independent theories." The Maximum Stress Theory refers to stress as ordinarily defined and is entirely unrelated to the Maximum Strain Theory. Throughout the remainder of this discussion the writer proposes to use the words "stress" and "strain" in their usual and accepted meanings.

The paper presents two conclusions which warrant close examination: (1) There is no such thing as shearing stress or shearing strength; and (2) the failure of materials under combined stress is to be predicted by the Maximum Strain Theory.

1. The assertion that there is no such thing as shearing stress or shearing strength easily could wander off into the fields of metaphysics, but a few definite statements can be made with comparative safety. First, the concept of shear is clear enough: If the resultant stress acting across a plane within a body is inclined to that plane, it can be resolved into a normal component called normal stress and a tangential component called shearing stress. Second, failure may occur along a plane (*a*) if the resistance of the material to rupture is less than the normal stress or (*b*) if the resistance of the material to sliding or plastic flow is less than the shearing stress.

The author's proof of the nonexistence of shear is found to proceed something like this: If shearing stress exists on one plane, then there must be normal stresses on planes inclined at 45° to this plane. Therefore (?) the normal stresses on these inclined planes are real and resist external loads, whereas the shearing stress on the plane first considered is redundant and unnecessary.

---

[13] Designer, Howard, Needles, Tammen & Bergendoff, New York, N. Y.

The fallacy of this argument is not difficult to discover. The fundamental requirement of equilibrium of stress at a point is that the stresses on the various faces of any small figure surrounding the point shall be in equilibrium. The author has failed to recognize this requirement. In effect, instead of declaring that the stress on one face of a small triangular prism must balance the stresses on the other two (the ends being free of stress), he has stated that stress on one face may be replaced by stresses on the other two.

TABLE 4.—COMPARISON OF FAILURE THEORIES FOR ALUMINUM SHAFTS

(In Pounds per Square Inch)

Test No.	Reported Stress		Principal Stresses			Ratio $\frac{p_0}{23{,}400}$			
	s	p	p_1	p_2	p_3	Maximum stress	Maximum strain	Maximum shear	Octahedral shear
3SA3	10,300	15,700	20,800	0	−5,100	0.89	0.96	1.11	1.01
3SA24	10,300	14,900	20,150	0	−5,250	0.86	0.94	1.08	0.99
3SA15	11,060	13,200	19,500	0	−6,300	0.83	0.92	1.10	1.00
3SA18	11,060	13,800	19,900	0	−6,100	0.85	0.93	1.11	1.10
3SA21	11,060	14,300	20,300	0	−6,000	0.87	0.95	1.12	1.02
3SA4	12,860	11,700	19,950	0	−8,250	0.85	0.97	1.20	1.07
3SA20	12,860	14,100	21,700	0	−7,600	0.93	1.03	1.25	1.13
3SA10	12,720	10,300	18,850	0	−8,550	0.81	0.93	1.17	1.04
3SA17	12,720	9,950	18,600	0	−8,700	0.79	0.92	1.17	1.03
3SA7	13,100	0	13,100	0	−13,100	0.56	0.75	1.12	0.97

2. The failure of materials under combined stress is to be predicted by the Maximum Strain Theory. Table 4 has been prepared to compare the failure stresses reported in Table 1, for some aluminum shafts, with the failure stresses predicted by four common theories. This has been done by calculating a simple axial tension assumed to be equivalent to the combined stress reported and by comparing it with the known tensile strength of 23,400 lb per sq in. The three principal stresses were first obtained:

$$p_2 = 0 \qquad (25a)$$

$$\left.\begin{matrix} p_1 \\ p_3 \end{matrix}\right\} = \frac{1}{2}\,p \pm \sqrt{\left(\frac{p}{2}\right)^2 + s^2} \qquad (25b)$$

in which p_1, p_2, p_3 = principal stresses, in descending order of magnitude, p = reported tensile stress, and s = reported shear stress. The equivalent simple tension, p_0, was then obtained in terms of p_1, p_2, and p_3, according to the rules for each of the four theories: For Maximum Stress Theory—

$$p_0 = p_1 \qquad (26a)$$

for Maximum Strain Theory—

$$p_0 = p_1 - \mu\,(p_2 + p_3) \qquad (26b)$$

for Maximum Shear Theory—

$$p_0 = (p_1 - p_3) \qquad (26c)$$

and, for Octahedral Shear Theory—

$$p_0 = \sqrt{\frac{(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_1 - p_3)^2}{2}} \quad \ldots\ldots\ldots (26d)$$

As has been observed generally for ductile materials, the octahedral shear formula gave conspicuously more accurate results than any of the others. Also, it will be noted that the Maximum Strain Theory erred consistently on the side of danger.

Tests on Concrete Cylinders.—In further support of the Maximum Strain Theory the paper cites compression tests performed on concrete cylinders and prisms with lubricated ends. Significant results of the tests were: The specimens failed by splitting; failure occurred under a stress producing lateral extension approximately equal to the elongation at which specimens fail in tension; and failure occurred at substantially lower stresses than in specimens tested in the ordinary manner. The explanation offered by the author to account for the difference between the behavior of lubricated and unlubricated specimens is that the latter are prevented, by end friction, from expanding laterally and thereby are prevented from failing. The answer may not be as simple as this. In particular, the lateral restraint at the ends is a more or less local effect. Therefore, according to the author's hypothesis, (*a*) failure of unlubricated specimens should originate at the middle, where the strengthening effect is least, instead of at the ends as was observed, and (*b*) the strength of restrained specimens should approach that of lubricated specimens as the length is increased.

A condensed summary of the results of the tests is given in Table 5. The anticipated convergence of strength values for increase of length seems to be indicated by the table, although the results are erratic, and the mortar cylinders vary in exactly the opposite manner. Table 5 also shows so great a difference between strengths of plain and lubricated specimens as to suggest that some other factor may have been at work to reduce the strength of the lubricated specimens. The specimens were lubricated by sheets of blotting paper saturated with grease. Conceivably this method may have had two possible effects: There may have been a hydraulic pressure gradient across the ends,

TABLE 5.—EFFECT OF LENGTH AND END CONDITIONS ON STRENGTH OF MORTAR COMPRESSION SPECIMENS[2]

Length diameter	Compression Strength for Plain Ends / Compression Strength for Greased Ends				
	1 : 3 Mortar		Neat Cement		Average all specimens
	2-in. diameter cylinder	4-in. by 4-in. prism	2-in. diameter cylinder	4-in. by 4-in. prism	
1	1.11	1.55	1.83	1.95	1.61
1.5	1.20	1.75	1.80	1.56	1.58
2	....	1.40	1.78	1.28	1.49
3	1.44	....	....	....	1.44
4	1.40	1.05	1.20	1.12	1.19

resulting in a maximum pressure exceeding the average; and the lubricant may have hastened failure by transmitting hydraulic pressure to all cracks as fast as they formed. These explanations are only speculative, but they do serve to emphasize the precariousness of a combined stress theory based on a comparison of greased and dry concrete columns, instead of on tests of the kind exemplified by Tables 1 and 4.

The evidence presented in support of the maximum strain failure theory thus sums up to two items:

I. Combined shear and tension tests on aluminum shafts which clearly show the superiority of another theory; and

II. A comparison of cement mortar columns with and without greased ends which, at best, points toward the Maximum Strain Theory as a plausible possibility.

The paper concludes with the suggestion that compression tests of concrete should be made on greased specimens, or a factor should be introduced to give equivalent values. Since all concrete column tests invariably include the making and testing of control specimens to measure concrete strength, it would appear that all necessary factors are included automatically.

Roberto Contini,[14] Jun. Am. Soc. C. E.—Except for conclusions (*a*) and (*b*), the "Conclusions" of this paper are altogether correct. However, they are among the most elementary results of the Theory of Elasticity and have been known for decades. Moreover, the general approach is not correct.

For example, the definition of stress as given by the author does not have any physical meaning. In fact the stress, f, defined as (see heading, "Stress and Intensity of Force") "* * * the magnitude of internal force normal to unit area which is accompanied by, and proportional to, strain in the same direction * * *," is merely the strain in any direction multiplied by Young's modulus E. It is a quantity proportional to strain by definition and not because of any physical fact and, except for very particular cases, does not represent any actual force per unit area existing in the material.

The author states that it is common to conceive, and sometimes to define, stress as the normal component of the force known to be transferred across a unit area of cross section of a material. In fact, this is the definition of normal stress as used in the Theory of Elasticity and seems to be the only possible definition of normal stress.

A misinterpretation of Hooke's law that "strain is proportional to stress" seems to have occurred in this paper. It must be made clear that stress and strain are two entirely distinct physical concepts, and that each one is defined independently. For elastic materials they are related by Hooke's law, which, correctly quoted, states that strain components are linear functions of stress components. This does not mean at all that there is a constant of proportionality between stress and strain in any direction. The mathematical development of Hooke's law leads to the concepts of Young's modulus and Poisson's ratio, which are taken for granted in this paper.

---

[14] Eng. Asst., Vega Aircraft Corp., Burbank, Calif.

When the author states (see heading, "Fundamental Problems of Stress in Terms of Strain"): "* * * If stress is proportional to strain * * *," he seems to establish some kind of Hooke's law for the material that he studies. However, this is only apparent because his "stress" is proportional to strain by definition, and a law between two interdependent quantities cannot have any physical meaning.

Using the notation and definitions of the author, it easily can be seen that the statement (see heading, "Stress and Intensity of Force") that "* * * f may be greater or less than p by a percentage not greater than Poisson's ratio, * * *" is not correct. In the case of Fig. 4, for $\alpha \doteq 0$, $p_a = 0$ from Eq. 11*a*, and $f_a = -\mu p$ from Eq. 8*b*. The difference $p_a - f_a = \mu p$, expressed as a percentage of f_a, is 100, and as a percentage of p, infinity.

Referring to Fig. 3, it is very possible that some materials tested in compression fail when the transverse strain is equal to the strain in the direction of the load at failure for a tension test. This amounts to stating that in such cases strain, and not stress, determines the failure of the material. However, the material does not fail in tension, because there is no tensile force transmitted through planes parallel to the direction of compression. Perhaps the author should refer to failure by elongation.

Under the heading, "Shear and Stress," the author states that "* * * (*a*) in materials strained by exterior forces including shearing forces, only two kinds of stresses are set up—compressive stresses and tensile stresses—and (*b*) shear is accompanied by and resisted by internal forces of tension and compression." Under the heading, "Shear and Stress," the well-known fact is shown that (in two-dimensional stress distributions) there exist at any point two planes, perpendicular to each other, where no shear stresses exist and where normal stresses are maximum and minimum. From this it cannot be concluded, as the author does, that external forces are resisted within the material only along the two aforementioned planes (principal planes of stress). The principal planes are only two of an infinity of planes passing through a point. Stresses, shear stresses as well as normal ones, act along each of the infinite planes, and the material resists along each of the infinite planes. The principal planes are particularly interesting, but the planes of maximum shear (at 45° with the principal planes) are no less important. Shear stresses are neither "imaginary" nor "inexplicable" but are just as real as the principal stresses, and there is no proof, in this paper or elsewhere, that the material must fail along one of the principal planes. It may or it may not. It is perfectly conceivable, and verified by tension tests in steel, that materials may fail along the planes of maximum shear. Therefore conclusion (*b*), which should embody the main result of this paper, is incorrect.

Conclusion (*a*) is correct for stresses as defined by the author. However, since such stresses actually represent only strains, conclusion (*a*) would be more correctly stated if "strains" were read instead of "stresses." It has been proved in fact that stresses (as defined by the Theory of Elasticity) do not depend on Poisson's ratio, in two-dimensional stress distributions, except in very particular cases.

In conclusion, not only does this paper present no new results, but it is extremely confused, and at odds with the fundamentals of the Theory of Elasticity.

I. NELIDOV,[15] M. AM. SOC. C. E.—The essential claims raised in this paper may be summarized briefly as follows:

1. Poisson's ratio should be considered in computations;
2. Shearing stress and strength are misnomers; and
3. Failure occurs when material is overstressed in tension.

In reading this paper, it is impossible to judge from its title or content whether the problem treated by the author refers to all materials or just to a particular one. It follows from the text, however, that, since the theory deals with the compression force acting on a prism, one may judge that the paper is intended for concrete. Some of the author's theses and conclusions seem to refer to concrete only, or, more generally, to brittle materials. However, since the author does not limit the scope of the paper, the result is confusing.

Referring to claim 1, one may regret that Poisson's ratio sometimes is unduly neglected in computations, and it is likewise without dispute that some engineers even raise their eyebrows at a mention of the so-called "true stresses." The term "true stress" is used herein according to the definition[16] of a stress formulated by the late Mansfield Merriman, M. Am. Soc. C. E., including the effect of Poisson's ratio, which the author calls simply "stress" and in contradistinction to Mr. Merriman's "apparent stress," which the author calls "force intensity." The terms chosen by Mr. Merriman are preferred by the writer because the term ("force intensity") used by the author is a somewhat more complicated one, whereas the term "stress" is so deeply rooted in terminology that to call it "apparent stress" or "true stress" will not be difficult. The stress is "apparent" because it results only from division of a force by an area without considering other factors. The "true stress" considers the effect of actual deformations.

However, a term is only a matter of choice of words. The principal thing should be the cognizance of the facts. From this point, the author's terminology will be used.

The importance of considering Poisson's ratio has been stated by the writer elsewhere,[17] at which time the "stress" in tension in a high buttress in the example chosen was found to be as high as 120 lb per sq in. in tension. The significance of neglecting this effect cannot be underestimated.

Eq. 10 and the effect of the Poisson's ratio,[18] as shown in Fig. 10, can be obtained directly from Mohr's circle.

---

[15] Senior Engr., U. S. Engr. Dept., Sacramento, Calif.

[16] "Mechanics of Materials," by Mansfield Merriman, 11th Ed., 1914, John Wiley & Sons, Inc., New York, N. Y., p. 359.

[17] *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1326.

[18] "Mohr's Circle with Poisson's Ratio," by I. Nelidov, *Western Construction News and Highways Builder*, November 25, 1932, p. 666.

Thus, $f_n = f_{\max} - 2\,s_{\max}\cos^2\alpha$. From the problem by the author: $f_{\max} = p$; and $f_{\min} = -\mu\,p$. From Mohr's circle: $s_{\max} = \dfrac{f_{\max} - f_{\min}}{2} = \dfrac{1+\mu}{2}\,p$, or

$$f_n = p\left[1 - (1+\mu)\cos^2\alpha\right]\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

It is evident that Eq. 27 is applicable only to materials that have a limit of proportionality of stress and strain and not to materials that have a modulus of elasticity which is a curved line from the beginning of loading. In this connection one wonders what the author has meant by his statement following Eq. 10 that "* * * Eq. 10 is applicable for all materials of construction up to the strains at which rupture occurs."

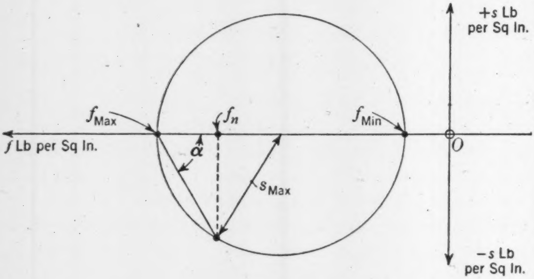


FIG. 10

In regard to claim 2, stated at the beginning of this discussion, the writer wishes to declare that, from a theoretical and conventional standpoint, one cannot dispose so easily of the terms "shearing stress" and "shearing strength," because tangential forces and resistance will always exist, disregarding the designer's wishes to be rid of them. It is true that in actual practice the true shearing force or stress is never approached because a case of true shear will call for two forces acting one against the other and nearly collinear. In actual problems shearing forces will have a lever arm always, thus forming a couple; or they will have a bending moment, with resulting tensile stress at right angles to the direction of shear. The reason that the term "shearing strength" exists is explained in the same way as the reason for the existence of the term "compressive strength." Neither of these terms has any real meaning, because both are obtained from the simplest possible way of loading a specimen, one in shear, another in direct compression. If one is desirous of finding the correct factor of safety for a material, one will have to test a specimen under the same conditions of lateral support in regard to such tension, compression, or shear as will exist in the full-sized structure and thus will determine its strength. Because of the complications involved in this kind of test, it is very seldom made; instead, judgment is supported by simple compressive, tensile, or other tests.

That definite shearing stresses exist if Poisson's ratio is considered can be proved from Fig. 3, concerning which the author quotes the well-known fact that, "if friction [at the ends of a prism under compression] be eliminated by effective lubrication, the diameter of the entire cylinder will expand and ultimately the material will fail by cracking parallel with the direction of compression * * *." What actually happens in this case is that the tensile strength of the material is greater than the shearing strength, which is zero at both ends. The specimen fails in shear at both ends due to lateral expansion long before it fails "in tension." However, the shearing failure is not conspicuous and is not noted, but the tensile failure is manifest and thus is wrongly named. If the ends are cast monolithically with the end supports, or even just restrained, as in ordinary compression tests, then tensile strength will become smaller than the shearing strength and failure will occur in tension on inclined planes, which again is erroneously called "shear failure." Shearing stress or force intensity is inseparable from tensile or compressive stress because it simply manifests angular distortion just as tensile or compressive stress manifests axial deformation.

Therefore, the writer disagrees with Mr. Woodard in regard to claim 2 and thinks that, conventionally, theoretically, and actually, the author's shearing "force intensity" and "stress" do exist and are not misnomers. In regard to claim 3 the following may be said:

There is danger in making quick conclusions regarding the stress condition and possible cause of failure based on the assumption of a uniform distribution of a stress. To illustrate, consider the prism used by the author of finite dimensions and under compressive load. His reasoning applies only to a theoretical case of a unit area wherein a force p is acting on an elementary cube under consideration. In an actual prism, a total force P is not even equally distributed on the bearing ends, and much less within the prism. Therefore, the generalization by the author derived from his elementary formula to explain the cause of failure of a finite body may be considered as a first step only. A further step would be to determine a true distribution of stresses and to observe the points where various stresses are excessive. Failure may begin at any of these points, and it will always be progressive. Usually, since the speed of progress of a failure is great, it is considered that the entire section failed at once, and some "average" stress is computed which caused this failure. This may be practical, but it is incorrect and the discrepancy is demonstrated best by failures of machine parts. These fail in many instances when the "average" stresses appear to be perfectly safe.

The best illustration of the writer's attitude on the causes of failure may be given in the following quotation from S. Timoshenko[19] (to which the writer has nothing to add):

> "In the case of brittle materials such as cast iron, fracture occurs without appreciable plastic deformation and on a cross-section perpendicular to the direction of tension. This is a separation failure. A specimen of a ductile material such as mild steel undergoes considerable plastic deformation and reductions in cross-sectional area due to sliding along planes inclined 45° to the axis of the specimen before fracture occurs. This is sliding failure. * * * If the resistance to sliding is greater than the re-

---

[19] "Strength of Materials," by S. Timoshenko, Pt. II, 1930 Ed., p. 671.

> sistance to separation, we have a brittle material and fracture will occur as a result of overcoming cohesive forces without appreciable deformation. If the resistance to separation is larger than resistance to sliding, we have a ductile material. * * * The relation between the resistance to separation and the resistance to sliding does not remain constant for the same material. It depends very much upon the velocity of deformation and upon the temperature at which a test is made. There are evidences that the resistance to sliding increases as the velocity of deformation increases and as the temperature is lowered. At the same time the resistance to separation is not affected to the same degree by these two factors. This would explain why a bar of a metal such as zinc can be bent like a ductile material under slow loading while the same bar fractures without plastic deformation if the loading is applied suddenly. * * * The type of fracture depends also on the manner of testing. If the loading is of such a nature that fracture due to separation is prevented, a considerable plastic deformation may be obtained in a material usually considered brittle. * * * Likewise, a ductile material may have a fracture of the brittle type if the form of the specimen or the type of stress distribution is such that plastic deformations, due to sliding, are prevented."

Based on his own previous statements and the foregoing quotation, the writer does not agree with Mr. Woodard's conclusions condensed in claim 3.

To summarize, the writer is of the opinion that the author has raised an important issue in the use of Poisson's ratio, but his conclusions about shear stresses and causes of failure cannot be verified.

BERNARD L. WEINER,[20] M. AM. SOC. C. E.—The laws of static equilibrium hardly can be questioned, at this late date. Although the mathematical theory of elasticity is not so well known, it is too well recognized to be questioned lightly. Finally, the ellipse of stress has been accepted for so long that it can be questioned only by offering the most exhaustive proof. The author's work is in direct contradiction of all three and, without exhaustive proof, is suspect. There can be little doubt that his work is in error.

Mr. Woodard proceeds to find the strains normal to a plane at an angle to an axial force acting on a rectangle and then argues that since stress is proportional to strain in the same direction, the stress is E times the strain. Let it be said once, and for all, that stress and strain in the same direction are not directly proportional to each other—except in one special case. The general case, which can be found in any standard text on the mathematical theory of elasticity and in texts on advanced mechanics of materials, is:

$$\left.\begin{aligned} \delta_1 &= \frac{p_a}{E} - \frac{\mu}{E}(p_b + p_c) \\ \delta_2 &= \frac{p_b}{E} - \frac{\mu}{E}(p_a + p_c) \\ \delta_3 &= \frac{p_c}{E} - \frac{\mu}{E}(p_b + p_c) \end{aligned}\right\} \cdots\cdots\cdots\cdots (28)$$

---

[20] Designer, Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

and

$$\left.\begin{aligned} \phi_1 &= \frac{s_a}{G} \\ \phi_2 &= \frac{s_b}{G} \\ \phi_3 &= \frac{s_c}{G} \end{aligned}\right\}\cdots\cdots\cdots\cdots(29)$$

in which s = unit shear; G = the shear modulus of elasticity; ϕ = angle of shear strain; and δ = unit elongation.

If only one force and its reaction are acting, it is true that stress and strain parallel to the force are proportional; but, even in this case, strain perpendicular to the force exists without stress. If the stresses are unknown but the strains are given, the former can be found by solving the general equations of the stress-strain relationship. If the strains in this simple case were substituted in these equations, p_a would be the only one having a finite value equal to p_a, of course, and p_b and p_c would equal zero. This is a matter of simple algebra—and no mysterious theories are involved.

What the author is really doing is to revive the old exploded theory of "virtual stress" as against "real stress." This conception has been renamed "intensity of force" and "intensity of stress" but is otherwise the same idea derived from the fallacy that stress and strain in the same direction are directly proportional. In his "Mechanics of Materials,"[21] Mr. Merriman made the same error probably more than half a century ago, and others have followed in his footsteps from time to time. Many "odd" results can be obtained, of course, by the use of Eq. 8b, but only one is necessary to show the error. If, in this equation, angle α is set equal to zero, $f_n = -(1 + \mu)\,p$; but since the plane that makes a zero angle with the only applied force is parallel to, and may be, an external surface, it cannot have any stress on it. Other results, although not so obvious, are equally incorrect.

Since the equation $\frac{\delta_n}{\delta} = \frac{f_n}{p}$ is incorrect, the results derived for the intensity of stress have no meaning. The author should have taken warning that something was radically wrong with his theory. No matter what else is involved, the fundamental laws of static equilibrium still must be satisfied under any and all conditions and a summation of the stresses on a plane must equal zero (taking external forces, if any, into account). As a matter of fact, for the simple case treated in Fig. 4, the intensity of force is also the true and synonymous intensity of stress and the author's version is merely a fallacy that does not exist.

There is no need to go into this part of the paper any further since the true equations have been developed in standard texts on the mathematical theory of elasticity and in texts on advanced mechanics of materials. It may be well to point out, however, that, for the general case, the stresses can be found only by solving partial differential equations. The fact that this is usually the case is rather unfortunate because, in the present state of knowledge of this type of equations, most of them cannot be solved directly. John Prescott,[22] in his "Applied Elasticity," makes this point very clear.

---

[21] "Mechanics of Materials," by Mansfield Merriman, 11th Ed., John Wiley & Sons, Inc., New York, N. Y., 1914.

Most of the author's equations are merely variations of the ellipse of stress derived by very laborious reasoning; and, when he does not tie them up with the fallacy of virtual stress (intensity of stress), they are correct. The derivations are arrived at much more simply in standard texts. However that may be, although the author's equations—without the virtual stress fallacy—are correct, his interpretations of the meaning of the equations are incorrect. Shear does exist and the reasoning which leads to the ellipse of stress simply states that a force which produces tension on one plane, compression on another, may, and usually does, produce shear on the same or other planes. Instead of denying the existence of shear, the real interpretation is that the material must be strong enough to resist the stresses in all directions, whatever they may be —tension, compression, or shear, or a combination of direct stress and shear. It might just as well be argued that compression does not exist in certain members of a truss, because the forces really are resisted, of course, by tension in other members. In Fig. 4, if line A-A were a cleavage plane, the upper part would certainly slide over the lower and it would be a sliding or shear failure.

According to the author's discussion in connection with Fig. 3, the block under compression would fail in tension—if the bearings were lubricated. Quite the contrary—as is well known in statically indeterminate stress analysis, only when "normal" deformation is prevented, are stresses produced. Deformation itself, unrestrained, does not produce any stress. The old illustration of the unrestrained bar which is heated and of the same bar which is confined between rigid abutments and also heated is sufficient to prove the fallacy of the author's contention. In Fig. 3, the very fact that the expansion of the diameter as measured by Poisson's ratio is prevented by the friction between the bearings and the block produces the stress at right angles to the force. The stress induced, with some modification that is unnecessary to go into for the present purpose, is the same as if the unstressed block were deformed by a system of radial, transverse forces by an amount equal to the Poisson expansion. Here again, of course, the fallacy lies in the incorrect assumption that stress and strain in the same direction are always directly proportional to each other.

Although the discussion in connection with Fig. 3 is in error, it does call attention to a problem which needs to be solved. The lateral stresses produced by the restraint will produce other stresses, in turn, and the solution, the writer ventures to guess, will not be either simple or easy. Further study is necessary to solve this problem, which may yield valuable results.

Since most of the paper is merely a laborious variation of the ellipse of stress erroneously combined with Poisson's ratio, most of the conclusions are invalid. The results derived in Table 1 are invalid also, but, by accident, they are probably not very seriously in error because of the fact that the applied tensions and shears are of the same order of magnitude. This fact tends to cancel the error. Hence, whatever meaning the data may have, they are very likely approximately correct.

---

[22] "Applied Elasticity," by John Prescott, Longmans, Green, London, 1924.

This paper is merely a compilation of old and new fallacies; and it controverts established fact.

Richard W. Albrecht,[23] **Jun. Am. Soc. C. E.**—A refinement of the Maximum Strain Theory of failure is presented in this paper. The usual method of treating this theory is to find the unit strain due to the maximum principal stress, modify this value by the strain caused by the stress at right angles to the maximum principal stress, and call the resulting value the maximum strain. The paper presents the method of first finding the unit strain in terms of the normal stress and the stress at right angles to this stress, and then maximizing the result to obtain the maximum unit strain. Although the latter method is theoretically more accurate, the difference between the values obtained in this manner and the values obtained in the usual manner is negligible and does not justify the use of the more cumbersome equations evolved. The evaluation of Poisson's ratio and the modulus of elasticity are more approximate than the assumption made in the usual derivation of the Maximum Strain Theory, which makes the more accurate assumption useless except for a mathematical derivation of the theory. A comparison of the values of the angle β and the maximum tensile and compressive unit stresses as obtained by the use of Eqs. 21 and 15 (which differ from the tabulated values in the paper) and as obtained by the usual Maximum Strain Theory, is given in Table 6. All differences are within the limits ordinarily to be expected when a slide rule is used.

TABLE 6.—COMPARISON OF MAXIMUM AND MINIMUM STRESSES, IN POUNDS PER SQUARE INCH, COMPUTED BY EQS. 21 AND 15 AND BY THE USUAL MAXIMUM STRAIN THEORY

(For Values of Applied Shear and Tension, See Table 1)

Test No.	Eqs. 21 and 15			Usual Maximum Strain Theory		
	β	Computed Stress		β	Computed Stress	
		Tensile	Compressive		Tensile	Compressive
3SA3	63° – 39′	22,480	11,950	63° – 40′	22,470	11,970
3SA24	62° – 56′	21,910	11,930	62° – 57′	21,890	11,900
3SA15	60° – 24′	21,560	12,715	60° – 26′	21,540	12,690
3SA18	60° – 59′	21,950	12,720	60° – 59′	21,930	12,700
3SA21	61° – 27′	22,280	12,720	61° – 27′	22,290	12,720
3SA4	57° – 14′	22,720	14,880	57° – 15′	22,700	14,850
3SA20	59° – 22′	24,220	14,780	59° – 23′	24,220	14,760
3SA10	56° – 0′	21,700	14,800	56° – 0′	21,730	14,820
3SA17	55° – 41′	21,520	14,850	55° – 41′	21,510	14,855
3SA7	45° – 0′	17,420	17,420	45° – 0′	17,420	17,420

In Conclusion (*b*) the author states:

> "Shear intensity is not a stress but is always accompanied and resisted by diagonal tensile or compressive stresses or both."

---

[23] Instr., School of Technology, College of City of New York, New York, N. Y.

This statement is correct only when two specific planes at right angles to each other are considered. There are an infinite number of other planes on which a tendency to slide exists. If the material is unable to furnish the necessary resistance to sliding but is at the same time able to furnish the necessary resistance to compression or tension, a sliding failure will occur along the plane on which the maximum shearing stress exists. Such failure occurs in a brittle material subjected to compression alone. Theodor von Kármán, M. Am. Soc. C. E., and Robert Böker conducted extensive studies of the theories of failure of a brittle material. Their results indicate that the failure of a material may be controlled by a dual law, a tensile or splitting failure occurring with certain stress combinations and a shearing or sliding failure occurring with other combinations.[24]

The author (see heading "Stress and Intensity of Force") "enlists with the partisans of the Maximum Stress and Maximum Strain Theories of failure as opposed to the Maximum Shear Theory, the Maximum Distortion Energy Theory, and the Maximum Strain Energy Theory." Can it be said that these theories are opposed to each other? Rather, should it not be said that these theories are various attempts to explain on a rational basis the failure of engineering materials? As no one of these theories deals with the atomic or molecular structure of the material, each theory makes certain basic assumptions which may or may not be exactly correct. The best test of any theory is its agreement with observed data. The five theories of failure under discussion are as follows:[25]

Maximum Stress Theory.—

$$\sigma_w = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \quad \ldots\ldots\ldots\ldots (30a)$$

Maximum Shear Theory.—

$$\sigma_w = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \quad \ldots\ldots\ldots\ldots (30b)$$

Maximum Strain Theory.—

$$\sigma_w = (1 - \mu)\left(\frac{\sigma_x + \sigma_y}{2}\right) + (1 + \mu)\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \ldots (30c)$$

Strain Energy Theory.—

$$\sigma_w = \sqrt{(\sigma_x)^2 - 2\,\mu\,\sigma_x\,\sigma_y + (\sigma_y)^2 + 2\,(1 + \mu)\,(\tau_{xy})^2} \quad \ldots\ldots (30d)$$

Distortion Energy Theory.—

$$\sigma_w = \sqrt{(\sigma_x)^2 - \sigma_x\,\sigma_y + (\sigma_y)^2 + 3\,(\tau_{xy})^2} \quad \ldots\ldots\ldots\ldots (30e)$$

---

[24] "Die Mechanik der bleibenden Formänderung in kristallinisch aufgebauten Körpern," by Robert Böker, *Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Vol. 176 and 176, 1915 (for reference to this subject, see "A Study of the Failure of Concrete under Combined Compressive Stresses," by F. E. Richart, A. Brandtzaeg, and R. L. Brown, *Bulletin No. 185*, Univ. of Illinois, Urbana).

[25] "Mechanical Properties of Materials and Design," by Joseph Marin, McGraw-Hill Book Co., Inc., 1942.

For the illustration used in the paper, in which $\sigma_y = 0$ and $\mu = 0.33$, Eqs. 30 reduce to the forms:

Maximum Stress Theory.—

$$\sigma_w = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} \qquad (31a)$$

Maximum Shear Theory.—

$$\sigma_w = 2\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} \qquad (31b)$$

Maximum Strain Theory.—

$$\sigma_w = 0.335\,\sigma_x + 1.33\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} \qquad (31c)$$

Strain Energy Theory.—

$$\sigma_w = \sqrt{(\sigma_x)^2 + 2.66\,(\tau_{xy})^2} \qquad (31d)$$

Distortion Energy Theory.—

$$\sigma_w = \sqrt{(\sigma_x)^2 + 3\,(\tau_{xy})^2} \qquad (31e)$$

A comparison of the values of the failure stresses, σ_w, as found by the various theories, and the ratio of this stress to the proof stress, σ_p, is given in Table 7.

TABLE 7.—COMPARISON OF FAILURE STRESSES COMPUTED BY VARIOUS FAILURE THEORIES

(For Values of Applied Shear and Tension, See Table 1)

Test No.	$\frac{\sigma_x}{\tau_{xy}}$	FAILURE STRESS, σ_w (LB PER SQ IN.)					$\frac{\text{FAILURE STRESS}}{\text{PROOF STRESS}} = \frac{\sigma_w}{\sigma_p}$				
		Stress theory	Shear theory	Strain theory	Strain energy theory	Distortion energy theory	Stress theory	Shear theory	Strain theory	Strain energy theory	Distortion energy theory
3SA3	1.135	20,800	25,900	22,470	22,800	23,800	0.889	1.107	0.960	0.984	1.018
3SA24	1.445	20,150	25,400	21,890	22,300	23,250	0.861	1.085	0.935	0.952	0.996
3SA15	1.192	19,470	25,740	21,540	22,350	23,250	0.832	1.100	0.920	0.955	0.996
3SA18	1.249	19,920	26,040	21,930	22,700	23,600	0.839	1.113	0.936	0.970	1.009
3SA21	1.293	20,320	26,340	22,290	23,000	23,950	0.867	1.125	0.952	0.982	1.023
3SA4	0.920	19,960	28,220	22,700	24,200	25,400	0.853	1.205	0.970	1.033	1.085
3SA20	1.098	21,700	29,300	24,220	25,250	26,350	0.926	1.252	1.035	1.079	1.125
3SA10	0.820	18,890	27,480	21,730	23,200	24,400	0.806	1.173	0.928	0.992	1.042
3SA17	0.782	18,655	27,360	21,510	23,000	24,200	0.797	1.168	0.918	0.983	1.034
3SA7	0	13,100	26,200	17,420	21,400	22,700	0.560	1.120	0.745	0.915	0.970

In Fig. 11, this ratio is plotted as ordinate against the ratio of applied tension to applied shear as abscissa. An attempt has been made to plot probable curves for the comparisons of the various theories of failure they may give, although data are lacking for enough points to give conclusive curves. These curves show the most conservative theory for the test conditions to be the Maximum Shear Theory, and the least conservative to be the Maximum Stress Theory.

The theory that gives results closest to the observed data is the Distortion Energy Theory.

The application of these theories to the data given in the paper is of doubtful value as they purport to explain only the failure of elastic action, whereas the stresses given in the paper are apparently for complete failure, being considerably higher than the 17,500 lb per sq in. yield point of the material.

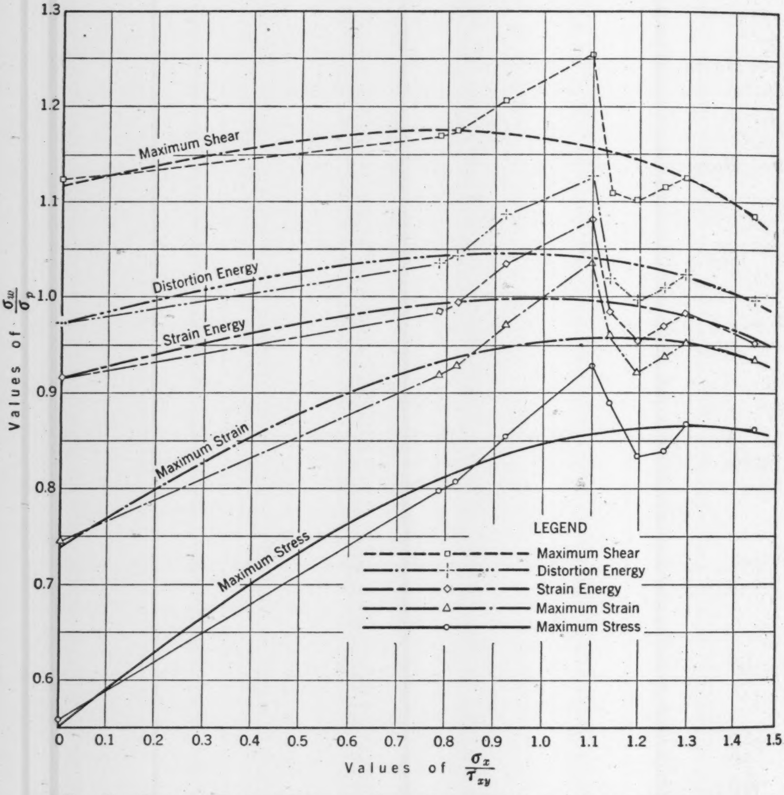


FIG. 11.—RATIO OF FAILURE STRESS TO PROOF STRESS BY VARIOUS FAILURE THEORIES

DUFF A. ABRAMS,[26] M. AM. SOC. C. E.—It should not be forgotten that strength of materials is a "land of make believe." Approaching the subject from the mathematical viewpoint, one must assume a material with idealized properties and load conditions that have little relation to anything in nature. Assumptions generally include isotropic material, constant elastic properties, and simple axial stress within the elastic range. Approaching the subject from the physical viewpoint, one soon encounters distinct limitations to experimental methods that lead to many contradictions. Probably no one knows the actual strength of any material. Determinations that have been made are based on arbitrary, and frequently unskillful test methods, all of which conceal important assumptions or fallacies that have been generally overlooked or ignored.

---

[26] Cons. Engr., New York, N. Y.

Most tests have been made in tension; yet a cursory consideration will show that pure and uniform tension is never developed. If such questions as boundary conditions, surface imperfections, localized stresses, sustained, reversed or repeated loads are considered, all the usual concepts of strength of materials must be modified or discarded.

As early as 1858 A. Wöhler[27] made fatigue tests on steel bars that failed at loads much lower than their strength as determined by once-applied loads. Investigations of fatigue of modern steels[28,29] showed that with 100,000,000 reversals of stress failure occurred at 40% to 70% of the usual tensile strength.

In 1907 J. L. Van Ornum,[30] M. Am. Soc. C. E., showed that both concrete prisms in compression and reinforced concrete beams in flexure failed under 10,000 to 32,000 repetitions at about 50% of the load they would withstand if loaded to their ultimate strength at once.

In 1920, A. A. Griffith[31] showed that the tensile strength of a certain glass varied enormously with the diameter of the specimen. For diameter 0.04 in. the strength was 24,900 lb per sq in.; and, for a diameter of 0.00013 in., it was 491,000 lb per sq in. Griffith estimated that if this glass were drawn to molecular diameter, it would have a tensile strength of 1,600,000 lb per sq in. Tests of metal wires have shown somewhat similar results.

The foregoing examples emphasize some of the properties of materials that are concealed by the "factor of ignorance," usually referred to as a "factor of safety."

Effect of Elastic Beds on Compressive Strength.—During the past century, considerable literature has accumulated on the effect of the end condition of test specimens on the compressive strength of cast iron, stone, concrete, brick, and similar materials. This problem is not so simple as indicated by Mr. Woodard's analysis; the evidence indicates that in addition to the end condition, the method of failure of short compression specimens is influenced by: (*a*) The rigidity of the material under test; (*b*) the size of specimen; and many other factors.

In 1773, C. A. Coulomb[32] concluded that a material loaded in compression will fail along a series of planes at 45° with the horizontal; in the case of a cube, six equal tetrahedra are formed, each having as its base one face of the original cube. Observation shows that, in general, this so-called pyramidal fracture occurs only in materials of comparatively low strengths, such as the weaker rocks, or mortars and concretes at early ages.

---

[27] "Zeitschrift für Bauwesen," by A. Wöhler, Vol. 8–20, 1860–1870 (summary in *Engineering*, Vol. 4, 1867, and Vol. 11, 1871).

[28] "The Engineer," by H. J. Gough, London, August 12, 1921; also "The Fatigue of Metals," by H. J. Gough, Scott, Greenwood & Son, London, 1924.

[29] "An Investigation of the Fatigue of Metals," by H. F. Moore and J. B. Kommers, ***Bulletin No. 124***, Univ. of Illinois Eng. Experiment Station, Urbana 1921.

[30] "The Fatigue of Concrete," by J. L. Van Ornum, *Transactions*, Am. Soc. C. E., Vol. LVIII (June, 1907), p. 294.

[31] "Phenomena of Rupture and Flow in Solids," by A. A. Griffith, *Philosophical Transactions*, Royal Soc. of London, Vol. 221, 1920, p. 181.

[32] *Mémoirs* of the Royal (French) Academy of Sciences, 1773, p. 343.

Mr. Woodard states with reference to this method of failure (see heading, "Fundamental Problems of Stress in Terms of Strain"):

> "* * * if friction be eliminated by effective lubrication, the diameter of the entire cylinder will expand and ultimately the material will fail by cracking parallel with the direction of compression, as indicated in Fig. 3."

In making early compression tests of building stones, it was customary to place thin sheets of lead under and over the test cube. The weakening effect of lead sheets was brought prominently to the attention of American engineers by the report of the Federal Commission of 1851 that was appointed to study marbles, etc., in connection with the extension of the U. S. Capitol; details are given in the architect's report that accompanied the message of President Millard Fillmore to the 32d Congress, 2d Session, in 1852. With reference to these tests it may be appropriate to quote from a paper dated 1873 by C. B. Richards:[33]

> "This Commission discovered the somewhat remarkable fact that the effect of pieces of sheet lead placed between the stone samples and the steel surfaces of the testing apparatus, was to occasion the failure of the specimen with about half the load it would sustain if pressed directly by the steel surfaces; that is, with the steel and stone surfaces in contact."

The findings of the Commission of 1851 on the use of lead sheets in testing building stone should be compared with the somewhat similar results in Mr. Woodard's compression tests of cement and mortar specimens with lubricated ends.

The effect on the strength of various end treatments of test specimens was brought out for stone cubes by Q. A. Gillmore,[34] the late W. C. Unwin,[35] Hon. M. Am. Soc. C. E., G. P. Merrill,[36] and by the tests made by the Chief of Engineers, U. S. Army[37]; and for concrete cylinders by D. V. Terrell, M. Am. Soc. C. E.,[38] and H. F. Gonnerman,[39] M. Am. Soc. C. E.

The effect of lubrication of the type used by Mr. Woodard on the strength and type of failure may be very different with different materials. Mr. Woodard's tests covered only the use of greased blotting paper. Sufficient data are available to throw much doubt on the author's conclusions when applied to the general case of short specimens in compression. Lubrication of the ends of specimens is not necessary to produce failure by "cracking parallel with the direction of compression." In 1859, W. Fairbairn and T. Tate[40] reported compression tests of glass cubes that showed this type of failure. They stated:

---

[33] "Experiments on the Resistance of Stones to Crushing," by C. B. Richards, *Transactions*, Am. Soc. C. E., Vol. II (1874), p. 192.

[34] "Notes on the Compressive Resistance of Freestone, Brick Piers, Hydraulic Cements, Mortar and Concrete," by Q. A. Gillmore, John Wiley & Sons, Inc., New York, N. Y., 1888.

[35] "The Testing of Materials of Construction," by W. C. Unwin, 1888, p. 416.

[36] "Stones for Building and Decoration," by G. P. Merrill, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1910.

[37] "Building Stones," Rept. of Chf. of Engrs., U. S. Army, Appendix II, 1875.

[38] "Irregular Ends on Concrete Test Cylinders," by D. V. Terrell, *Engineering and Contracting*, August 1, 1923.

[39] "Effect of End Condition of Cylinder on Compressive Strength of Concrete," by H. F. Gonnerman, *Proceedings*, A.S.T.M., Vol. 24, Pt. II, 1924.

[40] "On the Resistance of Glass Globes and Cylinders to Collapse from Internal Pressure, and on the Tensile and Compressive Strength of Various Kinds of Glass," by W. Fairbairn and T. Tate, *Proceedings*, Royal Soc. of London, May 12, 1859.

> "The specimens were crushed almost to powder by the violence of the concussion, when they gave way; it, however, appeared that the fracture occurred in vertical planes, splitting up the specimen in all directions. Cracks were noticed to form some time before the specimen finally gave way; then these rapidly increased in number, splitting the glass into innumerable irregular prisms of the same height as the cube. Finally, these bent or broke, and the pressure, no longer bedded on a firm surface, destroyed the specimen."

From 1900 to 1912 there was a lively discussion of the effect of elastic substances interposed between the loading surfaces in compression in the *Proceedings* of the International Association for Testing Materials.[41,42,43]

Capping of Concrete Cylinders.—An elaborate investigation of methods of capping the tops of 6-in. by 12-in. concrete cylinders, conducted under the writer's direction in 1923, showed that the strength of concrete was reduced by practically all of the yielding materials used. The principal data of these tests are given in Table 8, rearranged from Mr. Gonnerman's report.[39] The standard method consisted of a neat cement cap about $\frac{1}{8}$ in. thick applied to the fresh concrete 2 to 6 hours after molding the cylinders, by means of a planed cast-iron cover plate.

The values at the tops of the columns of strength ratios in Table 8 are the compressive strengths for standard capping method; each of these was based on 49 to 58 tests. Some of the conclusions are:

(*a*) Cylinders capped just prior to test with gypsum or mixtures of cement and gypsum gave sensibly the same strength as the standard method using neat cement.

(*b*) In general, compressible materials caused a loss in strength; the greatest loss was in the strongest concrete.

(*c*) The loss in strength from the use of sheet lead was about the same for thicknesses varying from 1/16 in. to 3/16 in., and was sensibly the same as for blotting paper or cork. For the strongest concrete (4,500 lb per sq in. at 28 days) the strength ratios from these materials was about 0.75.

(*d*) The greatest reduction in strength was found with sheet rubber; for the 1/16-in. thickness the ratios for the strongest concrete were 0.47 to 0.65.

The strength ratios for blotting paper in these tests and for those of Mr. Woodard using grease and blotting paper are as follows (length of specimens, two diameters):

Tests by	Weakest specimens	Strongest specimens
Gonnerman	0.92	0.77
Woodard	0.72	0.66

---

[41] "Baumaterialeinkunde," 1900, by A. Föppl, p. 129, and by F. Kick, p. 177.

[42] "On the Resistance of Stone to Compression, with Elastic Substances Interposed between the Surfaces in Compression," by S. C. Pace, *Paper B. 10e*, Brussels Cong., International Assn. for Testing Materials, 1906.

[43] "On the Question of Friction between the Parallel End Surfaces of a Body and the Plates through which Pressure Is Applied," by H. I. Hannover, *Proceedings*, VIth Cong., International Assn. for Testing Materials, Paper $XXVII_7$, 1912.

It should be noted that in Mr. Gonnerman's tests only the tops of the cylinders were affected by the various capping materials and methods; if the bottoms had been treated in the same manner, the observed reduction in strength would probably have been nearly doubled.

TABLE 8.—COMPRESSION TESTS OF CONCRETE CYLINDERS, USING DIFFERENT CAPPING MATERIALS

Capping material	Time[a]	Thickness of caps (in.)	STRENGTH RATIOS, LB PER SQ IN.[b]		
			1,080	2,150	4,500
Concrete mix by volume[c]	....	....	1 : 7	1 : 5	1 : 3.5
Neat cement; standard cap	....	....	1.00	1.00	1.00
Top smoothed by trowel; no cap	....	....	0.96	0.94	0.80
Top smoothed by grinder; no cap	....	....	1.00	1.06	1.03
Gypsum	0.25	....	0.97	1.04	1.01
	0.50	....	0.95	0.99	1.02
	1.00	....	0.96	1.01	1.03
	3.00	....	0.96	0.95	1.00
	6.00	....	0.97	1.01	0.99
	24.00	....	0.95	1.01	1.01
Cement-gypsum (1 : 1)	0.50	....	1.02	1.04	0.97
	1.00	....	1.02	1.00	0.98
	3.00	....	1.03	1.00	0.97
	6.00	....	1.00	0.98	1.00
	24.00	....	1.01	1.02	0.98
Cement-gypsum (3 : 1)	6.00	....	1.02	1.02	0.99
Cement-gypsum (1 : 3)	6.00	....	1.01	0.99	1.02
Beaver board	....	3/16	0.98	1.01	0.91
White pine	....	1/8	0.92	0.99	0.87
Millboard	....	1/16	1.01	0.92	0.81
Leather	....	9/32	0.96	0.91	0.82
Blotting paper	....	3/32	0.92	0.97	0.77
Cork carpet	....	5/16	0.93	0.87	0.70
Sheet lead	....	1/16	0.95	0.88	0.74
	....	1/8	0.91	0.90	0.74
	....	3/16	0.89	0.88	0.77
Sheet rubber	....	1/16	0.81	0.71	0.47
	....	1/8	0.89	0.77	0.65
	....	1/4	0.93	0.80	0.63

[a] Time of capping, in hours before test. [b] Cylinders were made of a mixture of four brands of portland cement purchased in Chicago, Ill. Values given are the averages of ten tests made on different days. [c] Flow about 220.

Tensile Strength of Mortar Briquets.—Mr. Woodard states (see heading, "Fundamental Problems of Stress in Terms of Strain"):

> "* * * the known tensile strength, divided by the observed compressive stress at time of failure equals Poisson's ratio as nearly as it can be determined."

This conclusion was based on tensile strengths of cement and mortar briquets of the form that are standard for cement tests,[44] and on compression tests of cylinders and prisms. If the quotient of these strengths equals Poisson's ratio, it was probably accidental. Several different methods of determining the tensile strength of mortars and concretes have been used. Each method gives a different tensile strength; and the briquet is probably the most unsatisfactory form of all. Since mortars fail at an elongation of about 0.001, and the most-stressed section of a briquet is only a fraction of an inch long, it is obvious that there is little opportunity for the stress to become uniformly distributed across the minimum section. The briquet represents a clear case of eccentric loading.

---

[44] "Standard Methods of Sampling and Physical Testing of Portland Cement," A.S.T.M. Standards, Pt. II, 1942, p. 33.

C. L. Durand-Claye[45] in 1895 gave an analytical method of calculating the variations of stress across the minimum section of a briquet. A. Föppl[7] in 1896 showed wide variations in the stresses in a briquet by measurements on an india rubber model. The late J. B. Johnson,[46] M. Am. Soc. C. E., in 1897 applied Durand-Claye's method of analysis to briquets of the form used in the United States. E. G. Coker,[47] in 1912 showed the wide range in these stresses by means of photoelastic observations on glass or celluloid models; and similar methods were used in 1939 by F. O. Anderegg, Royal Weller, and B. Fried.[48] These studies showed a high concentration of stresses at the edges of the waist of the U. S. Standard briquet, as follows:

Authority	Date	Ratio of maximum to average stress
Johnson	1897	1.54
Coker	1912	1.70
Anderegg	1939	1.78
Average		1.67

The failure of the briquet results from a tearing action rather than from a true tensile stress. As the cement becomes older and stiffer, the discrepancy between the indicated and the true strength becomes more and more pronounced. This accounts for phenomena that have been observed many times —namely, a rapid increase in tensile strength of cement mortars to 1 to 2 months followed by a gradual falling off, until the indicated briquet strength at 1 year was generally less than at 7 days, whereas the compressive strength of the same mortars and of concrete from the same cement continued to increase indefinitely, as long as the specimens did not dry out.

The briquet is a carry-over from the infancy of the art. It was adapted by John Grant[49] in London in 1859 from the practice of stone testing, and has undergone little change during the succeeding 84 years. In Germany the briquet was abandoned in favor of the cube for cement mortar tests a generation ago. The inadequacies of the briquet have been commented on by a number of American writers in addition to those already referred to.[50,51,52] Erroneous attempts to reconcile the composition of cement and the behavior of concrete with the falling-off in briquet strength are responsible for much of the confusion that has characterized the history of cement and concrete technology in the United States.

---

[45] *Annales des Ponts et Chaussées*, Durand-Claye, June, 1895.

[46] "Materials of Construction," by J. B. Johnson, 1st Ed., 1897, Wiley, New York, N. Y., p. 437.

[47] "Distribution of Stress at the Minimum Section of a Cement Briquette," by E. G. Coker, *Proceedings*, VIth Cong., International Assn. for Testing Materials, Paper XXVIII$_4$, 1912.

[48] "Tension Specimens Shape and Apparent Strength," by F. O. Anderegg, Royal Weller, and B. Fried, *Proceedings*, A.S.T.M. 1939, p. 1261.

[49] Experiments on the Strength of Cement," by John Grant, *Proceedings*, Institution of Civ. Engrs., London, 1865–66, and 1870–71; also book of same title, 1875.

[50] "Notes on Cement Testing," by W. P. Taylor, paper before 2d annual convention, Assn. of American Portland Cement Mfrs., December, 1904.

[51] "Data Considered by A.S.T.M. Committee on Cement," Pt. VII, 1916, p. 74.

[52] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941.

Tensile and Flexural Strength of Concrete.—The difficulties of determining the tensile strength of concrete may be demonstrated by data from a paper[53] by H. F. Gonnerman and E. C. Shuman, Assoc. M. Am. Soc. C. E., in which an attempt was made to determine the relation between compressive, flexure, and tensile strength of the same concrete. Table 9 gives compressive strengths of 6-in. by 12-in. cylinders for three water-cement ratios at ages from 3 days to 1 year, together with values for modulus of rupture and tension expressed as fractions of the compressive strength. Tension tests were made by applying loads to 6-in. by 18-in. cylinders by means of heavy cylindrical shackles clamped to the ends of the cylinder. This method is subject to the same criticism as the briquet, in that it does not apply a uniform tensile stress across the section of the cylinder. Flexural tests were made by loading plain concrete beams (7 in. by 10 in. by 38 in.) at the 1/3 points of a 36-in. span.

TABLE 9.—COMPRESSION, TENSION, and FLEXURE OF PLAIN CONCRETE[a]

Water-cement ratio	AGE AT TEST				
	3 Days	7 Days	28 Days	3 Months	1 Year
(*a*) COMPRESSIVE STRENGTH, POUNDS PER SQUARE INCH					
0.68	2,000	3,100	4,710	6,460	7,420
0.76	1,560	2,620	4,460	5,710	6,720
0.87	1,070	1,900	3,560	5,130	6,000
Average...	1,540	2,540	4,240	5,770	6,710
(*b*) RATIO OF TENSION TO COMPRESSION					
0.68	0.105	0.089	0.081	0.073	0.077
0.76	0.112	0.092	0.081	0.073	0.078
0.87	0.126	0.116	0.089	0.081	0.072
Average...	0.114	0.099	0.084	0.076	0.076
(*c*) RATIO OF MODULUS OF RUPTURE TO COMPRESSION					
0.68	0.177	0.154	0.135	0.114	0.114
0.76	0.214	0.170	0.135	0.132	0.125
0.87	0.248	0.196	0.164	0.141	0.132
Average...	0.213	0.173	0.145	0.129	0.124
(*d*) RATIO OF TENSILE STRENGTH TO MODULUS OF RUPTURE					
....	0.54	0.56	0.58	0.59	0.61

[a] Mix, Portland cement to total aggregates was 1 to 4 by volume. Specimens were moist cured and each value is the average of five tests.

The modulus of rupture of beams and the tensile strength varied with both water-cement ratio and age; at 28 days the average modulus of rupture was 0.145 and the average tensile strength was 0.084 of compressive strength. According to Mr. Woodard's analysis, all of these values represent Poisson's ratios; but it is seen that the values varied from 0.072 for the 1-yr concrete in tension to 0.248 for the 3-day concrete in bending. The flexural strength ratios more nearly correspond to the generally-accepted values for Poisson's ratio.

---

[53] "Compression, Flexure and Tension Tests of Plain Concrete," by H. F. Gonnerman and E. C. Shuman, *Proceedings*, A.S.T.M., Vol. 28, Pt. II, 1928, p. 527.

Particular attention is called to the values in Table 9(*d*), which show that the average tensile strength ranged from 0.54 of the modulus of rupture at 3 days to 0.61 at 1 year. These values correspond closely to the value reported herein under "Tensile Strength of Mortar Briquets," where it was shown that the maximum stress at the edge of a briquet was 67% higher than the average, and indicate that Mr. Woodard's value of Poisson's ratio as computed from the tensile strength of briquets and compression tests of prisms is 67% too high.

Standard Compression Test of Concrete.—Since 1921 American engineers have followed generally the method of compression tests of concrete that was adopted in that year by the American Society for Testing Materials,[54] in which the load is applied directly to the plane ends of the cylinders. For the usual sizes of aggregates, cylinders 6 in. in diameter and 12 in. long are used; for larger aggregates 8-in. by 16-in., or larger, cylinders are used; the U. S. Bureau of Reclamation has used 36-in. by 72-in. cylinders in making compression tests of concrete containing gravel graded up to 9 in.[55]

Mr. Woodard suggests (see "Conclusions") that this method:

> " * * * should be revised to eliminate friction which can be done easily, or a correction factor should be determined to apply to the results of tests as now made."

This suggestion seems to the writer of doubtful merit for several reasons: (*a*) Basing design stresses on the 28-day compressive strength of concrete is purely arbitrary; (*b*) concrete in structures is seldom subjected to the type of compressive stress encountered in a standard test cylinder; (*c*) testing concrete in the form of 6-in. by 12-in. cylinders that have been moist cured for 28 days at room temperature is conventional; quite different strengths would be obtained if other sizes of cylinders or if another ratio of length to diameter were used, or if the age, curing conditions, or rate of loading were changed; (*d*) other factors being equal, the early strength of concrete depends to a large extent on the composition and fineness of the Portland cement.

Effect of Size of Concrete Cylinder.—Compression tests by the U. S. Bureau of Reclamation[55] on concrete cylinders with length equal to two diameters showed the following strength ratios, considering the 6-in. by 12-in. cylinder as 1.00:

Diameter of cylinder, in inches	Strength ratio
2	1.09
3	1.06
6	1.00
8	0.96
12	0.91
18	0.86
24	0.84
36	0.82

---

[54] "Standard Method of Test for Compressive Strength of Concrete," *A.S.T.M. Standards*, Pt. II, 1942, p. 328.

[55] "Concrete Manual," U. S. Bureau of Reclamation 1942.

This gives an extreme range from the lowest to the highest of 33%. Doubling the diameter was accompanied by a loss of 7% in terms of the "standard" strength of 6-in. by 12-in. cylinders.

Composition and Fineness of Cement.—The way in which the early strength of concrete is increased by improved compound composition and greater fineness is illustrated by parallel tests by E. Gruenwald[56] on normal and high-early-strength Portland cements of 1938. Typical results are summarized in Table 10. These tests were made on 6-in. by 12-in. cylinders using five sacks of cement per cubic yard, 2-in. slump, and water-cement ratio 0.88, by volume.

TABLE 10.—STRENGTH OF CONCRETE USING TYPICAL NORMAL AND HIGH-EARLY-STRENGTH PORTLAND CEMENTS

Portland cement	Tricalcium silicate (%)	Surface area (sq cm per g)	COMPRESSIVE STRENGTH CONCRETE (LB PER SQ IN.)				
			1 Day	3 Days	7 Days	28 Days	3 Months
Normal	47	1,745	480	2,180	3,075	4,675	4,915
High-early	58	2,440	1,795	3,720	4,360	4,915	5,415

An unpublished study of the literature by the writer shows that the average 28-day strength of moist-cured concrete made of American Portland cements, of water-cement ratio 0.80 by volume (0.53 by weight), was about 2,500 lb per sq in. in 1918, but that it increased to 5,000 lb per sq in. in 1940. This resulted from a general improvement in the compound composition of the cement (more uniform burning at higher temperatures) and to finer grinding. This increase would no doubt have continued to the present, had it not been for the lower quality of Portland cement ordered by the War Production Board in August, 1942.[57]

It is well known that the accuracy of the plane surfaces that are exposed to the loading apparatus has an important effect on the compressive strength of stone and similar materials. It was noted by Mr. Merrill,[36] in 1910, that the reported compressive strengths of rocks showed a remarkable increase during 60 years. He properly attributed this to greater refinements in the preparation of the ends of the specimens. However, it does not explain the aforementioned 100% increase in the average strength of concrete, since there have been no significant changes in the methods of mixing, molding, curing, or loading the standard test cylinders. Separate studies of the effect of the compound composition of Portland cement (especially the quantity of tricalcium silicate) and of the fineness of grinding show that the increase of 100% in average concrete strength from 1918 to 1940 can be fully accounted for by these factors. Failure to recognize this notable rise in the "concrete strength level" of Portland cements has been responsible for much confusion of thought, both in scientific studies of concrete mix design and in application of these principles to construction problems.

---

[56] "Lean Concrete Mixes," by E. Gruenwald, *Proceedings*, A.S.T.M., 1939, p. 810.

[57] "Limitation Order 179, Part 3033—Portland Cement," WPB, August 3, 1942; amended August 24, September 24, 1942, February 16 and March 23, 1943.

Standard Test Cylinders Represent Idealized Conditions.—The foregoing paragraphs show that any strength test of concrete is purely relative and that nothing would be gained by changes in the test method that merely lower the strength by a fixed percentage, such as lubricating the ends of the cylinders, as suggested by Mr. Woodard.

Although the average 28-day concrete strength increased 100% from 1918 to 1940, there was no commensurate increase in the design working stresses in concrete; nor should there have been. The composition and fineness of the cement largely affect the 7-day and 28-day strength of concrete, but they do not exert proportional effects on the strength at 1 year, or 5 years. The normal Portland cement in Table 10 gave a strength at three days and the high-early-strength cement at one day that was equivalent to the 28-day strength of concrete at the time the 28-day strength was arbitrarily chosen as a basis for design stresses.

The compressive strength of concrete cylinders represents idealized conditions that rarely exist in the structure. There are always questions as to whether the concrete in the structure is as carefully placed and whether it receives a curing treatment that will enable it to gain strength, even at 28 days, comparable to that of the test cylinders made under standard conditions. This fact, however, does not minimize the importance of concrete tests made in connection with building operations, since these idealized cylinders do give a measure of the potential concrete quality. Whether this potential quality is realized on the job depends on many factors, most of which are well understood by those skilled in the art. In most applications the strength of concrete must be subordinated to durability. Durability of exposed concrete is more dependent on the use of a plastic mix of a low water-cement ratio than on the early strength. The evidence indicates that the concrete that gave 2,500 lb per sq in. at 28 days in 1918 probably is as durable as the concrete that gave 5,000 lb per sq in. in 1940.

Summary.—The writer maintains that Mr. Woodard:

(1) Completely misinterpreted his briquet tests and in doing so overlooked or ignored information that had been available for nearly 50 years. This mistake of underestimating by about 67% in the true tensile strength of mortars nullifies practically everything that follows.

(2) In his conclusions on greased blotting paper between load surfaces of mortar prisms, he overlooked similar results from using lead, as determined by tests on stone in 1851; and overlooked concrete tests made in 1923 in which blotting paper alone gave the same result.

(3) Mistakenly, he has recommended modifying the standard compression test of concrete to require lubricated ends.

C. A. P. TURNER,[58] M. AM. SOC. C. E.—Illogical deductions (see heading, "Shear and Stress") presented in Mr. Woodard's paper are that:

> "* * * (*a*) in materials strained by exterior forces including shearing forces, only two kinds of stresses are set up—compressive stresses and

---

[58] Cons. Engr., Columbus, Ohio.

tensile stresses—and (*b*) shear is accompanied by and resisted by internal forces of tension and compression. In other words, there is no such thing as shearing stress or shearing strength as distinguished from compressive and tensile stresses and compressive and tensile strength."

Axial forces cause elongation: Positive if in tension, negative if in compression. The unit stress under these conditions is $s = \frac{P}{A}$. In simple elongation, rotational strains accompany elongation. These are best visualized by a strip of rubber 1 in. in width marked off in 1-in. squares. By drawing diagonal lines from corner to corner of the squares with white lead, a kind of pantagraph frame is formed by the white lines. If the strip is stretched, these diagonal lines rotate about their intersection, disclosing the angular distortion of two-dimensional displacement. These rotational actions differentiate rotational shear strain from simple elongation.

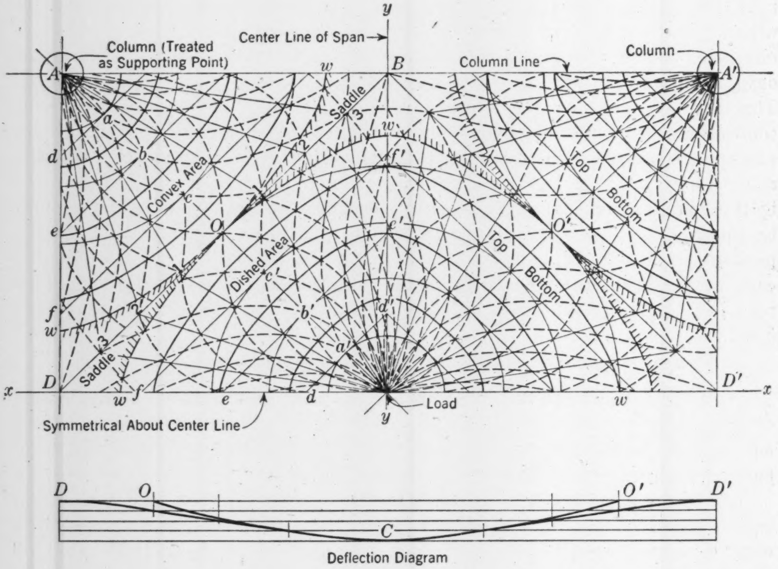

y
Column (Treated as Supporting Point)
Center Line of Span
Column Line
Column
A
w
B
A′
a
Saddle
3
2
w
d
b
f′
Convex Area
1
Top
Bottom
c
e
O
e′
O′
Top
Bottom
Dished Area
c′
1
2
f
b′
d′
w
3
a′
Saddle
D
D′
x
x
w
f
e
d
w
Load
y
Symmetrical About Center Line
D
O
O′
D′
C
Deflection Diagram

Fig. 12.—Stress-Line Analysis of a Column Supported, Centrally Loaded, Flat Slab

Pure shear is found in the torsion of the shaft or the twist in a square flat plate under load supported on four sides, or in the case of continuous plate supported by spaced columns. Fig. 12 represents a stress-line analysis of the latter type of plate, and the traces of the spiral right and left shears on top and bottom surfaces are presented in the diagram. The coordinates of these shears are the planes of zero shear and greatest vertical shear and of zero shear and greatest twist. These shears fix the contours of curvature under load by their intersections. The contours are principal stresses at the middepth of the plate. The twisting shears which develop the contours make variable angles with the principal strain far different from 45° as given by the author. Warping of the plate generates twisting couples about the axis of x, y, and z. The sign of the twisting couples changes at the point of contraflexure and at the principal planes.[59]

Shears and Principal Stresses in Beam Action.—When a load is placed on a simply supported beam, the beam bends down under the load. The top fibers are shortened and the bottom fibers are stretched, causing a trapezoidal distortion on the vertical face of the beam. The reaction of these positive and negative deformations causes shearing stresses in the material as illustrated by the deportment of two planks of the same length, one on top of the other, with the ends in vertical planes. As the planks bend under the load, the lower corners of the upper plank slide by the upper corners of the lower plank; and, when arranged in this way, the combined stiffness and load capacity is that of two planks. If, instead of allowing the planks to slide one upon the other, they are glued together, the strength is then equal to that of four single planks and the stiffness is increased to that of eight single planks, sliding the one on the other.

The stress brought on the glue is a shearing stress, and its relative intensity at any point is proportional to the amount of sliding that would occur were the planks not so glued together. This shear causes rhombic distortion, which, in a simply-supported beam, increases from zero at the center to a maximum at the end and, in a restrained beam, increases from the center to the point of inflection, decreasing from that point to zero at the support.

The vertical cross section at midspan is the critical section resisting horizontal elongation of bending, whereas the horizontal longitudinal cross section at middepth resists the horizontal shearing stress. Because the latter cross section is far greater in area than the former, the deflection from the shear strain to which it is subjected is so small in a rectangular section when the length is greater than twelve times the depth that it is neglected in the common or approximate theory of flexure.

The building codes of all North American cities recognize three kinds of beam failure: (1) Compression failure caused by crushing the top fibers under maximum moment; (2) tension failure caused by breaking the bottom fibers in tension; and (3) shear failure caused by combination of rhombic and trapezoidal shear strains. The latter are measured by the sharpness of curvature.

For safety, building codes provide limiting fiber stresses for tension and compression, and safeguard against shear failure by specifying fixed limits for the value of the quantity, external shear force, divided by the area of the beam at the support.

Thus, by observation, building code authors recognize shear failure in beam resistance and are a step in advance of Mr. Woodard's thesis. However, the safeguard which the building codes provide against shear failure is founded on complacent ignorance as to the locus[60] of greatest strain; also there is wide[60] divergence as to its magnitude under the same external shear, depending on the distribution of load and the restraint at the support.

---

[59] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 286–292.

In treating nonhomogenous concrete by the same theory as homogenous material, Mr. Woodard falls into an interesting error. The neutral plane of homogenous and concrete beams is the locus of consecutive states of right shearing stress, functioning as centers of rotation for each elemental section. Incidentally, in the homogenous beam, this locus is where tension and compression balance, and principal tension and compression make angles of 45° with the plane. Otherwise, the locus is in the reinforced concrete beam,[61] because the tension of the steel at the bottom equilibrates the compression in the concrete by bond shear at the surface of the steel, instead of at the neutral plane. As a result, the states of right shearing stress as centers of rotation in the neutral plane are twisted 22.5° [61] from their position in the homogenous beam. Consequently, the principal compression and horizontal shear intersect the neutral plane at 22.5°, and the curve of vertical shear and principal tension intersects that plane at 67.5°.[61] Therefore, elastic deflection of the concrete beam increases more rapidly than the load, because the neutral plane is compressed under load.

Mr. Woodard's treatment of torsion of a shaft is erroneous. The torsional moment is coordinated readily with the shear stress on a transverse section without reference to tension or compression in the material. The simplest solution is that of shear stress volume times lever,[62] which agrees with Rankine's analysis.[63]

The distribution[64] of tensile and compressive stresses and their reverse summation precludes the possibility of coordinating the torsional moment with their reactions. On a round shaft a neutral cylinder[64] (radius, 0.7071 r) is developed, on which every element is under no strain; but inside this cylinder every fiber parallel to the axis is stretched, and outside it every fiber is compressed. In the cross section all radial lines are under compression, increasing from the outer fiber to the axis; and all circular lines are under tension, which is at a maximum at the outer fiber and decreases toward the axis. Pure shear on the cross-sectional area has the same unit intensity from the axis to the surface. The spiral tensions and compressions on the surface of the shaft and each annular cylinder within intersect the cross section of pure shear at 45° only at the neutral cylinder. Their obliquity changes from 45° as radically as the twisting shears in the plate problem of Fig. 12.

The complexity of the three-dimensional strain of torsion precludes coordination of the reaction of tensile and compressive stresses with the torsional moment.

---

[60] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 166–167, 192–194.

[61] *Ibid.*, pp. 234–235.

[62] *Ibid.*, pp. 11–12.

[63] "Applied Mechanics," by W. J. M. Rankine, Arts. 320–321, pp. 353–355.

[64] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 239–251.

Mark W. Huggins,[65] **Esq.**—Although the writer cannot agree with the suggestions presented in this paper, he feels that its publication is certain to fulfil a useful purpose, if only by stirring up further discussion of this most important problem. The importance of this subject does not appear to be realized fully by engineers in general. The truth of this statement will be apparent to any one who studies any of the modern specifications for allowable stress. So far as the writer is aware, no specification attempts to outline any procedure which may be followed in determining safe stresses in cases of combined stress. The same stress, which is considered safe in the case of pure tension also may be used in designs in which a complex state of stress exists, provided only that the maximum principal stress does not exceed the specified limiting value for pure tension. Numerous tests, such as those published in Table 1 of the paper and in many texts on strength of materials[66] illustrate the inaccuracy of this procedure.

Any suggestion for changing the standard nomenclature of mechanics of materials is certain to lead to nothing but confusion. It has been accepted generally that the term "stress" shall apply to true forces and that "strain" shall apply to true deformation. The use of the term "stress" to define some artificial quantity, which is always proportional to strain, is not advisable as the same result might be attained just as well by setting an upper allowable limit on strain rather than on stress.

The author states that there is no need to consider "an imaginary and inexplicable thing called shear stress" in obtaining an answer to the question of how shear is resisted. He maintains that stresses are limited to tensile and compressive stresses. This hypothesis does not give a very satisfactory interpretation of the cup-cone type of failure which occurs in tension test pieces of mild steel, in which the effect of the ends would not appear to have any considerable effect on the type of failure, provided that the length of the reduced-area section is sufficiently large. The final failure is obviously of a shear type and occurs across a plane for which the normal strain is less than that in the axial direction. The initial cause of failure, in this case, is sliding along 45° planes, which produces necking. At a certain stage in the loading, the cohesive resistance of the middle part of the bar is overcome, causing a crack of the brittle type, but, at the same time, yielding by sliding continues near the surface and the final sliding fracture occurs near the boundary.[67]

A. A. Eremin,[68] **Assoc. M. Am. Soc. C. E.**—It is true, as the author states, that friction resistance at the ends of a short concrete test prism results in the appearance of an increased compressive strength in the concrete. More exact results may be obtained if the test block is made higher. Josef Fritsche[69] has shown that the compressive strength of concrete determined by a compressive test of a concrete block, with a ratio of height to width varying from 4 to 6, differs only about 20% from its true value. M. Möller[70] and later D. C. Henny,[71] M. Am. Soc. C. E., developed a method of computing the strength of concrete based on shear strength determined with a concrete test prism. The results of shear strength tests were correlated with strength tests of a concrete column and the error was found to be less than 20%.

---

[65] Asst. Prof. of Civ. Eng., Univ. of Toronto, Toronto, Canada.

[66] "Strength of Materials," by John Case, Edward Arnold & Co., London, 1938, 3d Ed., pp. 85–96.

[67] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 418–421.

[68] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[69] *Beton und Eisen*, April 5, 1935, p. 113.

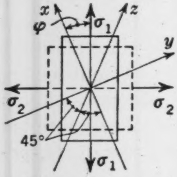


FIG. 13.—PRINCIPAL STRESSES

In the formulas for stresses and strains at an oblique section of a prism, Mr. Woodard has considered the case in which only direct stresses were applied. In practice, however, the principal stresses are generally considered along two axes. An interesting method of computing principal stresses in terms of measured strains ϵ_x, ϵ_y, and ϵ_z along three axes x, y, and z at angles of 45° was presented in 1934.[72]

In Fig. 13 the principal stresses σ_1 and σ_2 are,

$$\sigma_1 = \frac{E}{1 - \mu^2} (\epsilon_1 + \mu \, \epsilon_2) \qquad (32a)$$

$$\sigma_2 = \frac{E}{1 - \mu^2} (\epsilon_2 + \mu \, \epsilon_1) \qquad (32b)$$

and

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} \qquad (32c)$$

in which

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2 \cos 2 \phi} \qquad (33a)$$

$$\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2 \cos 2 \phi} \qquad (33b)$$

and

$$\tan 2 \phi = \frac{2 \, \epsilon_z - \epsilon_x - \epsilon_y}{\epsilon_x - \epsilon_y} \qquad (33c)$$

With the known modulus of elasticity E, Poisson's ratio μ, and Eqs. 32, stresses and strains at any oblique section may be computed.

In conclusion, Mr. Woodard states that Poisson's ratio in the more accurate computations must be considered. It is to be noted that, in some cases, reliable results may be obtained only if Poisson's ratio is considered.

The term "shear stress," as used by Mr. Woodard, is a misnomer. It is true that the conception of shear stress is based on assumptions that differ from those on which direct stresses are based. However, this fact entails no difficulties in computations of structural materials energy and deformations. In the more accurate computations, important consideration should be given to the limitations resulting from the assumption of straight-line variation of stresses and strains and also the neglect of plastic flow.

---

[70] *Beton und Eisen*, December 20, 1930, p. 435.

[71] "Stability of Straight Concrete Gravity Dams" by D. C. Henny, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1041.

[72] "La Détermination par Points, à l'Aide des Élastomètres, de la Répartition des Tensions dans les Pièces Métalliques," *Le Génie Civil*, January 13, 1934, p. 40.

SILAS H. WOODARD,[73] M. AM. SOC. C. E.—Most of the critical discussions of the paper (nearly all are adverse criticisms which are not in agreement with each other) are flat rejections of the theory advanced in the paper because it does not agree with some other theory or because it does not fit the discusser's mental concept of the nature of stress. With one exception there is no citation of observed facts or recorded behavior to oppose the theory. The exception is the citation by Mr. Abrams of the relative compression tests of concrete cylinders ranging from 2 in. to 36 in. in diameter. However, the writer cannot accept those results as conclusive evidence: (1) Because the cylinders were tested with restrained ends and were not lubricated; and (2) because the material tested was concrete with coarse aggregate in the large cylinders and fine aggregate in the small cylinders. Even the mortar in concrete with aggregates 9 in. in diameter must have differed from the mortar in concrete with ½-in. maximum size of aggregate. Such differences would occur in relative workability or in water-cement ratio, or the density and workability of the entire mass would differ for different sizes of samples.

As to the nature of stress, the writer has canvassed enough engineers and physicists to learn that, although mental concepts of stresses are not all of the same pattern, his own are not entirely unique. The common practice of using the term "stress" as a ratio should not be confusing because it is still an expression of force per unit area. To be of use the computed stress should be a measure of distress. The engineer wants to know the margin of safety in his design.

For tensile stresses, the writer's concept is that, between each pair of adjacent particles of a solid, there is an elastic force which resists separation or nearer approach of the particles. So long as no other forces are acting, the distance between the particles remains constant, but other forces may be applied to separate the particles further. This applied force is resisted by the elastic force tying the particles together. If the forced separation becomes too great, the elastic resisting force reaches its limit and lets go; or, in other words, each particle passes outside the field of action of the resisting force.

When and if the physicists discover the exact nature of this elastic resisting force, the engineer will probably still rely largely on stress calculations.

The chief purpose of the paper was to emphasize the necessity for the use of Poisson's ratio in more refined stress computations. The distortion phenomenon expressed by Poisson's ratio is believed to induce stress not covered by the simple equation of external forces. The possibility of these induced stresses should not be dismissed simply by saying that they are inconceivable.

In an amorphous solid the particles are presumably arranged in a haphazard manner like marbles in a bag so that, if a force, tension or compression, is to travel from a point or particle on one side of the solid to an equal and opposite force on the other side of the solid, it must follow a zigzag course from particle to particle. At all the angle points of the zigzag line, components of force must be resisted either by external or internal force; if no external force exists, the resisting force must be internal. In the interior of the solid the components of adjacent zigzag lines, of course, will tend to balance each other, but not so at its surface. The net sum of all of the components of the principal forces or stresses is an induced stress normal to the direction of the principal stress.

---

[73] Cons. Engr., New York, N. Y.

If the principal stress is compression, the induced stress will be a tension normal to the principal stress and in that direction; the dimension of the solid will be increased following the law of Poisson's ratio; the particles in that dimension will be less closely spaced; and, if the limit of tensile strength is reached, they will separate, resulting in a tensile failure. This cannot be called inconceivable especially as brittle amorphous solids do behave that way, and the writer cannot conceive of particles being torn apart except by tensile stress.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2215

AERATION OF SPILLWAYS

BY G. H. HICKOX,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOE W. JOHNSON, J. W. HOWE AND CLAUD C. LOMAX, JR., J. C. STEVENS, AND G. H. HICKOX.

SYNOPSIS

A method of computing the size of air vent needed for aeration of spillways, based on all of the available data, is presented in this paper. The effect of insufficient aeration on the reduction of pressure beneath the nappe and on the discharge is discussed. Formulas and diagrams for calculation of these effects are included.

INTRODUCTION

The aeration of spillways and measuring weirs has apparently not been given the attention it deserves. The direct result of insufficient aeration of overfall weirs is a reduction of pressure beneath the nappe due to the removal of air by the overfalling jet. This reduction of pressure in turn causes several undesirable secondary effects. The first is an increase of pressure difference on the weir itself. The reduction of pressure on the downstream side may increase the resultant load on the structure to the point of failure. In the case of dam A, spillway discharge flows over a gate 20 ft high. This gate was designed for a water load 20 ft over its crest, or 37,400 lb per ft of length. During tests on the aeration of this spillway, negative pressures under the nappe of 9.7 ft of water were measured. This was equivalent to an additional load of 12,100 lb per ft of length, or an increase of 32%. The significance of this increase in load becomes more apparent when it is noted that this was by no means the maximum pressure reduction that could have been obtained by closing the aeration vents completely. In this particular case, it represented the maximum extra load which it was thought the gate could support with safety.

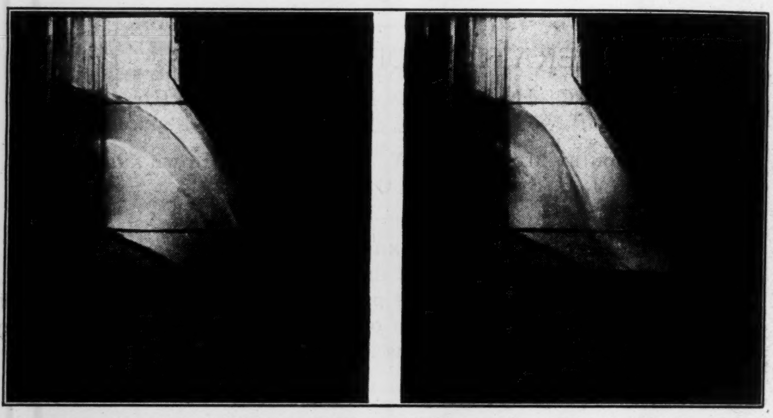
The second effect is a change in the appearance of the nappe. Because the difference in pressure acts across the nappe itself, the nappe is depressed below its normal position. In the case of large spillways, the upper surface becomes ridged in the direction of flow and begins to entrain air instead of

---

NOTE.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

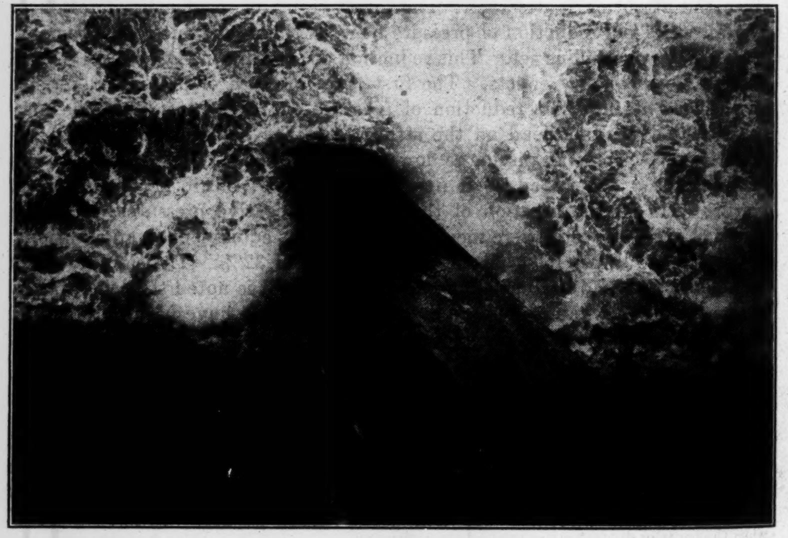
[1] Senior Hydr. Engr., TVA Hydraulic Laboratory, Norris, Tenn.

being smooth as for normal flow conditions. For thin nappes and considerable pressure reduction, the nappe may break, allowing air to pass through it. As soon as this occurs, the nappe springs back to its normal position and the process of air exhaustion and breaking of the nappe is repeated. Depression of the nappe because of insufficient aeration is illustrated on a small scale model by Figs. 1(a) and 1(b) and for a large spillway by Fig. 1(c).



a) Discharge Over Model (1 : 25 Scale) of Lower Gate Leaf; Full (or Complete) Aeration

(b) Discharge Over Model (1 : 25 Scale) of Lower Gate Leaf; No Aeration



(c) Prototype: Gate at Left Aerated; 2-Ft Vacuum Under Nappe at Right

Fig. 1.—Depression of the Nappe Because of Insufficient Aeration, Dam A

A third result of the reduction of pressure beneath the nappe is an increase in the pressure difference causing flow over the weir, and a corresponding increase in the discharge coefficient. For a given head, the discharge is increased. If the discharge is fixed, the measured head is reduced.

Reduction of pressure beneath a spillway nappe is also undesirable from the standpoint of the performance of hydraulic models. Fig. 1(*b*) illustrates a pressure reduction of about 1.5 ft in a 1 : 25 scale model, or 37 ft in the prototype; a result which is clearly not possible. In some cases, the performance of a stilling basin may be completely altered because of the difference in the manner in which the spillway discharge enters it, due entirely to lack of aeration of the nappe.

Where a weir is to be used for measuring purposes, lack of aeration results in an increased discharge, and sometimes, as mentioned, a fluctuation or pulsation of the nappe, which may be very objectionable.

There is a definite need for information on the effect of lack of aeration of weirs. Such information is necessary (*a*) to prevent excessive loads on spillway gates; (*b*) to insure the reliability of model tests, with respect both to the pressure reduction to be expected in the prototype, and the performance of the stilling basin; and (*c*) to provide a criterion for the necessary aeration of measuring weirs.

NOMENCLATURE

The letter symbols used in this paper are defined where they first appear in the paper and are assembled for reference in the Appendix.

EXPERIMENTAL DATA

The problem of aeration became important with the construction of dam A. There are twenty-two spillway gates at this dam, each built in two sections 20 ft by 40 ft, operating in the same vertical plane. Discharge is regulated by first lifting the upper leaves and allowing the water to flow over the tops of the lower leaves. The lower leaves are not raised until after all the upper leaves are open. The design of the piers is such that self-aeration is impossible and aeration vents are necessary. The same gate design and similar operation were provided for dams B, C, and D. At dam D, the problem was complicated by the fact that, under certain conditions, water might discharge beneath the lower gates as well as above them. A knowledge of the size of aeration vents required was necessary in designing these projects,

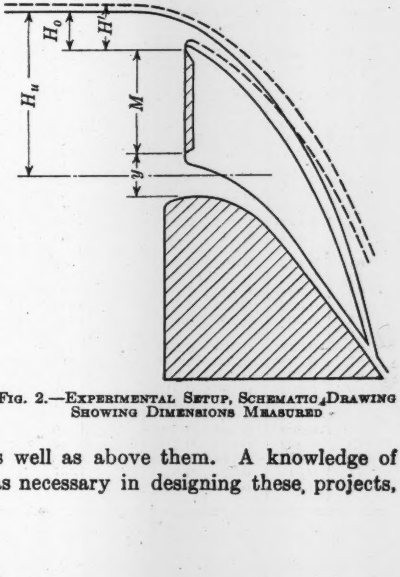


FIG. 2.—EXPERIMENTAL SETUP, SCHEMATIC DRAWING SHOWING DIMENSIONS MEASURED

TABLE 1.—AERATION OF A SHARP-CRESTED WEIR (VARIOUS STRUCTURES)

Test No.	FEET		CUBIC FEET PER SECOND			p (ft)[b]	$\frac{H_e}{H'}$	$\frac{p}{H_e}$	α	β
	H'	H_e	Q_e	q_e	q_a					
(1)	(2)	(3)	(4)	(5)	(6)[a]	(7)	(8)	(9)	(10)	(11)
(a) DATA REPORTED BY JOE W. JOHNSON, ASSOC. M. AM. SOC. C. E.										
1	0.306	0.295	0.59	....	401	0.0672	0.964	0.228	....	....
2	0.295	0.284	0.56	....	415	0.0695	0.963	0.245	....	....
3	0.276	0.268	0.506	....	351	0.0588	0.971	0.219	....	....
4	0.252	0.246	0.44	....	271	0.0454	0.976	0.185	....	....
5	0.222	0.219	0.361	....	131	0.0219	0.986	0.100	....	....
6	0.306	0.297	0.59	....	367	0.0615	0.970	0.207	....	....
7	0.273	0.267	0.498	....	245	0.0411	0.978	0.154	....	....
8	0.245	0.239	0.420	....	208	0.0349	0.975	0.146	....	....
9	0.228	0.225	0.376	....	157	0.0272	0.987	0.121	....	....
14	0.255	0.250	0.446	....	176	0.0295	0.981	0.118	....	....
18	0.306	0.299	0.59	....	390	0.0653	0.977	0.218	....	....
19	0.306	0.297	0.59	....	458	0.0769	0.970	0.259	....	....
20	0.306	0.302	0.59	....	294	0.0482	0.986	0.160	....	....
22	0.284	0.276	0.528	....	332	0.055	0.972	0.199	....	....
23	0.284	0.277	0.528	....	263	0.044	0.975	0.159	....	....
24	0.284	0.277	0.528	....	247	0.0413	0.975	0.149	....	....
25	0.284	0.281	0.528	....	93	0.0156	0.990	0.056	....	....
26	0.253	0.247	0.441	....	224	0.0375	0.976	0.152	....	....
27	0.253	0.247	0.441	....	180	0.0301	0.976	0.122	....	....
28	0.253	0.249	0.441	....	131	0.0221	0.985	0.089	....	....
30	0.239	0.235	0.404	....	145	0.0243	0.983	0.103	....	....
31	0.239	0.235	0.404	....	118	0.0198	0.983	0.084	....	....
32	0.239	0.237	0.404	....	81	0.0135	0.992	0.057	....	....
33	0.205	0.201	0.320	....	40	0.0335	0.980	0.167	....	....
36	0.307	0.299	0.59	....	304	0.051	0.975	0.171	....	....
37	0.307	0.300	0.59	....	244	0.041	0.978	0.137	....	....
38	0.307	0.304	0.59	....	102	0.017	0.990	0.056	....	....
44	0.308	0.299	0.59	....	441	0.074	0.971	0.248	....	....
45	0.308	0.300	0.59	....	364	0.061	0.974	0.204	....	....
46	0.308	0.301	0.59	....	312	0.052	0.977	0.173	....	....
47	0.308	0.305	0.59	....	96	0.0162	0.990	0.053	....	....
48	0.284	0.276	0.528	....	380	0.0635	0.971	0.230	....	....
49	0.284	0.276	0.528	....	297	0.0488	0.971	0.177	....	....
50	0.284	0.276	0.528	....	250	0.0424	0.971	0.153	....	....
51	0.284	0.279	0.528	....	104	0.0174	0.982	0.062	....	....
52	0.260	0.252	0.46	....	344	0.0576	0.970	0.229	....	....
53	0.260	0.253	0.46	....	260	0.0434	0.973	0.172	....	....
54	0.260	0.255	0.46	....	185	0.0309	0.981	0.121	....	....
55	0.227	0.223	0.376	....	138	0.023	0.983	0.103	....	....
56	0.227	0.224	0.376	....	120	0.020	0.987	0.089	....	....
57	0.227	0.223	0.376	....	84	0.014	0.983	0.063	....	....
58	0.227	0.224	0.376	....	44	0.0073	0.987	0.033	....	....
59	0.193	0.191	0.297	....	73	0.0122	0.990	0.064	....	....
60	0.193	0.191	0.297	....	60	0.0099	0.990	0.052	....	....
61	0.193	0.192	0.297	....	42	0.0071	0.995	0.037	....	....
62	0.308	0.295	0.597	....	577	0.0966	0.957	0.328	....	....
63	0.308	0.296	0.597	....	490	0.082	0.961	0.277	....	....
64	0.308	0.299	0.597	....	330	0.0552	0.970	0.185	....	....
65	0.308	0.297	0.597	....	502	0.084	0.964	0.283	....	....
66	0.308	0.299	0.597	....	435	0.0728	0.970	0.244	....	....
67	0.308	0.300	0.597	....	310	0.0515	0.974	0.172	....	....
68	0.284	0.272	0.528	....	440	0.0740	0.958	0.272	....	....
69	0.202	0.196	0.313	....	51	0.0422	0.970	0.215	....	....
70	0.202	0.201	0.312	....	21	0.0174	0.995	0.087	....	....
71	0.202	0.200	0.312	....	19	0.0157	0.990	0.079	....	....
72	0.202	0.198	0.312	....	14	0.0118	0.980	0.060	....	....
73	0.175	0.171	0.253	....	33	0.0271	0.976	0.159	....	....
74	0.313	0.304	0.61	....	337	0.0564	0.971	0.186	....	....
75	0.286	0.279	0.534	....	274	0.0458	0.976	0.164	....	....

TABLE 1.—(*Continued*)

Test No.	FEET		CUBIC FEET PER SECOND			p (ft)[b]	$\frac{H_e}{H'}$	$\frac{p}{H_e}$	α	β
	H'	H_e	Q_e	q_e	q_a					
(1)	(2)	(3)	(4)	(5)	(6)[a]	(7)	(8)	(9)	(10)	(11)
(*a*) DATA REPORTED BY JOE W. JOHNSON, ASSOC. M. AM. SOC. C. E.—*Cont.*										
76	0.243	0.239	0.414	....	192	0.0321	0.984	0.134	....	....
83	0.312	0.299	0.608	....	918	0.093	0.958	0.311	0.0774	0.249
84	0.286	0.276	0.533	....	775	0.078	0.965	0.283	0.0801	0.284
85	0.259	0.254	0.459	....	400	0.041	0.981	0.161	0.0736	0.456
86	0.226	0.222[c]	0.371	....	220	0.022	0.984	0.100	0.0725	0.732
87	0.188	0.185[c]	0.281	....	24	0.019	0.984	0.103	0.0299	0.291
88	0.199	0.196	0.306	....	21	0.0176	0.985	0.0898	0.0270	0.301
89	0.217	0.213	0.349	....	27	0.0225	0.982	0.106	0.0264	0.250
90	0.240	0.235	0.408	....	49	0.040	0.979	0.170	0.0279	0.164
91	0.253	0.246	0.441	....	63	0.052	0.973	0.211	0.0284	0.134
92	0.269	0.263	0.486	....	240	0.040	0.978	0.152	0.0551	0.363
93	0.292	0.282	0.551	....	420	0.070	0.966	0.248	0.0596	0.240
94	0.312	0.302[c]	0.607	....	690	0.07	0.967	0.232	0.0712	0.307
95	0.290	0.282	0.545	....	530	0.053	0.972	0.188	0.0717	0.381
96	0.256	0.251[c]	0.449	....	310	0.032	0.981	0.127	0.0697	0.547
97	0.223	0.220[c]	0.364	....	170	0.017	0.987	0.0773	0.0686	0.888
98	0.223	0.220[c]	0.364	....	206	0.021	0.985	0.0955	0.0723	0.757
99	0.126	0.125[c]	0.156	....	46	0.0078	0.989	0.0624	0.0769	1.23
100	0.166	0.163[c]	0.234	....	110	0.018	0.983	0.110	0.0742	0.672
101	0.184	0.180[c]	0.273	....	135	0.023	0.981	0.128	0.0695	0.544
102	0.204	0.200[c]	0.318	....	150	0.025	0.981	0.125	0.0645	0.516
103	0.249	0.242[c]	0.43	....	300	0.051	0.970	0.210	0.0632	0.300
104	0.268	0.261[c]	0.483	....	475	0.048	0.973	0.184	0.0751	0.408
105	0.285	0.277[c]	0.53	....	535	0.054	0.972	0.195	0.0729	0.374
106	0.311	0.300[c]	0.605	....	715	0.072	0.966	0.240	0.0727	0.303
(*b*) EIGHT-FOOT WEIR IN HYDRAULIC LABORATORY										
1	0.904	0.874	....	2.88	2.72	0.202	0.967	0.231	0.0375	0.163
2	1.003	0.952	....	3.38	3.40	0.322	0.949	0.338	0.0341	0.101
3	0.939	0.902	....	3.05	2.99	0.246	0.961	0.273	0.0361	0.133
4	0.717	0.700	....	2.03	2.05	0.113	0.977	0.161	0.0469	0.290
5	0.602	0.589	....	1.562	1.63	0.0701	0.979	0.119	0.0559	0.469
6	0.547	0.543	....	1.357	1.27	0.0428	0.992	0.0788	0.0605	0.767
7	0.483	0.478	....	1.128	0.965	0.0246	0.990	0.0515	0.0689	1.34
8	0.328	0.327	....	0.636	0.682	0.0123	0.997	0.0376	0.101	2.68
9	1.060	0.970	....	3.68	3.93	0.440	0.915	0.454	0.0333	0.0735
10	0.813	0.787	....	2.45	2.09	0.118	0.968	0.150	0.0417	0.278
11	0.894	0.867	....	2.83	2.19	0.130	0.970	0.150	0.0378	0.252
12	1.038	1.012	....	3.56	10.03	0.171	0.975	0.169	0.0645	0.382
13	0.970	0.960	....	3.21	5.71	0.0538	0.990	0.0560	0.0688	1.23
14	0.826	0.811	....	2.51	7.04	0.0816	0.982	0.101	0.0814	0.809
15	0.676	0.668	....	1.860	5.13	0.0434	0.988	0.0650	0.0988	1.52
16	0.434	0.430	....	0.962	3.32	0.0182	0.991	0.0424	0.153	3.62
17	0.815	0.803	....	2.47	6.45	0.0686	0.985	0.0855	0.0822	0.962
18	0.942	0.924	....	3.07	9.74	0.161	0.981	0.174	0.0708	0.406
19	1.012	0.984	....	3.43	9.94	0.168	0.968	0.170	0.0666	0.390
20	0.258	0.258	....	0.446	1.03	0.00174	1.000	0.00674	0.0256	37.9
21	0.725	0.724	....	2.07	32.8	0.00896	0.998	0.0124	0.342	27.6
22	0.837	0.836	....	2.57	43.6	0.0159	0.999	0.0190	0.296	15.5
23	0.884	0.880	....	2.79	49.3	0.0203	0.996	0.0231	0.281	12.2
24	0.994	0.990	....	3.33	52.0	0.0226	0.996	0.0228	0.250	10.9
25	1.097	1.086	....	3.88	71.6	0.0429	0.990	0.0395	0.228	5.78
26	0.386	0.387	....	0.809	32.0	0.0086	0.997	0.0223	0.635	28.6
27	0.815	0.814	....	2.47	30.8	0.0079	0.999	0.0097	0.304	31.2
28	0.942	0.942	....	3.07	38.6	0.0125	1.000	0.0133	0.262	19.8
29	1.012	1.011	....	3.43	51.6	0.0222	0.999	0.0219	0.244	11.1
30	0.258	0.258	....	0.446	14.3	0.00171	1.000	0.00663	0.958	14.4

TABLE 1.—(*Continued*)

Test No.	FEET		CUBIC FEET PER SECOND			p (ft)[b]	$\frac{H_e}{H'}$	$\frac{p}{H_e}$	α	β
	H'	H_e	Q_e	q_e	q_a					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*b*) EIGHT-FOOT WEIR IN HYDRAULIC LABORATORY—*Cont.*										
31	0.860	0.841	....	2.56	....	0.109	0.978	0.130	....	....
32	0.797	0.783	....	2.38	....	0.0903	0.982	0.115	....	....
33	0.698	0.686	....	1.95	....	0.0608	0.983	0.0887	....	....
34	0.605	0.597	....	1.58	....	0.0382	0.988	0.0636	....	....
35	0.498	0.494	....	1.18	....	0.0156	0.992	0.0316	....	....
36	0.408	0.406	....	0.870	....	0.0121	0.995	0.0298	....	....
37	0.269	0.267	....	0.474	....	0.0104	0.993	0.0390	....	....
38	0.920	0.899	....	2.96	....	0.141	0.977	0.157	....	....
39	0.983	0.953	....	3.28	....	0.188	0.970	0.197	....	....
40	1.062	1.019	....	3.68	....	0.290	0.958	0.285	....	....
41	0.815	0.781	....	2.46	....	0.175	0.959	0.224	....	....
42	0.942	0.910	....	3.07	....	0.352	0.966	0.387	....	....
43	1.012	0.963	....	3.42	....	0.375	0.952	0.390	....	....
44	0.258	0.258	....	0.446	....	0.00174	1.000	0.00675	....	....
45	0.395	0.393	....	0.836	....	0.00783	0.995	0.0199	....	....
46	0.646	0.632	....	1.74	....	0.0712	0.979	0.112	....	....
47	0.822	0.819	....	2.50	....	0.0420	0.996	0.0513	....	....
48	0.950	0.940	....	3.11	....	0.0604	0.989	0.0642	....	....
49	1.038	1.026	....	3.56	....	0.0615	0.989	0.0599	....	....
50	1.036	0.993	....	3.55	....	0.266	0.959	0.268	....	....
51	0.929	0.795	....	3.00	....	1.53	0.856	1.925	....	....
52	0.823	0.715	....	2.50	....	1.065	0.869	1.49	....	....
53	1.036	0.944	....	3.55	....	0.728	0.911	0.771	....	....
(*c*) DATA ON DAM A SPILLWAY										
1	....	11.59	....	....	2.21	0.970	....	....	0.0545	0.650
2	....	11.59	....	....	0.990	2.34	....	....	0.0291	0.144
3	....	11.59	....	....	2.58	0.775	....	....	0.0623	0.931
4	....	11.59	....	....	3.31	0.532	....	....	0.0773	1.68
5	....	15.69	....	....	5.00	1.22	....	....	0.0571	0.735
6	....	15.74	....	....	3.84	1.73	....	....	0.0455	0.414
7	....	15.79	....	....	3.38	2.22	....	....	0.0401	0.286
8	....	15.84	....	....	1.52	5.90	....	....	0.0210	0.0563
9	....	19.64	....	....	7.32	1.66	....	....	0.0506	0.600
10	....	19.54	....	....	6.21	1.84	....	....	0.0461	0.490
11	....	19.52	....	....	4.83	2.76	....	....	0.0367	0.260
12	....	19.51	....	....	4.18	3.50	....	....	0.0320	0.179
13	....	19.49	....	....	2.12	9.68	....	....	0.0179	0.0359

[a] Values of q_a in Table 1(*a*) are in millionths. [b] Feet of water.

[c] These values of H_e were not measured. They were computed with the assistance of the curve in Fig. 9.

and laboratory tests were undertaken to supply the needed information. Fig. 2 is a schematic diagram showing the dimensions which were measured in the various experimental setups.

The only previous experimental data which could be found were those referred to by J. W. Johnson,[2] Assoc. M. Am. Soc. C. E. His experiments were performed on a weir 1 ft long under heads varying from 0.171 to 0.305 ft (see Table 1(*a*)). In tests 1 to 76 of Table 1(*a*), air was allowed to recirculate beneath the nappe, giving rather uncertain values of q_a. In tests 83 to 106, recirculation was prevented by a sloping baffle beneath the nappe. Mr.

---

[2] "The Aeration of Sharp-Crested Weirs," by Joe W. Johnson, *Civil Engineering*, March, 1935, pp. 177–178.

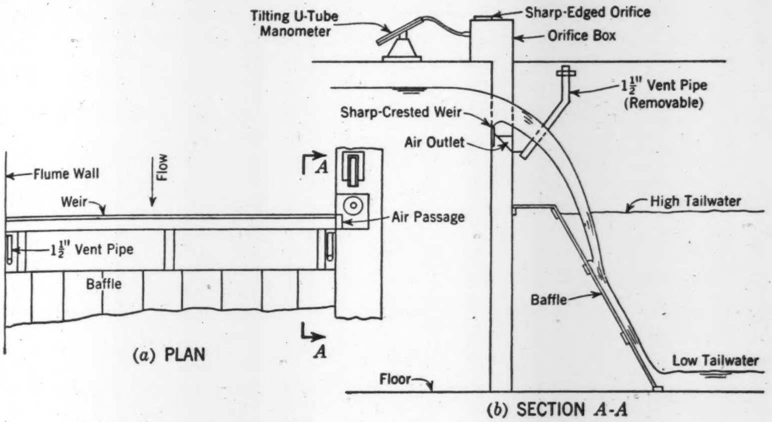

Tilting U-Tube Manometer
Sharp-Edged Orifice
Orifice Box
1½" Vent Pipe (Removable)
Sharp-Crested Weir
Air Outlet
High Tailwater
Baffle
Low Tailwater
Floor
Flume Wall
Flow
Weir
A
Air Passage
1½" Vent Pipe
Baffle
A
(a) PLAN
(b) SECTION A-A

Fig. 3.—Experimental Setup, 8-Ft Weir

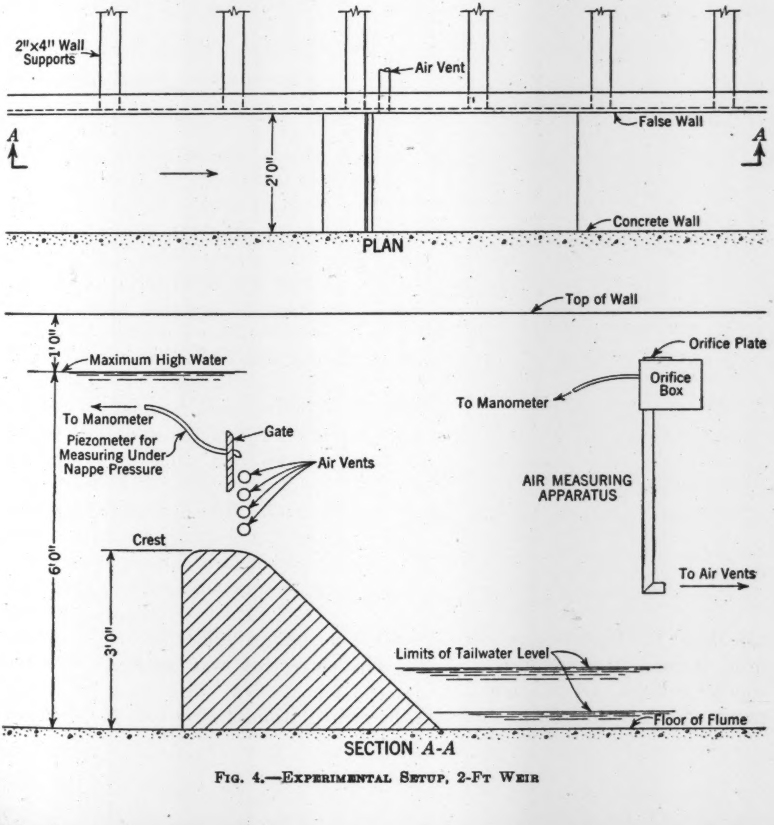

2"×4" Wall Supports
Air Vent
False Wall
A
A
2'0"
Concrete Wall
PLAN
Top of Wall
1'0"
Maximum High Water
Orifice Plate
Orifice Box
To Manometer
To Manometer
Piezometer for Measuring Under-Nappe Pressure
Gate
Air Vents
AIR MEASURING APPARATUS
6'0"
Crest
3'0"
To Air Vents
Limits of Tailwater Level
Floor of Flume
SECTION A-A

Fig. 4.—Experimental Setup, 2-Ft Weir

Johnson obtained an expression for the quantity of air required per foot length of weir as

$$q_a = 0.0019\,(q_o)^{2.1} \quad \ldots\ldots (1)$$

—for heads less than 0.4 ft; and

$$q_a = 0.0096\,(q_o)^{2.1} \quad \ldots\ldots (2)$$

—for heads greater than 0.4 ft. In view of the variation of the coefficient at low heads, and the dimensional nature of the equations, it was thought advisable to conduct additional experiments before applying these equations to gates having crests 40 ft long and operating under heads of 20 ft.

TABLE 2.—AERATION OF A TWO-FOOT, SHARP-CRESTED WEIR

(Various Gate Heights, Gate Opening $y = 0$)

Test No. (1)	H_o (ft) (3)	q_a (6)[a]	p (ft)[b] (7)	α (10)	β (11)
(*a*) GATE 0.5 FT HIGH					
85	1.143	42.1	0.0132	0.222	19.2
87	0.521	33.1	0.0079	0.494	32.6
90	0.512	7.58	0.0963	0.128	0.682
216	1.071	110	0.006	0.148	26.5
217	0.787	110	0.067	0.110	1.30
219	0.373	3.6	0.022	0.176	2.98
220	0.762	7.8	0.170	0.0760	3.41
221	0.769	11.5	0.223	0.0851	0.294
224	1.271	3.78	0.024	0.0515	2.73
225	0.243	1.90	0.006	0.272	11.0
(*b*) GATE 1.0 FT HIGH					
29	0.497	41.8	0.0102	0.544	26.5
30	0.704	40.6	0.0176	0.331	13.2
32	1.252	54.2	0.0215	0.204	11.9
34	0.392	23.9	0.0046	0.638	54.3
35	0.472	36.0	0.0095	0.542	27.0
39	1.246	144	0.0113	0.392	43.3
42	0.372	3.16	0.0171	0.176	3.82
43	0.469	3.14	0.0150	0.143	4.49
44	0.707	3.75	0.0232	0.0933	2.84
45	1.046	2.47	0.0105	0.0625	6.22
(*c*) GATE 2.0 FT HIGH					
143	1.051	15.2	0.0070	0.164	24.7
144	0.715	12.2	0.0067	0.226	24.2
150	0.963	36.8	0.0100	0.265	25.5
151	0.962	140	0.0093	0.525	54.3
152	0.738	130	0.0065	0.725	82.4
153	0.467	98.5	0.0037	1.15	145.0

[a] In millionths of a cubic foot per second. [b] Feet of water.

Accordingly, the laboratory conducted a series of tests on a larger weir to extend the range of Mr. Johnson's data. An 8-ft suppressed weir available at the laboratory was fitted with an arrangement for measuring the air required. Fig. 3 illustrates the apparatus. Full aeration was obtained by two 1.5-in. pipes, placed one at each end of the weir, as shown. These pipes caused two openings about 2 in. wide the entire height of the nappe. When full aeration was not desired, the pipes were removed and the air admitted under the nappe was measured by orifices 0.25 in., 0.5 in., and 2 in. in diameter mounted on a box about 1 ft square. The air was conducted under the nappe through a duct placed in the slot supporting the weir bulkhead. The differential pressure on the orifice was measured with a tilting water manometer. The pressure under the nappe was not measured directly but was calculated as the pressure drop across the measuring orifice plus the pressure drop through the connecting duct. The correction was negligible for the smaller orifices and was not large for the 2-in. orifice. A sloping baffle was provided below the weir to eliminate the recirculation of air which Mr. Johnson found in his experiments. The head on the weir varied from 0.258 ft to 1.097 ft. The tailwater level varied through the approximate range indicated in the figure. Discharges were measured by allowing the water to flow over the weir with the nappe completely ventilated. An accurate volumetric calibration for this condition of flow was available. The air requirement for this discharge was then measured with each of the orifices. A few runs were made with no air at all supplied under the nappe. Air and water temperatures were recorded. The data are given in Table 1(*b*).

In connection with the aeration requirements of dam D, additional tests were made to determine the effect of discharge beneath the gate as well as over it. A 2-ft weir, on which the head could be raised to 1.4 ft, was used for these tests. To prevent the back circulation of air drawn beneath the nappe, the lower face of the bulkhead was sloped so that the nappe fell on it. Three gates, 0.5 ft, 1.0 ft, and 2.0 ft high, were used with openings beneath them of 0, 0.1, 0.5, and 1.0 ft. The head over the crest varied from 0.095 ft to 1.409 ft. The air required was measured in the same way as that for the tests on the 8-ft weir. Discharges were measured by the calibrated 8-ft weir. Fig.

TABLE 3.—AERATION OF A TWO-FOOT, SHARP-CRESTED WEIR WITH GATE 0.5 FT HIGH

(Variable Openings y)

Test No. (1)	H_o (ft) (3)	H_u (ft) (3′)	q_a (6)[a]	p (ft)[b] (7)	α (10)	β (11)	ζ (12)
(*a*) y = 0.086 FT							
208	0.311	0.854	4.92	0.002	0.447	69.5	0.458
211	0.727	1.270	51	0.018	0.358	14.4	0.156
212	0.728	1.271	....	0.006	....	....	0.156
213	1.173	1.713	....	0.011	....	....	0.0890
214	1.409	1.952	70.5	0.028	0.194	9.75	0.0717
215	1.261	1.804	49.8	0.014	0.216	19.5	0.0816
(*b*) y = 0.10 FT							
66	0.446	0.996	11.6	0.202	0.152	0.336	0.334
67	0.218	0.768	4.22	0.0289	0.304	2.30	0.862
68	1.235	1.785	3.50	0.0208	0.0530	3.15	0.974
69	1.129	1.679	2.98	0.0151	0.0530	4.34	0.108
70	0.980	1.530	3.44	0.0200	0.0670	3.28	0.127
71	0.980	1.530	47.8	0.0164	0.262	15.7	0.127
73	0.795	1.345	51.6	0.0189	0.324	13.6	0.163
74	0.469	1.019	39.2	0.0109	0.549	23.6	0.314
77	0.637	1.187	120	0.0054	0.841	99.3	0.215
78	0.900	1.450	362	0.0727	0.542	6.70	0.141
(*c*) y = 0.50 FT							
52	0.912	1.662	62.0	0.0272	0.283	9.50	0.741
53	0.749	1.499	60.0	0.0225	0.356	11.8	0.943
54	0.416	1.166	41.0	0.0131	0.606	19.2	2.02
55	0.095	0.845	5.5	0.0003	2.51	794	15.7
56	0.189	0.939	....	0.0011	....	....	5.89
59	0.888	1.638	168	0.0153	0.553	32.1	0.765
60	1.019	1.769	224	0.0271	0.481	18.1	0.646
61	1.013	1.763	5.00	0.0384	0.0662	1.75	0.652
62	0.886	1.636	4.53	0.0343	0.0742	1.92	0.767
63	0.704	1.454	4.29	0.0319	0.0925	2.04	1.021
64	0.441	1.191	3.42	0.0198	0.149	3.31	1.845
65	0.160	0.910	2.27	0.0087	0.410	7.54	7.47
(*d*) y = 0.577 FT							
171	1.262	2.050	81.0	0.702	0.104	0.187	0.584
173	1.362	2.150	415	0.093	0.360	5.28	0.533
176	0.348	1.136	53.4	0.021	0.733	12.2	3.00
177	0.271	1.059	46.7	0.012	1.01	22.8	4.21
178	0.310	1.098	13.8	0.020	0.426	6.60	3.51
179	0.618	1.406	14.1	0.040	0.181	2.80	1.41
181	1.022	1.810	28.4	0.075	0.132	1.80	0.752
183	1.268	2.056	459	0.076	0.429	7.15	0.580
184	1.110	1.898	402	0.056	0.490	9.80	0.680
185	0.831	1.619	402	0.087	0.590	5.65	0.969
186	0.597	1.385	385	0.080	0.822	6.14	1.48
(*e*) y = 0.934 FT							
188	1.011	1.978	542	0.139	0.502	3.65	1.29
189	0.885	1.852	520	0.147	0.552	3.32	1.52
190	0.666	1.633	470	0.120	0.734	4.07	2.20
200	0.320	1.287	15	0.024	0.406	5.41	5.85
(*f*) y = 1.00 FT							
95	0.388	1.388	4.59	0.0357	0.169	1.84	4.87
96	0.197	1.197	1.86	0.0059	0.333	11.1	12.5
99	0.604	1.604	151	0.16	0.428	1.61	2.70
100	0.777	1.777	199	0.24	0.345	1.12	1.95
101	0.796	1.796	466	0.1109	0.625	4.49	1.89

[a] In millionths of a cubic foot per second. [b] Feet of water.

4 shows the general arrangement of the apparatus. Tables 2 to 5 give the data obtained.

TABLE 4.—AERATION OF A TWO-FOOT, SHARP-CRESTED WEIR WITH GATE 1.0 FT HIGH

(Various Openings y)

Test No. (1)	H_e (ft) (3)	H_u (ft) (3′)	q_a (6)[a]	p (ft)[b] (7)	α (10)	β (11)	ζ (12)
(a) y = 0.1 FT							
15	1.135	2.185	172	0.0160	0.432	30.6	0.122
16	0.810	1.860	136	0.0128	0.569	36.0	0.187
17	0.386	1.436	86.2	0.0056	1.17	80.5	0.500
18	0.222	1.272	78.6	0.0036	2.16	133	1.08
19	0.219	1.269	3.44	0.0182	0.306	3.68	1.10
20	0.569	1.619	3.91	0.0272	0.114	2.38	0.297
21	0.821	1.871	3.37	0.0202	0.0790	3.21	0.184
22	1.063	2.113	3.31	0.0189	0.0615	3.46	0.133
23	1.239	2.289	4.24	0.0315	0.0525	2.06	0.110
24	1.242	2.292	63.7	0.0285	0.207	9.02	0.110
26	0.907	1.957	45.3	0.0154	0.280	16.5	0.162
27	0.547	1.597	53.8	0.0170	0.494	15.9	0.312
28	0.216	1.266	32.2	0.0079	1.17	31.9	1.12
(b) y = 0.5 FT							
4	0.113	1.363	1.5	0.0044	0.558	14.3	15.3
5	0.113	1.363	6.4	0.0040	1.19	33.5	15.3
11	0.941	2.192	255.4	0.0417	0.500	11.3	0.812
12	0.743	1.993	230	0.0323	0.640	14.8	1.10
13	0.508	1.758	168	0.0200	0.904	22.9	1.82
14	0.239	1.489	78.4	0.0168	1.37	19.5	5.23
46	0.172	1.422	27.3	0.0053	1.49	48.5	8.35
47	0.383	1.633	54.0	0.0208	0.671	12.3	2.69
48	0.565	1.815	73.3	0.0340	0.469	7.79	1.59
49	0.770	2.020	92.1	0.0608	0.335	4.25	1.05
50	1.038	2.288	83.5	0.0575	0.239	4.31	0.716
(c) y = 1.0 FT							
105	0.680	2.180	491	0.135	0.715	3.60	2.63
108	0.776	2.276	541	0.1736	0.619	2.77	2.20
109	0.767	2.267	186	0.28	0.326	0.894	2.24
110	0.601	2.101	154	0.1769	0.423	1.44	3.11
111	0.420	1.920	104	0.0739	0.620	3.52	5.09
112	0.196	1.696	35.8	0.0090	1.32	28.7	15.0
114	0.406	1.906	6.55	0.0619	0.168	1.10	5.34
115	0.625	2.125	11.4	0.2312	0.103	0.280	2.95
117	0.798	2.298	59.2	0.41	0.160	0.312	2.12
118	0.413	1.913	29.6	0.0921	0.318	1.42	5.22

[a] In millionths of a cubic foot per second. [b] Feet of water.

In addition to the laboratory tests, it was possible to make observations of the actual air requirement of the dam A spillway. Fig. 5 shows the method of aerating this spillway. Each bay received air through two 16-in. pipes which terminated at their upper ends in a chamber at the downstream face of the pier. Piezometers were provided so that the pressure under the nappe could be measured. The air supplied was measured by placing orifice plates over the entrances to the chambers at each end of the bay and sealing the pipes aerating the adjacent bays. A pressure connection on the orifice plate permitted the rate of air flow to be calculated from the pressure difference across the orifice. Fig. 6 shows the orifice plate and pressure connection. All pressures were measured by means of water or mercury manometers located on the spillway deck. The quantity of air required and the reduction of pressure beneath the nappe were varied for a given head on the gate crest by changing the orifice plates. The crest of the weir was 40 ft long, the heads varied from 11.59 ft to 19.04 ft, and the reduction of pressure beneath the nappe from 0.53 ft to 9.68 ft. Table 1(*c*) gives the data.

ANALYSIS OF TEST DATA

Air Required for Aeration.—The quantity of air supplied beneath the nappe in all of the experiments except Mr. Johnson's, both in the laboratory and in the field, was measured by sharp-edged circular orifices and calculated by the hydraulic formula modified by the coefficient of Bean, Benesh, and Buckingham. For the case of discharge through an orifice from the free atmosphere, the equation reduces to

$$Q_a = (0.597 - 0.00844\,h) \times A \sqrt{2\,g\,h\,\frac{w_w}{w_a}} \ldots\ldots (3)$$

in which: Q_a is the rate of air supply in cubic feet per second at atmospheric pressure; h = head on the orifice in feet of water; A = area; g = acceleration due to gravity; w_a = unit weight of air in pounds per cubic foot; w_w = unit weight of water; and B = length of weir crest, in feet.

The rate of air discharge in Mr. Johnson's experiments (Table 1(*a*)) was measured by means of a capillary tube.

TABLE 5.—AERATION OF A TWO-FOOT, SHARP-CRESTED WEIR WITH GATE 2.0 FT HIGH

(Various Openings y)

Test No. (1)	H_o (ft) (3)	H_u (ft) (3′)	q_a (6)[a]	p (ft)[b] (7)	α (10)	β (11)	ζ (12)
(*a*) y = 0.1 FT							
119	1.027	3.077	24.8	0.0296	0.155	5.38	1.68
120	0.606	2.656	23.3	0.0080	0.355	26.9	3.46
127	0.982	3.032	274	0.0130	0.662	50.0	1.78
(*b*) y = 0.5 FT							
131	0.614	2.864	320	0.0283	0.945	20.5	1.76
137	0.455	2.705	56.5	0.0040	0.873	99.3	2.68
155	0.614	2.864	24.0	0.059	0.215	2.24	1.76
156	0.627	2.877	23.2	0.056	0.209	2.34	1.71
157	0.469	2.719	14.4	0.027	0.267	4.64	2.57
158	0.330	2.580	9.7	0.010	0.397	13.1	4.24
159	0.330	2.580	10.2	0.011	0.397	11.9	4.24
162	0.332	2.582	11.0	0.013	0.395	10.1	4.19
163	0.423	2.673	38.8	0.0106	0.610	24.4	2.97
164	0.587	2.837	71.5	0.037	0.438	6.95	1.87
165	0.580	2.830	69.5	0.049	0.405	4.80	1.90
166	0.445	2.695	69.5	0.027	0.616	10.2	2.76
167	0.243	2.493	49.3	0.007	1.33	46.1	6.60
168	0.656	2.906	92.8	0.048	0.742	10.15	1.60
169	0.654	2.904	89.7	0.056	0.398	4.65	1.61
170	0.654	2.904	23.6	0.059	0.200	2.22	1.61

[a] In millionths of a cubic foot per second. [b] Feet of water.

The wide range of the variables involved in the experiments made it desirable to use a dimensionless form of representation. It was found that the relationship between pressure reduction, air required, and head over the top of the gate could be represented by plotting

$$\alpha = \frac{(q_a)^{0.5}}{g^{0.25}\,p^{0.25}\,H_o} \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

against

$$\beta = \frac{(q_a)^{0.5}}{g^{0.25}\,p^{1.25}} \ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

in which: q_a = the rate of air supply per foot length of crest, $\frac{Q_a}{B}$; p = reduction of pressure beneath nappe, in feet of water; and H_o = head over the top of the gate, corresponding to the pressure reduction p. The length of crest was eliminated by assuming that each foot of length of overfalling nappe was independent of every other foot of length and that the total air required was proportional to the length. Fig. 7 is a plot of these variables for all tests in which discharge occurred only over the top of the gate. It will be seen that the results obtained with the different experimental arrangements agree with each other within the limits of accuracy of the tests. It is particularly noticeable that Mr. Johnson's data on a 1-ft weir fall on both sides of the points obtained on the dam A spillway. The effect of the height of the overfalling nappe on the air required was investigated by comparing the tests made with different heights of gate on the 2-ft weir. The tailwater elevation depended on the discharge, but since the channel was wide, this did not vary greatly. Fig. 8 is a plot of $\frac{(q_a)^{0.5}}{g^{0.25}\,p^{0.25}\,H_o}$ against $\frac{(q_a)^{0.5}}{g^{0.25}\,p^{1.25}}$ for these data. No definite trend can be established. Any variation due to the height of the nappe is masked by the uncertainties of the experimental data.

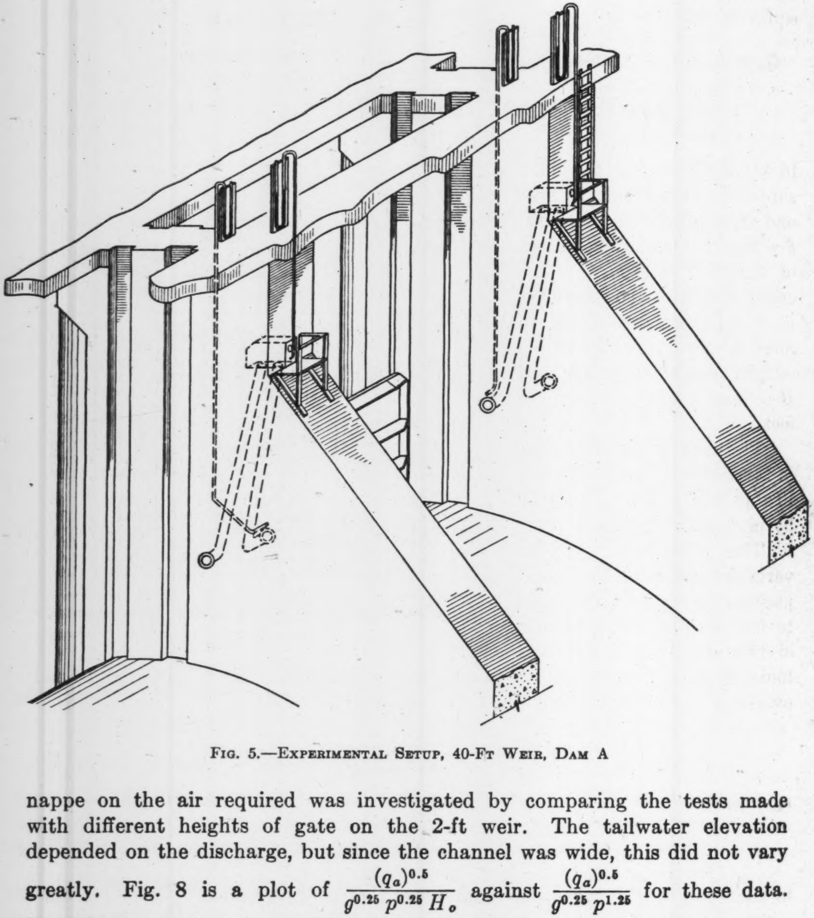


FIG. 5.—EXPERIMENTAL SETUP, 40-FT WEIR, DAM A

It was found that the results for the tests made with discharge beneath the gate could be represented satisfactorily by the dimensionless groups used for discharge over the gate, as long as the ratio of discharge beneath the gate to discharge over the gate remained constant. Since it was difficult to measure the two discharges separately with the experimental setup used and since such separation would be practically impossible in field installations, the ratio of discharges was represented as follows: Let Q_o and Q_u be the discharge over and beneath the gate, respectively; then:

$$Q_u = C_u\, B\, y\, \sqrt{H_u} \,..\, (5a)$$

and

$$Q_o = C_o\, B\, (H_o)^{1.5} \,...\, (5b)$$

and the ratio of discharges is

$$\zeta' = \frac{Q_u}{Q_o} = \frac{C_u\, y\, \sqrt{H_u}}{C_o\, (H_o)^{1.5}} \,..\, (6)$$

If it may be assumed that C_u and C_o are constant, the ratio may be expressed as

$$\zeta = \frac{y\, \sqrt{H_u}}{(H_o)^{1.5}} \,.....\, (7)$$

This form has the advantage of being composed of terms which are easily measured.

The ratio ζ was calculated for each test in which flow beneath the gate occurred (see Tables 3 to 5). All the data were then arranged in groups covering small ranges of ζ, as follows:

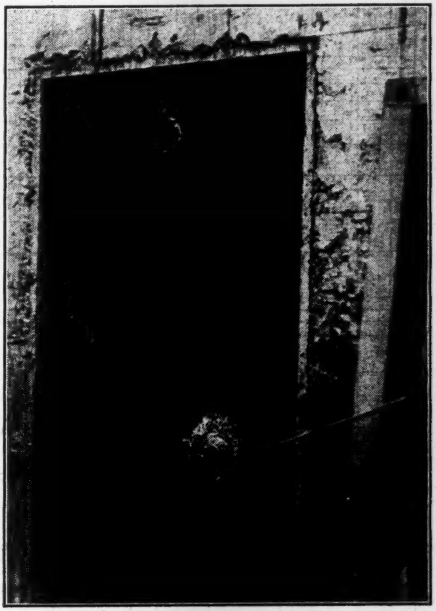


FIG. 6.—ORIFICE PLATE AND PRESSURE CONNECTION FOR MEASURING AIR SUPPLY AT DAM A

Range of ζ	Average ζ	No. of tests
0	0	93
0-0.4	0.17	20
0.4-1.2	0.81	24
1.2-2.3	1.79	24
2.3-3.3	2.81	10
3.3-4.5	4.0	6
4.5-5.5	5.1	5
>5.5	....	9

The data were plotted on separate sheets, using the groups $\frac{(q_a)^{0.5}}{g^{0.25}\, p^{0.25}\, H_o}$ and $\frac{(q_a)^{0.5}}{g^{0.25}\, p^{1.25}}$ as ordinates and abscissas, respectively, and the best straight line was drawn through each group. It was found that slight adjustments would make all the lines parallel with a slope of 0.45. Accordingly, the equa-

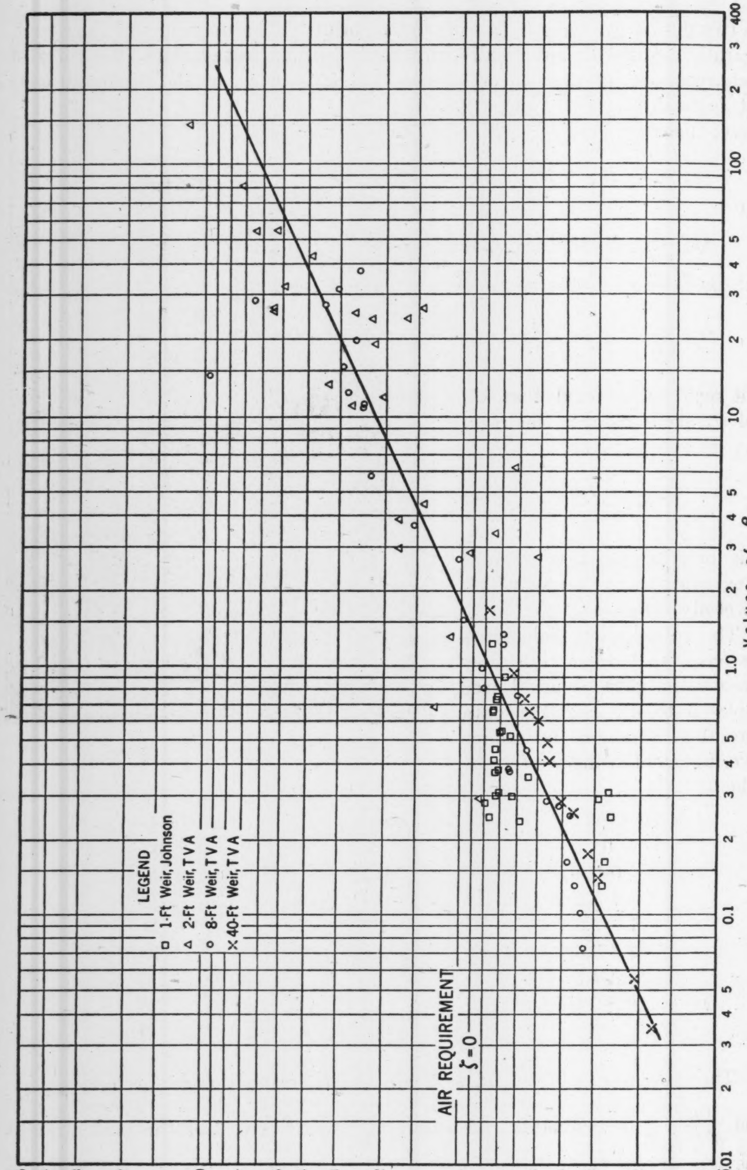


Fig. 7.—Effect of Weirs for ζ = 0

tion of each line could be written

$$\frac{(q_a)^{0.5}}{g^{0.25}\, p^{0.25}\, H_o} = C \left[\frac{(q_a)^{0.5}}{g^{0.25}\, p^{1.25}}\right]^{0.45} \qquad \text{.................. (8)}$$

in which C is a constant for each value of ζ. Fig. 9 shows the relation between C and ζ. The value of C increases rapidly for small values of ζ, but apparently reaches a maximum value of 0.225 at $\zeta = 2.5$. Further increase in ζ does not increase C.

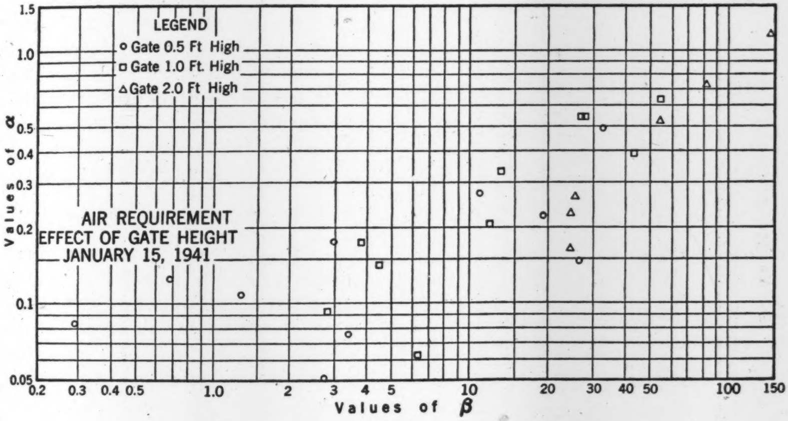


FIG. 8.—EFFECT OF GATE HEIGHT ON THE AERATION OF WEIRS

Eq. 8 may be solved for q_a:

$$q_a = \frac{(C\, H_o)^{3.64}\, g^{0.5}}{p^{1.14}} \qquad \text{.................. (9)}$$

Eq. 9 gives the quantity of air required per foot of length for a weir operating under specified conditions of gate opening, head, and pressure reduction beneath the nappe.

Effect of Pressure Reduction on Discharge.—The reduction of pressure beneath the nappe increases the effective head on the weir and causes the discharge for a given head to be greater than would be the case if the nappe were fully aerated. Similarly, for a given discharge, reduction in pressure decreases the measured head on the weir. The experimental data described in the preceding section provided a means of evaluating this effect. Fig. 10 is a plot of the ratios p/H_o and H_o/H'. A smooth curve has been drawn through the points.

The increase in discharge for a given head, caused by reduction of pressure beneath the nappe, can be calculated from these data. Suppose that the weir discharge at H is Q and at a slightly larger head H' is Q'. By reducing the pressure under the nappe, it would be possible to produce the discharge Q' with the head H. The increase of discharge at head H is then $Q' - Q$ and the ratio of this increase to the fully aerated discharge at head H is $\frac{Q' - Q}{Q}$. If the discharge coefficient of the weir is assumed to be constant, the ratio of increase in discharge may be written as $\frac{(H')^{1.5} - H^{1.5}}{H^{1.5}}$ or $\left(\frac{H'}{H}\right)^{1.5} - 1$. Fig. 10 gives the value of H_o/H' for any value of p/H_o. With this information it is a simple matter to calculate the increase in discharge. Fig. 10 also shows the percentage increase of discharge in terms of p/H_o. This curve is particularly useful in evaluating the effect of insufficient aeration on discharge where it is not possible to aerate the weir properly.

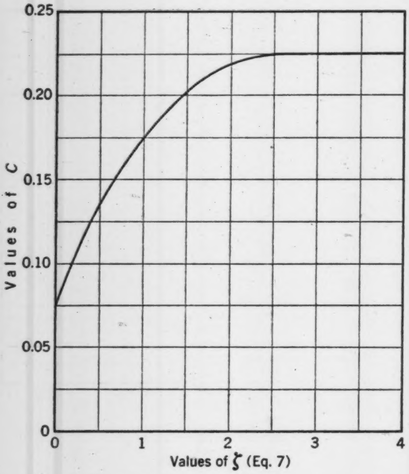


FIG. 9.—RELATION BETWEEN C AND ζ

APPLICATION OF RESULTS

Calculation of Aeration Vent Required.—Eq. 9 and Fig. 9 give sufficient information to determine the quantity of air required to maintain the pressure reduction beneath the nappe at any desired value. As an example, consider a spillway gate 40 ft long with water discharging over it under a head of 15 ft. Since there is no discharge beneath the gate, $\zeta = 0$. Fig. 9 shows that C for this condition is 0.077. It will be assumed that it is desired to limit the pressure reduction beneath the nappe to a maximum of 2 ft. Eq. 9 gives the air requirement for these conditions as $q_a = \frac{(0.077 \times 15)^{3.64} g^{0.5}}{2^{1.14}}$ = 4.35 cu ft per sec per ft of length of weir or 174 cu ft per sec for the 40-ft length of gate.

The vent diameter required to carry this amount of air may be computed by the use of the ordinary hydraulic equations. The effect on the area caused by neglecting compressibility is about 4% for a pressure reduction of 5 ft. In terms of diameter of vent, this difference is negligible. In calculating the flow of air, the effective head must be stated in terms of feet of air rather than feet of water. The value of p as given must be multiplied by the ratio of the unit weight of water to that of air. For air at 70° F, this ratio is approximately 830. In the example given, suppose that the most convenient method of venting is by the use of a pipe 30 ft long, with two right-angle bends and a sharp-cornered entrance. It is obvious that the total pressure drop through the vent pipe is equal to the pressure reduction beneath the nappe. A preliminary calculation of the pressure drop may be made with an assumed pipe diameter, say, 1 ft, and an air temperature of 70°. For these conditions, the

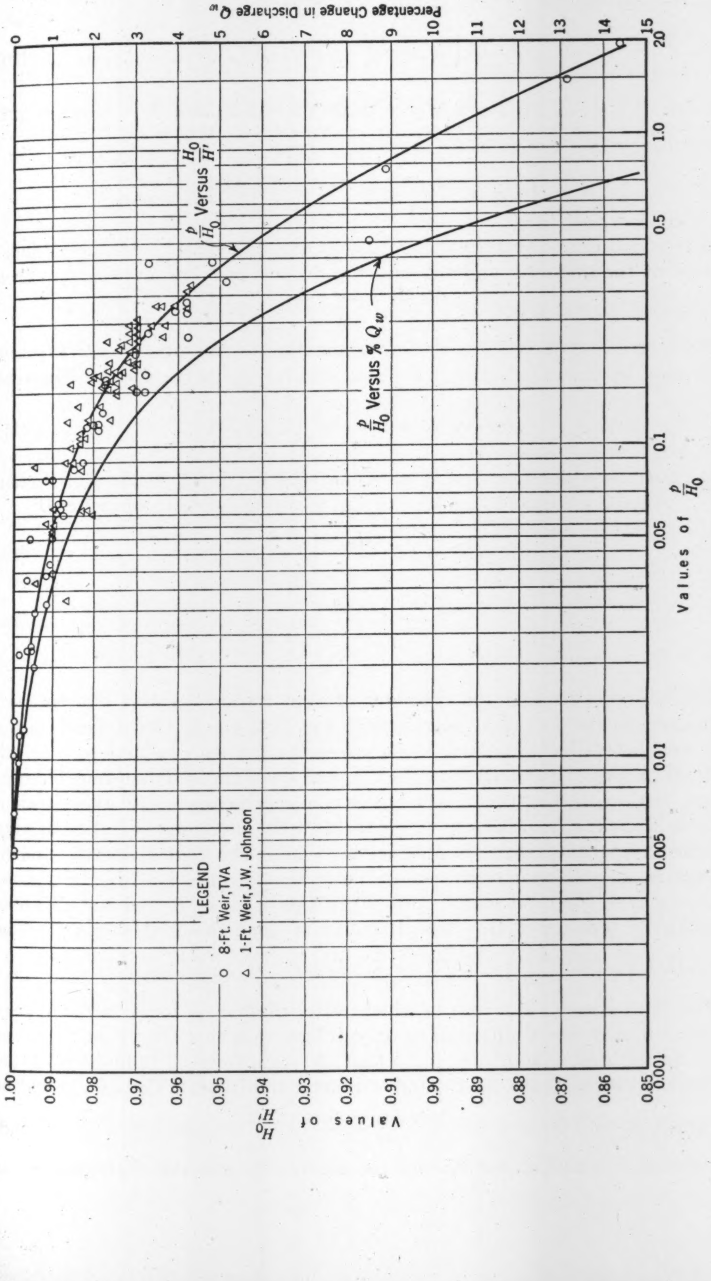


FIG. 10.—RELATION BETWEEN $\frac{p}{H_0}$ AND $\frac{H_0}{H'}$

pressure drop for the amount of air required is

$$p = \frac{w_a}{w_w}\left(C_e + \frac{f\,L}{D} + 2\,C_b + 1\right)\frac{V^2}{2\,g} \tag{10}$$

in which: $\frac{w_a}{w_w} = \frac{1}{830}$; $C_e = 0.5$; $f = 0.02$; $C_b = 0.5$; and $V = \frac{174}{0.785} = 222.$ Substituting these values in Eq. 10,

$$p = \frac{1}{830}\left(0.5 + \frac{0.02 \times 30}{1} + 1 + 1\right)\frac{222^2}{2\,g} = 2.85 \text{ ft.}$$

Since the friction loss coefficient is relatively small, it may be considered independent of the diameter and the pressure drop is proportional to D^4. Then the required diameter is $D = \left(\frac{2.85}{2}\right)^{0.25} \times 1 = 1.09$ ft.

There is a relation between the vent diameter and the other variable factors which may be found by writing the equation for discharge through the vent

$$Q_a = q_a\,B = \frac{C'\,\pi\,D^2}{4}\sqrt{2\,g\,p\,\frac{w_w}{w_a}} \tag{11}$$

in which C' is a coefficient that combines the entrance, head, and friction losses. Solving for q_a and equating to Eq. 9, it is possible to solve for D:

$$D = C''\,\frac{(H_o)^{1.82}\,B^{0.50}}{p^{0.82}} \tag{12}$$

in which

$$C'' = \frac{2\,C^{1.82}}{\sqrt{C'\,\pi}\,\sqrt[4]{2\,\frac{w_w}{w_a}}} \tag{13}$$

Eq. 12 indicates that the diameter of vent required varies directly with the square root of the crest length, with the 1.82 power of the head on the crest, and inversely with the 0.82 power of the pressure reduction. (It will be noted in Eqs. 11 and 12 that since q_a has been given dimensions of cubic feet per second, B must be considered as dimensionless.) With this relationship established, it is necessary to go through the foregoing calculations only once for any particular weir or gate installation. The vent diameter required for any reasonable variations can be determined from Eq. 12. In the example, suppose the crest length were 50 ft, the head 20 ft, and the allowable reduction of pressure 3 ft. The diameter of vent required would then be $D = 1.09\left(\frac{20}{15}\right)^{1.82}\left(\frac{50}{40}\right)^{0.50}\left(\frac{2}{3}\right)^{0.82} = 1.47$ ft.

The procedure for the case in which water discharges beneath the gate as well as over it is very similar. The only difference is that the value of ζ is not zero. In the first example, suppose that the gate were 20 ft high and lifted 6 ft, the elevation of the headwater remaining the same. Then H_o would be 9 ft, y would be 6 ft, and $\zeta = \frac{y\,\sqrt{H_u}}{(H_o)^{1.5}} = \frac{6 \times \sqrt{32}}{9^{1.5}} = 1.26$. The corresponding value of C from Fig. 8 is 0.190. Eq. 9 gives the amount of air required as

$q_a = \frac{(0.190 \times 9)^{3.64} g^{0.5}}{2^{1.14}}$ = 18.1 cu ft per sec per ft of length or 724 cu ft per sec for the entire gate. If the same method of venting is used as in the first example, the vent diameter required is 2.17 ft. This condition should always be investigated where there is a possibility of its existence. The reduction of pressure which would be developed with the 1.09-ft vent can be found by rewriting Eq. 12 in terms of pressure

$$p = C''' \frac{H^{2.22} B^{0.61}}{D^{1.22}} \qquad (14)$$

from which $p = 2\left(\frac{2.17}{1.09}\right)^{1.22}$ = 4.63 ft of water, which is more than double the allowable reduction of 2 ft.

Care should be taken to see that Eqs. 12 and 14 are used only in cases where the value of C is constant as this is a necessary condition of their derivation.

Calculation of Discharge for Incomplete Aeration.—Referring again to the first example, it is desired to know the actual discharge over the gate when the pressure reduction beneath the nappe is 2 ft of water. It may be known, from model tests, for example, that the discharge for a fully aerated nappe is 8,000 cu ft per sec. The ratio $\frac{p}{H_o} = \frac{2}{15} = 0.133$. Fig. 10 shows that the corresponding increase in discharge is 2%, or 160 cu ft per sec, making the total discharge 8,160 cu ft per sec for these conditions.

The data available are not sufficient to determine the increase in discharge when water is flowing both over and beneath the gate.

Conclusions

All of the available data on aeration of spillways have been collected and analyzed for the purpose of providing a workable criterion for the size of aeration vents required. The dimensionless groups selected do not reflect the effect of height of fall or angle of impact of overfalling nappe on spillway face or tailwater, although these factors would seem to be involved. Nevertheless, the agreement between small spillways 1 ft long at a head of 0.4 ft and a large spillway 40 ft long at a head of 19 ft is close enough for design purposes.

An interesting and useful by-product of the investigation was the effect of reduction of pressure beneath the nappe on the relation between head and discharge.

It is hoped that the discussion will bring out other data that will either support or disprove the results presented.

APPENDIX

Notation

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:[3]

A = area, in square feet;
B = length of weir crest, in feet;
C = (see Eq. 8);
C_b = coefficient of bend loss;
C_e = coefficient of entrance loss;
C_o = discharge coefficient for flow over the gate;
C_u = discharge coefficient for flow under the gate;
C' = coefficient combining the entrance, head, and friction losses;
C'' = (see Eq. 13);
C''' = (see Eq. 14);
D = diameter of pipe, in feet;
f = coefficient of pipe friction in the Fanning formula;
g = acceleration of gravity, in feet per second^2;
H = head on the weir, in feet;
H_o = head over the top of the gate corresponding to a pressure reduction p, in feet;
H_u = head on the center of the opening below the gate, corresponding to a pressure reduction p, in feet;
H' = head over the top of the gate for full aeration, in feet;
h = head on the orifice, in feet of water;
L = length of pipe, in feet;
p = reduction of pressure beneath the nappe, in feet of water;
Q = discharge over a fully aerated weir at head H, in cubic feet per second;
Q_a = rate of air supply, in cubic feet per second, at atmospheric pressure;
Q_o = discharge over the top of the gate, in cubic feet per second;
Q_u = discharge beneath the gate, in cubic feet per second;
Q' = discharge over a fully aerated weir at the head H', in cubic feet per second;
q = discharge per unit width of weir, in cubic feet per second;
q_a = rate of air supply per foot length of crest, Q_a/B, in cubic feet per second;
q_o = discharge over the top of the gate per foot length of crest, Q_o/B, in cubic feet per second;
V = mean velocity, in feet per second;
w = weight per unit volume:
    w_a = unit weight of air, in pounds per cubic foot;
    w_w = unit weight of water, in pounds per cubic foot;
y = height of the opening below the gate, in feet;
α = a dimensionless group defined by Eq. 4*a*;
β = a dimensionless group defined by Eq. 4*b*;
ζ = a dimensionless ratio, $\frac{y\sqrt{H_u}}{(H_o)^{1.5}}$: ζ' = ratio of discharges Q_u/Q_o.

---

[3] ASA—Z10.2—1942.

DISCUSSION

Joe W. Johnson,[4] **Assoc. M. Am. Soc. C. E.**—Air is withdrawn from the underside of a nappe by the falling sheet acting as a friction pump. The air withdrawn is replaced from two sources: (1) The air that flows through air vents to the space under the nappe, and (2) that which, upon being drawn into the downstream pool by both sides of the falling sheet, strikes the channel bottom and circulates back and upward to the space under the nappe. The observations[5] of the writer on a 1-ft weir indicated that the quantity of air withdrawn was proportional to the velocity of the falling sheet or, within certain limits, to the discharge. The only effect of the position of the downstream water level is the variation of the quantity of air circulated. If a deflector is placed near the underside of the nappe, as shown by Mr. Hickox in Fig. 3(*b*) and by the writer in Fig. 11, air is prevented from circulating backward into the space under the nappe. Therefore, all air that is finally withdrawn must first enter through the air vents. A direct measure of the power of the falling sheet to withdraw air is thus determined by the insertion of a deflector. The data from the writer's experiments on a 1-ft weir with heads never exceeding 0.4 ft are shown in Table 1(*a*), tests 76 and 83 to 106. A plot of these data is shown in Fig. 11 from which Eq. 1 is obtained.

In passing, it may be well to discuss briefly the reason for the differences between Eqs. 1 and 2. Upon examination of these equations, the reader is inclined to conclude that a "break" occurs in the relationship between q_a and q_o at a head of 0.4 ft. Actually, Eq. 1 was based on only the laboratory experiments plotted in Fig. 11, and Eq. 2 was based on both the laboratory experiments and some data on a large weir in the San Joaquin Valley. The extensive data collected since the writer's experiments have permitted a conclusive analysis of the problem of partial aeration as well as of the design of air vents. Thus, by utilizing all available data from both the laboratory and the field, Mr. Hickox has obtained excellent correlation between the various observations by means of dimensionless groups. In view of the wide range of conditions covered by this analysis, the author's procedure for designing air vents should be considered as superseding all previous methods.

To show the effect of different positions of the downstream channel bed and the effect of different depths of pools, several experiments were performed where conditions existed as shown in Figs. 11(*b*) and 11(*c*). The quantity of air, as indicated by the various curves, is that delivered beneath the nappe through the air vents. Fig. 11 shows that, for a particular position of the downstream channel bed, the volume of air taken in through the vents increases as the depth of pool decreases. In other words, the volume of air circulated decreases as the depth of pool is decreased since the volume drawn outward is the same for all depths.

---

[4] Asst. Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

[5] "The Aeration of Sharp-Crested Weirs," by Joe W. Johnson, a thesis presented to the Univ. of California, Berkeley, Calif., in May, 1934, in partial fulfilment of the requirements for the degree of Master of Science in Engineering (unpublished).

When a deflector is placed to prevent air from entering into the space under the nappe by any method except through the vents, the quantity of air withdrawn is given by Eq. 9, which is based on all available laboratory and field data. However, if air is allowed also to enter the underside of the nappe by circulation, less air is taken in through the vents and the relationship between flow of air through the vents and the other variables is different from that shown by Eq. 9. The quantity of air flowing through the vents depends on

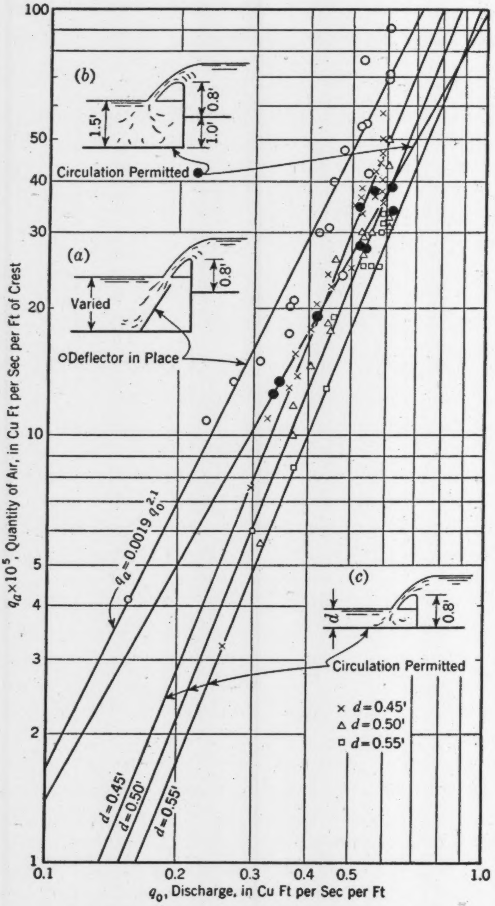


FIG. 11.—RELATION BETWEEN DISCHARGE AND QUANTITY OF AIR DRAWN THROUGH INLETS TO THE UNDERSIDE OF THE NAPPE OF A SHARP-CRESTED WEIR WITH VARIOUS CONDITIONS OF BOTTOM POSITION AND DEPTH IN THE DOWNSTREAM POOL

the position of the bed and the depth of the downstream pool with respect to the weir crest.

A reduction in the pressure under the nappe depresses the nappe inward and reduces the head on the weir by a certain amount. The effect of depressing the nappe generally expands the cross-section area of the nappe at the crest and thus increases the discharge by making the head more effective. The net result is to increase the weir coefficient.[6] For example, Fig. 12 shows the variation of the weir coefficient with the degree of aeration for the 1-ft weir tested by the writer. For a certain head the effect of preventing free access of air to the underside of the nappe gives a discharge greater than that obtained by complete aeration. A maximum increase in discharge of 23% resulted with an "adhering" nappe, and the "wetted underneath" and "depressed" conditions gave increases of 18% and 6%, respectively.

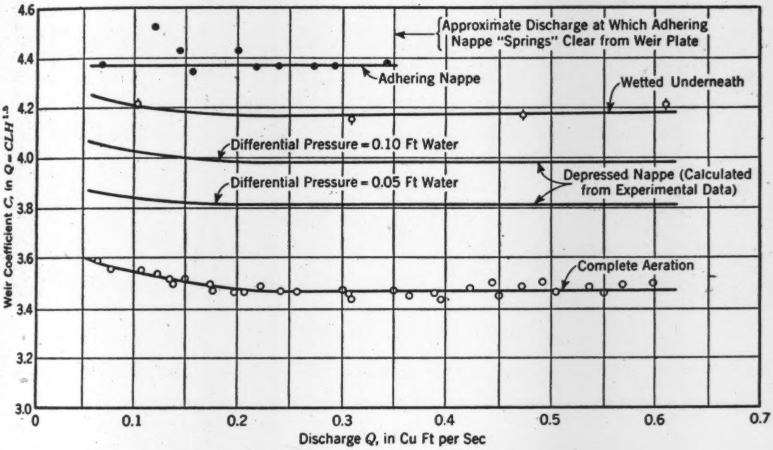


FIG. 12.—WEIR COEFFICIENT FOR VARIOUS CONDITIONS OF AERATION

J. W. HOWE,[7] ASSOC. M. AM. SOC. C. E., AND CLAUD C. LOMAX, JR.,[8] JUN. AM. SOC. C. E.—A practical problem which long has been neglected is comprehensively discussed in this paper. Also, the serious overload that can be produced by insufficient aeration of a gate or spillway is emphasized. The material presented should be welcome indeed to designers who must provide aeration ports in hydraulic structures.

Several years ago a rectangular weir in the laboratory of the Iowa Institute of Hydraulic Research at Iowa City was found to have an inconsistent head-discharge relation. Edward Soucek, Assoc. M. Am. Soc. C. E., believed this behavior was due to insufficient aeration, and later proved his point by enlarging the aeration ports, thereby obtaining a consistent relation between head and discharge. Remembering this incident, the writers undertook an investigation of the effect of insufficient aeration upon the discharge over a weir.[9]

---

[6] "Hydraulic Laboratory Practice," by John R. Freeman, A.S.M.E., 1929, p. 147.

[7] Prof. and Head, Mechanics and Hydraulics Dept., State Univ. of Iowa, Iowa City, Iowa.

[8] Ensign, U. S. Naval Reserve, David Taylor Model Basin, Washington, D. C.

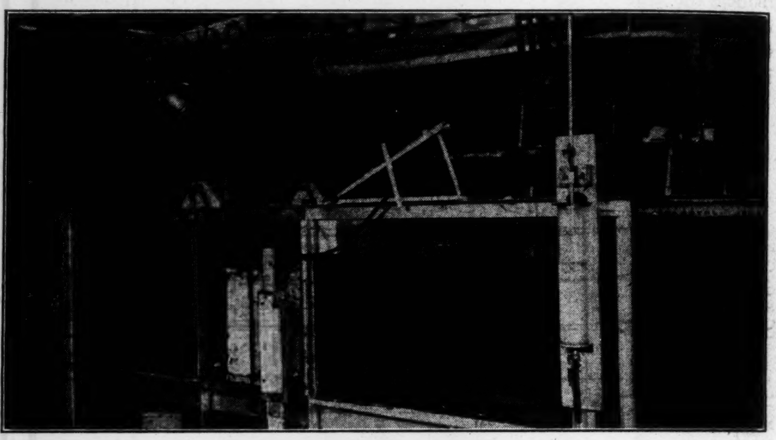


FIG. 13.—VIEW OF EXPERIMENTAL WEIR

TABLE 6.—TRANSFORMATIONS OF AUTHOR'S DATA

VALUES TAKEN FROM FIG. 10		COMPUTED VALUES	
p/H_o	H_o/H'	p/H'	$\Delta H/H'$
0.007	0.9995	0.0070	0.0005
0.01	0.998	0.0100	0.002
0.02	0.996	0.0199	0.004
0.03	0.994	0.0298	0.006
0.04	0.993	0.0397	0.007
0.05	0.991	0.0496	0.009
0.07	0.988	0.0692	0.012
0.10	0.984	0.0984	0.016
0.20	0.972	0.1944	0.028
0.30	0.958	0.288	0.042
0.40	0.948	0.379	0.052
0.50	0.937	0.468	0.063
0.60	0.928	0.557	0.072
0.80	0.912	0.730	0.088
1.00	0.899	0.899	0.101
1.50	0.872	1.309	0.128
2.00	0.852	1.704	0.147

Arrangements were made to measure, by means of small orifices, the air requirements of a 2.5-ft rectangular suppressed weir 1.67 ft high whose discharge could be determined from previously calibrated supply sources. No attempt was made to prevent the return of air to the underside of the nappe as it discharged freely into the tailwater. This procedure was in contrast to that followed by Mr. Hickox who did not allow air to return to the underside of the nappe. The apparatus permitted a variation of discharge between 0.52 and 3.60 cu ft per sec per foot of crest, a height of fall between 0.07 and 1.57 ft, and pressure reductions p up to 0.65 ft of water. Aeration was varied from none to complete by interchanging entrance orifices of different sizes on the aeration port (see box near center of Fig. 13). The pressure reduction was limited either by the complete evacuation of the air or by air breaking through the nappe.

---

[9] "The Effect of Aeration Rates Upon the Discharge Over a Suppressed Weir," by Claud C. Lomax, Jr., a thesis presented to the State Univ. of Iowa, Iowa City, in 1942, in partial fulfilment of the requirements for the degree of Master of Science in Hydraulic Engineering.

Direct comparison of the author's and the writers' results is possible only in the relation of pressure reduction to the consequent change in head, a striking agreement existing in this respect. The writers used a relation involving $\Delta H/H'$ and p/H' rather than H_o/H' and p/H_o as used by the author. However, multiplication of p/H_o by H_o/H' gives p/H'; and $\Delta H/H'$ equals $1 - H_o/H'$. These transformations (see Tables 6 and 7) permit the comparison shown in Fig. 14. The line whose equation is

$$\frac{\Delta H}{H'} = 0.115 \left(\frac{p}{H'} \right)^{0.85} \quad \ldots (15)$$

was located on the basis of the writers' experiments rather than from the points (encircled) taken from Fig. 10.

TABLE 7.—TRANSFORMATIONS OF WRITERS' DATA

VALUES FROM FIG. 14 (SEE EQ. 15)		COMPUTED VALUES	
$\Delta H/H'$	p/H'	H_o/H'	p/H_o
0.00105	0.004	0.999	0.004
0.0023	0.01	0.998	0.010
0.0042	0.02	0.996	0.020
0.0075	0.04	0.992	0.040
0.0120	0.07	0.988	0.071
0.0164	0.10	0.984	0.102
0.0355	0.25	0.964	0.260
0.064	0.50	0.936	0.534
0.090	0.75	0.910	0.825
0.116	1.00	0.884	1.132
0.153	1.40	0.847	1.652

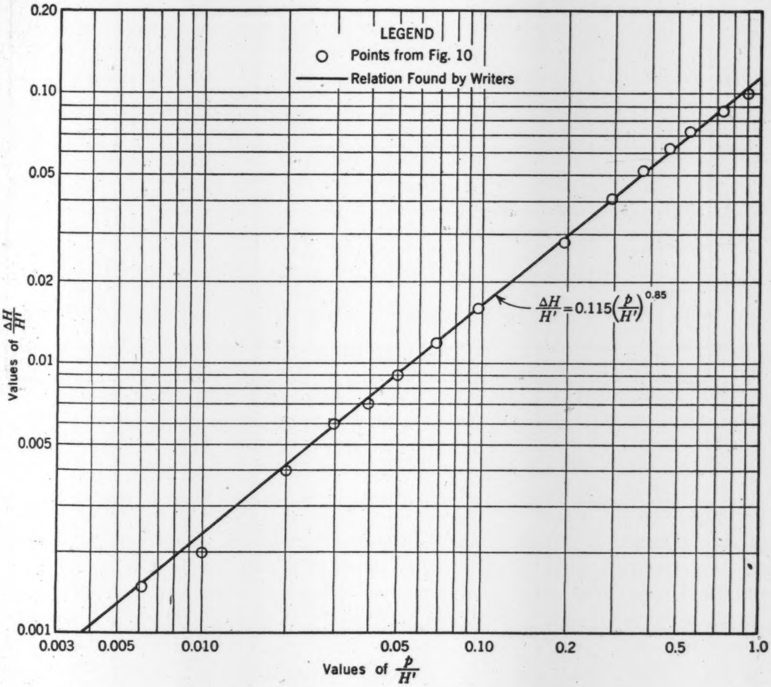


FIG. 14.—CHANGE OF HEAD WITH PRESSURE

The agreement between the two sets of data is again illustrated by Fig. 15 in which the writers' data are plotted on Fig. 10, precise agreement being evident up to $p/H_o = 0.9$. This independent evidence substantiates the author's relation between pressure reduction and discharge and emphasizes, as well, the importance of adequate aeration in the measurement of water. The highest value of p/H' observed by the writers was 0.9 which caused a change $\Delta H/H'$ of 0.105. A change of this magnitude indicates the possibility of an error in discharge computed from the observed head of approximately $3/2 \times 0.105 \times 100 = 16\%$.

Although the return of air to the underside of the nappe greatly complicates the estimation of air demand, it is commonly encountered in the use of weirs. The writers feel that such factors as the discharge, the roughness of the crest, the height of fall, the depth of tailwater, the conformation of the bed, and perhaps even the pressure reduction are involved. Some qualitative evidence of the influence of certain of these factors was found.[9] Empirical expressions relating the air discharge, the water discharge, and the height of fall were obtained, but these were not dimensionally homogeneous and depended upon the diameter of the air orifice used, making them of doubtful value in another situation. The author is reluctant to impute significance to the scattered points of Fig. 8, but it is evident that, in general, the data for the 2-ft gate

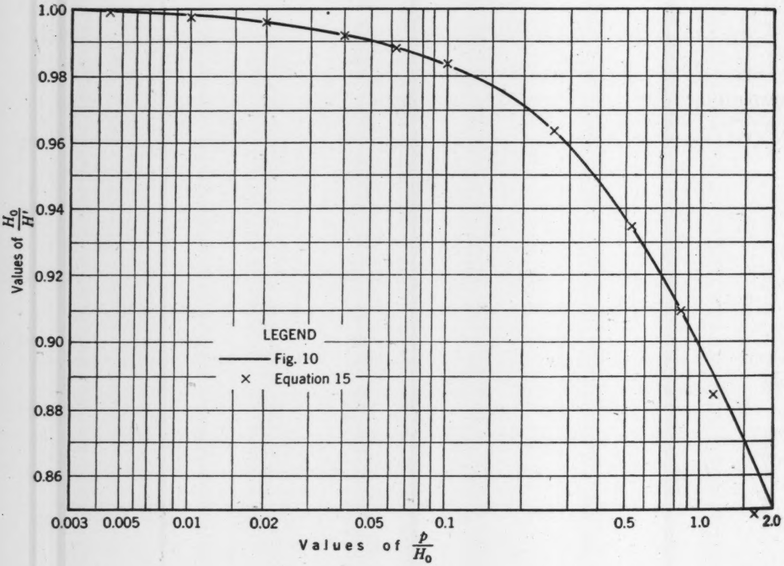


FIG. 15.

show a greater air demand than do those of the 1-ft gate with its shorter height of fall.

There is still need for considerable study of the air requirements of measuring weirs discharging naturally into the tailwater.

J. C. STEVENS,[10] M. AM. SOC. C. E.—The data and formulas presented in this paper will aid greatly not only in designing spillways but in ascertaining the flow characteristics of existing spillways. The profession's thanks are due the author for his contribution.

The author apparently assumes that it is always desirable to prevent subatmospheric pressures beneath the nappe. For vertical-lift gates designed for major overflow heads, this is doubtless true in order to prevent vibration. With other types of gates or on overflow crests either without gates or with undershot gates, this may not be so apparent. For example, drum gates, such as those on the crest of Grand Coulee Dam or those on the spillway crest at Boulder Dam, may withstand the effects of considerable subatmospheric pressure without serious effect and with some resulting benefits of increased discharge.

On overflow spillways without crest gates, advantage should be taken of the fact that the efficiencies of such crests can be increased by not aerating the underface of the nappe, and it is that phase of the subject the writer wishes to discuss briefly.

At flood-control dams it has been the practice to design for great overflows under high heads. The cost of the spillway thus becomes a large part of the total developmental cost. Adding some portion of an atmospheric head to the water head, therefore, presents opportunities for more economic design. Of course, the increased load on the spillway section, resulting from subatmospheric pressures beneath the overflowing sheet, must be taken into account, but it will be found they need give little concern in the gravity type of overflow dams.

Suppose a crest profile is adopted that will be just at the limit of form to produce no subatmospheric pressures in the interface between nappe and concrete for a given design head, H_d. Then if the head is increased to some greater value H_o, the discharge coefficient will also be increased because of the creation of subatmospheric pressures at the interface, and a consequent addition of some atmospheric head to the water head.

The magnitude of these increases was investigated[11,12] by Hunter Rouse, M. Am. Soc. C. E., and Lincoln Reid, Assoc. M. Am. Soc. C. E. It was shown that the design head might be increased several times with continually increasing coefficient before separation of the nappe. These increases check fairly well with the increases in discharge given in Fig. 10, as the following example will show.

---

[10] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[11] "Model Research on Spillway Crests," by Hunter Rouse and Lincoln Reid, *Civil Engineering*, January, 1935, p. 10.

[12] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., p. 316.

The No. II profile of overflow crest of the Rouse-Reid experiments[13] was used. A design head of 6.0 ft was selected for which $C = 3.51$ and $q = 52$ cu ft per sec per lin ft of spillway crest. The percentage increase in C then becomes the percentage increase in Q due to subatmospheric pressures at the interface. Table 8 shows the calculations, which are explained by the notes. Comparing the quantities in Cols. 4 and 11, they are seen to check fairly well with the limit of the curve in Fig. 10.

TABLE 8.—INCREASE IN DISCHARGE FROM REDUCED PRESSURES BENEATH THE NAPPE

Explanation: Col. 3, from Rouse-Reid experiments; Col. 4, same as increase of C (in Col. 3) over $C_d = 3.51$; Col. 5, calculated from $q = C\,(H_e)^{3/2}$; Col. 7, $H' = \left(\frac{q}{3.51}\right)^{2/3}$ (this would be the head if there were no subatmospheric pressure at the interface); Col. 9, read from curve of Fig. 10 with $\frac{H_e}{H'}$ as the argument; Col. 10 = Col. 9 × Col. 1; and Col. 11, read from curve of Fig. 10 with $\frac{p}{H_e}$ as the argument.

Head H_e (1)	Ratio $\frac{H_e}{H_d}$ (2)	C (3)	Increase in Q (%) (4)	q (5)	$\frac{q}{3.51}$ (6)	H' (7)	$\frac{H_e}{H'}$ (8)	$\frac{p}{H_e}$ (9)	Feet of water p (10)	Increase in Q_2 (%) (11)
6	1.00	3.51	0	52	14.8	6.00	1.000	0	0	0
8	1.33	3.65	4	83	23.6	8.23	0.971	0.20	1.6	4.7
10	1.67	3.79	8	120	34.2	10.5	0.953	0.35	3.5	7.8
12	2.00	3.91	11	162	46.1	12.9	0.931	0.57	6.8	12.0
14	2.33	4.02	14	211	60.0	15.3	0.915	0.75	10.5	14.8
16	2.67	4.12	17	264	75.1	17.8	0.904	0.96	15.4	....
18	3.00	4.19	19	320	91.2	20.3	0.887	1.05	18.9	....
20	3.33	4.26	21	381	108.5	22.8	0.878	1.08	21.6	....
22	3.67	4.30	23	444	126.5	25.2	0.873	1.10	23.2	....

It is obvious from the foregoing that the savings in cost are important enough to warrant further development of the thesis that spillway discharges can be increased materially and safely by designing for subatmospheric pressures beneath the nappe.

This would be particularly useful for flood-control dams. The design head would produce no subatmospheric pressures; and hence all ordinary floods would be passed with atmospheric pressure beneath the nappe. For the 1-in-100-yr or 1-in-1,000-yr flood, subatmospheric pressures of as much as $p = 30$ ft of water (which would increase discharges about 25%) could be provided for. This would permit either (1) reducing the spillway length to 80% of what would otherwise be required for the same head; (2) lowering the height of dam by 16% of the maximum overflow head and thereby reducing flood damages upstream; or (3) increasing the height of spillway crest for greater storage and power head if the pool level be fixed at some definite upper limit.

Little or no damage to the concrete, by pitting, could result from such intermittent use even if the pressure at the interface should fall to that of the water vapor and cavitation should result. If pitting should occur during a protracted flood, repairs could readily be made before the next flood would occur.

---

[13] "Model Research on Spillway Crests," by Hunter Rouse and Lincoln Reid, *Civil Engineering*, January, 1935, p. 11, Fig. 2.

It would seem, therefore, that those agencies concerned with the design and construction of flood-control spillways would see that the experiments described by the author were extended to include a further study of spillways without crest gates. It would be a decided advantage, in such studies, if the models could be enclosed and the atmospheric pressure within adequately controlled.

G. H. Hickox,[14] M. Am. Soc. C. E.—The discussions of this paper leave very little to be added in closing.

The writer is not quite in agreement with Professor Johnson that the increase of discharge occurring with reduction of pressure beneath the nappe is caused by expanding the cross section of the nappe. The cross section may expand, but the writer prefers to attribute the increase in discharge largely to the increased velocity, which must exist because of the larger pressure difference on the lower side of the nappe.

Mr. Johnson and Messrs. Howe and Lomax remark on the lessened air requirement when air circulates beneath the nappe. The amount of air circulating depends principally on the size of the eddy in the tailwater beneath the nappe. The size of this eddy is affected by the depth of tailwater, the height of fall, and the slope of the downstream face of the spillway. In the spillways for which the experiments were made, the nappe fell on the sloping face of the spillway and no air circulated. Provision of aeration vents for the maximum air requirement will always be safe and ordinarily will not be excessively expensive.

The agreement between the writer's experiments and the independent work of Messrs. Howe and Lomax with respect to the effect of pressure reduction on discharge, as shown in Figs. 14 and 15, is very gratifying. The use of Eq. 15 will eliminate the need for Fig. 10. The data of Messrs. Rouse and Reid, cited by Mr. Stevens,[11,12,13] also show good agreement with Fig. 10.

It was not assumed, as Mr. Stevens suggests, that it is always desirable to prevent subatmospheric pressures. In the case of the dams for which the studies were made, it was necessary to prevent the pressure beneath the nappe from being lowered to a point that would endanger the safety of the gate. In the case of the field tests described, the average hydrostatic head on the lower leaf was increased from a normal value of 29.5 ft to 39.2 ft. If the pressure beneath the nappe could have been reduced to the vapor pressure of the water, the average hydrostatic head on the gate would have amounted to more than 60 ft. This is twice the maximum load for which the gate was designed and does not consider any secondary effects that might be caused by vibration.

Mr. Stevens' suggestion that spillways be overloaded to increase their discharge capacity appears to have merit, if the overload condition is provided for peak discharges only and not designed for regular operation. In the present state of the knowledge of cavitation phenomena it is not safe to assume that cavitation damage will not occur, and until further knowledge of this subject is available it would seem wise not to invite it. Excessive damage should not occur during short periods of operation, however, and if the spillway face remains dry for long intervals, it would be possible to make any needed repairs. It would seem that the possibility of effecting economics by overloading spillways for short-period peak discharges could well be investigated in future designs.

---

[14] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

The writer wishes to thank those who have contributed the valuable confirming data and the suggestions contained in their discussions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2216

CONCRETE RESERVOIRS OF THE VERTICAL-BEAM TYPE

By C. Maxwell Stanley, Jr.,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Lewis A. Schmidt, Jr., Victor H. Cochrane, R. E. Koon and J. C. Gearhart, Gustavo Pérez Guerra, and C. Maxwell Stanley, Jr.

Synopsis

Covered concrete storage reservoirs can be designed by a "vertical-beam" method that offers advantages over conventional types. The method, which is presented herein, utilizes side-walls constructed as simple vertical beams with supports arranged at top and bottom to resist the internal horizontal pressure. The designation "vertical beam" is taken from this wall construction.

With such a design, all of the concrete in contact with the contained water is in compression. Simple methods are afforded for minimizing temperature and shrinkage stresses. These features reduce the formation of cracks on the interior face of the structure and thus, by preventing seepage into the wall, increase the resistance of the structure to deterioration. These advantages are obtained without increase in construction cost over conventional types.

Design Problem

Reservoir Experience.—The design of concrete storage reservoirs is difficult, particularly in climates having severe winters. Many cases of unfortunate experiences with concrete storage reservoirs have resulted from a lack of appreciation of the fundamental design principles.

The designer must obtain a watertight, structurally safe reservoir at a minimum cost. The wall must act both as a membrane preventing the flow of water from the reservoir and as a structural member resisting internal pressure. To fulfil the first function, the walls must be impervious and free from cracks. As it fulfils the second, it must deform to resist stresses and, as it deforms, it will crack wherever appreciable tensile stresses occur. Even though these cracks do not result in objectionable leaks, they permit saturation of the concrete which may lead to rapid damage from freezing and thawing. Even the use of

---

Note.—Published in December, 1942, *Proceedings*. Positions and titles are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr. (Stanley Engineering Company), Muscatine, Iowa.

low steel stresses will not eliminate cracks entirely. If the designer can select the design in which the concrete surface presented to the water remains in compression and is free from uncontrolled cracks, he will have done much toward the elimination of seepage into the concrete, with its ensuing deterioration.

Criteria of Wall Design.—The following are listed as criteria of an ideal wall design that will minimize and control cracks:

(1) Concrete in contact with water must be in compression;

(2) Walls must be poured in sections sufficiently small to avoid excessive shrinkage cracks;

(3) The entire structure must be so designed that expansion and contraction may be controlled to avoid cracking of surfaces in contact with water; and

(4) Construction joints must be kept to a minimum, and every construction joint must be treated as an expansion joint.

The failure of conventional types of reservoirs to measure up to these criteria led to the development of the "vertical-beam" design.

VERTICAL-BEAM TYPE

General Description.—As the name implies, the side-wall (see Fig. 1) acts as a simple vertical beam resisting the internal water pressure, with the horizontal reactions taken by suitable supports. The bottom of the beam sets in a recess in the footing which is designed so that the wall may rock slightly with changes in temperature and loading, and thus avoid rigidity.

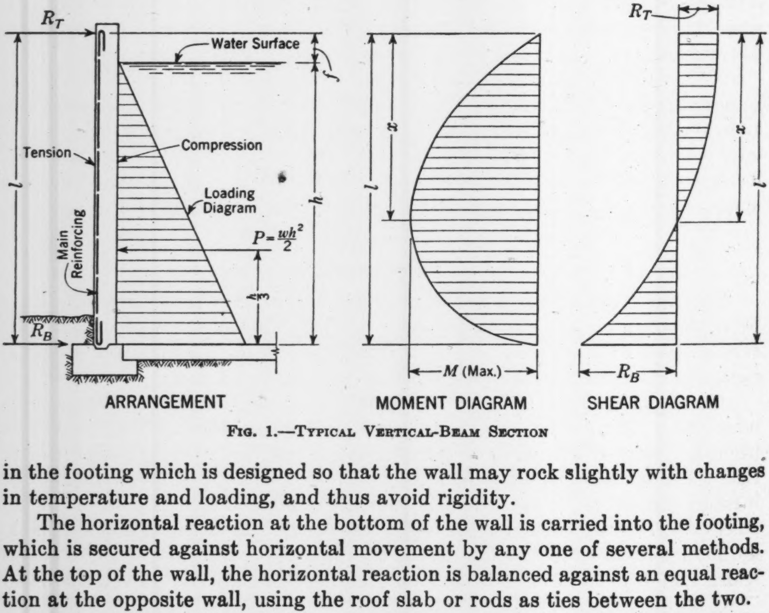


FIG. 1.—TYPICAL VERTICAL-BEAM SECTION

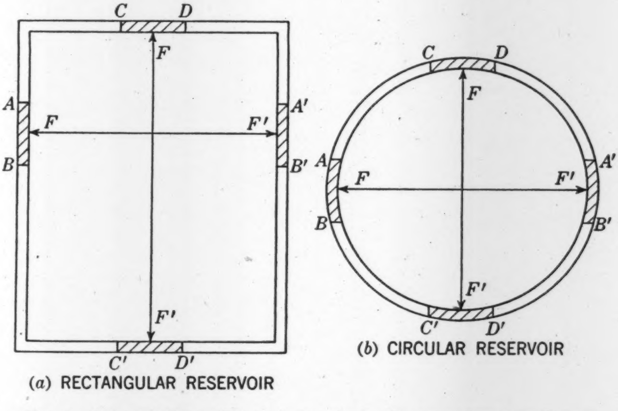
The horizontal reaction at the bottom of the wall is carried into the footing, which is secured against horizontal movement by any one of several methods. At the top of the wall, the horizontal reaction is balanced against an equal reaction at the opposite wall, using the roof slab or rods as ties between the two.

Sections of the side-wall are constructed in one pour from the footing to the top, thus eliminating all horizontal construction joints. The length of wall sections between vertical joints is kept reasonably short to limit the size of pour and to avoid extreme shrinkage or temperature stresses. All vertical construction joints are treated as expansion joints and are carefully sealed against leakage.

The main steel reinforcement is placed vertically near the outer face of the wall. Temperature steel (not shown in Fig. 1) may be placed horizontally near the outer face and vertically and horizontally near the inner face. The steel from the footing into the beam (if used) is placed in the plane of the main reinforcing near the outer face of the wall.

This design affords a construction in which cracking is minimized by placing in compression all concrete in contact with the water and by carefully providing for temperature and shrinkage stresses. The stresses in the wall may be computed readily by simple structural analysis that need not be outlined or discussed herein.

Support of Top.—Horizontal support of the top of the vertical beam relies upon the symmetry of the structure, which permits the balancing of horizontal forces on opposite sides of the reservoir as shown in Fig. 2. The forces F and F' will be equal in amount and opposite in direction whether the reservoir is circular, square, or rectangular. The two forces may be balanced by providing a suitable connecting member between the opposite walls. Three different methods are shown in Fig. 3(*a*). The top support will move, due to the expansion or contraction of the tie member and to the strains resulting from the internal load. This movement is taken care of by permitting the vertical beam to rock upon its base.



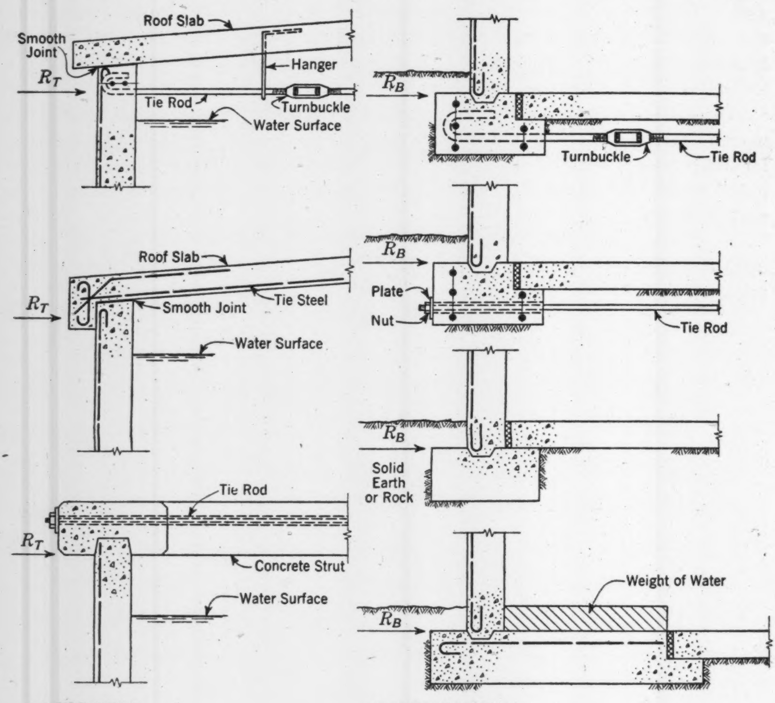
(*a*) RECTANGULAR RESERVOIR

(*b*) CIRCULAR RESERVOIR

FIG. 2.—VERTICAL-BEAM WALLS ARRANGED TO BALANCE OPPOSING FORCES

Support of Bottom.—The bottom of the vertical beam may be supported either by balancing opposing forces, as described for the top of the beam, or by utilizing a horizontal force resulting from the reaction of the wall on the earth or rock or from friction on the base.

Four different arrangements for supporting the bottom of the beam are shown in Fig. 3(*b*). In each of these designs the bottom of the wall is detailed so that it will be free to rock about its base. If tie rods are used between the footing and the wall, they are located in the plane of the main steel reinforcement near the outer wall to avoid tension at the bottom of the slab. A seal (not shown in Fig. 3(*b*)) must be provided to prevent leakage through the horizontal joints where the wall joins the footing. All of these arrangements avoid tension in the surface of the wall that is exposed to the water.


Roof Slab
Smooth Joint
R_T
Hanger
Tie Rod
Turnbuckle
Water Surface
R_B
Turnbuckle
Tie Rod
Roof Slab
R_T
Smooth Joint
Tie Steel
Water Surface
R_B
Plate
Nut
Tie Rod
R_B
Solid Earth or Rock
Tie Rod
R_T
Concrete Strut
Water Surface
Weight of Water
R_B

(*a*) TOP WALL (*b*) BOTTOM WALL

FIG. 3.—METHODS OF SUPPORTING THE WALLS

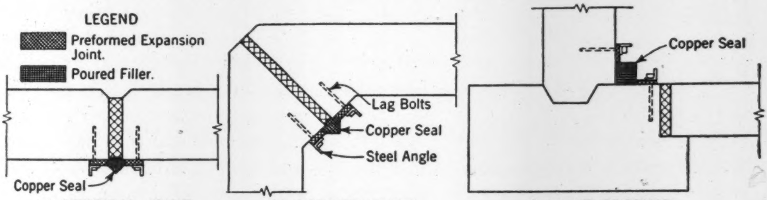
Provision of Expansion and Contraction.—Expansion and contraction are taken care of by limiting the dimensions of each pour and by designing the structure with a flexibility that will permit movement of the walls without damage to the structure.

Horizontal construction joints are eliminated and each vertical wall section is limited to such length as will avoid shrinkage or contraction cracks. In the original designs, sections of 30 ft to 40 ft were used, in keeping with customary practice in other concrete structures. However, vertical cracks have developed in sections of this length and are apparently the result of continued contraction. Therefore, more recent designs have used shorter lengths of wall (less than 20 ft) to eliminate such cracking. Temperature steel may be used as required in the faces of the wall, but should not extend through the vertical construction joints. Expansion joints are used between adjacent wall sections.

Corners of square or rectangular reservoirs are constructed with expansion joints arranged so that movement, or rocking of the wall, will not be resisted by the corners. Changes in temperature will result in the expansion and contraction of the roof slab or supporting members holding the top of the vertical beam. When this occurs, the vertical beam will rock slightly on its base, without changing the stress relationship within the member, as there is no rigidity in the connections between the wall, footing, and roof. Differences in temperature between the inside and outside of the reservoir will cause differential expansion or contraction of the inner and outer surfaces, and the wall will tend to assume a slightly curved section. However, as the wall is not restrained at either top or bottom, it may be distorted in this manner without harm.

Waterproofing.—Waterproofing is required on the wall at the vertical expansion joints and at the horizontal joints between footing and wall. The walls require a waterproofing treatment to fill the pores and increase imperviousness. Several kinds of waterproofing may be used, but very satisfactory results have been obtained with a cement grout containing iron filings.

The expansion joints are composed of poured or premolded material of suitable thickness and require some type of noncorrosive metal steel. After some unsatisfactory experiences with copper seal of conventional design, an externally mounted seal has been developed. The seal, shown in Fig. 4(*a*), is installed after the wall is poured and is thus accessible during installation and for maintenance. Its cost is probably a little more than a conventional seal, but any extra cost is fully warranted by its simplicity and dependability (see also Fig. 4(*b*) for a seal at the corner joint).



(*a*) VERTICAL JOINT (*b*) CORNER JOINT (*c*) AT FOOTING

FIG. 4.—TYPICAL SEALS

The joint between the wall and footing also must be made tight, and this can be accomplished either by a modification of Figs. 4(*a*) and 4(*b*) or by the use of a more conventional copper seal (Fig. 4(*c*)). If the head of water within the reservoir is small, the copper seal may be omitted and the joint can be sealed with bituminous material.

Backfilling.—Although the vertical-beam design is intended primarily for reservoirs located above ground, the side-walls may be partly backfilled without any modification of the design. Under such conditions, the simple beam acts as a cantilever with the main reinforcement carrying the tension and with the inner face of the wall remaining in compression. Dowels must then be used near the outside face to tie the wall to the footing; and, at the top of the wall, a type of support must be used in which the wall is free to move inward. A considerable depth of backfill may be placed without requiring any increase in wall section. The footing must be of adequate design to prevent the wall from overturning.

EXPERIENCE WITH VERTICAL-BEAM RESERVOIR

Actual construction and use are the final test of any design. A critical examination of experiences with the vertical-beam design reveals its advantages as well as the problems that have arisen. Both difficulties and advantages call attention to needed changes in details of design and construction. Data on three such reservoirs that have been constructed and used are shown in Table 1.

TABLE 1.—DATA ON VERTICAL-BEAM RESERVOIRS

No.[a]	Year built	Capacity (gallons)	AVERAGE DIMENSIONS (FEET)			
			Plan	Water depth	Wall height	Spacing[b]
1	1935	3,000,000	122 by 90[c]	18	20	40[d]
2	1939	120,000	41 by 41	9	10	41
3	1941	100,000	34 by 34	9.5	10.5	17

[a] Corresponding to project numbers in Fig. 5. [b] Spacing of expansion joints. [c] Each of two compartments. [d] Joint spacing 30 ft to 40 ft.

Typical details of wall construction and wall support for these three reservoirs are shown in Fig. 5. Experiences during the construction of the reservoirs, and observations after they had been placed in service, have disclosed defects that have been remedied in subsequent designs.

The arrangement of tie rods and struts in the first project (Fig. 5(*a*)) was very effective in preventing the movement of footings, but it was costly and was slow to construct. Even before the job was completed, the simpler arrangements for bottom support shown in Figs. 5(*b*) and 5(*c*) were developed for use in subsequent designs. These arrangements permit the tightening of tie rods after all other construction is complete.

The method of top wall support in the first project (slab rigidly connected to wall) hindered the free action of the wall. Cracks developed immediately over each vertical wall joint and extended back into the roof slab. Also, diagonal cracks developed across each of the roof slabs some 8 ft or 10 ft back from the intersection of the outside wall. This crack was matched by vertical cracks in each of the walls a like distance from the corner. All these cracks seemed to be the direct result of restricting the free action of the wall and roof by a rigid interconnection. This led to the development of other methods of top support (see Fig. 3(*a*)) that permitted free action of roof and wall, and these methods were adopted in subsequent designs.

Trouble with the expansion joints used on the first project developed when the reservoir was filled. Efforts to repair them were generally unsatisfactory until a seal similar to that of Fig. 4(*a*) was used. This type of seal has been so

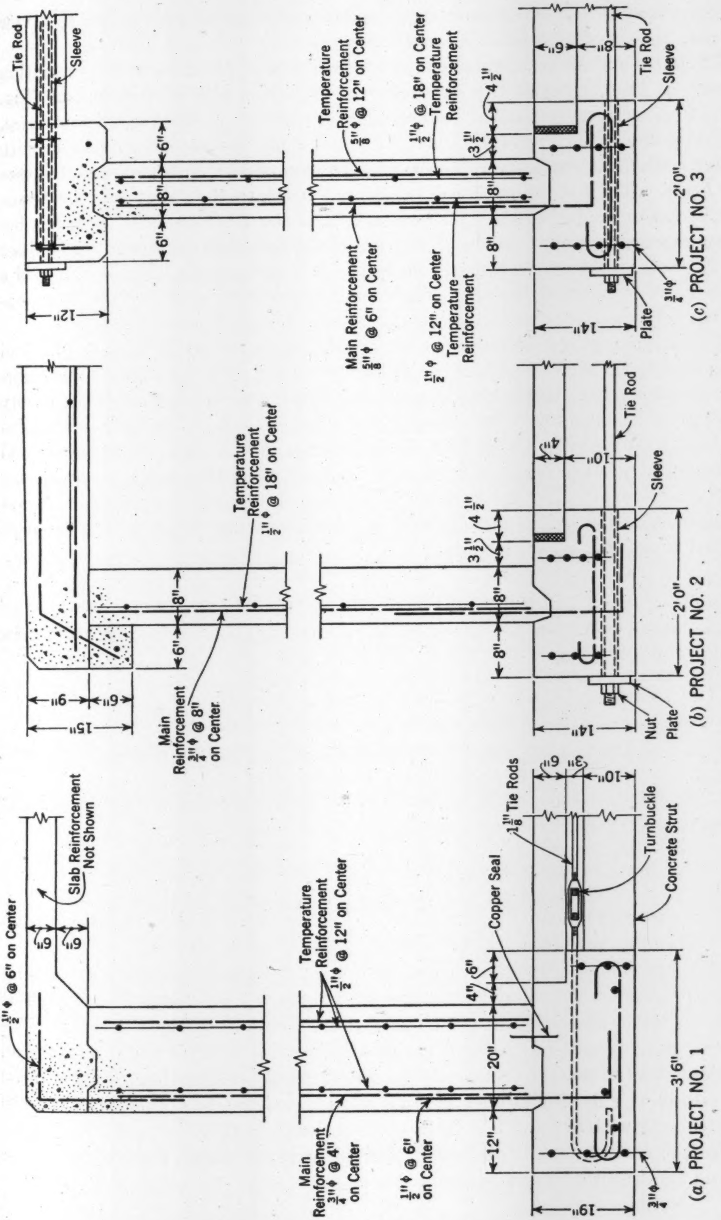

(a) PROJECT NO. 1
$\frac{1}{2}$"ϕ @ 6" on Center
Slab Reinforcement Not Shown
6"
6"
Temperature Reinforcement $\frac{1}{2}$"ϕ @ 12" on Center
Main Reinforcement $\frac{3}{4}$"ϕ @ 4" on Center
$\frac{1}{2}$"ϕ @ 6" on Center
Copper Seal
12"
20"
4"
6"
3'6"
19"
$\frac{3}{4}$"ϕ
$1\frac{1}{8}$" Tie Rods
Turnbuckle
Concrete Strut
6"
3"
10"
(b) PROJECT NO. 2
Temperature Reinforcement $\frac{1}{2}$"ϕ @ 18" on Center
Main Reinforcement $\frac{3}{4}$"ϕ @ 8" on Center
9"
6"
15"
6"
8"
8"
$3\frac{1}{2}$"
$4\frac{1}{2}$"
2'0"
14"
4"
10"
Tie Rod
Sleeve
Nut
Plate
(c) PROJECT NO. 3
Tie Rod
Sleeve
12"
6"
8"
6"
Temperature Reinforcement $\frac{5}{8}$"ϕ @ 12" on Center
Main Reinforcement $\frac{5}{8}$"ϕ @ 6" on Center
$\frac{1}{2}$"ϕ @ 18" on Center Temperature Reinforcement
$\frac{1}{2}$"ϕ @ 12" on Center Temperature Reinforcement
$3\frac{1}{2}$"
$4\frac{1}{2}$"
8"
8"
2'0"
6"
8"
14"
Tie Rod
Sleeve
Plate
$\frac{3}{4}$"ϕ

FIG. 5.—TYPICAL DETAILS OF SIDE-WALLS

effective that it has been generally applied not only on successive reservoir designs, but on other hydraulic structures.

The most serious problem encountered in the first designs was the formation of vertical cracks between construction joints. On the first structure (Fig. 5(*a*)), these cracks were not apparent when the structure was placed in service, but have developed gradually over the years. On the second reservoir (with lighter walls), the cracks were observed before construction was completed (see Fig. 5(*b*)). Their location and development indicate that they are the result of contraction and shrinkage stresses and that the 30-ft to 40-ft intervals between expansion joints are too large for the differential temperature stresses which occur in an exposed reservoir wall. The last design (Fig. 5(*c*)) used a much smaller interval between joints, and no vertical cracks have been observed.

The aforementioned troubles, although annoying, have not been serious and are probably no more than may be expected with any new design. The reservoirs that have been constructed have demonstrated the soundness of the theory of design. The construction is simple, and costs are low. The walls have acted as planned and have rocked with the expansion of the roof. The corner and intermediate vertical joints have reacted as expected. The reservoirs are giving satisfactory service and the troubles encountered have only served to improve subsequent design. The same principle of vertical-beam construction has been applied to several other structures.

CONVENTIONAL DESIGNS

The more common designs that have been applied to storage reservoirs include the following:

(1) Circular walls acting as shells to resist internal bursting pressure.
  (*a*) Conventional type with tension steel in concrete.
  (*b*) A pre-stressed reinforcement type developed by William S. Hewett, with tension steel outside the concrete.

(2) Retaining walls utilizing the weight of the impounded water to resist the outward pressure.
  (*a*) Cantilever.
  (*b*) Buttress or counterfort.

A description of each of these types and a statement of their obvious advantages and disadvantages follow.

Circular Walls.—Both the conventional and pre-stressed type of circular walls are based on the same fundamental theory. The walls are designed as shells to resist the bursting pressure of the water within the reservoir. The tension at any point on the wall varies with the diameter of the reservoir and the depth of the water. As concrete cannot resist tensile stress of substantial magnitude, circular hoops of steel must be used. The area of steel required will vary directly with the depth. In the conventional design, the steel is embedded within the concrete wall, whereas in the pre-stressed design the steel is placed around the wall and is pre-stressed.

The placing of the steel within the wall in the conventional design creates a serious problem, as the forces acting on the wall tend to increase its circumference, and the concrete, having limited elasticity, is cracked. Nothing can be done to prevent this cracking although the size of cracks may be controlled. Attempts often are made to avoid cracking by limiting the stress in the steel to about 9,000 or 10,000 lb per sq in., but even under these conditions some cracking is likely to occur. Furthermore, other cracks result from horizontal and vertical construction joints and from temperature and shrinkage stresses.

The disadvantages of the conventional circular reservoir are superbly overcome in the pre-stressed type, which has been quite widely used in recent years since this design separates the membrane from the structural member. This is accomplished by placing the hoop steel outside an inner membrane composed of narrow, vertical, concrete slabs without horizontal joints. The slabs are pulled together by tightening turnbuckles in the circular hoops, thus pre-stressing both slabs and hoops to such a point that, even with the reservoir filled, the slabs are still in compression. This design, with its pre-stressing, readily cares for shrinkage and temperature stresses and achieves the goal of all reservoir design—an inner concrete membrane placed in compression and free from cracking. Its only disadvantages are its mandatory circular shape, and the relatively high cost for certain ratios of depth and diameter.

Retaining Wall Types.—Wall designs of the retaining wall type depend primarily on the weight of the water within the reservoir to offset the overturning moments exerted by the water pressure on the side-wall. Either the cantilever or buttress type may be used. With the cantilever type, the entire inner face of the vertical wall is placed in tension, together with the top surface of the footing within the reservoir. With either buttresses or counterforts, the vertical wall and the base slab are designed as horizontal beams and are usually constructed continuously past two or more supports, thus placing a part of the inner face of the vertical wall and the top face of the footing in tension.

Thus, when the principles of a retaining wall are used with either type of design, a part, or all, of the concrete in contact with the water will be in tension. Such design also requires a rigid structure that does not lend itself readily to the elimination of horizontal construction joints, or to the minimizing of cracking due to temperature or shrinkage stresses.

Appraisal of Vertical-Beam Design

The vertical-beam method of reservoir construction has been developed and tested so that its advantages and disadvantages can be established reasonably well. Reservoirs may be designed successfully, using the vertical-beam method, provided that the details which have given trouble are avoided. The following conclusions may be drawn:

(1) The basic theory of the "vertical-beam" reservoir is sound. The tendency for concrete to crack is minimized by placing all wet surfaces in compression and by providing sufficient flexibility in the structure to reduce the tendency toward cracking from temperature movements and stresses. None of the conventional designs (except the pre-stressed concrete type) accomplish this.

(2) The vertical-beam type of reservoir offers extreme simplicity of both design and construction. Structural analysis is direct and not complicated. Construction is concerned only with plain sections and surfaces.

(3) The construction costs of vertical-beam reservoirs compare favorably with the conventional types. The flexibility of the vertical-beam type of construction and the absence of rigid connections permit the efficient use of concrete and steel and allow the selection of relatively thin sections. These facts, together with the simplicity of construction, permit economy.

(4) The design is most aptly suited to rectangular, or square, reservoirs. Although it may be applied to circular reservoirs, the latter must be of large diameter and the vertical sections must be made sufficiently short so that horizontal bending stresses will not be developed in the curved sections.

(5) The design has been limited to depths such that the vertical walls can be poured in one lift. It may be possible to use horizontal construction joints with suitable seals, but it seems that a better design will be obtained if this is avoided.

(6) This type of construction is limited as to the amount of backfill that may be used on the outside. Under backfill conditions, the vertical wall acts as a cantilever, and its strength as such is limited.

(7) This type of design is also probably limited to those cases where good foundations are available. If a reservoir must be constructed upon soft foundations, a continuous base slab probably will be dictated.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the suggestions that have been made by Arthur E. Stanley, Assoc. M. Am. Soc. C. E., and by Herman S. Smith and Allen H. Dunton, Juniors, Am. Soc. C. E., who have been associated with him in the preparation of the detailed designs of these reservoirs.

DISCUSSION

Lewis A. Schmidt, Jr.,[2] M. Am. Soc. C. E.—An interesting argument for vertical-beam type reservoirs is contained in this paper. Without doubt there are many applications for this construction.

Regarding vertical cracks developing in project 1 and project 2, Fig. 5, and not in project 3, it is quite likely that the added percentage of horizontal reinforcement (from 0.17% and 0.14% to 0.34%, respectively) contributed as much to the reduction of these cracks as did the shortening of the length of the pour. There is a decided construction economy in the reduced number of pours, because of the decreased number of joint seals that must be placed. Where comparatively thin walls are exposed to atmosphere on one or both sides, a proportion of $p = 0.5\%$ in each direction for temperature reinforcement is not unusual. This amount of steel would probably be justified by the aforementioned economies, and would perform the same function.

The concrete placement in the very thin walls of projects 2 and 3 is also diametrically opposed to achieving least shrinkage, since so thin a wall, 10 ft high, and with steel on one or both faces, almost dictates the use of high slump concrete. Concrete which can be placed with lower slumps gives added assurance of less shrinkage cracks, so some thought might well be given to a minimum wall thickness of about 12 in., which could be built with a drier concrete without danger of honeycombing and with less water to be dehydrated. It seems probable that the disparity of roof thicknesses between projects 1 and 2 contributed to the cracking in project 1 as much as the restraint offered by the rigid connection.

The author mentions that conventional copper seals have not proved to be satisfactory. These have been used so often in heavy construction, particularly in conjunction with premolded joints, that it would be desirable to set forth their deficiencies in the reservoirs.

Mr. Stanley advisedly calls attention to limitations of the design, and undoubtedly the seventh one would be very important in this case, especially where the bottom slab rests on the footing as well as on the foundation. Differential pressures at this point due to the wall load could contribute some unequal settlement and cracking, particularly of the bottom slab.

This type of construction may be expected to show up to best advantage in square or rectangular rather than circular structures. Within its limitations it would be interesting to see a comparative cost analysis of various designs.

Victor H. Cochrane,[3] M. Am. Soc. C. E.—The clear presentation of the advantages and limitations of the vertical-beam type of reservoir and the discussion (based on the author's experience) of both desirable and faulty details of construction make this paper a valuable contribution.

A reservoir of this type, built in the spring of 1942 for a large army cantonment in the Southwest, was designed under the direction of the writer. The capacity of 3,000,000 gal is sufficiently large to permit the elimination of conventional steel elevated-tank or standpipe storage. As the reservoir is located on a hill of suitable elevation, the shape of the limited area on top dictated the use of a circular reservoir. Earth was used for backing and for a roof cover 18 in. in thickness in order to improve the quality of the stored water, to minimize temperature variations, and to effect concealment of the structure. The inside diameter is 164 ft and the depth of water, 20 ft. The rock face of the foundation trench is relied upon to resist that part of the bottom reaction of the wall sections not taken care of by the earth backing. The support of the top of the wall is similar to that shown in the middle detail of Fig. 3(*a*). There are twelve vertical joints at intervals of 43 ft. The weight of reinforcement required was found to be only about one half as great as for a conventional hoop-tension reservoir.

---

[2] Acting Constr. Engr., TVA, Hales Bar Dam, Chattanooga, Tenn.

[3] Cons. Engr., Tulsa, Okla.

This structure departs from the author's recommendations in the following particulars: (1) It is circular in plan, for the reason previously stated; (2) it is covered and backed with earth; and (3) the length of the wall sections is greater than recommended. In general, these departures may be explained by the fact that the reservoir was expected to be in use not more than five years, thus making the requirements less severe than for a permanent structure.

From the structural point of view the earth covering is advantageous in some respects as it greatly reduces the temperature range and, at the same time, protects the concrete from the effects of freezing and thawing. Since the inward earth pressure is about one half as great as the outward water pressure with full reservoir, the inner face of the wall will remain in compression except when the tank is empty or drawn down to an unusual extent. If some small cracks should appear under these conditions, they should close when the reservoir is filled above the normal drawdown level.

Experience has shown that, unless extraordinary measures are employed, a concrete wall resting on earth or rock and exposed to extreme ranges of temperature tends to crack at intervals of 15 ft to 20 ft, and therefore the statement that the wall sections should be less than 20 ft long between joints is correct. As the author suggests, horizontal bending stresses may be set up in long sections because the vertical-beam elements are somewhat restrained from bending radially. The section is strengthened by this action unless cracking occurs. In the present instance it was felt that the earth protection and the short period of use justified the joint spacing adopted.

The structure has been in use since June, 1942, and no defects of consequence have been reported. The circular structure is free from the undesirable conditions occurring at the corners of rectangular reservoirs. It will be interesting to note whether any objectionable cracking occurs.

In his discussion of retaining wall types (see heading, "Conventional Designs: Retaining Wall Types"), the author omits reference to the arch-wall design, such as that developed by the writer for a 1-million-gal clear well of the water treatment plant near Fort Smith, Ark. Several descriptions of this reservoir have been published.[4] This type of structure seems to eliminate all the objectionable features listed by the author, and, in addition, it has some advantages.

---

[4] *Engineering News-Record*, September 17, 1936, p. 400; also *Water Works Engineering*, August 19, 1936, p. 1068.

The Fort Smith reservoir is circular, with a 20-ft depth of water, and the wall consists of twelve arches of 25-ft span, 8 in. in thickness, springing from radial buttresses with counterfort extensions. Foundations are in hardpan, and the structure is earth covered. The arches preferably are poured in stages, the forms being raised as the concreting progresses. The entire wall, including the joints between arches and buttresses, is in compression. Experience with multiple-arch dams indicates that such joints are not likely to leak, and, since there is no tendency for the joint to open, simple and inexpensive stopwater elements may be used. The only joint requiring special treatment is that between arches and footings. Temperature effects are taken care of by the flexibility of the arches. The roof is of beam and slab construction, with exterior columns located at the heel of the counterforts to lend stability to the wall.

A design was also made for a pre-stressed tank with a domed roof. The quantities required for the two designs were found to compare as follows:

	Arch-wall design	Pre-stressed wall design
Reinforcement, lb............	94,000	112,000 (including turnbuckles)
Concrete, cu yd..............	672	650

For an aboveground reservoir of the arch-wall type the quantities required would be considerably less than the foregoing. The Fort Smith tank has been in service since 1936, and no leakage has been observed and no defects have developed.

TABLE 2.—ECONOMIC DEPTHS FOR SQUARE ARCH-WALL RESERVOIRS

Reservoir	Cost C_1 (dollars per sq ft)	Constant k	CAPACITY, IN MILLION GALLONS:							
			0.1		1.0		3.0		10.0	
			Depth h	Length[a] l	Depth h	Length[a] l	Depth h	Length[a] l	Depth h	Length[a] l
Volume V (cu ft)...	....	....	13,400		133,700		401,000		1,337,000	
Covered...........	1.50	1/10	9.9	37	15.7	93	19.5	144	24.8	232
Open..............	0.50	1/12	....	....	10.9	111	13.6	172	17.2	279

[a] Length of side; all dimensions are in feet.

This kind of reservoir is unlimited as to size and virtually unrestricted as to shape. The roof may be of any construction, or it may be omitted entirely. The greater the capacity, the less will be the cost per unit of storage. If a cover is provided, a square or rectangular shape is preferable.

The economic depth for square arch-wall reservoirs is given by the formula

$$h = \left(\frac{C_1}{k}\right)^{2/5} \frac{V^{1/5}}{2} \qquad (1)$$

in which h = depth of water in feet; C_1 = cost of floor, roof, and roof covering, in dollars per square foot of reservoir area; V = volume of storage in cubic feet; and k is a constant.

The cost of the wall varies approximately as the square of the height, and $k = \frac{C_2}{h^2}$ (assumed), C_2 being the cost of the arches, buttresses, and backfill (if any) per linear foot of wall. Table 2 shows typical variations in economic depth for covered and uncovered reservoirs for assumed values of C_1 and k. The depth may be varied either way from the most economic value without greatly increasing the cost.

R. E. KOON,[5] M. AM. SOC. C. E., AND J. C. GEARHART,[6] ASSOC. M. AM. SOC. C. E.—Having, presumably, investigated the merits and deficiencies of covered concrete reservoirs built according to "conventional design," the author concludes that such structures can be built, to advantage, with "vertical-beam" side-walls. This conclusion was reached as the result of theoretical analyses and actual construction experience. Only the vertical-beam type is detailed in the paper, but it may be inferred from the text that the "conventional" type had been tried with unfavorable results. The author classes as "conventional" cantilever walls or buttress-supported walls and also hoop-reinforced walls of round reservoirs except those wherein the steel is pre-stressed. This definition seems adequate and is accepted for purposes of reference herein.

This discussion deals with certain of the author's conclusions which the writers cannot accept as well founded in theory, or established as the result of their own practical experience.

General Comments.—The data presented in support of the author's conclusions are limited, in all practical aspects, to observations on one reservoir of moderate size (3 million gal) in which several defects appeared, and to two quite small reservoirs (0.12 million gal and 0.10 million gal, respectively) in which changes in details of design were made to correct for defects observed in the earlier structure.

In general, the writers have favored the author's so-called "conventional design." An exception has been made in the case of relatively small structures, mainly settling basins ahead of mechanical filters, where the lengths of tie beams have been rather short. As a consequence of these experiences, this paper favoring vertical-beam construction for reservoir walls has been read with much interest.

Adverse Conclusions.—The writers' conclusions are quite contrary to those reached by Mr. Stanley. The importance of his assumptions of deterioration of concrete-embedded steel also is questioned. Supporting and explanatory discussion will follow, but an outline of these adverse conclusions is summarized thus:

---

[5] Cons. Engr. and San. Engr. (Stevens & Koon), Portland, Ore.

[6] Ensign, CEC-V(S), U.S.N.R., Camp Peary, Williamsburg, Va.; formerly Asst. Engr., Stevens & Koon, Portland, Ore.

1. Except under certain unusual conditions, there will be no advantage in the "vertical-beam" walls either structurally or economically;
2. To assume that defects which appeared in a 3-million-gal reservoir have proved to be corrected fully by changes made in design of later structures, each of which is less than one twenty-fifth the size of the original structure, is not convincing;
3. Structures of "conventional" design are not more subject to leakage than are those of the vertical-beam type;
4. It is not necessary to apply surface waterproofing to walls of "conventional" type which have been built properly (this is also true of walls of the vertical-beam type);
5. No evidence has been produced by the author, nor have the writers found it in literature, to the effect that water entering the minute cracks (which theoretically appear in properly stressed concrete walls) will cause injury to reinforcing steel;
6. The use of vertical-beam walls in place of cantilever walls does not effect a simplification of design; nor does it afford appreciable advantage to the builder;
7. Steel tie rods required by vertical-beam construction, unless specially protected, are more vulnerable to deteriorating action than concrete-encased reinforcement as used in walls of "conventional" design;
8. The writers find that an analysis of relative costs favors "conventional" design in the ratio of about 5 : 4;
9. Application of the vertical-beam theory to round reservoirs would develop extraordinary problems in tie-rod arrangement which should preclude any attempt to use such a design; and
10. Although the design of a reservoir with vertical-beam walls may be theoretically sound, the possibilities of major structural failures due to defective workmanship, particularly with respect to tie-rod placing and adjustment, seem very great as compared with an equally well-designed structure with walls of the "conventional" type.

Basic Theory.—The author's theory, that the use of an adequately supported vertical beam to carry the load due to water pressure in a reservoir is feasible, cannot be disputed; but the application of this theory cannot be made properly to reservoirs of all ordinary sizes and shapes. In some instances, a reservoir must be built (*a*) of small capacity, (*b*) with necessarily high vertical walls, (*c*) square or rectangular in plan, and (*d*) where the steel tie members can be encased in good concrete. In these cases, the writers agree with the author as to the soundness of application of the theory but only if its use will show economy in construction cost as compared to a design of "conventional" type. In short, a good reservoir can be built using either type of wall design. Architectural requirements or foundation conditions may be determining factors in the selection of the type of wall to build. In another case the designer's preference, influenced by his personal appraisal of the relative merits of compression or tension in the wetted wall face, may determine the final choice.

Few reservoirs, however, must conform in all respects to the foregoing stipulations. Usually the designer has several options: The reservoir may be relatively shallow and consequently may cover a greater area; it may be made round in plan and relatively shallow or deep; and it may be built with a considerable part, or all, of the water capacity below or above natural ground level. All, or some, of these alternates may be, and usually are, available for the designer's choice, and his determination to give predominant weight to any special condition perhaps will be influenced by many factors. In some instances, certain unalterable facts—the elevation of the available site in relation to a fixed requirement for elevation of the water, the nature of the foundation or of the material to be excavated, architectural requirements, and perhaps others—may be conclusive in dictating the type of reservoir wall to be built; but such circumstances are rare. For purposes of this discussion, it is understood that certain basic requirements only remain unchanged: (*a*) The reservoir must be built of reinforced concrete; (*b*) it is to be provided with a roof—presumably of concrete, but not necessarily so; (*c*) it must have the required capacity; and (*d*) it must be a well-built structure, free from leakage.

Only rarely will there be such a combination of all circumstances as clearly will favor the designer's selection of the vertical-beam type of wall; and, consequently, the application of this "basic theory" of vertical-beam construction to all concrete-covered reservoirs or, indeed, to more than a negligible few, is not sound.

Effect of Cracks in Concrete.—It is regrettable that so little information has been published on the appearance of cracks, either visible or microscopic, in reinforced concrete members under normal loads. The stresses within a vertical wall subjected to hydraulic loading can be calculated very precisely, and, consequently, the deformation of such a wall can be determined with reasonable accuracy. Since the tensile strength of concrete alone is assumed as zero in most reinforced concrete design, it follows that minute cracks must occur in any member that suffers deformation; and deformation will occur, of course, as soon as any load is applied.

The author's principal argument in favor of the vertical-beam type of wall for reservoirs seems to be that the entire concrete surface in contact with the water is thus in compression and therefore free of cracks which might permit moisture to attack the reinforcing steel. Although the writers agree that minute cracks will develop in the inside face of a cantilever wall, they are not convinced that these cracks are of sufficient magnitude to permit water to enter and thus, perhaps, to be detrimental to the steel, unless such a wall is loaded appreciably above that allowable in normal design.

If it is assumed, because there is no evidence to the contrary, that the cracks in a cantilever wall will be large enough to permit water to contact the steel, it is evident that similar cracks of equivalent magnitude must appear on the outer or tension side of the wall designed as a vertical beam. Such cracks, although not exposed to water from within the reservoir, would be vulnerable to the attacks of rain, air, and frost if the wall is exposed to the atmosphere and perhaps even to worse conditions if the wall supports a backfill. These agents may be much more severe in their effect on the reinforcement and concrete than the reservoir water alone.

By the use of sufficient reinforcing steel, all cracks can be eliminated or so reduced in size as to remove any chance of their being deep enough to reach the steel. There is no evidence, however, to indicate that such extravagance in steel could be justified, in any way, whether the wall be of the "conventional" or the vertical-beam type.

Choice of Design.—The use of walls of the vertical-beam type in large reservoir construction may be new, but this does not signify that they have not been considered for such service. The experienced reservoir designer customarily has in mind several alternate types of structures or methods of construction for each such project; and, since there are frequent occasions to use vertical beams to good advantage in ordinary structures, it is not reasonable to assume that their possibilities for reservoirs would be overlooked. It appears, therefore, that the chief reason why this type of design is not usual, and consequently "conventional," is because safer or more economical types have been chosen.

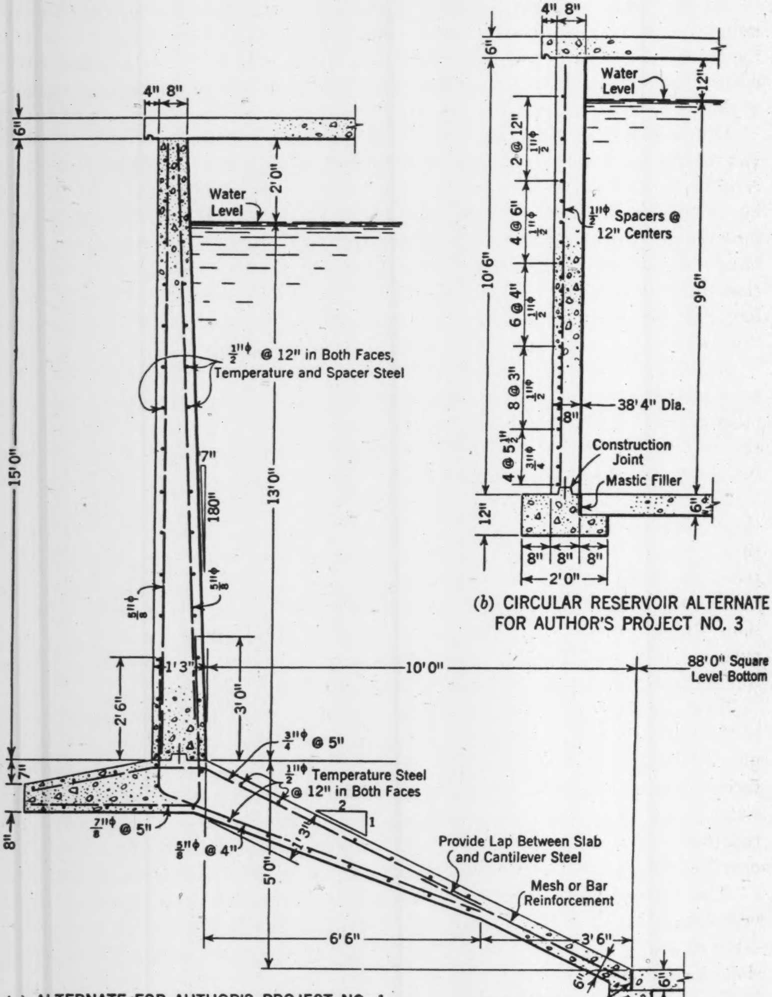
Relative Cost of Construction.—It is stated in the paper that reservoirs with vertical-beam type of walls have been constructed without increase in cost over other types. A few calculations made by the writers do not appear to bear out this statement. Alternate designs for the author's project No. 1 and project No. 3 are submitted herewith in Fig. 6.

Since the author does not indicate the roof or floor slab reinforcement, nor the column supports for his roof in project No. 1, it is assumed that the quantities of steel would be substantially the same with either type of wall and, therefore, these quantities have been neglected in the comparisons made. In making the comparative estimates, it also has been necessary to assume dimensions which appear to be reasonable for structural details not given by the author. Likewise, it will be necessary to make similar assumptions for details which the writers could not show in their sketches.

The roof slab in Fig. 6 is shown, with a thickness of 6 in., to be comparable to the author's design for estimating purposes. This type of wall design requires slightly greater roof area. The vertical steel bars in the outside face, carried to varying elevations, are adequate for earth backfill as high as the water level, and should be omitted if there is no backfill or if they are not required as spacer bars to support temperature steel. The principal reinforcement bars in the wall and in the footing vary in lengths, as the stresses require.

The writers' alternate for project No. 1 (Fig. 6(*a*)) has a capacity of 1.5 million gal in each of two square sections, with a common wall between. This division wall is similar to the side-wall detail except that the floor has a 1-on-2 slope for a distance of 10 ft on each side.

The concrete quantity in project No. 1 (Fig. 5(*a*)), including walls, footings, and floor and roof slabs, approximates 1,895 cu yd. The reinforcing steel, neglecting that in the roof and floor slabs and assuming that no extra steel is added in the roof slab to restrain the top of the wall, is calculated to be 183,000 lb. If the concrete is estimated to cost \$40 per cubic yard and the steel \$0.06 per pound, then the total cost of these parts of the reservoir is approximately \$87,000.


4" 8"
6"
2'0"
Water Level
½"φ @ 12" in Both Faces, Temperature and Spacer Steel
15'0"
13'0"
7"
180"
⅝"φ
⅝"φ
1'3"
10'0"
88'0" Square Level Bottom
2'6"
3'0"
¾"φ @ 5"
½"φ Temperature Steel @ 12" in Both Faces
2
1
7"
8"
⅞"φ @ 5"
⅝"φ @ 4"
1'3"
5'0"
Provide Lap Between Slab and Cantilever Steel
Mesh or Bar Reinforcement
6'6"
3'6"
6"
6"
Expansion Joint
4" 8"
6"
Water Level
12"
2 @ 12"
½"φ
4 @ 6"
½"φ
½"φ Spacers @ 12" Centers
10'6"
9'6"
6 @ 4"
½"φ
8 @ 3"
½"φ
38'4" Dia.
8"
4 @ 5½"
¾"φ
Construction Joint
Mastic Filler
12"
6"
8" 8" 8"
2'0"

(*b*) CIRCULAR RESERVOIR ALTERNATE FOR AUTHOR'S PROJECT NO. 3

(*a*) ALTERNATE FOR AUTHOR'S PROJECT NO. 1

FIG. 6

For the alternate design indicated in Fig. 6(*a*), a reservoir of equivalent capacity, with the same items of steel neglected, the concrete is estimated at 1,480 cu yd and the reinforcing steel at 131,000 lb. Using the same unit prices, the cost of these similar parts of the alternate reservoir is approximately $67,000, thus showing a saving of $20,000 in these items.

This comparison does not present quite a true picture, however. If the two reservoirs were located on identical sites, the alternate reservoir must be 5 ft lower than the all-vertical-wall type to obtain the same foundation medium. Approximately 4,000 cu yd of additional excavation would be required for the proposed alternate, but it still would show a saving of probably $16,000, or more than 18% in construction cost. Since local conditions affecting the available site elevation are as likely to favor one design as the other, this feature as related to probable cost cannot be considered here without undue extension of this discussion. However, by placing a series of foundation walls under the slope slab and wall footing and by increasing the thickness of the sloping slab to carry the load between these walls, the alternate reservoir could occupy any site suitable for the vertical-beam type; but there then would be little difference in cost of the two types. It should be noted here that the 131,000 lb of steel used in the alternate design include about 35,000 lb in the outer face of the cantilever wall to permit backfill to the full wall height. If such backfilling is not required, then an additional saving of $2,100 would be effected in favor of the "conventional" design.

In project No. 3 (Fig. 5(*c*)), the author again gives adequate details only of the wall, footing, and wall cap. Therefore, the quantities involved in these units are the only ones included in the following comparison. From the details given, the capacity of this reservoir is only a little more than 82,000 gal. There are 51.7 cu yd of concrete in the walls, footings, and cap, and the weight of the reinforcing steel (neglecting tie rods for which no data are given) is 9,200 lb. Using the same unit prices as before for comparative estimates, the cost of these parts of the reservoir would be $2,620.

For a circular reservoir of the same depth and capacity as shown for project No. 3 (Fig. 6(*b*)), the concrete, excluding roof and floor slabs, is 39.6 cu yd and the total weight of the reinforcement, with $f = 12{,}000$ lb, is 3,250 lb. The cost of wall and footing alone then would be $1,779 or 32% less than for these same features of project No. 3. The unit cost of concrete might be higher for the circular wall, but, even if this proved to be true, the round type has such distinct advantages for so small a structure that the writers certainly would select this type unless some very unusual local conditions should dictate another choice. In addition to the quantities compared, there is also the problem of tie rods and expansion joints in the vertical-beam-wall type, and their cost undoubtedly would be such as to permit the designer to double the quantity of reinforcement in the circular tank and stress the steel as low as 6,000 lb without showing an unfavorable cost comparison for the round type.

Vertical Construction Joints.—The author states that some difficulty with leaks through vertical shrinkage or contraction cracks between the construction joints occurred on his projects Nos. 1 and 2, and that, as a result, the subsequent design satisfactorily employed construction joints at 20-ft centers or less. However, since the final project listed had walls only 34 ft long, there is no practical reason to conclude that construction joints at 20-ft intervals are necessary or desirable. The writers are certain, as a result of their own experience, that such short wall sections would accomplish nothing in guarding against reservoir leakage, would increase costs, and perhaps actually would result in inferior work.

Many structural design authorities use round figures of 0.20% or 0.25% for temperature reinforcing in ordinary slab design. However, extensive tests and experiments by C. P. Vetter[7] and Walter Dreyer,[8] Members, Am. Soc. C. E., indicate that 0.60% to 0.75% of temperature steel is necessary to eliminate all possibility of cracks large enough to permit leakage from a concrete reservoir. Under conditions where maximum air temperature ranges are as much as 100° F, the writers have used materially smaller percentages of steel than here recommended, with entire success.

In project No. 1, the author used longitudinal temperature steel to the extent of only 0.17% of the cross-sectional area of the wall and, in project No. 2, it was approximately 0.14%. These low percentages of temperature reinforcing steel point quite definitely to a reason for the troubles experienced.

In the design of a 15-million-gal reservoir (built in 1939) with a 20-ft water depth and wall section somewhat similar to that shown in Fig. 6, the writers used approximately 0.35% temperature steel in main wall sections, 58 ft 8 in. long between construction joints. After this reservoir had been in use for approximately three years, there were neither visible cracks nor leakage in any of these relatively long wall sections, and a face, 475 ft long, was exposed for its entire height. Experience on this and other reservoirs of somewhat similar design has led the writers to believe that the distance between construction joints can be almost any convenient length, within reason, as long as the amount of temperature steel is adjusted accordingly. A design feature of this reservoir, which may have contributed materially to the elimination of shrinkage cracks, was the separation of all major wall sections by short closing sections which were poured not less than 20 days after the adjacent wall sections were built.

The troubles experienced by the author with shrinkage or contraction cracks in his project No. 1 are definite evidence of the undesirability of making a rigid connection between wall and roof in a structure so large. From the incomplete details given, the reader must assume that the roof slab is one continuous piece, approximately 125 ft by 185 ft on its sides. A 100° range in temperature will produce material changes in slab dimensions and, consequently, transmit very severe stresses to the wall sections. In all cases the walls will be affected much less by changes in air temperatures than the roof slab and thus will not expand or contract in harmony with it. The best solution is to support the roof slab entirely free of the walls so that movements in one will not transmit stress into the other; at least this is true in structures of such size that vertical expansion joints are required in the walls.

Tie-Rod Use Questionable.—The use of unprotected tie rods seems ill-advised when they perform so important a function in the stability of the structure. Corrosion surely will be severe when bars are placed in earth or gravel under a reservoir floor or when they are suspended below the roof slab in what is always an extremely humid atmosphere. Such steel cannot be expected to approach the life of concrete-encased reinforcement. If so used, the safety factor should be increased materially, and this, in turn, increases the steel quantity. In some way, these tie rods may be encased or protected, but construction costs will increase with any such design provisions. The difficulties of properly placing and adjusting these tie rods, together with the uncertainties and hazards attendant on their use, seems so great as to preclude use of a design which requires them even if a substantial construction cost saving could be realized.

---

[7] "Stresses in Reinforced Concrete Due to Volume Changes," by C. P. Vetter, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1039.

[8] *Ibid.*, p. 1058.

Surface Waterproofing.—The author states that the walls require a waterproofing treatment to fill the pores in the concrete, thus making it impervious to water. Test results have convinced the writers that this is not necessary and adds considerably to the over-all cost of the reservoir.

The 15-million-gal reservoir previously mentioned is approximately 235 ft by 475 ft in plan, with a dividing wall to provide two square units. Although there is a 20-ft water depth when full, the floor slab is only 6 in. thick and the slope and wall sections are quite similar to those shown in Fig. 6(*a*). All joints between adjacent floor-slab panels, as well as wall sections, have flexible metal water stops embedded in the concrete. Definite artificial cracks, about ½ in. wide, were provided at the water stops in the floor, and these were filled with hot mastic after completion of the roof. (It is important that the roof be completed before the mastic joint-filler is poured as, otherwise, during and for some time after the joints are completed, the floor slabs are subject to expansion and contraction greatly out of proportion to that which occurs after the roof is in place. Unless this procedure is followed an inferior job of joint construction is quite certain to occur.) No waterproofing of any kind was applied to the concrete mix or its finished surfaces. The reservoir is underdrained completely to an open pit. At no time since the reservoir has been in service has there been any flow whatever from these drains. The soil under the reservoir is practically impervious, and, consequently, any leakage would be evidenced by flow in the drains.

Upon its completion, the reservoir was filled to capacity and subjected to a prescribed leakage test. As measured by hook gage, the water level dropped ⅛ in. in 48 hr, thus indicating a loss of approximately 4,250 gal per day, a truly negligible part of the capacity. This test was repeated after the reservoir had been in service three years, and there was no measurable change in the rate of loss. Since there have been three cycles of seasonal changes in temperature, it seems reasonable to assume that this minute loss is established permanently and is certainly undetectable through visual observation of exposed walls or open underdrains. This case is cited thus because it proves on a rather large scale that even relatively great areas of thin concrete slabs are watertight if suitable concrete is used and, particularly, if adequate care is taken to insure its proper deposition and cure.

GUSTAVO PÉREZ GUERRA,[9] JUN. AM. SOC. C. E.—The information and ideas contained in this paper should prove of great value to all members of the profession engaged in the design and construction of concrete reservoirs. In addition to considerations of watertightness (which are vital in such an undertaking), the economy in the use of concrete and the more logical design which takes full advantage of all parts of the structure point to a general adoption of this type of reservoir in the water works field.

---

[9] Asst. Engr., Parsons, Klapp, Brinckerhoff & Douglas (formerly Asst. Designer in Water Works Div. of the Ministry of Public Works), Caracas, Venezuela.

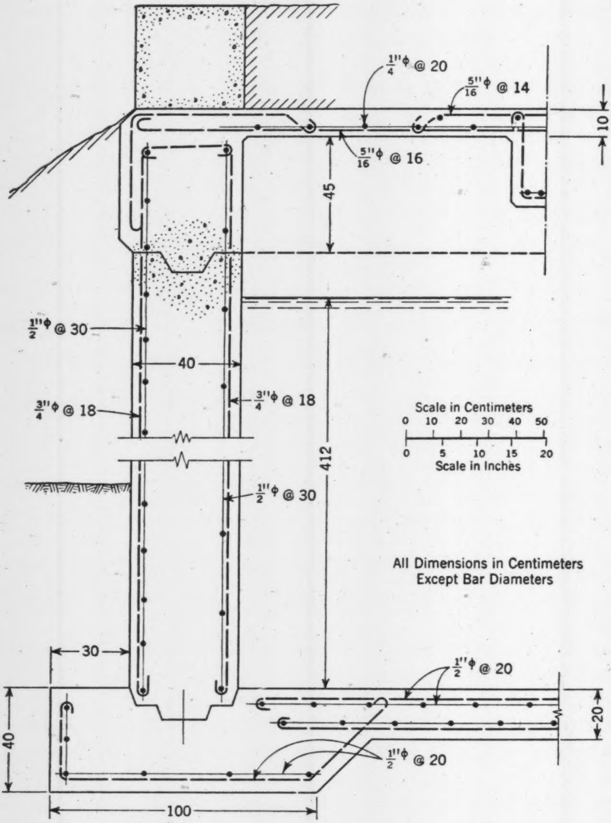

$\frac{1}{4}''\phi$ @ 20
$\frac{5}{16}''\phi$ @ 14
10
$\frac{5}{16}''\phi$ @ 16
45
$\frac{1}{2}''\phi$ @ 30
40
$\frac{3}{4}''\phi$ @ 18
$\frac{3}{4}''\phi$ @ 18
412
Scale in Centimeters
0 10 20 30 40 50
0 5 10 15 20
Scale in Inches
$\frac{1}{2}''\phi$ @ 30
All Dimensions in Centimeters
Except Bar Diameters
30
$\frac{1}{2}''\phi$ @ 20
20
40
$\frac{1}{2}''\phi$ @ 20
100

FIG. 7.—TYPICAL DESIGN

As far as the writer knows, the first tank of the type described in this paper built in Venezuela was designed by the Water Works Division of the Ministry of Public Works (M.O.P.), early in 1939. Since then the original design has been modified only slightly. About ten such reservoirs have been built by the M.O.P., in sizes ranging from 56 to 1,200 cu m (14,800 to 317,500 gal) and with water heads varying from 2 to 4.12 m (6.56 to 13.52 ft). A typical design for a head of 4.12 m is shown in Fig. 7, in which both the floor and the roof slabs are tension members supporting the bottom and the top of the wall. The wall acts as a simply supported vertical beam for both the water and the earth loads. The bottom slab, designed as a tension member, avoids the use of tie rods, which are not desirable in Venezuela because of difficulties in inspection and maintenance.

On the other hand, it is true that no expansion joints are provided; but, since the reservoirs are intended primarily for distribution purposes and in most cases have a relatively large fire reserve, the temperature change seems to be of minor importance. Furthermore, the use of the wall as a simple beam to resist the thrust of the earth cover creates tension in the inner face of the tank walls thus discounting one of the advantages of Mr. Stanley's design. This is possible because there are no winter problems, and the cracks formed do not jeopardize the efficiency of the structure. Because of the absence of snow, the roofs of tanks in Venezuela are almost flat (slope, 1%) and are built with a parapet for the purpose of holding an earth backfill 35 cm (1 ft 2 in.) thick. Most of the tanks are partly underground and are covered by backfill, later planted with grass to improve the general appearance of the structure. However, some have been built aboveground, with no backfill. Table 3 gives data on several of the structures built in Venezuela.

Horizontal construction joints are eliminated and the walls poured in lengths of 4 to 6 m (13.1 to 19.7 ft), and in all joints 8-in. copper strips are used as water stops. The construction industry has not resorted to the elaborate and resourceful treatment, described by the author, for watertightness and expansion because local experience with this type of design has shown none of the difficulties reported in the paper. Fortunately, the absence of low temperatures reduces the importance of minute cracks in the inner face of walls. In addition, the narrow range of temperature variation during the year reduces the danger of cracking, as contraction is far more troublesome than expansion. Some temperature reinforcement is provided according to general design practice.

TABLE 3.—DATA ON VERTICAL-BEAM RESERVOIRS IN VENEZUELA

No.	Year built	CAPACITY		WATER DEPTH		AREA	
		Cubic meters	Gallons	Meters	Feet	Square meters	Square feet
1	1940	1,254	3,320,000	4.12	13.52	304.5	3,278
2	1939	553	1,460,000	4.12	13.52	134.2	1,442
3	1939	743	1,960,000	4.12	13.52	180.2	1,940
4	1941	1,239	3,275,000	4.12	13.52	300.0	3,229
5	1939	56	148,000	2.00	6.56	28.0	301

Summarizing, customary design of this type of structure in Venezuela does not comply with all the ideal wall criteria listed by the author. However, its efficiency is proved by the several units constructed, and it gives satisfactory service with only routine maintenance. The design of such tanks, irrespective of geographic or climatic conditions, will be improved greatly as a direct result of this paper which contains such valuable suggestions for designers. The favorable local experience with the original design, quite different from that described by Mr. Stanley, corroborates the known fact that tropical conditions, although producers of hardships in practically every other aspect of life, really simplify the task of the hydraulic engineer.

C. MAXWELL STANLEY, JR.,[10] M. AM. SOC. C. E.—Some of the discussers favor the vertical-beam type of construction for concrete reservoirs and some question the value of this design. Both comments are needed for a proper evaluation of any new method of design.

The examples of vertical-beam type construction given by Messrs. Cochrane and Pérez Guerra apply to conditions somewhat different from those encountered in the reservoirs described by the writer. The reservoir designed by Mr. Cochrane is of particular interest: It illustrates the application of the method to a circular reservoir of large diameter as suggested in the paper; and it extends the method to a greater degree of backfilling than the writer has considered desirable. As Mr. Cochrane states, the internal pressure normally will exceed the external pressure and cracks that may occur in the inner face of the wall when the tank is empty will tend to close when the reservoir is filled above the normal drawdown level. The arch-wall design described by Mr. Cochrane is a proper addition to the retaining wall type listed under the heading of "Conventional Design: Retaining Wall Types."

Messrs. Schmidt and Koon and Gearhart comment on the desirable percentage of horizontal reinforcement. The writer agrees that the amount of horizontal reinforcement in projects No. 1 and No. 2 is less than that desirable. Two reservoirs designed and constructed since the preparation of the paper (December, 1942) have had approximately 0.4% horizontal reinforcement which is comparable to that of project No. 3. No indication of cracking appeared on either of these reservoirs as late as fourteen months after construction.

The discussion of Messrs. Koon and Gearhart is an able presentation of the objections that may be raised against the vertical-beam design. Comments on these objections are in order. There are many factors that affect the selection of the type of reservoir, and, where those factors are such that the vertical-beam type of wall cannot be applied, obviously some other design must be used. Although the vertical-beam design is not universally applicable, it has certain advantageous features that may be utilized where it is applicable. The vertical-beam design need not be eliminated from consideration because a part of the water capacity is below natural grade, for in such cases a design similar to that shown in Fig. 8 may be used with the bottom of the wall supported by the frictional resistance resulting from the weight of the water on the base. This construction is only a modification of the method of bottom wall support shown in the lower example of Fig. 3(*b*). The vertical-beam design is not applicable to circular reservoirs, except those of relatively large diameter, but seldom is a circular reservoir required by local conditions. Rather the circular reservoir is more often the result of the type of design selected. There are few instances in which a square or rectangular shape cannot be used if desired.

---

[10] Cons. Engr. (Stanley Engineering Company), Muscatine, Iowa.

In general, the vertical-beam design may be applied where any of the "conventional designs" are considered, provided the height of wall is not too great for the wall to be poured in one lift. Moreover, local conditions as well as the ratio of depth and area will vary cost relationships between the various designs.

The hypothesis, presented by the writer, that a better design is obtained when the surfaces in contact with the water are in compression can probably be proved or disproved only by research which has not yet been undertaken. The writer believes thoroughly that having compression concrete in contact with the water is advantageous. This belief is based on his observation of numerous concrete structures and is confirmed by observed experiences with the Hewitt type of pre-stressed circular reservoir.

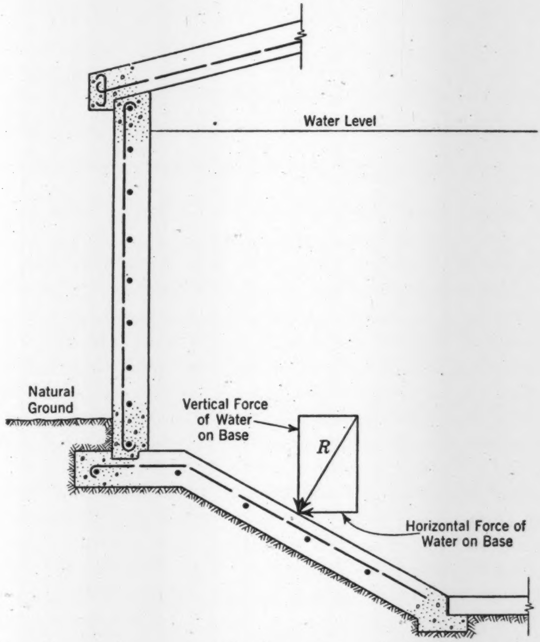

Water Level
Natural Ground
Vertical Force of Water on Base
R
Horizontal Force of Water on Base

FIG. 8.—APPLICATION OF VERTICAL-BEAM DESIGN TO RESERVOIR WITH PART OF STORAGE BELOW NATURAL GRADE

Cost comparisons are extremely difficult. Those made by Messrs. Koon and Gearhart assume the same unit price for concrete for the two types of construction—namely, project No. 1, Fig. 5(*a*) and the alternate Fig. 6(*a*); and, likewise, for project No. 3, Fig. 5(*c*) and the alternate Fig. 6(*b*). The unit prices for the two types of construction should not be the same. Moreover, the comparison made by Messrs. Koon and Gearhart in Fig. 6(*a*) assumes a design with only a 15-ft wall height for the cantilever type. If such an arrangement is acceptable for the cantilever type, it may also be used for the vertical-beam type, employing a construction similar to that shown in Fig. 8.

For these reasons the writer questions the cost comparison made by Messrs. Koon and Gearhart. It has been his experience, as indicated in the paper, that the cost of the vertical-beam reservoir compares favorably with conventional types. Table 4 shows the construction cost of several projects adjusted to January, 1942, prices using the *Engineering News-Record* (ENR) cost index. These costs seem self-explanatory.

TABLE 4.—COST DATA[a] ON VERTICAL-BEAM RESERVOIRS

Project	Contract cost[b]	Date awarded	ENR cost index	Contract cost[c]	Capacity (gallons)	Adjusted cost[d]
1	$39,750	August, 1935	93.82	$54,800	3,000,000	$18,300
2	4,375	November, 1938	112.69	5,040	120,000	42,000
4	2,710	March, 1942	129.65	2,710	60,000	45,200

[a] Costs do not include land, engineering, or inspection. [b] Cost of project No. 1 includes all piping for inlet, outlet, overflow, etc.; the cost of projects No. 2 and No. 4 does not include piping. [c] Adjusted to March, 1942. [d] Dollars per million gallons.

If construction joint spacing of greater than 20 ft is adequate, then this should be used, and may be used with the same advantage for the vertical-beam type of construction as for any of the conventional types.

The writer agrees that the use of unprotected tie rods is ill-advised, and, on the projects described, adequate means have been taken to protect them. The methods have varied, but for tie rods buried under the floor the following procedure is suggested: (1) Design the bars over size to allow for some corrosion; (2) give the bars two or more coats of good asphaltic waterproofing material and wrap them in burlap, sisal-craft, or similar material to protect the waterproofing; and (3) place and backfill the bars carefully with sand (for protection and drainage) so as to shield them by the slab placed above. These means of protection are not dissimilar to those used on steel pipe and if carefully applied and inspected will give adequate protection. However, if one is skeptical of the use of tie rods, they may be eliminated and the vertical-beam design may still be used by placing reinforcement in the top slab (middle illustration in Fig. 3(*a*)) to support the top of the wall and by using the weight of the water to resist the horizontal thrust at the bottom of the wall (bottom illustration in Fig. 3(*b*)).

Objection to the use of surface waterproofing on the inner walls of reservoirs is largely a matter of opinion; it applies equally to all types of design and need not be argued.

In conclusion the writer wishes to reaffirm his statement that the vertical-beam method of reservoir construction has been developed and tested so that its advantages and disadvantages can be established. In those instances where conditions permit its use, it has certain advantages and affords a simple and economical type of construction.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2217

FLOW AROUND BENDS IN STABLE CHANNELS

By C. A. Mockmore,[1] M. Am. Soc. C. E.

With Discussion by Messrs. J. M. Robertson, Joe W. Johnson and M. A. Selim, I. Nelidov, S. M. Woodward and J. W. Howe and C. J. Posey, and C. A. Mockmore.

Synopsis

The purpose of the project discussed herein was to make careful flow measurements under laboratory conditions, to record, and to study the phenomenon of flow around a stable curved channel.

An open channel, 18 in. wide and 10 in. deep, was built of transparent pyralin sections so they could be fitted together in combinations of tangents and curves. Motion pictures were taken of the flow phenomena as indicated by objects floated on the water surface, dragged or rolled on the stream bed, or held in suspension. The latter was accomplished by use of drops of a mixture of n. Butyl Plithalate and technical Xylol, colored with a deep red oil dye and having the same specific gravity as that of water. The water velocities were measured at various cross sections from which velocity contours were prepared.

Two different sets of assumptions were made as to the nature of the filamental velocities of flow around a bend, and a mathematical analysis was made of the streamlines, accelerations, and velocity components by use of the laws of hydrodynamics.

The Laboratory Experiments

Complications are inherent with the flow of water in any channel where the particles of water are forced to undergo a general change in direction. If there were streamline flow of equal filament velocities at the entrance to the bend and if friction could be eliminated so the water would act as a perfect fluid, no spiral motion could develop. The two principal causes of spiral flow in either closed or open channels are:

(*a*) The existence of friction on the channel walls, resulting in higher filamental velocities near the center of the channel than near the walls; and

---

Note.—Published in March, 1943, *Proceedings*. Positions and titles are those in effect when the paper or discussion was received for publication. The author of this paper is a cooperating member of the Committee on Hydraulic Research. He accepted this subject as a research project at the request of the Committee and submits the results of his studies in this paper.

[1] Prof. and Head, Dept. of Civ. Eng., Oregon State College, Corvallis, Ore.

(*b*) The centrifugal force required to deflect the particles of water from straight-line motion.

The existence of a condition of spiral flow around bends in open channels was first enunciated by Prof. James Thomson[2] of England in 1876.

A study of the flow conditions in the bend of a natural stream would involve many complexities, not only because of the difficulty in obtaining accurate measurements of the filamental velocities at specified points in a large stream, but also because of the many irregularities in, and the instability of, the stream bed. The laboratory is the most logical place, therefore, to study such fundamentals.

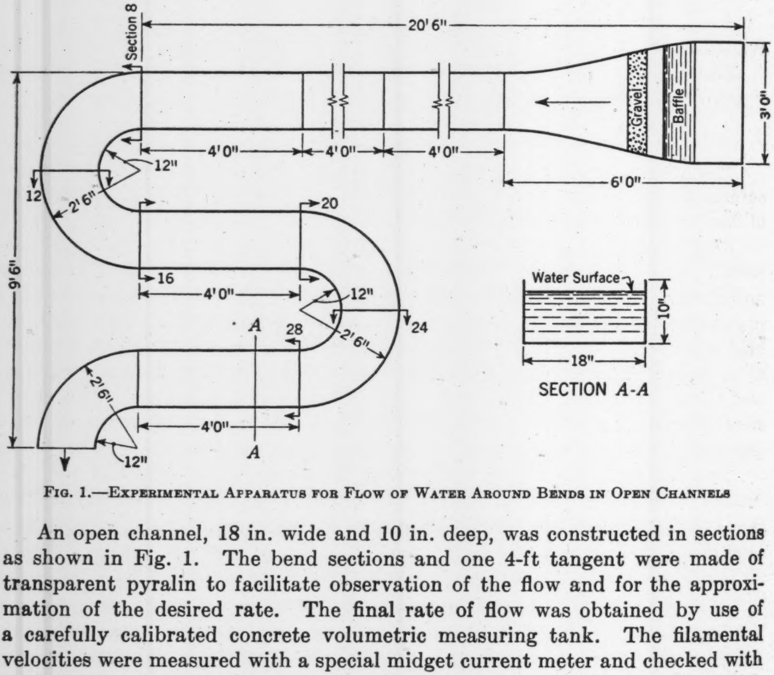


FIG. 1.—EXPERIMENTAL APPARATUS FOR FLOW OF WATER AROUND BENDS IN OPEN CHANNELS

An open channel, 18 in. wide and 10 in. deep, was constructed in sections as shown in Fig. 1. The bend sections and one 4-ft tangent were made of transparent pyralin to facilitate observation of the flow and for the approximation of the desired rate. The final rate of flow was obtained by use of a carefully calibrated concrete volumetric measuring tank. The filamental velocities were measured with a special midget current meter and checked with a pitot tube, the latter being one used in numerous other experiments, which had been carefully calibrated.[3,4,5]

The channel was set on a slope of 0.005% and the depth of the water was controlled by a tail gate at the outlet of the channel. In some experiments the gate caused a backwater curve which gave a different depth of water at each section of the channel, but this was not of enough consequence to require its elimination.

---

[2] "On the Origin and Winding of Rivers in Alluvial Plains, with Remarks on the Flow Around Bends in Pipes," by James Thomson, *Proceedings*, Royal Soc. of London, March, 1876, pp. 5–8.

[3] "Flow Characteristics in Elbow Draft Tubes," by C. A. Mockmore, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 412.

[4] "Hydraulic Laboratory Practice," J. R. Freeman, Editor, A.S.M.E., 1929, p. 712.

[5] "Flow of Water Around Bends in Pipes," by D. L. Yarnell and F. A. Nagler, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1020.

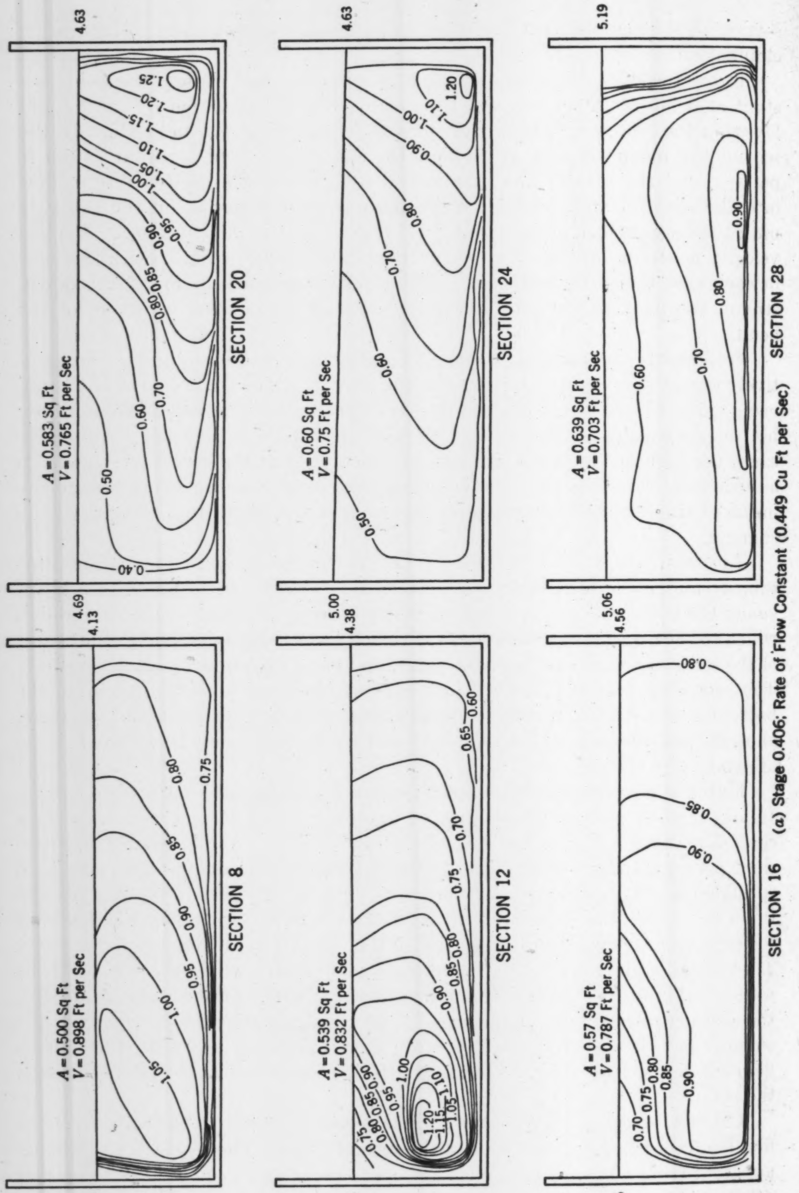
Two series of tests were conducted. In one the rate of flow was held constant at 0.449 cu ft per sec, and the mean velocity was varied by raising and lowering the water surface in the channel by means of the tail gate. In the second the mean velocity at Section 16, Fig. 1, was held constant at 0.5 ft per sec, and the rate of flow was varied by admitting a desired rate of flow into the channel and then adjusting the micrometer screws on the tail gate until the mean velocity as computed for Section 16 was 0.5 ft per sec. Typical velocity contours for these two experiments are shown in Fig. 2 with the view in each case facing downstream. All velocities recorded in Fig. 2 are downstream components, tangent to circles concentric with the curvature of the bend.

Preliminary to each test considerable effort was made to obtain a comparatively even velocity distribution over the section at the beginning of the bend, using various kinds of screens and floats at the channel inlet below the weir box; but in every stage of the flow the effect of the bend was reflected upstream to cause the highest filamental velocities to occur, not at the center of the stream as ordinarily would be the case in a straight channel, but somewhat toward the inside of the bend, depending upon the ratio of the width to the depth of the channel.

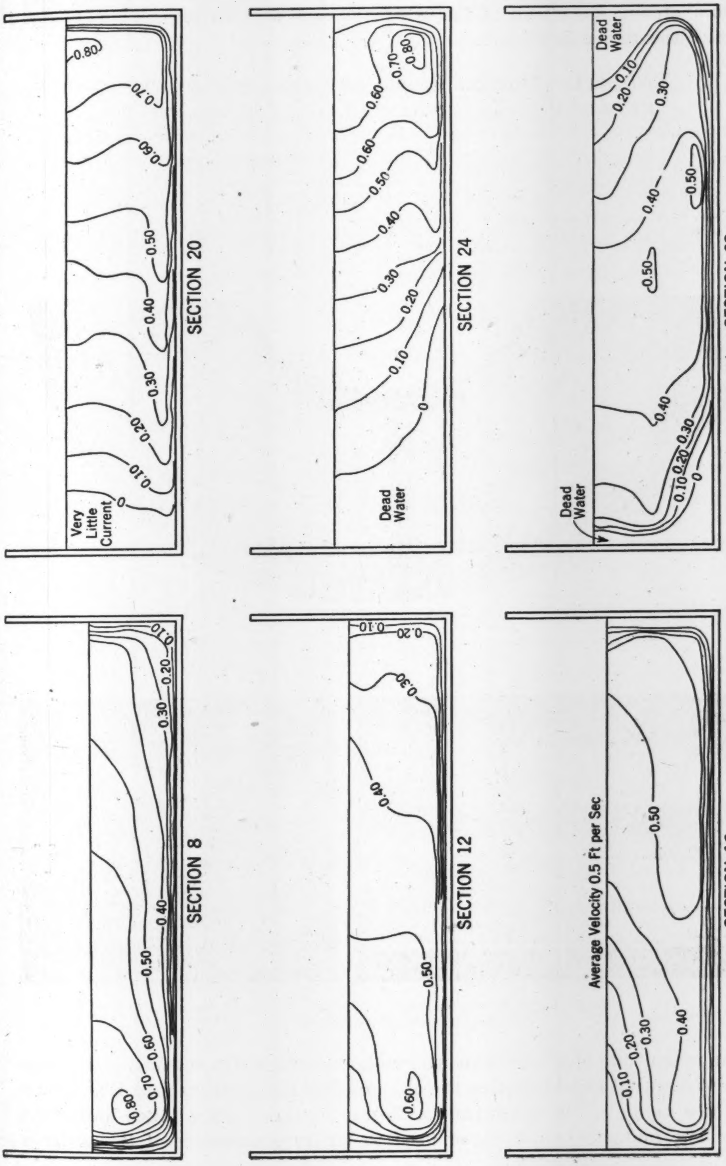
To obtain the data for velocity contours such as those in Fig. 2, the pitot tube or midget current meter was placed in a clamp and fastened to a beam across the top of the channel. Five verticals were selected as suitable through which to raise and lower the meter for velocity measurements, one at the center of the stream, one at each quarter point, and two at 0.5 in. from each side-wall. The procedure was as follows: The meter was lowered to within 0.5 in. of the bottom of the stream, a velocity measurement taken, then raised 0.5 in. more, another measurement taken, etc. The stage heights were all referred to an arbitrary zero datum.

Fig. 3 is a composite photograph made by taking several frames from a 16-mm motion picture of the first or upstream bend when the average flow conditions were as shown for Stage 0.600, in Table 1. Some crystals of potassium permanganate, which formed dark patches in the photograph, had been sprinkled on the bottom of the channel. The broken lines follow the path of the water along the extreme bottom of the channel. It will be evident that the streamlines crossed the center line of the channel at an angle of 45° or more. The unbroken lines are the paths taken by confetti strewn on the surface of the water, and indicate that the streamlines on the surface of the water edged across the channel toward the outside wall. The average downstream velocity in the channel in this case was slightly more than 0.5 ft per sec, with the highest filamental velocities near the inside wall and about one fourth the way around the bend.

Confetti strewn on the water surface seemed to edge to the outside of the first bend, but hugged the inside of the second bend. The reason for this is not apparent from a study of the velocity contours (Fig. 2), but the cause may be attributed to the short tangent between the two bends. If the tangent had



(a) Stage 0.406; Rate of Flow Constant (0.449 Cu Ft per Sec)



(*b*) Stage 0.030; Mean Velocity Constant (0.5 Ft per Sec)

FIG. 2.—TYPICAL VELOCITY CONTOURS

been longer, perhaps the surface currents of the second bend would have had time to overcome the effects of the upstream bend and perform as did the surface velocities in the first bend.

TABLE 1.—AVERAGE FLOW CONDITIONS, STAGE 0.600; RATE OF FLOW IS CONSTANT (0.449 CU FT PER SEC)

Description	SECTION NUMBER:					
	8	12	16	20	24	28
Area of section (sq ft)	0.811	0.843	0.88	0.90	0.91	0.963
Average velocity (ft per sec)	0.61	0.532	0.51	0.494	0.49	0.466

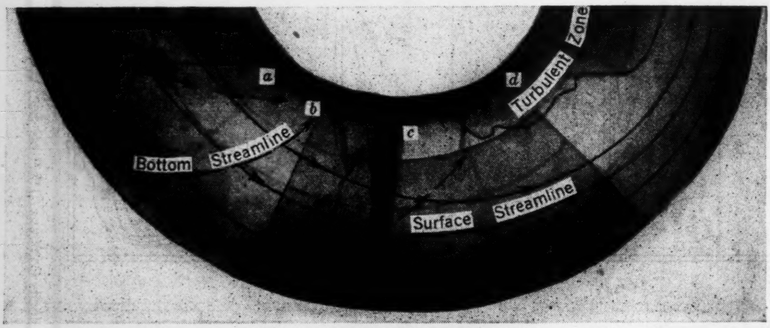


FIG. 3.—COMPOSITE PICTURE MADE FROM SEVERAL FRAMES OF A 16-MM MOTION PICTURE FLOW AROUND BEND

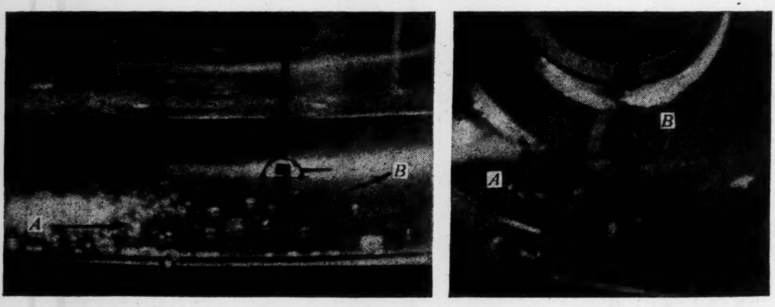
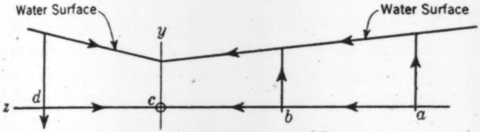


FIG. 4.—RICE GRAINS MOVED RAPIDLY ACROSS THE CHANNEL FROM A TO B, WHERE A BAR WAS FORMED

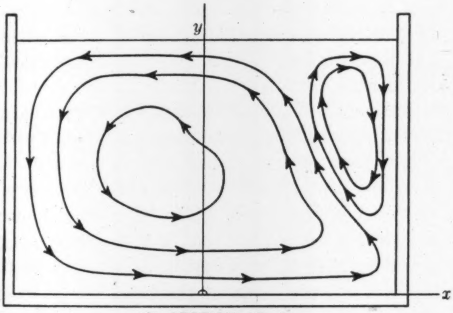
The magnitude of the bottom streamline velocities was considerable. Rice grains were swept quickly across the channel (see Fig. 4) from A to B to form a bar. The views in Fig. 4 are from a 16-mm motion picture taken during the tests. The midget current meter used to measure velocities of the water is shown.

On the convex bank in the last half of both bends there existed (see Figs. 2, 3, and 4) regions of unsteady flow in which, at times, the water actually moved upstream. At slightly more than halfway around the bend and above mid-depth the streamlines appeared to leave the inside wall of the channel. This is indicated by the darkened region of turbulent water in Fig. 3, caused by the accumulation of dissolved potassium permanganate being swept along by the streamlines nearest the bottom of the channel.

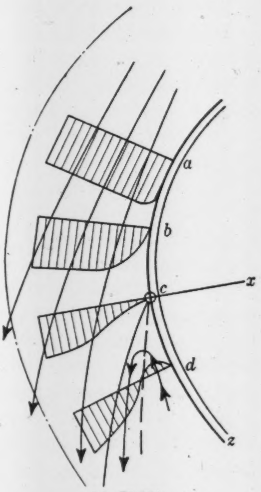
If, for the moment, the velocity component of a thin layer of fluid along the water surface in the vicinity of a, Fig. 5, is considered, it will be apparent that particles not only are moving downstream but also are edging toward the outside of the bend. Then as the particles pass on to c they experience a kind of diverging channel effect, and at d the streamlines diverge from the inside wall causing a backflow somewhat as shown. This effect was admirably demonstrated by filling the channel with still water, liberally covering the stream bed with water-soaked sawdust, and suddenly opening the tail gate wide. As the water accelerated, spiral flow developed, and the finely divided sawdust particles were carried along the bottom of the channel to the region of d, Fig. 5, where they were churned about violently in the turbulent eddy. The sawdust was then replaced with crystals of potassium permanganate, and the colored water followed the same pattern as the transported sawdust (Fig. 3).



(a) ELEVATION



(b) SECTION AT d



(c) PLAN

FIG. 5.—DEVELOPMENT OF TURBULENCE AT INSIDE OF BEND

In the lower half of the section at d, Fig. 5, the water particles had a velocity component toward the inside of the bend. This prevented the region of turbulence from developing to the same extent as it did nearer the water surface. The velocity contours (Fig. 2) give evidence of this phenomenon.

To gain information on the nature of the streamlines around the bend, use was made of a mixture of n. Butyl Plithalate and technical Xylol colored with a deep red oil dye and having the same specific gravity as water. The mixture was admitted into the stream through capillary tubes, and it came from the tubes in globules about the size of a small pea, providing a ready means of tracing the streamlines. Motion pictures were taken giving an excellent record of the existence, extent, and nature of the spiral flow. When the globules passed the section halfway around the bend and happened to be along the bottom of the channel and midway between the side-walls, their path crossed the center line of the channel at an angle of about 45°, indicating a strong current toward the convex or inner bank of the stream.

Floats consisting of ⅜-in. hedgewood cubes boiled in paraffin and marked with white crosses for identification were used to study the angular velocity of the water surface. Although the blocks showed an angular velocity about the vertical axis only, they indicated that, if an open channel turns to the left, the cubes in general take a rotational motion in the opposite direction, except in cases where a cube was trapped in the region of turbulence, such as near d, Fig. 5.

The motion of rice grains, shoved along the bottom of the channel, or chemical globules held in suspension by the water, progressed with an accelerated velocity from the outside of the bend toward the center of the channel and then decelerated as they approached the inside of the bend.

It is difficult to make any comparison of the streamlines followed by a particle of water starting near the outside at the entrance of the bend with the streamlines of a particle starting near the inside wall at the entrance of the bend because of the differences in filamental velocities and in the lengths of path. A particle starting at the entrance of the bend and on the water surface near the outside wall would move downstream slowly with a gradual lowering below the water surface; after reaching about mid-depth would accelerate toward the center of the channel; when halfway around the bend would cross the center line near the bottom of the channel at an angle of nearly 45°; and would end in the turbulent region. A particle starting at the entrance of the bend and on the water surface near the inside wall would move swiftly downstream with a strong velocity component outward, upon approaching the outside wall would gradually be lowered below the water surface, and would leave the bend near the bottom of the channel. On the average the liquid completed half a turn of the spiral in flowing around the 180° bend. However, the time required for the different particles to move through their allotted courses differed considerably, the shorter period being for those starting near the inside wall.

The transverse profile of the water surface at Section 12, Stage 0.406 (Fig. 2(*a*)), shows that the water is higher at the outside wall than at the inside wall. This condition, due to the centrifugal force of the water flowing around the bend, is not enough in itself to induce the spiral flow, but it is one of the two essential conditions, the other being a difference between the filamental velocities of the water near the surface and near the bottom of the channel.

NOMENCLATURE

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

THEORY OF SPIRAL FLOW

The theory of the induced spiral has been discussed admirably by Sherman M. Woodward,[6] Hon. M. Am. Soc. C. E., whose reasoning may be explained by reference to Fig. 6. If the layer of water E–F moves in a curve, a centrifugal force of

$$F = \frac{m\,V^2}{r} \qquad (1)$$

must be directed toward the center of curvature. In Eq. 1, m is the mass of the water; V, the velocity in feet per second; and r, the radius of curvature. The excess of pressure on the face D–H over that on C–G causes the water to be deflected toward the center of curvature on the side toward A. This excess of pressure can exist only when the water surface at D is higher than at C. The layer near the bottom G–H has a lower velocity than the central layer E–F. Since the pressure difference between F and E is the same as that between G and H, it follows that the layer G–H has a shorter radius of curvature than the layer E–F. A spiral flow inevitably results, in this case clockwise when facing downstream. If the velocity of the layer E–F were lower than that of G–H, the spiral would be counterclockwise.

Since the vertical curve of the velocity variation approximates a parabola in shape, it is natural that the crosscurrents of the spiral be most pronounced near the bottom of the stream. At Section 12, Stage 0.406, the water surface at the outside and inside of the bend had a difference of elevation of 0.13 in. This is sufficient to induce toward the inside of the bend a theoretical velocity of 0.8 ft per sec, which is nearly as large as the average downstream velocity in the section as a whole. Friction losses greatly reduce the theoretical velocity,

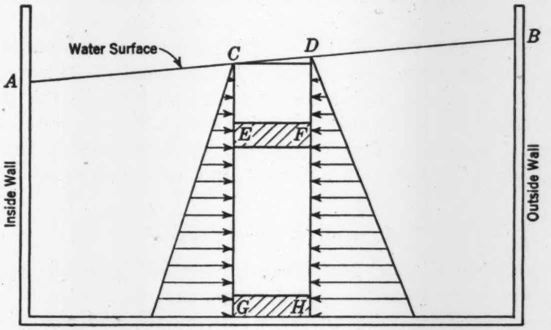


FIG. 6.—CROSS SECTION OF A BEND OF OPEN CHANNEL, FACING DOWNSTREAM

but the motion pictures taken verify the existence of a crosscurrent throughout the entire 180° bend, with the most pronounced crosscurrents near the very bottom of the stream.

---

[6] "Hydraulics of the Miami Flood Control Project," by S. M. Woodward, *Technical Reports*, Miami Conservancy Dist., Dayton, Ohio, Pt. VII, 1920, p. 264.

If the channel under consideration happens to be rather wide and comparatively shallow, if the water is flowing through a bend, such as may exist in a natural river or model of a river, and if it encounters channel irregularities, there undoubtedly will be localized turbulence which may tend to distort the spiral flow pattern. The localized turbulence and eddy currents in such a channel may so obscure the spiral flow pattern as to cause doubt in the minds of some as to possibility of its existence in the bend of any open channel.[7] However, if the essential driving forces exist at a bend in the stream, with higher downstream velocities near the water surface than near the stream bed, a general spiral motion as first enunciated by Thomson[2] is inevitable. A careful weighing of the basic cause of the spiral will usually suffice to convince the skeptic.[8]

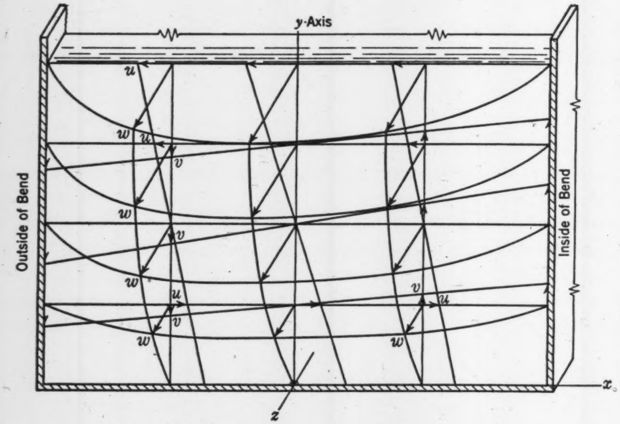


FIG. 7.—CHANNEL SECTION SHOWING SOLID OF FLOW AND ASSUMED VELOCITY COMPONENTS

MATHEMATICAL ANALYSIS

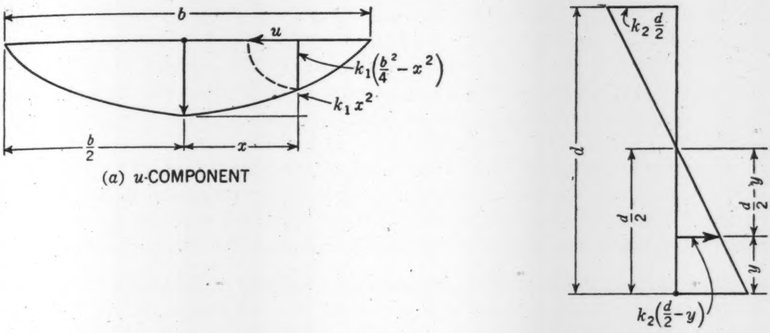
Suppose an open channel of rectangular section be considered, and that assumptions are made regarding the u-component, v-component, and w-component of the velocity parallel to the x-axis, y-axis, and z-axis as shown in Fig. 7 for a frictionless fluid as follows:

1. Parabolic variation of the u-component of the velocity, starting with zero at each side-wall and becoming a maximum on the y-axis midway between the two side-walls. Straight-line variation of the u-component, starting with

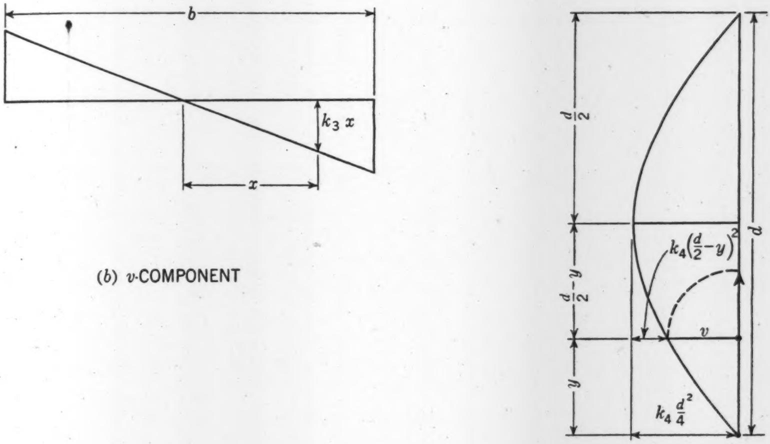
---

[7] "Flow in River Bends," by Herbert D. Vogel and Paul W. Thompson, *Civil Engineering*, May, 1933, pp. 266–268.

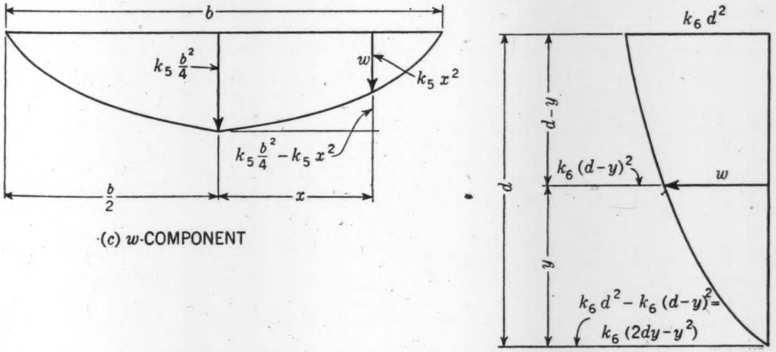
[8] "Existence of Helicoidal Flow," by Herbert D. Vogel and Paul W. Thompson, *ibid.*, July, 1934, pp. 370–371.


b
u
k₁(b²/4 − x²)
k₁x²
b/2
x
k₂ d/2
d
d/2
d/2 − y
y
k₂(d/2 − y)

(*a*) *u*-COMPONENT


b
k₃x
x
d/2
d
k₄(d/2 − y)²
d/2 − y
v
y
k₄ d²/4

(*b*) *v*-COMPONENT


b
k₅ b²/4
w
k₅x²
k₅ b²/4 − k₅x²
b/2
x
k₆d²
d − y
d
k₆(d − y)²
w
y
k₆d² − k₆(d − y)² =
k₆(2dy − y²)

(*c*) *w*-COMPONENT

FIG. 8.—VELOCITY COMPONENTS

zero at mid-depth of the channel and becoming a maximum at top and bottom of channel, with opposite signs.

2. Straight-line variation of the v-component of the velocity, starting with zero on the y-axis and becoming a maximum at each side-wall, with opposite signs. Parabolic variation of the v-component starting with zero at top and bottom of channel, with a maximum value at mid-depth.

3. Parabolic variation of w-component of the velocity, starting with zero at the bottom of the channel and becoming a maximum at the water surface. Parabolic variation of w-component, starting with zero at each side-wall and becoming a maximum on the y-axis midway between the two side-walls.

Then, if b and d, respectively, represent the breadth of channel and depth of water, the velocity components at a particular cross section of the channel may be written in terms of x, y, b, d, and certain coefficients, which in this discussion will be called k_u, k_v, and k_w, for the u-component, v-component, and w-component of the velocity. Referring to Fig. 8,

$$u = k_1 k_2 \left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right) = k_u \left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right) \ldots\ldots (2a)$$

$$v = k_3 x \left[k_4 \frac{d^2}{4} - k_4 \left(\frac{d}{2} - y\right)^2 \right] = k_3 k_4 x (y d - y^2) = k_v x (y d - y^2) \ldots (2b)$$

and

$$w = k_5 \left(\frac{b^2}{4} - x^2\right) k_6 (2 y d - y^2) = k_w \left(\frac{b^2}{4} - x^2\right)(2 y d - y^2) \ldots (2c)$$

In Eqs. 2, k_u and k_v have dimensions of $L^{-2} T^{-1}$, and k_w has dimensions of $L^{-3} T^{-1}$. Assuming for simplicity that $k_u = k_v = k_w = 1$, Eqs. 2 become

$$u = \left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right) \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

$$v = x (y d - y^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

and

$$w = \left(\frac{b^2}{4} - x^2\right)(2 y d - y^2) \ldots\ldots\ldots\ldots\ldots\ldots (3c)$$

Streamlines.—If streamlines are defined as curves the tangents to which everywhere give the direction in which the fluid is moving, then the equation of the streamlines is[9] $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, and

$$\frac{dx}{\left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right)} = \frac{dy}{x (y d - y^2)} \ldots\ldots\ldots\ldots\ldots (4a)$$

Separating terms and integrating,

$$\log \left(\frac{b^2}{4} - x^2\right) = - \log (y d - y^2) + C_1 \ldots\ldots\ldots\ldots (4b)$$

---

[9] "An Introduction to Mathematical Physics," by R. A. Houstoun, Longmans, Green, 1925, Chapter II.

or

$$\left(\frac{b^2}{4} - x^2\right)(y\,d - y^2) = C \tag{5}$$

Eq. 5 should be the formula of the streamlines as projected on the x–y plane, which is the plane of the channel cross section. For the y–z plane,

$$\frac{dz}{\left(\frac{b^2}{4} - x^2\right)(2\,y\,d - y^2)} = \frac{dy}{x\,(y\,d - y^2)} \tag{6a}$$

but, from Eq. 5,

$$x = \left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5} \tag{6b}$$

then

$$\frac{dz}{\left(\frac{C}{y\,d - y^2}\right)(2\,y\,d - y^2)} = \frac{dy}{\left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}(y\,d - y^2)} \tag{6c}$$

and

$$z = \int \frac{C\,(2\,y\,d - y^2)\,dy}{\left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}(y\,d - y^2)^2} + C_2 \tag{7a}$$

If the coefficients k_v and k_w are retained, and Eq. 7*a* is made a definite integral,

$$z = \frac{k_w}{k_v}\int \frac{C\,(2\,y\,d - y^2)\,dy}{\left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}(y\,d - y^2)^2} \tag{7b}$$

Eq. 7*b* may be evaluated as follows:
First step: Simple reductions put the integral in the form:

$$z = \frac{k_w\,C}{k_v}\int \frac{(2\,d - y)\,dy}{(d - y)\sqrt{(y\,d - y^2)\left[\frac{b^2}{4}(y\,d - y^2) - C\right]}} \tag{8a}$$

Second step: Let $y = \left(\frac{d}{2}\right)(1 + s)$; $k^2 = 1 - \left(\frac{16\,C}{b^2\,d^2}\right)$; and $M = \frac{4\,k_w\,C}{k_v\,b\,d}$.
Then

$$z = M\int \frac{(3 - s)\,ds}{(1 - s)\sqrt{(1 - s^2)(k^2 - s^2)}} \tag{8b}$$

Third step: Break up the integrand in three terms as follows:

$$\frac{3 - s}{(1 - s)\sqrt{(1 - s^2)(k^2 - s^2)}} = \frac{1}{\sqrt{(1 - s^2)(k^2 - s^2)}} + \frac{2}{1 - k^2}\sqrt{\frac{1 - s^2}{k^2 - s^2}} - 2\,\frac{d}{ds}\left[\frac{\sqrt{(1 - s^2)(k^2 - s^2)}}{(1 - k^2)(1 - s)}\right] \tag{8c}$$

This step can be verified by differentiating the right-hand term of Eq. 8*b* as indicated, reducing all terms to a common denominator and combining. Fourth step: From Eqs. 8*b* and 8*c*,

$$z = M\left[\int \frac{ds}{\sqrt{(1-s^2)(k^2-s^2)}} + \frac{2}{1-k^2}\int\sqrt{\frac{1-s^2}{k^2-s^2}}\,ds - 2\,\frac{\sqrt{(1-s^2)(k^2-s^2)}}{(1-k^2)(1-s)}\right] \ldots\ldots\ldots (8d)$$

Fifth step: In the two integrals on the right in Eq. 8*d*, make $s = k \sin\phi$. Then

$$z = M\left[\int \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} + \frac{2}{1-k^2}\int \sqrt{1-k^2\sin^2\phi}\,d\phi - \frac{2\sqrt{(1-s^2)(k^2-s^2)}}{(1-k^2)(1-s)}\right] \ldots\ldots\ldots (8e)$$

The two integrals are now the standard elliptic integrals of the first and second kind, respectively. Hence

$$z = M\left[F(\phi,k) + \frac{2}{1-k^2}E(\phi,k) - \frac{2\sqrt{(1-s^2)(k^2-s^2)}}{(1-k^2)(1-s)}\right] + C_2 \ldots (9)$$

in which C_2 is the constant of integration.

To compute values of y and z the best method is to use ϕ as a parameter. Then, since $s = k \sin\phi$, Eq. 9 becomes

$$y = \frac{\sqrt{b^2 d^2 - 16\,C}}{2\,b}\sin\phi + \frac{d}{2} \ldots\ldots\ldots (10)$$

Then y and z are calculated[10] from Eqs. 9 and 10 respectively, using tables of $F(\phi,k)$ and $E(\phi,k)$. The streamlines have been plotted in Fig. 9, showing three views—section, plan, and elevation. Here the value $k_v = k_w = 1$ was used, merely to determine the curve forms, since any other relationship between k_v and k_w would only change the distance that a particle would have to move downstream to complete one cycle of the curves, but would not change their general shape.

Angular Velocity.—According to the laws of hydrodynamics the components of the angular velocity, ξ, η, and ζ, about the x-axis, y-axis, and z-axis are, assuming time does not affect the velocities at a given section:

$$\xi = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = (d-y)\left(\frac{b^2}{4} - x^2\right) \ldots\ldots\ldots (11a)$$

$$\eta = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = x\,(2\,y\,d - y^2) \ldots\ldots\ldots (11b)$$

and

$$\zeta = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(y\,d - y^2) + \left(\frac{b^2}{4} - x^2\right) \ldots\ldots\ldots (11c)$$

---

[10] "Formules et Tables Numeriques aux Fonctions Circulaires, Hyperboliques, Elliptiques," by L. Potin, 1925, pp. 753–827.

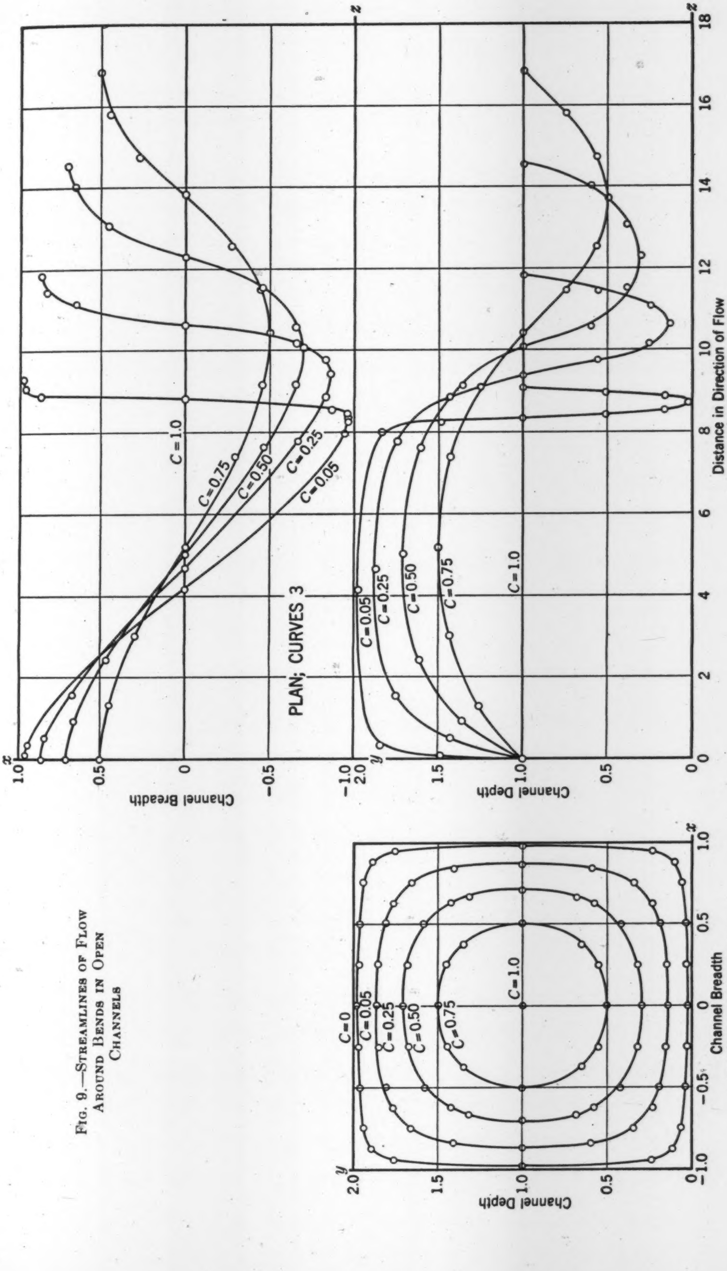


Fig. 9.—Streamlines of Flow Around Bends in Open Channels

Then the angular velocity in radians per second is:

$$\omega = \sqrt{\xi^2 + \eta^2 + \zeta^2}$$

$$= \sqrt{\left[(d-y)\left(\frac{b^2}{4}-x^2\right)\right]^2 + [x\,(2\,y\,d-y^2)]^2 + \left[\frac{1}{2}(y\,d-y^2) + \left(\frac{b^2}{4}-x^2\right)\right]^2} \quad ..(12)$$

A check on the accuracy of Eq. 12 may be had by noting that the divergence of the angular velocity must be zero: That is, a divergence of

$$\omega = \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} = 0 = \frac{\partial}{\partial x}\left[\,(d-y)\left(\frac{b^2}{4}-x^2\right)\right] + \frac{\partial}{\partial y}[x\,(2\,y\,d-y^2)],$$

or

$$\omega = -2\,x\,(d-y) + 2\,x\,(d-y) = 0 \ldots\ldots\ldots\ldots\ldots(13)$$

In Eq. 12 it should be noted that the η-component is not affected by the breadth of the channel.

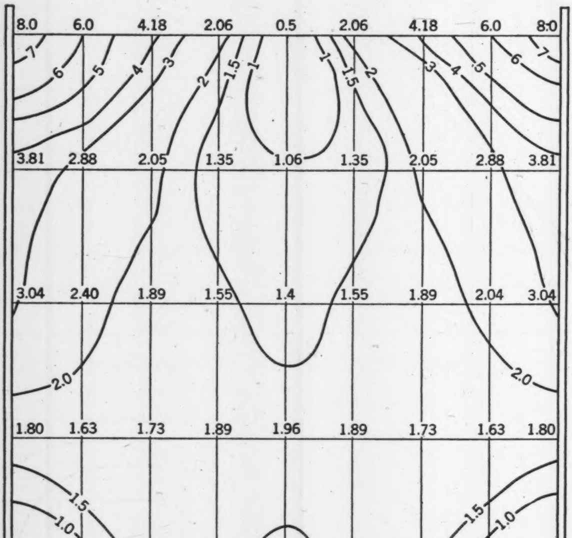
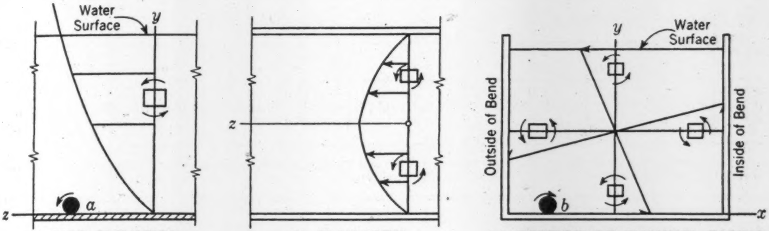


FIG. 10.—LINES OF EQUAL ANGULAR VELOCITY

Fig. 10 shows lines of equal angular velocity in the channel section, assuming $b = d = 2$, and $k_u = k_v = k_w = 1$. It is worthy of note that at values of $y = \frac{d}{4}$, the vorticity is fairly uniform all across the channel. The symmetry of the diagram is due to the original assumptions as to velocity components.

The direction of rotation of the components of the angular velocity can be seen by reference to Fig. 11. The direction of rotation in the y–z and y–x planes is always the same regardless of what point in the section is selected, whereas rotation in the z–x plane changes direction as a particle moves from the inside half of the bend through the center of the channel to the outside half of the bend.



(*a*) SIDE ELEVATION y - z PLANE (*b*) PLAN VIEW z - x PLANE (*c*) SECTION OF CHANNEL FACING UPSTREAM, y - x PLANE

FIG. 11.—DIRECTION OF ROTATION OF ANGULAR VELOCITY COMPONENTS

Acceleration.—Components of the acceleration parallel to the x-axis, y-axis, and z-axis are given by the equations:

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -2x\left(\frac{d}{2} - y\right)^2\left(\frac{b^2}{4} - x^2\right) - x\left(\frac{b^2}{4} - x^2\right)(y\,d - y^2) \qquad \text{(14a)}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right)(y\,d - y^2) + x^2(y\,d - y^2)(d - 2y) \qquad \text{(14b)}$$

and

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -2x(2y\,d - y^2)\left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right) + 2xy(d - y)^2\left(\frac{b^2}{4} - x^2\right) \qquad \text{(14c)}$$

$$A = \sqrt{a^2_x + a^2_y + a^2_z} = \left\{\left[-2x\left(\frac{d}{2} - y\right)^2\left(\frac{b^2}{4} - x^2\right) - x\left(\frac{b^2}{4} - x^2\right)(y\,d - y^2)\right]^2 + \left[\left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right)(y\,d - y^2) + x^2(y\,d - y^2)(d - 2y)\right]^2 + \left[-2x(2y\,d - y^2)\left(\frac{d}{2} - y\right)\left(\frac{b^2}{4} - x^2\right) + 2xy(d - y)^2\left(\frac{b^2}{4} - x^2\right)\right]^2\right\}^{0.5} \qquad \text{(15)}$$

Table 2 shows components of the acceleration parallel to the x-axis, y-axis, and z-axis for different values of x, y, and z in terms of b and d, assuming

$k_u = k_v = k_w = 1$. It should be noted that, at all points in the section, a_x varies as $b^3\,d^2$; a_y varies as $b^2\,d^3$; and a_z varies as $b^3\,d^3$. This is significant where, as in most natural streams, breadth is greater than depth, and hence the acceleration across the channel and parallel to the x-axis is much greater than that in the vertical direction parallel to the y-axis. Likewise, the maximum u-component of the velocity, parallel to the x-axis, would exceed the v-component, parallel to the y-axis, and be in direct ratio of width to depth of stream.

TABLE 2.—COMPONENTS OF ACCELERATION, a,

Values of x[a]	$y = 1.00\,d$			$y = 0.75\,d$			
	a_x	a_y	a_z	a_x	a_y	a_z	a_x
$0.50\,b$	0	0	0	0	$-3\,b^2\,d^3/128$	0	0
$0.25\,b$	$3\,b^3\,d^2/128$[b]	0	$-3\,b^3\,d^3/64$[b]	$-3\,b^3\,d^2/1{,}024$[b]	$-33\,b^2\,d^3/1{,}024$	$-24\,b^3\,d^3/1{,}024$[b]	$3\,b^3\,d^2/256$[b]
0[c]	0	0	0	0	$-3\,b^2\,d^3/256$	0	0

[a] Values of x in terms of channel width, b. [b] All values in this table are for the outside half of the indicated by the footnote index. [c] Center line of channel.

On the outside half of the channel the velocity components parallel to the x-axis, and in the lower half of the section, are directed toward the inside of the bend and are increasing in magnitude as the particle moves toward the center of the channel. If a piece of gravel or sand should start moving along the bed of the stream in this vicinity, it would tend to continue and, after passing the middle of the stream, would tend to be stopped due to the decelerating components of the filamental velocities directed toward the inside of the bend.

The components of the angular velocity are helpful in understanding the movement of detritus along the bed of the stream. If, in Eq. 12, y is taken as zero, the angular velocity becomes:

$$\omega = \sqrt{\left[d\left(\frac{b^2}{4} - x^2\right)\right]^2 + 0 + \left[\frac{1}{2}\left(\frac{b^2}{4} - x^2\right)\right]^2} \quad \ldots\ldots\ldots(16)$$

Hence, under the assumptions made, there is no tendency for the gravel on the stream bed to rotate about a vertical axis, but there are components of angular velocity about the x-axis and z-axis. The rotation about the x-axis is the same as the direction a piece of gravel would be obliged to roll along the stream bed, and, although the rotation about the z-axis, parallel to the axis of the center line of the channel (Figs. 11(a) and 11(b)), is retrograde to the direction a piece of gravel would be obliged to rotate, its magnitude varies as $\frac{b^2}{8}$ compared to $\frac{b^2\,d}{4}$ for the value of $x = 0$ at the center of the stream. When $b = d = 2$ and $k_u = k_v = k_w = 1$, the one component is four times the other in value. However, in a natural channel it is likely that the maximum cross-currents are at some distance above the bed of the stream, in which case the angular velocity would not be retrograde to the direction that a piece of gravel would roll.

The coefficients accompanying the components of the acceleration in Table 2 are based on the assumption $k_u = k_v = k_w = 1$, which is not necessarily true, although it can be shown that $k_u = k_v$, since the divergence of

AT VARIOUS POINTS IN THE CHANNEL SECTION

$y = 0.50\ d$		$y = 0.25\ d$			$y = 0$			Values of $x^{(a)}$
a_y	a_z	a_x	a_y	a_z	a_x	a_y	a_z	
0	0	0	$3\ b^2\ d^3/128$	0	0	0	0	$0.50\ b$
0	$-3\ b^3\ d^3/256^{(b)}$	$15\ b^3\ d^2/1{,}024^{(b)}$	$15\ b^2\ d^3/1{,}024$	$-3\ b^3\ d^3/1{,}024^{(b)}$	$3\ b^3\ d^2/128^{(b)}$	0	0	$0.25\ b$
0	0	0	$b^2\ d^3/64$	0	0	0	0	$0^{(c)}$

channel. Values for the inside half are symmetrically similar except that the sign is changed in the cases

the velocity $= 0 = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}$; and that $\dfrac{\partial}{\partial x}\left[k_u\left(\dfrac{d}{2} - y\right)\left(\dfrac{b^2}{4} - x^2\right)\right]$ $= -\dfrac{\partial}{\partial y}\left[k_v\, x\,(y\, d - y^2)\right]$; or $k_u = k_v$.

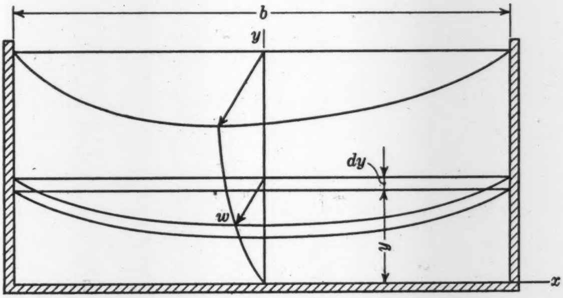


FIG. 12.—VELOCITY COMPONENT ALONG THE z-AXIS

The relation of these coefficients to k_w depends upon how far downstream a particle must move to complete a cycle of the streamlines shown in Fig. 9. However, some notion of the value of k_w may be had from the following analysis: From Fig. 12 it is apparent that when $x = 0$, $dQ = \dfrac{2}{3}\, b\, w\, dy$ $= \dfrac{2}{3}\, b\, k_w \left(\dfrac{b^2}{4} - x^2\right)(2\, y\, d - y^2)\, dy$. When $x = 0$,

$$dQ = \frac{k_w\, b^3}{6}\,(2\, y\, d - y^2)\, dy \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Integrating,

$$Q = \frac{k_w\, b^3}{6}\int_0^d (2\, y\, d - y^2)\, dy = \frac{k_w\, b^3\, d^3}{9} \ldots\ldots\ldots\ldots (18)$$

By use of Eq. 18 it is possible to evaluate k_w if the rate of flow Q, breadth of channel b, and depth of channel d are known.

At this point it is of interest to note the ratio of the maximum to the average velocity, w, in the direction of the z-axis. The maximum w-component is: When $x = 0$, and $y = d$,

$$w\,(\text{max}) = k_w \left(\frac{b^2}{4} - x^2 \right) (2\,y\,d - y^2) = \frac{k_w\,b^2\,d^2}{4} \ldots\ldots\ldots \text{(19a)}$$

$$w\,(\text{ave}) = \frac{Q}{b\,d} = \frac{k_w\,b^2\,d^2}{9} \ldots\ldots\ldots \text{(19b)}$$

and

$\dfrac{w\,(\text{max})}{w\,(\text{ave})} = \dfrac{k_w\,9\,b^2\,d^2}{k_w\,4\,b^2\,d^2} = \dfrac{9}{4} = 2.25.$ This would indicate that the maximum value of the velocity along the axis of the stream is 2.25 times as large as the average, on the basis of the assumptions made in this paper.

FREE VORTEX FLOW

Now suppose the w-component of the velocity complies more nearly with the law of a free vortex as shown in Fig. 13. Referring to Figs. 13(*b*) and 13(*c*), $k_7\,d^2 - k_7\,(d - y)^2 = k_7\,(2\,y\,d - y^2)$ and $w_i\,r_i = w_o\,r_o =$ a constant.

At the water surface

$$w = \frac{k_8}{r} \ldots\ldots\ldots \text{(20a)}$$

and at any point—

$$w = \frac{k_7\,k_8\,(2\,y\,d - y^2)}{r_c - x} = \frac{k_w\,(2\,y\,d - y^2)}{r_c - x} = \frac{k_w\,(2\,y\,d - y^2)}{1 + \frac{b}{2} - x} \ldots\ldots \text{(20b)}$$

If u and v remain the same as before, and $k_u = k_v = k_w = 1$, the components of the filamental velocities parallel to the x-axis, y-axis, and z-axis will be:

$$u = \left(\frac{d}{2} - y \right) \left(\frac{b^2}{4} - x^2 \right) \ldots\ldots\ldots \text{(21a)}$$

$$v = x\,(y\,d - y^2) \ldots\ldots\ldots \text{(21b)}$$

$$w = \frac{2\,y\,d - y^2}{1 + \frac{b}{2} - x} \ldots\ldots\ldots \text{(21c)}$$

Streamlines.—In the x–y plane the streamlines are the same as those computed under the first assumptions. The general equation is obtained as follows:

$$\frac{dz}{2\,y\,d - y^2} \div \left(1 + \frac{b}{2} - x \right) = \frac{dy}{x\,(y\,d - y^2)} \ldots\ldots\ldots \text{(22a)}$$

and

$$dz = \frac{(2\,d - y)\,dy}{x\,(d - y) \left(1 + \frac{b}{2} - x \right)} \ldots\ldots\ldots \text{(22b)}$$

Substituting from Eq. 5, $x = \left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}$ and

$$z = \int \frac{(2\,d - y)\,dy}{\left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}(d - y)\left[1 + \frac{b}{2} - \left(\frac{b^2}{4} - \frac{C}{y\,d - y^2}\right)^{0.5}\right]} + C_3 \ldots (23)$$

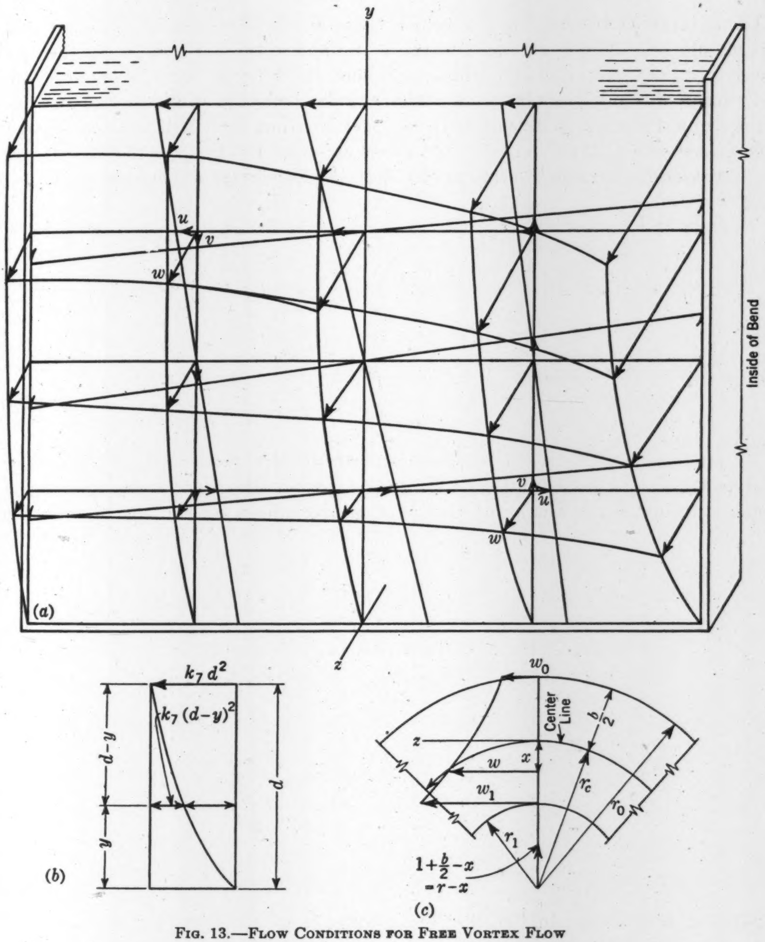

y
u
v
w
v
u
w
Inside of Bend
(a)
z
$k_7 d^2$
$k_7 (d-y)^2$
d-y
y
d
(b)
w_0
Center Line
$\frac{b}{2}$
z
w
x
w_1
r_c
r_0
r_1
$1+\frac{b}{2}-x$
$=r-x$
(c)

FIG. 13.—FLOW CONDITIONS FOR FREE VORTEX FLOW

It appears difficult to evaluate this integral, and perhaps it is not necessary since some notion of the nature of the streamlines may be had by reference to those for the first assumptions.

Angular Velocity.—For the revised assumptions as to velocity distribution, the general equation for angular velocity becomes:

$$\omega = \sqrt{\left[\frac{d-y}{1+\frac{b}{2}-x}\right]^2 + \left\{-\frac{y}{2}\left[\frac{2\,d-y}{\left(1+\frac{b}{2}-x\right)^2}\right]\right\}^2 + \left[\frac{y}{2}(d-y)+\frac{1}{2}\left(\frac{b^2}{4}-x^2\right)\right]^2} \quad ..(24)$$

The divergence of this angular velocity was checked and found to be zero, as it should be. The component in the y–x plane is the same as under the first assumptions, and, although the component in the y–z plane is changed, its direction has not been altered. The angular velocity in the z–x plane has the same direction at all points in the section; that is, it will not change sign from one side of the channel to the other as under the first assumptions.

Acceleration.—For the free vortex flow the acceleration becomes:

$$\begin{aligned} a = \Bigg\{ & \left[-2\,x\left(\frac{d}{2}-y\right)^2\left(\frac{b^2}{4}-x^2\right) - x\left(\frac{b^2}{4}-x^2\right)(y\,d-y^2)\right]^2 \\ & + \left[\left(\frac{d}{2}-y\right)\left(\frac{b^2}{4}-x^2\right)(y\,d-y^2) + x^2\,(y\,d-y^2)\,(d-2\,y)\right]^2 \\ & + \left[\frac{\left(\frac{d}{2}-y\right)\left(\frac{b^2}{4}-x^2\right)(2\,y\,d-y^2)}{\left(1+\frac{b}{2}-x\right)^2} + \frac{2\,x\,(y\,d-y^2)\,(d-y)}{1+\frac{b}{2}-x}\right]^2\Bigg\}^{0.5} \quad . (25) \end{aligned}$$

The components of the acceleration parallel the x-axis and y-axis, and, as given by Eq. 25, have not been changed over what they were under the original assumptions; hence the projection of the streamlines on the z–y plane is the same as that shown in Fig. 9. The z-component of the acceleration, parallel to the axis of the stream, does not now have as large values as before. If time and care were taken to compare velocities and accelerations as indicated by equations here developed, a suitable comparison between the streamlines of Fig. 9 and those indicated by Eq. 23 might be made.

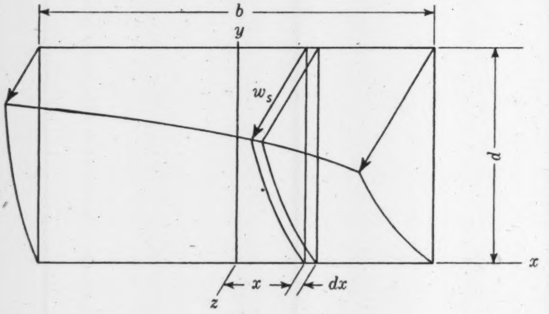


FIG. 14

It is possible to analyze the relation between the maximum and the average w-component of the velocity, similar to that under the original assumptions. If w_s is taken as the w-component of the velocity at the water surface, then (see Fig. 14):

$$dQ = \frac{2}{3}\, d\, w_s\, dx = \frac{2}{3}\, d\, k_w \left(\frac{2\,y\,d - y^2}{r_c - x}\right) dx \tag{26a}$$

When $y = d$

$$dQ = \frac{2\,k_w\,d^3}{3\,(r_c - x)}\, dx \tag{26b}$$

and

$$Q = \frac{2}{3}\, k_w\, d^3 \int_{-0.5\,b}^{0.5\,b} \frac{dx}{r_c - x} = -\,\frac{2}{3}\, k_w\, d^3 \log_e \left(\frac{2\,r_c - b}{2\,r_c + b}\right) \tag{27}$$

The maximum value of the w-component of the velocity occurs when $y = d$ and $x = \frac{b}{2}$; then

$$w = \frac{k_w\,(2\,y\,d - y^2)}{r_c - x} \tag{28a}$$

$$w\,(\max) = \frac{k_w\,d^2}{r_c - 0.5\,b} = \frac{2\,k_w\,d^2}{2\,r_c - b} \tag{28b}$$

and

$$w\,(\text{ave}) = \frac{Q}{b\,d} = -\,\log_e \frac{2\,r_c - b}{2\,r_c + b} \left(\frac{2\,k_w\,d^2}{3\,b}\right) \tag{28c}$$

Finally,

$$\frac{w\,(\max)}{w\,(\text{ave})} = \frac{-\,3\,b}{(2\,r_c - b)\log_e \dfrac{2\,r_c - b}{2\,r_c + b}} \tag{29}$$

In the laboratory experiment where $r_c = 1.75$ ft and $b = 1.5$ ft, the ratio indicated by Eq. 29 has the value of 2.457, and is about 10% higher than under the first set of assumptions as to filamental velocities. It is of interest to note that Eq. 29 is not affected by the depth of the channel, the only variables in the right-hand member being breadth of channel and radius of curvature.

Eq. 27 can be used to evaluate k_w when the rate of flow and the breadth, depth, and radius of curvature of channel are known. For the laboratory experiment when the rate of flow $Q = 0.449$ cu ft per sec, $b = 1.50$ ft, $d = 0.467$ ft, and $r_c = 1.75$ ft, k_w has the value of 7.22. Using this value of k_w, computations were made for the theoretical velocities at 0.6 d and plotted against values scaled from velocity contours obtained from laboratory data as shown in Fig. 2(a). The laboratory data were for a section of the channel taken one-half way around a 180° bend.

Some justification may be had for assuming the w-component of the filamental velocities to vary according to the law of a free vortex. (Fig. 15 shows values of the velocity scaled from Fig. 16, Section 12, Stage 0.496, at 0.60 depth, and $k_w = 7.2$.) The computed values of the velocities are higher than the actual values near the water surface, and are smaller near the bottom of the channel. This difference perhaps could have been made less by taking the origin of the parabola, Fig. 15, Section A–A, at the bottom of the channel instead of at the water surface as was done in the original assumptions; but there seems to be no really typical vertical velocity curve for all open channels,[11] and if one were to be selected for a rectangular channel in actual service it

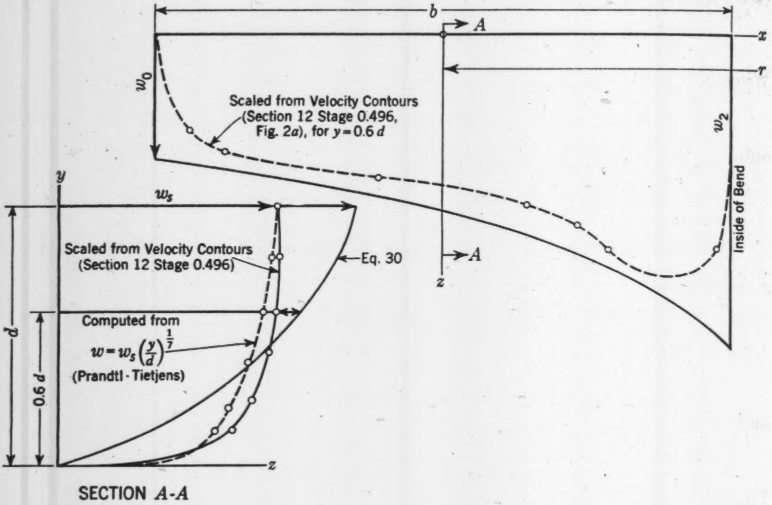


FIG. 15.—COMPARISON OF ACTUAL TO COMPUTED VALUES OF *w*-COMPONENTS OF VELOCITIES

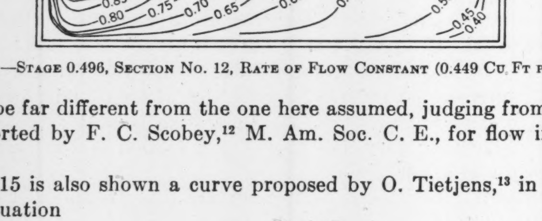


FIG. 16.—STAGE 0.496, SECTION NO. 12, RATE OF FLOW CONSTANT (0.449 CU FT PER SEC)

would not be far different from the one here assumed, judging from the actual curves reported by F. C. Scobey,[12] M. Am. Soc. C. E., for flow in irrigation canals.

In Fig. 15 is also shown a curve proposed by O. Tietjens,[13] in accordance with the equation

$$w = w_s\left(\frac{y}{d}\right)^{1/7} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

---

[11] *Water-Supply Paper No. 868-A*, U. S. Geological Survey, Plate 4.

[12] "The Flow of Water in Irrigation Canals," by Fred C. Scobey, *Bulletin No. 194*, U. S. D. A., 1915, p. 18.

[13] "Hydro- und Aeromechanik," by L. Prandtl and O. Tietjens, *Zweiter Band*, Springer, Berlin, 1931, p. 87.

Although this compares favorably with the actual velocity measurements, it does not exactly fit, and to make use of such a law for the mathematical definition of the velocity components would lead to an equation of the streamlines that would be most difficult if not impossible to evaluate.

The curve for Eq. 30, for $x = 0$, $r_c = 1.75$ ft, $d = 0.467$ ft, $b = 1.50$ ft, and $k_w = 7.2$, is also included in Fig. 15.

CONCLUSIONS

This research was undertaken as a study of the flow conditions in the bend of an open channel. Many observations have indicated that the spiral exists in the bends of both open and closed channels, and that in the latter the spiral is usually double. An attempt has been made to observe the direction and magnitude of the angular velocities of particles in suspension as the water moved through the channel bend, and to compare these with theoretical analyses made under assumed flow conditions. The observed rotation of the suspended particles behaved in general with the theoretical analysis based on the assumed conditions. The path taken by a particle on its way around a bend in an open channel is extremely complicated and varied, but in general it must follow the pattern of a spiral.

In the experimental channel the crosscurrents moving along the stream bed from outside to inside of bend were amazingly strong compared to the average velocities in the stream. Evidence of this phenomenon was recorded in a motion picture for reference.

The following conclusions seem justified by the research:

1. Spiral flow in the bends of open channels as enunciated by James Thompson[2] certainly does exist, although the spiral pattern is an exceedingly complex one.
2. The downstream components of the filamental velocities in the first half of the bend are greater near the convex bank than at the concave bank and vary across the channel in close agreement with the law of a free vortex.
3. The angular velocities and accelerations inherent to the spiral motion constitute strong contributing factors to the movement of bed load, not only in a downstream direction but toward the inside of bends.
4. At about three fourths the way around the bend there is a tendency for the development of an eddy, or slack water, along the inside bank due to the spiral motion, which is conducive to the deposition of suspended matter and the formation of a bar.

ACKNOWLEDGMENTS

Special acknowledgment is due W. E. Milne, head of the department of mathematics at Oregon State College, Corvallis, Ore., for the solution of the streamline equation, and to Alton E. Alspaugh and William B. Matlock for plotting the curves for the streamlines. The writer also wishes to give credit to Frank E. Lucas, Jun. Am. Soc. C. E., for help in the construction of the laboratory apparatus, and to all other individuals whose advice and counsel were so kindly given.

The funds for the purchase of expendable laboratory apparatus and miscellaneous materials for the project were furnished by the Engineering Foundation through J. C. Stevens, M. Am. Soc. C. E., chairman of the committee of the Society's Hydraulic Division on Hydraulic Research.

APPENDIX

NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:[14]

a = acceleration for free vortex flow;
b = breadth of water stream;
C = constants of integration, C_1, C_2, etc.;
d = depth of stream;
F = a centrifugal force;
k = modulus of the elliptic integral: k_u, k_v, and k_w = velocity coefficients, referring to the u-component, v-component, and w-component of velocity;
L = length;
m = mass of water;
Q = rate of flow;
r = radius of curvature: r_c = radius of curvature at center of stream; r_i = inside radius; and r_o = outside radius;
T = time;
u = component of velocity, parallel to the x-axis;
V = average velocity across one given section;
v = component of velocity, parallel to the y-axis;
w = component of velocity, parallel to the z-axis;
x = lateral distance, measured parallel to the x-axis, across the channel;
y = vertical distance, measured parallel to the y-axis;
z = horizontal distance, measured parallel to the z-axis, along axis of channel;
ζ = angular velocity about the x-axis;
η = angular velocity about the y-axis;
ξ = angular velocity about the z-axis;
ϕ = amplitude of the elliptic integral;
ω = angular velocity.

---

[14] ASA—Z10.2—1942.

DISCUSSION

J. M. ROBERTSON,[15] JUN. AM. SOC. C. E.—The conclusions reached in this paper appear to be in essential agreement with those of previous investigators. For instance, the author states (see Conclusion 2) that "The downstream components of the filamental velocities in the first half of the bend are greater near the convex bank than at the concave bank * * *." Essentially the same conclusion was reached by the late D. L. Yarnell,[16,17,18] M. Am. Soc. C. E., at the University of Iowa at Iowa City, in connection with his extensive study of the flow of water around bends in open and closed channels. In the paper in which he and Mr. Woodward report on the flow in open and closed 180° bends the following conclusions are reached:[17] "* * * with substantially uniform velocity distribution in the approach channel to the bend, the velocities of the filaments along the inside wall of the bend are increased while the velocities of the filaments along the outside wall are reduced, * * *" and, "as the water flows around a bend the effect of wall friction is to produce an unstable relation between the filaments flowing at different velocities, with the result that the filaments of highest velocity tend through a secondary circulation to drift toward the outer wall of the bend." This second statement pertains to the flow near the end of a 180° bend. In spite of information such as this, the writer can refer to several well-known books on hydraulics in which data are presented and statements are made, inferring that, as water flows around a bend, the filament of maximum velocity is always at the outside of the bend. Actually, as may be seen from the author's study, the filament of maximum velocity is initially, and, for most of the way around the bend, near the inside of the bend.

The flow of water or any fluid around a bend in an open or closed channel is affected by many factors; it is unfortunate that Professor Mockmore did not investigate more of them. Besides the primary variable of the rate of flow (or the Reynolds number) the flow conditions in a bend should depend upon the relative curvature of the bend, the shape of the channel cross section, and the length of the bend. Also, when one bend follows another, the flow conditions in the second bend are modified greatly by the relative proximity of the first bend. The writer will attempt to evaluate the effect of some of these variables on the flow in open channel bends through reference to data obtained mainly in closed conduit bends. If one keeps in mind that, at least in most respects, the flow in an open channel is similar to that in the lower half of a closed channel of twice the depth, it is not difficult to translate these observations to open channel bends, at least qualitatively.

In one part of his mathematical analysis the author assumes that the downstream component (w-component) of the velocity follows the free vortex law, according to which the velocity at any point is inversely proportional to the radius to that point. As was shown by Fig. 15, this is a good assumption. It would be expected from this law that the sharper the curvature of the bend the more pronounced would be the velocity distribution. Several years ago in connection with a study of the flow of water around bends in pipes at the Hydraulics Laboratory of the University of Illinois at Urbana, under the direction of W. M. Lansford, M. Am. Soc. C. E., the writer measured the velocity distribution in several 90°, cast-iron, flanged, bends in a 4-in. pipe line.[19] The bends were preceded by some 450 diameters of straight pipe, thus

---

[15] Asst. Prof., Dept. of Eng. Mechanics, Pennsylvania State College, State College, Pa.

[16] "Flow of Water Around Bends in Pipes," by David L. Yarnell and Floyd A. Nagler, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1018.

[17] "Flow of Water Around 180-Degree Bends," by D. L. Yarnell and S. M. Woodward, *Technical Bulletin No. 526*, U. S. D. A., October, 1936.

[18] "Flow of Water Through 6-Inch Pipe Bends," by D. L. Yarnell, *Technical Bulletin No. 577*, U. S. D. A., October, 1937.

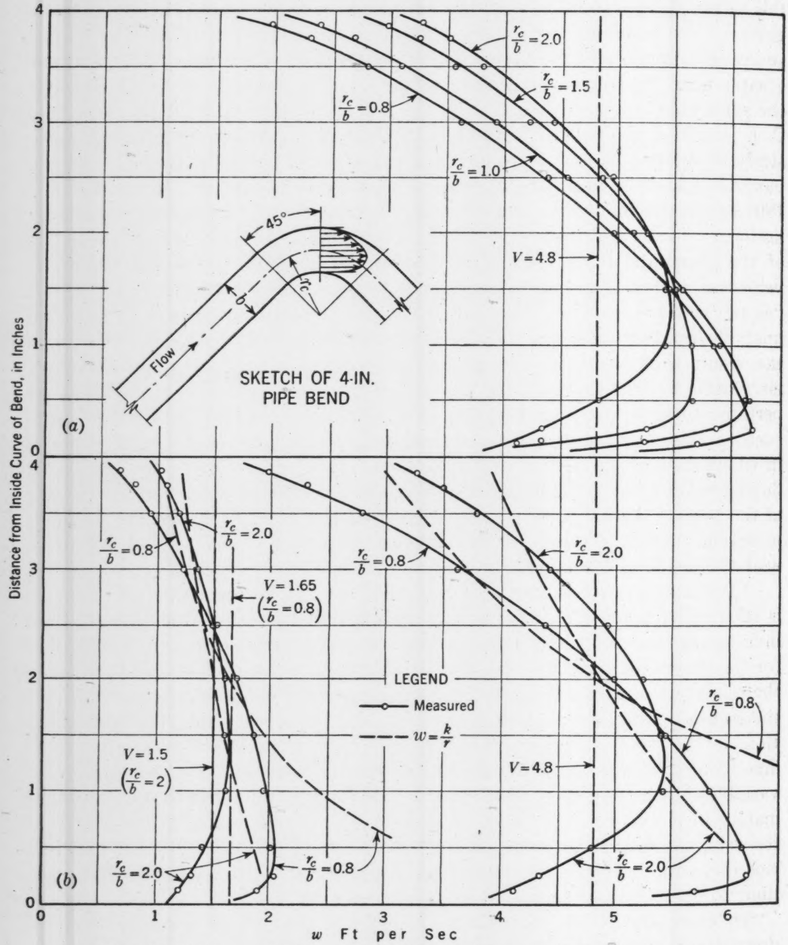


FIG. 17.—EFFECT OF CURVATURE ON VELOCITY DISTRIBUTIONS IN PIPE BENDS

assuring a normal velocity distribution above them; and the measurements were made across a diameter, 45° around the bend, in the plane of the bend. Using the ratio $\frac{r_c}{b}$ (r_c is the radius of the center line of the bend, and b is the channel breadth or pipe diameter) as indicative of the relative curvature of the bend, the curvatures of the bends studied were 0.8, 1.0, 1.5, and 2.0. Two bends of each curvature were studied. Typical velocity distributions obtained are shown in Fig. 17(*a*) for a mean flow velocity V of about 4.8 ft per sec. The effect of curvature on velocity distribution is quite apparent. A rough comparison of the results obtained with the free vortex law prediction is shown in Fig. 17(*b*) for two rates of flow in the bends of least and greatest curvatures. The constant in the free vortex law relation for each case was chosen arbitrarily so that

---

[19] "A Study of the Flow of Water Around Bends in Pipes," by J. M. Robertson, thesis for the degree of Bachelor of Science in Civil Engineering, College of Eng., Univ. of Illinois, Urbana, 1938.

$$w\,r = k = V\,r_c \qquad (31)$$

The general trends predicted by the free vortex law appear to be verified, but the actual distributions are modified greatly due to the effect of the velocity distribution in the approach pipe. In general, it appears that the sharper the curvature the greater will be the maximum velocity and the nearer the filament of maximum velocity will be to the inside of the bend.

Another case in which the curvature plays an important rôle is in connection with the superelevation of the water surface as the water flows around the bend. As the author states, this is due to the centrifugal action of the water rounding the bend. In a closed conduit this phenomenon manifests itself in a difference in pressure between the inner and outer curves of the bend or elbow. As has been shown by Professor Lansford[20] and others,[16,17,21] this difference in pressure allows the bend to be used as a flow meter. Starting with the basic force relation given by Eq. 1, the expression for this difference in pressure Δh, in feet of fluid flowing, in terms of the velocity head $\frac{V^2}{2\,g}$ in the pipe line (V is the average velocity) becomes,[6,20]

$$\Delta h = \frac{2\,b}{r_c}\,\frac{V^2}{2\,g} = K\,\frac{V^2}{2\,g} \qquad (32)$$

For a circular pipe, b is the diameter; for a rectangular channel, it is the breadth. In the study on the flow in 4-in. pipe bends previously referred to,[19] the writer verified the relation between the coefficient K in Eq. 32 and the bend curvature. The results are shown in Fig. 18. The pressure differences all were measured across a diameter 45° around the bend in the plane of the bend. The values shown for K were found to apply for velocities from 2.5 to 9.5 ft per sec. Below 2.5 ft per sec the coefficient increased somewhat, probably due to a Reynolds number effect such as is observed for other differential head meters. Also shown on this plot are data taken by Herbert Addison[21] for 4-in. and 8-in. circular cast-iron pipe bends, by Mr. Yarnell[18] for a circular 6-in. celluloid bend, by Professor Lansford[20] for various sizes of bends up to 24 in., and by Mr. Addison[21] for two cast-iron and one brass, square (b = 6 cm), closed channel bends. Considering the fact that all of these elbows and bends may not have had such ideal approach conditions as did the writer's, the agreement is excellent.

---

[20] "The Use of an Elbow in a Pipe Line for Determining the Rate of Flow in the Pipe," by W. M. Lansford, *Bulletin No. 289*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1936.

[21] "The Use of Pipe Bends as Flow Meters," by Herbert Addison, ***Engineering*** (London), March 4, 1938, pp. 227–229.

The effect thus shown of the curvature on the pressure difference between the inside and outside curves of a bend should apply equally well to the superelevation of the water surface around a bend in an open channel. From the data presented by Messrs. Yarnell and Woodward[17] on the flow around 180° open and closed rectangular channels, the writer has computed the coefficient K for two different open channel bends. The computations were made from plotted results given for conditions 45° around a 10-in. and a 5-in. wide channel. These data are plotted in Fig. 18, and the agreement with the pipe data is seen to be fair. Finally, the writer has estimated a point from data in Fig. 2(*a*) by means of considerable interpolation. From this figure at the start of the bend (Section 8) Δh is 0.25 in. and V is 0.898 ft per sec, and at 90° around the bend (Section 12) Δh is 0.13 in. and V is 0.832 ft per sec. Assuming the average of these values to be those which apply at a station 45° around the bend, a value of K equal to 1.36 is obtained. For this bend $\frac{r_c}{b}$ is 1.16 and K should be 1.7. In view of the assumptions made this is probably a fair check.

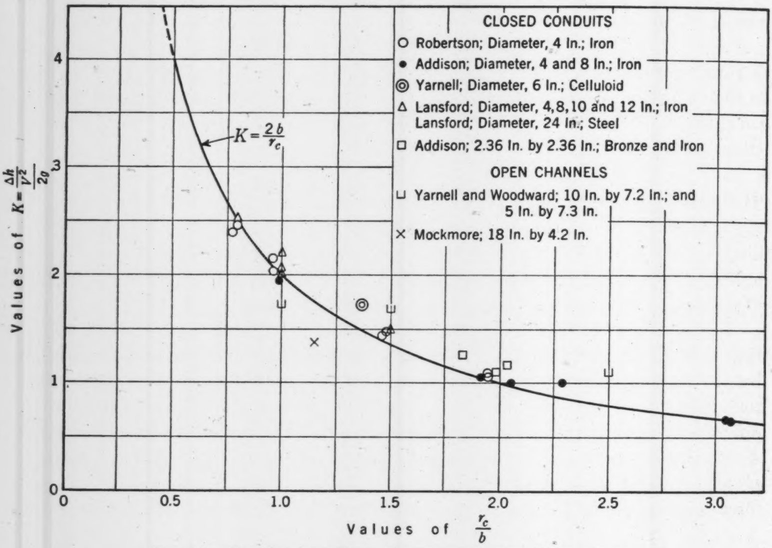


FIG. 18.—VARIATION IN BEND COEFFICIENT WITH CURVATURE

As to the other factors which affect the flow conditions around bends in open and closed channels, the writer has no data of his own to quote. However, he would like to mention the observations of others which may be pertinent to the discussion in connection with the effect of shape of the channel on flow conditions. Mr. Yarnell[18] notes that "the strength of the secondary currents in bends is greater in wide channels than narrow channels." In an excellent discussion[22] of the flow around curved channels and the occurrence of secondary currents, reference is made to some observations of A. Betz[23] on the flow in rectangular channels of different shapes. Fig. 19 indicates the nature of the flow observed in a square and in a narrow rectangular channel showing the regions in which an accumulation of retarded fluid tends to occur. In each case the plan view depicts conditions in the middle of the section and the sectional view conditions at the end of the bend. The explanation for the conditions in the square section are rather simple. As many observers have noted, because of the secondary motion the faster moving fluid in the central part is moving outward, pushing the fluid in the boundary layer at the outer wall to the top and bottom of the channel and thence back toward the inner wall of the bend where it accumulates.

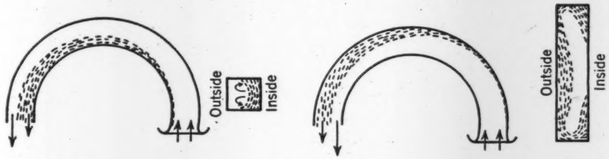


Fig. 19.—Flow Conditions in Square and Deep Rectangular Sectioned Bends

In a narrow, deep curved channel almost the reverse seems to occur as shown in the right side of Fig. 19. Here the secondary currents are confined to the top and bottom parts of the section where they tend to act just as in the square channel. The middle part of the section is too far from the top and bottom for the secondary currents to affect it greatly. The effect of the centrifugal forces on the fluid elements in this region tends to accentuate the retarded layer at the wall.[22] Although this type of action probably rarely occurs in open channels as they are more generally wide and shallow, the explanation given tends to show why (as Mr. Yarnell noted) the secondary currents are stronger in wide channels. Reference should also be made to the observations of V. M. Hegly[24,25] on the flow around a bend in a model canal of compound trapezoidal section. This canal consisted of a deep section and an adjacent shallow section of about one third the depth and about the same width. The velocity distributions are shown to be considerably different when the deep, instead of the shallow, region is against the concave bank.

Joe W. Johnson,[26] Assoc. M. Am. Soc. C. E., and M. A. Selim,[27] Jun. Am. Soc. C. E.—A mathematical analysis of the problem of flow around a bend has been presented in this paper. Some rather extensive and elaborate experimental measurements also are given. The velocity distribution included in the experimental data is sufficiently complete to permit the determination of the velocity-head correction factor α and the momentum factor β,[28] although these terms apparently have not been computed by the author.

---

[22] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford Press, Vol. 1, 1938, pp. 84–87.

[23] "On Turbulent Frictional Layers at Curved Walls," by A. Betz, Vorträge aus dem Gegiete der Aerodynamik und verwandter Gebiete, Aachen 1929, A. Gillis, L. Hopf, and Theodor von Kármán, editors, Berlin, Julius Springer, 1930, pp. 10–18.

[24] "Note sur l'Écoulement de l'Eau dans un canal à Profil Complexe," by V. M. Hegly, *Annales des Ponts et Chaussées*, Mémoires et Documents, Vol. 106, No. 5, 1936, pp. 445–528.

[25] "Flow in Earthen Canals of Compound Cross-Section," by V. M. Hegly, translated and abstracted by Chilton A. Wright, Abridged Translations of Hydraulic Papers, *Proceedings*. Am. Soc. C. E., November, 1937, p. 36.

[26] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[27] Irrig. Dept., College of Eng., Fouad First Univ., Giza, Egypt; formerly Instr. in Mech. Eng., Univ. of California, Berkeley, Calif.

From the velocity distributions given in Fig. 2(*a*), values of α and β have been computed and are summarized in Table 3(*a*). These data show the variation of α and β with the location of the various sections. Although not conclusive, the values of the coefficients appear to be highest at the entrance end of the bends and to be least at the exit end.

TABLE 3.—VALUES OF α AND β

Item	(*a*) VARIOUS CROSS SECTIONS						(*b*) SECTION 12		
	Station 8	Station 12	Station 16	Station 20	Station 24	Station 28	Fig. 2(*b*)	Fig. 2(*a*)	Fig. 16
Stage			0.406				0.030	0.406	0.496
Q			0.449 (see Fig. 2(*a*))				....	0.449	0.449
α	1.12	1.13	1.05	1.62	1.37	1.15	1.46	1.13	1.31
β	1.05	1.11	1.02	1.25	1.14	1.04	1.16	1.11	1.14

Another item of interest that could be obtained from the various velocity distributions is a comparison between the values of α and β for a particular section at various stages and discharges. Data from the velocity distributions shown in Figs. 2(*a*), 2(*b*), and 16 for Section 12 are summarized in Table 3(*b*). Unfortunately, the data for only three stages are not enough to warrant any conclusions.

Values of the α and β coefficients for all the velocity distributions observed by the author might show an interesting trend of the data.

TABLE 4.—VALUES OF α FOR A STRAIGHT FLUME

Slope	DEPTH OF FLOW, IN FEET					
	0.8	0.6	0.4	0.3	0.2	0.1
0.0002	1.037	1.068	1.060	....	1.068	....
0.00045	....	1.033	....	....	....	....
0.0005	....	....	1.040	....	1.052	1.045
0.00010	....	....	1.048	1.056	1.063	1.048
0.00015	....	....	1.054	1.098	1.061	1.089
0.00020	....	....	....	1.057	1.071	1.072
0.00040	....	....	....	....	1.057	1.063

Of interest in this latter respect are the values of α observed in a straight flume at the U. S. Waterways Experiment Station.[29] Observation was by pitot tube on the velocity distribution at a section near the center of a flume 48 ft long and 2.31 ft wide, with various depths and slopes. In all observations the water-surface slope was parallel to the bed slope. The values of α for various runs are summarized in Table 4.

Examination of Table 4 shows that, for a particular slope, α tends to increase slightly with decreasing depth. Actually, the range in variation is so slight that for practical purposes the average of the values for all depths and slopes represents a reasonable value of α for a straight flume. Thus, in a reach of a channel with no bends or obstructions the values of α, for most practical purposes, can be assumed equal to 1.

---

[28] "Velocity-Head Correction for Hydraulic Flow," by M. P. O'Brien and J. W. Johnson, *Engineering News-Record*, August 16, 1934, p. 214.

[29] "The Effect of Geometric and Slope Distortion on the Distribution of Energy and Tractive Force in Stream Cross-Sections," *Technical Memorandum No. 32–2*, U. S. Waterways Experiment Station, Vicksburg, Miss., 1936.

I. Nelidov,[30] M. Am. Soc. C. E.—So far as it is known to the writer, there has been no clear exposition of the mechanics of flow in curved channels. Professor Mockmore's accomplishment therefore is that he produced an analysis of the components of the velocities on three orthogonal planes within the body of the current.

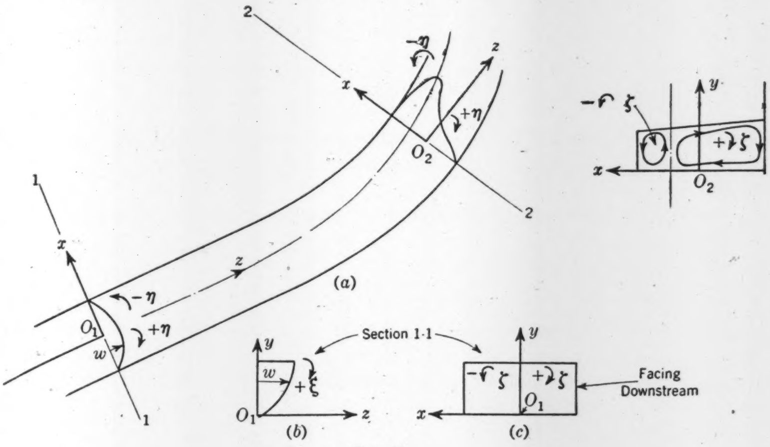


Fig. 20

The writer wishes to illustrate this problem from the standpoint of mechanical analogy to gyroscopic motion. From the theory of gyroscopic[31] motion, if a body is acted on by two orthogonal couples, there will exist, as a result of this action, a third couple orthogonal to the first two couples. Consider a straight part of a channel as shown in Fig. 20(*a*), Section 1–1. The approximate velocity distribution in plan and vertical cross section is known and shown in Figs. 20(*b*) and 20(*c*). This distribution of velocities results in the couples η and ζ. Since these couples have a negative and a positive sign, the water will flow so that, in a straight reach of channel, there will be two spirals, one clockwise and the other counterclockwise.

It may be noted here that M. Möller[32] quotes experiments in the Ruhr River which indicated the reverse direction of spirals from the one in Fig. 20(*c*). However, he maintains that two spirals will exist in a straight reach of channel.

In the curved reach of channel, the relations will be as indicated in Fig. 20(*d*), which is in general agreement with Fig. 5(*b*). If there are reversals in distribution of the velocity *w* either in plan or in a vertical section, there will be, as a result of this condition, some additional couples which, if of opposite sign, may create "dead" water. Therefore, before the direction of spiral rotation is established, a rather complete picture of the distribution of velocity w should be obtained. Considering a reversal of velocities w near the point b in the flow indicated in Section 2–2 in Fig. 20 near the surface (as shown in Fig. 5(c)) and no reversal at the bottom, the rotations as viewed facing downstream will be as shown in Figs. 21(a) and 21(b).

---

[30] Senior Engr., U. S. Engr. Dept., Sacramento, Calif.

[31] "Engineering Mechanics," by S. Timoshenko and D. H. Young, 1937 Ed., Pt. II, Dynamics, p. 255.

[32] "Hydraulic Laboratory Practice," by J. R. Freeman, article by M. Möller, 1929 Ed., p. 70.

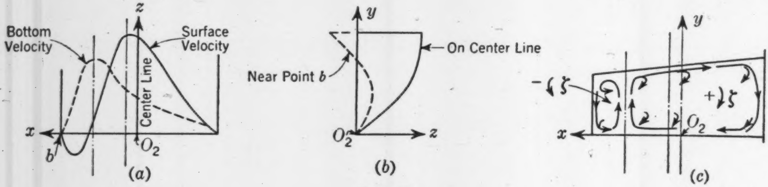


FIG. 21

Professor Mockmore has presented a very clear picture of mechanics of spiral flow.

S. M. WOODWARD,[33] HON. M. AM. SOC. C. E., J. W. HOWE,[34] M. AM. SOC. C. E., AND C. J. POSEY,[35] ASSOC. M. AM. SOC. C. E.—The mechanism of flow around bends in open channels is extremely complicated, and the resultant difficulties of experimentation and observation have kept the number of published investigations small in spite of the pressing practical need for information. This paper contains a lucid description of the spiral motion and several other interesting phenomena, as observed in an experimental channel of adequate dimensions. Use of a simple rectangular channel permits comparison with the results of mathematical analyses based on different assumptions for the distribution of three components of velocity throughout the cross section.

According to the author, the fundamentals of flow around bends in open channels are best studied in the laboratory, rather than in natural streams, because of the irregularity and instability of the stream bed. That the first of these difficulties is not insurmountable has been shown by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield,[36] who made a detailed study of a full-sized river bend, and who observed several of the interesting phenomena discovered by the author in his laboratory investigation. Their report probably had much to do with convincing the skeptics referred to by the author.[8]

The most important advantages of the laboratory approach come from the comparative ease with which the main variables can be changed. Thus, otherwise identical runs can be made with different depth-width ratios, with different central angles, and with different ratios of radius of bend to width of channel. Although such a testing program would be lengthy and expensive even in the laboratory, it might yield answers to some of the important unsolved problems in this field.

---

[33] Cons. Planning Engr., TVA, Knoxville, Tenn.

[34] Prof. and Head, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

[35] Res. Engr., Iowa Inst. of Hydraulic Research; Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa, Iowa City, Iowa.

[36] "Flow Around a River Bend Investigated," by F. L. Blue, Jr., J. K. Herbert, and R. L. Lancefield, *Civil Engineering*, May, 1934, p. 258.

The author's data show that in the bend tested the spiral was strong enough for bottom velocities to carry particles across the center line of the channel at an angle of nearly 45°. How does the "strength" of the spiral vary around the curve? Does it persist in the tangent downstream? How does its strength vary with variation in the dimensions of the bend and of the cross section? What is a satisfactory measure of the strength of the spiral motion?

The author's mathematical analyses both apparently assume that the spiral flow comes suddenly into existence at the point of tangency and remains constant around the bend. Replacement of this assumption with a more reasonable one would add considerably to the complications of the mathematical treatment, and yet would not permit a complete solution to be approached much more closely, for other factors, such as the vertical velocity distribution, still remain to be brought into agreement with observations.

Should the velocity distribution at the point of tangency be assumed to be the normal velocity distribution of the stream in a long, straight portion of its channel? Consideration of the water-surface profile through the beginning of the curve indicates that it is not normal—a deduction[37] made some years ago which is verified by the author's tests.

A question of great practical importance is that of the energy loss due to bends. A. A. Humphreys, Hon. M. Am. Soc. C. E., and Henry L. Abbot give the following formula, based on measurements on the lower Mississippi River:[38]

$$h = \frac{V^2 \sin^2 a}{134} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which h is the loss of head in feet, V is the velocity in feet per second, and a is a complicated variable, with only slight claim to rationality, introduced by L. G. Dubuat. It depends upon the radius, central angle, and width of the stream. Its computation is explained in detail by J. F. D'Aubuisson.[39]

An experimental study of a bend of the Iowa River, made in 1913–1914 by B. C. Boer and R. H. Urick, at Iowa University, Iowa City, showed that

$$\text{Loss of head} = \frac{0.0256\, V^2 \times \text{width of stream}}{\text{Geographic radius of curvature}} \quad \ldots\ldots\ldots (34)$$

On the basis of tests of the Tiger Creek flume, F. C. Scobey, M. Am. Soc. C. E., suggests that the value of Kutter's n be increased 0.001 for each 20° of curvature in 100 ft of flume.[40]

C. H. Yen found the head loss in a small 90° bend, rectangular in cross section and having an 11-in. width and 5-ft radius, to be $0.38 \frac{V^2}{2g}$.[41] His measurements of secondary currents indicated that the velocity of the spiral flow was much higher at the bottom of the cross section than elsewhere. Modifications of the geometric proportions of the cross section resulted in large changes in the head loss.

---

[37] "Hydraulics of the Miami Flood Control Project," by S. M. Woodward, *Technical Reports*, Miami Conservancy Dist., Dayton, Ohio, Pt. VII, 1920, p. 266.

[38] "Report upon the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, Lippincott, Philadelphia, Pa., 1861, p. 315.

[39] "Hydraulics," by J. F. D'Aubuisson, translated by Joseph Bennett, Little, Brown & Co., Boston, Mass., 1852, p. 215.

[40] "The Flow of Water in Flumes," by Fred C. Scobey, *Technical Bulletin No. 393*, U. S. D. A., December, 1933.

[41] "Effects of Channel Shape on Losses in a Canal Bend," by C. H. Yen and J. W. Howe, *Civil Engineering*, January, 1942, p. 28.

Many features of flow around bends in open channels remain to be understood. Both laboratory investigations and measurements on natural streams will be necessary before the needed fundamental relationships have been formulated and evaluated.

C. A. Mockmore,[42] M. Am. Soc. C. E.—Very properly, Professor Robertson records that well-known textbooks on hydraulics present data and make statements inferring that the filament of maximum velocity is always at the outside of the bend and that, in fact, "the filament of maximum velocity is initially, and, for most of the way around the bend, near the inside of the bend." The maximum velocity filament is at the inside of the bend initially, but is not there near the end of the bend, either in an open or a closed channel. In addition, Professor Robertson presents the results of some very fine experiments which he conducted at the University of Illinois in Urbana with closed pipe bends. Fig. 18 is especially interesting and valuable.

An attempt was made by the writer to evaluate α and β as presented by Messrs. Johnson and Selim. The reason for leaving this study out of the paper is given in their comment—"the data for only three stages are not enough to warrant any conclusions."

Mr. Nelidov applies the theory of gyroscopic motion to flow of a fluid around a bend. The theory works well with solid bodies, but it is difficult to form a clear picture of its application to a stream of water flowing around a bend where there are myriads of exceedingly small particles, molecular in size and not rigidly affixed to one another.

Several interesting questions on the "strength" of the spiral in an open channel are formulated by Messrs. Woodward, Howe, and Posey. It is a delight to contemplate the questions which these discussers would have asked had they been present when the apparatus shown in Fig. 1 was in operation. All the answers must await further researches.

There seems to be a slight misconception of the assumptions for the mathematical analyses. The writer does not assume "that the spiral flow comes suddenly into existence at the point of tangency and remains constant around the bend," but that a particular flow condition exists at a cross section of the bend of the stream, such as that in Fig. 7. Then the curves in Fig. 9 give a graphical representation of the paths various particles of water would take if they were permitted to continue that behavior in a tangent to the bend of the channel. At any other downstream section an entirely new set of flow conditions may obtain.

The writer has read the article by Messrs. Blue, Herbert, and Lancefield,[36] describing a detailed study of a full-sized river bend, and appreciates not only the value of the work but also the hard labor and the occasional hazards incident to collecting such data, since on three different occasions he was spilled out of a boat into a mountain stream (once at flood stage) while attempting to collect river flow data. The writer firmly believes that the fundamentals of stream flow can best be studied in the laboratory, and second best in the field.

---

[42] Prof. and Head, Dept. of Civ. Eng., Oregon State College, Corvallis, Ore.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2218

EFFECT OF TURBULENCE ON SEDIMENTATION

By William E. Dobbins,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. A. M. Gaudin, John S. McNown, H. A. Einstein, Thomas R. Camp, Garbis H. Keulegan, C. W. Thornthwaite, A. A. Kalinske, E. R. Van Driest, R. Schuhmann, Jr., H. B. Southwood, L. Standish Hall, and William E. Dobbins.

Synopsis

The basic theory of turbulent flow has been applied successfully to explain the observed vertical distribution of suspended matter in streams in which the suspended load is in equilibrium with the bed material. This paper presents the results of studies made in an effort to extend the theory to explain the concentration changes in a stream or settling basin under nonequilibrium conditions. The general differential equation expressing the concentration changes during turbulent sedimentation in an infinitely wide stream is derived, and the complete solution for a certain simplified case is presented. Evidence of the validity of this solution is provided by the results of experiments which are described herein.

Introduction

An attempt to set up a mathematical theory of sedimentation should take into account both the character of the sediment and the character of the fluid motion.

The sediment characteristics to be considered are not only the settling velocities of the individual particles but also the question of whether or not they retain their individualities as they settle. When two particles of a nonflocculent material come together the only effect is the momentary mutual interference; if two particles of a flocculent material come together they may lose their individual identities and become one. Thus, in a suspension of nonflocculent material, the sediment characteristics are constant, whereas in a suspension of flocculent material they are continuously changing.

The characteristics of the fluid motion under consideration are not only the motion of the fluid as a whole but also the momentary fluctuations from the average motion at every point in the fluid mass. To take into account the mo-

---

Note.—Published in February, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Engr., Industrial Div., Stone and Webster Eng. Corp., Boston, Mass.

tion of the fluid as a whole, the boundaries of the flow, the average point velocities, and the variations of these with time should be studied. Besides these factors, all secondary motions, such as convection or displacement currents and the fluctuations which are characteristic of turbulent flow, must be considered if the fluid motion is to be completely defined.

A complication arises through the fact that there is not complete independence between the sediment and the fluid flow characteristics. In the case of a flocculent material, the rate at which and the extent to which coagulation takes place are influenced by the nature of the flow. Any mixing of the fluid will increase the chances of contact between the individual particles, and hence the speed of coagulation. On the other hand, the shear stresses set up in the particles by the velocity gradients tend to limit the size to which the particles can grow.

In a horizontal flow tank, any upward flow caused by a secondary motion at some point must be compensated by a downward flow at the same rate at some other point. This is obviously necessary if the fluid mass is to remain continuous. In a suspension of particles which are settling out, the suspended matter tends to become more concentrated toward the bottom than above, although the suspension may be clearing throughout the entire depth. An upward flow, then, carries fluid from a region in which the concentration of suspended matter is high into one in which it is relatively low. Likewise, a downward current carries fluid with a low concentration into a region where the concentration is high. Thus, one effect of the secondary motion is to create an upward transport which tends to retard the rate at which the suspension clears. On the other hand, the secondary motion might increase the rate of coagulation and there also tend to increase the rate of clearance. It is clear, then, that whether or not a suspension of flocculent material will settle more rapidly in a turbulent fluid than in a quiescent fluid will depend upon the relative magnitudes of the effects of the turbulence in increasing the rate of coagulation and in transporting the material upward. In a suspension of discrete particles, the upward transport is the only effect of turbulence and hence the rate of clearance is retarded.

The complexities involved in the analysis of the secondary motions themselves combined with those involved in the analysis of the coagulation phenomenon make the analysis of the sedimentation problem very difficult. However, an understanding of the separate effects of secondary motions and of coagulation on sedimentation would be helpful in understanding the entire phenomenon.

One of the earliest and most widely known attempts to set up a mathematical theory for settling in a sedimentation basin was made by the late Allen Hazen, M. Am. Soc. C. E. (1).[2] He distinguished between quiescent and "turbulent" flow and restricted his analysis to suspensions of discrete particles having a single settling velocity. The principal contribution which he made was to demonstrate that the extent of the removal of discrete particles in a quiescent basin is independent of the depth of the basin. His analysis of sedimentation in turbulent basins is incorrect because it is based on incorrect assumptions.

---

[2] **Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix 1).**

The case of sedimentation of discrete particles of varying settling velocities in a quiescent basin has been presented by T. R. Camp, M. Am. Soc. C. E. (2).

The study described herein concerns the case of the sedimentation of discrete particles of uniform size in a stream in which turbulence is fully developed. By "fully developed turbulence" is meant a condition such that, although the velocity at every point is in a continuous state of flux, there is a statistical average velocity that remains constant. Modern developments in fluid mechanics have led to an understanding of the phenomenon of turbulence which has enabled a rational attack to be made on the sedimentation problem.

BASIC THEORY OF SEDIMENT SUSPENSION BY TURBULENCE

Notation.—The letter symbols introduced in this paper are defined where they first appear and are assembled for convenience of reference in Appendix 2.

The Principle of Turbulent Transport.—In the modern theory of turbulence it has been shown that the intensity of shear τ at a point in a moving fluid is

$$\tau = \rho |\bar{v}'_y| \, l \, \frac{\partial \bar{v}_x}{\partial y} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which ρ is the fluid density; l is the Prandtl (3) "mixing length"; $|\bar{v}'_y|$ is the absolute magnitude without regard to sign of the temporal mean value of the fluctuation velocity in the y-direction; and $\frac{\partial \bar{v}_x}{\partial y}$ is the temporal mean velocity gradient. Eq. 1 expresses the fact that the rate of transport of momentum per unit volume across any stream line is proportional to the gradient of the momentum per unit volume in the direction of the transfer. Theodor von Kármán, M. Am. Soc. C. E. (4), showed that the rate of transport of any local characteristic of the fluid, such as its heat content, salinity, or silt content, can be expressed by an equation similar to Eq. 1. If c represents the magnitude per unit volume of such a characteristic and $|\bar{v}'_s|$ is the mean numerical value of the velocity fluctuations of the fluid in any direction s, the rate of transport of the local characteristic in the direction of s will be given by

$$-\beta |\bar{v}'_s| \, l \, \frac{\partial c}{\partial s} = -\epsilon \frac{\partial c}{\partial s} \dots\dots\dots\dots\dots\dots\dots (2)$$

the minus sign indicating that the net transport is in the direction of decreasing c, and β being a factor whose value depends upon the characteristic being considered. The quantity ϵ is called the exchange or mixing coefficient. If c represents the momentum per unit volume and s is taken in the direction normal to the flow, the value of β is unity.

If c represents the magnitude or intensity of some inherent characteristic of the fluid, such as its heat content or color, it seems likely that the value of β would also be unity, since the characteristic would be transported by the transport of the fluid itself. This has been confirmed by A. A. Kalinske, Assoc. M. Am. Soc. C. E., and J. M. Robertson, Jun. Am. Soc. C. E. (5), in studying the diffusion of droplets of dye which had the same density as the water in which the diffusion took place.

In case c is the concentration of sediment in suspension, it is to be expected that the value of β might differ somewhat from unity, because the presence of the sediment increases the density of the fluid as a whole and thus alters the pattern of the turbulence, and also because the mixing of the suspended particles might not be the same as for the fluid masses. In studying suspensions of sand grains in water, Hunter Rouse, M. Am. Soc. C. E. (6), found that the value of β was the same for all suspensions in which the grain sizes were less than $\frac{1}{16}$ mm in diameter but that it changed for suspensions of larger grains. He was dealing with concentrations up to about 3% by weight. Since he found no systematic deviations in the value of β for material of any one size when the concentrations were varied, he attributed the variations with grain size to the fact that the mixing of the larger grains was not the same as that of the fluid itself. He had no means of determining the actual values of β, but simply the degrees of variation. Experiments by V. A. Vanoni, Assoc. M. Am. Soc. C. E. (7), indicated a value of β close to unity when dealing with relatively low concentrations of small grains. It is probable that the value of β can be taken as unity in most practical cases.

The Theory of Sediment Suspension.—In a turbulent stream the eddies dig into the bed and carry material into suspension. This action is opposed by the settling of the particles due to their own weight. If steady conditions prevail, there will be an equilibrium between the rate at which sediment is raised from the bed by the turbulence and the rate at which it settles on to the bed again. At every elevation above the bed, a similar equilibrium will result between the rate of upward transport due to the turbulence and the rate of fall due to the weight of the particles. Thus the concentration will remain constant at every elevation, although the particles of which the suspension is composed will be changing continuously.

If the concentration at any elevation, y, be denoted by c, and the settling velocity of the particles by w, the rate at which the material settles through a unit area at the elevation y is $w\,c$. If the state of equilibrium exists, this rate will be balanced by the rate of turbulent transport upward, and

$$w\,c = -\,\epsilon\,\frac{\partial c}{\partial y} \qquad (3)$$

This equation was first used by W. Schmidt (8) in studying the distribution of dust particles in air and was applied to the problem of silt suspension by Morrough P. O'Brien, M. Am. Soc. C. E. (9). Integration of Eq. 3 yields an expression for the concentration, c, in terms of the concentration, c_a, at some arbitrary elevation, a:

$$\log_e \frac{c}{c_a} = -\,w \int_a^y \frac{dy}{\epsilon} \qquad (4)$$

To evaluate the integral in Eq. 4 it is necessary to express ϵ as a function of y.

The general validity of Eq. 4 was established by Professor Rouse (6) in experiments in which a uniform intensity of turbulence was imposed on a suspension of sand in water by means of an agitating mechanism. Under such conditions, ϵ is independent of y and Eq. 4 becomes

$$c = c_a \, e^{-w\epsilon^{-1}(y-a)} \quad \text{.......................} \quad (5a)$$

in which e is the base of Naperian logarithms. If it is assumed that ϵ is constant down to the mean level of the top of the bed material, a can be assumed zero and Eq. 4 reduces to

$$c = A \, e^{-wy\epsilon^{-1}} \quad \text{.........................} \quad (5b)$$

in which A is the concentration at $y = 0$. In such a distribution, the total sediment, W, held in suspension per unit area between any two elevations, y_1 and y_2, is given by the expression

$$W = \int_{y_1}^{y_2} c \, dy = \frac{c_1 \, \epsilon}{w} \left(1 - \frac{c_2}{c_1} \right) \quad \text{..................} \quad (6)$$

If the values of the concentrations at any two elevations, y_1 and y_2, are known, the value of $\frac{w}{\epsilon}$ can be computed from the equation,

$$\frac{w}{\epsilon} = \frac{1}{y_2 - y_1} \log_e \frac{c_2}{c_1} \quad \text{.......................} \quad (7)$$

In an actual stream, ϵ varies with the distance from the bed and also from point to point across the section. If the flow is unsteady and nonuniform it will also vary from section to section and from time to time. If the discussion is limited to the case of steady uniform flow in the central region of a very wide stream, ϵ will be a function of y alone. In such a case, the value of the shear, τ, at any distance y from the bottom is given by the expression,

$$\tau = \tau_0 \left(1 - \frac{y}{h} \right) \quad \text{........................} \quad (8)$$

in which h is the total depth and τ_0 is the bottom shear. If this is combined with Eq. 1—

$$\epsilon = \beta \, \frac{\tau_0}{\rho} \frac{1 - \dfrac{y}{h}}{\dfrac{dV}{dy}} \quad \text{........................} \quad (9)$$

in which $\bar{v}_x$ has been replaced by V. It has been shown by G. H. Keulegan (10) and also by Mr. Vanoni (11) that the von Kármán universal logarithmic velocity law for pipes can be applied to the case of uniform two-dimensional open-channel flow, yielding the expression

$$V = \overline{V} + \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \left(1 + \log_e \frac{y}{h} \right) \quad \text{...............} \quad (10a)$$

from which

$$\frac{dV}{dy} = \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \frac{1}{y} \quad \text{.......................} \quad (10b)$$

Using this value for the velocity gradient, Eq. 9 becomes

$$\epsilon = \beta \kappa \sqrt{\frac{\tau_0}{\rho}} \left(1 - \frac{y}{h}\right) y = \beta \kappa \overline{V} \sqrt{\frac{f}{8}} \left(1 - \frac{y}{h}\right) y$$

$$= \beta \kappa \sqrt{g h S} \left(1 - \frac{y}{h}\right) y \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In these expressions, κ is the von Kármán universal constant having a value approximately equal to 0.40; $\overline{V}$ is the mean velocity of flow; f is the Darcy friction factor for the bed; g is the acceleration of gravity; and S is the slope of the energy gradient.

Eq. 11 indicates a value of ϵ which is a maximum at $y = \frac{h}{2}$ and which approaches zero as y approaches both zero and h. This general trend has been verified by Messrs. Kalinske and Robertson (5), who made measurements of ϵ by observing the spread of droplets of dye which were injected into a rectangular channel at various points in the cross section. If the value of ϵ, as given by Eq. 11, is used in Eq. 4, the expression for the distribution of sediment becomes

$$\frac{c}{c_a} = \left(\frac{\frac{h}{y} - 1}{\frac{h}{a} - 1}\right)^z \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which

$$z = \frac{w}{\kappa \beta \sqrt{\frac{\tau_0}{\rho}}} = \frac{w}{\kappa \beta \overline{V} \sqrt{\frac{f}{8}}} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Experiments by Mr. Vanoni (7) have indicated close agreement between the actual distribution in a turbulent channel and the distribution as expressed by Eqs. 12 and 13 when the values of κ and β were taken as 0.40 and unity, respectively. He found that the agreement was close throughout the major range of the depth.

It would seem that the value of ϵ cannot be absolutely zero at either the bed or the surface because a value of zero at the surface would indicate that the surface is absolutely smooth, and a value of zero at the bed would indicate no pickup. The experimental measurements of Messrs. Kalinske and Vanoni indicate, however, that ϵ follows the functional trend of Eq. 11 as close to the bed and the surface as measurements are practically possible.

The foregoing expressions refer the value of c at any elevation, y, to its value at some arbitrary elevation, a. Thus, these expressions determine the shape of the concentration curves but give no indication of the absolute magnitudes. They also apply only to the case of sediments made up of particles having a single settling velocity.

The absolute magnitudes of the concentrations should depend upon the bottom conditions. In experiments described subsequently in this paper a uniform mixing was imposed upon a suspension of particles of a fairly uniform size in a cylindrical container. The resulting equilibrium concentration for a given depth and a given degree of mixing depended only upon the distance of the mixing apparatus from the bottom, provided that the bottom was completely covered with the sediment when the equilibrium condition was reached.

Although many variables determine the flow pattern in a stream, a given combination of these variables will result in a definite flow condition. Hence, it seems logical to assume that there will be a definite rate of pickup of a given type of sediment. At the time of this writing the analysis of the problem has not progressed to the point where the rate of pickup can be expressed as a function of the variables involved.

DEVELOPMENT OF THE THEORY OF SEDIMENTATION FOR TURBULENT CONDITIONS

Statement of the Problem.—The equilibrium condition described in the previous chapter is a condition which is approached but is never fully reached. The silt content of a natural stream at some section might be more or less than the ultimate transport capacity because of a sudden change in the cross section immediately upstream. If steady uniform flow prevails downstream, sediment will be deposited or picked up as the new equilibrium condition is approached. In a settling tank the content of suspended matter at the inlet end is much greater than the carrying capacity, and so the material settles out. It is with the changes in concentration from section to section that this study is mainly concerned. The discussion is confined to the two-dimensional case corresponding to the conditions of steady uniform flow in the central region of a wide channel.

Derivation of the Differential Equation for Sedimentation in a Turbulent Stream.—With reference to Fig. 1, the x-axis is taken at the bottom of the channel in the direction of the flow. For the sake of generality, the concentration is assumed to increase with both x and y and the velocity with y. Consider the point m (x,y) and the element mm′nn′ having the dimensions dx and dy in the x,y-plane and unity in the normal direction, and let c be the concentration of suspended matter at any time t at the point m.

If the amount of suspended matter carried out of the element across the face nn′ by turbulence and settling and across the face m′n′ by turbulence and the flowing stream is subtracted from the weight carried into the element across the opposite faces, it may be shown that

$$\frac{dW}{dx\,dy\,dt} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + \frac{\partial \epsilon_y}{\partial y}\left(\frac{\partial c}{\partial y} + \frac{\partial^2 c}{\partial y^2}dy + \frac{\partial^2 c}{\partial x\,\partial y}\frac{dx}{2}\right) + \epsilon_x \frac{\partial^2 c}{\partial x^2} + \frac{\partial \epsilon_x}{\partial y}\frac{\partial^2 c}{\partial x^2}\frac{dy}{2} + w\frac{\partial c}{\partial y} - \left(V + \frac{\partial V}{\partial y}\frac{dy}{2}\right)\frac{\partial c}{\partial x} \;\ldots\ldots\; (14a)$$

in which dW is the net increased amount of suspended matter in the element in time dt.

For the steady state, the concentration at a fixed point remains unchanged with time. Therefore, $\dfrac{dW}{dx\,dy\,dt} = 0$. The other terms which contain the factors dy and dx also become zero as the element $dx\,dy$ approaches a point. Then the final equation becomes

$$V\frac{\partial c}{\partial x} = \epsilon_y\frac{\partial^2 c}{\partial y^2} + \left(w + \frac{\partial \epsilon_y}{\partial y}\right)\frac{\partial c}{\partial y} + \epsilon_x\frac{\partial^2 c}{\partial x^2} \qquad (14b)$$

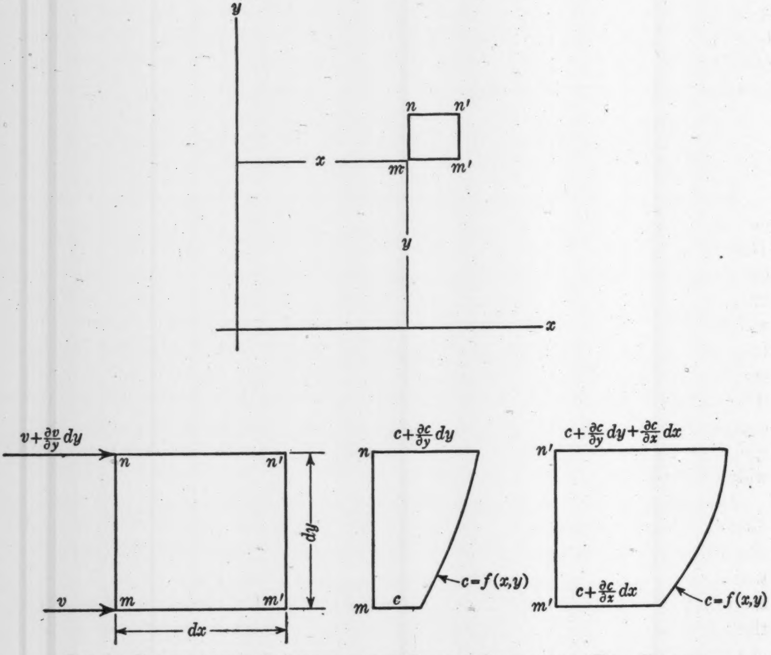

y
n
n'
x
m
m'
y
x
$v+\frac{\partial v}{\partial y}dy$
n
n'
v
m
m'
dy
dx
$c+\frac{\partial c}{\partial y}dy$
n
m
c
$c=f(x,y)$
$c+\frac{\partial c}{\partial y}dy+\frac{\partial c}{\partial x}dx$
n'
m'
$c+\frac{\partial c}{\partial x}dx$
$c=f(x,y)$

FIG. 1.—CONCENTRATION GRADIENTS IN THE CENTRAL REGION OF A WIDE CHANNEL

Eq. 14b is the general differential equation for the concentration changes in the two-dimensional case. An equation similar to 14b, but which does not take into account the term $\frac{\partial \epsilon_y}{\partial y}\frac{\partial c}{\partial y}$, has been presented by Professor Kalinske (14). Before a solution for Eq. 14b can be found, the factors V, ϵ_y, and ϵ_x must be expressed as functions of y. Eq. 10a expresses the necessary relationship for V and Eq. 11 gives the proper function for ϵ_y. It has already been stated that the value of ϵ_x is probably equal to or at least proportional to ϵ_y.

Simplifications of the General Differential Equation.—No solution has been obtained for the general two-dimensional case as expressed in Eq. 14b. However, the equation can be greatly simplified by making certain assumptions. If the velocity distribution were assumed to follow a parabolic law instead of the logarithmic law expressed in Eq. 10*a*, the velocity equation could be written (see Fig. 2).

$$V = V_s - k\,(h - y)^2 \qquad \text{(15a)}$$

in which V_s is the velocity at the surface and k is the constant of the parabola. Then

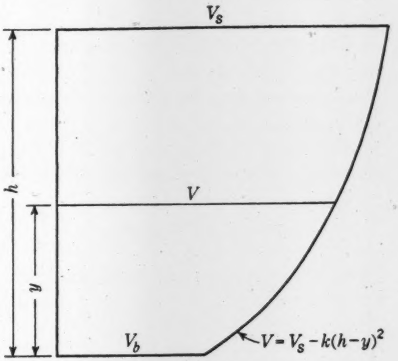


FIG. 2.—PARABOLIC VELOCITY DISTRIBUTION

$$\frac{dV}{dy} = 2\,k\,h\left(1 - \frac{y}{h}\right) \qquad \text{(15b)}$$

From Eq. 9, assuming β to be unity,

$$\epsilon_y = \frac{\tau_0}{\rho}\,\frac{1 - \dfrac{y}{h}}{\dfrac{dV}{dy}} \qquad \text{(16a)}$$

Combining Eqs. 15*b* and 16*a* gives

$$\epsilon_y = \frac{\tau_0}{2\,k\,h\,\rho} \qquad \text{(16b)}$$

which is a constant for any particular flow condition. If it is further assumed that $\epsilon_x \dfrac{\partial^2 c}{\partial x^2} = 0$ (that is, there is no diffusion of the suspended matter in the x-direction), the differential equation reduces to

$$[V_s - k\,(h - y)^2]\,\frac{\partial c}{\partial x} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \qquad \text{(17a)}$$

A further simplification is obtained if it is assumed that V is constant throughout the depth, $\epsilon_x \dfrac{\partial^2 c}{\partial x^2}$ is zero, and ϵ_y is constant. In this case, Eq. 17*a* becomes

$$V\,\frac{\partial c}{\partial x} = \epsilon\,\frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \qquad \text{(17b)}$$

For the case in which the body of the liquid has no motion as a whole but has a uniform turbulence imposed on it throughout, the independent variables become y and t, and the differential equation becomes

$$\frac{dW}{dx\,dy\,dt} = \frac{\partial c}{\partial t} = \epsilon\,\frac{\partial^2 c}{\partial y^2} + w\,\frac{\partial c}{\partial y} \qquad \text{(17c)}$$

Eqs. 17*b* and 17*c* are identical mathematically, because if V is constant, $V\dfrac{\partial c}{\partial x}$ is equivalent to $\dfrac{\partial c}{\partial t}$, in which t is equal to $\dfrac{x}{V}$.

It is to be realized that the simplifying assumptions involved in Eqs. 17 may introduce large errors when applied to the case of a natural stream. The assumption of a parabolic velocity distribution need not introduce large errors in the values of V but it does introduce considerable error in the value ϵ_y. The equation for this case was presented to show that the assumption of a constant value for ϵ_y is equivalent to assuming a parabolic velocity distribution. The assumption that the value of $\epsilon_x \dfrac{\partial^2 c}{\partial x^2}$ equals zero would not introduce a very large error if the concentration gradient in the x-direction were small compared to the gradient in the vertical direction. This would be the case when the concentrations do not differ greatly from the final equilibrium concentrations.

Boundary Conditions.—The solution for any practical case must satisfy not only the differential equation for that case but also the boundary conditions. The latter are four in number, two of which specify the conditions that prevail at the surface and at the bed, and two of which specify the original and the final distributions of the suspended matter. The rate of transport of sediment across any plane, $y = y_1$, is given by the expression

$$\text{Transportation rate} = \epsilon_y \left(\frac{\partial c}{\partial y}\right)_{y_1} + w\, c_{y_1} \qquad (18)$$

There is no transport of material across the surface, in any case, and so the first boundary condition which must be satisfied is

$$\text{(I)} \qquad \left(\frac{\partial c}{\partial y}\right)_h = -\frac{w}{\epsilon_y} c_h$$

Since c_h varies with x or t, the concentration gradient at the surface is not a constant.

The rate at which material is picked up from the bottom by the turbulence is $-\epsilon_y \left(\dfrac{\partial c}{\partial y}\right)_{y=0}$. The minus sign accounts for the fact that the pickup is positive when the concentration gradient is negative. The rate of pickup of a given type of sediment depends only upon the characteristics of the flow and so, for steady uniform flow, the pickup will be constant. At the equilibrium condition the rate of pickup equals the rate of deposit, $w\, c_0$, and so the second boundary condition is

$$\text{(II)} \qquad \epsilon_y \left(\frac{\partial c}{\partial y}\right)_{y=0} = -w\, c_{\substack{y=0\\t=\infty}} = -w\, A$$

The third boundary condition is simply that the concentration at time $t = 0$ is some function of y between the limits of zero and h:

$$\text{(III)} \qquad c = f(y) \quad \text{when} \quad 0 < y < h, \quad \text{and} \quad t = 0$$

The fourth boundary condition is

(IV)............The concentration distribution approaches the equilibrium distribution as t approaches infinity.

For a channel this is expressed by Eq. 12 and for the case of a uniform turbulence throughout the fluid by Eqs. 5.

Solution of the Differential Equation for the One-Dimensional Case.—Eq. 17*c*, and its equivalent Eq. 17*b*, are the only forms of the differential equation which have yielded to an analytical solution. The problem in this case is to find a solution to the differential equation, Eq. 17*c*, in which ϵ and w are constants, that satisfies the boundary conditions as defined by I, II, III, and IV.

A particular solution to the differential equation can be obtained by assuming that the solution is a product of two functions—one a function of y alone and the other a function of t alone. Then

$$c = Y(y) \times T(t) \tag{19}$$

By Eq. 17*c*,

$$\frac{T'}{T} = \frac{\epsilon\, Y'' + w\, Y'}{Y} \tag{20}$$

Since each side of this equation is a function of only one of the independent variables, it can hold only if each member is equal to some constant, say $-\,k^2$. Then

$$T' + k^2\, T = 0 \tag{21a}$$

and

$$Y'' + w\, Y' + k^2\, Y = 0 \tag{21b}$$

The solution for T is found directly as

$$T = C_1\, e^{-k^2 t} \tag{22}$$

in which C_1 is an arbitrary constant. The equation involving Y is a second-order linear differential equation which has the well-known solution

$$Y = e^{-0.5\, w y \epsilon^{-1}}\, (C_2 \cos \alpha\, y + C_3 \sin \alpha\, y) \tag{23}$$

in which C_2 and C_3 are arbitrary constants, the exponent of e is $-\,\frac{w\, y}{2\,\epsilon}$, and

$$\alpha^2 = \frac{k^2}{\epsilon} - \frac{w^2}{4\,\epsilon^2} \tag{24}$$

Then the equation,

$$c = e^{-k^2 t}\, e^{-0.5\, w y \epsilon^{-1}}\, (C_1 \cos \alpha\, y + C_2 \sin \alpha\, y) \tag{25}$$

in which the constants are redefined, satisfies the differential equation.

It can be shown easily that the boundary condition for $t = \infty$ also satisfies Eq. 17*c*, and so

$$c = A\, e^{-w y \epsilon^{-1}} + e^{-k^2 t}\, e^{-0.5\, w y \epsilon^{-1}}\, (C_1 \cos \alpha\, y + C_2 \sin \alpha\, y) \tag{26}$$

satisfies both the differential equation and the $t = \infty$ condition. In Eq. 26, A = the concentration at $y = 0$ and $t = \infty$.

To satisfy the boundary conditions for $y = 0$ and $y = h$ it is necessary to differentiate Eq. 26 with respect to y and to compare the result with conditions I and II. Performing the differentiation, there results:

$$\begin{aligned}\frac{\partial c}{\partial y} &= -\,\frac{w}{\epsilon}\, A\, e^{-w y \epsilon^{-1}} + e^{-k^2 t}\, e^{-0.5\, w y \epsilon^{-1}} \\ &\times \left[\sin \alpha\, y \left(-\, C_1\, \alpha - \frac{w}{2\,\epsilon}\, C_2\right) + \cos \alpha\, y \left(C_2\, \alpha - \frac{w}{2\,\epsilon}\, C_1\right)\right]\end{aligned} \tag{27a}$$

When $y = 0$, this reduces to

$$\frac{\partial c}{\partial y} = -\frac{w}{\epsilon} A + e^{-k^2 t}\left(C_2\,\alpha - \frac{w}{2\,\epsilon} C_1\right) \qquad (27b)$$

By comparing this with the boundary condition for $y = 0$ (Condition II) it is seen that

$$C_2\,\alpha - \frac{w}{2\,\epsilon} C_1 = 0 \qquad (28a)$$

and

$$C_2 = \frac{w}{2\,\epsilon\,\alpha} C_1 \qquad (28b)$$

Comparing Eq. 27*a* with Condition I in which the value of C from Eq. 26 has been inserted, it is seen that

$$-\frac{w}{\epsilon}(C_1 \cos \alpha\, h + C_2 \sin \alpha\, h) = \left(C_2\,\alpha - \frac{w}{2\,\epsilon} C_1\right)\cos \alpha\, h - \left(C_1\,\alpha + \frac{w}{2\,\epsilon} C_2\right)\sin \alpha\, h \qquad (29)$$

from which

$$\tan \alpha\, h = \frac{C_2\,\alpha + \dfrac{w\,C_1}{2\,\epsilon}}{C_1\,\alpha - \dfrac{w\,C_2}{2\,\epsilon}} \qquad (30)$$

By combining Eq. 30 with Eq. 28 and changing it into a more convenient form, the following is obtained:

$$\eta = 2 \cot h\,\alpha = \frac{h\,\alpha}{\dfrac{w\,h}{2\,\epsilon}} - \frac{\dfrac{w\,h}{2\,\epsilon}}{h\,\alpha} \qquad (31)$$

Then, a solution of the differential equation which satisfies all of the boundary conditions except Condition III ($t = 0$) is given by

$$c = A\,e^{-wy\epsilon^{-1}} + e^{-0.5\,wy\epsilon^{-1}}\,e^{-k^2 t}\,C_1\left(\cos \alpha\, y + \frac{w}{2\,\epsilon\,\alpha}\sin \alpha\, y\right) \qquad (32a)$$

in which α is defined by Eq. 31, and k^2 is defined by Eq. 24. Inspection of Eq. 31 will reveal that for any given values of w, h, and ϵ there is an infinite number of real positive roots for α, any one of which could be used in Eq. 32*a*. Then, because of the linear nature of the differential equation, the most general solution is

$$c = A\,e^{-wy\epsilon^{-1}} + e^{-0.5\,wy\epsilon^{-1}} \times \sum_{n=1}^{\infty} e^{-(\alpha^2{}_n\epsilon + 0.25\,w^2\epsilon^{-1})t}\,C_n\left(\cos \alpha_n\, y + \frac{w}{2\,\epsilon\,\alpha_n}\sin \alpha_n\, y\right) \qquad (32b)$$

To satisfy Condition III, C_n must now be chosen so that $c = f(y)$ when $t = 0$ and $0 < y < h$. Taking t as zero and c as $f(y)$ in Eq. 32*b*:

$$f(y) = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\, wy\epsilon^{-1}} \sum_{n=1}^{\infty} C_n \left(\cos \alpha_n y + \frac{w}{2\, \epsilon\, \alpha_n} \sin \alpha_n y \right) \ldots (33a)$$

and

$$[f(y) - A\, e^{-wy\epsilon^{-1}}]\, e^{+0.5\, wy\epsilon^{-1}} = \sum_{n=1}^{\infty} C_n\, Y_n \ldots\ldots\ldots\ldots (33b)$$

in which

$$Y_n = \cos \alpha_n y + \frac{w}{2\, \epsilon\, \alpha_n} \sin \alpha_n y \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

The expression for C_n can be derived in a manner presented by H. S. Carslaw (15). The complete derivation for C_n which is on file in the Engineering Societies Library (see "Acknowledgments") yields the following:

$$C_n = \frac{2\, \alpha^2_n}{\left(\alpha^2_n + \frac{w^2}{4\, \epsilon^2} \right) h + \frac{w}{\epsilon}} \int_0^h [f(y) - A\, e^{-wy\epsilon^{-1}}]\, e^{0.5\, wy\epsilon^{-1}}\, Y_n\, dy \ldots (35)$$

Then the final solution to the problem is

$$c = A\, e^{-wy\epsilon^{-1}} + e^{-0.5\, wy\epsilon^{-1}} \sum_{n=1}^{\infty} e^{-(\alpha^2_n \epsilon + 0.25\, w^2 \epsilon^{-1})\, t}\, C_n\, Y_n \ldots\ldots (36)$$

in which Y_n is defined by Eq. 34, C_n by Eq. 35, and $\alpha_1, \alpha_2, \cdots \alpha_n$ are the successive real positive roots of the transcendental formula (Eq. 31).

The roots of Eq. 31 are found most easily by the graphical method illustrated in Fig. 3. The values of $h\, \alpha$ are plotted as abscissas and values of the expressions on each side of the equation as ordinates. The values of $h\, \alpha$ corresponding to the points of intersection of the curves are the values to be used in Eq. 36. It is to be noted that the curves of $2 \cot h\, \alpha$ corresponding to the successive values of α are the same for all cases, but that the proper intersecting curve to use depends upon the value of the ratio $\frac{w\, h}{2\, \epsilon}$. Since ϵ is a product of a velocity and a length, the ratio $\frac{w\, h}{2\, \epsilon}$ is dimensionless and Fig. 3 is a dimensionless plot.

Solutions of Eq. 36 for Various Cases.—If the distribution of sediment at time $t = 0$ is represented by the equation

$$f(y) = c_0\, e^{-By} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

in which c_0 is the concentration at $y = 0$, $t = 0$, and B is in general not equal to $\frac{w}{\epsilon}$, the expression for C_n becomes

$$C_n = \frac{2\, \alpha^2_n}{\left(\alpha^2_n + \frac{w^2}{4\, \epsilon^2} \right) h + \frac{w}{\epsilon}} \int_0^h (c_0\, e^{-By} - A\, e^{-wy\epsilon^{-1}})\, e^{0.5\, wy\epsilon^{-1}}\, Y_n\, dy \ldots (38)$$

The integral in Eq. 38 may be evaluated so that

$$C_n = \frac{2\,\alpha^2{}_n}{\left(\alpha^2{}_n + \dfrac{w^2}{4\,\epsilon^2}\right) h + \dfrac{w}{\epsilon}}$$

$$\times \left\{ \frac{c_0 \left[B + \left(\dfrac{w}{\epsilon} - B \right) e^{(0.5\,w\,\epsilon^{-1} - B)h}\, H_n \right]}{\left(\dfrac{w}{2\,\epsilon} - B \right)^2 + \alpha^2{}_n} - \frac{A\,\dfrac{w}{\epsilon}}{\alpha^2{}_n + \dfrac{w^2}{4\,\epsilon^2}} \right\} \dots\dots (39a)$$

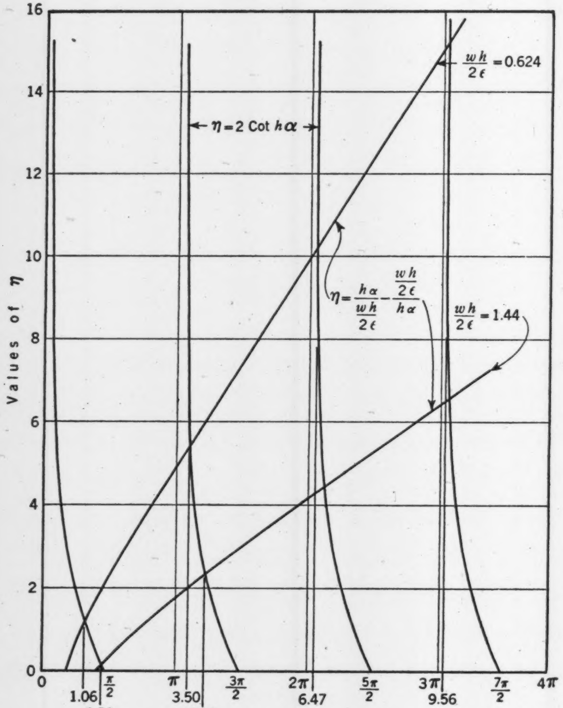


FIG. 3.—GRAPHICAL SOLUTION OF EQ. 31

in which

$$H_n = \cos \alpha_n h + \frac{w}{2\,\epsilon\,\alpha_n} \sin \alpha_n h \dots\dots\dots\dots\dots (39b)$$

With the aid of Eq. 31, it can be shown readily that the value of H_n is ± 1, the plus sign applying for odd values of n and the minus for even values of n.

If the value of B is zero, the concentration at time $t = 0$ is uniform and equal to c_0. The expression for C_n in this case is

$$C_n = \frac{2\,\alpha^2{}_n\,w\,(c_0\,e^{0.5\,wh\epsilon^{-1}}\,H_n - A)}{\left(\alpha^2{}_n\,\epsilon + \frac{w^2}{4\,\epsilon}\right)\left[\left(\alpha^2{}_n + \frac{w^2}{4\,\epsilon^2}\right)h + \frac{w}{\epsilon}\right]} \quad \ldots\ldots\ldots\ldots (40a)$$

If $B = \frac{w}{\epsilon}$, Eq. 39*a* reduces to

$$C_n = \frac{2\,\alpha^2{}_n\,w\,(c_0 - A)}{\left(\alpha^2{}_n\,\epsilon + \frac{w}{4\,\epsilon}\right)\left[\left(\alpha^2{}_n + \frac{w^2}{4\,\epsilon^2}\right)h + \frac{w}{\epsilon}\right]} \quad \ldots\ldots\ldots\ldots (40b)$$

and Eq. 36 becomes

$$c = A\,e^{-wy\epsilon^{-1}} + (c_0 - A)\,e^{-0.5\,wy\epsilon^{-1}}$$

$$\times \sum_{n=1}^{\infty} \frac{2\,\alpha^2{}_n\,w\,e^{-(\alpha^2{}_n\epsilon + 0.25\,w^2\epsilon^{-1})\,t}\,Y_n}{\left(\alpha^2{}_n\,\epsilon + \frac{w^2}{4\,\epsilon}\right)\left[\left(\alpha^2{}_n + \frac{w^2}{4\,\epsilon^2}\right)h + \frac{w}{\epsilon}\right]} \quad \ldots\ldots\ldots\ldots (41)$$

Eq. 41, with c_0 taken as zero, is the solution for the case of scour in which the original concentration is zero for all values of y. With A taken as zero, it is the solution for the case of settling out from the equilibrium concentrations to zero concentration at all depths. It is to be noted that the solution represented by the complete equation is the sum of these two solutions.

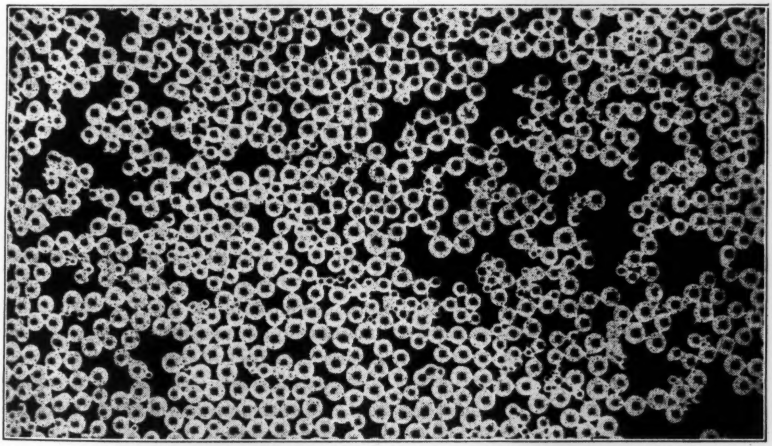


FIG. 4.—ENLARGED VIEW OF MOLDING POWDER PARTICLES

EXPERIMENTAL WORK

Scope and Purpose.—The experimental work was confined to a study of the one-dimensional case and had the primary object of verifying the equations developed in the previous section.

Description of the Suspensions Used in the Experiments.—The material used for the sediment in all of the experiments was the molding powder of a methyl methacrylate resin called "lucite." This material was obtained in the unplasticized and unlubricated state. The powder is made up of transparent spherical particles, most of them ranging in diameter from one tenth of a millimeter to one quarter of a millimeter. An average value of 1.194 g per cu cm was obtained for the density when measured by the pycnometer bottle method and by computation from the results of settling velocity measurements.

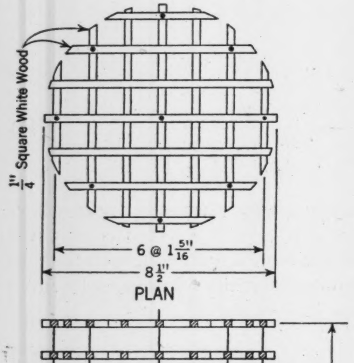


FIG. 5

Fig. 4 illustrates the material used in the experiments. This material, which was separated from the powder by sieves and a wind tunnel, was analyzed to determine the size and settling velocity distributions. The diameters of two hundred particles, which were chosen at random, were measured and the settling velocities in water at 10° C were then computed. In the computations the value of the drag coefficient was obtained from the equation by C. W. Oseen (16). The diameters varied from 0.138 mm to 0.216 mm and the corresponding settling velocities from 0.150 cm per sec to 0.344 cm per sec. The arithmetical average value of the settling velocity was 0.264 cm per sec. This value was used for w in all of the subsequent computations.

Suspensions of these particles in distilled water were used in all of the experiments. To prevent the particles from attaching themselves to air bubbles it was necessary to use a wetting agent. About 1 cu cm of an agent known as "Ultrawet No. 40" was used in each liter of the suspension. It was found that this amount had no measurable effect on the settling velocities of the particles.

Apparatus and Methods of Measurement.—The experimental setup was similar in its essentials to that used by Professor Rouse (6). In all of the runs the suspension was contained in a cylinder 9 in. in diameter by about 20 in. high. The mixing was accomplished by the lattice structure shown in Fig. 5. This was oscillated vertically in simple harmonic motion, thus producing an essentially uniform degree of mixing throughout the fluid. The cylinder was set in a thermostatically controlled water bath by which the temperature of the water in the cylinder was maintained at 10° ± 0.2° C.

The concentrations were measured by syphoning off samples of the suspension and obtaining the dry weight of the particles. In the sampling operation an amount equal to two or three times the volume of the siphon was wasted before the sample to be measured was collected. Each sample was collected in a 15-cc graduated centrifuge tube. When the lucite particles settled to the bottom of the tube they were picked up by an eye dropper and deposited in a small weighing dish which was then dried and weighed on an analytical balance.

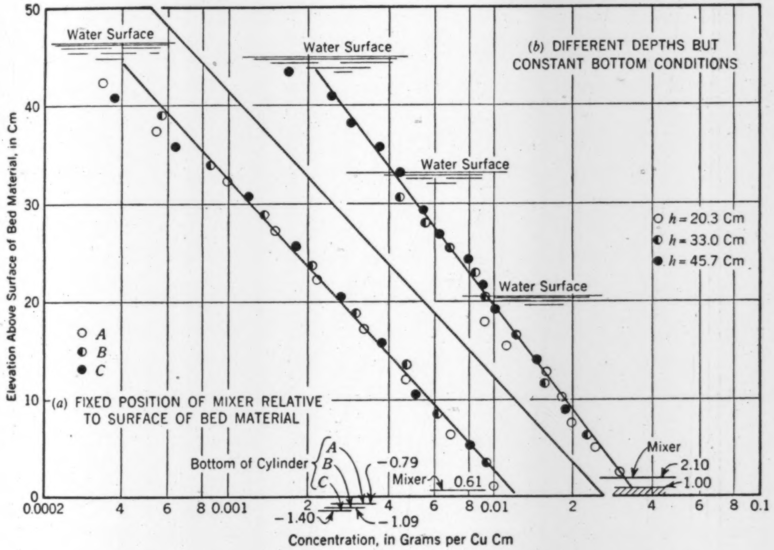


FIG. 6.—EQUILIBRIUM CONCENTRATIONS AT A MIXER SPEED OF 1.65 RPS

Procedure.—Measurements were made of the equilibrium concentrations with various total amounts of the powder, various mixing speeds, and various positions of the mixer with respect to the bottom of the cylinder. All of the results obtained were in agreement with the equilibrium distribution theory as expressed in Eq. 5*a*. Ample evidence of the validity of Eq. 5*a* has already been presented by Professor Rouse (6) and so all of the results will not be presented herein. Typical results are shown in Figs. 6 and 8. In each case the computed average thickness of the bottom deposit has been indicated after the equilibrium condition had been reached. The quantity of material on the bottom was determined as the difference between the volume in suspension and the total known amount used in the test. The volume in suspension was computed by Eqs. 6 and 7, the values of the necessary factors being taken directly from the linear plot. In these computations, the ratio of the dry weight to the volume occupied under water was taken as 1 g per 1.40 cu cm. This factor had been determined by measuring the volumes occupied by various known weights of material when settled out under water in a graduate.

Two runs were made for the purpose of checking the theory developed in the previous section. The first one was confined to the case in which the distribution of suspended matter at time $t = 0$ is defined by Eq. 5*b* and in which the pickup is zero. The solution for this case is given by Eq. 41 when A is taken as zero.

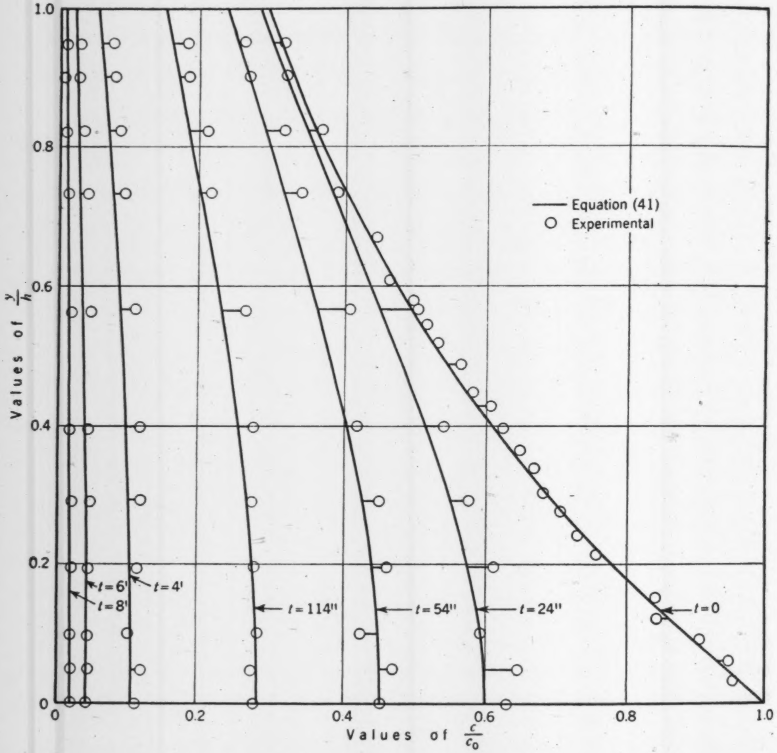


FIG. 7.—RUN IV. CONCENTRATION CHANGES WITH NO PICKUP

$\left(\frac{w}{\epsilon} = 0.030 \text{ cm}^{-1}; \text{and } h = 41.6 \text{ cm} \right)$

To satisfy these conditions physically, it was necessary to impose the condition of no pickup suddenly upon the equilibrium condition. This was accomplished by the setup shown diagrammatically in Fig. 9. The cylinder was supported at the top, as shown in Fig. 9(*a*), with a clearance of about 2 in. between the bottom of the cylinder and the floor of the water bath. The grid shown in Fig. 9(*c*) rested on the bottom of the cylinder and was attached to a framework external to the cylinder. Sufficient lucite was added to fill the grid completely and extend about 1 cm above the top of the grid. When the mixer was set in motion, the lucite above the grid was picked up and distributed according to the logarithmic law. This satisfied the condition for the original distribution.

To impose the condition of no pickup suddenly, the supporting bars at the top were swung outward, allowing the cylinder to drop to the floor of the water bath. This was accomplished in a fraction of a second. The particles of lucite settled through the grid into the space below, from which they could not be picked up. At various times after the cylinder was dropped, samples were drawn off through the siphon. After most of the material had settled, the cylinder was lifted back to its original position, the siphon was set to a different value of y, and the entire process was repeated.

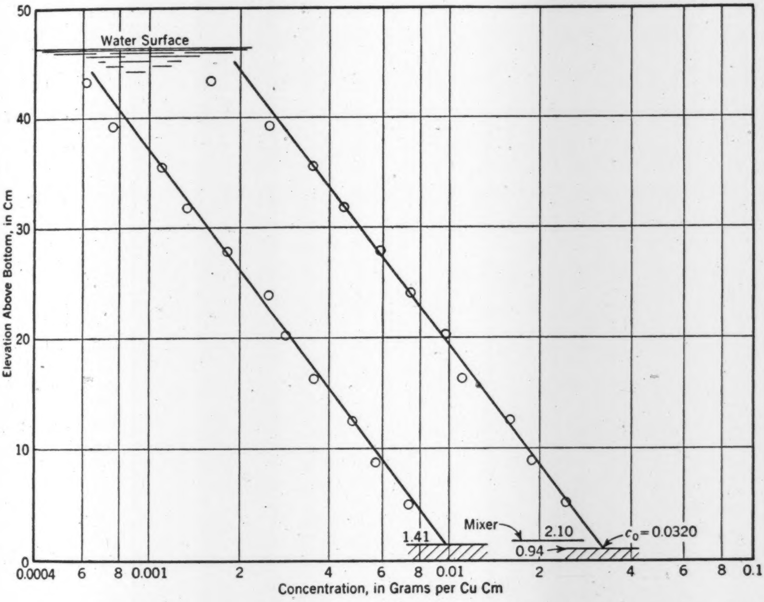


Fig. 8.—Initial and Final Equilibrium Concentrations; Run V; Mixer Speed, 1.65 RPS

The average sample size was about 14 cu cm and required about 6 sec for its collection. The volume of the siphon was found to be 13.8 cu cm and hence the time of passage through the siphon was also about 6 sec. To take this into account, the value recorded for the sampling time was 6 sec less than the time at which the center of the sample entered the collecting tube. A comparison between the theoretical and measured results is shown in Fig. 7.

The next run was made in order to check the theory for the case in which the pickup is not zero but has a constant value. An equilibrium condition was taken for the initial distribution of suspended matter, and another equilibrium condition with the same value of ϵ but with smaller absolute concentrations was taken as the final distribution.

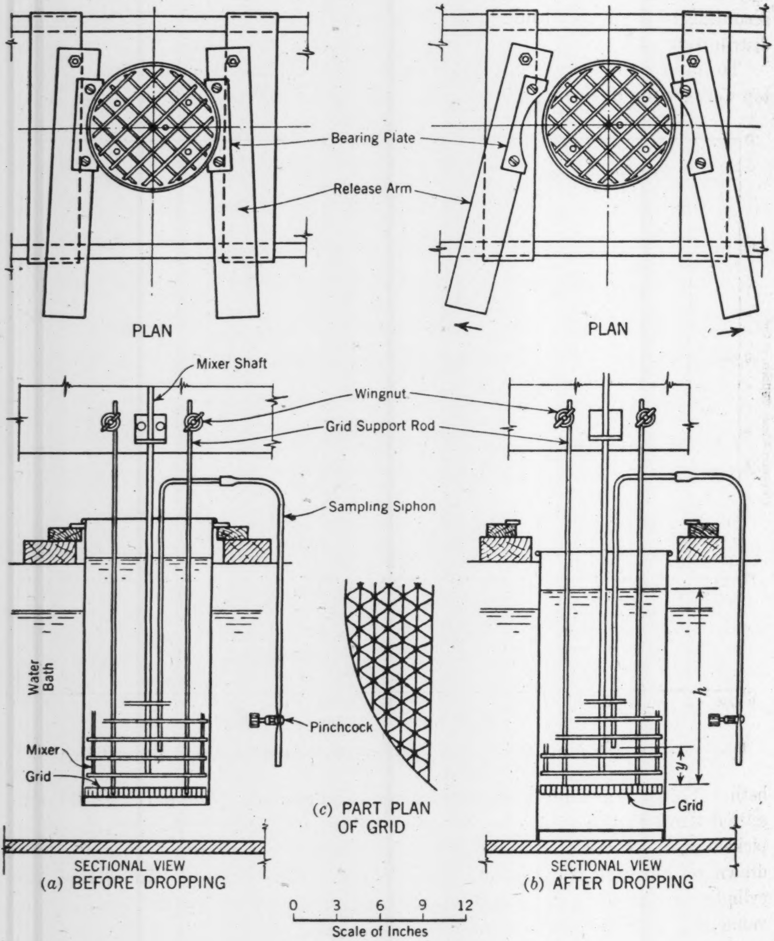

PLAN
Bearing Plate
Release Arm
PLAN
Mixer Shaft
Wingnut
Grid Support Rod
Sampling Siphon
Water Bath
Mixer
Grid
Pinchcock
(c) PART PLAN OF GRID
h
y
Grid
SECTIONAL VIEW
(a) BEFORE DROPPING
SECTIONAL VIEW
(b) AFTER DROPPING
0 3 6 9 12
Scale of Inches

FIG. 9.—EXPERIMENTAL SETUP FOR RUN IV

The experimental setup is shown diagrammatically in Fig. 10. The cylinder could be raised or lowered by turning the bolt at the right—one com-

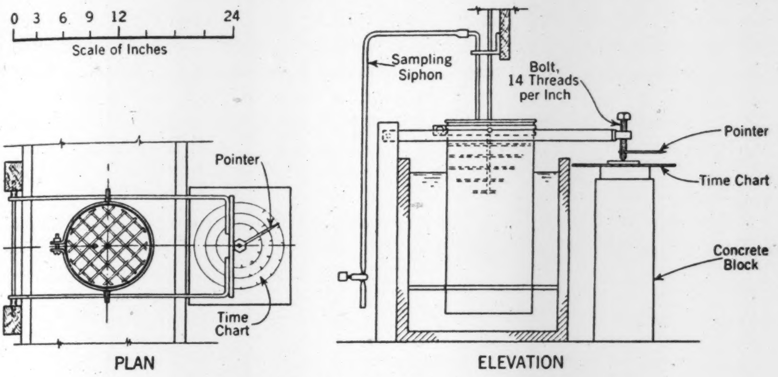

0 3 6 9 12 24
Scale of Inches
Pointer
Time Chart
PLAN
Sampling Siphon
Bolt, 14 Threads per Inch
Pointer
Time Chart
Concrete Block
ELEVATION

FIG. 10.—EXPERIMENTAL SETUP FOR RUN V

plete turn had the effect of raising or lowering the cylinder 0.825 mm. Since the position of the mixer remained unchanged, the space between the mixer and the bottom of the cylinder was decreased or increased by the same amount.

A total of 660 cu cm of the lucite was used, and for the initial condition the cylinder was set so that the bottom was 14.4 mm below the lowest position of

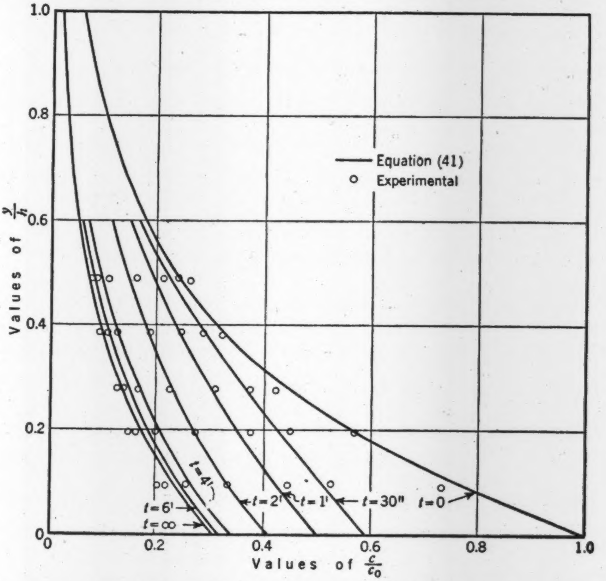


FIG. 11.—RUN V. CONCENTRATION CHANGES WITH CONSTANT RATE OF PICKUP $\left(\frac{w}{\epsilon} = 0.0638 \text{ cm}^{-1}; \text{ and } h = 45.2 \text{ cm}\right)$

the mixer. For the final condition the cylinder was lowered 6.60 mm, which was accomplished by making eight complete turns of the bolt. The measured concentrations for both of these conditions are shown in Fig. 8. Computations based on these curves show that the average depth of the bottom deposit was 9.4 mm in the initial condition and 14.1 mm in the final condition. Therefore, the distance between the surface of the bottom material and the lowest position of the mixer was 5.0 mm for the initial condition and 6.9 mm in the final condition.

To make the time study, it was necessary to impose upon the initial distribution a sudden rate of pickup equal to that in the final condition and to maintain this rate of pickup constant as the concentrations decreased. The results in Fig. 6(*a*) indicated that the rate of pickup depends upon the relative positions of the mixer and the surface of the bottom deposit, provided that the bottom is saturated. Therefore, to satisfy the pickup requirement, it was necessary to lower the cylinder suddenly until the distance between the lowest position of the mixer and the surface of the bottom deposit was equal to 6.9 mm, and then gradually to lower the cylinder so as to maintain this value as the suspended matter settled out.

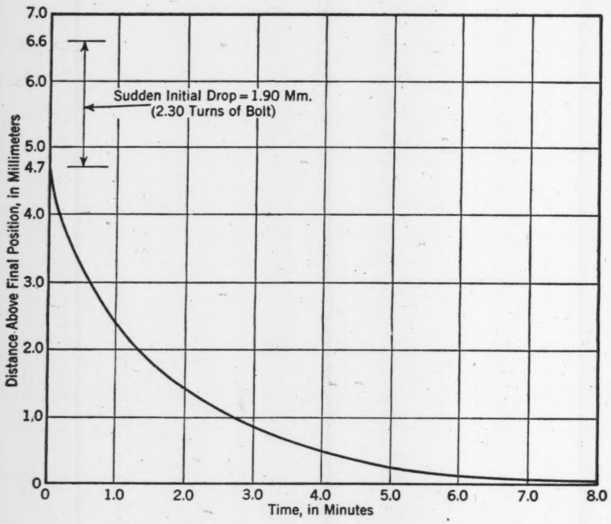
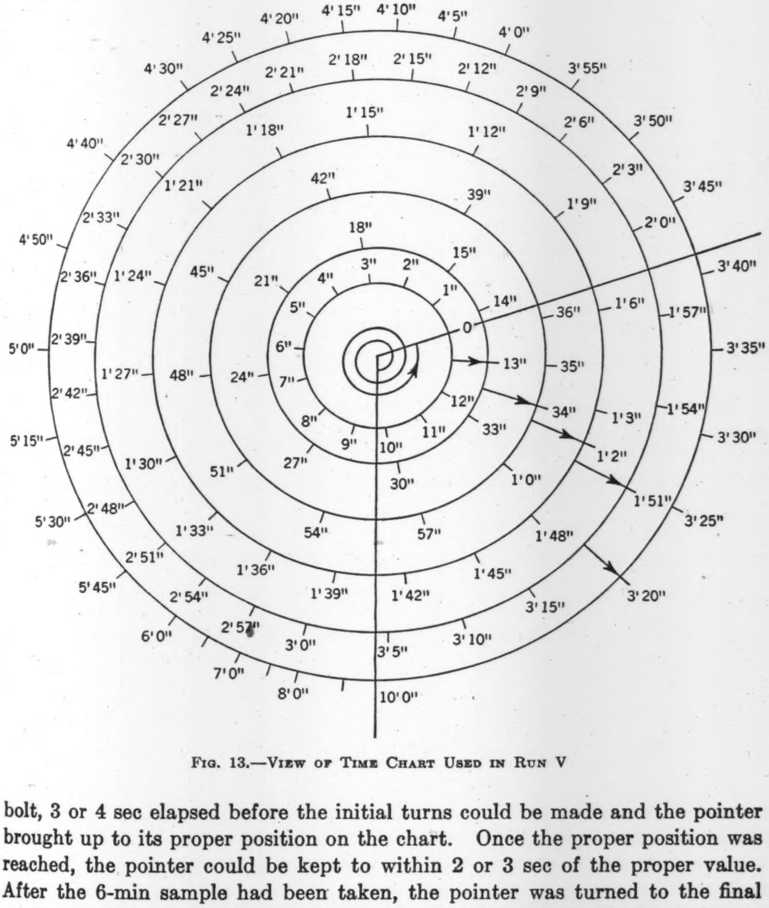


FIG. 12.—RATE OF DROP OF CYLINDER FOR RUN V

The sudden initial drop was 1.9 mm, which was equivalent to 2.3 turns of the bolt. From the theoretical concentration curve for any time after the initial drop (Fig. 11), the total amount of material deposited on the bottom up to that time could be computed, and hence the necessary total drop for the cylinder up to that time could be ascertained. By this means was prepared the curve shown in Fig. 12, which gives the necessary drop of the cylinder as a function of the time after the start. With the aid of Fig. 12, the time chart illustrated in Fig. 13 was prepared. In making the run, this chart was placed directly under the bolt and the bolt was turned so as to keep the pointer over the proper value of the time.

In the actual execution of the run, the services of two operators were required—one to turn the bolt and the other to take the samples. In turning the bolt, 3 or 4 sec elapsed before the initial turns could be made and the pointer brought up to its proper position on the chart. Once the proper position was reached, the pointer could be kept to within 2 or 3 sec of the proper value. After the 6-min sample had been taken, the pointer was turned to the final position and about 10 min were allowed to elapse before the final sample was drawn. After a complete series of samples had been taken for one value of y, the position of the siphon was changed and the entire procedure repeated. The results for this run are shown in Figs. 8 and 11.



0
1"
2"
3"
4"
5"
6"
7"
8"
9"
10"
11"
12"
13"
14"
15"
18"
21"
24"
27"
30"
33"
34"
35"
36"
39"
42"
45"
48"
51"
54"
57"
1'0"
1'2"
1'3"
1'6"
1'9"
1'12"
1'15"
1'18"
1'21"
1'24"
1'27"
1'30"
1'33"
1'36"
1'39"
1'42"
1'45"
1'48"
1'51"
1'54"
1'57"
2'0"
2'3"
2'6"
2'9"
2'12"
2'15"
2'18"
2'21"
2'24"
2'27"
2'30"
2'33"
2'36"
2'39"
2'42"
2'45"
2'48"
2'51"
2'54"
2'57"
3'0"
3'5"
3'10"
3'15"
3'20"
3'25"
3'30"
3'35"
3'40"
3'45"
3'50"
3'55"
4'0"
4'5"
4'10"
4'15"
4'20"
4'25"
4'30"
4'40"
4'50"
5'0"
5'15"
5'30"
5'45"
6'0"
7'0"
8'0"
10'0"

FIG. 13.—VIEW OF TIME CHART USED IN RUN V

DISCUSSION OF THE RESULTS

The results show that the absolute magnitudes of the concentrations in suspension depend upon the quantity of material on the bottom and the hydraulic conditions at the bottom. The factors affecting the latter are the speed of the mixer and its position relative to the surface of the bottom deposit. The greater the mixer speed and the closer it is to the bed material, the greater are the concentrations of material in suspension.

The results demonstrate that, for a given speed of the mixer and a given position of the mixer relative to the bottom deposit, there is a definite concentration at any value of y, provided that the bottom is saturated. Fig. 6(a) shows conditions in which the depth of the bottom deposit varies and Fig. 6(b) shows conditions in which the depth of the fluid varies. In all cases, the concentration curves coincide when the bottom is saturated and the speed of the mixer and its position relative to the surface of the bed material are fixed.

In all of the experiments it was found that the final equilibrium concentrations were independent of the manner in which the lucite was added. In some cases all of the material had settled to the bottom before the mixer was set in motion; in other cases the mixer was already in motion as the material was added.

All of the results show that the absolute magnitude of the equilibrium concentration at a given elevation above the surface of the bed material is independent of the total amount of material in suspension and depends only upon the hydraulic conditions at the bed, provided that the bottom is saturated with the material. At the equilibrium condition, the rate of pickup must equal the rate of deposit, $w\,c$. Therefore, the results show that the rate of pickup of a given material is independent of the total amount of material in suspension and depends only upon the hydraulic conditions at the surface of the bed material. It seems logical to assume that the rate of pickup of a given material in a stream would also depend only upon the hydraulic conditions at the bed. Of course, in a natural stream the distribution of the particle sizes in the bed material would have to be taken into account.

The results of the time studies shown in Figs. 7 and 11 provide definite verification of the validity of Eq. 41. The trends of the experimental results follow those of the theoretical results quite closely. The validity of the assumptions regarding the boundary conditions as well as the validity of the time factor involved in the equation is demonstrated.

In Fig. 7 the plotted points lie consistently to the right of the theoretical curves, although in no case is the error any larger than 5% of the original bottom concentration. These results show that the rate at which the material settled was slightly less than the theoretical rate. Two factors are offered as an explanation of this. At the start of the run, when the cylinder was dropped, the effect on the suspension was the same as if the mixer had been suddenly lifted upward. This may have had the effect of transporting material upward so as to result in an original concentration at every value of y which was slightly greater than what was assumed. Probably most important, however, was the fact that the pickup was not absolutely zero as the material settled out, since the grid provided some surface on to which the particles could fall and from which they could be picked up again.

The results in Fig. 7 show quite conclusively that the concentration gradient at the surface is proportional to the magnitude of the surface concentration and that the gradient at the bottom is zero when there is no pickup. The results in Fig. 11 demonstrate that the concentration gradient at the bottom is constant when the pickup is constant. Verification of the formulas that express boundary conditions I and II at the surface and at the bottom is thereby obtained.

CONCLUSIONS

The principal conclusions that can be deduced from the experimental results may be summarized as follows:

1. Further evidence of the validity of the logarithmic law for the vertical distribution of sediment at the equilibrium condition is provided;
2. The equilibrium concentrations are independent of the method by which the material is added to the suspension;
3. Other things being equal, the equilibrium concentrations depend upon the amount of material available on the bottom, up to the point of saturation;
4. When the bottom is saturated with a given material, the equilibrium concentrations depend upon the hydraulic conditions at the bed;
5. The equilibrium concentration at any elevation above the bed is independent of the depth of the fluid provided that the bottom is saturated;
6. The rate of pickup for unsteady conditions is independent of the total amount in suspension and depends only upon the hydraulic conditions at the bed and the characteristics of the bed material, provided that the bottom is saturated with the material; and
7. The one-dimensional case of the theory of turbulent sedimentation for discrete particles is verified.

ACKNOWLEDGMENTS

This paper was condensed from a thesis presented by the writer in partial fulfilment of the requirements for the degree of Doctor of Science in Sanitary Engineering from the Massachusetts Institute of Technology, Cambridge, Mass., in October, 1941. Copies of this thesis are on file at Massachusetts Institute of Technology and the Engineering Societies Library. The experimental work was done in the Institute's Sanitary Engineering Laboratory. The writer gratefully acknowledges his indebtedness to the members of the staff of the Department of Civil and Sanitary Engineering for their interest and help throughout the work.

Special acknowledgment is due to Professor Camp, under whose direction the study was made, for his advice and encouragement during the study. P. C. Stein, Assoc. M. Am. Soc. C. E., gave many valuable suggestions and assisted in the preparation of some of the figures, and Prof. G. P. Wadsworth of the Department of Mathematics advised the writer concerning some of the mathematical aspects of the study.

APPENDIX 1

BIBLIOGRAPHY

(1) "On Sedimentation," by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 45.

(2) "A Study of the Rational Design of Settling Tanks," by Thomas R. Camp, *Sewage Works Journal*, Vol. VIII, No. 5, September, 1936, p. 742.

(3) "Bericht über Untersuchungen zur ausgebildeten Turbulenz," by L. Prandtl, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 2, 1925, p. 136.

(4) "Some Aspects of the Turbulence Problem," by T. von Kármán, *Proceedings*, Fourth International Cong. for Applied Mechanics, Cambridge, Mass., 1934.

(5) "Turbulence in Open Channel Flow," by A. A. Kalinske and J. M. Robertson, *Engineering News-Record*, Vol. 126, No. 15, April 10, 1941, p. 53.

(6) "Experiments on the Mechanics of Sediment Suspension," by H. Rouse, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938.

(7) "Some Experiments on the Transportation of Suspended Load," by V. A. Vanoni, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1941, p. 608. (*a*) p. 613.

(8) "Der Massenaustausch in freier Luft und verwandte Erscheinungen," by W. Schmidt, *Die Wasserwirtschaft*, No. 5–6, 1932 (Mitteilung 10, Inst. für Wasserbau, Tech. Hochschule, Berlin).

(9) "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1933, p. 487.

(10) "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP1151*, *Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 707.

(11) "Velocity Distribution in Open Channels," by Vito A. Vanoni, *Civil Engineering*, June, 1941, p. 356.

(12) "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1939, p. 637.

(13) "Laws of Transportation of Sediment by Streams: Suspended Load," by H. Rouse, *Reprint No. 21*, State Univ. of Iowa, Iowa City, Iowa, Reprints in Engineering, 1939.

(14) "Suspended-Material Transportation Under Non-Equilibrium Conditions," by A. A. Kalinske, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1940, p. 613.

(15) "Introduction to the Mathematical Theory of the Conduction of Heat in Solids," by H. S. Carslaw, Macmillan, London, 1921.

(16) "Neuere Methoden und Ergebnisse in der Hydromechanik," by C. W. Oseen, Akademische Verlagsgesellschaft, Leipzig, 1927.

(17) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 561.

(18) **"Measurement of Evaporation from Land and Water Surfaces,"** by C. W. Thornthwaite and B. Holzman, *Technical Bulletin No. 817*, U.S.D.A., May, 1942.

(19) **"Wind Influences on Transportation of Sand,"** by Helmut Landsberg, Second Hydraulics Conference, Univ. of Iowa, Iowa City, June, 1942.

(20) **"Measurement of Evaporation from Land and Water Surfaces,"** by C. W. Thornthwaite and Benjamin Holzman, *Technical Bulletin No. 817*, U. S. D. A., 1942.

(21) **"Note on the Variation of Wind with Height in the Layer near the Ground,"** by C. W. Thornthwaite and Maurice Halstead, *Transactions*, Am. Geophysical Union, 1942, pp. 249–255.

(22) **"Wind-Gradient Observations,"** by C. W. Thornthwaite and Paul Kaser, *ibid.*, 1943, p. 166.

(23) **"Experiments on Eddy-Diffusion and Suspended-Material Transportation in Open Channels,"** by A. A. Kalinske and C. L. Pien, *ibid.*, p. 350.

(24) **"A Study of Transportation of Fine Sediments by Flowing Water,"** by C. H. Hsia, thesis presented to the Department of Mechanics and Hydraulics, Univ. of Iowa, Iowa City, in 1943 in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

(25) **"Applied Fluid Mechanics,"** by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York, 1937, p. 189.

(26) **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** *Report No. 1* entitled "Field Practice and Equipment Used in Sampling Suspended Sediment," St. Paul Engr. Dist. Sub-Office, Hydr. Laboratory, Univ. of Iowa, Iowa City, August, 1940, pp. 85 and 90.

APPENDIX 2

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

A = concentration at $y = 0$, $t = \infty$;
a = reference elevation for suspended-load function;
B = constant used in a function describing the initial distribution of suspended matter;
C = constant: C_1, C_2, C_3 = arbitrary constants; C_n = arbitrary constant in Eq. 32*b*;

---

[3] ASA—Z10.2—1942.

c = concentration of sediment in suspension: c_0 = concentration at $t = 0, y = 0$;
e = base of Naperian logarithms;
f = Weisbach-Darcy friction factor;
g = acceleration of gravity;
H_n = a function of h defined by Eq. 39b;
h = total depth of fluid;
k = a constant defined by Eqs. 21 and 24; when specifically defined, it is also used as a constant in the formula of a parabola (Eqs. 15);
l = mixing length;
n = any integer;
S = slope of energy gradient;
s = distance in any direction (see Eq. 2);
T = function of t;
t = time of settling;
V = mean velocity at a point;
$\overline{V}$ = mean velocity for the entire cross section;
v = instantaneous velocity at a point;
$\bar{v}$ = mean velocity at a point;
v' = fluctuation velocity at a point: $\bar{v}'$ = temporal mean value of the fluctuation velocity; $|\bar{v}'|$ = temporal mean value of the absolute magnitude of the fluctuation velocity;
W = total amount of sediment held in suspension per unit horizontal area;
w = settling velocity of the particles;
x = horizontal distance in the direction of flow;
Y = a function of y: Y_n = a function of y defined by Eq. 34;
y = elevation above surface of bed material;
z = exponent in a suspended-load function;
α = constant defined by Eq. 31;
β = proportionality factor;
ϵ = kinematic turbulence coefficient = $\beta|\bar{v}'|l$;
η = a function of $h\,\alpha$ defined in Eq. 31;
κ = von Kármán universal velocity distribution constant;
ρ = mass density of the fluid;
τ = intensity of fluid shear: τ_0 = intensity of fluid shear at $y = 0$.

DISCUSSION

A. M. GAUDIN,[4] ESQ.—A particularly significant attempt to analyze the problem of sedimentation under turbulent conditions is contained in the paper by Mr. Dobbins. The discussion of the distribution of particles in a vessel in which the turbulence varies along a vertical coordinate only is convincing indeed in the agreement of theory and practice. Of course, the broader problem of the distribution of particles in a flowing stream is what the author is looking toward, but has not yet undertaken to solve. It is to be hoped that this will be done in subsequent papers.

Systems in which particles of various sizes, specific gravities, and shapes occur simultaneously have been excluded from analysis; yet they represent the more usual condition. Further work should include such studies.

In concluding these very brief comments, the writer wishes to congratulate Mr. Dobbins for this well-documented and splendid contribution to the knowledge of the movement of solids in fluids.

JOHN S. MCNOWN,[5] JUN. AM. SOC. C. E.—It is a real pleasure to find an engineer who is both willing and able to carry through a difficult mathematical analysis of the type presented in this paper. The study of turbulence and sedimentation is an outstanding example of that unfortunately small phase of engineering in which a theoretical analysis leads to a complete solution. Despite the necessary simplifying assumptions and the tedium of the numerical computations, this approach doubtless will prove more valuable than empirical results from experimental data. Engineers must resort continually to empirical or trial-and-error methods, but scientific advancement must include satisfactory theoretical treatments.

As in so many physical problems, the definition and application of the proper boundary conditions present the most difficult part of this solution. The surface and bottom boundary conditions for combined liquid and solid motion are not only difficult to apply, but also difficult to predetermine. The surface conditions may be approximated by a plane surface with zero turbulence. However, as the author states, this condition is not accurate as the surface is generally irregular. Measurements of open-channel flow conducted by the writer at the St. Anthony Falls Hydraulic Laboratory at the University of Minnesota gave evidence of a turbulent velocity near the surface which was not small compared to turbulent velocities in other parts of the channel. In one instance the vertical component of the turbulent velocity in the upper 1% of the depth was as much as half the maximum vertical turbulence velocity occurring at the lower one-quarter point. The turbulent diffusion coefficient decreased near the surface because the mixing length decreased. Nevertheless, the concept of a zero turbulence coefficient at the surface does not agree with the physical picture of turbulent open-channel flow.

The artificial bottom conditions introduced by the author served nicely in furnishing a boundary condition that could be evaluated in the solution. For sediment transportation in open channels the accurate evaluation of this boundary condition provides the principal stumbling block to a solution of this troublesome problem. The relationships between bed load, suspended load, and the flow characteristics are dependent on this condition primarily.

---

[4] Dept. of Metallurgy, Mass. Inst. Tech., Cambridge, Mass.

[5] Research Associate, Div. of War Research, Univ. of California, San Diego, Calif.

The differential equation for this problem (Eq. 14*b*) is incomplete as witnessed by the lack of symmetry in x and y. The term, $\frac{\partial \epsilon_x}{\partial x} \frac{\partial c}{\partial x}$, should be added to the right-hand side of the equation. The derivation of Eq. 14*b* through the rather involved relationships expressed in Eq. 14*a* and Fig. 1 is less direct and, hence, more subject to errors of omission than a vector presentation of the same equations. Further simplification is obtained by expressing the relationships in two fundamental equations—the general expression for the current or transport in a turbulent medium and the equation of continuity. The vector current, **q** (of sediment in this case), is the vector sum of the field velocity, **u**, times the concentration and the turbulent diffusion which is the product of the gradient of the concentration and the coefficient of diffusion. Thus, in vector-operator form,

$$\mathbf{q} = \mathbf{u} c - \epsilon \operatorname{grad} c \text{ .... (42)}$$

The equation of continuity is expressed by equating the divergence of the current at any point to the rate of decrease of concentration with time, thus:

$$\operatorname{div} \mathbf{q} = - \frac{\partial c}{\partial t} \text{ .... (43)}$$

If the divergence of Eq. 42 is taken, the resulting expression is

$$\operatorname{div} \mathbf{q} = \operatorname{div} (\mathbf{u} c) - \operatorname{div} (\epsilon \operatorname{grad} c)$$

or,

$$\nabla \mathbf{q} = \mathbf{u} (\nabla c) - \nabla (\epsilon \nabla c) \text{ .... (44)}$$

if **u** is considered to be a constant. Combining Eq. 43 and Eq. 44 and performing the indicated operations for a two-dimensional problem, the equation may be written as follows:

$$\frac{\partial c}{\partial t} = - u_x \frac{\partial c}{\partial x} - u_y \frac{\partial c}{\partial y} + \frac{\partial \epsilon_x}{\partial x} \frac{\partial c}{\partial x} + \frac{\partial \epsilon_y}{\partial y} \frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial x^2} + \epsilon_y \frac{\partial^2 c}{\partial y^2} \text{ ... (45a)}$$

The velocities u_x and u_y are the components of the field velocity tending to move the sediment continuously in the x-direction and y-direction. In the author's notation they are V, the stream velocity, and $- w$, the settling velocity, respectively. In the steady-state problem $\frac{\partial c}{\partial t} = 0$. The final equation to be compared with Eq. 14*b* is,

$$V \frac{\partial c}{\partial x} = w \frac{\partial c}{\partial y} + \frac{\partial \epsilon_x}{\partial x} \frac{\partial c}{\partial x} + \frac{\partial \epsilon_y}{\partial y} \frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial x^2} + \epsilon_y \frac{\partial^2 c}{\partial y^2} \text{ .... (45b)}$$

The only difference between Eqs. 14*b* and 45*b* is the added term $\frac{\partial \epsilon_x}{\partial x} \frac{\partial c}{\partial x}$, as pointed out in the foregoing. Admittedly, the variation of ϵ with x is not important in the problem under consideration, and would introduce no change in the simplified form, Eq. 17*b*. This derivation and the correction have been presented to give complete generality to the basic equation, so far as intended, and to show the nicety of the vector-operator derivation.

Unfortunately, this problem entails a long and involved mathematical solution, even after a number of simplifying assumptions. The complications of the boundary conditions and parameter variations in most practical problems are formidable. In many cases some simplifications of the numerical computations would be justifiable, particularly if the boundary conditions and the parameter values were not determined accurately. The solution of Eq. 30 may be simplified for small values of $\frac{w\,h}{2\,\epsilon}$. The first value of α could be determined by trial and error or graphically, and all others could be approximated by the relationship,

$$\alpha_n = n\,\pi \quad \text{(46)}$$

with n taking the values of 1, 2, 3, $\cdots$ as far as required. The discrepancy between this simplified solution and that used by the author was compared by the writer in a similar set of computations and found to be only a few per cent.

The author has not mentioned direct measurements of the settling velocity of the particles. Such measurements would give a further check on the degree of accuracy of the results. The values computed from predetermined coefficients may be sufficiently accurate, but the opportunity for the direct measurement of a critical parameter, such as the settling velocity, should be taken wherever possible. The use of an arithmetic mean value of the settling velocities for values of w ranging from 0.150 cm per sec to 0.344 cm per sec is of doubtful accuracy. This range of more than a factor of two becomes magnified for large time values because of the nature of the equation. The concentration of smaller particles after a comparatively long time may well have been many times greater than the concentration of the larger particles. Any information on such a change in the statistical spread of the various sizes would be interesting in this connection. It is difficult to observe in the author's results whether this or other effects introduced errors into the computations at large values of the time variable. A semilogarithmic plot for Fig. 7 and Fig. 11 (as in Fig. 8) would enable the reader to compare the theoretical curves with the plotted points more readily.

An interesting by-product of this experiment is the indirect determination of turbulence parameters. The data available serve to evaluate the turbulence diffusion coefficient, ϵ. From the data at the bottom of Fig. 7 and Fig. 11 and the settling velocity, $w = 0.264$ cm per sec, ϵ can be computed. In the two instances presented $\epsilon = 8.8$ and 4.1 cm per sec. The comparison between these values and the quantities which determine the mixing length and the root-mean-square turbulent velocity might be of interest. The grid spacing, the vertical displacement, and the velocity of the lattice structure are the values most likely to be effective in determining ϵ. In a simplified form this type of experiment could be used to study the effect of the mechanical motion on the scale of turbulence. The determination of the concentration of sediment at various levels could be compared with the steady-state Eq. 5*a*. Good agreement with this equation would furnish a constant value of ϵ. A lack of agreement could be analyzed from the standpoint of a variable ϵ.

H. A. EINSTEIN,[6] ASSOC. M. AM. SOC. C. E.—Ostensibly resulting from the study of sedimentation problems in settling basins, this paper is especially interesting to the hydraulic engineer who is not familiar with this special problem. Artificial settling basins, as studied and applied by the sanitary engineer, usually are characterized by very small average flow velocities of the water. Professor Camp (2) showed that under certain circumstances an increase of turbulence in such basins will shorten the settling time if this additional mixing affects coagulation thus increasing, materially, the settling velocity of small particles. The hydraulic engineer studying sedimentation in natural streams finds those conditions only in rare cases. Usually the turbulent mixing in natural streams is sufficient to prevent the clay from settling independently of the stage of flocculation. In many special cases, however, where high sediment concentration and salt content favor flocculation, this effect may explain otherwise incomprehensible situations.

Most significant in Mr. Dobbins' paper is the emphasis given to what he calls "pickup"—the exchange of particles between bed and suspension. In another connection (17), the writer has showed the paramount importance of this same exchange of particles in the description of bed-load movement, and probably this same exchange or pickup will some day make it possible to calculate the rates of bed material carried in suspension without the use of any concentration measurements.

In his experiments Mr. Dobbins used, exclusively, "artificial turbulence," that is, turbulence created by a system of baffles rather than by friction along the sediment deposit. The intensity of the resulting turbulence is not measured, and, therefore, no comparison with conditions in a natural river is possible. As a check of the formulas, the experiments are excellent and prove the formulas to be correct although somewhat complicated for practical use. The exceptionally clear presentation of this intricate subject makes Mr. Dobbins' paper an outstanding contribution to the literature on sedimentation in general.

THOMAS R. CAMP,[7] M. AM. SOC. C. E.—A significant contribution to current knowledge of sedimentation principles, this paper demonstrates clearly that suspended sediment is not held in suspension in a turbulent stream; but rather that it is settling out continuously and is being thrown back continuously into the stream by scour from the bottom. The concentration of suspended matter at any depth in a stream at equilibrium is thus shown to depend primarily upon the rate of scour from the bottom. When the rate of scour is equal to the rate of settling out, equilibrium prevails at all levels. When the rate of scour exceeds the rate of settling out, the concentration is increasing at all depths. When the rate of scour is less than the rate of settling out, the stream is clearing. If scour is prevented, the stream will clear completely; but the time required for clearing depends upon the settling velocity of the particles, the depth, and the magnitude of the turbulence.

---

[6] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

[7] Associate Prof., San. Eng., Mass. Inst. Tech., Cambridge, Mass.

The equilibrium condition characterized by Eqs. 3 and 4 has been widely used in the study of suspended-load distribution. It has been applied also to the measurement of evaporation of moisture from land and water surfaces (18), and to the movement of sand dunes by wind (19). In the light of Mr. Dobbins' studies, it is evident that, in all three problems, the equilibrium condition is only approximated in nature. The suspended matter in a stream and the sand in the air will be at the equilibrium concentration only when the rate of pickup equals the rate of settling out. The moisture in the air will be at the equilibrium concentration only when the rate of transport upward is the same across all horizontal planes.

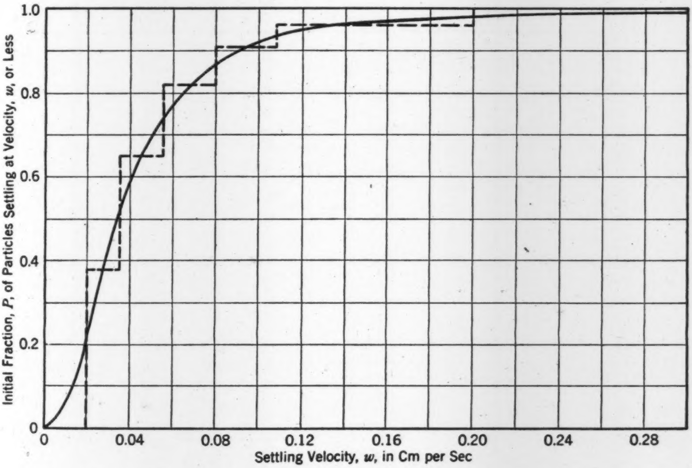


FIG. 14.—SETTLING VELOCITY ANALYSIS OF SUSPENSION OF DISCRETE PARTICLES

The studies described by Mr. Dobbins were undertaken as a part of a larger program aiming at a rational approach in the design of settling tanks. In a settling tank the rate of scour from the bottom is always less than the rate of settling out. In most types of settling tanks, there should be no scour from the bottom if the tank is functioning properly. The effect of turbulence is to retard the rate of clearing. The immediate objective of the author's studies was to obtain some estimate of the magnitude of this retarding effect.

Eq. 41 is an expression for the concentration at any height y and any time t for the one-dimensional case, the initial concentration being uniformly distributed and equal to c_0 from top to bottom. This corresponds to a value of ∞ for the mixing coefficient ϵ at the entrance to a settling tank, followed immediately by a finite value of ϵ within the tank. It is convenient to assume a uniform distribution of suspended matter at the start in order to compare the removal with that obtained under similar conditions but in the absence of turbulence.

In the absence of turbulence and scour, starting with uniform distribution of suspended matter, the removal after any time t of all particles whose settling velocity w is less than w_o is

$$r = \frac{w}{w_o} \qquad (47)$$

in which $w_o = \frac{h}{t}$ = the "overflow rate." In the sanitary engineering field, the "overflow rate" is usually defined as the discharge per unit of surface area. It is also the settling velocity required for a particle to settle throughout the depth h in time t. Therefore all particles with settling velocities exceeding w_o will be 100% removed in time t.

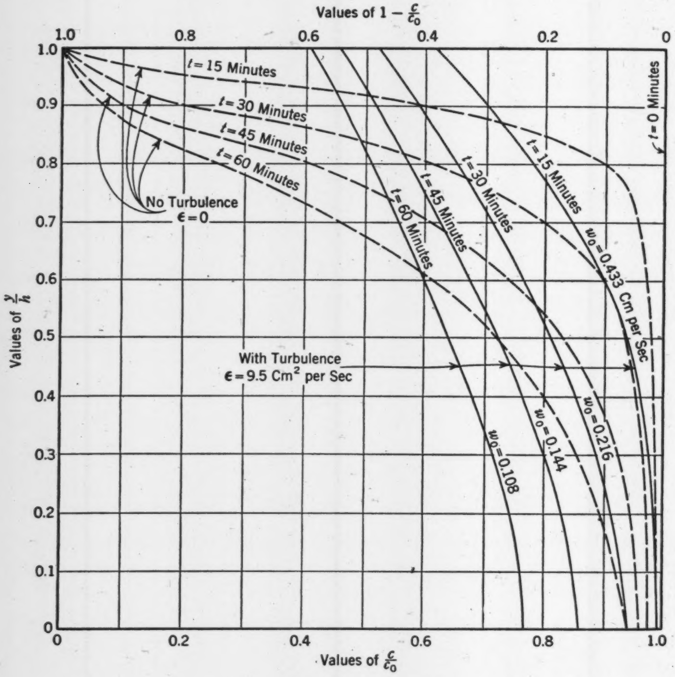


FIG. 15.—EFFECT OF TURBULENCE IN TANK 12.8 FT DEEP UPON SETTLING OF SUSPENSION OF FIG. 14

In the absence of scour, A is zero in Eq. 41. For this condition, Eq. 41 can be rewritten in the following dimensionless form:

$$\frac{c}{c_0} = \frac{w\,h}{2\,\epsilon}\,e^{i} \sum_{n=1}^{n=\infty} \frac{(h\,\alpha_n)^2\,H_n\,Y_n\,e^{-j}}{\left[\left(\frac{w\,h}{2\,\epsilon}\right)^2 + (h\,\alpha_n)^2 + \frac{2\,w\,h}{2\,\epsilon}\right]\left[\left(\frac{w\,h}{2\,\epsilon}\right)^2 + (h\,\alpha_n)^2\right]} \qquad (48)$$

in which, to simplify typography,

$$i = \frac{w\,h}{2\,\epsilon}\left(1 - \frac{y}{h}\right) \dots\dots\dots\dots (49a)$$

and

$$j = \left[\left(\frac{w\,h}{2\,\epsilon}\right)^2 + (h\ \alpha_n)^2\right] \frac{1}{\frac{2\,w\,h}{2\,\epsilon}} \frac{w}{w_o} \dots\dots\dots\dots (49b)$$

The relative concentration $\frac{c}{c_0}$ in Eq. 48 is a function of only three independent variables, $\frac{y}{h}$, $\frac{w\,h}{2\,\epsilon}$, and $\frac{w}{w_o}$, all of which are dimensionless.

The application of Eq. 48 to a suspension of particles of varying settling velocity requires that the analysis of the suspension in terms of settling velocity be known. Fig. 14 is a typical settling-velocity analysis of a suspension of discrete particles. In order to apply Eq. 48, the suspension should be fractionated as shown by the broken line. The equation is applied separately to each fraction, assuming that all particles in each fraction have the same settling velocity as shown; and the total relative concentration then is computed by adding the fractional values of $\frac{c}{c_0}$. Fig. 15 shows the results of such a computation. The corresponding values of $\frac{c}{c_0}$ in the absence of turbulence are shown by broken lines for comparison.

The average relative concentration throughout the depth h may be computed from the integral $\int_0^1 \frac{c}{c_0}\ d\left(\frac{y}{H}\right)$ in which the value of $\frac{c}{c_0}$ is taken from Eq. 48. This integral is evaluated readily. The removal from suspension by settling in time t is the difference between this integral and the initial concentration. Since the relative initial concentration must be taken at unity throughout the depth h, the relative removal with turbulence is

$$r = 1 - 8\left(\frac{w\,h}{2\,\epsilon}\right)^2 e^{(w\ h)/(2\ \epsilon)}$$

$$\times \sum_{n=1}^{n=\infty} \frac{(h\,\alpha_n)^2\,H_n\,e^{-i}}{\left[\left(\frac{w\,h}{2\,\epsilon}\right)^2 + (h\,\alpha_n)^2 + \frac{2\,w\,h}{2\,\epsilon}\right]\left[\left(\frac{w\,h}{2\,\epsilon}\right)^2 + (h\,\alpha_n)^2\right]^2} \dots\dots (50)$$

The term Y_n which appears in Eq. 48 has been eliminated in Eq. 50 by integration over the depth h. The relative removal in Eq. 50 is a function of only two independent variables, $\frac{w\,h}{2\,\epsilon}$ and $\frac{w}{w_o}$, both of which are dimensionless.

Numerical computations by means of Eqs. 48 and 50 are extremely tedious and cumbersome. Some problems of practical significance require fifteen or more terms for convergence of the series. Moreover, for such problems, the graphical evaluation of $h\,\alpha$ as illustrated by the author in Fig. 3 is not sufficiently accurate. It is convenient, therefore, to have a graph for solution of numerical problems; and the writer has prepared such a graph for Eq. 50. Fig. 16 is a dimensionless plot of r against $\frac{w\,h}{2\,\epsilon}$ for various values of $\frac{w}{w_o}$. The successive values of $h\,\alpha_n$ required for each computed point on this graph were determined to six decimal places by trial-and-error solutions of Eq. 31. Convergence of the series is rapid for low values of $\frac{w\,h}{2\,\epsilon}$, only one term being required for a value of 0.1. However, fourteen terms were required for $\frac{w\,h}{2\,\epsilon} = 30$, and no solution was practicable for $\frac{w\,h}{2\,\epsilon} = 100$.

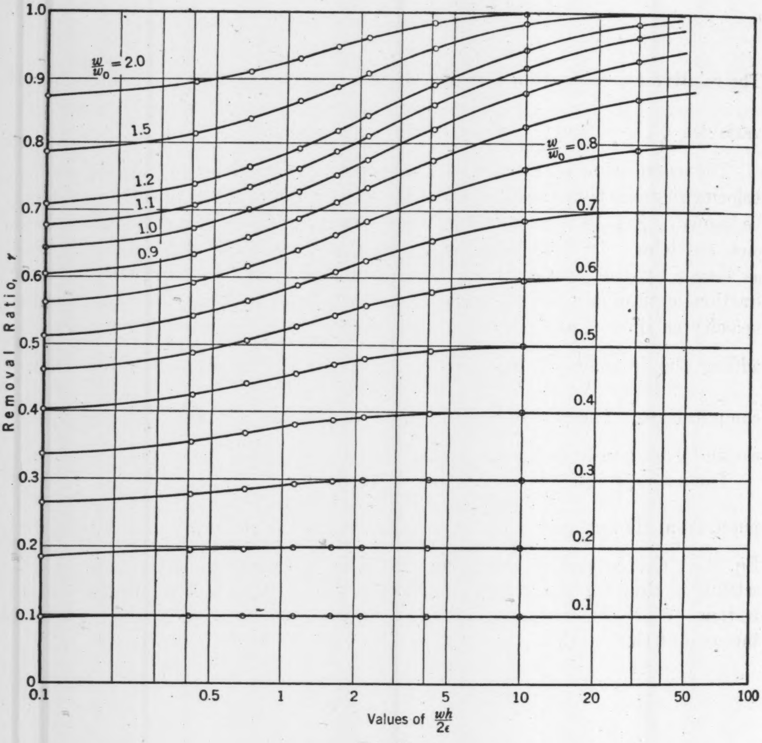


Fig. 16

The effect of turbulence in retarding settling for the one-dimensional case is apparent from Fig. 16. When turbulence is relatively great with a high value of ϵ, the value of $\frac{w\,h}{2\,\epsilon}$ is low and removal is reduced. For example, for particles that would be just 100% settled without turbulence (that is,

$\left.\frac{w}{w_o} = 1.0\right)$, the removal is only 64% if $\frac{w\,h}{2\,\epsilon} = 0.1$; but it is 72% if $\frac{w\,h}{2\,\epsilon} = 1.0$; and it is 94% if $\frac{w\,h}{2\,\epsilon} = 40$. The effect of turbulence on the settling out of particles is much less if the removal is less—that is, for low values of $\frac{w}{w_o}$.

Eqs. 48 and 50 and Fig. 16 are correct solutions for only the one-dimensional case. However, they may be used for two-dimensional flow in a stream or settling tank to obtain approximate solutions. The following bold assumptions are required in the adaptation to two-dimensional flow:

1. The fluid velocity is the same at every point in the channel; and
2. The mixing coefficient ϵ has the same value at every point in the channel.

By reference to Eq. 16*b*, it will be noted that the second assumption corresponds to a parabolic velocity distribution as illustrated by Fig. 2. Obviously, then, the two assumptions are contradictory; but they permit the use of Eqs. 17*b* and 17*c*, the only form of the general differential equation for which a solution has been obtained.

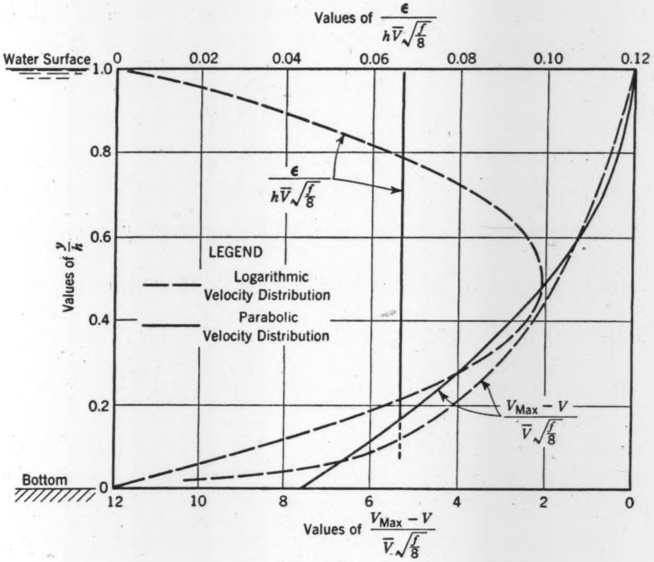


FIG. 17

The value of ϵ at any height y for two-dimensional open-channel flow and the logarithmic velocity distribution is given by Eq. 11, which may be rewritten in dimensionless form as follows:

$$\frac{\epsilon}{h\,\bar{V}\,\sqrt{\frac{f}{8}}} = \kappa\,\frac{y}{h}\left(1 - \frac{y}{h}\right) \qquad (51)$$

This value of ϵ and the corresponding velocity distribution are shown with broken lines in Fig. 17. The mean value of ϵ over the depth h is given by

$$\frac{\epsilon}{h\,\bar{V}\sqrt{\frac{f}{8}}} = \frac{\kappa}{6} = 0.0667 \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

in which the value of κ is taken at 0.4. Eq. 52 is shown by the solid vertical line in Fig. 17. If this value of ϵ is assumed to obtain from top to bottom, the corresponding parabolic velocity distribution is as shown by the solid curve in Fig. 17.

The value of $\frac{w\,h}{2\,\epsilon}$ for two-dimensional open-channel flow may be expressed in terms of the mean velocity of the stream by means of Eq. 52 as follows:

$$\frac{w\,h}{2\,\epsilon} = \frac{3\,w}{\kappa\,\bar{V}\sqrt{\frac{f}{8}}} = 7.5\,\frac{w}{\bar{V}\sqrt{\frac{f}{8}}} \ldots\ldots\ldots\ldots\ldots\ldots (53a)$$

Also, since by similar triangles $\frac{w_o}{\bar{V}} = \frac{h}{L}$ (in which L is the length of travel at the mean velocity $\bar{V}$ in time t)—

$$\frac{w\,h}{2\,\epsilon} = \frac{3}{\kappa\sqrt{\frac{f}{8}}}\,\frac{h}{L}\,\frac{w}{w_o} = \frac{7.5}{\sqrt{\frac{f}{8}}}\,\frac{h}{L}\,\frac{w}{w_o} \ldots\ldots\ldots\ldots\ldots (53b)$$

As an example, let it be required to find the effect of turbulence on the settling of particles which would be just 70% removed $\left(\frac{w}{w_o} = 0.70\right)$ in the absence of turbulence in a 200-ft stretch of tank 10 ft deep $\left(\frac{h}{L} = 0.05\right)$ if $f = 0.032$. From Eq. 53*b*, $\frac{w\,h}{2\,\epsilon} = 4.15$. From Fig. 16, it is found that the removal is 65.7%—a reduction due to turbulence of $\frac{4.3}{70}$ or 6.1% in the removal.

As another example, let it be required to find the effect of turbulence on the removal of particles whose settling velocity w is 0.22 cm per sec, settling to take place in a 200-ft settling zone, 10 ft deep in which the mean velocity is 10 ft per min and f is 0.032. The detention period, t, is $\frac{200}{10} = 20$ min or 1,200 sec. The overflow rate $w_o = \frac{10 \times 30.48}{1{,}200} = 0.254$ cm per sec. The removal in the absence of turbulence is $\frac{w}{w_o} = \frac{0.22}{0.254} = 0.866$ or 86.6%. From Eq. 53*a*, $\frac{w\,h}{2\,\epsilon} = 7.5\,\frac{0.22}{10 \times \frac{30.48}{60}\sqrt{\frac{0.032}{8}}} = 5.13$. From Fig. 16 it is found that the removal is about 77.2%, the effect of turbulence being to reduce the removal about 10.8%.

Mr. Dobbins has tackled a difficult problem with ingenuity and resourcefulness. He has overcome many obstacles in mathematical analysis and experimental technique and has evolved a sound answer which must serve as a basis for future work in this field. The writer contributed the idea which initiated this program and helped with advice from time to time. It is gratifying to have been associated with the author in this small way. Even the kibitzer is pleased when the hand is won.

GARBIS H. KEULEGAN,[8] ESQ.—The statement of the boundary conditions relating to the solution of the differential equation of the one-dimensional case requires examination.

Apparently it is the author's understanding that the solution of the basic equation, Eq. 17*c*, requires four boundary conditions. The equilibrium distribution which finally is established is taken as one of these. Obviously, the final disposition of a physical process is a consequence and cannot be preassigned as a boundary condition. Indeed the description of the final disposition must result from the analysis itself. Actually, only three conditions are necessary for the desired solution.

The mathematical expression of the argument is much simplified if Eq. 17*c* is transformed into a relation involving dimensionless quantities only. Suppose that t is measured in terms of T, and y in terms of h; thus, let $\tau = \frac{t}{T}$; $\eta = \frac{y}{h}$; $T = \frac{h^2}{\epsilon}$, and $\beta = \frac{w}{h\,\epsilon}$. Then, Eq. 17*c* takes the form

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial \eta^2} + \beta \frac{\partial c}{\partial \eta} \text{......................} (54)$$

Suppose that a solution is sought, subject to the following three boundary conditions:

First, the transport of the material across the upper surface into the liquid below is brought about in a preassigned manner. Mathematically,

$$\frac{\partial c}{\partial \eta} + \beta\, c = \phi(\tau); \qquad \eta = 1, \quad \tau > 0 \text{................} (55)$$

Second, the rate of pickup from the bottom is controlled in a preassigned manner; or, what is the same thing, the gradient of concentration at the bottom is controlled in a preassigned manner. Mathematically,

$$\frac{\partial c}{\partial \eta} = \theta(\tau); \qquad \eta = 0, \quad \tau > 0 \text{................} (56)$$

Third, the initial concentration is specified. Mathematically,

$$c = f(\eta); \qquad 0 > \eta > 1, \quad \tau = 0 \text{..................} (57)$$

It will be noted that the conditions assumed are of the most general type.

---

[8] Physicist, National Bureau of Standards, Washington, D. C.

If a solution of Eq. 54 conforms to the end conditions, Eqs. 55, 56, and 57, then this solution is unique.

In order to prove this statement, a theorem of expansion is needed. Consider the sequence of functions

$$\Theta_n = 2\,\delta_n \cos \delta_n\,\eta + \beta \sin \delta_n\,\eta; \qquad n = 1, 2, \cdots \quad \ldots\ldots\ldots(58)$$

in which δ_n is one of the positive roots of the relation

$$4\,\delta\,\beta \cos \delta = (4\,\delta^2 - \beta^2) \sin \delta \ldots\ldots\ldots\ldots\ldots\ldots(59)$$

These functions have the properties

$$\int_0^1 (\Theta_n)^2\, d\eta = \frac{(2\,\delta_n + \beta)^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots(60a)$$

and

$$\int_0^1 \Theta_n\,\Theta_m\, d\eta = 0; \qquad n \neq m \ldots\ldots\ldots\ldots\ldots\ldots(60b)$$

Because of these properties (ordinarily referred to as the properties of orthogonality) any bounded function $F(\eta)$ may be expanded in terms of Θ_n in the interval, $0 \overline{\overline{<}} \eta \overline{\overline{<}} 1$, provided that, if the function has discontinuities, these are finite in number. Mathematically,

$$F(\eta) = \sum (C_n\,\Theta_n) \ldots\ldots\ldots\ldots\ldots\ldots(61)$$

in which $C_n = \dfrac{2}{(2\,\delta_n + \beta)^2} \displaystyle\int_0^1 F(\eta)\,\Theta_n\, d\eta$. In addition to the discontinuities, the expansion may fail at the end points, $\eta = 0$ and $\eta = 1$, depending on the character of F. Similar and equivalent expansions were used by the author.

Now suppose that the solution of Eq. 54, conforming to the boundary conditions, Eqs. 55, 56, and 57, is not unique. Let C_1 be one solution and C_2 another. Consider the difference,

$$q = C_1 - C_2 \ldots\ldots\ldots\ldots\ldots\ldots(62)$$

Since C_1 and C_2 individually satisfy the boundary conditions, Eqs. 55, 56, and 57, it can be shown that q satisfies certain simpler boundary conditions, which are derived from Eqs. 55, 56, and 57:

$$\frac{\partial q}{\partial \eta} + \beta\, q = 0; \qquad \eta = 1, \quad \tau > 0 \ldots\ldots\ldots\ldots(63a)$$

$$\frac{\partial q}{\partial \eta} = 0; \qquad \eta = 0, \quad \tau > 0 \ldots\ldots\ldots\ldots(63b)$$

and

$$q = 0; \qquad 0 \overline{\overline{<}} \eta \overline{\overline{<}} 1, \quad \tau = 0 \ldots\ldots\ldots\ldots(63c)$$

Again, since C_1 and C_2 individually satisfy the linear differential equation, Eq. 54, q also satisfies the same equation,

$$\frac{\partial q}{\partial \tau} = \frac{\partial^2 q}{\partial \eta^2} + \beta\,\frac{\partial q}{\partial \eta} \ldots\ldots\ldots\ldots\ldots\ldots(64)$$

Obviously, the solution of Eq. 64 conforming to the boundary conditions, Eqs. 63*a* and 63*b*, is

$$q = \sum (C_n \, e^{\gamma\tau} \, e^{-0.5\beta^2\eta} \, \Theta_n) \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

in which $\gamma = -\frac{\beta^2}{4} - \delta^2$. It remains to ascertain C so that the boundary condition given by Eq. 63*c* is also satisfied. When $\tau = 0$,

$$q = \sum (C_n \, e^{-0.5\beta^2\eta} \, \Theta_n) \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

Since $q = 0$ at the points $\eta = 0$ and $\eta = 1$, an application of the theorem underlying Eq. 61 yields

$$C_n = 0; \qquad n = 1, 2, \cdots \quad \ldots\ldots\ldots\ldots\ldots (67a)$$

and, therefore,

$$q = 0; \qquad \tau > 0 \ldots\ldots\ldots\ldots\ldots\ldots (67b)$$

Accordingly $C_1 = C_2$. This proves the proposition that the solution of Eq. 54 conforming to the boundary conditions, Eqs. 55, 56, and 57, is unique.

Thus, the fourth boundary condition stated by the author is redundant for the problem since the solution finally arrived at is correct. That the solution is correct may be established by modifying the arguments of the author so that only the first three conditions are used to obtain the final solution; or another method of solution may be adopted, using the same three conditions and leading to the same results. Both of these methods were followed by the writer.

C. W. THORNTHWAITE,[9] ESQ.—In connection with studies directed toward the objective of measuring the transport of moisture from a natural surface into the atmosphere, through the medium of turbulent mass exchange, the writer and his associates have found it necessary to make detailed and careful observations of the variation of wind velocity with height in the layer within 30 ft of the ground (20)(21)(22). To obtain observations that would reveal the true nature of the wind gradient it was first necessary to select anemometers that were identical as to starting and stopping speeds, and running speed at all velocities. A number of matched anemometers were mounted one above the other on a 28-ft tower and connected electrically to a photographic recorder that makes simultaneous recordings of all at 15-min intervals.

The data indicate that the familiar logarithmic law of wind velocity is of restricted validity. This law applies very well when the temperature lapse rate is adiabatic (or within this thin layer, isothermal), but it fails when the lapse rate is superadiabatic and also at times of inversions. Gradients of wind movement for a series of hours on July 25 and 26, 1942 (Fig. 18), illustrate the dependence of wind distribution on stability. The data are plotted on a semilogarithmic scale; in order for the logarithmic law to be verified the data for individual hours would form straight lines; but only two series made in the evening of July 25 (lines F and G) approach straight lines. The daytime observations plot along lines having concave curvature; those made near noon show the greatest departure from a straight line. Observation J made near midnight displays strong convex curvature. This is the only one of the series in the figure that does so, but it represents a tendency that is characteristic of most observations made in the early hours of the morning before sunrise.

---

[9] Chf., Climatic and Physiographic Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

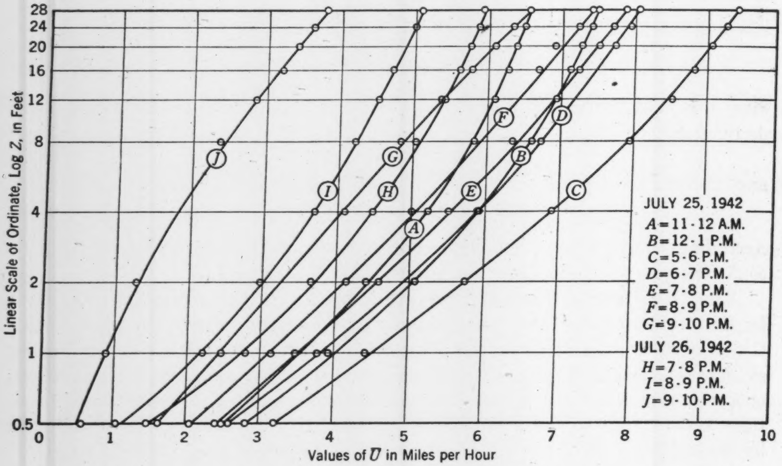


Fig. 18.—Gradients of Wind Movement

These observations suggest a reasonable physical picture of turbulent mixing in the atmosphere. When the thermal lapse rate is adiabatic, as for brief periods in the morning and again in the evening, and the logarithmic law is fulfilled, Prandtl's theory (Eq. 1) is valid, and the coefficient of turbulent mixing, designated ϵ by the author, may be obtained by Eq. 9. Turbulent mixing is mechanical only.

In the middle of the day, when thermal convection is active, convective turbulence is superimposed upon mechanical turbulence and ϵ becomes greater—possibly much greater than is indicated by Eq. 9. The concave curvature of the wind gradients is a consequence of this increase in the coefficient of turbulent mixing. As momentum is transported downward more actively the tendency for differences in velocity in adjacent layers to be equalized increases, and the wind profile becomes more nearly vertical except very close to the ground surface.

At night when the ground, having been cooled by radiation, cools the adjacent air, the lowest layer of air becomes cooler than that above it and a stable lapse rate is created. The air next to the ground, being cooler, is denser than that above and, if moved mechanically upward into a faster moving layer, would tend to move down again instead of mixing with the air in the new environment. In this instance ϵ becomes less than is indicated by Eq. 9. Momentum is transported downward less actively, the tendency for velocity differences in adjacent layers to be equalized diminishes, and the velocity profile departs more than ever from the vertical except within a foot or two of the ground.

This physical picture of the variation of ϵ with stability is in accord with the well-known phenomenon that the diurnal march of wind velocity a few hundred feet aloft (on the top of the Eiffel Tower, for example) is opposite to that close to the ground.

The suspension of sediment in water will tend to make the fluid more stable. The experiments by Mr. Vanoni (7) confirm this. Mr. Vanoni's semilogarithmic plots of velocity profiles at the center of a flume depart quite widely from the logarithmic law and display the same convex curvature that has been observed in a stable atmosphere. This is contrary to his statement that, "The data follow the logarithmic law very closely except near the bottom where the measured velocities exceed those predicted from the law" (7*a*).

Mr. Vanoni reports that the velocity of sediment laden water is greater than that of clear water, which fact he correctly ascribes to a decrease in ϵ. He thinks that the decrease in ϵ is due to a decrease in the value of K, the von Kármán universal turbulence constant. Of course, ϵ will decrease with increasing density (see Eq. 9) but at least in the free air density differences in the profile of stable air are so small that their effect on ϵ must be negligible. The increase of density due to sediment suspension likewise falls far short of being sufficient to account for the decrease in ϵ.

The data plotted in Fig. 7 suggest a variation in ϵ from top to bottom of the tank that may be due to a changing value of the mixing length with changing sediment concentration. Turbulence was imposed artificially by a "lattice structure" and the author assumes that the fluid was mixed uniformly. Actually the sediment concentration near the bottom of the cylinder was about three times as great as at the top at the beginning of the experiment. Therefore, in line with Mr. Vanoni's findings, the value of ϵ near the bottom would be less and that near the top would be greater than the mean value. Then, when the apparatus is set in motion, mixing near the bottom is less than expected and the sediment concentration does not diminish at the rate called for by Eq. 41. Conversely, near the top of the cylinder, mixing is greater and the concentration of sediment is in excess of that predicted. Only when the sediment is nearly uniformly distributed through the fluid do the author's data verify his theory.

An interesting experiment would be to simulate, in a flume, the unstable conditions that prevail in the atmosphere near the ground on warm sunny days. Heating coils in the bottom of the flume would serve the purpose, and it is to be expected that plots of the velocity gradient on semilogarithmic paper would display the same concave curvature that has been observed in the wind velocity profiles.

The writer is of the opinion that Prandtl's theory of turbulence will need to be modified before it can be applied to the study of sediment transport in streams, since there seems to be considerable evidence that the mixing length varies with height in a sediment laden fluid, according to a law that is different from the one that has been applied successfully to a fluid that is homogeneous throughout.

A. A. KALINSKE,[10] ASSOC. M. AM. SOC. C. E.—By analyzing an especially difficult problem mathematically and then testing the validity of his development by making experiments under certain controlled boundary conditions, the author has presented a very fine contribution. His analyses and experiments lead him to certain definite conclusions. These all seem to be warranted by the facts at hand and the writer is in entire agreement with them. Of particular significance to the writer are the author's conclusions relating to the influence of the bed composition on the suspended material. This item is of basic importance in developing a relation among the absolute value of suspended material concentration, the bed composition, and the hydraulic conditions at the bed.

In connection with the author's assumption of similarity in turbulent flow between the momentum and sediment transfer processes ($\beta = 1$) the results of further studies that have been made along this line may be of interest (23). A detailed study was made of the value of the mass-transfer coefficient in turbulent flow in open channels, and its directly measured value was used to calculate sediment distribution curves. The calculated sediment distribution curves agreed very well with actual sediment concentration measurements. The turbulence diffusion coefficients were measured by determining the spread in the turbulent flow of an injected mixture of hydrochloric acid and alcohol, the proportions being adjusted so the density of the mixture was equal to that of the water. Point samples of water were taken to determine the spread of the mixture, and the concentration of the chlorides in the samples was determined by titration. This provided a quick and accurate means of obtaining data for direct measurement of the turbulence diffusion coefficient.

The author's comments on the "pickup" of material from the bed and the relation between suspended and bed material were of considerable interest to the writer. This is one of the most important problems relating to suspended sediment transportation. There is no doubt but that, for equilibrium conditions, there exists, for any material size, a definite relation between the amount of that material on the bed, the amount in suspension above the bed, and the hydraulic conditions at the bed. The first attempt to formalize this relation was made by E. W. Lane, M. Am. Soc. C. E., and the writer (12), and certain field data were presented indicating the existence of such a relation. Since 1939, laboratory experiments have been made which have verified this further and have indicated more definitely the parameters that enter into the relation. Experiments with very fine materials also have indicated the existence of such a relation (24). The general functional formula is as follows:

$$\frac{c_0}{c_b} = \phi\left(\frac{w}{\sqrt{\frac{\tau_0}{\rho}}}, \frac{D\sqrt{\frac{\tau_0}{\rho}}}{\nu}\right) \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

in which c_0 and c_b are the concentrations in weight per unit volume of the sediment and liquid mixture for sediment characterized by fall velocity, w, and size, D, in suspension and in the bed, respectively. Laboratory data for materials ranging in size from 0.001 mm to 0.30 mm have been obtained and these data follow the functional relationship indicated by Eq. 68.

---

[10] Associate Director, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

Another item that seems to merit some consideration is the possibility of obtaining solutions to complex differential equations, such as enter into the problem considered by the author, by methods of numerical integration. The integration of Eq. 17*c*, even under simplified boundary conditions and assuming a constant value for ϵ, is a most formidable mathematical problem. Also, the final solution is so complex as to discourage its use. Methods of numerical integration have been used with success in many physical problems such as, for instance, unsteady heat flow; and, although sometimes the calculations are laborious, they are always simple and can be done by nontechnical workers.

To illustrate how numerical integration may be applied to this problem the solution to Eq. 17*c* for the two boundary conditions will be indicated. Consider a point y distance above the bottom and two points which are (Δy) above and below and which will be designated a and b, respectively. Then the values of $\frac{\partial c}{\partial y}$, $\frac{\partial^2 c}{\partial y^2}$, and $\frac{\partial c}{\partial t}$ at the point y can be written in increment form as follows:

$$\frac{\Delta c}{\Delta y} = \frac{c_a(t) - c_y(t)}{\Delta y} \quad \text{.....................} (69a)$$

$$\frac{\Delta^2 c}{\Delta y^2} = \frac{c_a(t) - 2\, c_y(t) + c_b(t)}{(\Delta y)^2} \quad \text{.................} (69b)$$

and

$$\frac{\Delta c}{\Delta t} = \frac{c_y(t + \Delta t) - c_y(t)}{\Delta t} \quad \text{..................} (69c)$$

Substituting Eq. 69 into Eq. 17*c* and letting $\Delta t = \frac{(\Delta y)^2}{2\,\epsilon}$:

$$(c_y)' = \frac{c_a + c_b}{2} + \frac{w\,\Delta y}{2\,\epsilon}(c_a - c_y) \quad \text{...............} (70)$$

in which $(c_y)'$ is the concentration at point y at a time interval Δt later than the concentrations c_a, c_b, and c_y. To use Eq. 70 the values of concentration must be known at $t = 0$ and the increment (Δy) selected; the magnitude of (Δy) to be used will be determined by the accuracy desired. In checking the curves shown in Fig. 7 the writer selected $\Delta\left(\frac{y}{h}\right) = 0.20$ which made $\Delta y = 8.32$ cm, and $\Delta t = 3.94$ sec.

Eq. 70 cannot be used for the points at the top and bottom since for these points the special boundary conditions that apply must be used when substituting into Eq. 17*c*. Thus, for the curves shown in Fig. 7, for the bottom point, $\frac{\partial c}{\partial y} = 0$. The increment equation for the bottom point then becomes:

$$(c_0)' = c_1 \quad \text{.........................} (71)$$

in which, again, $(c_0)'$ is the concentration at the bottom at $(t + \Delta t)$ and c_1 is the concentration at a point (Δy) above the bottom at time t.

For the top point, since $\frac{\Delta c}{\Delta y} = \frac{-\,w\,c_h}{\epsilon}$, the increment equation is:

$$(c_h)' = c_{h-1} - c_h\left[\frac{w\,\Delta y}{\epsilon} + \frac{w^2\,(\Delta y)^2}{2\,\epsilon^2}\right] \quad \text{..............} (72)$$

in which c_{h-1} is the concentration at a point (Δy) below the top at time t. Using Eqs. 70, 71, and 72, the values of sediment concentrations were computed for points having relative values of $\frac{y}{h}$ equal to 0, 0.2, 0.4, etc., for fourteen time increments or a total time of $14 \times 3.94 = 55.16$ sec. The values obtained checked the curve given in Fig. 7 for $t = 54$ sec as closely as this plotting could be read. For $t = 24$ sec the check was not quite as good, but adequate for practical purposes.

For the curves shown in Fig. 11, Eqs. 70 and 72 still apply; however, the bottom conditions are different since in this case $\frac{\partial c}{\partial y_0} = -\frac{w\,A}{\epsilon}$, in which A is the concentration at $y = 0$, $t = \infty$. The increment equation to use for obtaining values for the bottom point is then:

$$(c_0)' = c_1 + A\left[\frac{w\,\Delta y}{\epsilon} - \frac{(\Delta y)^2\,w^2}{2\,\epsilon^2}\right] \ldots\ldots\ldots\ldots\ldots (73)$$

in which c_1 is again the concentration at a point (Δy) above the bottom at time t. To obtain concentration values for the conditions shown in Fig. 11 the value of Δy is taken as $0.2\,h$, or 9.04 cm, and since $\Delta t = \frac{(\Delta y)^2}{2\,\epsilon}$ the values of c are obtained for time increments of 9.87 sec. Computations for twelve increments or 118.5 sec checked the values given for $t = 120$ sec in Fig. 11 fairly well. Again, the check is not as good for the first few time increments.

One of the principal advantages in solving a differential equation by numerical computation is that complex boundary conditions can be handled, and, also, in the present problem, for instance, the value of ϵ could be varied if necessary with y or t without unduly complicating the problem. Numerical methods, although inelegant and sometimes laborious, nevertheless have proved to be tremendously powerful tools in many fields of work and have provided answers to problems that could not have been solved by more formal methods.

E. R. VAN DRIEST,[11] ASSOC. M. AM. SOC. C. E.—An excellent example of the mathematical analysis of an engineering problem is presented in this paper. It clearly demonstrates the procedure followed in such a treatment; namely, the establishment of the basic assumptions, the derivation of the fundamental differential equation describing the physical phenomenon involved, the application of the boundary conditions, and the finding of a solution of the differential equation that satisfies the boundary conditions. Furthermore, the paper presents experimental data to substantiate the basic assumptions and the assigned boundary conditions.

The writer is not in full accord with the derivation of Eq. 17*b*. The assumption that $\epsilon_x \frac{\partial^2 c}{\partial x^2} = 0$ does not mean that $\frac{\partial c}{\partial x} = 0$, or that there is no diffusion of suspended matter in the x-direction. However, if it is assumed that the mean concentration is constant all along the x-axis, then there will be no diffusion in that direction; whereupon both $\frac{\partial^2 c}{\partial x^2}$ and $\frac{\partial c}{\partial x}$ will be zero and Eq. 17*b* will become, for the required steady state,

---

[11] Associate Prof., Dept. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

$$\epsilon \frac{\partial^2 c}{\partial y^2} + w \frac{\partial c}{\partial y} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

which of course leads back to the basic Eq. 3. Eq. 17*b* is a case of steady state, but this necessitates a diffusion in the x-direction. Furthermore, the statement concerning the mathematical identity of Eq. 17*b* and Eq. 17*c* is rather difficult to follow. Certainly the solutions of these equations are independent of one another since c in the one case is a function of x and y whereas c in the other is a function of y and t. However, the statement could be interpreted so that the effect of the right-hand member of Eq. 17*b* in maintaining steady conditions is the same as the effect of the right-hand member of Eq. 17*c* in causing growth.

As a matter of generality, it can readily be shown, upon consideration of the flux of sediment into and out of a small element of volume, that the general three-dimensional equation for diffusion in a liquid stream is given by

$$\frac{\partial c}{\partial t} + U \frac{\partial c}{\partial x} + V \frac{\partial c}{\partial y} + W \frac{\partial c}{\partial z}$$

$$= \xi \frac{\partial c}{\partial x} + \eta \frac{\partial c}{\partial y} + \zeta \frac{\partial c}{\partial z} + \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\epsilon_z \frac{\partial c}{\partial z}\right) \ldots (75)$$

in which U, V, and W are the stream velocities in the x-direction, y-direction, and z-direction and ξ, η, and ζ are the settling velocity components in the directions of $-x$, $-y$, and $-z$, respectively. Hence, it is seen that, for horizontal flow parallel to the x-axis with uniform turbulence and sediment concentration a function of y and t only, the y-axis being vertical—

$$\frac{\partial c}{\partial t} = \epsilon \frac{\partial^2 c}{\partial y^2} + \eta \frac{\partial c}{\partial y} \ldots\ldots\ldots\ldots\ldots\ldots (76)$$

Eq. 76 is the same as Eq. 17*c* when w is substituted for η. It will be noted that in the one-dimensional case the concentration is independent of the stream velocity. The fact that Eq. 75 becomes the general heat conduction equation when ξ, η, and ζ are placed equal to zero is interesting.

The author has very cleverly simplified the boundary condition at the bottom of the cylinder by requiring constant hydraulic conditions to prevail at the bed. That the pickup, in the case of run V, for such conditions is constant with time for small concentrations near the bed is probably true as can be inferred from the physical picture, but the pickup would undoubtedly change for larger concentrations near the bed because of the effect of the sediment on the turbulence mixing and the mutual interference of the solid particles. It seems that this action should be somewhat analogous to that of the transfer of heat from a wall of fixed temperature to a turbulent fluid; here the eddies coming in contact with the wall carry away heat at a rate depending upon the mean temperature of the eddies near the wall. The fact that the experimental data agree closely with the theoretical curves proves the author's choice of this bottom condition to be valid.

The author should be commended for his perseverance and ingenuity in accomplishing the theoretical analysis and performing the experimental work.

R. SCHUHMANN, JR.,[12] ESQ.—The treatment of the effect of turbulence on sedimentation, presented in this paper, will be of widespread general value because, both experimentally and theoretically, the simplest possible turbulent-sedimentation system was considered. That is, the particles were in a dispersed state and of one size and one settling velocity; the suspensions were dilute; agitation was uniform with respect to depth; and there were no horizontal currents. Because of the complicating effects of variable particle size, flocculation, concentrated suspensions, variable turbulence, etc., only empirical data can be obtained in practicable sedimentation systems.

The sedimentation systems with which the writer is familiar occur in separation operations for dressing minerals. These operations pertain not only to the separation of suspended matter from fluids but also to the grading of particles according to size, and to the separation of particles of one mineral from those of another when the minerals differ in specific gravity. Turbulence, induced both by flow and by mechanical agitation, undoubtedly plays a large rôle in some of these operations, and Ensign Dobbins' methods seem to afford a good starting point for fundamental study.

The writer is particularly interested in the observations on pickup, since various devices for separating minerals require maintaining conditions of selective pickup at the bottom of a stream. Gold, cassiterite, and other heavy minerals are separated from quartz and other light gangue minerals in this way. In some respects the "rheolaveur-launder" process for separating coal and slate is also of the same nature.

H. B. SOUTHWOOD,[13] ESQ.—Although limited in scope by the necessity for simplification in order to arrive at equations possible of solution, the author's study provides additional verification and extension of principles forming the basis for the largely qualitative analyses on which the engineer must often depend when he deals with problems of open-channel flow involving sediment transportation and deposition. The definition of any of the factors relating to these phenomena, however circumscribed may be the conditions under which the analysis is made, is a step forward and eliminates at least some part of the guesswork included in the process generally termed "engineering judgment" or "engineering intuition." The author's rational approach, resulting in an expression of the relations between such factors as concentration at any point, pickup, hydraulic conditions at the bed, and characteristics of the bed material, certainly constitutes such a forward step. Direct application of the results to the solution of practical problems must necessarily await the formulation of methods whereby turbulence can be classified, evaluated, and measured. Further investigations of turbulent suspensions may result in the establishment of relations that will permit the measurement of turbulence by means of sediment observations. The development of a relatively simple field technique based on such relations would be an invaluable tool in the study of natural streams and other open channels.

---

[12] Asst. Prof., Mineral Dressing, Dept. of Metallurgy, Mass. Inst. Tech., Cambridge, Mass.

[13] Chf., Hydr. Section, Dept. of Public Works, State of Louisiana, Baton Rouge, La.

Regarding the observed lack of conformity between experimental data and theoretical values as plotted in Fig. 7, the two factors mentioned by the author (upward transportation of material when the cylinder was dropped, and pickup of the material lodging on the grid surface) are considered to have had the described effects. However, at least two additional factors are believed to have contributed to the existence of observed concentrations in excess of plotted theoretical values.

(1) The use of an arithmetical average value for settling velocity in the computations may result in erroneous positions of the theoretical curves. A range in values of from 0.150 cm per sec to 0.344 cm per sec appears rather wide. Analyses of individual samples obtained during the course of the run probably would have given some insight into the effect of this factor.

(2) The method of sampling is subject to errors which may be sufficiently great to account for most of the deviation noted in Fig. 7. The description of sampling procedure indicates that flow through the siphon was in progress for 12 sec to 18 sec prior to beginning of collection of each sample. This should be a sufficient period for the establishment of a field or zone of undetermined lateral and vertical extent in which the pattern of turbulence would be distinctly altered by the velocities of the converging flow entering the siphon tube. Since these sustained velocities undoubtedly had substantial upward components throughout the major portion of the affected field, it follows that concentrations of suspended material entering the sampling tube would be in excess of those existing at the same elevation in other portions of the container.

L. Standish Hall,[14] M. Am. Soc. C. E.—Although the tests and theory described in this paper are primarily intended for application in the design of settling basins, the author indicates that the same principles may be used for studies of sediment load in natural streams. A modification of the formulas would be required before such a general application could be made. The author states that "the simplifying assumptions involved in Eqs. 17 may introduce large errors when applied to the case of a natural stream." This is undoubtedly true.

The settling rate of suspended particles conforms with Stokes' law (25). The lucite used by Ensign Dobbins in the experiments had a specific gravity of 1.194 as compared with a specific gravity of 2.65 for normal stream sediment. The diameter of the lucite particles ranges from 0.138 mm to 0.216 mm. By Stokes' law, these diameters would correspond to diameters of 0.048 mm to 0.075 mm for stream sediment of the usual specific gravity.

Suspended sediment in stream channels normally is composed predominantly of silty loam with some percentage of clay and some percentage of fine sand. Medium or coarse sand, gravel, and boulders ordinarily move on the stream bottom as bed load. The analysis of suspended-stream sediment indicates that normally 70% to 80% is of a smaller diameter than the equivalent of the lucite; also, that from 10% to 20% is of larger diameter. Therefore, the tests are indicative of the performance of larger sizes of suspended sediment in streams.

---

[14] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

In the transportation of sediment in stream channels, the movement of the bed load absorbs a part of the momentum. This action undoubtedly influences the relation between the suspended load and the total load, so that the latter depends on the relative quantity of each type of load carried in the stream channel. Without some such explanation, it is difficult to interpret the results of actual measurements of silt load. For example, the maximum concentration of sediment usually occurs prior to the peak discharge; and the sediment load carried by a stream varies between a rising and a falling stage, being greater in the former, and also varies with the same discharge between different storms (26).

More study is required on the larger problem of the distribution of sediment particles in a natural stream channel because of the greater number of variables that must be introduced. Notwithstanding the limitations of the formulas, Ensign Dobbins' paper is a valuable contribution to the theory of sedimentation.

WILLIAM E. DOBBINS,[15] JUN. AM. SOC. C. E.—Many interesting and important suggestions have been revealed in the discussions of this paper. The ultimate solution to this difficult problem is still to be found. The principal difficulties lie in obtaining the solution to the general form of the differential equation and in finding the complete relationship between the rate of pickup from the bottom and the factors that control it. Professor Kalinske has reported progress on the latter problem.

If these difficulties can be overcome, the resulting equations undoubtedly will be very cumbersome. However, it is much better to introduce simplifications after the correct solution has been found than before. Long and tedious calculations need be made only once since the results can be preserved in the form of tables or charts. The writer hopes that he may have contributed some ideas that will help others who are working on this and other related problems.

---

[15] Ensign, CEC-V(S), U.S.N.R., Woburn, Mass.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2219

THE QUEENS MIDTOWN TUNNEL

By Ole Singstad,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Thomas W. Fluhr, Howard L. King, Francis V. Wagner, William J. Wilgus, S. A. Thoresen, H. G. Cruthers, Leo Geenens, Charles E. Conover, Edward Levy, Robert F. Legge, Orrin L. Brodie, A. A. Eremin, Arthur S. Tuttle, and Ole Singstadt

Synopsis

The engineering features of the Queens Midtown Tunnel, including its function from the standpoint of automotive transportation, its planning, design and construction, are discussed. An attempt has been made, within the permissible limits of space, to emphasize certain features of the project which presented more than the usual problems.

Introduction

The Queens Midtown Tunnel is a subaqueous highway crossing of the East River in New York, N. Y. It extends in an east-west direction between the

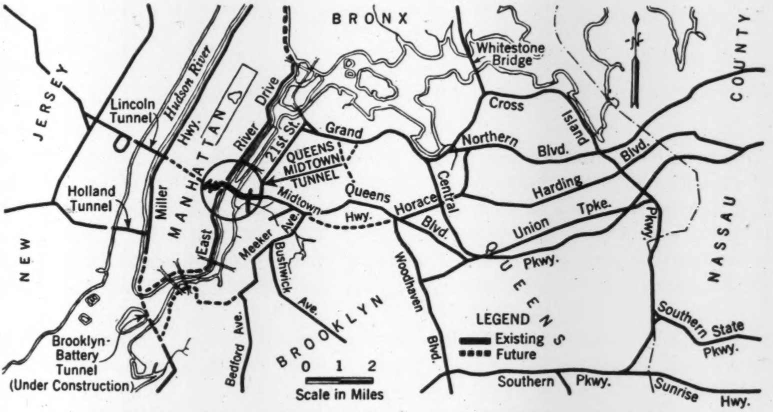

BRONX
NEW JERSEY
Lincoln Tunnel
Hudson River
Holland Tunnel
Miller Hwy.
MANHATTAN
East River Drive
21st St.
QUEENS MIDTOWN TUNNEL
Grand Central
Whitestone Bridge
Cross Island Blvd.
Northern Blvd.
Harding
Queens Midtown Hwy.
Horace Harding
Central
Queens Blvd.
Union Tpke.
Pkwy.
Meeker Ave.
Bushwick Ave.
Bedford Ave.
Brooklyn-Battery Tunnel (Under Construction)
BROOKLYN
QUEENS
Woodhaven Blvd.
NASSAU COUNTY
Southern State Pkwy.
Southern Pkwy.
Sunrise Hwy.
LEGEND
Existing
Future
0 1 2
Scale in Miles
N

Fig. 1.—Highways and Parkways Tributary to the Tunnel as of 1943

Note.—Published in March, 1943, *Proceedings*. Positions and titles are those in effect when the paper or discussion was received for publication.

[1] Chf. Engr., New York City Tunnel Authority, New York, N. Y.

east side of the midtown section of the Borough of Manhattan and Long Island City in the Borough of Queens. Its location, highway connections, and relation to the New York City metropolitan area are shown in Fig. 1, which is based on the present (1943) status of development of highways and parkways. The tunnel exits and entrances in Manhattan connect with the local street system and are conveniently situated with respect to the East River Drive, which extends along the east shore of Manhattan Island. In Queens, the principal connection to the tunnel is the Midtown Highway which, via the Connecting Highway, provides an express route to Queens Boulevard, the main central traffic artery of Queens. The latter, in turn, leads to and from the express parkway and highway system of Long Island. The Queens plaza also provides connections via 21st Street (Van Alst Avenue), 50th Avenue, and Borden Avenue to the local street system and areas in Brooklyn and Queens directly tributary to the tunnel.

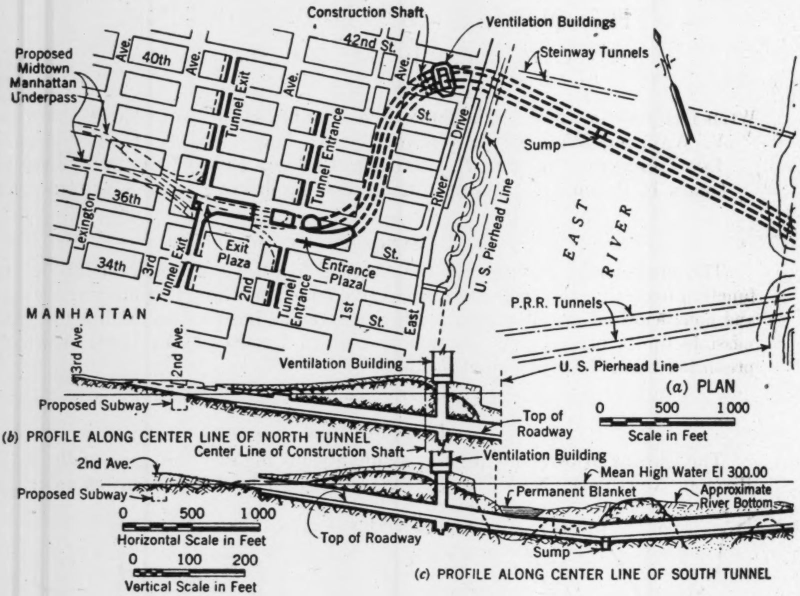
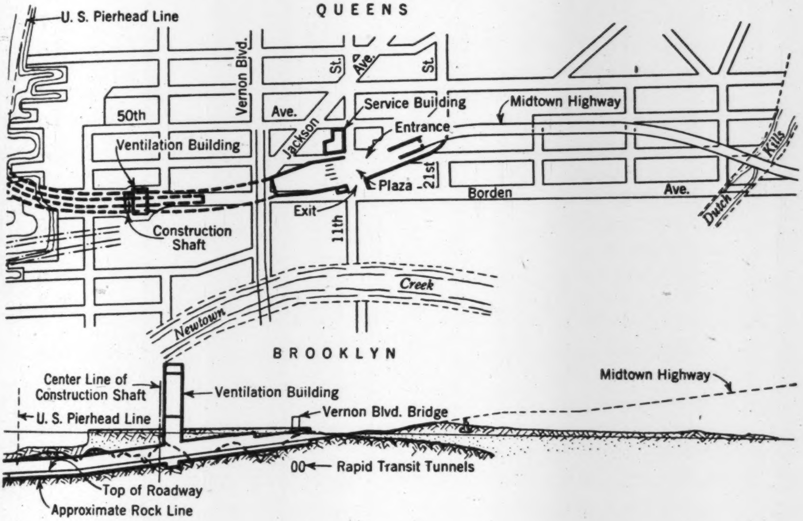

Construction Shaft
Ventilation Buildings
Steinway Tunnels
Proposed Midtown Manhattan Underpass
42nd St.
40th
Ave.
Ave.
Ave.
Ave.
Tunnel Exit
Tunnel Entrance
St.
Drive
River
Sump
36th
Lexington
U. S. Pierhead Line
EAST
RIVER
Exit Plaza
Entrance Plaza
St.
34th
3rd
Tunnel Exit
2nd
Tunnel Entrance
1st
St.
East
MANHATTAN
P.R.R. Tunnels
3rd Ave.
2nd Ave.
Ventilation Building
U. S. Pierhead Line
(a) PLAN
Proposed Subway
Top of Roadway
0 500 1000
Scale in Feet
(b) PROFILE ALONG CENTER LINE OF NORTH TUNNEL
Center Line of Construction Shaft
Ventilation Building
2nd Ave.
Mean High Water El 300.00
Proposed Subway
Permanent Blanket
Approximate River Bottom
0 500 1000
Horizontal Scale in Feet
Top of Roadway
0 100 200
Vertical Scale in Feet
Sump
(c) PROFILE ALONG CENTER LINE OF SOUTH TUNNEL

Fig. 2.—General

Fig. 2 shows the general plan and profile of the tunnel. Between portals, the tunnel structure includes two cast-iron-lined shield-driven tubes, 31 ft in diameter, 3,760 ft long, two structural steel-lined tubes in rock tunnel, 32 ft 10 in. in diameter, 1,051 ft long, and steel bent and concrete structure for a length of 387 ft in Manhattan and 1,051 ft in Queens. In all essential respects, the design of the tubes, interior details, and provisions for ventilation and lighting are of the same design as for the Holland Tunnel and the other modern tunnels for automotive traffic that have been designed and constructed according to the Holland Tunnel pattern. (The Holland Tunnel was built by the New York State Bridge and Tunnel Commission and New Jersey Interstate Bridge and Tunnel Commission under the Hudson River between Canal Street and Broome Street, Manhattan, and 12th Street and 14th Street, Jersey City, N. J. It was opened to traffic November 13, 1927.) The tubes have an interior lining of concrete with walls and ceilings finished in tile. The roadways, which are paved with brick, are 21 ft wide between curbs, increased to a maximum of 23.5 ft on curves, providing for two lanes of westbound traffic in the north tube and two lanes of eastbound traffic in the south tube. The roadway headroom is 13.5 ft.


U. S. Pierhead Line
QUEENS
Vernon Blvd.
St.
Ave.
St.
50th
Ave.
Service Building
Midtown Highway
Jackson
Entrance
Ventilation Building
Plaza
21st
Borden
Ave.
Exit
Construction Shaft
11th
Dutch Kills
Creek
Newtown
BROOKLYN
Center Line of Construction Shaft
Ventilation Building
Midtown Highway
U. S. Pierhead Line
Vernon Blvd. Bridge
Rapid Transit Tunnels
Top of Roadway
Approximate Rock Line

Plan and Profile

The tunnel plazas, paved with brick in areas adjacent to the portals and with concrete beyond those areas, widen from the two lanes at the portals to six lanes for each direction of travel. Ventilation equipment is housed in two buildings, one on each side of the river. Remote control of ventilation and lighting, offices for the supervisory staff, quarters for maintenance crew and tunnel police and equipment are provided for in the service building in Queens. A garage for the emergency wrecking tractors is located at each plaza. Booths (ten single and one double) for the collection of tolls in both directions of travel are situated on the Queens plaza, approximately 432 ft from the tunnel portals.

General data describing the tunnel are as follows:

Traffic.—Estimated maximum capacity, in vehicles per year	16,000,000
Dimensions.—Length of crossing between plazas, in feet (from street grade to street grade)—	
Westbound	7,865
Eastbound	7,400
Maximum Grades Between Portals (%).—	
Up	3.5
Down	4.0
Maximum Grades at the Plazas (%).—	
Manhattan entrance (down)	4.8
Manhattan exit (up)	4.7
Queens (up and down)	4.5
Structural Depths Below Mean High Water, in Feet.—	
At the Manhattan pierhead line	
Roadway	77.0
Top of tube	55.2
At the low point	
Roadway	94.3
Bottom of tube	103.5
At the Queens pierhead line	
Roadway	74.0
Top of tube	52.2
Cost.—	
Construction contracts	$38,888,701.38
Miscellaneous construction items	23,107.81
Operating equipment	195,897.89
Real estate (including acquisition expenses)	6,836,790.81
Interest during construction	2,858,888.90
Engineering, administration, legal, overhead, and miscellaneous expenses	5,269,946.54
Actual total cost of project	$54,073,333.33
Estimated cost of project	$58,365,000.00
Finance (U. S. Public Works Administration).—	
Grant in aid	$11,235,000.00
Forty-year loan, at 4%	$47,130,000.00

HISTORICAL

A preliminary plan for a tunnel between Long Island and mid-Manhattan, extending across town to the west side, was first officially prepared by the president of the Borough of Manhattan in January, 1926. The idea of such a tunnel was endorsed early in the same year by the Regional Plan Association, Inc. (then the Committee on the Regional Plan of New York and Its Environs). In 1923 this committee had included in its scheme for main highway routes in the New York district[2] an east and west traffic axis from Long Island to New Jersey across midtown Manhattan. This plan contemplated the use of the

---

[2] "Highway Traffic," *Regional Survey of New York and Its Environs*, Vol. III, p. 29.

Queensborough Bridge as the East River link in the route; but it was proposed that two new tunnels across the Hudson River be located between 59th Street and the Holland Tunnel, which was then being constructed.

From 1926 to 1935, the 38th Street-East River Tunnel (as it was first designated) was strongly advocated by many interested civic and business associations and groups as a means of:

(*a*) Relieving traffic congestion on existing East River bridges, particularly the Queensborough Bridge;

(*b*) Providing a necessary modern express link for vehicular traffic joining Manhattan Island with the rapidly growing Borough of Queens, the Borough of Brooklyn, and the highway system of Long Island; and

(*c*) Stabilizing the midtown Manhattan area as a center for business, manufacturing, shopping, amusements, hotels, and residence.

The Department of Plant and Structures of the City of New York made preliminary engineering studies and rendered a report on the project in June, 1929. In July of the same year, the Board of Transportation was assigned the work of preparing plans and specifications and starting construction with an initial appropriation of $2,000,000. The financial depression following the stock market crash of 1929 resulted in the rescinding of the greater part of this appropriation and the suspension of work on the project in 1931, at which time a plan for the East River crossing had been formulated.

The act of the New York State Legislature creating the New York City Tunnel Authority set up the East River crossing of the original project as separate from the east-west crossing of Manhattan Island; and in January, 1936, the Public Works Administration (PWA) allocated $58,365,000 for the construction of the East River crossing, or Queens Midtown Tunnel, with the understanding that the city would construct an express connection from the tunnel plaza in Queens to the main highway system of Long Island, to be completed by the time the tunnel was opened to traffic. The first work of the Tunnel Authority was a restudy of the tunnel plan and the development and adoption of the plan shown in Fig. 2. Ground was broken for the project on October 2, 1936, by President Franklin D. Roosevelt, and, after a construction period of slightly more than four years, regular operation of the tunnel was started on November 15, 1940.

TUNNEL HIGHWAYS, CONNECTIONS AND TRAFFIC

In conformity with the promise made by the city at the time the tunnel project was financed, the scheme of connections and highways in Queens contemplated the completion, simultaneously with that of the tunnel, of the extension of Midtown Highway as far east as the junction of Horace Harding Boulevard, Queens Boulevard, and Woodhaven Boulevard. Although the construction of this extension of the Midtown Highway has been deferred temporarily, its necessity is apparent, as the existing alternate route via Connecting Highway and Queens Boulevard has insufficient capacity to provide for the normal growth of local, tunnel, and other interborough traffic using it, and is, in other respects, unsuitable and inadequate as part of the main east-west route of the tunnel.

In Manhattan, where the congestion which now exists on the crosstown streets of the midtown area west of Third Avenue and east of Eighth Avenue is a serious impediment to vehicular travel, the proposed Midtown Manhattan Underpass will provide an express route between the Manhattan plazas of the Queens Midtown Tunnel, the West Side Express Highway, and the Lincoln Tunnel. Its construction, with that of the highway improvements planned in Queens, is necessary to complete the express highway system joining Long Island, Manhattan, and New Jersey by a route through the center of the metropolitan area, of which the Queens Midtown Tunnel is a part.

The Queens Midtown Tunnel is the seventh East River crossing for vehicular traffic to be constructed and the first tunnel crossing of the East River for automotive vehicles. Vehicular traffic crossing the East River had increased twentyfold from 4,500,000 trips per annum in 1910 to 90,000,000 trips per annum in 1936. The Queensborough Bridge, which is the most heavily traveled of these crossings, increased in vehicular traffic from 769,000 trips in 1910 to a maximum of 36,000,000 in 1933, a toll-free volume approximately equal to that of all the Hudson River crossings from the Battery to Tarrytown. The Queens Midtown Tunnel, charging a base toll of 25¢ for passenger cars and light trucks, functions principally as an express alternate to relieve this load.

Maximum peak hour traffic recorded for the tunnel to date (March, 1943) occurred on Wednesday, December 24, 1941, between 8 a.m. and 9 a.m., when 2,075 cars used the westbound tube. This peak was easily handled with five of the toll booths on the Queens plaza, and smooth flow of traffic through the tunnel was maintained, giving a practical demonstration of the adequacy of entrance and exit plazas and approaches.

Tunnel Plan

The East River crossing, or the present plan of the Queens Midtown Tunnel, was authorized for construction in 1935, after careful consideration had been given to prior plans such as those of the president of the Borough of Manhattan (1926) and the Board of Transportation (1931). The Lincoln Tunnel under the Hudson River was already being built. Every one who had investigated these projects from the first inception had agreed that the East River and Hudson River crossings should be located so that an under-Manhattan connection could be conveniently provided. Therefore very little choice was possible in the location of the Queens Midtown Tunnel. The location was further limited by the Pennsylvania Railroad tunnels at 33d Street and by the Steinway Tunnel at 42d Street.

That this location was the correct one for both the Hudson and East River tunnels was established by traffic surveys made on both rivers showing that the midtown section was the one which was most urgently in need of traffic relief on both rivers; and, although the general location of the Queens Midtown Tunnel was established by legislative enactment, there was real justification for such location. The East River crossing received intensive study by the

Tunnel Authority, and a great many study layouts were made before the final plan was adopted. All of the earlier plans developed by other city departments showed approach plans for the tunnel on the Manhattan side which were deemed unsatisfactory from the point of view of the free and expeditious handling of traffic without congestion.

Conditions affecting the choice of plaza and approach locations in Manhattan included the character of the topography, real estate costs, and street and tunnel traffic considerations. These conditions limited practical locations of the Manhattan plazas to the area south of 40th Street between Third Avenue and the East River where the ground surface is comparatively flat and low in elevation, where values of real estate were generally within allowable limits, and where traffic on crosstown streets is not heavy. West of Third Avenue the street grades rise more rapidly into the Murray Hill section of Manhattan, increasing, in some instances, to a rate of almost 7%. The high value of real estate in certain blocks eliminated these from consideration as possible sites for the plazas, and, in general, the values of real estate increase, as the center of the midtown section is approached, to prohibitive sums in so far as the allowable costs of the tunnel project were concerned. Local traffic on the crosstown streets east of Third Avenue is comparatively light, whereas, if the tunnel plazas were built farther west, the great increase in volume of traffic would seriously affect the convenience of travel.

The War Department required certain minimum depths of channel and tunnel structure between U. S. pierhead lines, the prescribed minimum depth of the top of the tunnel tubes at the Manhattan pierhead lines being 48 ft below mean low water with a minimum depth of channel of 38 ft below mean low water. To provide a minimum cover of 13 ft over the tubes, the depth of the structure at the pierhead line was actually fixed at 51 ft below mean low water. The crown of the tunnel roadway is 21.77 ft below top of tube. It would have been impossible to reach street grade at the desired points with direct alinement from the depth of roadway so fixed at the pierhead line and stay within maximum permissible tunnel grades. Accordingly, a plan was adopted in which the line of the East River crossing from Borden Avenue, in Queens, extends to a point at the Manhattan pierhead line between the lines of 41st Street and 42d Street extended, the necessary distance being developed along First Avenue to the plaza locations between 36th Street and 37th Street. This crossing of the river is no greater in length between pierhead lines than the direct one.

The entrance and exit plazas in Manhattan are separated so as to avoid concentrating tunnel traffic in too small an area, and to avoid conflict between traffic bound for the tunnel and that from it. Wider separation was not practicable, as moving the exit plaza north would have resulted in steeper grades and greater real estate costs, whereas restriction on the use of park property prevented moving the entrance plaza farther south. The exit plaza passes under Second Avenue above the level of the proposed Second Avenue subway; it is bridged between First Avenue and Second Avenue to permit tunnel-bound traffic to reach the entrance plaza from points north of 37th Street.

The surface approaches are three lanes wide, those for exit extend between Second Avenue and Third Avenue connecting with all crosstown streets from 34th Street to 41st Street, inclusive, with the exception of 36th Street, where there is an underpass for southbound tunnel traffic to prevent conflict with traffic on 36th Street bound for the tunnel. The entrance approaches extend between First Avenue and Second Avenue and connect with all crosstown streets between 34th Street and 40th Street, inclusive. By this scheme of surface approaches connecting with the plazas, tunnel traffic is not concentrated on any one crosstown street, and does not conflict with the traffic on the heavily traveled north and south avenues.

The Manhattan plazas and approaches of the tunnel are so designed as to permit the construction, at any time in the future, of plazas and approaches for the Midtown Manhattan Underpass, with provision for direct connections between the underpass and the tunnel.

In Queens, real estate values were relatively low and the slope of the terrain was very gradual east from the river, reaching an elevation of 15 ft above mean high water at Borden Avenue and 21st Street.

Under these favorable conditions, the most advantageous location for the plaza was found north of Borden Avenue and east of Vernon Boulevard and Jackson Avenue (see Fig. 2), where connections to Midtown Highway and 21st Street (Van Alst Avenue) were conveniently made.

At the U. S. pierhead line on the Queens side, the War Department prescribed that, with due consideration for the thickness of permanent protective covering decided necessary, the depth of top of tubes below mean low water should be a minimum of 43 ft. Street level at the plaza was easily reached with suitable grades from a depth at the pierhead line 5 ft greater than the prescribed minimum. The lowering of the tunnel profile below the minimum by this amount provided additional thickness of rock cover in the full rock sections and permitted a greater length of tunnel to be excavated entirely in rock.

The results of studies for the location of ventilation shafts and buildings on either side of the East River placed the shaft and building in Queens in Borden Avenue about 50 ft east of Front Street and, in Manhattan, about 250 ft east of First Avenue between 41st Street and 42d Street. This original location of the Queens shaft contemplated the caisson method for its construction. When bids were received for this work, which was included in the contract for the river tunnels, it was found that a substantial saving could be effected by moving the shaft about 200 ft farther east where it could be constructed in the open-cut section, and, by agreement with the contractor, this change of location and design was made.

The low point of the tunnel is approximately 1,168 ft east of the Manhattan ventilation shaft in the rock reef near the middle of the East River, making it possible to construct the midriver sump and pump chamber entirely in rock between tubes.

On the line of the river crossing of the tunnels, the west channel of the East River reaches a depth of 66 ft below mean high water. This depth would have required lowering the tunnel profile considerably to maintain a suitable cover over the tunnel, thus increasing the air pressures necessary for construction. Furthermore, as this channel depth occurred near the Manhattan pierhead line, it would have increased the length of line necessary to reach the surface in Manhattan. The profile of the tubes at this point was made high enough to avoid these conditions, and for approximately 150 ft in each tube, the top is above the original bed of the river (maximum 8 ft). The War Department permit included provision for a permanent blanket of clay overlain with a protective layer of riprap of sufficient extent and depth to maintain a minimum cover of 13 ft over the tubes in this area.

Geologic Data

Test borings were made at the plaza and approach sites and along the line of the tunnels to determine the topography of the rock floor, the character and kind of rock and overburden, and, in the land areas, ground-water elevations. When rock was encountered, a core was taken of sufficient length to establish the material as ledge rock, generally a minimum of 10 ft.

Borings along the river crossing were staggered north and south of the tube locations and were generally not farther apart than 200 ft measured along the line of the tunnels. They were taken closer together where necessary, to develop the contour of the rock surface.

The rock floor was disclosed to be exceedingly irregular in both elevation and character, consisting of Manhattan schist under Manhattan and changing progressively to Inwood limestone, Fordham gneiss, Hell Gate dolomite, and Brooklyn injection gneiss along the tunnel line to Queens. There were zones of disintegrated rock lying at low points, generally at the contacts of one formation with another. The overburden was indicated to be mostly pervious glacial deposits consisting of sand, gravel, and numerous boulders. The face of tunnel excavation was shown to pass many times in and out of full rock, mixed ground, and soft ground.

Design of Structures

Lining for Shield-Driven River Tunnels.—Cast-iron lining of the usual design was chosen for the shield-driven tunnels as best adapted to resist shield thrusts and to provide rigidity and watertightness. The length of straight ring was 32 in., or 2 in. greater than the maximum length of ring heretofore used in the river tunnels around New York City. This longer ring was used with complete success and resulted in a saving of material and erection costs and in faster progress as compared with shorter ring lengths. A width greater than 32 in. was impracticable at the time because of foundry limitations; 1¼-in. and 2½-in. taper rings, used to provide for vertical and horizontal curvature and corrections to alinement and grade, were a maximum of 32 in. in length.

Each ring was made up of fourteen segments and a key, and two weights of cast-iron lining were used. The heavier ring, as shown in Fig. 3, with 14-in. flanges and weighing approximately 22.7 tons per ring, was used in soft ground and mixed face or where the character or thickness of rock cover required it; the lighter ring with 11-in. flanges and weighing approximately 12.5 tons was used in rock face where the cover was adequate.

The segments making up the tunnel ring were bolted together with four high-strength steel bolts in each cross flange, and adjacent rings were bolted together with eighty-five high-strength steel bolts in the circumferential flange, the bolts for the heavy rings being 1¾ in. in diameter and those for the light rings, 1⅝ in. in diameter. The specified yield point for the bolts was 85,000 lb per sq in., and the ultimate strength was 110,000 lb per sq in.

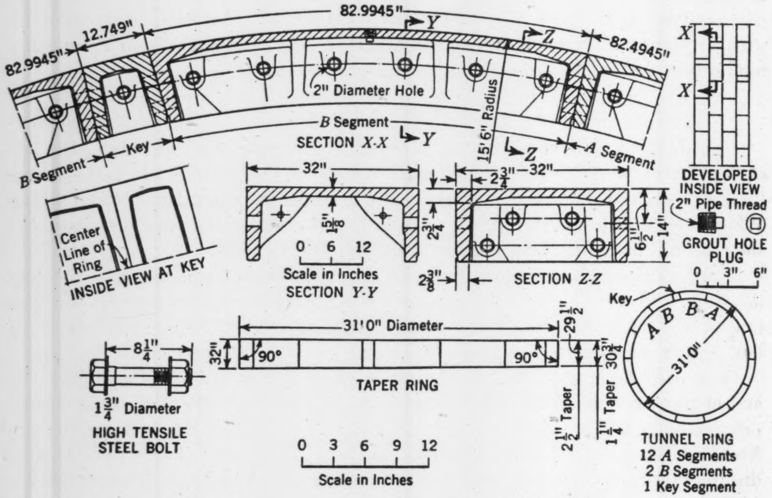

82.9945"
12.749"
82.9945"
Y
Z
82.4945"
X
X
2" Diameter Hole
B Segment
SECTION X-X
Y
15'6" Radius
Z
A Segment
B Segment
Key
DEVELOPED INSIDE VIEW
2" Pipe Thread
GROUT HOLE PLUG
0 3" 6"
32"
1⅝"
0 6 12
Scale in Inches
SECTION Y-Y
2¾"
2¾"
32"
2¾"
6½"
14"
2⅜"
SECTION Z-Z
Center Line of Ring
INSIDE VIEW AT KEY
Key
A B B A
31'0"
31'0" Diameter
29½"
8¼"
32"
90°
90°
30¾"
TAPER RING
1¾" Diameter
HIGH TENSILE STEEL BOLT
2½" Taper
1¼" Taper
0 3 6 9 12
Scale in Inches
TUNNEL RING
12 A Segments
2 B Segments
1 Key Segment

Fig. 3.—Heavy Cast-Iron Tunnel Lining Details

The load factors which were used in designing the heavy cast-iron lining are the weight of the cast-iron ring and that of the concrete lining; the full weight of earth and water above the horizontal diameter of the tunnel; lateral earth and water pressures; and the effect of buoyancy. The load factors used in designing the light cast-iron lining were the weight of the cast-iron and concrete linings, full hydrostatic pressure, and a load of 10 ft of rock packing above the top of the tunnel. The ring as thus designed was investigated for localized grout pressures.

The strength and efficiency of the segmental cast-iron ring depend on the capability of the bolts in the cross flanges to transmit the tunnel ring stresses between segments; and, with the cross joints of adjacent rings staggered, the bolts in the circumferential flanges connecting the adjacent rings, with proper bolt tightening, add materially to the strength of the cross joints by the splicing action provided.

An initial tension in the bolts of 25,000 lb per sq in. was specified. The bolts, which were erected with steel washers and hemp rope grommets, were tightened by hand, generally by two to three men on a ratchet wrench. A 5-ft wrench was used for the heavy cast-iron lining and a 4-ft wrench for the light lining. Actual bolt stresses, observed some time after the tunnel lining was erected, varied from intensities considerably less than 25,000 lb per sq in. to stresses approaching the yield point of the bolts. It is now considered good practice to make an allowance of about 5,000 lb per sq in. above the required bolt stress to offset any plastic yield effect in the entire bolt assembly under sustained loading.

Initial bolt stresses that are too low, and lack of uniformity in stress, may contribute to some leakage, especially in the case of the light cast-iron lining, as the flanges are thinner and more flexible than those of the heavy cast-iron lining and the surrounding material has no tendency to silt up any small crevices that may exist between flanges. Since even very small leaks may be objectionable as affecting the appearance of the tile-finished interior of a modern vehicular tunnel, it would be desirable to use machine tightening for the bolts in the light cast-iron lining. As a further safeguard against leakage, the light cast-iron lining may be designed with five bolts in the cross flanges. The fifth or center bolt of each cross flange has been omitted in recent tunnel design as theoretically carrying no stress.

The outside faces of all flanges of the Queens Midtown Tunnel lining were machined according to standard practice. The specified tolerance on the width of ring segments was very small (0 to $-\frac{1}{64}$ in.), in order to secure as tight a fit as possible along circumferential flanges. The lining was calked for watertightness with lead wire in the grooves formed at the inner edges of the flanges, the calking rebate on each segment being specified to have a maximum depth of $\frac{3}{16}$ in. so that a calking groove would be a maximum of $\frac{3}{8}$ in. in width when two segments were joined. Heavy machining on flange faces sometimes narrowed this groove so as to make calking difficult.

Permanent tunnel tie rods at ceiling and roadway levels were placed in the Queens Midtown Tunnel lining for short distances in both tubes under the Manhattan bulkhead area, where there might be changes in outside loadings after the tunnel was built which would possibly cause variations in the stresses in the lining. Permanent tunnel tie rods were also placed at the midriver sump, where certain segments of the tunnel lining were not erected because of the construction of the cross passage between tubes.

Midriver Sump and Pump Chamber.—The midriver sump and pump chamber are between the tunnels and consist essentially of two tubes, each one composed of segmented cast-iron rings with an interior lining of concrete. The axis of the pump chamber is at right angles to the main tunnels. The pump chamber rings are 16 ft in outside diameter, giving sufficient inside space, in addition to that required by the pumps, for a pedestrian cross passage between the main tunnels. The sump is 42 ft long, out to out, with rings of 14-ft outside diameter, and is located below the pump chamber with its axis parallel to, below and about midway between, the main tunnels. The pump chamber and cross passage are connected to the main tunnels by short rectangular structural steel frames and plates.

Rock Tunnels and Steel Bent and Concrete Structure, Manhattan.—In order to provide for the roadway widths which, because of the curved alinement, are increased to 23.5 ft in Manhattan, the metal lining, in the length of the rock tunnel sections extending from east of the Manhattan ventilation shaft to 38th Street, is designed with an outside diameter of 32 ft 10 in. These rock sections of tunnel were constructed as far as 41st Street under the contract for the river tunnels. Another contract was let for the structure from 41st Street to the tunnel portals, located just west of First Avenue between 36th Street and 37th Street, consisting of a continuation of the metal-lined rock tubes to 38th Street and steel bent and concrete structure from 38th Street to the portals. All of the metal lining, 32 ft 10 in. in diameter, was furnished under this contract, three alternative designs being submitted to the contractors for competitive bidding.

The three designs were based on the same strength factors, namely: To resist full hydrostatic pressure, plus a crown load of 10 ft of broken stone and grout estimated to amount to 1,500 lb per sq ft. One alternative was a lining 32 ft 10 in. in diameter, constructed of light cast-iron rings 32 in. long. The other two alternatives were linings of structural steel, designated as type 1 and type 2. Both type 1 and type 2 were rings of ten segments 5 ft in width and about 113 in. long and a key segment about 16 in. long. The weights of the two types were each about 2.2 tons per linear foot of tunnel. Type 1 was made of skin plates welded to circumferential and segmental flange plates 5¼ in. deep. The bulk of the stiffness and circumferential strength of the ring was provided by two split I-beams placed at the quarter points of the ring, bent to the tunnel radius, welded, and spliced with bolted connection plates at each cross joint to form a continuous circular rib. Type 2 also had segmental flange plates welded to the skin plate at each end, but the skin plate was bent to form circumferential flanges 9½ in. deep. One split I-beam ring was provided instead of two as on type 1.

Upon receipt of competitive bids, structural steel type 2 was found to be the least expensive of the three alternatives, and the contract was let on that basis. A third design in structural steel was submitted by the contractor (see Fig. 4). This design, which presented a saving in cost to the Tunnel Authority, differed from type 2 in that specially rolled bulb angles were provided for the circumferential flanges instead of bending up the skin plates. It was approved and installed.

The steel bent and concrete structure in Manhattan (Fig. 5(*a*)) extends from the circular tunnels, where the junction is made by bellmouths around the tubes, southwest to the plaza portals. The type of construction consists of structural steel bents spaced about 5 ft on centers down to the roadway level, below which the fresh air duct is of inverted reinforced concrete arch construction. Since the invert is in rock for its full length, this construction was found to be the most economical.

There is very little regularity to the steel bent and concrete structure in Manhattan. As the grade ascends to go into the plaza, the earth cover diminishes making it necessary to bring the exhaust air ducts from the top to one side of the structure. Special construction was necessary to form the junction between the circular tunnels and the rectangular section and also at the portals, where the structure is belled out to provide a gradual transition from tunnel lighting to daylight. Where the west tunnel passes under private property in the block between 38th Street and 37th Street, the structure was designed to support future 12-story storage warehouse buildings with floor live loads of 200 lb per sq ft.

Steel Bent and Concrete Structure, Queens.—The Queens land section was constructed entirely in trench from the construction shaft to the Queens plaza portal, a distance of 1,051 ft. For about one half of this length the structures for eastbound and westbound roadways were separated with a berm of earth and rock, and, for the remaining length, a single structure carries both roadways. The structural type is a steel bent spaced about 5 ft on centers, embedded in concrete and waterproofed with fabric and brick in asphalt mastic (see Fig. 5(*b*)).

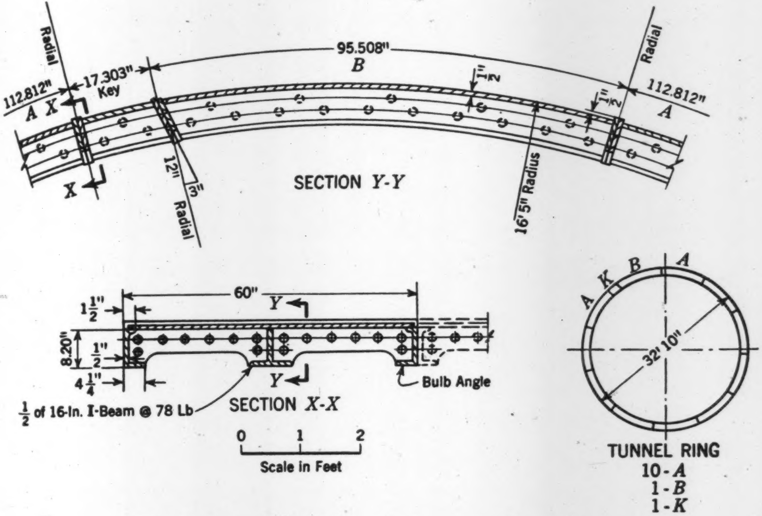
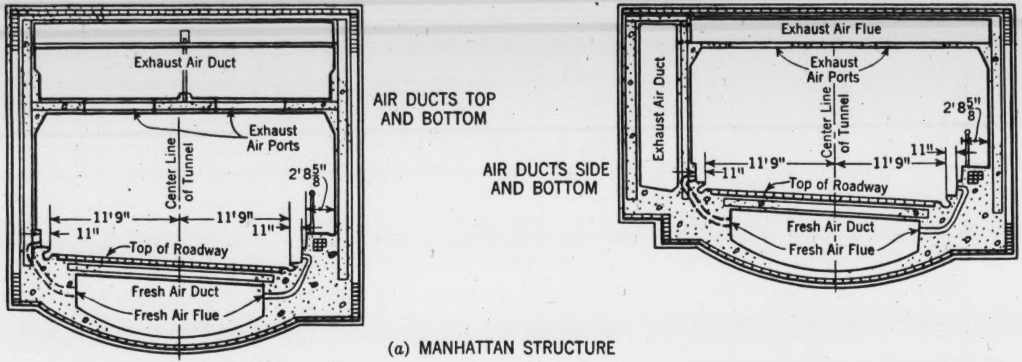


Fig. 4.—Steel Lining Details Designed for Use in Rock

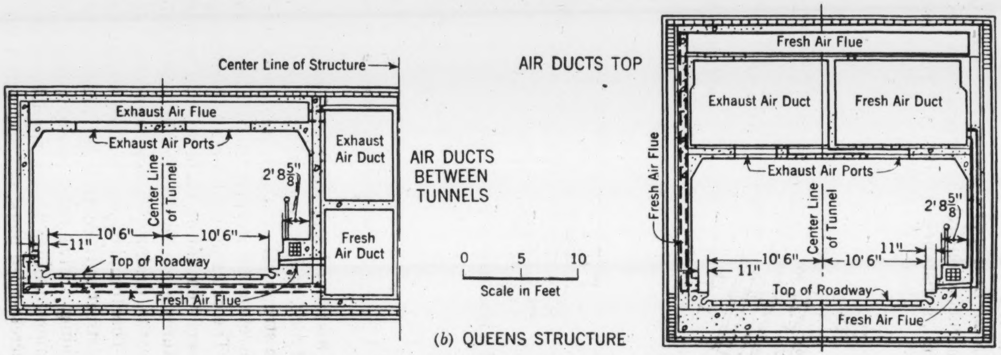
The extreme west end of this land portion serves as the foundation for the Queens ventilation building, and provision was made in its design for all of the features normally located in a ventilation shaft—air ducts, emergency stairs, elevator, drainage sump, pump chamber, cable splicing chambers, etc.

This foundation structure was brought up to an elevation about 3 ft above the street surface and was designed to form the first floor of the ventilation building. Five pairs of steel girders, 5 ft in depth, distribute the building column loadings to the tunnel structure below and to ten circular steel-shell concrete-filled piers, varying from 4 ft to 4.5 ft in diameter, five located on each side of the tunnel structure and extended down to rock.

The depth of the structure made it possible to effect a substantial economy by placing the fresh air ducts above the ceiling for a large portion of the section instead of in the normal position below the floor. Toward the portal end of the section, the shallow depth of cover over the structure entailed the transition of


Exhaust Air Duct
AIR DUCTS TOP AND BOTTOM
Exhaust Air Ports
Center Line of Tunnel
2'8$\frac{5}{8}$"
11'9"
11'9"
11"
11"
Top of Roadway
Fresh Air Duct
Fresh Air Flue
Exhaust Air Flue
Exhaust Air Duct
Exhaust Air Ports
AIR DUCTS SIDE AND BOTTOM
Center Line of Tunnel
2'8$\frac{5}{8}$"
11'9"
11'9"
11"
11"
Top of Roadway
Fresh Air Duct
Fresh Air Flue

(a) MANHATTAN STRUCTURE


Center Line of Structure
Exhaust Air Flue
Exhaust Air Ports
Exhaust Air Duct
Fresh Air Duct
AIR DUCTS BETWEEN TUNNELS
Center Line of Tunnel
2'8$\frac{5}{8}$"
10'6"
10'6"
11"
Top of Roadway
Fresh Air Flue
0
5
10
Scale in Feet
AIR DUCTS TOP
Fresh Air Flue
Exhaust Air Duct
Fresh Air Duct
Exhaust Air Ports
Fresh Air Flue
Center Line of Tunnel
2'8$\frac{5}{8}$"
10'6"
10'6"
11"
11"
Top of Roadway
Fresh Air Flue

(b) QUEENS STRUCTURE

FIG. 5.—STEEL BENT AND CONCRETE LAND SECTIONS.

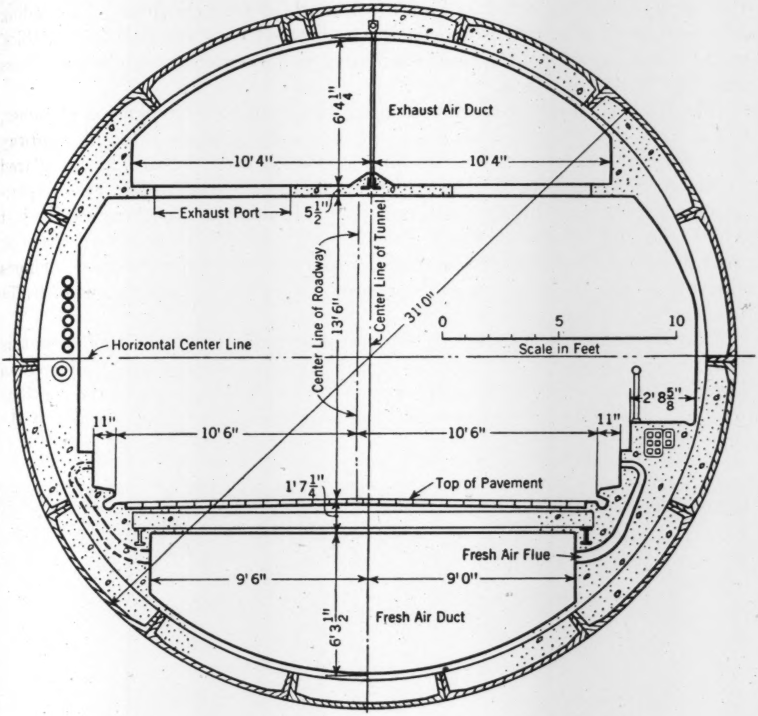

Exhaust Air Duct
6'4¼"
10'4"
10'4"
Exhaust Port
5½"
Center Line of Tunnel
Center Line of Roadway
13'6"
31'0"
0
5
10
Scale in Feet
Horizontal Center Line
2'8⅝"
11"
10'6"
10'6"
11"
1'7¼"
Top of Pavement
Fresh Air Flue
9'6"
9'0"
Fresh Air Duct
6'3½"

FIG. 6.—TYPICAL CAST-IRON-LINED TUNNEL SECTION IN EARTH

both the fresh air ducts and the exhaust ducts from above the ceiling to a location between the tubes.

Tunnel Interior.—The interior features of each tube include: (1) An interior concrete lining in the metal-lined sections of the structure; (2) roadway slab and paving; (3) a walk on the left of the direction of travel, approximately 2 ft 8 in. wide and 3 ft 6 in. above paving level; (4) a fresh air duct generally under the roadway, connected to the roadway space by fresh air flues; and (5) an exhaust air duct, generally above the ceiling, connected to the roadway space by openings in the ceiling. Fig. 6 shows a typical tunnel section.

The roadway slab consists of transverse 10-in. I-beams spaced 21 in. on centers with longitudinal reinforcing rods above and below, and transverse reinforcing bars between the I-beams. The roadway slab is designed in accordance with the theory advanced by H. M. Westergaard,[3] M. Am. Soc. C. E., in 1930. The roadway beams rest on concrete benches and when placed over fresh air flues are supported on 8-in. I-beams which span the flues. The ceiling slab, containing the exhaust ports, is a reinforced concrete slab 5.5 in. thick supported at the sides of the tunnel and hung at the center by stainless steel bars generally 10 ft 8 in. apart.

---

[3] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, 1930, Vol. 11, No. 1. pp. 1–23.

The fresh air flues are not terminated in a continuous expansion chamber, as has been the practice heretofore, but discharge directly into the roadway space through openings 5 ft 4 in. long and 3 in. wide, formed in the glazed architectural terra-cotta blocks at the front of the benches (see Fig. 7). Experience has shown that the distribution of fresh air along the roadway can be just as efficient and continuous with direct discharge into the roadway space as through a continuous expansion chamber, particularly when the fresh air flues on either side of the roadway are staggered, and when the movement of traffic furnishes longitudinal distribution.

Clay ducts under the sidewalk are used for low-tension cables as, from this location, wiring to control boxes for signals, telephones, etc., was simpler than if low-tension conduits were placed in the opposite wall as has been the previous practice. Continuous lighting-cable conduits are placed in the walls of the upper air duct so that no conduits of any kind are contained in the ceiling slab.

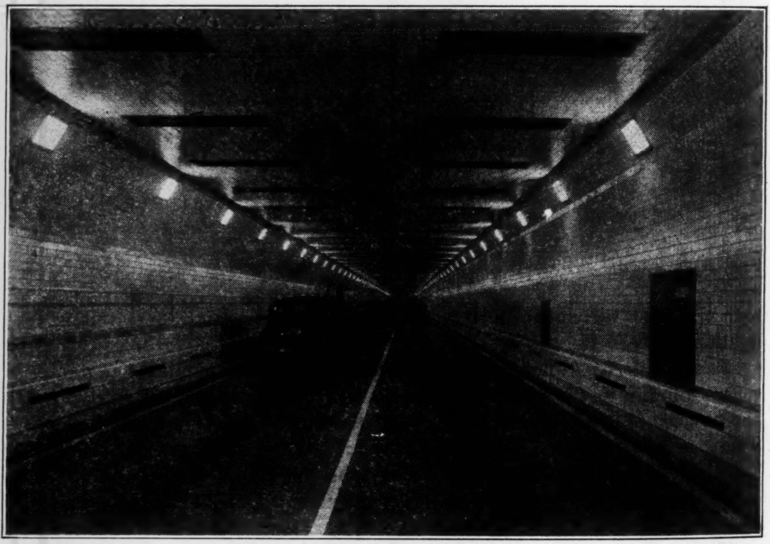


FIG. 7.—FINISHED INTERIOR OF TUNNEL

In each tunnel, between ventilation buildings, six 4-in. asbestos-cement conduits are provided in the wall opposite the sidewalk. The lower three conduits are so spaced as to be suitable for high-voltage transmission cables, and the upper three conduits are suitable for lower-voltage cables. The six conduits are available for rental, and are carried up the shafts to street-level splicing chambers in each ventilation building, where connections may be made to private transmission lines entirely independent of tunnel services.

Low-tension splicing chambers in the sidewalk bear a more or less constant relation to the police posts, which house the telephones and manual signal controls. These posts are spaced at such intervals as to provide visibility and easy access to the portions of the tunnel between them, in general about 250 ft. Two small niches, each holding one 15-lb carbon-dioxide fire extinguisher, are in the wall above the sidewalk at each control station and two additional small fire extinguisher niches are installed midway between control stations. This spacing makes fire extinguishers readily available to the tunnel policeman on duty at the control station. Larger fire extinguisher niches, each niche containing three 15-lb carbon-dioxide fire extinguishers, are installed in the wall on the side opposite the sidewalk. These niches are spaced at approximately 250-ft intervals for use by the tunnel police or by others. Niches containing 100 ft of racked fire hose attached to a $2\frac{1}{2}$-in. valved outlet from the 6-in. tunnel water service pipe are spaced about 125 ft apart. The chemical fire fighting equipment distributed through the tunnel (aggregating 350, 15-lb carbon-dioxide fire extinguishers) is ample for all small fires and is sufficient for the control of larger fires until such time as the emergency wrecking tractor and the police car (each of which is equipped with a foam generator type of fire extinguisher), which answer all fire calls, can arrive on the scene.

Other niches in the tunnel are:

Niches	Spacing (in feet)
Tools	500
Gate valve and clean-out for shutting off and cleaning out the 6-in. water service line	500±
Access to valves controlling outlets from the 6-in. water service pipe to the fresh air duct to flush the latter	500±
High-tension splicing chambers	460±

Roadway and ceiling manholes are spaced about 500 ft apart.

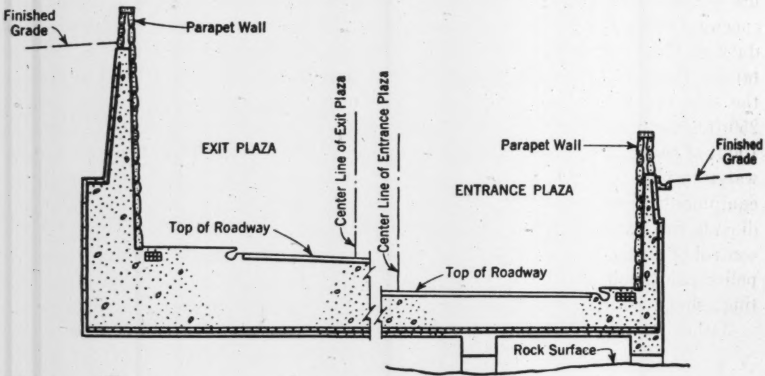
The tunnel roadway is paved with de-aired vertical fiber brick $3\frac{1}{2}$ in. in depth, laid transversely on a $\frac{3}{4}$-in. bituminous mastic cushion. Joints between bricks are filled with pure asphalt with 20% to 30% inert fine material added. The center line marker—a single continuous line on tangent and slight curves, and two continuous white lines about 8 in. apart on the sharper curves—is made with pure white-body vitreous bricks with hard bright, white glazed top surfaces and laid longitudinally end to end. Wherever double-lane markers are used, electrically illuminated amber prisms, protected by cast-steel hoods, are installed in the paving between the lanes at regular intervals.

The roadway surface is crowned 1 in. on tangent and superelevated on the outside of all curves. On curves of about 500-ft radius, the superelevation is a maximum of $\frac{3}{4}$ in. per ft. The transition from crowned to banked roadway is at a rate which will maintain a gutter grade of not less than 0.5% nor more than 6%. In the circular tunnel sections the side-walls are normal to the transverse slope of the banked roadway; and, in the steel bent and concrete sections, the side-walls are vertical and side clearances are increased as necessary.

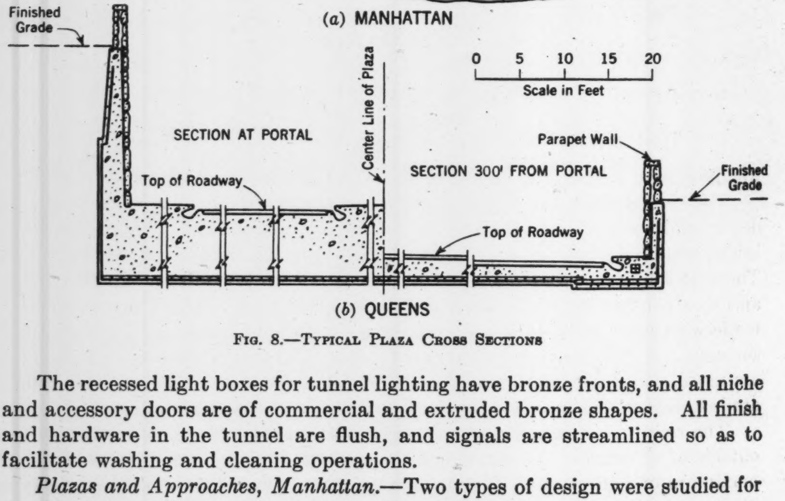
The walls of the tunnel are finished with ivory-tinted, ceramic, vitreous, semi-mat glazed tile with a diffused light reflection factor of 0.78. There is a continuous border of ming green tile at the ceiling.

The tunnel ceiling is finished with 6-in. by 6-in. light ivory-tinted pebbled glass tile set in copper alloy grippers in the ceiling slab.

The tunnel sidewalk bench is faced with light, buff-tone, mottled glazed, architectural terra cotta. A sidewalk railing 2 ft 3 in. high is composed of cream-colored, porcelain enameled steel pipe 1½ in. in diameter.


Finished Grade
Parapet Wall
EXIT PLAZA
Center Line of Exit Plaza
Center Line of Entrance Plaza
Top of Roadway
Parapet Wall
Finished Grade
ENTRANCE PLAZA
Top of Roadway
Rock Surface

(*a*) MANHATTAN


Finished Grade
0 5 10 15 20
Scale in Feet
Center Line of Plaza
SECTION AT PORTAL
Parapet Wall
SECTION 300' FROM PORTAL
Finished Grade
Top of Roadway
Top of Roadway

(*b*) QUEENS

FIG. 8.—TYPICAL PLAZA CROSS SECTIONS

The recessed light boxes for tunnel lighting have bronze fronts, and all niche and accessory doors are of commercial and extruded bronze shapes. All finish and hardware in the tunnel are flush, and signals are streamlined so as to facilitate washing and cleaning operations.

Plazas and Approaches, Manhattan.—Two types of design were studied for the Manhattan plazas. One type employed a drainage system under a comparatively thin invert slab, through which the underground water would be conveyed to a main drainage gallery and from there into the sumps to be removed by pumping. The other type of design utilized the weight of walls and heavy invert slab to counterbalance hydrostatic uplift, and, as construction costs would have been about the same in either case, this latter design was adopted as the more reliable.

In general, the typical cross section of each plaza is that of a U-shaped reinforced concrete structure in a waterproofing envelope with concrete protection outside the waterproofing (see Fig. 8). Invert slabs vary in thickness from 8 ft to 1 ft; the retaining wall height above the roadway is a maximum of 22.5 ft at the entrance portal and 31 ft at the exit portal. Invert subgrade is partly in rock cut and partly above rock surface elevations. In practically all cases, retaining walls were extended down to rock. For an area of the entrance plaza under which the overburden of rock contained layers of peat, the invert slab is supported on piers carried down to the rock. The invert slabs of both plazas are divided by expansion joints placed at 50-ft intervals; these joints are filled with premolded cork filler and provided with copper water stops.

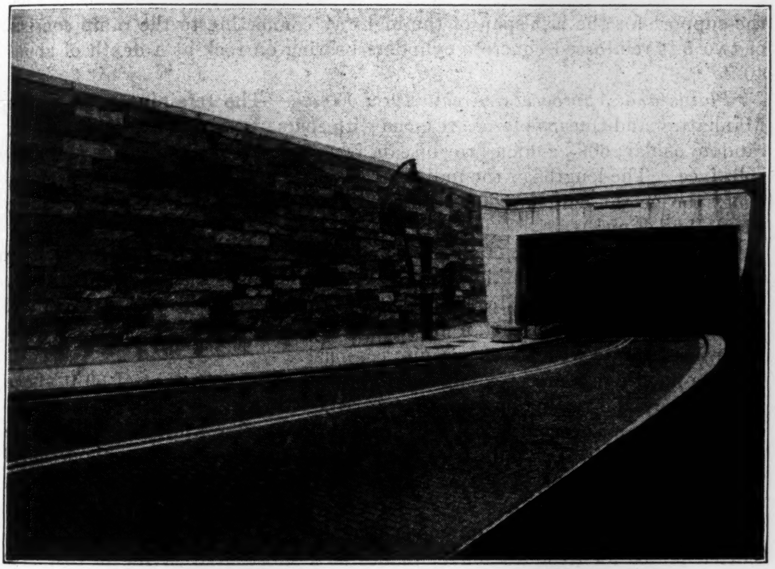


FIG. 9.—PLAZA WALL FINISH AT MANHATTAN ENTRANCE

The exit plaza underpass at Second Avenue is of the steel bent and concrete subway type of structure, as is the underpass under 36th Street.

The north and south tunnel entrance and exit avenues in Manhattan provide 32-ft roadways and a sidewalk generally 4 ft 4 in. wide on the side to the right of the direction of travel of each roadway. When practicable, the foundation walls of the demolished improvements were utilized for foundations of the parapet walls. In other instances the walls had to be built to retain high ground of adjacent areas.

Plazas and Approaches, Queens.—The Queens plaza may be considered structurally as consisting of two sections. The west section between the portal at Vernon Boulevard and 11th Street, where the roadway comes to grade, is similar in design to the Manhattan plazas. The east section of the plaza, starting at 11th Street and continuing to 21st Street and 50th Avenue, is constructed on compacted backfill contained by L-shaped reinforced concrete walls located on the south side; the roadway pavement is a reinforced concrete slab 10 in. thick laid directly on the backfill.

The east section of the plaza includes a ramp leading to the elevated express highway. Because the bearing capacity of the soil was found to be unreliable, the support for the last span of the highway connecting to the ramp consists of two 5-ft reinforced concrete cylinders bearing on rock at a depth of about 40 ft.

Plazas and Approaches, Architectural Finish.—The retaining walls of the Manhattan and Queens plazas are faced with coarse-texture, seam-face, granite, random ashlar; 60% golden-brown seam face, 20% bright seam face, and 20% split face. The lengths of the individual stones are as much as 8, and even 10, times the height, emphasizing the horizontal lines of the plazas (see Fig. 9: Plaza Wall Finish).

The tunnel portals are of six-cut Moose-a-bec granite with polished granite cheeks extending 20 ft into the tubes, and finishing flush with the tunnel tiling. The Queens portal is a twin design with exceptionally large pieces of granite. The soffits of the Manhattan portals are arched. Parapet walls are solid seam-face granite with six-cut granite copings.

The flood lights are of commercial type, placed at a height of 60 ft above the roadway and resting on granite pedestals.

Buildings.—The ventilation buildings of the Queens Midtown Tunnel (see Fig. 10) are classified as industrial power house structures. They are designed with structural steel skeleton frames and are of fireproof construction throughout.

A distinct departure from previous designs was the development of masonry-slotted jambs for fresh air louver blades at a substantial saving in cost over the all-bronze louver construction. The louver blades are ebony asbestos cement slabs impregnated with waterproof compound of fluxed natural asphalt.

The Manhattan ventilation building is 110 ft by 144 ft and is 124 ft high to the top of the two exhaust, or evasé stacks, which rise 14 ft above roof level. The building, which has an octagonal shape, is partly supported on the Manhattan ventilation shaft structure and partly on piers based on rock. This building houses twelve fresh air supply fans and twelve exhaust fans, a large switchboard room, transformer compartments, carbon-monoxide analyzer room, chemical room, maintenance rooms, electrical work shop, pressure-tank room, and elevator motor room. Two enclosed stairways and an elevator in the building provide access from the tunnel roadways to all floors. The cubic-foot cost of the building was $0.439, not including the cost of foundations or rough first floor slab.

The Queens ventilation building, which is partly supported on the bent structure of the tunnel and partly on concrete-filled steel cylinders, is 107 ft by 113 ft in plan with a height of 121 ft to the top of the evasé stacks, which rise about 30 ft above the roof level. Its structural and architectural treatments are similar to those of the Manhattan building. Eleven fresh air supply fans and eleven exhaust fans are housed on three floors, and other equipment and facilities are as for the Manhattan building. Exclusive of foundations and rough first floor slab, the cost of this building was $0.404 per cu ft.

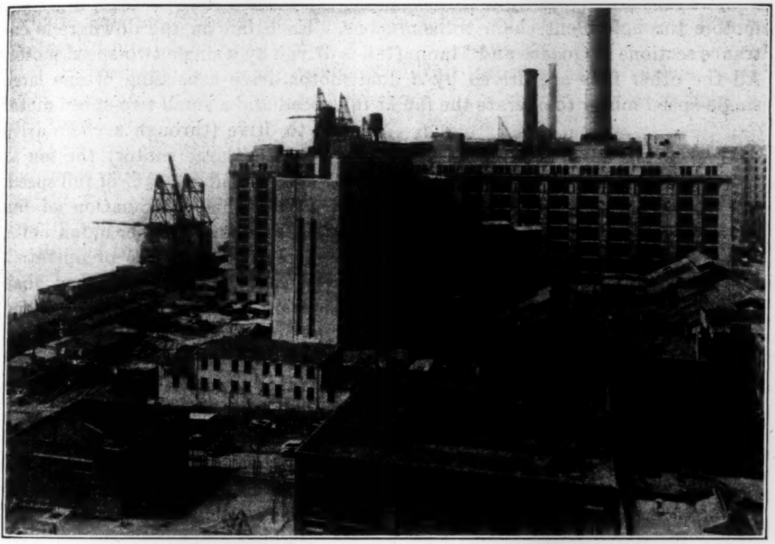


FIG. 10.—MANHATTAN VENTILATION BUILDING, NEARING COMPLETION

The service building in Queens is approximately 100 ft by 185 ft in plan, and is a single-story structure housing shops and a garage for maintenance equipment and two stories and basement housing the rooms for supervisory control, electrical equipment, police quarters, detail room, and maintenance and operating offices. The cubic-foot cost of this building was $0.543.

TUNNEL VENTILATION

The method of ventilating the Queens Midtown Tunnel follows closely that developed for the Holland Tunnel. A continuous supply of fresh air is introduced to the driveway just above the curb through fresh air flues generally spaced at intervals of 10 ft 8 in. on upgrades and level grades, and 16 ft on downgrades, on each side of the roadway, locations on opposite sides of the roadway being staggered. These fresh air flues are fed from a longitudinal supply duct under the roadway. The vitiated air is removed through exhaust air ports placed at similar intervals in the roadway ceiling slab, above which is the longitudinal exhaust air duct.

Each tunnel is divided into four separate ventilating sections. With the exception of a short ventilating section on the Queens end of the north tunnel, which has two fresh air fans and two exhaust air fans, all the ventilating sections are connected to three fresh air fans and three exhaust air fans.

For normal maximum ventilation requirements, fifteen fresh air supply fans and fifteen exhaust air fans are operated. The remaining sixteen fans are stand-by spares.

The fans are driven by totally enclosed, fan-cooled, squirrel-cage, induction motors through silent chain transmissions. Each fan on the downgrade entrance sections in Queens and Manhattan is driven by a single two-speed motor. All the other fans are driven by a dual motor drive consisting of one large single-speed motor to operate the fan at full speed and a small two-speed motor to drive (through a chain drive to the large motor) the fan at $66\frac{2}{3}\%$ and at $33\frac{1}{3}\%$ of full speed. With this combination of fan speeds and number of fan units, greater economies of operation have been accomplished than were possible at the Holland Tunnel where the fans are driven by slip-ring motors of the wound rotor type with external resistance for varying the speed.

During 1941 when approximately 4,400,000 vehicles were carried by the tunnel, of a total electric power bill of $33,412.35 (which was mostly for lighting), only $4,704.30 was for ventilation, or a rate of about 0.1¢ per vehicle. Table 1 is a comparison of the salient features of the ventilating system of the Holland Tunnel and the Queens Midtown Tunnel.

TABLE 1.—COMPARISON OF VENTILATION; QUEENS MIDTOWN AND HOLLAND TUNNELS

(Twin Tunnels in Each Case)

No.	Description	Holland	Midtown
	Length of Tunnel, in Feet:		
1	North	8,557	6,414
2	South	8,371	6,272
3	Total ventilated	16,928	12,686
	Number of:		
4	Ventilation sections	14	8
5	Ventilation equipment, buildings	4	2
6	Fresh air fans	42	23
7	Exhaust air fans	42	23
8	Electric motors	84	82
9	Air changes per hour	42	42
	Volumes, in Cubic Feet per Minute:		
10	Fresh air supplied	3,765,200	2,889,000
11	Air exhausted	3,915,600	2,954,000
	Fan Brake Horsepower Installed:		
12	Total	6,260	4,118
	Normal Maximum Operating Fan Brake Horsepower:		
13	Total	4,173	2,725

TUNNEL LIGHTING

The tunnel is lighted by means of individual luminaires along the walls of the tunnel, immediately below the ceiling. Each luminaire contains a 150-watt and a 100-watt incandescent lamp on independent circuits. Three levels of illumination are used according to the intensity of daylight outside the tunnel. Additional illumination during the daytime inside the entrance portals is provided by 1,000-watt and 500-watt enclosed fixtures suspended in open bays in the ceiling of the tunnel, graduated from five 1,000-watt fixtures per bay at the portal to three 500-watt fixtures per bay at 160 ft inside the portal. The graduation of light at the portals, produced by this additional illumination in combination with the three levels of tunnel illumination, has eliminated the discomfort and blinding effects experienced by motorists on entering other vehicular tunnels.

Lighting circuits are arranged so that a motorist is always within the light of luminaires fed from four sources of power regardless of whether the 150-watt lamps or the 100-watt lamps are burning.

TUNNEL OPERATION

The central point of control of operation is in the service building, which contains the electrical control board from which ventilation fans are started and stopped and their speed changed, from which the traffic signals and alarms for both fires and wrecks are piloted and controlled, and from which the intensity of tunnel lighting is varied.

The board also carries the carbon-monoxide recorders of the various ventilation sections of the tunnel and indicators of the rate of flow of vehicles in each tunnel.

The personnel attending to the movement of vehicles functions as a private uniformed police force supervised by a captain who reports to the superintendent. The mechanical, electrical, and structural maintenance force reports directly to the superintendent. The operating staff was organized under the direction of the chief engineer and, after six weeks of successful operation, was turned over to the direction of the administration department.

The arrangement of the organization, which was an innovation over other tunnel projects, has proved itself to be sound and economical. The cost of operation during 1941 was $587,916.22, or less than 40% of the cost of operating the Holland Tunnel.

CONSTRUCTION

Construction Program and Costs.—The construction work of the project was divided into thirty-one contracts, including three for test borings; and, in addition, there was a miscellaneous group of contracts and open-market orders for operating equipment. A coordinated program for preparation of plans, contracts and specifications, advertising, opening bids, awarding, and times for completion of these contracts, was prepared when work began on the project. This schedule, of course, was subject to some modification and revision from time to time to suit varying requirements and conditions as the work progressed, but the essential principle of the program was successfully adhered to—that is, to complete the construction of the project in the minimum time practicable, with an accumulation of payments to contractors for the work as completed, which would result, as nearly as possible, in a minimum total for interest during construction.

All work was let under open competitive bidding and the contracts were awarded to the lowest responsible bidder. In all cases, this was the low bidder, with the exception of contract No. 2-A for borings at the site of the Manhattan ventilation shaft where the low bidder was unable to start work as ordered because of inability to obtain labor of the required type.

Table 2 gives a list of the contracts for the project, together with final costs, and the dates of beginning and completing the field work. The dates for completion are the actual dates when all work under each contract was fully completed and accepted, in some cases extending beyond the date of opening the tunnel.

The first construction contract to be let was No. 1, and the second, No. 3. The individual letting of these contracts permitted work to progress on them while plans, contract, and specifications were completed for contract No. 4, the principal contract of the project. When contract No. 4 was let, the construction shaft in Queens had been completed and was ready to serve as the point of operations for tunneling from Queens; tunneling operations from Manhattan were started in the ventilation shaft excavated under contract No. 3 and, later, were transferred to construction shafts sunk to the west of the ventilation shaft, thus permitting work on the Manhattan ventilation building to proceed. With the letting of contracts Nos. 7 and 8, the entire main structure of the project from portal to portal was placed under construction. All remaining contracts were coordinated to contracts Nos. 4, 7, and 8. Work on the Manhattan ventilation building was suspended from August 14, 1939, to November 1, 1939, and on the Queens ventilation building from February 17, 1940, to May 27, 1940, while ventilation equipment was being installed.

TABLE 2.—TIME OF START AND FINISH AND COST OF CONTRACTS ON THE QUEENS MIDTOWN TUNNEL

Contract No.	Description of work	Started	Finished	Total cost
1	Queens construction shaft	Oct. 5, 1936	June 9, 1937	$ 201,074.06
2	Land and river borings	Oct. 13, 1936	Jan. 13, 1937	49,585.17
2A	Manhattan shaft borings	Oct. 2, 1936	Oct. 10, 1936	3,644.57
2B	Manhattan plaza borings (modified to include Queens plaza borings)	Oct. 4, 1937	Nov. 30, 1937	4,477.55
3	Excavation for Manhattan ventilation shaft	Jan. 4, 1937	July 29, 1937	299,332.34
4	River tunnels and shafts	July 14, 1937	June 3, 1941	22,133,304.32
4A	Dredging and pile driving	Sept. 14, 1938	Nov. 17, 1938	63,449.60
5	Manufacture, storage, and delivery of cast-iron tunnel lining	Jan. 4, 1937	Oct. 31, 1939	3,333,587.58
6	Manufacture, storage, and delivery of bolts, nuts, and washers	Mar. 8, 1937	Sept. 19, 1939	191,781.14
7	Queens land section	Apr. 25, 1938	Aug. 21, 1940	2,173,613.45
8	Manhattan land section	June 8, 1938	Nov. 14, 1940	3,387,484.07
9	Manhattan ventilation building	Sept. 12, 1938	Sept. 30, 1940	658,000.00
10	Queens ventilation building	May 26, 1939	Dec. 12, 1940	548,093.66
11	Manufacture and installation of fans, motors, and transmissions	Dec. 22, 1937	Dec. 18, 1940	284,689.00
12A	Manhattan plazas	July 24, 1939	May 13, 1941	1,870,128.95
12B	Manhattan surface approaches	Mar. 7, 1940	May 15, 1941	257,706.28
12C	Demolition of buildings for Manhattan plazas	Mar. 7, 1939	July 19, 1939	43,495.00
13	Manufacture, storage, and delivery of ceiling tile	Oct. 5, 1938	July 22, 1940	238,876.41
14	Queens plaza	May 15, 1939	Oct. 25, 1940	643,001.91
14A	Demolition of buildings for Queens plaza	Jan. 24, 1939	Apr. 22, 1939	16,821.00
15A	Electrical installation	Aug. 14, 1939	Feb. 13, 1941	584,077.60
15B	Manufacture, storage, and delivery of switchboards	Sept. 22, 1939	Oct. 9, 1940	427,750.00
15C	Electrical fixtures	June 20, 1940	Feb. 24, 1941	98,339.00
16	Tunnel finish	Mar. 25, 1940	Feb. 5, 1941	339,411.87
17	Pumping equipment	May 22, 1940	Dec. 11, 1940	46,120.00
18A	Toll booths	June 11, 1940	Feb. 14, 1941	90,425.44
18B	Queens plaza finish	June 19, 1940	Jan. 10, 1941	88,154.48
18C	Manhattan portal finish	Aug. 22, 1940	Dec. 30, 1940	73,141.76
18D	Manhattan plaza finish	Nov. 20, 1940	June 19, 1941	249,196.91
19	Service building and Queens emergency garage	May 23, 1939	Oct. 30, 1940	296,911.36
20	Tunnel and plaza paving	July 12, 1940	Dec. 10, 1940	193,026.67
	Total contract costs	....	....	$38,888,701.38

Work on the site of the project was conducted under the rules and regulations of the PWA specifying minimum wages and maximum hours of work—the 30-hr week, applying to contracts Nos. 1 to 3, inclusive, and, by special dispensation, the 40-hr week to contracts Nos. 4 to 20, inclusive.

Total contractors' labor for the project amounted to approximately 9,193,000 man-hr at the site, at an average hourly wage of $1.61. For contract No. 4 alone, the total man-hours were 5,942,000 and the average hourly wage was $1.78. These figures for man-hours and average hourly wages are based on the actual working hours and hourly wages paid, including the net work-shift hours of compressed-air workers whose wages were on a daily basis.

Shafts.—In Queens, tunneling operations for the river tunnels were conducted through a working shaft located on the center line of tunnels 260 ft east of the east building line of Second Street. The shaft was 86 ft 9 in. by 33 ft 9 in. and was excavated to subgrade depth at 67.4 ft below street surface. The shaft structure was of steel and concrete. Two octagonal openings, exposing the rock wall of the excavation, were provided on 45-ft centers in the west wall of the shaft through which to drive the tunnel shields.

The Manhattan ventilation shaft was excavated to net lines 60 ft 2 in. by 111 ft 8 in. in plan and an average depth of 112 ft below ground surface, the excavation being mostly in rock. The shaft was used for initial tunneling operations and, as a precaution during this period, lined with a designed minimum thickness of 4 in. of protective concrete reinforced with wire mesh, and the walls were braced temporarily with timber frames.

The two construction shafts in Manhattan, to which tunneling operations were transferred, were 25 ft by 25 ft in plan, and centered on the tunnel lines an average distance of 112 ft west of the center of the ventilation shaft. The shafts were excavated to a depth of 4 ft below the tops of the respective tunnels, and the rock headings from the ventilation shaft driven through them.

River Tunnels.—Construction of the river tunnels only will be discussed in this paper as the limitation of space will not permit any further discussion of the other contracts of the project.

In the New York area, three large diameter tubes comparable in size to those of the Queens Midtown Tunnel—Holland Tunnel (two tubes, 29.5 ft in diameter) and the south crossing of the Lincoln Tunnel (one tube, 31.0 ft in diameter)—had been driven successfully in the bed of the Hudson River. On the other hand, the experiences in tunneling under the East River, to 1937, had been gained mainly with rapid transit tunnels, 18 ft 3 in. or less in diameter, and with the four tubes for the Pennsylvania Railroad,[4] 23 ft in diameter. The latter provided the best available basis for comparison in anticipating construction conditions in the Queens Midtown Tunnel. Uncertainties in connection with driving the substantially larger 31-ft tubes through the porous overburden and varying rock conditions of the bed of the East River contributed to a narrowing of the field of competition for contract No. 4, for which only two bids were received. The Walsh Construction Company, low bidder, was awarded the contract and completed the work well within schedule.

Contract No. 4 extended between limits 4,086 ft apart in the south tunnel and 4,111 ft apart in the north and comprised all of the shield-driven, cast-iron-lined tunnel of the project (3,772 lin ft in the south tunnel and 3,756 lin ft in the north); part of the rock tunnel lined with 32 ft 10 in. structural steel

---

[4] "The New York Extension of the Pennsylvania Railroad," *Transactions*, Am. Soc. C. E., Vols. LXVIII and LXIX (1910).

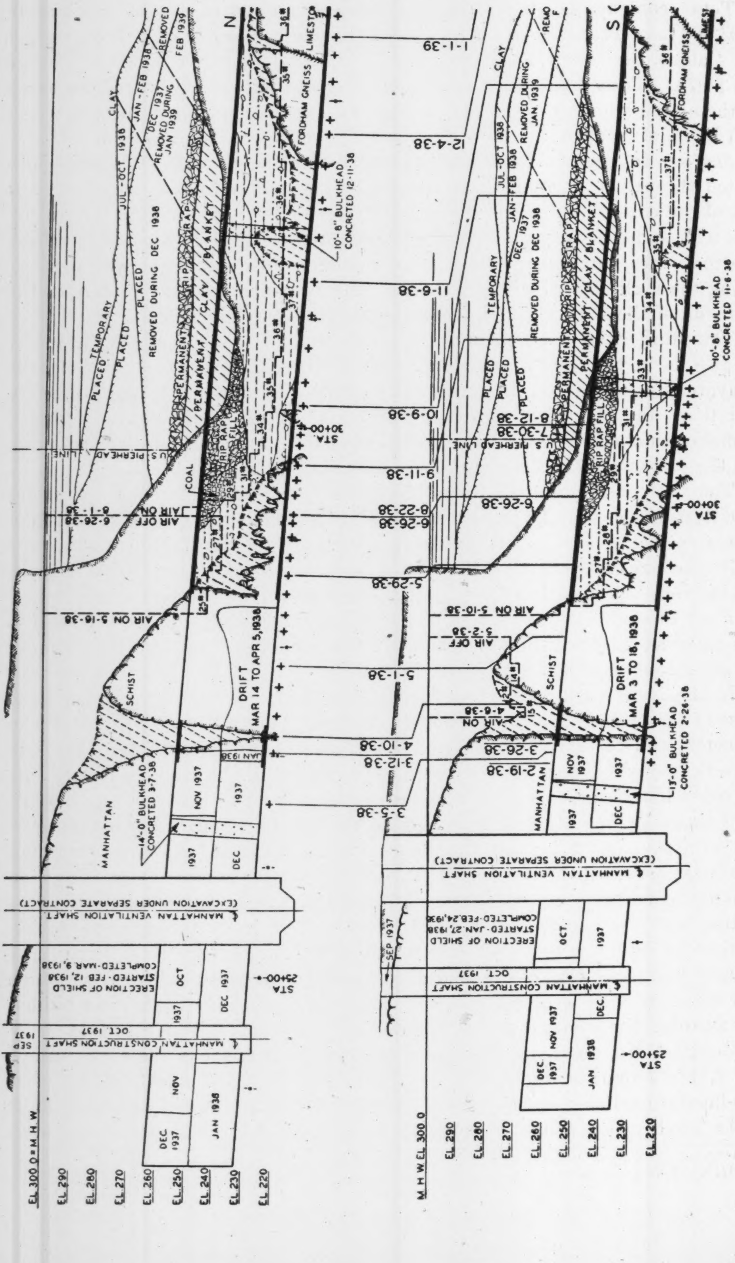

N
S
EL 300.0 = M H W
EL 290
EL 280
EL 270
EL 260
EL 250
EL 240
EL 230
EL 220
M H W EL 300.0
STA 25+00
STA 30+00
℄ MANHATTAN CONSTRUCTION SHAFT
℄ MANHATTAN VENTILATION SHAFT
(EXCAVATION UNDER SEPARATE CONTRACT)
ERECTION OF SHIELD STARTED - FEB 12, 1938 COMPLETED - MAR 9, 1938
ERECTION OF SHIELD STARTED - JAN 27, 1938 COMPLETED - FEB 24, 1938
SEP 1937
OCT. 1937
DEC 1937
NOV 1937
JAN 1938
MANHATTAN
SCHIST
DRIFT MAR 14 TO APR 5, 1938
DRIFT MAR 3 TO 18, 1938
14'-0" BULKHEAD CONCRETED 3-7-38
13'-0" BULKHEAD CONCRETED 2-26-38
10'-6" BULKHEAD CONCRETED 12-11-38
10'-6" BULKHEAD CONCRETED 11-6-38
AIR ON 5-16-38
AIR OFF 6-26-38
AIR ON 8-1-38
AIR ON 4-6-38
AIR OFF 5-2-38
AIR ON 5-10-38
3-5-38
3-12-38
4-10-38
2-19-38
3-26-38
5-1-38
5-29-38
6-26-38
8-22-38
9-11-38
7-30-38
8-12-38
10-9-38
11-6-38
12-4-38
1-1-39
U.S. PIERHEAD LINE
COAL
RIP RAP FILL
PERMANENT CLAY BLANKET
PERMANENT RIP RAP
PLACED
TEMPORARY
REMOVED DURING DEC 1938
CLAY
JUL-OCT 1938
JAN-FEB 1938
DEC 1937
REMOVED DURING JAN 1939
REMOVED FEB 1939
FORDHAM GNEISS
LIMESTONE
14#
15#
25#
27#
28#
29#
31#
33#
34#
35#
36#
37#
38#

FIG. 11A.—LONGITUDINAL SECTIONS SHOWING PROGRESS OF RIVER TUNNELS; STATIONS 25+0 AND 30+0

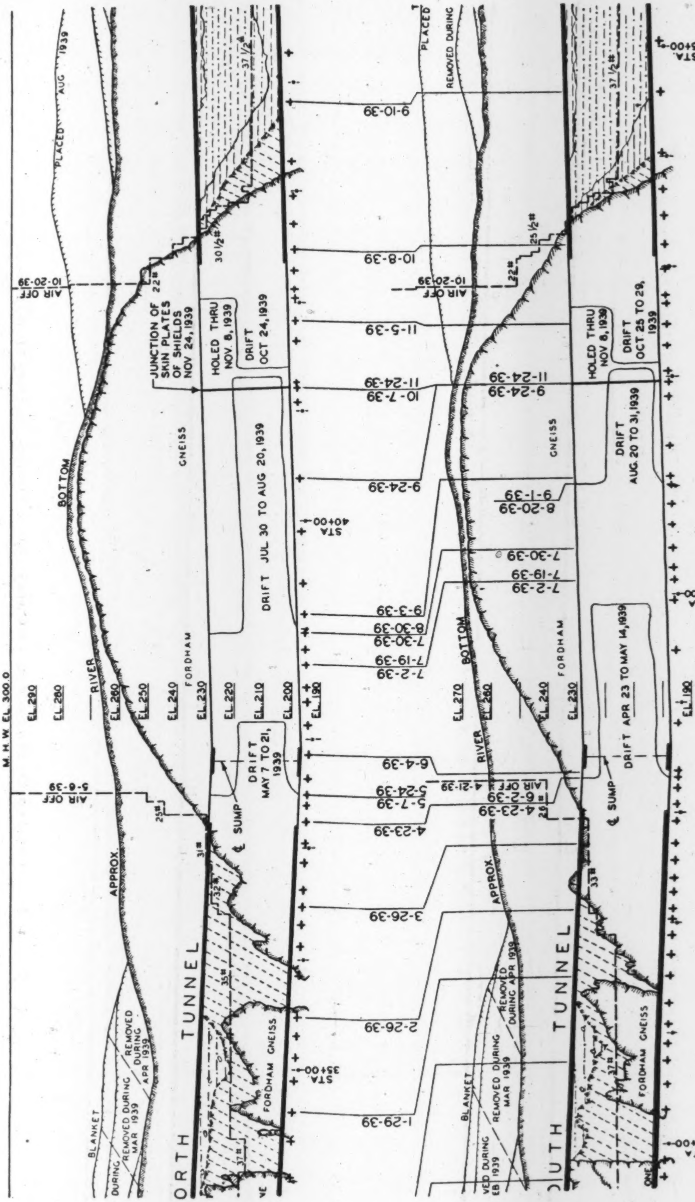
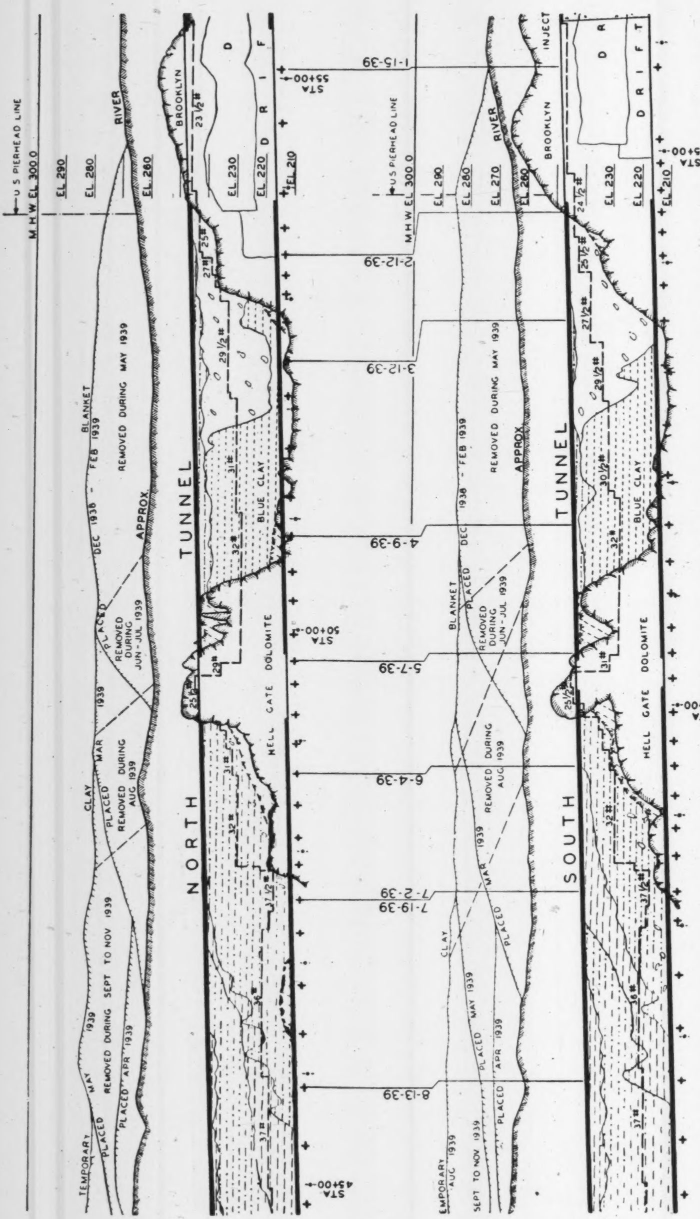

M.H.W. EL 300.0
EL 290
EL 280
EL 270
EL 260
EL 250
EL 240
EL 230
EL 220
EL 210
EL 200
EL 190
ORTH TUNNEL
OUTH TUNNEL
RIVER BOTTOM
APPROX.
BLANKET
DURING REMOVED DURING MAR 1939
REMOVED DURING APR 1939
PLACED AUG 1939
PLACED
REMOVED DURING
VED DURING EB 1939
REMOVED DURING MAR 1939
REMOVED DURING APR 1939
GNEISS
FORDHAM
FORDHAM GNEISS
℄ SUMP
AIR OFF 5-6-39
AIR OFF 10-20-39
AIR OFF 4-21-39
JUNCTION OF SKIN PLATES OF SHIELDS NOV 24, 1939
HOLED THRU NOV. 8, 1939
DRIFT OCT 24, 1939
DRIFT JUL 30 TO AUG 20, 1939
DRIFT MAY 7 TO 21, 1939
DRIFT APR 23 TO MAY 14, 1939
DRIFT AUG. 20 TO 31, 1939
DRIFT OCT 25 TO 29, 1939
STA 35+00
STA 40+00
STA 45+00
1-29-39
2-26-39
3-26-39
4-23-39
5-7-39
5-24-39
6-2-39
6-4-39
7-2-39
7-19-39
7-30-39
8-20-39
8-30-39
9-1-39
9-3-39
9-10-39
9-24-39
10-7-39
10-8-39
11-5-39
11-24-39
37#
35#
32½#
31#
25#
22#
30½#
37½#
33#
26#
25½#

FIG. 11B.—LONGITUDINAL SECTION SHOWING PROGRESS OF RIVER TUNNELS; STATIONS 35+0 AND 45+0



NORTH TUNNEL
SOUTH TUNNEL
U S PIERHEAD LINE
M H W EL 300.0
EL 290
EL 280
EL 270
EL 260
EL 230
EL 220
EL 210
RIVER
BROOKLYN
DRIFT
INJECT
BLUE CLAY
HELL GATE DOLOMITE
APPROX.
STA 55+00
STA 50+00
STA 45+00
1-15-39
2-12-39
3-12-39
4-9-39
5-7-39
6-4-39
7-2-39
7-19-39
8-13-39
BLANKET PLACED DEC 1938 - FEB 1939
REMOVED DURING MAY 1939
CLAY PLACED MAR 1939
REMOVED DURING JUN-JUL 1939
REMOVED DURING AUG 1939
TEMPORARY PLACED MAY 1939
REMOVED DURING SEPT TO NOV 1939
PLACED APR 1939
TEMPORARY AUG 1939
23 1/2#
24 1/2#
25#
25 1/2#
27#
27 1/2#
29#
29 1/2#
30 1/2#
31#
32#
36#
37#
37 1/2#

FIG. 11C.—LONGITUDINAL SECTION SHOWING PROGRESS OF RIVER TUNNELS; STATIONS 50+0 AND 55+0

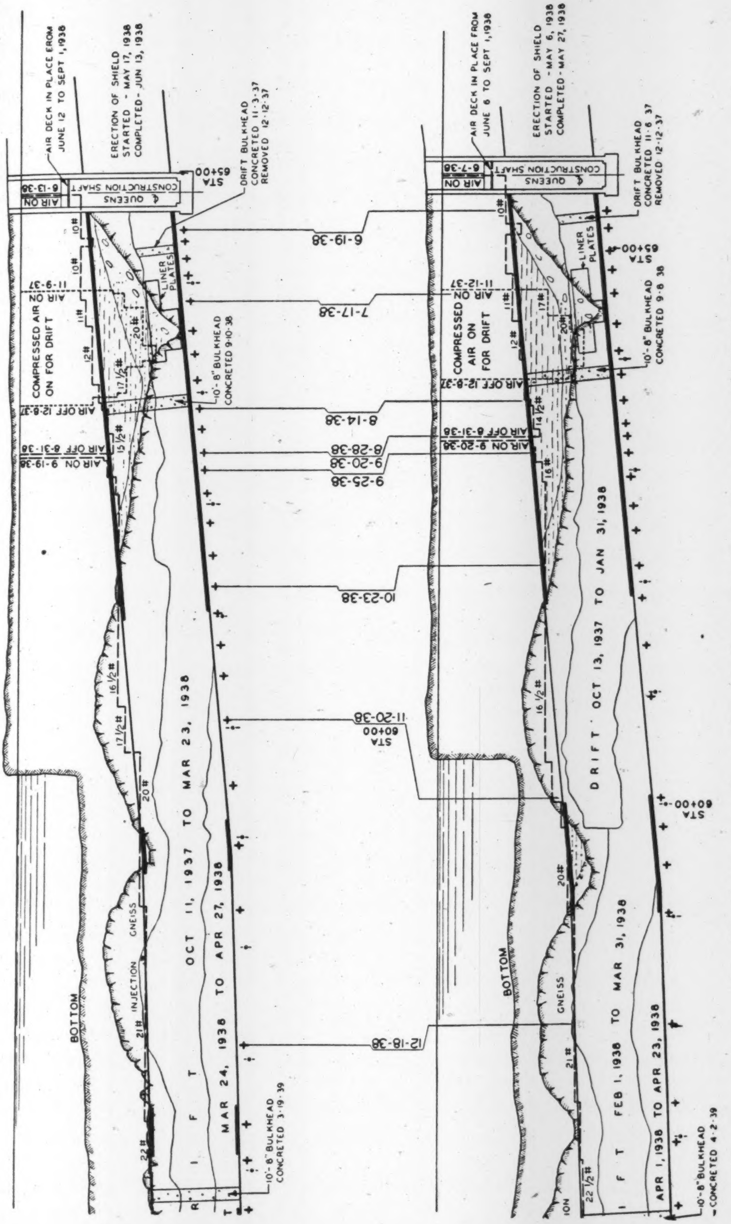

AIR DECK IN PLACE FROM JUNE 12 TO SEPT 1, 1938
ERECTION OF SHIELD STARTED - MAY 17, 1938 COMPLETED - JUN 13, 1938
AIR ON 6-13-38
℄ QUEENS CONSTRUCTION SHAFT
STA 65+00
DRIFT BULKHEAD CONCRETED 11-3-37 REMOVED 12-12-37
LINER PLATES
COMPRESSED AIR ON FOR DRIFT
AIR ON 11-9-37
AIR OFF 12-8-37
10'-8" BULKHEAD CONCRETED 9-10-38
AIR OFF 8-31-38
AIR ON 9-19-38
10#
11#
12#
17 1/2#
20#
15 1/2#
16 1/2#
21#
22#
BOTTOM
INJECTION
GNEISS
DRIFT OCT 11, 1937 TO MAR 23, 1938
MAR 24, 1938 TO APR 27, 1938
10'-8" BULKHEAD CONCRETED 3-19-39
6-19-38
7-17-38
8-14-38
8-28-38
9-20-38
9-25-38
10-23-38
11-20-38
STA 60+00
12-18-38
AIR DECK IN PLACE FROM JUNE 6 TO SEPT 1, 1938
ERECTION OF SHIELD STARTED - MAY 6, 1938 COMPLETED - MAY 27, 1938
AIR ON 6-7-38
℄ QUEENS CONSTRUCTION SHAFT
DRIFT BULKHEAD CONCRETED 11-6-37 REMOVED 12-12-37
STA 65+00
10'-8" BULKHEAD CONCRETED 9-8-38
AIR ON 11-12-37
COMPRESSED AIR ON FOR DRIFT
AIR OFF 12-8-37
AIR OFF 8-31-38
AIR ON 9-20-38
14 1/2#
17#
22 1/2#
DRIFT OCT 13, 1937 TO JAN 31, 1938
FEB 1, 1938 TO MAR 31, 1938
APR 1, 1938 TO APR 23, 1938
STA 60+00
10'-8" BULKHEAD CONCRETED 4-2-39

FIG. 11D.—LONGITUDINAL SECTION SHOWING PROGRESS OF RIVER TUNNELS; STATIONS 60+0 AND 65+0

(225 lin ft in the south tunnel and 265 lin ft in the north); completion of the Manhattan ventilation shaft structure to surface line; and the necessary work in the Queens construction shaft. Prices for tunnel excavation east of the Manhattan ventilation shaft under this contract were as follows: Earth, $62.00 per cu yd; rock, $52.50 per cu yd; and mixed face, $96.00 per cu yd. These are the highest unit prices for tunnel excavation in any corresponding quantities that have come to the writer's attention and reflect the uncertainties and difficulties anticipated in connection with driving such large-diameter tubes under the adverse ground conditions of this part of the East River.

Figs. 11 show progress in tunneling, together with materials penetrated, air pressures, point of balance of hydrostatic head, and sequence of placing and removing clay blanket. Weekly progress is shown for the advance of the shields. The junction between steel and cast-iron lining is 115 ft east of the Manhattan ventilation shaft in the north tunnel and 85 ft east in the south. The legend for Figs. 11 is as follows:

LEGEND

17# 16#	Gage Pressure of Compressed Air, Lb per Sq In., and Elevation at Which Theoretical Hydrostatic Head Was Balanced.
+ + +	Indicates Weekly Progress of Shield Station of Cutting Edge
━━━	Heavy Cast Iron Tunnel Lining
───	Light Cast Iron Tunnel Lining
▨	Rock
▨	Decomposed Rock
▨	Sand, Gravel, Clay, Silt and Boulders
▤	Sand, Gravel, Silt, Some Clay and Boulders, Stratified
☰	Silt and Clay, Stratified

Note: Elevation 300.00 is the Datum of the New York City Tunnel Authority Which is 2.653 Feet Above Mean Sea Level at Sandy Hook, N.J.

In Manhattan, the procedure was to drive the rock headings from the ventilation shaft (Fig. 11A), west to the limits of the contract and east to the beginning of cast-iron-lined tunnel. The shields were then erected and equipped at the bottom of the ventilation shaft and advanced in the excavated headings a sufficient distance to permit construction of the tunnel bulkheads. With the tunnel bulkheads in place, bottom drifts were excavated in free air as far as rock cover would permit. Then, the shields were advanced completing the full face of excavation and erecting the permanent cast-iron lining, compressed air being placed on the headings when test drilling encountered soft material. Operations were transferred to the construction shafts on March 31, 1938.

From the Queens side (Fig. 11D), bottom drifts were advanced from the construction shaft for 1,119 ft in the north tunnel and 1,056 ft in the south tunnel (to about Station 55+0, Fig. 11C), where the rock floor dropped off in elevation rapidly. Compressed air was required in short stretches in each drift just west of the construction shaft, because of a dip in the rock floor there, the soft ground in the upper part of the drift being supported by liner plates. The remaining lengths of the drifts were excavated in free air and driven as high as safe rock cover would permit. The drifts were excavated to subgrade and concrete cradles were constructed on which to advance the shields, which were erected and equipped in the construction shaft. Shield driving, excavating full section, and erecting cast-iron lining were started under air pressure, work being conducted through an air deck placed on the construction shaft. When the shields were advanced sufficiently, the headings were bulkheaded with 12-in. concrete walls placed against the breasting, air pressures were reduced to normal, the shaft air deck was removed, the tunnel bulkheads and locks were placed, air pressure again was placed on the headings, and shield driving was resumed.

Excavation from the Manhattan side was stopped in the south heading on September 26, 1939, and in the north heading on October 7, 1939. Excavation from the Queens side was continued until the holing through in both headings on November 8, 1939. The shields were brought together and all but the steel skins removed, closure rings erected, tunnel lining cleaned and calked, and the interior concrete lining placed, progressing with concreting from Manhattan to Queens.

It was necessary to deposit a temporary clay blanket on the bed of the river in both the west and east channels, in advance of the headings. The riprap shoulders of the permanent blanket in the west channel were placed first. When tunneling was completed, all temporary clay was removed, and dredging was carried down so as to permit placing 5 ft of riprap at required channel depths to complete the permanent blanket.

Altogether 289,330 cu yd of material were placed in the west channel. Some of this was dredged and reused in the east channel, where the total clay deposited amounted to 125,000 cu yd. The blanket was approximately 140 ft wide at the top, with slopes of 1 on 4, or flatter at the sides, averaging about 285 ft in width, toe to toe, with a maximum of 480 ft.

Each of the four tunnel shields was 31 ft 8 in. in outside diameter, the lower half having an over-all length of 15 ft 10 in. from end of tail to cutting edge. Fig. 12 is a view of the shield for the south tunnel, Manhattan. Above the horizontal diameter at the front end, the hood projected 2 ft 10 in., making the upper half of the shield 18 ft 8 in. long. The inside diameter of the tail of the shield was 31 ft 2 in., providing a 1-in. clearance all around the outside of the 31-ft tunnel lining. The thickness of tail was 3 in., and the clearance from the face of the shoving jacks to the end of the shield was 6 ft 4 in.

Ring girders, 5 ft $2\frac{3}{4}$ in. on centers, and horizontal and vertical struts formed the intermediate section of the shield which was divided into thirteen pockets. Above the springing line there were eight working pockets with sliding platforms, each of which was operated by a 96-ton jack. Below the springing line a large center pocket 10 ft $9\frac{7}{16}$ in. high and 8 ft 8 in. wide permitted access of a mucking machine to the face of excavation. The upper two of the remaining four pockets were provided with sliding platforms and jacks as were those above the springing line. Each shield was equipped with sixteen hydraulic face jacks, twelve of 50-ton capacity and four of 96-ton capacity; and there were two 96-ton hydraulic jacks carrying a waler beam for bracing the face at about midheight of the center bottom pocket. Twenty-eight hydraulic jacks 10 in. in diameter were provided for shoving the shield, generally operated at a pressure of 5,000 lb per sq in., and a hydraulic erector arm was provided at the axis of the shield for erecting the lining segments. The face jacks and sliding platforms of the shields were effective in supporting the face of excavation under very difficult conditions, while the large bottom pocket permitted machine mucking in all materials encountered.

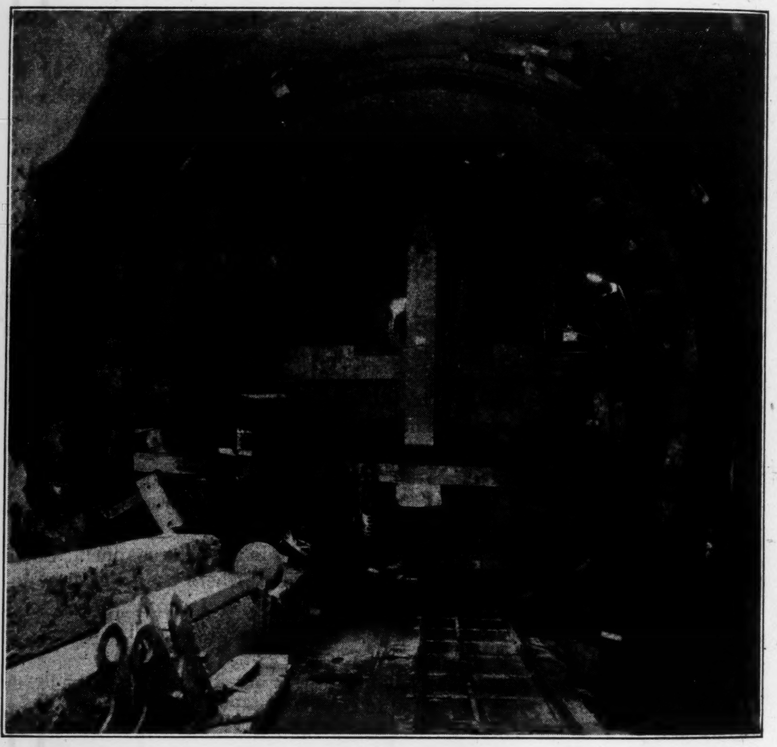


FIG. 12.—REAR VIEW OF SHIELD, SOUTH TUNNEL, MANHATTAN

Tunneling operations were generally conducted on a 24-hr-per-day basis, six days per week.

Whenever practical, bottom drifts were driven in rock in advance of the shields (see Figs. 11). Bottom drifts on the Manhattan side ranged in size from

15 ft wide by 8 ft high to 22 ft by 22 ft; in Queens, it was possible to enlarge the drifts in advance of the shields to full section in some instances.

The tunnels were supplied with low-pressure, compressed air through 14-in. pipes, extending in duplicate from the compressor plants on the surface to each heading. The equipment of the Manhattan compressor plant was typical of both plants and consisted of seven low-pressure air compressors with a total capacity of 40,930 cu ft per min of free air, and three high-pressure air compressors with a total capacity of 6,000 cu ft per min. The high-pressure supply could be used to supplement the low-pressure supply and in the specifications—which required a minimum low-pressure air capacity of 45,000 cu ft per min of free air at a gage pressure of 50 lb per sq in., for each side of the river—80% of the high-pressure air capacity was allowed in computing the low-pressure air capacity of the plant. Three independent sources of power were provided in Manhattan with three separate 13,800-volt electric feeders. In Queens, there were three separate feeders of 27,000 volts each. Adequate capacity and continuity of supply in the low-pressure compressed air plants were vital factors in the successful completion of tunneling operations.

Operations in mixed face consisted, in general, of excavating and breasting the face at the plane of the front edge of the hood down to rock surface. The breast boards were 2-in. or 3-in. planks and were supported by vertical "soldiers," face jacks, and sliding tables. The rock face was excavated sufficiently by drilling and blasting to permit the cutting edge at the bottom of the shield to advance a ring length. If rock extended above the springing line, it was drilled and shot out around the hood just before making the shove. When there was no bottom drift, the shield cradle was made up of concrete-filled burlap bags.

In full face of earth, depending on the nature of material in the lower quarter of the face, the breasting was extended down generally from 6 ft to 12 ft below the springing line, averaging about 7.5 ft. When boulders were encountered in the lower part of the face, the breasting had to be extended down almost to the bottom of the heading. The face was breasted down in the plane of the edge of the hood, and, when the shove was started, the top boards were gradually removed and the material broken down to permit the advance of the hood cutting edge.

The most unfavorable conditions were encountered on the Manhattan side where riprap and other very porous materials in the upper part of the face permitted the compressed air to escape in large quantities, particularly in the area just outside the river bulkhead, where, in order to avoid difficulties with piles, the wharf over each tube had been demolished, the piles pulled out, and a heavy clay blanket placed prior to tunneling. Here, the shields were being advanced in mixed ground, excavating full face with each shove. The consumption of compressed air became dangerously near the full capacity of the plant, including that of the high-pressure air supply. Power consumption was at the rate of approximately 70,000 kw-hr per day. (The power bill for the plant, under these conditions, averaged approximately $4,900 per week.) Air blows through the blanket in the area of the demolished wharf were serious, and so eroded the blanket as to require its constant replacement. The north tunnel was shut down from June 16, 1938, to June 21, 1938, and the heading grouted up with volcanic clay, but the loss of air upon resuming operations in both headings was as great as ever.

On Sunday, June 26, 1938, at 8 p.m., a fire of undetermined origin was discovered in the north heading on the Manhattan side. The north and south headings were about abreast at this time (Fig. 11A). The fire was extinguished by lowering the air pressure and thereby flooding the tunnel at 8:30 a.m., June 27. Before successfully recovering the heading, the material adjacent to and in front of the north shield was grouted through eight 2½-in. casings driven from the river surface, using 4,157 bags of mortar cement in a 1 : 1 mix with beach sand. The heading was dewatered on August 8, 1938, disclosing an inflow of grout and muck at a maximum depth of 14 ft at the shield and running back a distance of 208 ft in the tunnel. After cleaning up, repairing equipment, and grouting the face thoroughly, progress was resumed on August 18, 1938.

Slow progress in the south tunnel (Manhattan end) was continued while the north tunnel was shut down, a total advance of 85 ft being made from June 26 to August 18. Volcanic clay and a volcanic clay-sawdust mixture were used in grouting through pipes in the heading and also through the tunnel lining to consolidate the face and cut down the escape of compressed air. On Sunday, July 17, 1938, the south tunnel was flooded to within 9 ft of the roof after a movement of the bracing at the heading resulted in rapid lowering of air pressure. From August 1 to August 10, inclusive, while the face was grouted (Fig. 11A), no advance was made in the south shield.

Tunneling conditions improved on the Manhattan side as the shields entered the permanent blanket area of the west channel. Tunnel progress from Queens, as shown on Figs. 11D and 11C, was considerably faster than from Manhattan, the material encountered being less pervious, and the air consumption much less than plant capacity; at maximum, compressed air consumption in Queens amounted to 17,250 cu ft per min of free air for the two headings (about April 5, 1939—Fig. 11C).

Excluding shutdowns and abnormally fast or slow progress, normal average progress in the shield headings, in feet per 24 hr, was:

Material	Progress
Earth	7.4
Rock	3.2
Mixed face	3.7

Based on total elapsed time, the average progress for tunneling in the river headings was 18 ft per week per heading. This over-all average includes the time for erecting shields and transferring to shield operations from rock tunneling operations, as well as all other suspensions of progress, including those due to labor disputes. (One period of suspension due to a jurisdictional dispute involving out-of-town work which had no relation to the project resulted in no progress from 8 a.m., Monday, July 3, 1939, to 8 a.m., Wednesday, July 19, 1939.)

The haulage system in the tunnels and on the surface was by storage-battery locomotives, operating on narrow gage tracks. Steel cages, shaft ways, and headframes were installed at the construction shafts, two cages, 8 ft wide by 20 ft long with a 100-cu-yd muck hopper between them, being located on each side of the river. In the tunnel heading, the mucking machine loaded directly into 2½-cu-yd muck cars. The contractor had planned to load from the mucking machines into specially designed conveyors, which would have permitted continuous loading of the cars. However, very little use was made of the conveyors, because of union stipulations as to their operation to which the contractor would not agree and which apparently outweighed their advantages. The tunnel cars dumped directly from the cages into the hoppers; disposal was by truck.

The cast-iron lining segments for each ring were loaded on flat cars, cleaned on the surface, and delivered to the heading with the necessary grommets, washers, and bolts. The segments were lifted from the flat cars by monorail hoist behind the grouting platforms and skidded along the track rails to the erector. The lining bolts were tightened by hand. Before shoving the shield for the next ring, all bolts in the two preceding rings erected were re-tightened. The annular space between the tail of the shield and the last ring erected (a source of considerable air leakage) was calked with burlap bags.

As soon as practicable, during the erection of each ring, a 2-in. turnbuckle was placed on the back flange of the ring at the horizontal diameter. Turnbuckles were removed when observations indicated that any tendency of the ring to flatten had stopped.

The turnbuckle brackets also served as supports for the working platforms, which were towed behind the shield. Each of these platforms had two levels, one about 6 ft above the other, and, by means of the platform adjacent to the shield which was 15 ft long, access to the tunnel lining for erection, bolting, grouting, and graveling operations was obtained. The second platform, about 30 ft long, followed a few feet behind the first and carried grout pans, gravel pans, and materials, and served in connection with these operations. The monorail hoist, which was used to unload the cast-iron-lining segments, also served for carrying materials from cars to the working platforms and shield.

When the tunnel was in soft ground or mixed face, gravel was ejected from the grout holes of the lining to fill the annular space left by the tail of the shield, or the space between the rock and lining, starting at the bottom and working up on both sides of the tunnel to the three-quarter points or to top of rock when rock extended higher, except in cases where special treatment was desirable. This was followed by grouting up to top of rock, generally using a 1 : 1 mix of portland cement and sand. Both pea and ½-in. gravel were used. In full rock sections, the void outside the lining was completely filled with gravel and grout.

Observations showed that, in general, the heavy lining in full face of soft ground tended to flatten, increasing horizontal diameters over a range of from zero to 1¾ in. (averaging about 1 in.); the opposite was true of the light lining, where horizontal diameters generally decreased after erection over a range of from 0 to 2 in. (averaging about 1 in.). This decrease occurred, as would be expected, when the gravel and grout were placed outside the lining.

Modern cast-iron lining is manufactured very nearly true to theoretical form, and practice in erection would be improved by erecting the lining as nearly to its designed shape (that is, circular) as is practically possible. This would tend to eliminate the localized strains and possible open joints which may result from distorted erection; and, with a uniformly high initial bolt stress, it would be a step in improving the waterproof qualities of the calked cast-iron tunnel ring.

Working conditions and safety precautions in compressed air and tunnel construction were in strict accordance with the best standard practice which included conformity with the Industrial Code.[5] Second tunnel bulkheads were installed in the four tunnels to provide for double-stage decompression (see Figs. 11). Three locks were generally provided in each bulkhead, the muck locks being 10.5 ft in diameter and 35 to 44 ft in length. Man locks were generally 6.5 ft in diameter and 34 ft long, and emergency locks, 5 ft in diameter and 30 ft long. Additional service locks, 5 ft in diameter and 20 ft long, were installed in each of the two first bulkheads on the Queens side.

The maximum air pressure used in tunneling was 37½ lb per sq in. There were 745,480 decompressions with 1,148 reported cases of compressed air illness (mostly mild) with no deaths or permanent ill effects. This record may be compared with that of the Pennsylvania Railroad (East River tunnels), constructed from 1906 to 1908, in which there were 557,000 decompressions and 3,692 cases of compressed air illness, with 20 cases resulting in death.[6]

Upon removal of compressed air and completion of excavation, the tunnel lining was cleaned and calked, and concreting operations, begun on October 17, 1939, were completed on July 11, 1940. Sections of the tunnel prepared for concreting were required to remain absolutely watertight after cleaning and calking for at least one week immediately prior to placing the concrete.

Conclusions

Extended research is no longer necessary in planning and designing tunnels for highway traffic, and, with proper coordination of contracts, such projects can be completed in a practical minimum of time. The operation of existing highway tunnels will continue to provide data for improvements in design which will add to the safety, utility, and economical operation of future tunnels. Although it is possible to construct large-diameter subaqueous tunnels with safety under adverse ground conditions, a field remains for the development of improvements in construction methods which would increase tunneling progress, and reduce construction cost.

Ackowledgments

The Queens Midtown Tunnel was built, and is owned and operated as a toll facility, by the New York City Tunnel Authority, a public benefit corporation created on January 6, 1936, by act of the Legislature of the State of New York. It is authorized to construct and operate, on a self-supporting and self-liquidating basis, certain specified vehicular tunnel projects within the limits of the City of New York. The three commissioners of the Authority (Alfred B. Jones, chairman, Albert T. Johnston (died February 9, 1943), and William H. Friedman, who serve without pay, and under whose tenure of office the Queens Midtown Tunnel has been financed, constructed, and placed in operation) were originally appointed by Mayor LaGuardia in 1935 as the commissioners of the Queens Midtown Tunnel Authority, which was superseded in 1936 by the New York City Tunnel Authority. The writer served as consulting engineer to the Tunnel Authority from 1935 to April, 1936, when he was appointed chief engineer, and in that capacity directed the planning, design, and construction of the project.

---

[5] *Bulletins Nos. 22 and 25*, as amended, Dept. of Labor, State of New York; effective May 1, 1922.

[6] "Compressed Air Illness," by Frederick L. Keays, *American Legislation Review*, 1912.

During all phases of the work the Tunnel Authority has had the helpful cooperation of the municipal departments and utility companies involved, as well as of the U. S. District Engineer's Office and the representatives of the PWA. This, together with the loyalty and efficiency of the more than three-hundred members of the engineering department and the skill and cooperation of the contractors' organizations, contributed in no small measure to the completion of the project ready for operation ahead of schedule.

For the Tunnel Authority, W. McK. Griffin, M. Am. Soc. C. E., is deputy chief engineer; James H. Dugan, M. Am. Soc. C. E., engineer of design; Jacob Mechanic, Assoc. M. Am. Soc. C. E., engineer of construction; John H. Quimby, M. Am. Soc. C. E., assistant engineer of design; Leo Geenens, electrical engineer; Herbert G. Cruthers, mechanical engineer; Erling Owre, architect; John J. Nanry, M. Am. Soc. C. E., and Denzil W. Coe, M. Am. Soc. C. E., resident engineers; George L. Lucas, M. Am. Soc. C. E., materials inspection engineer; Norman D. Richardson, M. Am. Soc. C. E., engineer of contracts; David G. Baillie, Jr., M. Am. Soc. C. E., assistant to chief engineer; Louis C. White, assistant engineer of construction. Principal consultants were Jesse B. Snow, M. Am. Soc. C. E., consulting engineer; Edward Levy, M.D., medical consultant; and Charles P. Berkey, Hon. M. Am. Soc. C. E., consulting geologist. The J. G. White Engineering Corporation and Gibbs and Hill, Inc., were electrical consultants.

In the preparation of this paper the writer has had the cooperation of many of the key members of his engineering staff, and he is particularly indebted to Mr. Baillie for his valuable assistance.

This paper has been condensed from a more extended one which has been filed at the Engineering Societies Library in New York.

DISCUSSION

Thomas W. Fluhr,[7] Assoc. M. Am. Soc. C. E.—Attention is called in this paper to the difficulties of tunnel work beneath the East River as contrasted with those beneath the Hudson River. This results from fundamental geological differences in the two places.

A gorge, which at some points reaches depths of 300 ft or more below sea level, has been eroded in the bedrock beneath the Hudson River so that, except at the shores, rock lies too deep to affect tunnels of this type. The gorge is filled partly with glacial deposits above which there is a considerable thickness of plastic river silt. A shield penetrating the silt can be bulkheaded off and "shoved blind," displacing most of the silt in its course and allowing only a small amount to enter the tunnel.

In the East River, on the contrary, erosion has not cut so deeply into the bedrock. Moreover, there are several varieties of rock present, differing in hardness and in resistance to erosion. On the Manhattan side of the river there is a hard schist and, on the Queens side, a hard gneiss. A ridge of gneiss also occupies the center of the river, forming, where it reaches the surface, Welfare Island and its associated reefs. The east and west channels of the river are underlain by softer and more easily eroded limestone. In addition to these differences, faulting has produced weakened zones along which erosion has occurred. Hence the elevation of the rock floor varies so that at tunnel level the excavation passed alternately through hard rock, soft rock, mixed ground, and soft ground.

Moreover the overburden in the East River, in contrast to that of the Hudson River, consists largely of compact glacial deposits together with some artificial fill, necessitating complete excavation and breasting before each shove of the shield. Thus progress in East River tunnels is slower and more costly than in Hudson River tunnels.

As an interesting side light, it may be noted that originally it was proposed to locate the Manhattan ventilation shaft and building close to the Manhattan bulkhead. A zone of soft rock, resulting from decay along a fault (see Fig. 11A), was discovered by test borings. In consequence the shaft and building were shifted to a point farther west where the tunnel curves to the south, thus skewing the building in reference to the street system and building lines. In order to avoid the incongruity of a rectangular building at a skew to existing structures, the architect modified the original building design, which resulted in its rather unusual octagonal form.

Howard L. King,[8] M. Am. Soc. C. E.—An invaluable record of a great achievement in design and construction is presented in this paper. Its value is emphasized by the regrettable fact that the Society has published no similar report on the Holland Tunnel or the Lincoln Tunnel.

---

[7] Eng. Geologist, New York, N. Y.

[8] Chf. Engr., Mason & Hanger Co., Inc., New York, N. Y.

The paper bears witness to the thoroughness and wise forethought that characterized all matters of design. The record of construction is not as complete as it might advantageously have been, but this lack is offset by the excellent longitudinal sections showing progress on the underriver section.

The author points out the difficulty of making the light tunnel iron perfectly watertight and attributes this leakage in part, at least, to the fact that the exterior material had no tendency to silt up any small spaces between the abutting iron flanges. The use of a machine is suggested to tighten the bolts in the light iron and thus to reduce the chance of crevices. Another proposal might prove more effective—to supply an outside material that will choke up the crevices.

Light iron is used where the tunnel is located in weak and wet rock. The excavated tunnel is necessarily irregular and is likely to be a foot or more larger than the iron diameter. The annular space between the exterior of the iron and the irregular surface of the excavation is backfilled with half-inch gravel and with portland cement grout or with grout alone. This grout is ejected at a moderate pressure through the tunnel lining; it contains much water; and probably, when it sets, it shrinks and cracks, providing excellent channels for seepage water to follow and to reach the lining. It would not be practicable to regrout with a thin neat cement grout to fill these cracks, using perhaps 500-lb pressure, because such a pressure might crack the iron.

It would be of interest to experiment with backfill of a soft plastic material, such as Hudson River silt or a clay that is free of stones. Probably the material should be delivered in the tunnel in dry, powdered shape. Then it would be mixed in the grout machine to a liquid mass whose water content would be just on the wet side of the liquid limit, and finally the material would be forced through a hose in the same manner as grout. Once outside the tunnel the clay fill would crack if it dried, but, if the section were a wet one, it would remain wet and should act as an effective waterproofing. Perhaps a small admixture of bentonite would be of value.

FRANCIS V. WAGNER,[9] JUN. AM. SOC. C. E.—The relatively meager literature on subaqueous tunneling has been well supplemented by this valuable and comprehensive paper. Inasmuch as its very scope precluded extensive treatment of any one phase, the writer wishes to enlarge on one aspect of the job. The author mentions the unfavorable conditions for compressed air tunneling that existed in the extremely porous ground beneath the Manhattan River bulkhead. When the heading was in this region, the consumption of compressed air increased to an alarming rate, and the bill for electric power to supply it reached the relatively high average of $4,900 per week. This sum, of course, is the result of an unusual situation. A weekly electric bill of $1,750 would be more representative of average conditions. While he was an employee of the New York City Tunnel Authority, a study was undertaken of power-house costs for the construction of the Queens Midtown Tunnel.

Functions of the Power House.—As stated by the author, the primary function of the power house is to supply sufficient air to the heading to maintain the desired pressure. This is known as "low air." However, the power house has several other functions: It must supply compressed air to operate tools and machinery in the tunnel. This is known as "high air" and, since these tools and motors exhaust into the heading, it must be at a pressure of 60 to 80 lb per sq in. greater than that existing in the heading. The power house must also supply sufficient water under a pressure great enough to operate the hydraulic jacks that propel the shield. It must supply direct current for charging the batteries of the electric locomotives used in construction operations and for miscellaneous power.

---

[9] Senior Structural Engr., Brewster Aeronautical Corp., Johnsville, Pa.

Power-House Costs.—The total power-house costs will be the sum of the fixed charges for the entire power house, and the operating costs for each of its functions.

The cost of the land is the first fixed charge that must be considered. If the land is leased, the rent will be a fixed charge. If it is bought, taxes will be in the same classification. The largest charge, of course, is the amortization of the original investment and interest on the capital tied up in it. Although this actually might be done by any of several methods, for the purposes of this study it was considered that this charge was distributed evenly over the entire period during which the power house was supplying low air to the tunnels. Therefore, it was constant.

The cost of insurance, of course, was constant over the period to be studied. Although the charges for light, heat, and incidental maintenance to the building varied seasonably, their relative weight was negligible, and it was considered sufficiently accurate to compute them as fixed charges.

Operating Costs.—The operating costs of the individual functions are composed of the charges for power, and for labor for both operation and maintenance. Some of these costs will be constant and others will be variable.

The cost of power will vary according to the amount of the particular item supplied. The cost of labor will vary according to the amount of man-hours necessary to supply the item. It will not vary, necessarily, with the amount of the item supplied. The usual force in the power house was an engineer, an oiler, a gage tender, and a hydraulic maintenance man. This force was sufficient for all operations and for all ordinary, routine maintenance. It devoted a certain amount of time to each function—supplying low air, high air, hydraulic pressure, and electric current. The fact that the high air compressors were running at three-quarters speed and the low air compressors at half speed, or vice versa, did not, necessarily, alter the distribution of their time between these machines. No records were kept of the actual time distribution of these men. Inasmuch as they were responsible for the entire power house, and distributed their time as they saw fit, it was impossible to allocate this labor cost to each function. Therefore, it was considered as a constant charge.

The only variable charges therefore were the costs of power to supply the various functions of the power house. To consider them in the inverse order of their importance: Direct current was supplied for charging the batteries of the electric locomotives and for other miscellaneous power. When the job was underway, the amount consumed was a fairly constant one. No matter what the type of work in the tunnel, the electric locomotives were kept busy hauling materials and muck, and, consequently, needed recharging at a fairly constant period. Direct current consumption for other miscellaneous power was small and may be considered to be constant. The error, accordingly, will be negligible, if the cost for supplying direct-current power be considered a constant charge.

Variable Costs.—Hydraulic power must be supplied to actuate the various jacks on the shield. This water was stored in two hydraulic receivers, which were kept charged by three 100-hp high-pressure hydraulic pumps. The quantity of water used varied with the number of times the jacks were used. With the exception of a few times when the smaller "face" or "platform" jacks were used, jacks were used only during a shove. The cost of supplying this hydraulic power, therefore, varied with the frequency of shoves. This water was pumped continuously, however, and the amount of power consumed by the pumps was small in comparison with that used for the air. There were two other variable costs: The power to supply high air and the power to supply low air.

High Air Consumption.—An important function of the power house was to supply high air to operate power tools all over the job. Some of this was used on the surface and outside the compressed air chambers and was exhausted to the atmosphere. Much of it was used in the heading, and was exhausted therein, thus augmenting the supply of low air. This quantity, then, was used twice—once as high air for power, and once as low air. Although the amount that was thus doubly utilized varied, on the average it is believed that it amounted to about 70% of the high air compressed.

The high air used on the surface powered various tools in the machine shop and yard. Routine maintenance and repairs required a relatively constant amount. However, when there was much rock in the heading, the sharpening of drill bits increased the consumption of high air in the machine shop.

In the heading high air was used for power for a monorail hoist and for various "tuggers." Inasmuch as these machines were used in routine fashion, the amount of high air consumed in their operation was assumed to be constant. The power wrenches for tightening the bolts in the tunnel lining consumed some high air. The amount used in this manner was governed by the frequency of ring erection, which, of course, is the same thing as the frequency of shoves. This was a very small amount. Somewhat larger consumers of high air in the heading were the power spades and the rock drills; but the largest users of high air were the grout and gravel machines. These were used most, when the heading was in rock. The high air consumption in the tunnel might be expected, accordingly, to vary with the amount of rock in the heading.

The only important factor that might be expected to govern the variation of the cost for supplying high air is the amount of rock in the tunnel heading.

Low Air Consumption.—The amount of low air consumed was equal to that which escaped from the tunnel. As has been noted, the air in a tunnel can escape by four paths. The quantity that escapes by all of those ways is, of course, directly proportional to the pressure existing in the tunnel at the time. The amount that escapes by each of these methods, however, is determined by several other factors as well.

The first way in which low air can escape is by means of the air locks through which men and materials enter and leave the tunnel. The amount lost in this manner is dependent upon the number of times the lock is used.

The second avenue of escape for the low air is through the joints in the iron lining. The volume that leaks through these joints is dependent upon the tightness of the joints; but once through the joints, the air must escape through the ground surrounding the tunnel. The porosity of this ground, therefore, is also a governing factor.

The third way in which this air can escape is through the annular opening between the tail of the shield and the last completed ring of lining. The quantity that escapes in this manner is dependent upon the condition of the opening. It is usually kept "bagged up"—that is, stuffed full of old burlap bags—which reduces the leakage considerably. Again, as in the case of joint leakage, the air must escape through the river bed, and that being the case the porosity of the ground is the determining factor.

The last path of escape for the low air is through the face of the heading itself. Once more it is evident that the condition of the opening governs the volume of air that escapes. If the breasting boards are tight and well packed with bags and salt hay and plastered with clay, leakage will be less than through a more open face. As in the last two methods of escape, however, the air must leak through the ground itself. The character of the soil, then, is once more the dominant factor in any study of the volume of leakage.

It must be evident from the foregoing that any study of the factors governing the consumption of low air must take considerable account of the type of ground traversed by the tunnel. With the exception of that volume lost through the air locks, all the low air consumed escapes through this ground. A consideration of the size of the openings through which the air leaks would lead to the conclusion that most of it escapes in the vicinity of the shield. This view is confirmed by observation of the river surface during the construction of the tunnel. Directly over the shield the surface of the water was in constant agitation due to the escaping air. The limits of the area in which bubbles of air were breaking the surface could be noted plainly from a rowboat, and were even more clearly defined from an elevation. Usually, elsewhere along the line of the tunnel, however, no air leakage was discernible at the surface. It is safe to assume that most of the air that escaped through the ground did so near the face of the heading. The type of ground in the heading was one of the most important factors that governed the consumption of low air. It is to be expected, therefore, that, the other factors being equal, if the heading were in impervious material, for instance, the amount of low air consumed would be less than when the heading was in loose ground.

Factors Influencing Variable Costs.—It has been shown, then, that the variable costs for the power house depended, in the main, upon the consumption of high and low pressure air. The factors which governed such consumption would govern the variable costs. Accordingly, the factors that influenced the variable costs of the power-house operations may be listed. They included the air pressure in the tunnel, the frequency of use of the air locks, and the type of ground in the heading.

Statement of the Problem.—Inasmuch as these variable costs combined with certain fixed charges to comprise the operating costs of the power house, it follows that the operating costs of the power house are dependent upon the same factors. In the end, these factors make themselves felt upon the total costs of power-house operations. However, the influence of these factors may be great or it may be small. The first question which must be answered, therefore, is:

1. What is the extent of the variation in operating costs with the factors which influence them?

Once this question has been answered, a second one presents itself:

2. Is this variation in operating costs of sufficient magnitude to affect the total costs of the power house to any appreciable extent?

Should the answer to question 2 be in the affirmative, a study can be made of the variation in total power-house costs with the factors that influence them.

Method of Attack.—In order to answer questions 1 and 2 for the Queens Midtown Tunnel, a survey was made of the power-house facilities which were set up for the construction of the tunnel. That the headings which were driven from opposite sides of the river might be served, the contractor established a power house for each of the two construction shafts. The operations of the Manhattan power house were highly irregular, and fluctuated considerably due to several blows and fires which interfered with the prosecution of the work, and due to many sudden variations in the character of the material in the heading. On the Queens side, however, operations were fairly constant for considerable periods, and conditions were favorable for the study proposed. The Queens power house was chosen as a basis for the study. All information contained in this study and all data presented herein were obtained from the records of the Queens Field Office of the New York City Tunnel Authority.

First, a profile of the tunnel, similar to Fig. 11, was drawn, showing the variations in strata through which the face of the heading passed. On this was superimposed a progress chart of construction, which fixed the dates during which various conditions of face prevailed in the heading. From a consideration of the profile, and from a study of the Queens power-house records, certain periods were chosen which isolated each of the aforementioned variable factors. Then all the operating data of the power house were compiled into usable form for these selected periods. From a study of such data concerning the isolated factors, the extent of their influence upon the quantities involved and, therefore, upon the operating costs, was determined. The first question then was answered.

Once the influence of the governing factors upon operating costs was known, their influence upon total costs could be discovered, and the answer to the problem could be found.

Running Speed of Compressors.—The study disclosed that the plant rarely operated at more than one third of its full capacity of 64,152,000 cu ft per day. To obtain this quantity of low air, the compressors were run alternately, usually at one-half or three-quarters load, which was not the point of their highest efficiency. Occasionally, however, for short intervals, they were run at full load. It was this peak load which determined the power cost, and, at this peak, the machines were operating at their highest efficiency. The efficiency at which they operated for the remainder of the time was of no importance. In order to avoid all chances of a breakdown, then, they were run most of the time at the lower speeds.

Summary.—It was concluded from the study that, in the case of the Queens Midtown Tunnel, there was no appreciable variation in the quantity of high air compressed. No correlation could be discovered between the quantity of low air consumed and the frequency of use of the air locks, or between the quantity consumed and the pressure used in the heading. Nevertheless, the quantity of low air consumed varied directly with the porosity of the ground in the face of the heading, as measured by the pressure used therein. This is shown in Fig. 13. The unit operating costs of the power house decreased in such proportion to the quantity of low air supplied that the operating costs of the tunnel were constant while it was supplying between 12,000,000 and 20,000,000 cu ft of low air a day. This is shown in Fig. 14.

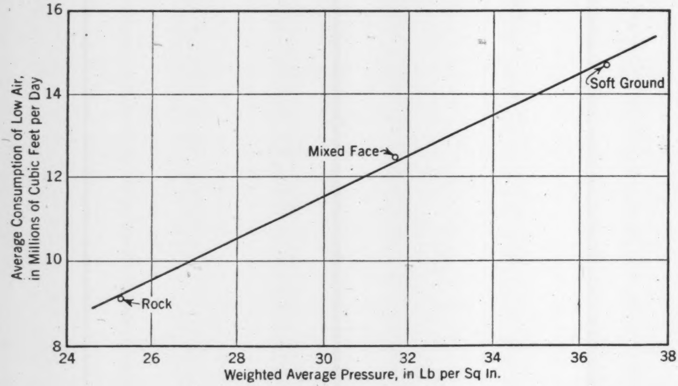


FIG. 13

Power-House Costs.—The total daily power-house cost was constant at $928.27 a day. On the basis of this value, the various elements which constitute it represent the following percentages of the total:

Item	Daily cost		% of total daily cost	
Land	$ 56.563		6.1	
Interest	1.107		0.1	
Amortization	442.638		47.7	
Insurance	6.640		0.7	
Total fixed cost		$506.95		54.6
Labor	178.84		19.3	
Power	242.48		26.1	
Total operating cost		421.32		45.4
Total daily cost	$928.27		100.0	

Trends and Indications.—For the Queens Midtown Tunnel the daily fixed charges were about 20% greater than the average daily operating cost. It would require a fairly large percentage variation in operating costs to cause any appreciable percentage of variation in total power-house costs. This indicated that it is the fixed charges which are important in considering construction power-house costs, and that savings made in them are of more importance than economies in operating expenses. It must be emphasized, however, that the Queens side of this tunnel was in exceptionally good ground for tunneling. It consisted of very fine sand and clay which was not at all porous. In soft ground, therefore, air consumption was relatively low, and, what is more, peak loads were infrequent and were not excessively greater than the average.

Inasmuch as power costs could have been driven sharply upward by an unexpected peak load, the question of maintaining as even a power demand curve as possible is of paramount importance. It must not be forgotten, however, that the low air is a tool that must be used efficiently. Sometimes peak loads are vitally important for economies elsewhere in the work, and then they cannot be avoided. A critical study of the causes of peak loads, with a view to avoiding those which can be eliminated and minimizing those which are unavoidable, would pay handsome dividends in reducing operating costs.

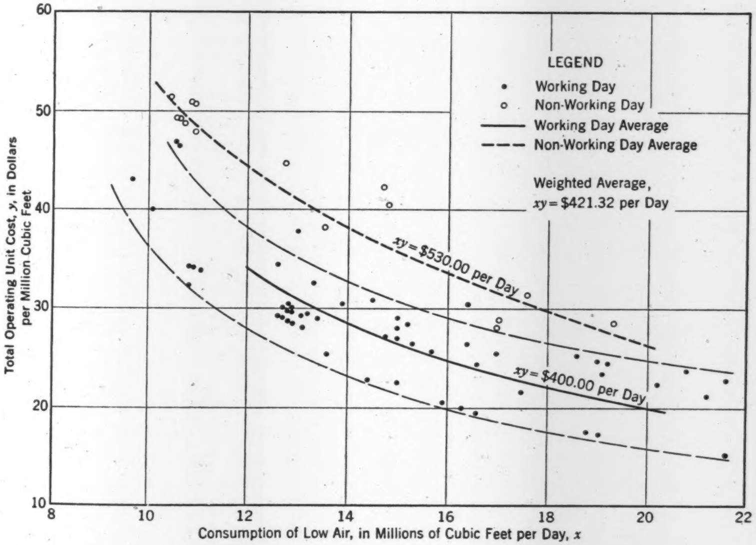


FIG. 14

Acknowledgments.—This study was made in partial fulfilment of the requirements for the degree of Master of Mechanical Engineering, at New York

University, New York, N. Y. The writer is indebted to Prof. A. C. Coonradt, head of the Department of Mechanical Engineering, for his advice and encouragement, and to numerous engineers of the New York City Tunnel Authority and of the Walsh Construction Company, without whose cooperation the study could not have been made. The results were compiled into a thesis: "A Study of Powerhouse Costs for the Construction of the Queens Midtown Tunnel," which is on file in the Library of New York University, and of which the present discussion is a brief summary.

WILLIAM J. WILGUS,[10] HON. M. AM. SOC. C. E.—This is indeed an admirable account of the birth of a star of the first magnitude in the galaxy of great engineering feats that grace the nation's most populous region. Seven monumental bridges for vehicular traffic now span the wide tidal waters encompassing the island of Manhattan on the east and west and between the Bronx and Queens; and, beneath the surface, pass three subaqueous tunnels that serve the same purpose, the one described in the paper being the latest. Between lower Manhattan and Brooklyn a fourth tunnel is in the making, and, beneath the Narrows between Brooklyn and Richmond, a fifth is in contemplation by a method differing from the others. It is to be hoped that more papers will come from the pen of the author, giving the profession the benefit of his remarkable experience in planning, constructing, and operating such public works.

The writer is interested particularly in the happy result that attended the freedom given the bidders, in the case of a part of the Queens Tunnel, to make a selection from specified alternative methods of construction, or, if they preferred, to offer their own solutions on which to base their bids. It happens that the writer took occasion to do this many years ago in the preparation of the preliminary plans and specifications both for the construction of the subaqueous section of the Detroit River railroad tunnel, which resulted in the adoption of his trench-and-tremie method, and for its electrification, which resulted in the selection of the direct-current, third-rail method. Jokingly, this course was referred to, by one of the tunnel bidders, as a "brain poultice," as was true. In a novel situation the bidders, governed by the determined location, gradients, dimensions, and qualities of materials and workmanship, thus are given an opportunity of basing their proposals as to time of performance and costs on methods they consider best qualified for the achievement of the desired end; and the principal is given the assurance of a successful outcome at a minimum of first cost and cost of operation.

In his closing remarks, perhaps the author will give comparisons of leakages and costs per cubic foot of internal content of the subaqueous sections of the Holland, Lincoln, and Queens tunnels—with an index of costs of labor and material predicated on a scale of 100 in the first-named instance. It would be a matter of interest to compare these costs with those for similar work in the past and a help in making rough preliminary estimates for like work in the future.

---

[10] Weathersfield, Ascutney P.O., Vt.

The inclusion of interest on money during construction in the author's statements of cost is of great significance; too often, this is omitted from estimates of the cost of public work to the injury of the taxpayer who thus is misled into believing that his burden will be far less than truth demands. Also, it is gratifying to know that the actual cost of this project was less than the estimated amount. The question may well be asked if the latter is the one that obtained at the outset or was prepared at a later date after unforeseen conditions and difficulties had developed. In public work a suitable provision for contingencies too often is omitted at the beginning because the enlarged figure may scare the public and discourage approval of the project. Utter frankness in these respects is due those who are to pay the bills, as has been shown so obviously in the paper.

In addition to the purpose of the tunnel as a utility, it is evident that sight has not been lost of the esthetic. The interior treatment is most attractive, and the facing of the walls of the approaches with stone gives to them a sense of solidity and comeliness as befits the dignity of a great public enterprise.

S. A. THORESEN,[11] M. AM. SOC. C. E.—The presentation of this valuable contribution reveals the principles of design and construction details that were produced by a staff of engineers with long experience in a specialized field of engineering and construction. The development of vehicular tunnels, being closely related to the increased use of motor vehicles, may be considered a twentieth century product of research and invention. When comparison is made of the modern type of construction with that of Blackwall Tunnel in England, built before the introduction of the automobile, it is apparent that careful thought has been given not only to invite the traveling public for a ride through "a hole in the ground" but also to enjoy an attractively finished, well lighted, pleasant, and comfortably ventilated subaqueous highway.

The construction of the Queens Midtown Tunnel through rock of variable character, with overburden of sand, gravel, and numerous boulders, presented a most difficult problem, and the engineers and contractors entrusted with its execution merit due credit.

Reference has been made to this tunnel as being the first tunnel crossing of the East River for automotive vehicles. In connection with this statement it may prove of interest to review certain historical facts dating back to the beginning of the century, when the first subaqueous rapid transit crossings under the East River were projected and built under the direction of the late William Barclay Parsons, Hon. M. Am. Soc. C. E.

The completion of the Battery Tunnel from South Ferry, Manhattan, to Joralemon Street, Brooklyn, inaugurated the first rapid transit service under the East River, to be followed a few years later by the construction of the Steinway Tunnel from 42d Street, Manhattan, to Long Island City, Queens.

Since the Steinway Tunnel route is in close proximity to that of the Queens Midtown Tunnel, the records kept during the period of construction show certain similarities with respect to the topography of the rock floor and to the deposits of sand, gravel, and boulder formations. On the Manhattan side the tunnel was driven through solid rock and the river section was driven from three shafts, one located at the foot of 42d Street, a second in the middle of the river, and the third in Long Island City. Rapid progress was made by attacking the river part from four faces, and the completion of the twin-tube tunnel during a two-year period from 1905 to 1907 reflects credit on the pioneer engineers and tunnel contractors of that time for a successful and speedy solution under difficult ground conditions full of hazards.

---

[11] Cons. Engr. (Parsons, Brinckerhoff, Hogan & Macdonald), New York, N. Y.

The enterprising spirit and economical prosecution of work in those days (when money was subscribed from private sources) are best demonstrated by actual records, when, in 1906, labor cost for rock excavation under 35-lb air pressure would average from $6 to $7 per cubic yard and the contract cost for this twin-tube rapid-transit tunnel from Grand Central Terminal to Long Island City, 8,500 ft long, was about $5,000,000. The cost of contracts as given by tabulation in the paper is of interest in showing the tendency toward an increase in cost of tunneling under present-day labor, material, and overhead charges, as compared with work performed years ago or during the economic depression. Any combination of construction methods that may reduce costs to a lower level will be accepted favorably, particularly when the total invested capital is supported by toll collection.

Since the construction of the Holland Tunnel (when ventilation research and experimental work were conducted on an extensive scale in collaboration with the U. S. Bureau of Mines), its ventilation system—namely, fresh air beneath the roadway and vitiated air above the ceiling slab—has been adopted in principle by all modern vehicular tunnels.

In Table 1 it is interesting to note the economy of operation brought about by introducing large and small motors for full and partial load conditions as a means of reducing power consumption according to traffic.

Any improvements that tend to reduce the construction cost, and correspondingly the fixed charges must be borne in mind by the tunnel engineer, if the subaqueous highway is to become the means for linking together cities, states, countries, or continents, as the case may be, thus meeting the demands of the future in competition with other methods of transportation.

Postwar problems may well include projects of this character which, in combination with the trench and pre-cast type of design, possess certain economical advantages where soil conditions may favor types of construction such as those adopted for the tunnels under the Detroit River or the Oakland-Alameda Estuary. The latter type of construction has not proved practical for tunnels projected under the Hudson and East rivers but, without doubt, will receive proper consideration in connection with other plans within the New York metropolitan area.

As a monumental structure, the Queens Midtown Tunnel is another contribution to the many underground facilities of Greater New York and, like all great achievements, is an example of organized talent dedicated by the engineering profession to the service of the general public.

H. G. CRUTHERS,[12] ESQ.—The cost of 0.1¢ per vehicle, to provide power for ventilation for the vehicles carried by the Queens Midtown Tunnel, as shown in Mr. Singstad's paper on that project, is extremely low compared to that for similar tunnels and reflects not only a high over-all efficiency for the ventilating equipment but a low cost of power.

For the first year of operation, the actual cost of power for all purposes was 0.9548¢ per kw-hr. This cost included all charges for energy, demand, and fuel and was made possible through purchase from the New York City pool.

The power used for ventilation during 1941, when 4,400,000 vehicles more or less were carried by the Queens Midtown Tunnel, was approximately 0.111 kw-hr for each vehicle. From data available, the power required to provide ventilation for the Holland Tunnel during 1936, when 11,499,000 vehicles were carried by that tunnel, was approximately 0.529 kw-hr per vehicle and for the Lincoln Tunnel during 1938, when 1,790,000 vehicles were carried by that tunnel, approximately 0.289 kw-hr per vehicle.

It has been estimated that, when the Queens Midtown Tunnel carries 11,499,000 vehicles per year, the power required for ventilation will be about 0.10 kw-hr per vehicle.

The number of possible air changes provided for each of these three tunnels is identical (42 per hour) so that the difference in the amount of power required per vehicle carried depends on the method of providing the air changes.

The Holland Tunnel (two tubes) is divided into fourteen ventilating sections, the average length of each being approximately 1,200 ft. The Lincoln Tunnel, a single tube, is divided into five ventilating sections, the average length of each being 1,640 ft. The Queens Midtown Tunnel (two tubes) is divided into eight ventilating sections, the average length of each being 1,590 ft. The areas of the air ducts of the Lincoln Tunnel and the Queens Midtown Tunnel are approximately, for the fresh air ducts, 25%, and for the exhaust air ducts, 20%, greater than those of the Holland Tunnel.

Since the power required to provide ventilation by the transverse method of a continuous rate of supply and exhaust varies inversely as the square of the areas and directly as the fourth power of the lengths of the ducts, it would appear that the power required to provide ventilation for each vehicle carried by either the Lincoln Tunnel or the Queens Midtown Tunnel should be much greater than the power required at the Holland Tunnel. The data show that this is not so—0.529 kw-hr per vehicle at the Holland Tunnel, 0.289 kw-hr per vehicle at the Lincoln Tunnel, and 0.111 kw-hr per vehicle at the Queens Midtown Tunnel. The reason for the lower values for the Lincoln Tunnel, as well as for the Queens Midtown Tunnel, is attributed to the use of higher motor efficiencies at the low-speed operation of the ventilating equipment.

Records from the Holland Tunnel for 1936 operation show that the ventilating equipment was operated for 90% of the time at fan speeds which supplied 29.5% of the normal maximum requirements. Maximum requirements are satisfied by the use of two fans on a duct operating at full speed, and 29.5% of this was obtained at the Holland Tunnel by one fan on a duct operating at approximately 48% of full speed.

---

[12] Mechanical Engr., New York City Tunnel Authority, New York, N. Y.

The records of operation also indicated that for a great part of the time the ventilation requirements were such that much less than 29.5% of maximum requirements would have been ample to provide good atmospheric conditions in the driveways so that, if the ventilating equipment of the Holland Tunnel could have been operated to provide something less than 29.5%, greater economies could have been effected.

With these records of operation as a guide, the Queens Midtown Tunnel was provided with equipment which would allow for operation such that one fan could be operated on a duct at one-third full speed, supplying approximately 20% of maximum ventilation requirements.

At this tunnel, as described by Mr. Singstad, dual-motor drives were installed to operate the fans—one large single-speed motor to drive the fan at full speed and one small two-speed motor to drive the fan at two-thirds' full speed and at one-third full speed. These motors were all totally enclosed fan-cooled squirrel-cage motors with average efficiencies of 92.5% for full speed, 90.7% for two-thirds of full speed, and 85.2% for one-third of full speed, the latter to operate for more than 90% of the time.

At the Holland Tunnel, one slip-ring, wound-rotor motor with external resistance for varying the fan speed was installed to drive each fan. The average efficiencies of the motors were as follows: For full-speed operation, 91%; for 87% of full speed, 78%; for 72% of full speed, 64%; and, for 48% of full speed, 38%.

The difference between motor efficiencies at the low-speed operation of 85.2% for the Queens Midtown Tunnel and 38% for the Holland Tunnel, together with the fact that for a great part of the time ventilation requirements are such that 20% of maximum requirements instead of 29.5% are sufficient to maintain good atmospheric conditions in the driveway, accounts in a large part for the much lower power requirements per vehicle carried—0.111 kw-hr for the Queens Midtown Tunnel as compared to 0.529 kw-hr for the Holland Tunnel.

Dual-drive, squirrel-cage motors also were installed to drive the fans at the Lincoln Tunnel. These motors had operating efficiencies comparable to those installed at the Queens Midtown Tunnel, but the average lengths of ventilating ducts at the Lincoln Tunnel were greater than those of the Queens Tunnel, and, since the Lincoln Tunnel is a single tunnel for two-way operation, the traffic conditions impose a greater load factor; and it is presumed that these two conditions alone account for the difference between the power requirements of 0.289 kw-hr per vehicle at the Lincoln Tunnel as compared to 0.111 kw-hr per vehicle at the Queens Midtown Tunnel.

Leo Geenens,[13] Esq.—The author reports a substantial reduction in the cost of tunnel operation. Approximately three quarters of the operation cost represents wages, mostly of policing, toll-collecting, and maintenance personnel. In general, the greater part of the savings in operating the Queens Midtown Tunnel as compared with the Holland Tunnel is the result of reducing payroll costs. In a large measure this was accomplished through skillful designing based on past operating experience.

---

[13] Electrical Engr., New York City Tunnel Authority, New York, N. Y.

During the design period, the selection and location of apparatus, the selection of materials, and the arrangement of rooms in the buildings were studied in relation to the organization chart for operation adopted at the start of the project. In this chart the personnel was reduced drastically. Man-hours could not be decreased by the elimination of any major operating function but rather by their curtailment on a very large number of relatively small operations. The following are a few examples of the saving obtained in this way in the electrical and mechanical fields:

1. Where possible, operations were made automatic; such was the use of automatic dial telephones to eliminate telephone switchboard operators. In other cases, single operations were made simpler and easier in order to permit employees to take on additional duties—for instance, in the case of the operators attending the main ventilation and traffic control switchboard. This function was combined with the administrative work of the police force and resulted not only in saving man-hours but also in producing a smoother working force with more centralized authority.
2. A reduction in the electrical and mechanical personnel was effected by judicious selection of the apparatus in order that it might function safely without attention for long periods of time. As many as eight operators per shift or a total of thirty-two men (counting relief men) were eliminated in this way. Also the number of maintenance men needed for fans, motors, and pumps was reduced.
3. Particular attention was given to the ever-present problem of corrosion, as painting is a large item of maintenance. Where economically possible, nonferrous metals or materials were used. For instance, as mentioned in the paper, asbestos cement boards were used for baffles and louvres in the ventilation system; all bolts, nuts, screws, washers, and small parts in electrical devices are of silicon bronze; electrical conduits, below ground, are silicon bronze or asbestos cement; electrical relays are sealed in glass containers; motors are totally enclosed; etc.

Many similar reductions were made. For instance, maintenance labor was reduced by a radical simplification of the carbon-monoxide analyzers and recorders. Savings in man-hours have resulted from improved toll-registering machines which have reduced the auditing time. For washing the tile walls in the tunnel, a new machine which does the work faster and with less labor was designed. In order to decrease payroll costs, every operating function was analyzed in the greatest detail.

CHARLES E. CONOVER,[14] M. AM. SOC. C. E.—Under the heading, "Historical," Mr. Singstad states: "In July of the same year [1929], the Board of Transportation was assigned the work of preparing plans and specifications [for the Queens Midtown Tunnel] and starting construction with an initial appropriation of $2,000,000."

---

[14] Cons. Engr., Pearl River, N. Y. (formerly Div. Engr. of Planning and Design, Board of Transportation, New York, N. Y.).

As all who have studied vehicular tunnels know, their success or failure depends largely on the plaza layouts together with the approaches on the assumption, of course, that the development leading up to the plazas warrants the expense of tunnel construction. By approaches is meant not only the streets in the immediate vicinity, but also the traffic arteries that may extend for miles beyond. To illustrate: In the case of the Holland Tunnel the New York City approaches are limited very largely to the lower and midtown areas of Manhattan via adjacent streets and to Brooklyn via Manhattan Bridge, whereas the New Jersey approaches may be considered to extend to, and connect with, every municipality in central New Jersey and even Philadelphia, Pa.

The location of any expensive vehicular traffic facility, whether bridge or tunnel, must be the subject of extensive preliminary investigation, and, as Mr. Singstad has indicated, the Regional Plan Association had made very extensive and comprehensive traffic studies covering midtown Manhattan especially. This research clearly showed the great need, as Mr. Singstad so aptly states, for an east and west traffic axis from Long Island to New Jersey.

The investigations and the plans prepared by the Board of Transportation led to the conclusion that, to render the maximum benefit to Manhattan and Queens and to serve the entire metropolitan area most advantageously, such a traffic axis should have ample and convenient plazas on both the east and west sides of Manhattan and should extend by fairly direct connections far into New Jersey and Long Island. It should provide for express highway service free of the shackles that "stop-and-go" lights impose on fast and continuous service. In practical terms the East River tunnel should so connect with the Hudson River, or Lincoln Tunnel (then being studied, and later constructed and operated, by the Port of New York Authority), that traffic between New Jersey and Long Island could operate across Manhattan without emerging into the street system or in any way interfering with traffic.

Through the splendid cooperation of the engineers of the Board of Transportation with the engineers of the Port Authority, a joint west side plaza was developed, and, through the cooperation of the engineers of the Board of Transportation with the engineers of the Borough of Manhattan, a joint east side plaza was developed.

Although the east side plaza, or the Manhattan plaza, of the New York City Tunnel Authority differed radically from the east side plaza as developed by the Board of Transportation, it retained the possibility of a direct connection with the west side plaza (the Manhattan plaza of the Lincoln Tunnel), and thereby provided the possibility of a direct New Jersey-Long Island highway, if and when the Manhattan tunnels were constructed.

It is to be hoped that the New York City authorities will plan these connecting tunnels as a postwar project. With that connection there should be considerable lessening of crosstown traffic in the midtown Manhattan area, and a considerable increase not only in the traffic of the Queens Midtown Tunnel but also in that of the Lincoln Tunnel. This trend is indicated clearly by the traffic studies of the Regional Plan Association. Further extension into and through New Jersey by express highway or highways also would increase the traffic and relieve local congestion.

In addition, the plans of the Board of Transportation provided an express connection extending several miles into the Borough of Queens, somewhat similar to that mentioned in the paper. That connection should be constructed as a postwar project, as should a viaduct from the Queens plaza crossing Newtown Creek and connecting with the Brooklyn crosstown highway which was under consideration in 1929 and 1930. These connections not only would increase the traffic through the East River tunnels, but would relieve, most assuredly, traffic at and near the Queens plaza.

A detail that should be studied, if and when the Manhattan tunnels are constructed, is the method of connecting those tunnels to the north and south auxiliary streets provided as a part of the Manhattan plaza of the Queens Midtown Tunnel, so that the traffic may move in one direction if reasonably possible. In other words, the auxiliary street east of Third Avenue should be an exit street, not only for the river tunnels, but also for the Manhattan tunnels. Likewise, the auxiliary street east of Second Avenue should be an entrance street for both facilities. The purpose of such an arrangement would be to simplify the flow of traffic at the intersections of the auxiliary streets with cross streets.

In the case of the Queens Midtown Tunnel, the lighting that was developed for the portals where special attention was given to the difficulty of passing between brilliantly sun-lighted plazas on clear days and the more dimly artifically-lighted tunnel portals undoubtedly has resulted in better driving conditions at the portals, than at the portals of the Holland Tunnel.

Under the heading, "Introduction," Mr. Singstad refers to "remote control of ventilation." It is suggested that the designers might have gone a step further and provided not only remote control, but automatic control, of ventilation.

A detail, not mentioned in the paper but which may have been considered, is adequate widening and suitable arrangement at curves to provide for the passage of fire-fighting apparatus. In the plans prepared by the Board of Transportation this matter was given careful attention.

EDWARD LEVY,[15] M. D.—The same high standards of achievement were set by Mr. Singstad for the control of safety and health of the men working under compressed air on the Queens Midtown Tunnel, as for the other features of this outstanding project. The experiences on the Queens Midtown Tunnel in this field have provided considerable important data which may help to establish improvements in methods affecting health, safety, and economy on future work.

As stated by the author, the medical record for work in compressed air on the Queens Midtown Tunnel shows great improvement over that of the East River tunnels of the Pennsylvania Railroad. These railroad tunnels were built at a time (1904–1908) when practical regulations for the prevention of compressed air illness were still in an experimental stage.[16] As shown in Fig. 2, the Queens Midtown Tunnel is located near the Pennsylvania Railroad tunnels, and, consequently, conditions encountered in construction were similar in many respects on the two projects. The comparison, denoting progress in the control of compressed air illness during the period from 1904 to 1939, is therefore a sound one, and the improvement shown is undoubtedly due to better regulations and better enforcement.

---

[15] Medical Consultant, New York City Tunnel Authority, New York, N. Y.

[16] "Caisson Disease and Its Prevention," by Henry Japp, *Transactions*, Am. Soc. C. E., Vol. 65 (1909), p. 1.

There were no legal regulations during the construction of the Pennsylvania Railroad tunnels. For gage pressures up to 32 lb per sq in., the working day was 8 hours long, with a half-hour rest period. For gage pressures of from 32 to 42 lb per sq in., there were two 3-hr work periods per day with a 3-hr rest period between them. An attempt was made to enforce a maximum decompression rate of 2 lb per min.

The first statutory regulations for work in compressed air to be adopted by New York State became effective on January 1, 1910. These have since been modified from time to time by reducing the hours of work. Projects constructed since 1925, including the 53d Street, Fulton, Rutgers-Jay, and Harlem River tunnels and part of the Holland Tunnel work, have been under the regulations shown in Table 3. The rest periods are ones actually adopted and are equal to or greater than the prescribed minimums.

The maximum rates of decompression required by these regulations are: For 0 to 15 lb per sq in., 3 lb per min; for 15 to 20 lb per sq in., 2 lb per min; for 20 to 30 lb per sq in., 3 lb per 2 min; and for 30 lb per sq in. or over, 1 lb per min. A stage method of decompression is used in which the pressure in "locking out" is dropped at a rapid rate (specified at 5 lb per min) to half the gage pressure, and the remaining decompression is at a uniform rate such that the total time of decompression, including that for the initial rapid drop in pressure, shall equal the time specified for the original maximum pressure.

The Queens Midtown and Lincoln tunnels both were built under these regulations with the modification that the schedule of 3 hours of work, 3 hours of rest, and 3 hours of work was adopted at 0 to 18 lb per sq in. as well as in the higher range from 18 to 26 lb per sq in. Also, on the Queens Midtown Tunnel, the decompression times in the higher pressures were increased in length from 2 to 8 min above those required by specified maximum rates.

TABLE 3.—REGULATIONS IN EFFECT IN THE STATE OF NEW YORK, GOVERNING THE MAXIMUM DAILY HOURS OF WORK PERMISSIBLE UNDER COMPRESSED AIR

Time sequence (hours)	GAGE PRESSURE, IN POUNDS PER SQUARE INCH						
	0–18	18–26	26–33	33–38	38–43	43–48	48–50
Work	4.0	3.0	2.0	1.5	1.0	0.75	0.5
Rest	0.5	3.0	4.0	4.5	6.0	5.25	7.0
Work	4.0	3.0	2.0	1.5	1.0	0.75	0.5

The Clark, Montague, 60th Street, and 14th Street tunnels were built under statutory regulations that permitted 8 hours of work in 24 hours in gage pressures of from 0 to 22 lb per sq in., two 3-hr work periods in from 22 to 30 lb per sq in., two 2-hr work periods in 30 to 35 lb per sq in., two 1.5-hr work periods in 35 to 40 lb per sq in., two 1-hr work periods in 40 to 45 lb per sq in., and two 0.75-hr work periods in 45 to 50 lb per sq in. Decompression was specified to be at the maximum rate of 3 lb per 2 min up to 36 lb per sq in., and 1 lb per min in pressures of 36 lb per sq in. and greater. Part of the Holland Tunnel work was performed under this schedule of hours of work with the exception that the first and second ranges of pressures were 0 to 21 and 21 to 30 lb per sq in., respectively.

It is noteworthy that there have been no deaths due to compressed air illness since the construction of the 60th Street tunnel, built from 1917 to 1921. In fact, the record shows quite conclusively that, although legislation is periodically demanded to further shorten the lengths of the working shifts, a practical limit already has been reached if not surpassed in this regard. Naturally it is to be expected that the shorter the time of exposure in compressed air the less the incidence of compressed air illness will be, but this can be carried to a point at which the cost of compressed air tunneling would be prohibitive. Although the practice, on the Queens Midtown Tunnel and other recently constructed tunnels, of limiting the working time to a maximum total of 6 hours in 24 hours is sound both from medical and operational standpoints, it is evident from the record that further reduction in the length of the working shift is not warranted.

Work under compressed air at the Queens Midtown Tunnel was supervised most carefully, and, in some cases, hourly limitations of work and decompression requirements were even more rigorous than required by the code. As stated by the author, working conditions and safety precautions were strictly in accordance with the best standard practice. There were almost three quarters of a million decompressions, most of them from pressures of from 26 to 37.5 lb per sq in. The reporting of cases of compressed air illness was encouraged and the records were kept carefully. It was the intention to approach the ideal as closely as possible. In this the Tunnel Authority had the full cooperation of the contractor and his staff, and also that of the members of the Compressed Air, Tunnel, Caisson, Subway, Cofferdam and Sewer Construction Workers Union.

Having in mind the idea of using oxygen while decompressing, so as to hurry the desaturation of the body fluids and tissues of excess nitrogen, the help of the Division of Industrial Hygiene of the National Institute of Health, U. S. Public Health Service, and U. S. Bureau of Mines was enlisted in a field study along these lines conducted from July, 1938, to May, 1939.

Theoretically, at least, this use of oxygen should have been successful. In conjunction with the investigation,[17] blood and urine tests were made on men who had suffered from attacks of compressed air illness, but no causative reason was found to show why these men should have been attacked while others, in whom there was no physiological difference, were not. Briefly, the tests with oxygen breathing were successful when conducted with smaller groups under the direct supervision of the representatives of the government agencies. When continued on a more extensive scale under the contractor's supervision, they were somewhat disappointing. The investigation indicated that the use of oxygen in decompressing would be practicable for the smaller groups of men, especially when they had been subjected to the higher pressures.

---

[17] "Administration of Pure Oxygen to Compressed Air Workers During Decompression," by R. R. Jones, J. W. Crosson, F. E. Griffith, R. R. Sayers, H. H. Schrenk, and Edward Levy, *Journal* of Industrial Hygiene and Toxicology, Vol. 22, No. 10, December, 1940, pp. 427–444.

As a by-product of the oxygen-breathing tests conducted on the Queens Midtown Tunnel work, the use of helium-oxygen mixtures for the alleviation of ear block was developed successfully.[18] As ear block is the cause of many compensation cases on compressed air work, as well as sometimes the cause of delay when a gang is locking into the tunnel, this use of a helium-oxygen mixture (for which a practical apparatus was perfected on the Queens Midtown Tunnel) is an important development.

It is interesting to note that on the Queens Midtown Tunnel work the onset of compressed air illness occurred in 38% of the cases during the first half hour after decompressing, 54% occurred during the first hour, 66% in the first hour and a half, and 72% during the first two hours after decompression. Corresponding percentages based on the record of a great number of cases on previous projects are, respectively, 44%, 64%, 73%, and 82%.

A comparison of the percentages of types of symptoms on the Queens Midtown Tunnel (A), the Pennsylvania Railroad East River tunnels (B), and 680 cases of compressed air illness in New York Public Service Commission Tunnel projects (C) is afforded by Table 4. These percentages are fairly consistent. As the joints, fibrous tissues, and fats of the body (which have poorer circulation than other parts of the body) desaturate more slowly during decompression, this accounts for the high percentage of symptoms which are classified as localized pain in the body.

TABLE 4.—INCIDENCE OF SYMPTOMS (PERCENTAGE OF TOTAL CASES)

Type of Symptom	A	B	C
Central nervous system	2.7	2.2	1.6
Localized pain in body	86.2	90.3	91.7
Vertigo	9.9	5.4	6.5
Chokes	1.0	1.6	0.1
Partial or complete unconsciousness, with collapse	0.2	0.5	0.1
Totals	100.0	100.0	100.0

A study of the blood of compressed air workers on the Queens Midtown Tunnel gave no clew to the susceptibility of some workers over others. However, there was a definite feeling that alcohol played a part in the occurrence of compressed air illness. Grover C. Emery, M.D., who was in direct medical charge of the contractor's employees has expressed this idea as related to dissipation following payday:[19] " '* * * a majority of cases occur following payday, and can be assumed to have dissipation as an etiological basis.' " However, there is no particular evidence, other than by casual observation, apparently, that there was an increase in dissipation following payday. Table 5 does show a marked increase in the incidence of compressed air illness after exposure on Sunday, which was covered by small watch shifts with little work to do. On the basis of such working conditions, a minimum incidence of compressed air illness would be expected rather than double the average incidence, as actually experienced. Could this phenomenon be due to the fact that Sunday follows Saturday night so closely?

---

[18] "Helium-Oxygen Mixtures for Alleviation of Tubal and Sinus Block in Compressed Air Workers," by J. W. Crosson, Roy R. Jones, and R. R. Sayers, *Public Health Reports*, Vol. 55, No. 33, August 16, 1940, pp. 1487–1496.

[19] "Engineering Problems Related to the Health of Workers in Compressed Air," by Ole Singstad, 6th Pacific Science Cong., Vol. VI, p. 82.

TABLE 5.—INCIDENCE OF COMPRESSED AIR ILLNESS ON QUEENS MIDTOWN TUNNEL, BY DAYS OF THE WEEK (FOR 20 TO 37.5 LB PER SQ IN. GAGE PRESSURES)

Day	Total decompressions	Number of cases	Cases per 100 decompressions
Sunday (watch shifts)	15,715	64	0.407
Monday	90,303	112	0.124
Tuesday	100,567	177	0.176
Wednesday	100,239	194	0.194
Thursday (payday)	100,312	200	0.199
Friday	101,891	203	0.199
Saturday	99,637	193	0.194

The New York State regulations, under which the Queens Midtown Tunnel was constructed, provide that

> "Whenever the air pressure in a tunnel heading exceeds twenty-one pounds per sq. in. above atmospheric pressure, two air chambers shall always be in use, except for such time as may be necessary when headings are being started from shafts; and whenever practical the pressure in the outer chamber shall not exceed one-half the pressure in the heading."

This condition is established by constructing the necessary additional bulkhead and is generally worked out in conjunction with the maintenance of a bulkhead within a specified maximum distance of the heading, generally about 1,000 ft. Fig. 11A shows the location of the bulkheads forming the outer air chambers in the two Manhattan headings of the Queens Midtown Tunnel, and Fig. 11D shows the locations on the Queens side. Using the single bulkhead and no outer chambers, 270,672 decompressions were made from a 22 to 37.5 lb per sq in. range of pressure with 486 cases of compressed air illness, an incidence of 0.179 per 100 decompressions. Using the additional bulkheads and outer chambers, 316,595 decompressions were made from a 22 to 37.5 lb per sq in. range of pressures with 646 cases of compressed air illness, an incidence of 0.204 per 100 decompressions. As far as controlling compressed air illness is concerned, this record shows no advantage in the use of the outer air chambers; in fact, the record for the single air chamber is better. The use of the outer chamber has been looked on as a great aid in the prevention of compressed air illness but the results on the Queens Midtown Tunnel, comparing the aforementioned large number of decompressions, indicate that this may not be the case. As the cost of installation, operation, and removal of a tunnel bulkhead is a substantial one, further study of this regulation is advised as it appears that the use of an additional bulkhead solely to satisfy the requirement for an outer air chamber may be an unwarranted expense. This consideration, of course, is entirely apart from the use of additional tunnel bulkheads when necessary to satisfy other safety requirements.

In further regard to decompression regulations, exception could well be taken to the requirement that the stage method of decompression be used. Although there is nothing in the record of compressed air work in the New York area, including that of the Queens Midtown Tunnel, definitely to prove this assertion, there is, on the other hand, nothing to prove the efficacy of the stage method of decompression as far as tunnel work is concerned. In fact, it well may be that the fast initial drop of pressure employed in this method of decompression actually induces the beginning of the formation of the air bubbles that are the cause of compressed air illness. The initial drop to half the gage pressure, as practiced on the Queens Midtown Tunnel, was often slower than the rate of 5 lb per min required by the regulations. Nevertheless, it is possible that an even slower initial rate of decompression might have given better results. This should be investigated on the next compressed air tunnel project. It should be remembered that the stage method of decompression was developed in connection with diving, where the diver is raised slowly to the surface and is subject, meantime, to the low temperatures of the surrounding water. The stage method is designed to achieve what may be considered a safe decompression in the minimum time; and therefore it is a desirable method in deep water diving, for which it was developed, where the effect of cold increases the diver's susceptibility to compressed air illness. As tunnel workers decompress in a heated lock, a decompression method well may be used which sacrifices some small saving in time to greater safety.

It is interesting to note that, of the total of 1,148 cases of compressed air illness reported on the Queens Midtown Tunnel work, 194 or 16.8% occurred following the first of the two periods of daily exposure and that 954 cases or 83.2% occurred after the second period. Such a difference between the incidence of compressed air illness after the first and second periods of exposure may be explained in part by fatigue. The fact that some cases develop after the first period and are not reported until after the second (the men hoping to relieve themselves during the second exposure) may contribute slightly to the increased number of cases occurring after the second shift, but not to any great extent, as compressed air illness is painful and it is not likely many would wait several hours for relief.

In the lower pressures, especially up to 18 lb per sq in. and possibly higher, some consideration should be given to the use of one daily work shift of 6 hours, rather than two shifts of 3 hours. This modification would be in line with suggestions that have been made from time to time since the early days of the use of compressed air in construction, and the experience on the Queens Midtown Tunnel indicates that it might be practicable as there would be certain operational advantages without adverse effect on safety. This idea at least should receive further study.

Robert F. Legget,[20] **Assoc. M. Am. Soc. C. E.**—The literature of civil engineering has been enriched by this notable account of a notable undertaking, an account enlivened by attention to many unusual details of design and construction. As was to be expected for such a public work in the vicinity of New York, due attention was paid to the necessity for preliminary subsurface exploration. The costs of this work as given in Table 2, amounting to only 0.2% of the total contract cost, provide further evidence of the economic value of such exploratory work. The successful performance of the Queens Midtown Tunnel makes any discussion of the main parts of the paper of academic interest only. There are some details, however, that the writer would like to see amplified, if only briefly, and he therefore ventures to comment upon them.

(*a*) Under the heading "Design of Structures: Lining for Shield-Driven River Tunnels," the author gives some details of the fastening of the bolts used for securing the tunnel liner plates and it is noted that "Actual bolt stresses * * * varied from intensities considerably less than 25,000 lb per sq in. to stresses approaching the yield point of the bolts." Would Mr. Singstad advise whether the ratchet wrenches used for bolt tightening were merely standard wrenches or, alternatively, wrenches designed to give a fixed maximum torque? Use of wrenches of the latter type is standard practice in certain branches of aircraft work, great ingenuity having been displayed in their design. It is to be hoped that one of the many minor by-products of the present wartime industrial development will be the acceptance of such tools in civil engineering practice, particularly since bolts and turnbuckles form so important a part of many structures—being designed for definite initial stresses and yet all too often installed without reference to this requirement. The writer has in mind particularly the turnbuckles used for supporting bulkhead retaining walls as equivalent in importance to the bolts described in this paper.

(*b*) In dealing with the Manhattan plazas and approaches, the author states that one design for the plaza was not adopted since construction costs, for a thin invert slab and drainage system, were about the same as those for a very heavy slab design. This suggests that the drainage involved must have been unusually complicated. Could Mr. Singstad amplify this comparison of designs in order to make clear why drainage thus had to be neglected?

(*c*) Finally, in view of the "structural" use to which the clay in the river blanket was put, could Mr. Singstad give details of the type of clay used for the blanket, and also of the material referred to as "volcanic clay."

Orrin L. Brodie,[21] **M. Am. Soc. C. E.**—In his several allusions to the design of the Holland Tunnel, Mr. Singstad states that in all essential respects the design for the Queens Midtown Tunnel is the same design as that for the Holland Tunnel and that extended research is no longer necessary in planning and designing tunnels for highway traffic. The length of the cast-iron ring of the latter tunnel, after two decades, could be increased to 32 in., or 2 in.

---

[20] Asst. Prof., Dept. of Civ. Eng., The Univ. of Toronto, Toronto, Ont., Canada.

[21] Designing Engr., Board of Water Supply, New York, N. Y. (Mr. Brodie died on October 5, 1943).

over the earlier tunnel, thereby effecting considerable saving, while the number of segments (14) would remain unchanged. The initial bolt-tightening tension (25,000 lb per sq in.) is the same in the newer structure; but at the same time the author states (see heading, "Design: Lining for Shield-Driven River Tunnels") that "an allowance of about 5,000 lb per sq in. above the required bolt stress" is now made "to offset any plastic yield effect in the entire bolt assembly under sustained loading." From this statement the writer understands that 30,000 lb per sq in. is specified instead of the lower value.

Although the Holland Tunnel studies were based on a tube with an outside diameter of 29 ft, the sizes actually constructed were 29 ft 6 in. for the greater part and 30 ft 4 in. for a stretch in Jersey City in the northern tube, extending for 800 ft west of the New Jersey land shaft, to provide greater ventilation spaces and facilities for the long upgrade. This was the largest diameter shield-driven tunnel that ever had been constructed.

The author's statement that the operation of existing tunnels will continue to provide data for improvements in design is borne out in Table 1, which compares the ventilation of the Holland Tunnel with that of the Queens Tunnel.

The Holland Tunnel project presented problems unique in their time, both in design and ventilation, and called for pioneer work. It has been brought to the writer's attention, from several sources, that the studies for the Holland Tunnel have never been published. For this reason a report in January, 1921, to Mr. Singstad (who was then engineer of designs) on the studies for design of the Holland Tunnel ring is referred to herein. (The full report has been placed on file with Mr. Singstad's records for reference in Engineering Societies Library, New York, N. Y.) The conclusions still are applicable today to its prototype, the Queens Tunnel.

Analysis of Cast-Iron Tunnel Sections, Holland Tunnel.—Subaqueous shield-tunnel practice recognizes the uncertainty of absolute values of external loading, especially at the critical times during construction. A controlling consideration is the possibility of a state of collapse during construction rather than the realization of the safe working stresses that might be employed in direct designing.

Therefore, studies that are comparative are possible and were made on different sections of cast-iron tunnels, including sections proposed for the vehicular tunnel and sections of tunnels already built and in service under similar conditions. The various results were compared rationally through the different angles of repose (Rankine's theory) assumed for the material forming the bed of the Hudson River. The investigations included angles of repose ranging from 3° to 20°, which serve as an index common to all phases of the work, and also signify different relations of horizontal thrusts to vertical loadings, exterior to the tunnel.

The resultants of the forces acting on the tunnel lining include the weight of the tunnel ring and that of interior structural features together with compressed air within the tunnel and the exterior loads. The last named are least capable of precise definition; hence, varying the exterior forces throughout a comprehensive range yields more conclusive results than if attention is confined to any one set of forces, themselves of doubtful value. These exterior forces represent active thrusts based upon Rankine's theory of earth pressure. The horizontal, abutting resistances of the surrounding material could be developed only through the deformation of the tunnel ring. This deformation develops internal stresses, all predicated on the active thrusts. Stresses due to the active thrusts of the surrounding material at a given angle of repose may be found to equal the stresses due to the abutting power of the material at some other angle of repose. Therefore, for comparative purposes it should be sufficient to base these studies on active thrusts only and thus preclude the uncertainties of speculations as to what abutting resistances might be developed at a given angle of repose. At the same time conditions are approached that are admittedly more rigorous than those for the tunnel sections in their final state of completion, since the active thrusts provide a series of horizontal and vertical loads of increasing inequality that are more nearly representative of loads encountered during construction. Thus, in these studies comparative values are emphasized rather than any consideration of the forces assumed for the exterior loading at any given angle of repose.

Conclusions were drawn from comparisons of results which indicated the more favorable of alternative designs. Local points of weakness were developed and remedial measures were devised.

The analysis comprised the determination of true equilibrium polygons for different conditions of loading from which the resultant thrusts and bending moments in the tunnel rings were determined completely. From these thrusts and moments, the stresses in the cast iron were computed directly: First, by neglecting the effects of joints and bolts locally—that is, assuming a given ring to be continuous, acting alone, and with a cross section of corresponding segment design; and, second, by means of supplementary analysis, including the effect of the joints and bolts under stress. The results were plotted as curves, in terms of the angles of repose, in both cases, and conclusions were made.

The true equilibrium polygons were determined by the method of work and the method of deflections. The advantage of the latter method is that it makes graphics unnecessary, and thereby is more precise than the natural limitations of graphical analysis. The method of work, as applied herein, consists of a graphical adaptation of the principle of least work, and, although generally satisfactory for ordinary purposes, lacks sufficient precision for low angles of repose, where percentage errors in scaling thrust eccentricities increased. Wherever both methods were used, the deflection method checked back most accurately on results found independently by employing the principle of least work.

Supplementary analysis of the action of the bolts under stress and the modifications of resulting stress in the cast iron of the segments were made. In general, the joint and bolt analysis consisted of investigations of (1) the modifications of stress sustained by the cast iron caused by the action of the bolts in the end and circumferential flanges of a segment and, through the bolts in the latter, (2) the effect of the two adjacent rings on the given ring between them. The elasticity of the bolts, the bending of the short longitudinal flanges of the tunnel ring, the transfer of compression from the joint to the body of the segment, the tendency of a joint to open, and the friction between the circumferential flanges enter this part of the investigations. These calculations of bolt and joint stresses are also dependent upon the true equilibrium polygons, determined previously.

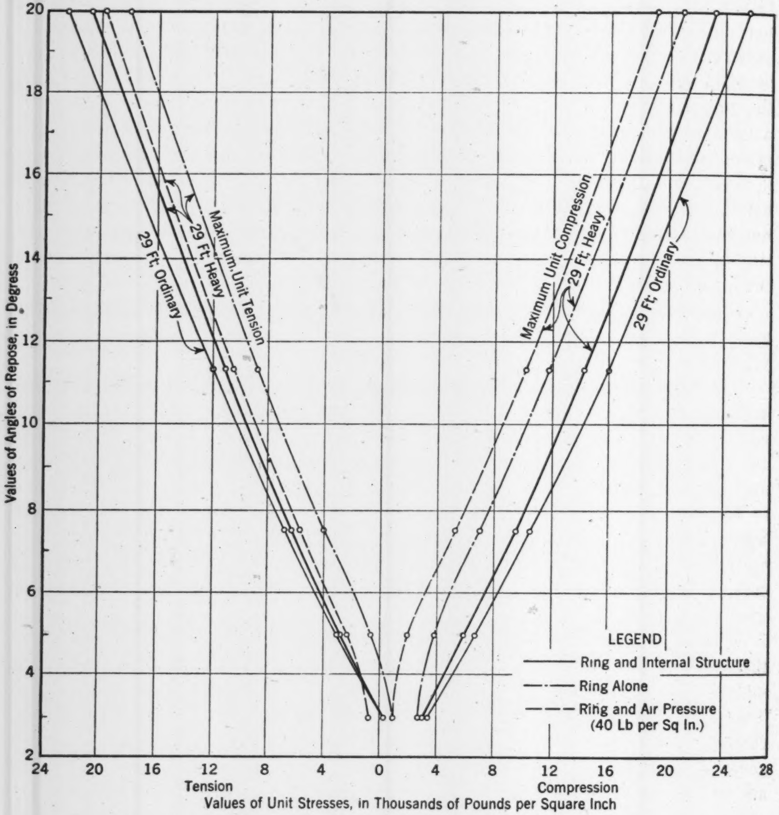


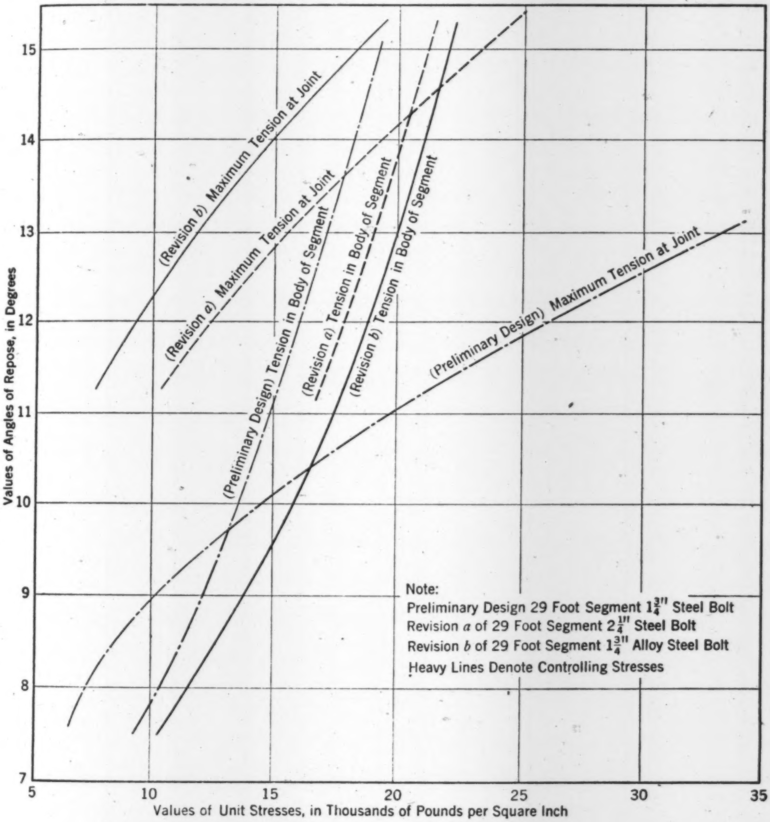
FIG. 15.—COMPARATIVE STRESSES, CAST-IRON RINGS OF THE HOLLAND TUNNEL

The effect on the bending moments of the ring manifested by horizontal ties at the roof and floor of the driveway of the vehicular tunnel, 29 ft in diameter, was also approximated.

The cast-iron ring with entire internal structure, the cast-iron ring alone, and the cast-iron ring subjected to compressed air within furnished three distinct cases for investigation. The concrete lining was considered only as contributing its dead weight.

The basis of computation was 1 ft of tunnel length; 38 ft of water on 31 ft of silt were assumed to rest on the top of the tunnel ring. The weights of water and silt were assumed, respectively, at 64 lb per cu ft and 104 lb per cu ft. The air pressure, for the 29-ft tunnel, was applied at 40 lb per sq in.

As cohesion is neglected in Rankine's theory of a "granular mass," vertical forces due to tunnel ring alone, and to the overlying material, in some cases are less than the upward active pressures computed according to this theory. In such cases it appears that the structure would tend to float. For the sake of equilibrium the vertical forces are equalized by adding the necessary difference as a uniform, overlying load, which may be considered either as due to extra fill or as a tacit recognition of the existence of cohesion in the overlying mass. During the construction of the Holland Tunnel the characteristics of



Note:
Preliminary Design 29 Foot Segment $1\frac{3}{4}''$ Steel Bolt
Revision *a* of 29 Foot Segment $2\frac{1}{4}''$ Steel Bolt
Revision *b* of 29 Foot Segment $1\frac{3}{4}''$ Alloy Steel Bolt
Heavy Lines Denote Controlling Stresses

FIG. 16.—PRELIMINARY DESIGN OF CAST-IRON RINGS, HOLLAND TUNNEL

the silt were found and its value in cohesive effect fully determined. The overlying depth of silt at one place, for the tunnel, showed no tendency to float although its buoyancy, in terms of simple displacement of water and silt, indicated otherwise. Similarly, where the downward forces exceed the upward active pressures, the uniformly distributed difference required to balance is applied on the lower half of the structure. This additional set of forces may be considered as a partial development of the vertical abutting resistance of the material below the tunnel. In this case, the resulting upward reaction is found to lie between the values of the forces derived from the active pressures and those that would result if the full abutting pressures could be developed, according to the theory. The assumed elastic limits were 40,000 lb per sq in. for steel bolts and 80,000 lb per sq in. for alloy ("high tensile") steel bolts. In all cases the stresses were computed for the so-called "heavy" section of segment, except for the stresses shown in Fig. 15, for the complete structure of the "ordinary" section of 29-ft tunnel. The web thickness of the heavy section was 2 in.; that of the ordinary section, $1\frac{7}{8}$ in.; and that of the final design for the Holland Tunnel, $1\frac{5}{8}$ in. However, the extent of the web thickening for the heavy section was 7 in. back from the inside corner of the flange, instead of the 6 in. of the studied, ordinary sections. The equilibrium or funicular polygons were derived for the "ordinary" section of the 29-ft tunnel, and the stresses calculated for the "heavy" section were based on these polygons as the differ-

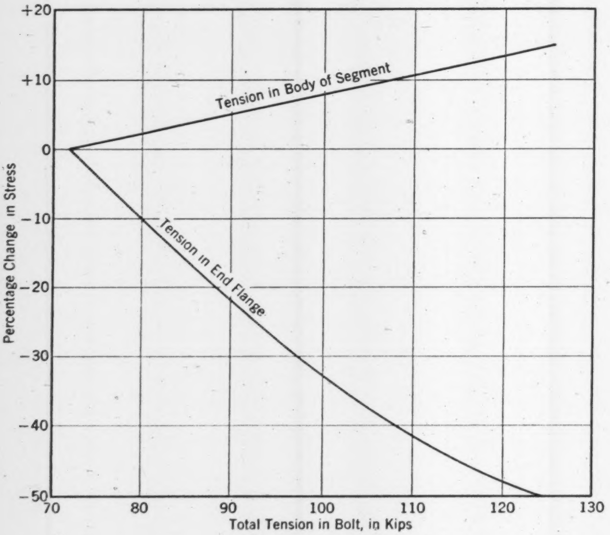


FIG. 17.—PERCENTAGE VARIATIONS IN STRESS FOR CHANGES IN TOTAL BOLT TENSION; HEAVY SECTION OF HOLLAND TUNNEL

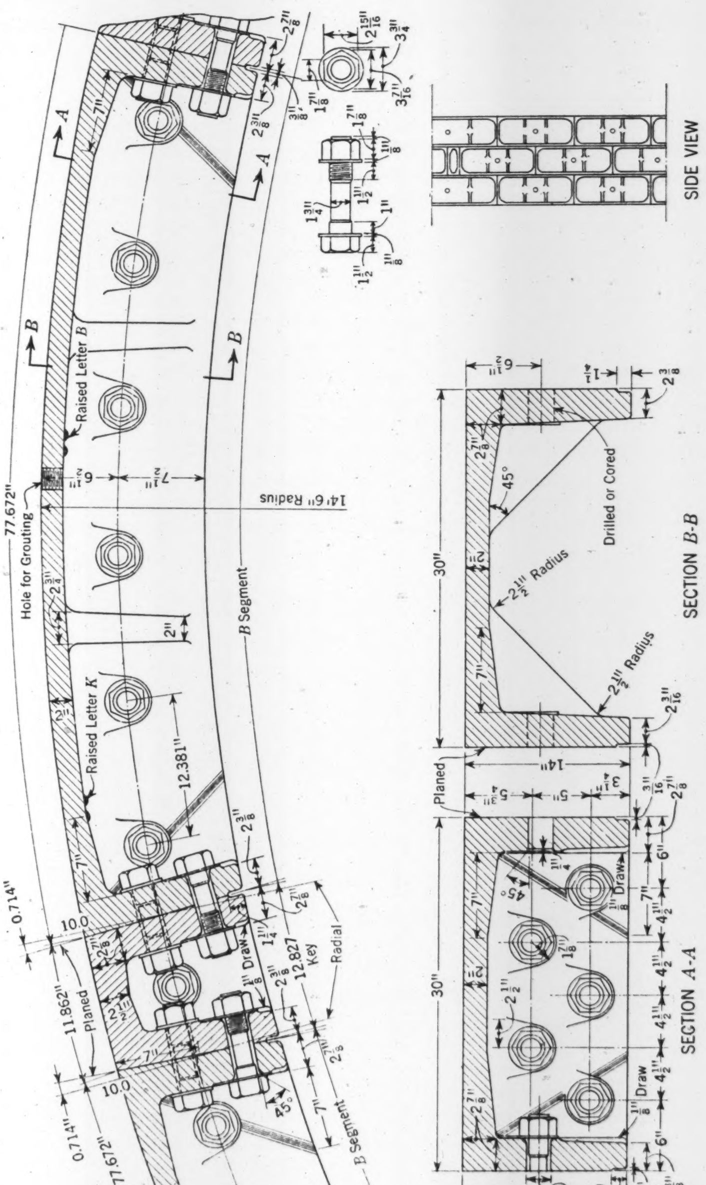

77.672"
Hole for Grouting
14'6" Radius
B Segment
Raised Letter B
Raised Letter K
12.381"
0.714"
10.0
11.862"
Planed
12.827
Key
Radial
Draw
45°
SECTION A-A
SECTION B-B
SIDE VIEW
Drilled or Cored
$2\frac{1}{2}$" Radius
30"
14"
A
B

FIG. 18.—CAST-IRON DETAILS, HEAVY, 29-FT SECTIONS, HOLLAND TUNNEL

ence between these two sections was such as to affect inappreciably—a small fraction of 1%—the resulting bending moments and thrusts.

Discussion of Analysis.—In the following, the terms "outer," "inner," "outside," and "within" refer to directions in the circular tunnel cross section from or toward its center, respectively.

Eight features of the analysis deserve comment in so far as they apply to a similar analysis of the Queens Midtown Tunnel: (1) Application of thrust; (2) the joints; (3) location of critical stress; (4) bolt tightening; (5) location of end flange bolts; (6) the nonutility of the middle end flange bolt; (7) flange thickening at corners and the web of the section; and (8) effect of tie rods.

(1) Inspection of the funicular polygons used in the analysis indicated that the thrusts in the ring may be outside or within the neutral axis of the ring—that is, positive or negative bending moments, besides direct thrusts in the ring may exist. Two cases thus may arise—Case I, with thrust outside, and Case II with thrust within, the neutral axis. The calculations showed that Case I resulted in maximum stresses in tension and Case II, in maximum stresses in compression; hence, Case I usually governs.

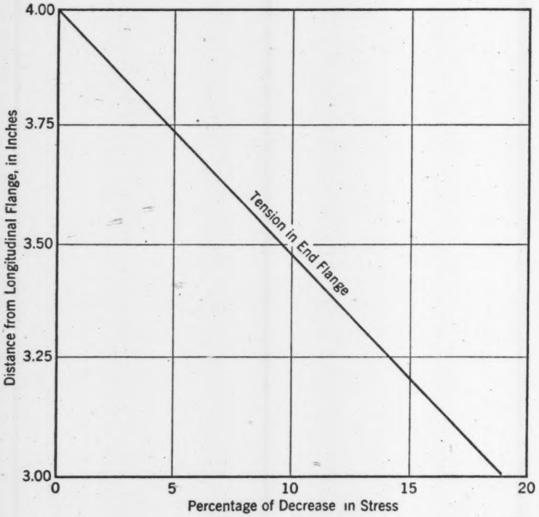
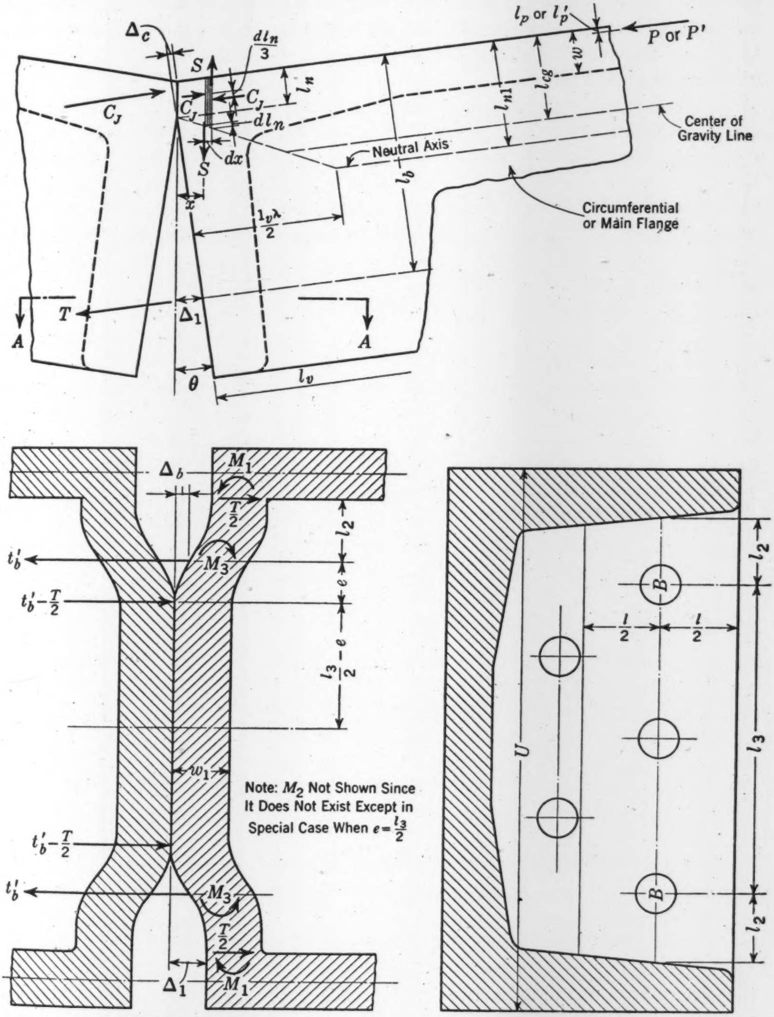


Fig. 19.—Percentage Stress Variation with Change in Bolt Position (Bolts Assumed Stressed to 40 Kips per Sq. In., the Elastic Limit, at Beginning)

(2) The joint, as may be expected, is the weakest feature of the tunnel ring. Comparison of the stresses in the 29-ft, cast-iron-segment, heavy-section, preliminary design, with those for the same section, but with joints neglected, indicates the increase in stress due to the joints. Hence, to strengthen the ring as a whole, it is necessary to increase the strength of holding at the joints.

In Fig. 15, only the cast-iron segments are assumed to withstand stresses, and the effects of joints and adjacent rings are neglected. In Fig. 16, on the other hand, the effects of bolts and joints are considered. The bolts are assumed to be stressed to the elastic limit at the beginning, and the maximum tension at the joint occurs in the segment near the end flange.


Δ_c
$\frac{dl_n}{3}$
l_p or l'_p
P or P'
S
C_J
C_J
C_J
l_n
w
l_{cg}
l_{n1}
dl_n
dx
S
Center of Gravity Line
Neutral Axis
l_b
Circumferential or Main Flange
x
$\frac{l_v\lambda}{2}$
T
Δ_1
A
A
θ
l_v
Δ_b
M_1
$\frac{T}{2}$
l_2
t'_b
M_3
e
$t'_b - \frac{T}{2}$
$\frac{l_3}{2} - e$
w_1
$t'_b - \frac{T}{2}$
t'_b
M_3
$\frac{T}{2}$
Δ_1
M_1
U
l_2
B
$\frac{l}{2}$
$\frac{l}{2}$
l_3
B
l_2

Note: M_2 Not Shown Since It Does Not Exist Except in Special Case When $e = \frac{l_3}{2}$

SECTION *A-A*

END FLANGE OF SEGMENT

FIG. 20.—STRESS CONDITIONS WITH THE CENTER OF PRESSURE NEAR THE EXTRADOS (CASE I)

(3) The critical stress in a segment is not in the joint itself, but immediately adjacent to it, at the intersection of the circumferential flange of the tunnel ring. This stress is due to the combination of the normal bending stress in the body of the segment, as this bending is modified by the circumferential bolt action, with the stress resulting from the local bending induced by the bolts of the end flange. These last bolts bind together the two successive segments of a ring at their ends.

(4) The tighter a joint is bolted, the greater the strength of that joint, and consequently, the ring. Fig. 17 indicates that the greater the stress which the bolts can carry, as initial stress (that is, in bolting operations) the greater the strength of the ring will be. Although a small increase in tension is apparent (due mainly to the circumferential bolts) in the body of the segment, where the tensions are usually lower than at the joints, this is inconsiderable when compared with the great decrease in stress in the critical part of the segment, the end flange, where the stresses are much higher. Fig. 16 shows a similar effect obtained by utilizing the higher elastic limit of alloy steel bolts of 1¾-in. diameter, instead of that of ordinary steel. The importance of keeping all bolts tightened constantly to their utmost strength, especially during construction when the tunnel ring receives its greatest punishment, is evident from these results alone and suggests the desirability of bolt tightening by means other than manual, if possible. Finally, the advantage of changing to the 1¾-in. alloy steel bolts is obvious. Fig. 18 shows this design.

(5) The nearer the outer two bolts in the end flange of a "plate," or segment, approach the web and the nearer the two side bolts of the inner three approach the circumferential flanges, the greater will be the reduction of local bending stresses, along with compression in the joint induced in the metal at the joint. Fig. 19 indicates the reduction in the case of the inner bolts.

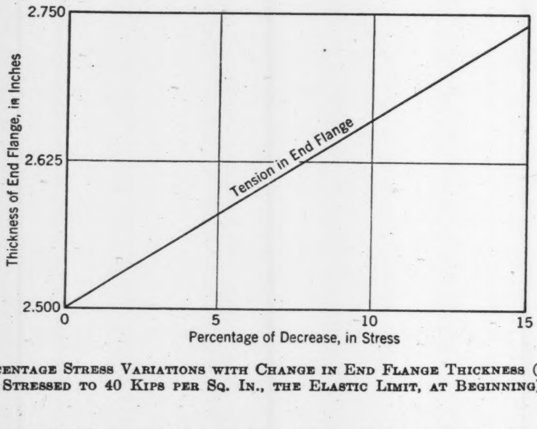


FIG. 21.—PERCENTAGE STRESS VARIATIONS WITH CHANGE IN END FLANGE THICKNESS (BOLTS ASSUMED STRESSED TO 40 KIPS PER SQ. IN., THE ELASTIC LIMIT, AT BEGINNING)

(6) The middle of the three inner bolts of the end flange (Fig. 18, Section A–A) cannot act with its two adjacent bolts, for, if the elasticity of the metal of

the bolts and joints is considered at all, and the elastic deformations are the essence of the studies, the action of the two inner bolts adjacent to this middle bolt puts the middle part of the end flanges of the two abutting segments meeting at this joint into compression. No possibility was found of a tension being induced in this bolt, except that caused by bolt tightening, unless the other bolts were stressed to failure, a condition that the middle bolt never could withstand. The only apparent advantage is the aid of the middle bolt in the process of tightening the remaining bolts—a practical consideration only. It is plainly easier to tighten up three bolts in line successively, back and forth, than two bolts alternately. As a factor contributing to tunnel-ring strength the value of the middle bolt is nil. This conclusion resulted from the analysis of bolt and joint stresses but may be suggested by regarding the cross section of such a segment as Section A–A of Fig. 18 along with Section A–A of Fig. 20.

(7) To strengthen the segment locally (Conclusion 2) it was further concluded that the metal of the flanges, inside, at the corners of the segment should be thickened as shown in the side view of Fig. 18. The comparative value of this local flange thickening is brought out in Fig. 21, where the end flange is thickened $\frac{1}{4}$ in. Subsequently, when the tunnel segments were being cast, it developed that this combined thickening at the corners resulted in better castings, since the casting with more metal than heretofore cooled at the corners so that a more uniform texture of metal was indicated as observed upon castings which were purposely broken for inspection.

(8) In addition, a study of the restraining effect of transverse tie rods in the roof and floor systems indicated that they would provide for a reduction of two thirds in the values of the maximum bending moments in the ring.

A. A. Eremin,[22] Assoc. M. Am. Soc. C. E.—Interesting details of design and construction for the Queens midtown subaqueous tunnel have been presented in this paper. In the "Conclusions" the author has stated that "Extended research is no longer necessary in planning and designing tunnels for highway traffic, * * *." Although it is true that considerable improvement in design and construction of subaqueous tunnels has occurred in the last two decades, further study is necessary.

Method of Construction.—It is well known that in tunneling soft ground shield driving is the most economic method. The protective blanket is generally used to prevent excessive loss of air and blowout accidents although the mechanical properties of the blanket assuring safety are still not definitely determined. On account of defects in the protective blanket, a serious blowout accident occurred during the construction of the Holland Tunnel. As a result, an improved type of blanket was used in the East Boston Tunnel, which, nevertheless, suffered considerable loss of air through the pile holes at the pierhead line. Similarly, much air was lost during construction of the Queens Midtown Tunnel, as reported by Mr. Singstad.

---

[22] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

The Posey Tunnel penetrating soft ground between Oakland and Alameda, Calif., although designed almost at the same time as the Holland Tunnel, was constructed by open-trench excavation and by floating pre-cast concrete segments. The shield-driving method was rejected because of the excessive cost of protective blanket.

Shield driving has many advantages, as has been pointed out in this paper:

(1) It causes very little disturbance in the ground. This is especially important because subaqueous tunnels are generally constructed at highly developed sites, and slight disturbance in the ground permits construction of twin tunnels close together. The Queens Midtown tunnels were 45 ft apart. Similar spacing of twin tunnels proved safe in driving the Holland and Lincoln tunnels.

(2) It may often result in saving of construction time. The speed of driving for the Queens Midtown tunnels was about 7.4 ft per day. The Lincoln tunnels were driven at the even higher rate of 45 ft per day.[23] Nevertheless, because it is an essential part of shield driving, further study of the mechanical properties of the protective blanket is highly desirable.

Loading.—An extensive study of loading on the subaqueous tunnel lining was made for the design of the Holland and Posey tunnels. As a result of this research, a circular rigid tube with steel ties at the roadway slab and ceiling was adopted for lining tunnels. During the construction of the Posey Tunnel at some sections the tie rods at the roadway slab were under compression instead of tension. The horizontal diameter was shortened during construction of the Lincoln Tunnel.[23] In the Queens Midtown Tunnel a flattening effect in the heavy cast-iron lining was observed, thus confirming the original study of loading. Mr. Singstad has shown important effects of grouting on distribution of loading and increasing resistance of the ground to the air losses. However, he states that (see heading, "Construction: River Tunnels") "The heading was dewatered on August 8, 1938, disclosing an inflow of grout and muck at a maximum depth of 14 ft at the shield and running back a distance of 208 ft in the tunnel." Thus, in shield driving, a further study of the methods of grouting is still essential. Correct grouting may control the load on the lining and the rate of shield driving as stated in the paper.

In the shield-driven tunnels various types of lining have been used. The most well-known linings are cast-iron, structural steel plate, and concrete. Mr. Singstad states that (see heading, "Design of Structures: Lining for Shield-Driven River Tunnels") "Cast-iron lining of the usual design was chosen for the shield-driven tunnels as best adapted to resist shield thrusts and to provide rigidity and watertightness" and that "* * * lack of uniformity in stress, may contribute to some leakage, * * *." The cast-iron segments have been proved rigid. However, their rigidity is decreased at the connections. Fig. 3 shows that the bolt holes in the segments are $\frac{1}{4}$ in. larger than the bolts. Evidently with such clearance pre-stressing and the use of a high strength steel for the bolts cannot make the connections as rigid as the segment units. Sufficient

---

[23] *Engineering News-Record*, June 24, 1937, p. 955.

rigidity and watertightness in the lining of the Lincoln and Detroit tunnels were obtained with the structural plate segments, which are generally three times lighter than cast-iron segments. To resist the direct thrust from the shield the structural plate liners generally are reinforced with old rails or H-sections.

Ventilation.—When the Holland Tunnel was being designed, very little was known about the ventilation of vehicular tunnels. In cooperation with the U. S. Bureau of Mines, formulas for the design of ventilation features in the tunnels were developed. Poisonous gas, carbon monoxide, was limited to 4 parts per 10,000. After the construction of the Holland Tunnel tests revealed that during maximum traffic the amount of carbon monoxide never was more than 1.60 parts per 10,000.[24] At first it was concluded that the Holland Tunnel and the Posey Tunnel (which was designed on a similar principle) were overequipped. However, it was decided later that greater circulation of fresh air purifies the odor and smoke resulting from combustion of oil in the vehicles. Nevertheless, in subsequent tunnels and in the Queens Midtown Tunnel, design of ventilation facilities was based on a limitation of the maximum amount of carbon monoxide. Evidently the physical properties of smoke and burned oil are different from those of the vitiated air containing carbon monoxide. Therefore, for an economic ventilation design, further study of the smoke problem is necessary. Separate fresh air outlets were used in the Broadway Tunnel in Berkeley, Calif. It is quite possible that the separate openings may require a greater air pressure at the bulkhead. For efficient design of ventilation a minimum loss of air pressure at the ventilating fans and at the outlets near the bulkhead is essential.

Transition of Light.—Various methods of lighting at the portals have been used to eliminate the discomfort and the blinding effect of daylight. It is interesting to determine the various possible structural details for regulating light. Mr. Singstad has shown that the belled-out approach structure provided a gradual transition of light. In the Broadway Tunnel in Berkeley, transition of light was improved with transparent roofing between the approach retaining walls.

Conclusions.—Vehicular tunnels have an important rôle in solving problems of growing interurban traffic. Therefore, the author should be highly commended for presenting such interesting information on the design and construction of the Queens Midtown Tunnel.

ARTHUR S. TUTTLE,[25] PAST-PRESIDENT AND HON. M. AM. SOC. C. E.—Essential data concerning the Queens tunnel project, with very complete information as to its inception, administration, design, construction, and cost are recorded in this paper. The profession is indebted to Mr. Singstad for making this information available not only as a matter of historical interest but also as a help in the planning of structures of similar character.

As is generally the case with all monumental projects undertaken by the City of New York, many years elapse between the need for such a facility and the completion of the final plan, during which period various pros and cons are given consideration, all of which must be appraised. The possibility of securing federal aid in 1936 was doubtless responsible for the city committing itself definitely to the project at that time. The 30% grant, with a 70% loan offered by the PWA, for meeting the entire cost of the project, was conditioned upon the presentation of evidence that the project was necessary and would be self-supporting and that construction would be begun without delay. The former condition was intended to insure against wasteful expenditures; and the latter, to relieve unemployment in accordance with the primary purposes of the congressional act under which the PWA was set up and provided with funds.

---

[24] *Engineering News-Record,* June 9, 1927, p. 934.

[25] Cons. Engr. (Tuttle, Seelye, Place & Raymond), New York, N. Y.

The tunnel is located between the nonrevenue producing Williamburg and Queensborough bridges, both of which are badly congested during hours of maximum traffic. Extensive traffic studies were presented by the Tunnel Authority to support the claim that the relief to the bridge traffic which would be afforded by the tunnel would bring enough business to justify the project as financially sound. These studies contemplated the provision, by the city, of adequate highway approaches in the Borough of Manhattan and the Borough of Queens, which were considered essential to the successful operation of the tunnel. Had the cost of these approaches been included in the project, it is more than likely that the project would have been disqualified for a loan on the ground that it would not be self-supporting.

Although plans have been developed for a much needed crosstown tunnel in Manhattan to relieve the narrow and congested streets, overworked by traffic destined for the west side and for New Jersey, no definite commitment has yet been made. In the Borough of Queens a part of the approach system has been constructed, but major links are still lacking. Until adequate feeders are provided, the traffic capacity of the tunnel will not be fully utilized, and revenues will be handicapped.

When the project was finally accepted by the PWA as a part of its program and the "go-ahead" signal was given to the Tunnel Authority, difficulties began to develop: State legislation was threatened which would substitute a bridge instead of a tunnel; civil service procedure for a time raised barriers to the selection of an engineering staff properly qualified for the work; and rapidly rising prices for labor and material occasioned a question as to the sufficiency of the available funds. Pending the solution of these difficulties, the federal government threatened to withdraw the loan and grant so that the funds could be diverted to projects which could be put under construction more speedily. During all of this troublous period, there was very close cooperation between the engineering staff of the Tunnel Authority and the engineering staff of PWA, the final outcome being a commitment to the river crossing, which necessarily placed the project on a list from which there could be no withdrawal.

The tunnel, as completed, undoubtedly represents the latest and best of engineering skill in subaqueous tunnel construction, and the author is to be complimented for his success in overcoming the many obstacles placed in his way as the project advanced.

Ole Singstad,[26] M. Am. Soc. C. E.—The discussions, and the questions raised in them, have contributed much to the value of the paper. The writer appreciates the complimentary remarks regarding the paper made by the discussers, and he is particularly gratified that no less than six of his former associates in the planning and building of the Holland Tunnel have taken part in the discussion.

Mr. Fluhr relates the fundamental differences between the geology of the East River and that of the Hudson River, describing the causes of the variations in character of rock and the porous nature of the overburden in the East River which account for the increased hazard, slower progress, and higher cost of building a tunnel under the East River as compared to the Hudson River.

Mr. King suggests an interesting experiment to determine whether a soft plastic backfill such as clay, with perhaps a small admixture of bentonite outside of the tunnel lining in rock, instead of grout and gravel, would make the lining more nearly watertight. If practicable, this would undoubtedly be an aid to the watertightness of the tunnel. This kind of fill, however, would greatly reduce the passive lateral support of the surrounding material. The light cast-iron lining is designed to withstand, in addition to the hydrostatic pressure, a load of 10 ft of loose rock above the top of the tunnel, assuming full support at the haunches by grout and gravel. If this full support is removed and a plastic material substituted under this loading condition, excessive tensile stresses and deflections are indicated. To meet such a contingency, permanent blocking or supports might be constructed between the outside of the lining and the rock as required, although this would not always be practicable.

Mr. Wagner makes an analysis of the operating costs of the Queens compressor plant of the Queens Midtown Tunnel. The observation is made that it is the fixed charges that are important in considering construction power-house costs and that savings made in them are of more importance than economies in operating expenses.

The tunnel engineer's chief responsibilities with regard to the power house are (1) to provide a compressor plant of sufficient low-pressure air capacity to be adequate for the worst conditions anticipated, and (2) to assure continuous operation. Economy of fixed costs and operating costs must be considered, but it is of secondary importance to these requirements. An experienced tunnel engineer may be able to make a reasonable estimate of the required average compressed air capacity of a proposed plant, but such a capacity would generally not be safe to install.

The low-pressure air capacities specified for the Queens Midtown Tunnel were determined primarily on the basis of the experiences during construction of the Pennsylvania Railroad Tunnels, which had been constructed near-by through somewhat the same materials as the Queens Midtown Tunnel. Even with this basis of comparison it was difficult to anticipate the Queens Midtown Tunnel requirements, as its diameter (31 ft) differed considerably from that of the Pennsylvania Railroad Tunnels (23 ft). To insure continuous operation, three independent power sources were provided for each compressor plant of the Queens Midtown Tunnel.

---

[26] Chf. Engr., New York City Tunnel Authority, New York, N. Y.

Actual experience in driving the Queens Midtown Tunnel demonstrated that the low-pressure air capacity provided on the Manhattan side was none too great, whereas the low-pressure air capacity of the plant on the Queens side might have been made less without jeopardizing the work. The writer cannot

TABLE 6.—COMPARISON OF COSTS AND CONSTRUCTION INDEXES

Item No.	Description	Holland Tunnel	Lincoln Tunnel		Queens Midtown Tunnel
			South tube	North tube	
	Contracts let	1922	1934	1937	1937
	(a) CAST-IRON LINED SHIELD-DRIVEN TUNNEL				
1	Dollars per linear foot	1,430	1,235	1,657	3,159[a]
2	Dollars per cubic foot	1.83	1.63	2.20	4.18
3	Percentage cost per cubic foot	100	89	120	228
	(b) BASIC LABOR RATES				
4	Muckers, dollars per day	7.00	10.00	11.00	11.00
5	Ironmen, dollars per day	8.00	10.00	11.00	11.00
6	Miners, dollars per day	8.50	11.00	12.00	12.00
7	Average, all sandhogs	8.00	10.50	11.50	11.50
8	Index number (Holland Tunnel, 100)	100	131	144	144
9	Tunnel laborer, dollars per hour	0.60	0.75	0.825	0.825
10	Form setter, dollars per hour	1.125	1.40	1.40	1.40
11	Average, cleaning and concreting (Items 9 and 10)	0.862	1.07	1.11	1.11
12	Index number (Holland Tunnel, 100)	100	125	129	129
	(c) BASIC MATERIALS				
13	Cement, in dollars per barrel[b]	2.43	2.48	2.07	2.07
14	Steel shapes, in dollars per hundred weight	1.70	1.78	2.21	2.21
15	Cast-iron lining, in dollars per ton delivered	35.00	56.80	61.20	60.70
	(d) INDEX NUMBERS, ENGINEERING NEWS-RECORD				
16	Construction cost (1913 = 100)	174.45	198.10	234.71	234.71
17	Construction cost (1922 = 100)	100	113	140	140
18	Materials cost (part of Item 16)	79	88	103	103
19	Materials cost (1922 = 100)	100	111.5	130.6	130.6
	(e) AVERAGE LABOR WAGE, NEW YORK CITY				
	Skilled Labor:				
20	Dollars per hour	1.17	1.52	1.71	1 71
21	Index number (1922 = 100)	100	130	146	146
	Common Labor:				
22	Dollars per hour	0.60	0.8475	0.76	0.76
23	Index number (1922 = 100)	100	141	127	127
	(f) TUNNEL COST INDEXES (HOLLAND TUNNEL = 100)				
24	Labor only	100	130	141	141
25	Material only	100	156	166	164
26	Labor and material	100	143	153.5	152.5

[a] The contractor's payrolls aggregated $1,287 per foot of cast-iron lined shield-driven tube, exclusive of shafts and including concrete lining placed under river contract on the Queens Midtown Tunnel, or as much per linear foot as the total cost of the same work on the south tube of the Lincoln Tunnel. [b] Delivered in New York City.

agree with Mr. Wagner's statement, however, that the "Queens side of this tunnel was in exceptionally good ground for tunneling," although the ground was much better than that encountered on the Manhattan side and generally somewhat better than had been anticipated. Most of the large-diameter, shield-driven tunnels constructed prior to the Queens Midtown Tunnel had been driven in what could correctly be described as exceptionally good ground for tunneling, consisting largely of impervious silt and clay, vastly superior for tunneling to the ground encountered on the Queens side of the Queens Midtown Tunnel. Among these tunnels the writer would include the Holland, Lincoln, East Boston, and Antwerp tunnels and the shield-driven sections of the Detroit-Windsor vehicular tunnel.

Predetermination of the required capacity of a compressor plant to meet exactly the low-pressure compressed air requirements is practically impossible, especially for a tunnel diameter as large as that of the Queens Midtown Tunnel and with similar ground conditions. The tunnel engineer's judgment, in determining the proper capacity to be installed, must be guided by considerations of a margin of safety to meet emergencies which are more important than the fixed and operating costs.

Colonel Wilgus raises several pertinent questions. The quantity of leakage during winter months is too small to be measured, and during the summer there is no evidence of any leakage. The objection to such a small amount of leakage in the tile-finished interior of a modern vehicular tunnel is largely, if not entirely, on the basis of appearance.

The estimated cost of the Queens Midtown Tunnel compared with the actual cost is the estimate made at the outset by the writer when acting as consulting engineer to the Tunnel Authority before the engineering staff was organized. It was based on preliminary plans and made at the time of application to the PWA for funds for the project.

The cost per linear foot of cast-iron lined shield-driven tunnel, exclusive of shafts and including concrete lining as placed on the river contracts, is given in Item 1, Table 6(*a*), for the Holland Tunnel, as of 1922, the south tube of the Lincoln Tunnel, as of 1934, the north tube of the Lincoln Tunnel, as of 1937, and the Queens Midtown Tunnel, as of 1937. The respective costs per cubic foot of tunnel are given in Item 2 and, on the basis of 100% for the Holland Tunnel, the respective percentage costs are given in Item 3, Table 6(*a*).

The index numbers in Table 6(*f*) are based, for material (Item 25), on the cost of cast-iron lining (Item 15, Table 6(*c*)) and concrete per cubic foot of tunnel and, for labor (Item 24), on the basic labor rates for sandhogs (Item 7, Table 6(*b*)) and for the labor used in cleaning the tunnel and concreting (Item 11). Using the index numbers in Item 26, Table 6(*f*), the costs of the Queens Midtown Tunnel, reduced to Holland Tunnel conditions (1922), were as follows:

	Dollars per cubic foot (Holland Tunnel basis)	Percentage comparison (1922 = 100)
Holland Tunnel	1.83	100
Lincoln Tunnel, south tube	1.135	62
Lincoln Tunnel, north tube	1.43	78
Queens Midtown Tunnel	2.74	150

In other words, if contracted for under labor and material rates prevailing at the time of the construction of the Holland Tunnel, the river section of the south tube of the Lincoln Tunnel would have cost 62% as much as the Holland Tunnel per unit, that of the north tube of the Lincoln Tunnel 78%, and that of the Queens Midtown Tunnel 150%.

It should be noted that these differences in cost, as adjusted for the labor and materials indexes, are applied to the shield-driven river sections only and can be accounted for practically in their entirety on the following basis:

The Holland and Lincoln tubes were constructed through substantially the same kind of material, namely, Hudson River silt. As the Holland Tunnel was the first large tunnel built under the Hudson River, there was much uncertainty as to the behavior and stability of such a large tunnel in this soft material during the construction period. Therefore, the bid prices were high, but in view of the uncertainties, not unduly high. During the driving of the Holland Tunnel, after considerable experimentation, it was found that only about 20% of the material displaced by the tunnel had to be taken in through the face of the shield, and approximately 80% could be pushed aside by advancing the shield through the silt. This had two distinct advantages—namely, fast progress with correspondingly reduced cost, and stability. It was found that by leaving the 20% of the displaced material in the bottom of the tunnel until shield driving was completed, the weight increased the stability of the tube which, during the disturbed condition attending the shield driving, had a buoyant tendency. When the south tube of the Lincoln Tunnel was being built, this exploratory or pioneering work on the Holland Tunnel was used to great advantage by the contractor in adapting his tunneling methods from the start to the experience gained on the former project. The contractor's manager-engineer on the Lincoln Tunnel was a former resident engineer in charge of building the Holland tube on the staff of the chief engineer of the Tunnel Commissions. In the building of the Lincoln Tunnel the contractor developed a mechanical bolt-tightening device which greatly increased progress, since no muck was transported out of the tunnel until after the tube construction was completed. The progress was then a function of the speed with which the iron rings could be erected and bolted together. If another similar tunnel in material such as Hudson River silt were to be built by experienced tunnel engineers and contractors, the cost of the under-river section of the Lincoln Tunnel south tube would be a truer basis for cost estimating than that of the Holland Tunnel.

In the case of the Queens Midtown Tunnel, which is the last one to be built, the cost as adjusted for the labor and material index (Item 26, Table 6(*f*)) is very much in excess of that of either of the Hudson River tunnels, and this can be ascribed in its entirety to the extremely difficult and hazardous ground conditions existing on a great part of the Queens Midtown Tunnel site, which impeded progress and caused higher air pressures, with correspondingly reduced daily working hours for labor with increased daily rates.

The contract for the north tube of the Lincoln Tunnel was let under approximately the same labor and material cost conditions as the Queens Midtown Tunnel shield-driven section. The fact that this tube was driven alongside the already completed south tube accounts for its higher adjusted cost as compared with the south tube.

Mr. Thoresen states that careful thought has been given to providing the traveling public with an "attractively finished, well lighted, pleasant, and comfortably ventilated subaqueous highway." The writer has long been an advocate of this kind of treatment of vehicular tunnel projects, as reflected in the development of the use of a tile finish on the walls of the Holland Tunnel. At the time of the building of that tunnel, no tile was manufactured in the United States which was deemed suitable for the severe moisture and temperature conditions that would prevail in a tunnel in this climate, and an extensive research program was conducted with the assistance of the late George H. Brown, professor and head of the Department of Ceramics at Rutgers University, New Brunswick, N. J. Extensive laboratory and field tests were made on a large number of tile samples manufactured in the United States and abroad. This resulted in specifications which produced a wall tile not only pleasing in appearance but easy to clean and durable. One half of the tile used on the walls of the Holland Tunnel was imported and the other half was manufactured in the United States. The research work on tile and the requirements specified for the Holland Tunnel have stimulated the domestic tile industry so that this high quality tile is now considered a standard product.

The trench and pre-cast type of tunnel design and construction, on a scale larger than any heretofore undertaken, is contemplated for the Narrows Tunnel between Brooklyn and Staten Island, where conditions indicate that this method is both practicable and economical.

Both Mr. Cruthers and Mr. Geenens correctly state some of the reasons for the greater economy in the operation of the Queens Midtown Tunnel as compared to the Holland Tunnel.

Mr. Conover very appropriately emphasizes the point that the Queens Midtown Tunnel is merely one link in an east-west express highway route running through mid-Manhattan and extending far out on Long Island and into the State of New Jersey. From its very inception, in 1926, all those who had made a comprehensive study of traffic conditions in the metropolitan area realized the necessity for such a through highway. This route comes under many jurisdictions—the state highway departments of the two states, the parkway authorities, the borough presidents of the various boroughs, the Department of Public Works of the City of New York, and the two authorities charged with the responsibility of financing and building the river crossings.

Although, as of 1944, the two principal links in this route—the Lincoln Tunnel under the Hudson River and the Queens Midtown Tunnel under the East River—have been in operation for a period of six and three years, respectively, very little else has been done to complete this through route. In fact, only about two miles of modern highway have been built—namely, that part immediately east of the Queens plaza of the Queens Midtown Tunnel which was completed by the City of New York simultaneously with the opening of that tunnel.

Mr. Conover expresses the hope that the New York City authorities will include a crosstown tunnel connecting the two river tunnels as a postwar project. The writer is in accord with this hope. It would be a rational development of the city's highway system. Other missing links in this through route, particularly extensions farther out on Long Island, should be included.

During the past decade, 1933–1943, New York City has been preoccupied with the building of parkways and high-speed roads for the exclusive use of "pleasure" traffic; that is, passenger cars. Commercial vehicles of every kind have been excluded. Scores of millions of dollars have been spent for this type of highway, with the result that there has not been enough money left to provide modern highway facilities for business traffic. Commercial traffic is still shackled with the stop-and-go lights almost as much as it was twenty years ago. It may have been assumed by those responsible for this policy that, by removing a large amount of passenger cars from the streets and placing them on express highways, there would be that much more room for commercial traffic on the streets with the stop-and-go lights. Experience has demonstrated, however, that this is only partly true because, with the new facilities for the fast movement of passenger cars, there is also a corresponding increase in "pleasure" driving. It is to be hoped that much more attention will be given in New York City to high-speed roads for mixed traffic than has heretofore been the case; otherwise the increased cost of doing business here due to traffic delays is certain to have adverse effects.

Mr. Conover refers to the West Side Plaza and the East Side Plaza developed for the Midtown Manhattan Tunnel by cooperation, in the first case by the engineers of the Board of Transportation and those of the Port of New York Authority, and in the second case by the engineers of the Board of Transportation and those of the Borough President of Manhattan. He states that the East Side Plaza adopted by the New York City Tunnel Authority differs radically from the plan developed by the Board of Transportation. This is also true of the West Side Plaza. The plan developed jointly by the engineers of the Port Authority and the Board of Transportation and recommended for adoption in April, 1931, by the chief engineer of the Port Authority located the four exits and entrances of the twin Hudson River tubes and of the Manhattan tubes on the easterly line of 10th Avenue in the two narrow blocks between 37th and 39th streets. The writer, who at that time (1931) was one of several consulting engineers to the Port Authority, after analyzing that plan, pointed out that such an arrangement would create unsatisfactory traffic conditions, with resultant congestion and confusion on the four tunnel approaches as well as on the adjacent city streets, particularly on 10th Avenue. The plan violated what the writer considers a fundamental principle—namely, that eastbound traffic and westbound traffic should be separated as much as possible. The writer also pointed out that the plan was unnecessarily costly. As a result of that analysis the plan was not adopted and a re-study was made by the staff of the chief engineer of the Port Authority. In September, 1931, that staff produced a new plan eliminating these objectionable features as well as effecting an estimated saving in cost of construction and real estate of more than $10,000,000. This new plan was endorsed by all concerned, including the writer, and the tunnel was built according to it.

Although the through route from Long Island to New Jersey has been slow in emerging in complete form, it is gradually developing step by step. Another section of highway about three miles long, to be built from the easterly end of the present highway extending from the Queens plaza of the Queens Midtown Tunnel, is included in New York City's postwar program. It was recognized from the start that each link should form a part in a through express highway, and the links so far built or authorized fit into this scheme.

The writer is in accord with the plans of all sections so far built or authorized, with one exception. The New Jersey approach to the Hudson River tunnel, involving on the main route a corkscrew spiral with a continuous lift of 268 ft from under the river to the top of the Palisades, coupled with curves having radii of less than 300 ft for as much as one half of the complete circular loop, is not up to the standard of modern highway requirements, especially for commercial traffic.

A more detailed description of the various elements involved in the complete east-west route (of which the Queens Midtown Tunnel is a link) and their development is given in the writer's more complete paper filed in the Engineering Societies library at New York.

It is true, as Mr. Conover states, that the success of a vehicular tunnel depends largely on the plaza layouts together with the approaches.

The suggestion that the tunnel avenues for the Mid-Manhattan Underpass should correspond, for exit and entrance traffic flow, to the new avenues for the Queens Midtown Tunnel was given considerable study. The conclusion reached was that this arrangement would be expensive and impracticable. Although it would provide a slightly simpler traffic flow at some intersections, the disadvantages outweighed the advantages.

Automatic control of ventilation for a large tunnel, such as the Queens Midtown, would be an added expense with no saving or advantage gained, because an operator would have to be on duty at the remote control buttons at all times. Peaks of short duration would result in a great deal of fan-speed shifting if automatically controlled. These peaks can be "ironed out" with manual operation. The Oakland-Alameda Tunnel has (or had) automatic control. Automatic control is deemed advisable only on short and less important tunnels where such operation actually will save manpower on the operating payroll.

City fire apparatus has often passed through the Queens Midtown Tunnel, not on fire duty, but for access to the repair shop in Long Island City. No difficulty has been experienced on account of clearances. One lane only is necessary for the city's longest hook-and-ladder equipment, even on the smallest radius curve near the Manhattan portal.

Dr. Levy discusses, in considerable detail, the precautions taken during the construction of the Queens Midtown Tunnel to protect the men against compressed air illness, and also certain experiments which were undertaken, having in mind the development of better protection of the men from a medical point of view.

As one result of the investigations made on helium-oxygen mixtures, the installation and use of apparatus for the alleviation of ear block is specified as a requirement on the compressed air work on the Brooklyn-Battery Tunnel, on which work is now (1944) temporarily suspended on account of war conditions. If the results prove to be as satisfactory as anticipated, it may well be that the installation and use of apparatus to alleviate ear block should be a standard requirement for future compressed air tunnel work.

On the basis of past experience, and especially the experience on the Queens Midtown Tunnel, Dr. Levy suggests further study in regard to the present requirement of the New York State rules for the installation of a second tunnel bulkhead when pressures exceed 21 lb per sq in. The double bulkhead method of decompression did not show any advantage over the use of the single bulkhead on the Queens Midtown Tunnel in so far as the incidence of "bends" cases is concerned, and it seems appropriate to suggest that the present regulations might be modified advantageously to require that a second bulkhead be used as required by considerations of safety depending upon the physical conditions rather than upon any given air pressure in the tunnel heading. On the Manhattan side of the Queens Midtown Tunnel, where the distance between bulkheads was approximately 430 ft in one tunnel and 470 ft in the other, the second bulkheads were placed primarily because of the requirement of the regulations. As the headings entered full rock section in the rock reef approximately only 1,000 ft from the first bulkhead, serious consideration could have been given to the omission of the second bulkhead had it not been for the code requirement.

The question as to whether, in decompressing, the full amount of the fast initial drop of pressure as required by the present regulations may or may not be beneficial, seems to the writer to be a matter for further experimentation. It appears that Dr. Levy makes a pertinent statement in this connection when he writes, "In fact, it well may be that the fast initial drop of pressure in this method of decompression actually induces the beginning of the formation of the air bubbles that are the cause of compressed air illness."

Dr. Levy also suggests that further study be given to working one shift of six hours rather than two shifts of three hours in low pressures up to 18 lb per sq in. and possibly higher. Although this would be an interesting experiment, there may be some practical objections to it in actual practice. It may be questionable whether a man working six hours continuously, regardless of whether or not he is working in compressed air, would produce a satisfactory six hours' work. There is a present-day tendency in certain industries to give employees short rest periods at various intervals during the day to offset fatigue. It may be questioned whether a man working six hours continuously would be operating at full efficiency during the latter part of the period, and this condition might be hazardous in tunnel work where alertness and efficiency are required at all times for reasons of safety.

Professor Legget has raised several pertinent questions: The ratchet wrenches used for bolt tightening were standard equipment. Fixed maximum-torque wrenches have been considered for use on the Brooklyn-Battery Tunnel, where the minimum bolt stress specified is 30,000 lb per sq in. It was found that there were no maximum-torque wrenches on the market of sufficient capacity for this specification and one was ordered to be constructed specially. It was impossible to get this wrench on account of war conditions. If the trial had proved successful it was contemplated that inspectors of the Tunnel Authority would be provided with torque wrenches and would spot-check the bolts for tightness. It was considered impracticable to use torque wrenches for the actual tightening.

In experimenting to determine the torque necessary to produce a given stress, it was observed that there was quite a range in the frictional behavior of different combinations of bolt, nut, and washer. This would be expected, especially since the bolts are forged and the threads rolled. In this process threads are sometimes burred in packaging and handling, and the nuts and washers have a relatively rough finish.

When conditions permit resumption of construction work on the Brooklyn-Battery Tunnel, the investigation of the use of torque wrenches to test bolts will be continued.

The drainage system considered for the Manhattan plaza of the Queens Midtown Tunnel involved excavating to expose the rock surface over the entire plaza area, the construction of a fairly complicated system of drainage pipes, galleries, sumps and pump chambers, and backfilling to subgrade of the invert slab with gravel. The heavy slab was estimated to cost little more than the drained plaza, which involved more excavation, the cost of gravel backfill, and higher unit costs of concrete than the massive slab, besides the drainage system itself. As the cost was practically the same as that for the heavy slab design and there was an unusual ground-water condition (ground water being as high as 19 ft above mean high water at one point) the heavy invert slab was adopted as involving no operation and maintenance costs.

Clay for blanket purposes is chosen because it is a dense, erosion-resistant material that can be obtained economically in the large quantities required. This generally results in a clay being obtained from a near-by source. The clay used for blanket purposes on the Queens Midtown Tunnel had a weight of 115 lb per cu ft and was dredged from Newark Bay, a previously used source of clay for blanket purposes.

The volcanic clay referred to is a form of bentonite described as a type of clay originating from volcanic eruptions and consisting chiefly of mineral montmorillonite, a hydrous silicate of aluminum. It is found almost exclusively in the Black Hills region of the United States (South Dakota). It possesses in high degree the qualities of absorption and swelling, and will re-swell after drying for an indefinite number of cycles. Its swelling properties are used in retarding or stopping the flow of water, and in the tunnel it was used as an aid in the attempt to reduce the escape of compressed air through porous material.

Mr. Brodie's analysis of the tunnel ring, together with the resulting design for the cast-iron lining of the Holland Tunnel which he describes, is still representative of fundamental principles in this art. Very important is his statement, "Subaqueous shield-tunnel practice recognizes the uncertainty of absolute values of external loading, especially at critical times during construction." There is sometimes a tendency for theorists not familiar with practical shield tunneling to lose sight of this principle.

The lining section actually used for the Holland Tunnel is not to be confused with Fig. 18, "Cast-Iron Details, Heavy, 29-Ft Sections, Holland Tunnel."

The outside radius of the adopted section was 14 ft 9 in. and in some cases 15 ft 2 in.; the web thickness was 1⅝ in. increasing to 2¾ in. at the flanges. Mr. Brodie made it clear in his discussion that the lining shown in Fig. 18 formed the basis of the design and analysis but that the modified sections, with the foregoing dimensions, were actually used. The figure alone, however, might be misleading.

Referring to feature (8) of Mr. Brodie's discussion, "effect of transverse tie rods," it may be pointed out that the firm character of the material surrounding the Queens Midtown Tunnel tubes made such permanent tie rods unnecessary, except for the short stretches under the Manhattan bulkhead and at the midriver sump, as stated in the paper.

In his letter of September 27, 1943, to the writer, written only eight days before his death, Mr. Brodie requested that in this discussion mention be made of those who assisted in his work on the design of the cast-iron lining for the Holland Tunnel, as follows: Henry G. Babcock, Assoc. M. Am. Soc. C. E., James H. Dugan, M. Am. Soc. C. E., Jacob Mechanic, Assoc. M. Am. Soc. C. E., and Morris Cohen.

Some of the statements made by Mr. Eremin call for further comments, and a few of them will be discussed briefly. The protective blanket acts primarily as a counterweight against rupture of the river bed due to the unbalanced air pressure in the upper part of the face of the tunnel heading. If the face of the tunnel is in porous material, the imperviousness of the blanket does not accomplish much in preventing the escape of air from the tunnel. As escaping air from the tunnel does find fissures and other such avenues of escape through the blanket, the blanket is eroded to an extent sometimes requiring constant replacement under these circumstances. If the blanket were entirely impervious, air would continue to escape from a tunnel heading in porous material and find its way out around the edges of the blanket. A dense and erosion-resistant material is indicated for blanket purposes.

If the tunnel heading is in impervious material, the required air pressure is generally less than theoretical hydrostatic head and there is no unbalanced air pressure tending to blow out the river bed. Under these conditions a blanket would not be necessary, even though the natural cover over the tunnel were not great. For this reason blankets and loss of air in shield tunneling are generally associated. To the best of the writer's information, the blanket used in the East Boston Tunnel was used to build up low spots in the river bed, thus providing a uniform total thickness of cover of only 15 ft over the tunnel. Altogether only 7,200 cu yd of clay were used for the blanket, most of it deposited at pierhead lines to consolidate the mud and increase cover from 9 or 10 ft to 15 ft, as a precaution against material running into the heading. There was no danger of a blowout as the material penetrated by the tunnel was largely impervious clay, plastic to stiff in consistency, and air pressures used were generally below theoretical hydrostatic head.

The use of grout to reduce air leakage at the face, and its use to fill the annular space around the tunnel lining left by the tail of the shield should not be confused. Its latter use is standard practice and has little, if any, effect on shield progress, whereas the effectiveness of grouting the face, which is done only under unusual circumstances, might have some effect on shield progress.

Mr. Eremin should not include the Lincoln Tunnel with his examples of structural steel-lined shield-driven tunnels. The shield-driven sections of the Lincoln Tunnel were lined with cast iron; the approach tunnels in full section of rock, built without the use of shields, were lined with steel, as were the Manhattan approach tunnels of the Queens Midtown Tunnel.

The writer had no intention of inferring that further study is not necessary in the planning and designing of tunnels for highway traffic. On the contrary, he is convinced that each such project requires a large amount of study and planning to meet the conditions peculiar to each location. The writer had in mind, however, that only twenty-five years ago (1919) long tunnels, designed for the exclusive use of motor-driven vehicles, were considered in the nature of an experiment and were looked upon with skepticism, not only by the general public but also by some engineers. During the intervening period extensive research work has been conducted on some of the important phases relating to this kind of project and much has been learned through operating experience of the several such tunnels which have been built and are now in operation.

When the Holland Tunnel was being planned, an extensive program of research was conducted, involving the expenditure by the Tunnel Commissions of more than $100,000 and by the cooperating agency, the U. S. Bureau of Mines, of a considerable amount. This research work was intensively promoted over a period of two years and since has been continued intermittently to provide supplementary information on related phases of the same problem. The uncertainty and apprehension with regard to this phase of vehicular tunnel design were effectively overcome by the striking success of the ventilation system developed as a result of that research work.

Another illustration of a question that was then considered uncertain by some engineers was the suitability of cast iron as the primary lining for such a large tunnel in a material like the Hudson River silt. The design of the lining for the Holland Tunnel, which involved a much larger diameter in Hudson River silt than had before been constructed, was quite properly the subject of open technical discussion. In one paper[27] the opinion was expressed that "In soils with a low angle of repose, for any specific case presented, there is a limiting inside diameter (which may be determined for that case) beyond which the primary lining, whether of cast iron or of concrete blocks, cannot be designed to stand without internal bracing." At considerable variance with the design and analyses of the Holland Tunnel engineers, and predicated on the line of reasoning on which the foregoing conclusion was based, it was determined in that paper that the inside diameter of a cast-iron lined tunnel not subject to non-elastic deformation in Hudson River silt would be limited to 26.2 ft with flanges more than 5 ft deep and an outside diameter of more than 36 ft, the lining segments at this diameter being unwieldy in size, number, and weight. However, with faith in their own determinations, the Holland Tunnel engineers designed and successfully constructed in Hudson River silt a cast-iron lining of

---

[27] "Notes on Tunnel Lining for Soft Ground," by S. Johannesson and B. H. M. Hewett, *Transactions* Am. Soc. C. E., Vol. LXXXIII (1919–1920), pp. 1845 and 1855.

29 ft 6 in. outside diameter and 14-in. flanges or 27 ft 2 in. inside diameter, the general features and details of which have subsequently set the pattern for cast-iron lining of 31 ft outside diameter in both the Hudson and East rivers. A synopsis of this design and analysis is mentioned in Mr. Brodie's discussion.

One more illustration of the extensive research work which was necessary heretofore is that referred to in the comments on Mr. Thoresen's discussion regarding the development of a suitable wall tile for this type of tunnel.

The writer is indebted to Mr. Tuttle for a concise statement of the historical background of the Queens Midtown Tunnel.

The adequate highway approaches to the tunnel, both in Manhattan and Queens, to which Mr. Tuttle refers, have to a large extent yet to be provided. That these approaches could not have been included, as part of the tunnel project on a self-liquidating and self-supporting basis, is self-evident. It is generally the practice of New York City to make its toll river crossings a real part of its free highway system. In the case of the Queens Midtown Tunnel this practice has met with serious delays which the writer hopes will soon be overcome.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2220

A SYMMETRICALLY LOADED BASE SLAB ON AN ELASTIC FOUNDATION

By Stanley U. Benscoter,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Robert O. Jameson, L. J. Mensch, Alfred L. Parmé, A. A. Eremin, A. E. Cummings, Harris Epstein and James R. Ayers, Jr., and Stanley U. Benscoter.

Synopsis

In the design of a base slab it is common practice to assume the reaction distribution and compute bending moments by statics, or by elastic theory if the slab is part of an indeterminate frame. In the latter type of case, fixed-end moments and stiffness values are computed as for an ordinary beam or slab with fixed load distribution. The object of this paper is to develop a method of analysis which takes account of the effect of the rigidity of the foundation upon the reaction distribution. The base slab extends between two vertical walls to which it is rigidly connected. The slab is considered to be acted upon symmetrically by thrust, shear, and moment from the vertical walls. Formulas and graphs for stiffness and fixed-end moment values are presented so that moments can be obtained by moment distribution.

Notation

The letter symbols introduced in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

Reaction-Displacement Relationship

A reasonable shape of the displacement and reaction diagrams for a base slab is obtained if the assumption is made that the upward reaction, w, at each point is proportional to the downward displacement, z, at that point. This assumption is expressed by

$$w = -k\,z \quad\text{(1)}$$

in which k is the foundation modulus. This simple manner of foundation behavior is assumed for convenience. The relationship is exact for a slab floating in a liquid and may be called the basis of the "floating slab" theory.

Note.—Published in May, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] The Engineer Board, Bridge Section, Fort Belvoir, Va.

If the foundation is elastic, isotropic, and homogeneous a correct solution could be obtained by replacing the constant k by the reciprocal of an influence function. At present, any assumed relationship between displacements and reactions for a slab resting on a soil subgrade must be regarded as an approximation.

The constant k may be obtained experimentally from a plate-loading test. A stress-displacement curve is plotted and the average slope of the curve in the working range is determined. This slope is the foundation modulus. It is essential that the loaded plate have at least 5 sq ft of area. Values of k for soil commonly range from 100 to 200 lb per cu in., although greater or less values are not unusual. The lower value of 100 lb per cu in. is recommended for design purposes when the results of plate-loading tests are not available. For more rigid foundations, such as shale or limestone, an estimate of a reasonable value of k for design is given (in pound-inch units) by:

$$k = \frac{E'}{100} \qquad (2)$$

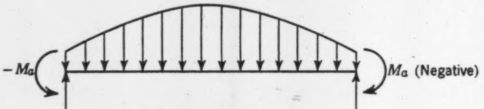
in which E' is the modulus of elasticity. The application of floating slab theory to such rigid foundations must be regarded as a temporary method of solution to be used only until a more exact method becomes available.

SLOPE-DEFLECTION EQUATION FOR A SYMMETRICAL BEAM

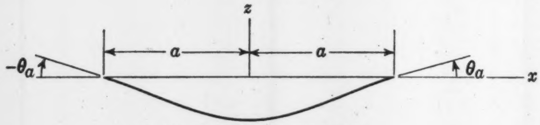
It is desirable to consider first the slope-deflection equation for a prismatic beam with symmetrical loading and symmetrical end moments, as shown in Fig. 1. The equation takes the following simplified form, $M_a = M'_a + \frac{E\,I}{a}\,\theta_a$; or

$$M_a = M'_a + \phi_0\,\theta_a \qquad (3)$$

in which M'_a is the value of M_a when the ends of the beam are fixed; E is



(a) FORCE DIAGRAM



(b) DISPLACEMENT DIAGRAM

FIG. 1.—SYMMETRICALLY LOADED PRISMATIC BEAM

the modulus of elasticity for an isolated beam or a modified modulus of elasticity $\frac{E'}{1 - \mu^2}$ if the beam is a slice of a slab; I is the moment of inertia of the beam or the moment of inertia of the slab per unit of width; ϕ_0 is the stiffness of the beam; and θ_a is the end rotation angle (or end slope). The signs of the terms in this equation are correct for the usual slope-deflection sign convention at either end of the beam. In a bending-moment sign convention the equations are correct only at the right end of the beam. Throughout this paper a bending-moment sign convention will be used, and the signs that appear in the equations will be such that the equations are applicable to the right half of the beam.

The coefficient ϕ_0 in Eq. 3 is a modified beam stiffness for the case of symmetrical end moments and may be obtained readily by multiplying the standard stiffness[2] value $\frac{4\,E\,I}{L}$ by the applicable end-rotation constant[3] $\frac{1}{2}$. It is apparent from Eq. 3 that ϕ_0 is the change in the end moment M_a caused by a unit change in θ_a, or a unit end rotation. Eq. 3 gives the end moment M_a as a linear function of the end slope θ_a. The convenience of the moment distribution process of analysis arises from the fact that M_a and θ_a are related linearly. It will be shown that this same linear relationship holds true for an elastic slab on an elastic foundation. The formulas for fixed-end moment and stiffness contain the foundation modulus as well as the flexural properties of the slab.

It is possible also to write the moment at any point of the beam in the same form as Eq. 3; thus:

$$M(x) = M'(x) + \phi_0\,\theta_a \qquad (4)$$

The fixed-end moment $M'(x)$ is a function of x. The stiffness ϕ_0 is a constant which, as before, is the change in the moment $M(x)$ caused by a unit end rotation. In the case of a beam on an elastic foundation, the change in moment at a point x, caused by a unit end rotation, is not a constant but a function of x. Hence, the constant ϕ_0 must be replaced by a function of x.

BEAM FUNCTIONS

The base slab under consideration is regarded as a plane strain problem and a slice of the slab one unit thick is considered in the analysis. This slice may be treated as a beam.

The beam functions to be determined are the displacement, slope, moment, and shear, each of which may be written in a form analogous to Eq. 4:

$$z(x) = z'(x) + \phi_1(x)\,\theta_a \qquad (5a)$$

$$\theta(x) = \theta'(x) + \phi_2(x)\,\theta_a \qquad (5b)$$

$$M(x) = M'(x) + \phi_3(x)\,\theta_a \qquad (5c)$$

and

$$V(x) = V'(x) + \phi_4(x)\,\theta_a \qquad (5d)$$

Eq. 5c is obtained from Eq. 4 by replacing ϕ_0 by $\phi_3(x)$. The function $\phi_3(x)$ contains ϕ_0 as a factor and may be written in the following forms,

$$\phi_3(x) = \phi_0\,S(x) = \frac{E\,I}{a}\,S(x) \qquad (6)$$

---

[2] "Continuous Frames of Reinforced Concrete," by H. Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 83.

[3] *Ibid.*, p. 119.

The function $S(x)$ remains to be determined. At the end of the beam, where $x = a$, the function ϕ_3 becomes,

$$\phi_3(a) = S_a \frac{E\,I}{a} \dots\dots\dots\dots (7)$$

It is the stiffness value $\phi_3(a)$ that is needed to distribute moments by the method of moment distribution. The fixed-end moment at the end of the beam M'_a is also needed.

In actual design practice it is not necessary to compute the end rotation angle θ_a. The moment distribution process gives the final end moment M_a without a computation of θ_a. If the change in end moment from M'_a to M_a is called ΔM_a, the beam functions can be expressed in terms of ΔM_a instead of θ_a. The moment at the end of the beam, from Eq. 5*c*, is:

$$M_a = M'_a + \phi_3(a)\,\theta_a \dots\dots\dots\dots (8)$$

Consequently,

$$\Delta M_a = M_a - M'_a = \phi_3(a)\,\theta_a \dots\dots\dots\dots (9a)$$

or,

$$\theta_a = \frac{\Delta M_a}{\phi_3(a)} \dots\dots\dots\dots (9b)$$

Substitution of this formula for θ_a into Eqs. 5 gives the beam functions in the following forms:

$$z(x) = z'(x) + \psi_1(x)\,\Delta M_a \dots\dots\dots\dots (10a)$$

$$\theta(x) = \theta'(x) + \psi_2(x)\,\Delta M_a \dots\dots\dots\dots (10b)$$

$$M(x) = M'(x) + \psi_3(x)\,\Delta M_a \dots\dots\dots\dots (10c)$$

$$V(x) = V'(x) + \psi_4(x)\,\Delta M_a \dots\dots\dots\dots (10d)$$

in which

$$\psi_i(x) = \frac{\phi_i(x)}{\phi_3(a)}; \qquad i = 1, 2, 3, 4 \dots\dots\dots\dots (11)$$

The fixed-end functions and the ϕ-functions or ψ-functions must be determined by solving the differential equation which governs the displacement of an elastic beam.

SOLUTION OF THE DISPLACEMENT EQUATION

The elastic base slab in Fig. 2 is resting on an elastic foundation. The forces and moments that act on the beam are shown and also the displacements caused by the acting forces. The reaction and displacement diagrams have the same shape and are proportional in agreement with Eq. 1. The displacement of a point on the elastic curve of a prismatic beam is governed by the following equation,

$$\frac{d^4z}{dx^4} = \frac{w}{E\,I} \dots\dots\dots\dots (12)$$

The load w on the beam in the present case is the reaction that has been expressed in terms of z by Eq. 1. Substitution of Eq. 1 into Eq. 12 gives

$$\frac{d^4z}{dx^4} + \frac{k}{E\,I}\,z = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The form of the solution of this equation can be simplified by the following substitution,

$$\frac{k}{E\,I} = \frac{4}{\lambda^4} \quad\ldots\ldots (14)$$

so that Eq. 13 becomes,

$$\frac{d^4z}{dx^4} + \frac{4}{\lambda^4}\,z = 0 \quad\ldots (15)$$

The fundamental length λ may be obtained by solving Eq. 14:

$$\lambda = \sqrt[4]{\frac{4\,E\,I}{k}} \quad\ldots\ldots (16)$$

In Eq. 16 the numerator of the fraction beneath the radical is the standard stiffness of a unit length of the beam, whereas the denominator is the stiffness of the foundation.

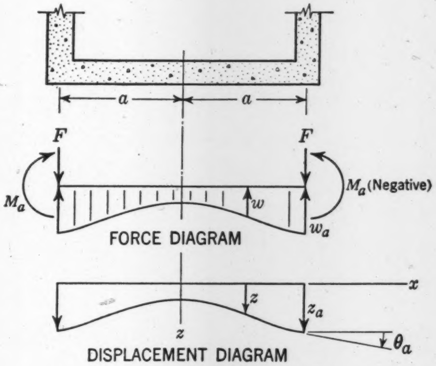


FIG. 2.—BASE SLAB WITH FORCE AND DISPLACEMENT DIAGRAMS

The solution of Eq. 15 may be written in the following form,

$$z = A \sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + B\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda} + C\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + D\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda} \quad\ldots (17)$$

in which A, B, C, and D are arbitrary integration constants. This solution contains four constants since it is the solution of a fourth-order differential equation. It is necessary to evaluate the constants of integration in such a manner that the desired boundary, or end, conditions of the beam are satisfied.

Since the displacements are symmetrical about the z-axis, the coefficients of the antisymmetrical terms may be set equal to zero.

$$C = D = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Since it is desired to obtain the beam functions in terms of θ_a the first derivative of z at the end of the beam must be set equal to θ_a. The end shear is $-F$. These two end conditions give the following equations for determining A and B.

$$\left[\frac{dz}{dx}\right]_a = \theta_a \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

and

$$\left[\frac{d^3z}{dx^3}\right]_a = -\frac{F}{E\,I} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

If the derivatives of Eq. 17 are substituted into Eqs. 19, two linear equations in A and B are obtained, which can be solved simultaneously to obtain,

$$A = \frac{v\,\lambda^3\,F}{2\,E\,I} + u\,\lambda\,\theta_a \qquad (20a)$$

and

$$B = \frac{u\,\lambda^3\,F}{2\,E\,I} - v\,\lambda\,\theta_a \qquad (20b)$$

in which

$$u = \frac{\sin\alpha\cosh\alpha + \cos\alpha\sinh\alpha}{\cosh 2\,\alpha - \cos 2\,\alpha} \qquad (21a)$$

and

$$v = \frac{\sin\alpha\cosh\alpha - \cos\alpha\sinh\alpha}{\cosh 2\,\alpha - \cos 2\,\alpha} \qquad (21b)$$

The functions u and v have been introduced for convenience. They are represented graphically in Fig. 3. If the formulas for A, B, C, and D are substituted into Eq. 17, the displacement z is obtained in the form of Eq. 5a; thus,

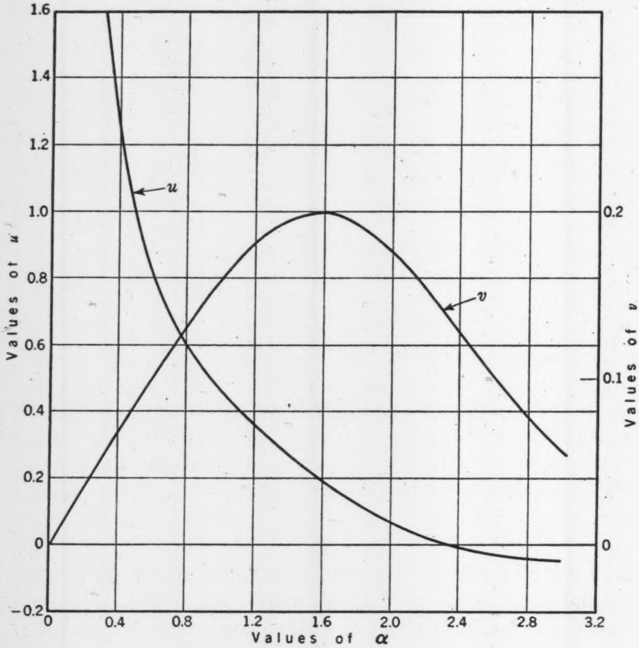


FIG. 3.—CONVENIENT FUNCTIONS OF α (SEE EQS. 21)

$$z = \frac{F\,\lambda^3}{2\,E\,I}\left(v\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + u\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) + \theta_a\,\lambda\left(u\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} - v\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \qquad (22)$$

The first term is the fixed-end function and the second term is the product of θ_a and the end-rotation, or ϕ, function.

FIXED-END FUNCTIONS

The slope, moment, and shear of an elastic, prismatic beam may be expressed in terms of the derivatives of z by

$$\theta = \frac{dz}{dx} \qquad (23a)$$

$$M = E\,I\,\frac{d^2z}{dx^2} \qquad (23b)$$

and

$$V = E\,I\,\frac{d^3z}{dx^3} \qquad (23c)$$

If the formula for z obtained from Eq. 22 by setting $\theta_a = 0$ is differentiated and substituted into Eqs. 23, the following fixed-end functions are obtained.

$$z'(x) = \frac{F\,\lambda^3}{2\,E\,I}\left(v \sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + u \cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \qquad (24a)$$

$$\theta'(x) = \frac{F\,\lambda^2}{2\,E\,I}\left[(v-u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u+v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \qquad (24b)$$

$$M'(x) = -F\,\lambda\left(u \sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} - v\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \qquad (24c)$$

and

$$V'(x) = -F\left[(u+v)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u-v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \qquad (24d)$$

END-ROTATION FUNCTIONS

The effect of a unit end rotation on deflections can be obtained from Eq. 22 by setting $F = 0$ and $\theta_a = 1$. The deflection formula thus obtained can be differentiated and substituted into Eqs. 23 to obtain the remaining end-rotation functions. These end-rotation functions are,

$$\phi_1(x) = \lambda\left(u \sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} - v\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \qquad (25a)$$

$$\phi_2(x) = (u+v)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u-v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda} \qquad (25b)$$

$$\phi_3(x) = \frac{2\,E\,I}{\lambda}\left(v \sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + u\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \qquad (25c)$$

and

$$\phi_4(x) = \frac{2\,E\,I}{\lambda^2}\left[(v-u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u+v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \qquad (25d)$$

It is interesting to note that the functions of x, which are contained in the parentheses or brackets of the fixed-end functions, appear again in the end-rotation functions but in different order. The fixed-end functions and the end-rotation functions now can be substituted into Eqs. 5 to obtain a complete statement of the beam functions.

DESIGN FORMULAS

From the formulas that have been derived the numerical values of the beam functions can be computed at any point x providing either ΔM_a, or θ_a, has been determined. The determination of ΔM_a rather than θ_a offers the preferable method of solution since the designer is vitally concerned with end-moment changes but has little interest in numerical values of end rotations. Whenever possible, a practicing engineer instinctively prefers to deal entirely with bending moment values in a structure analysis. If the end moment M_a can be determined by statics, then ΔM_a can be computed from its definition as given in Eq. 9*a*. An example of this type of structure is an open water channel constructed as a monolithic frame, such as a spillway channel or a stilling basin.

If the structure is indeterminate, as in the case of a closed conduit, the value of ΔM_a must be determined by elastic theory. Moment distribution offers a convenient method of obtaining this solution. The two quantities needed for the moment distribution computation are the "fixed-end moment" and the stiffness of the slab. The formula for the "fixed-end moment" can be obtained from Eq. 24*c* by setting $x = a$; thus:

$$M'_a = -\,C_a\,F\,\lambda \dots\dots\dots\dots\dots\dots\dots (26a)$$

in which

$$C_a = \frac{1}{2}\left(\frac{\sinh 2\,\alpha - \sin 2\,\alpha}{\cosh 2\,\alpha - \cos 2\,\alpha}\right) \dots\dots\dots\dots\dots (26b)$$

The coefficient C_a is represented graphically in Fig. 4 (curve 1). The stiffness of the slab may be obtained from Eq. 25*c* by setting $x = a$:

$$\phi_3(a) = S'_a\,\frac{E\,I}{\lambda} \dots\dots\dots\dots\dots\dots\dots (27a)$$

in which S'_a, the stiffness coefficient, is

$$S'_a = \frac{\sinh 2\,\alpha + \sin 2\,\alpha}{\cosh 2\,\alpha - \cos 2\,\alpha} \dots\dots\dots\dots\dots\dots (27b)$$

It is probably more desirable to express the stiffness in terms of $\frac{E\,I}{a}$ as given by Eq. 7. By comparing Eq. 7 and Eq. 27*a*, it becomes apparent that

$$S_a = \frac{a}{\lambda}\,S'_a = \alpha\,S'_a \dots\dots\dots\dots\dots\dots\dots (28a)$$

Substitution of Eq. 27*b* gives

$$S_a = \alpha\left(\frac{\sinh 2\,\alpha + \sin 2\,\alpha}{\cosh 2\,\alpha - \cos 2\,\alpha}\right) \dots\dots\dots\dots\dots (28b)$$

The coefficient S_a is represented graphically in Fig. 4 by curve 2. This coefficient reflects the effect of foundation rigidity upon the beam stiffness.

In a large proportion of base slab designs the only quantities needed for the actual design are the end shear, end moment, and center moment. The end shear is known. The end moment can be computed either from statics or by moment distribution. In either case the value of ΔM_a can be determined and the center moment computed from Eq. 10c by setting $x = 0$:

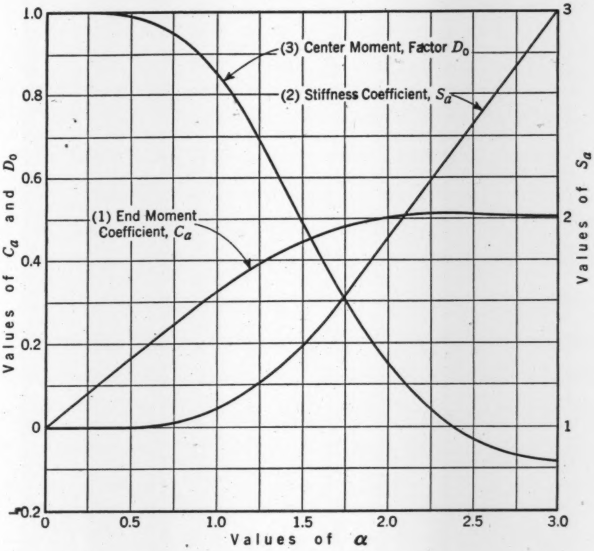


FIG. 4.—QUANTITIES USED IN MOMENT COMPUTATIONS

$$M_0 = M'_0 + D_0 \Delta M_a \qquad (29a)$$

in which

$$D_0 = \psi_3(0) \qquad (29b)$$

The term M'_0 is the center moment with ends fixed, and the factor D_0 is the change in center moment which accompanies a unit change in end moment. The formula for M'_0 can be obtained from Eq. 24c by setting $x = 0$:

$$M'_0 = v F \lambda \qquad (30)$$

The formula for D_0 can be obtained from Eqs. 11, 25c, and 27a:

$$D_0 = \psi_3(0) = \frac{\phi_3(0)}{\phi_3(a)} = \frac{2u}{S'_a} \qquad (31a)$$

Substitution of Eqs. 21*a* and 27*b* gives

$$D_0 = 2\left(\frac{\sin\alpha\cosh\alpha + \cos\alpha\sinh\alpha}{\sinh 2\alpha + \sin 2\alpha}\right) \qquad (31b)$$

The factor D_0 is represented graphically in Fig. 4 by curve 3.

PHYSICAL SIGNIFICANCE OF VALUES OF α

A conception of the practical significance of values of α can be gained by studying the curves for the fixed-end moment and stiffness coefficients. Referring to Fig. 4, curve 1, it may be noted that the initial slope of the graph is $\frac{1}{3}$ and that, for $0 < \alpha < 1$, the value of C_a is approximately $\frac{\alpha}{3}$. For $\alpha > 2$, the value of C_a is approximately one half. Consequently the following approximate formulas for the fixed-end moment can be written:
When $\alpha < 1$,

$$M'_a \approx -\frac{F\,a}{3} \qquad (32a)$$

and, when $\alpha > 2$,

$$M'_a \approx -\frac{F\,\lambda}{2} \qquad (32b)$$

For the range $0 < \alpha < 1$, the formula for M'_a may be converted to the well-known formula $\frac{W\,L}{12}$, in which W is the total base reaction and L is the span length ($W = 2\,F$, and $L = 2\,a$). For $\alpha > 2$ the value of M'_a is only slightly affected by the span length, being equal approximately to one half of the fundamental moment $F\,\lambda$. The three ranges of α might be classified as follows:

Range	Classification
$0 < \alpha < 1$	Rigid beam on flexible foundation
$1 < \alpha < 2$	Rigid beam on rigid foundation, or flexible beam on flexible foundation
$2 < \alpha < \infty$	Flexible beam on rigid foundation

An example of the first range of α is a reinforced concrete slab 3 ft thick and 20 ft long resting on soil. Examples of the second range are the same slab resting on shale, or a slab 1 ft thick and 20 ft long resting on soil. An example of the third range would be the latter slab resting on limestone.

Referring to Fig. 4, curve 2, and Eq. 7, the stiffness of the slab may be approximately expressed by the following formulas:
When $\alpha < 1$,

$$\phi_3(a) \approx \frac{E\,I}{a} \qquad (33a)$$

and, when $\alpha > 2$,

$$\phi_3(a) \approx \frac{E\,I}{\lambda} \qquad (33b)$$

Thus the stiffness, for $\alpha > 2$, is seen to be independent of the span length of the slab.

Another method of demonstrating the physical significance of values of α is to plot reaction diagrams. Reaction diagrams are shown in Fig. 5 for $\alpha = 1$ and $\alpha = 2$ with the ends of the slab fixed. The unit of reaction used in plotting the diagrams is the average reaction $\frac{F}{a}$. It may be seen that for $\alpha \leqq 1$ the reaction is approximately uniform, but for $\alpha = 2$ the variation from uniform distribution becomes a matter of practical significance.

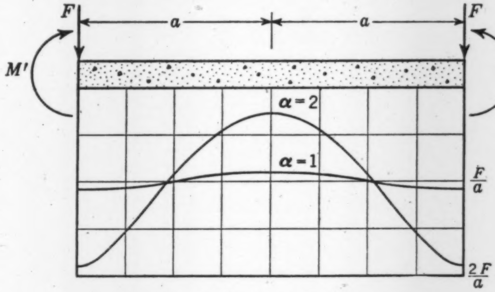


FIG. 5.—REACTION DIAGRAMS

ILLUSTRATIVE EXAMPLES

Fig. 6 shows a section of a water channel in which the floor slab is to be constructed without an expansion joint parallel to the direction of the channel. The weight of the slab itself has no effect upon moments or shears in the slab. The weight of the side-wall (12.4 kips per ft) has been computed using the weight of concrete as 150 lb per cu ft. The values of E for the concrete and k for the foundation are assumed as 4×10^6 lb per sq in. and 100 lb per cu in., respectively. The values of λ (= 138 in.) and α $\left(= \frac{240}{138} = 1.74 \right)$ may be computed. From the various figures in the paper the constants C_a, S_a, D_0, u, and v may be read as follows: $C_a = 0.48$; $S_a = 1.61$; $D_0 = 0.30$; $u = 0.14$; and $v = 0.195$.

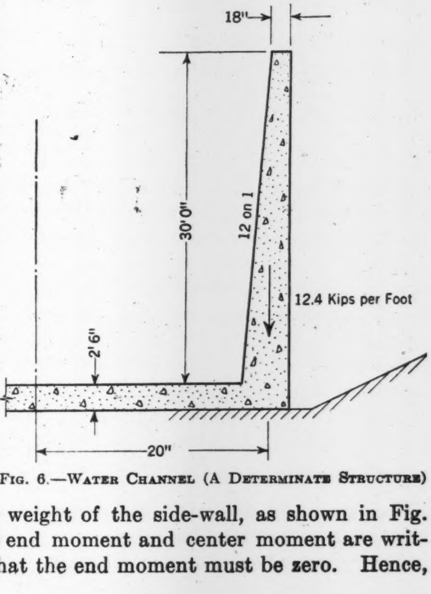
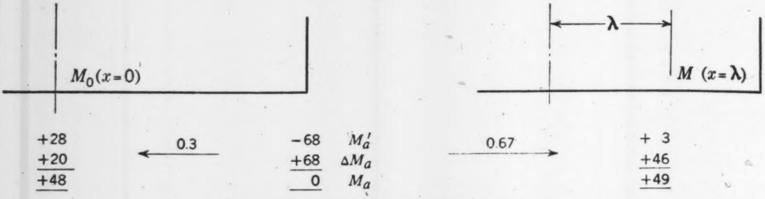
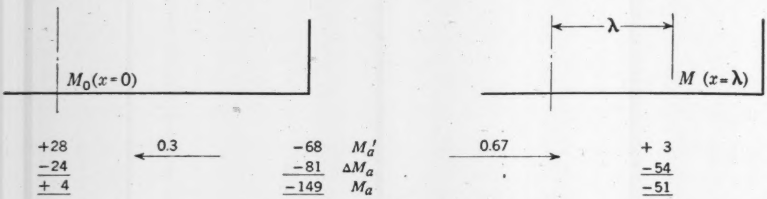


FIG. 6.—WATER CHANNEL (A DETERMINATE STRUCTURE)

It is first desired to compute the center moment caused by the weight of the side-wall, as shown in Fig. 7(a). The fixed-end values of the end moment and center moment are written first. From statics it is seen that the end moment must be zero. Hence, when the artificial restraint at the end of the slab is released, the change in end moment which occurs will be equal in magnitude to the fixed-end value, and of opposite sign. This change in end moment is then multiplied by the moment factor D_0 to obtain the change in center moment which occurs when the artificial end restraint is released. The final value of center moment then is found by addition to be 48 ft-kips. It is interesting to compare this value with that which would occur with uniform reaction distribution as given by



(a) MOMENTS CAUSED BY WEIGHT OF WALL



(b) MOMENTS WITH BACKFILL IN PLACE

FIG. 7.—COMPUTATION OF MOMENTS

$$M = \frac{W\,L}{8} = \frac{F\,a}{2} = 124 \text{ ft-kips} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Fig. 7(*a*) also shows the computation for determining the moment at an intermediate point $x = \lambda$, chosen for convenience. The fixed-end value of moment at this point was computed from Eq. 24*c*, and the moment factor, $\psi_3(\lambda) = 0.67$, was computed by using Eqs. 11 and 25*c*. The moment factor appears in the computations in a manner which resembles the carry-over factor of moment distribution.

It is also desired to compute the moments that will occur after the backfill is placed. This computation is shown in Fig. 7(*b*). The backfill has been assumed to be placed to the top of the wall and to exert a pressure equal to that of a liquid weighing 33 lb per cu ft. The moment at the base of the wall, which is also the moment at the end of the slab, may be computed from statics:

$$M = \frac{q\,H^3}{6} = \frac{33\,(30)^3}{6} = 149 \text{ ft-kips} \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which q is the density of the equivalent liquid and H is the wall height. Assuming an artificial restraint at the end of the base slab the fixed-end values of end moments and center moments are written first. When the artificial restraint is released the magnitude of the end moment will increase to 149. The change in end moment is then multiplied by the moment factor to obtain the simultaneous change in the center moment. A computation is made also for moment at the point $x = \lambda$.

In Fig. 8 a rectangular conduit is analyzed by moment distribution to illustrate an indeterminate structure. The relative I-values are assumed as shown on the left side of the center line. The stiffness values may be computed to be as shown on the right side. Due to symmetry the stiffness of the top slab includes the end-rotation constant $\frac{1}{2}$. This avoids the necessity of carrying across the center line in the moment balancing process. It has been assumed that the stiffnesses of the base slab and foundation are such as to give the value $\lambda = 24$ in. and hence to give $\alpha = 2$. The moments have been distributed in the usual manner after computation of the distribution factors. The total change in moment at the end of the base slab has been multiplied by a moment factor $D_0 = 0.15$ to obtain the change in center moment. A statical moment sign convention has been used in the moment distribution.

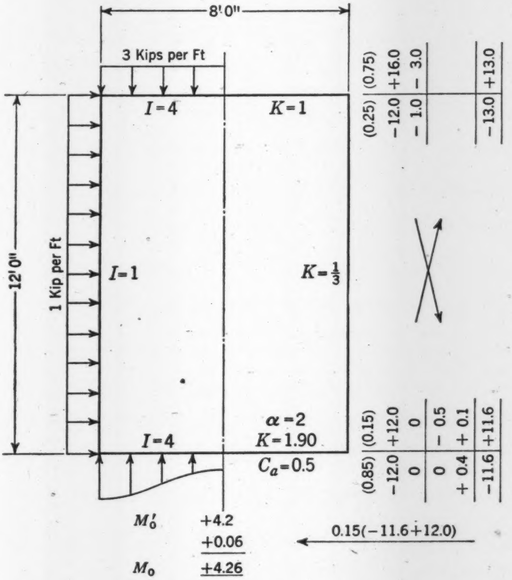


FIG. 8.—RECTANGULAR CONDUIT (AN INDETERMINATE STRUCTURE)

CONCLUSION

A method of determining shears and moments in a base slab resting on an elastic foundation has been developed by using a floating slab theory. Formulas have been developed for fixed-end moment and stiffness values to permit an analysis by moment distribution when the slab is part of an indeterminate frame. Graphs of those coefficients commonly needed in practice have been presented.

APPENDIX

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[4] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = an integration constant (also B, C, and D, see Eq. 17);
a = half-length of slab;
B = (see A);
C = (see A);
C_a = a fixed-end moment coefficient;
D = (see A);
D_0 = center moment factor;
E = modified modulus of elasticity $= \dfrac{E'}{1 - \mu^2}$;
E' = modulus of elasticity;
F = external force acting on a beam;
I = rectangular moment of inertia of a slab, per unit of width;
k = foundation modulus;
L = span length;
M = bending moment: M_a = bending moment at $x = a$;
S = stiffness function;
u = a convenient function of α defined by Eq. 21;
V = total shear;
v = a convenient function of α defined by Eq. 21;
W = total base reaction;
w = reaction pressure;
x = horizontal coordinate;
z = vertical displacement;
α = a fundamental ratio or angle, in radians, $= \dfrac{a}{\lambda}$;
θ = slope of elastic curve: θ_a = end slope or end-rotation angle;
λ = a fundamental length;
μ = Poisson's ratio;
ϕ = an end-rotation function: ϕ_0 = a beam stiffness $= \dfrac{E\,I}{a}$;
ψ = an end-rotation function.

Subscripts on z, θ, M, V, or S indicate the value of the functions when x equals the subscript. For example: z_a = displacement at $x = a$. Primes on z, θ, M, or V indicate the value of the functions when the ends of the beam are fixed.

---

[4] ASA—Z10a—1932.

DISCUSSION

Robert O. Jameson,[5] M. Am. Soc. C. E.—Although the subject of beams on elastic foundations has been treated at some length by other authors,[6,7] Mr. Benscoter has simplified the solution of this special case greatly by use of moment distribution. The formulas and graphs presented should prove valuable timesavers.

The complete formulas for slope, moment, and shear (compare Eqs. 24) are:

$$\theta(x) = \frac{dz}{dx} = \frac{F\,\lambda^2}{2\,E\,I}\left[(v - u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u + v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right]$$

$$+ \theta_a\left[(v + u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u - v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \ldots\ldots\ldots (36a)$$

$$M(x) = E\,I\,\frac{d^2z}{dx^2} = -\,F\,\lambda\left(u\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} - v\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right)$$

$$+ \frac{2\,E\,I\,\theta_a}{\lambda}\left(v\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + u\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \ldots\ldots\ldots\ldots (36b)$$

and

$$V(x) = E\,I\,\frac{d^3z}{dx^3} = -\,F\left[(u + v)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u - v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right]$$

$$+ \frac{2\,E\,I\,\theta_a}{\lambda^2}\left[(v - u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u + v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \ldots\ldots (36c)$$

Some designers prefer to plot two moment curves for a restrained beam: One representing positive moments due to vertical loads on the simply supported beam, and the other showing negative moments due to restraint at ends of the beam. The equations for finding these moments are as follows: Referring to Eq. 36(*b*) when $M_a = 0$,

$$\theta_a = F\,\frac{\phi_1(a)}{\phi_3(a)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Substituting Eq. 37 in Eq. 36*b*,

$$M(x) = C_1\,F\,\lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which

$$C_1 = \frac{1}{\lambda}\left[\frac{\phi_1(a)\,\phi_3(x)}{\phi_3(a)} - \phi_1(x)\right] \ldots\ldots\ldots\ldots\ldots (39)$$

---

[5] Care, U. S. Engrs., Southwestern Div., Dallas, Tex.

[6] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 1–24.

[7] *Engineering News-Record*, Vol. 93, p. 881 (letter to the editor by C. W. Pantke).

Values of C_1 are plotted in Fig. 9. Eq. 38 is used in finding the positive moments on the beam.

With no external loads on the beam, $F = 0$ in Eq. 36*b*, and

$$\theta_a = \frac{M_a}{\phi_3(a)} \quad \text{(40)}$$

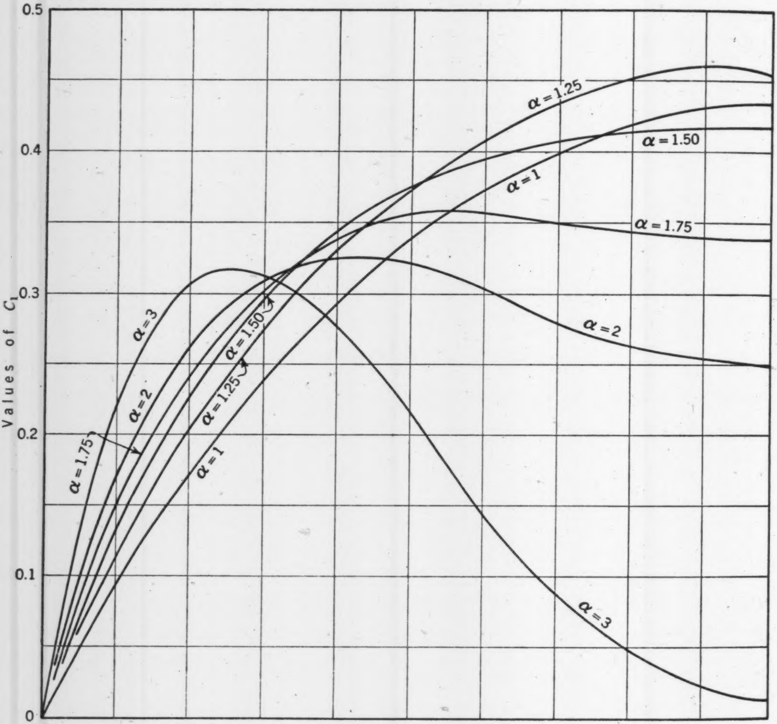


FIG. 9.—POSITIVE MOMENT FACTORS

Substituting Eq. 40 in Eq. 36*b* gives

$$M(x) = D(x)\, M_a \quad \text{(41)}$$

in which

$$D(x) = \frac{\phi_3(x)}{\phi_3(a)} \quad \text{(42)}$$

Values of $D(x)$ are shown in Fig. 10. These values may be used also in solving Eq. 10c. The moments due to end restraint only can be determined by Eq. 41.

The example shown in Fig. 7(a) in the paper is solved by use of Eq. 38 and Fig. 9 as follows: $\alpha = 1.74$; $x = 0$; $\lambda = 11.5$; $C_1 = 0.343$ (from Fig. 9); and—

$$M_0 = 0.343 \times 12.4 \times 11.5 = +48.9$$

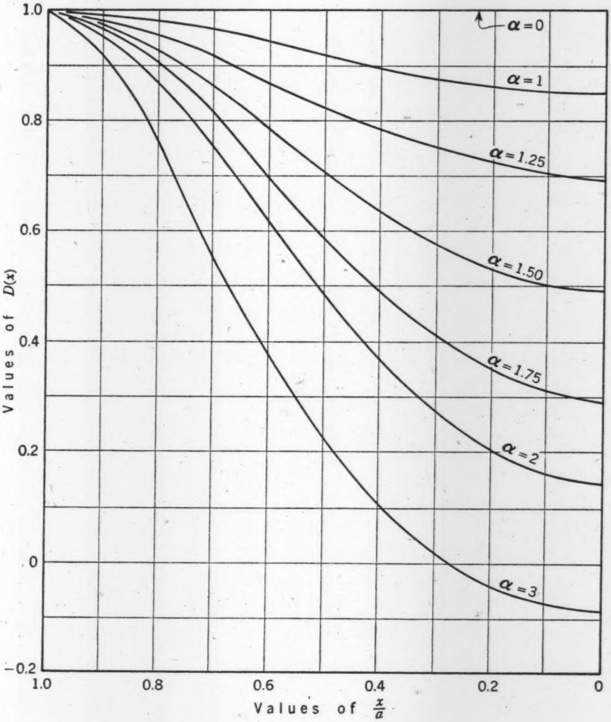


FIG. 10.—RESTRAINT MOMENT FACTORS

Again, for $x = \lambda$; and $C_1 = 0.347$:

$$M(\lambda) = 0.347 \times 12.4 \times 11.5 = +49.5$$

For solving the example shown in Fig. 7(b), Eq. 41 and Fig. 10 are used, thus: $\alpha = 1.74$; $x = 0$; $M_a = 149$; $D(x) = 0.306$; and—

$$\begin{aligned} -M_0 &= 0.306 \times 149 = -45.6 \\ +M_0 \text{ (from Fig. 7(a))} &= \underline{+48.9} \\ M_0 \text{ (net)} &= +\ 3.3 \end{aligned}$$

Also, $\alpha = 1.74$; $x = \lambda$; $D(x) = 0.673$; and—

$$\begin{aligned} -M(\lambda) &= 0.673 \times 149 = -100.3 \\ +M(\lambda) &= +49.5 \\ \hline M(\lambda) \text{ (net)} &= -50.8 \end{aligned}$$

For the example shown in Fig. 8, the center moment (for $\alpha = 2$; $C_1 = 0.248$; $D(x) = 0.144$; and $\lambda = 2$) is

$$\begin{aligned} +M_0 &= 0.248 \times 12 \times 2 = +5.95 \\ -M_0 &= 0.144 \times 11.6 = -1.67 \\ \hline M_0 \text{ (net)} &= +4.28 \end{aligned}$$

L. J. MENSCH,[8] M. AM. SOC. C. E.—Many able mathematicians have worked on the problem presented in this paper, and all of them have based their analyses on the assumption that Eq. 13 holds true. Any experienced engineer who seriously reflects on the requirements in such a case or makes the simplest kind of experiment with a card or wooden board must predict that this assumption plainly is not feasible except in rare instances. It did not apply in the case of a rail supported by ties as treated by the late A. N. Talbot,[9] Past-President and Hon. M., Am. Soc. C. E., although the error was probably less than 25%. Frost action will make k unsteady and reduce its value to possibly 30 to 50 lb per cu in. The mathematics are valid enough, but they do not prove the assumption.

Eq. 12 means neither more nor less than that the intensity of the forces acting on the slab per unit length equals w. It is identical with the fundamental equation of flexure in elementary textbooks:

$$E\,I\,\frac{d^2M}{d^2x} = w \qquad (43)$$

Once the computer can make a good guess at its distribution, the problem can be solved easily by simple mathematics familiar to busy structural engineers. The writer has shown elsewhere[10,11] that the distribution of w can be assumed as shown in Fig. 11(*a*) and that the deflection of the slabs will be approximately as in Figs. 11(*b*) and 11(*c*), and will rarely be as shown in Fig. 11(*d*). In most cases the slab will rise from contact with the soil in the central part, which makes Eq. 13 invalid.

The design of a water channel or large tank is by no means as simple as would appear from the paper, and serious mishaps with such structures have been encountered by engineers of standing in the profession. The engineer, experienced in this type of structures, will attack the problem in the author's "Illustrative Examples," as follows:

---

[8] Civ. Engr. and Contr., Chicago, Ill.

[9] Progress Report of the Special Committee to Report on Stresses in Railroad Track, *Transactions* Am. Soc. C. E., Vol. LXXXII (1918), p. 1207.

[10] "Joints for Concrete Pavements," by L. J. Mensch, Chicago, Ill., 1935.

[11] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1148.

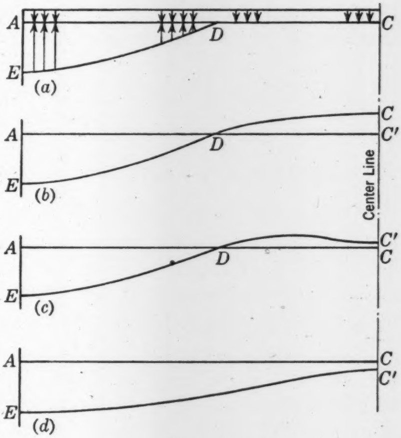


Fig. 11

1. He will ask himself what a heavy rainstorm of 2-in. to 6-in. precipitation in as many hours will do to the structure which is being constructed. Will the banks cave in at one or both sides of the channel at various places and trap the water so that an uplift is produced which may shove the structure sideways a few feet or lift it up entirely and break it? This phase of the project alone will involve quite some study and expense.

2. Suppose that the engineer is in a position to guard his structure against a highwater level not exceeding 15 ft above the subgrade. Then the base slab will have to be strong enough to withstand the moment of 148 ft-kips mentioned by the author, as the water pressure will just balance the weight of the structure.

3. Let it be now assumed that conditions are favorable and no hydraulic uplift has to be taken in consideration and that the engineer, induced by the paper, will proceed to investigate the stresses in the base slab due to the wall load alone. He or his assistants will not have the time or patience to check the mathematics of the paper, especially in the absence of any bibliography on this subject, and instead likely will make a simple test by laying a thin ruler about 1 ft long with the flat side on the pages of an open book and by pressing the ends down with a force of about 20 lb. They will notice a pronounced depression near the ends and a slight tendency of lifting up at the middle. Considering that it is good practice to cantilever the base slab beyond the wall from 6 in. to several feet, they may assume as a first approximation that the base slab deflects as shown in Fig. 12.

They may assume the line ED as a common parabola; line AE equals z_a in the author's nomenclature and, following the author's idea, they will assume the soil pressure, w, proportionate to the ordinates of the parabola on the length AD. The length of the half slab AC will be loaded by the uniform dead load of 375 lb per sq ft, the weight of the wall is 12,400 lb per lin ft; and therefore one can write, for the vertical loads—$12{,}400 + 26 \times 375 = \frac{2}{3} w_a \times 16$, from which $w_a = \dfrac{22{,}150}{10.67} = 2{,}070$ lb per sq ft or 14.4 lb per sq in.

The moments acting in the slab can be found by simple statics: $M_C = 22{,}150 \times (26 \text{ ft} - \frac{3}{8} \times 16 \text{ ft}) - 12{,}400 \times 22 \text{ ft} - 375 \times \dfrac{26^2}{2} = 43{,}000$ ft-lb (tension at bottom). The maximum negative moment will be found at the section where the shear is zero; it is considerably smaller than the positive moment. The point C′ closely approaches the point C.

No radical change will be found when the line of deflection is assumed to intersect the line AC nearer or farther from C than the 10 ft shown in Fig. 12.

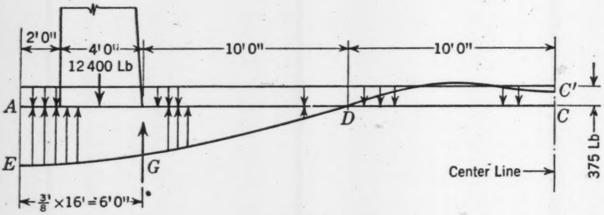


FIG. 12

4. The experienced engineer is accustomed to design the base of a water channel or tank for actual service conditions; namely, for the wall backfilled, and for the structure filled with water (in this example assumed to be to a height of 25 ft). When hydraulic uplift is not encountered, it is not economical in a structure of considerable width (sometimes several hundred feet wide) to make the base slab of uniform thickness, and, in order to avoid the use of piles in ground of medium carrying capacity, the experienced engineer most likely will assume the base slab to cantilever 4 ft beyond the wall and will make it 30 in. thick for only about 10 ft inside the structure. He will make the remaining slab only about 6 in., or perhaps 8 in., thick, and will connect it with the thicker part by a well-designed metal water stop, as shown in Fig. 13. The engineer is now confronted with a number of decisions before he can proportion the foundation slab of this structure. What assumption shall be made for the earth pressure acting on the wall? Shall he assume the customary value of 15 H^2 for the pressure per linear foot of wall, and the moment about the base as 5 H^3, or is the moment of 4 H^3 a better guess?

Some engineers, who have seen shrinkage cracks develop in backfills near the wall after heavy rains, claim that the earth pressure should not be counted upon to relieve the pressure on the wall from the water on the inside. Others go as far as to say that colloidal tension may exert a pull on the wall that will increase the effect of the water pressure. Others have observed considerable deflections in concrete walls, not tied on the top, and claim that passive earth pressure should be taken into account. When the backfill consists of good material, such as sand, gravel, stone, and slag, and is thoroughly rolled or tamped, it requires a movement of about $\frac{H}{200}$ at the top to develop a passive pressure about four times the active pressure, whereas when the fill simply is dumped from cars or grab buckets, it will take a movement of $\frac{H}{50}$ to $\frac{H}{100}$ to develop the aforementioned passive pressure. The movements given take into account time loading. Suppose the decision has been made that the combined water and earth pressure on the wall exerts a moment of 80,000 ft-lb at the base of the wall for each linear foot.

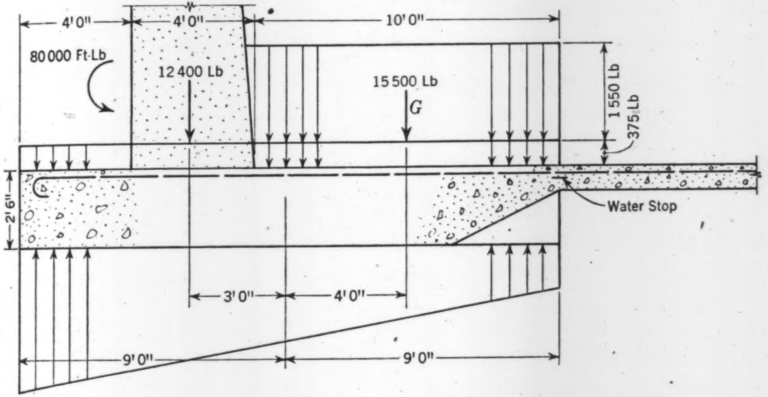


FIG. 13

The sum of all vertical loads on the foundation will be $12{,}400 + 375 \times 18 + 1{,}550 \times 10 = 34{,}650$ lb and the ideal distributed soil pressure $\frac{34{,}650}{18} = 1{,}925$ lb per sq ft. This pressure will be modified by the moment acting at the base of the foundation slab: $80{,}000 + 12{,}400 \times 3 - 15{,}500 \times 4 = 55{,}200$ ft-lb which produces an ideal plus-and-minus stress of $\frac{55{,}200}{\frac{1}{6} \times 18^2} = 1{,}022$ lb per sq ft, or the soil pressure at A $= 2{,}947$ lb per sq ft and that at D $= 903$ lb per sq ft. The differential pressure of 2,000 lb per sq ft will tilt the base about 0.14 in. for a soil modulus of 100 which will result in a horizontal movement of the top of the wall of about $\frac{1}{4}$ in. The elastic deformation of the wall will increase this movement to about $\frac{1}{2}$ in., or $\frac{H}{720}$, which will bring into action part of the passive pressure of the backfill.

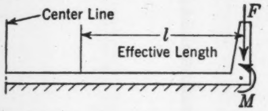
5. When the structure is built on wet clay it is possible that the outward pressure of the water (diminished by the earth pressure) will cause a sliding of the foundations, as shown in Fig. 13. This outward pressure has been assumed at about 10,000 lb per lin ft, whereas the weight on the foundation slab is 34,650 lb; and the coefficient of friction required $\frac{10{,}000}{34{,}650} = 0.3$. Conservative engineers do not allow more than 0.2 as the coefficient of friction on wet clay. It will be necessary to run reinforcing steel from the 30-in. slab into the thin slab across the contraction joint, a practice that many engineers condemn but that

has been used on some of the largest and most successful reservoirs built in the United States.

Alfred L. Parmé,[12] **Jun. Am. Soc. C. E.**—Commendation is due Mr. Benscoter on the masterly style in which he has presented the problem of a slab on an elastic foundation. The information and ideas contained in the paper should lead to a more economical design of concrete channels and conduits and should serve to stimulate a more rational approach to the problem of designing reinforced concrete walls.

A further problem arising in the design of wide U-channels is that of determining how far the influence of the corner moments extends. For this purpose, the writer has utilized the work done by the Portland Cement Association on the design of circular tanks to develop a simple and easily applied formula. A comparison of the differential equations for circular walls and for beams on an elastic foundation reveals that both follow the same law. For an unloaded beam (see Table 1):

TABLE 1.—Analysis to Determine Influence of Corner Moments

Moment = coefficient $\times \left\{\begin{array}{l}\text{edge moment}\\ \text{edge shear} \times l\end{array}\right.$										
	0.9 *l*	0.89 *l*	0.7 *l*	0.6 *l*	0.5 *l*	0.4 *l*	0.3 *l*	0.2 *l*	0.1 *l*	0.0 *l*
Edge moment...	−0.001	−0.043	−0.112	−0.189	−0.311	−0.464	−0.638	−0.798	−0.944	−1
Edge shear	+0.0	+0.017	+0.034	+0.060	+0.083	+0.106	+0.117	+0.109	+0.077	+0

$$\frac{d^4 z}{dx^4} + \frac{4\,z}{\lambda^4} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

in which $\lambda = \sqrt[4]{\frac{4\,E\,I}{k}}$. On the other hand, for a circular tank, the equation

$$\frac{d^4 z}{dx^4} + 4\,\beta^4\,z = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

expresses the behavior of an unloaded tank. In Eq. 45, $\beta = \sqrt[4]{\frac{3}{r^2\,t^2}}$. For $\frac{H^2}{D\,t} > 2$ the upper boundary is slightly affected by operations at the lower boundary.[13] If the relationship between the parameters is then established, a convenient way of determining the effective length of the slab will be available. Therefore, setting $\frac{l}{\lambda}$ equal to $\beta\,H$ and making the proper substitution:

$$\frac{l}{t}\sqrt[4]{\frac{3\,k\,t}{E}} = H\sqrt{\frac{3}{r^2\,t^2}} \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

---

[12] Bronxville, N. Y.

[13] "Circular Concrete Tanks Without Prestressing," *Concrete Information Sheet No. ST 57*, Portland Cement Assn.

and therefore

$$\left(\frac{l}{t}\right)^2 \sqrt{\frac{k\,t}{E}} = 2\left(\frac{H^2}{D\,t}\right) \qquad (46b)$$

With $\frac{H^2}{D\,t}$ equal to 2, it follows that $\left(\frac{l}{t}\right)^2 \sqrt{\frac{k\,t}{E}} = 4$; or,

$$l = t\sqrt[4]{\frac{E}{k\,t}} \qquad (47)$$

To illustrate the applicability of the formula, a specific example will be computed. Assume $k = 100$ lb per cu in.; $t = 1$ ft; and $E = 3{,}000{,}000$ lb per sq in. Substituting these numerical values in Eq. 47, $l = 4 \times 1\sqrt[4]{\frac{3{,}000{,}000}{100 \times 12}}$ = 28.3 ft. The value of t inside the radical must be in inches to conform with the dimensions of the other terms. The variation of the moment along the base due to an edge moment or shear is presented as an aid to detail analysis.

At this point a word of caution against the indiscriminate use of the method should be voiced. The designer should check the pressures exerted on the slab. The existence of negative pressures greater than the weight of the slab creates a condition that cannot be met in the field. Hence, it would have been desirable if the author had included the displacements due to the applied shear and moment. The inclusion of these values would immediately disclose whether the assumed conditions could be realized. In the case where the width of the channel exceeds the effective length as computed by Eq. 47, a shear greater than $\frac{2.57\,M}{l}$ will provide positive pressures throughout the slab.

A. A. EREMIN,[14] ASSOC. M. AM. SOC. C. E.—The foundation pressures under an elastic footing vary according to the elastic properties of the soil and the footing. Interesting cases of pressure distributions under footings were illustrated by D. P. Krynine[15] in 1937. Mr. Benscoter has considered the case of a footing with decreased foundation pressures at the midspan. He has determined the stresses in the footing in terms of a constant α which varies with the elastic properties of footing and foundation material. However, the range over which the constant α varies is too wide for exact computation.

The writer has found that the approximate method of computing stresses for the assumed distribution of foundation pressures is often sufficient for practical purposes. The foundation pressures under the elastic footing may be assumed to vary according to the sine curve in Fig. 14. The average foundation pressure in Fig. 14 is w_o and the reduction of foundation pressure at the midspan is w_1. Assume that the footing slab in Fig. 14 is supported by hinged supports at the ends; then the bending moment at midspan will be

---

[14] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[15] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 838.

$$M = \frac{L^2}{8}(w_o - 0.41\,w_1)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48a)$$

If the footing slab in Fig. 14 is supported by built-in ends, the bending moments at the built-in ends are

$$M' = \frac{L^2}{12}(w_o - 0.30\,w_1)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48b)$$

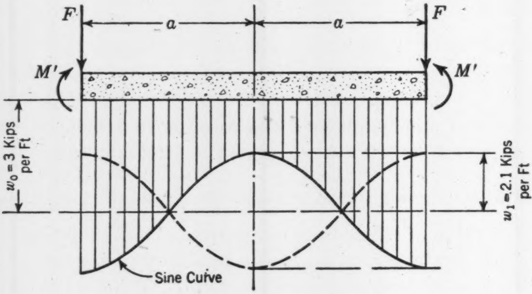

F
F
a
a
M'
M'
$w_0 = 3$ Kips per Ft
$w_1 = 2.1$ Kips per Ft
Sine Curve

FIG. 14.—FOUNDATION PRESSURE DIAGRAMS

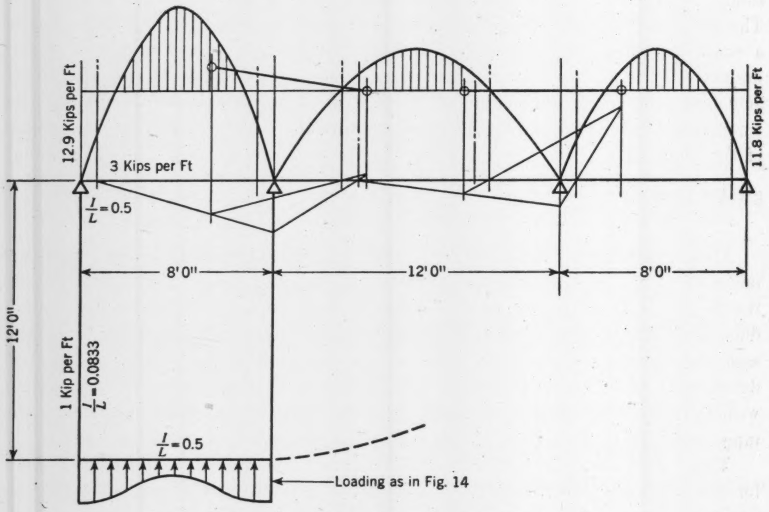

12.9 Kips per Ft
11.8 Kips per Ft
3 Kips per Ft
$\frac{I}{L} = 0.5$
8'0"
12'0"
8'0"
12'0"
1 Kip per Ft
$\frac{I}{L} = 0.0833$
$\frac{I}{L} = 0.5$
Loading as in Fig. 14

FIG. 15.—DISTRIBUTION OF MOMENTS IN RECTANGULAR CONDUIT

The example shown in Fig. 8 may be solved by applying Eqs. 48 as shown in

Figs. 14 and 15. The average foundation pressure in the footing slab (Fig. 14) is w_o = 3 kips per ft. Assume that the foundation pressures vary according to the sine curve, with a reduction of pressure at the midspan to w_1 = 2.1 kips per ft, or about 70% of the average pressure. The assumption that foundation pressures are reduced at the midspan may be verified easily by tests. The bending moments at the fixed ends of the footing slab (Fig. 14) computed by Eq. 48*b* are M' = 12.65 kip-ft. The bending moments in all members of the rigid frame were determined by the graphical method[16] as shown in Fig. 15. The difference between the resulting bending moments and those shown by the author is negligible.

Often the foundation pressures at midspan of the footing slab are increased, as shown by Professor Krynine.[15] In this case also the foundation pressures may be assumed to vary according to the sine curve, with the maximum pressure at the midspan, as shown by the dotted line in Fig. 15. Bending moments in the footing, in this case, may also be determined by Eqs. 48, except that the sign before the second term in the parentheses will be changed from minus to plus.

A. E. CUMMINGS,[17] M. AM. SOC. C. E.—As a background for this subject the paper needs some reference to the extensive literature already available. Some of the works dealing with the problems of beams and plates on yielding foundations are indicated by the following references:

(1) "Die Lehre von der Elasticität und Festigkeit," by E. Winkler, 1867, p. 182.

(2) Correspondence on "Iron Permanent Way," by J. W. Schwedler, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, Vol. LXVII, 1882, p. 95.

(3) "Über das Gleichgewicht Schwimmender Elastischer Platten," by H. Hertz, *Annalen der Physik und Chemie*, Vol. 22, 1884, p. 449.

(4) "Über die Berechnung von Trockendocks," by O. Franzius, *Zeitschrift für Bauwesen*, 1908, p. 475.

(5) Progress Report of the Special Committee to Report on Stresses in Railroad Track, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1191.

(6) "Über das Gleichgewicht von Elastischen Platten unter einer Einzellast," by H. Happel, *Mathematische Zeitschrift*, Vol. 6, 1920, p. 203.

(7) "Theorie des Trägers auf Elastischer Unterlage," by K. Hayashi, Julius Springer, Berlin, 1921.

(8) "Belastingsproeven ter Bepaling van de Grondelasticiteit," by B. Wolterbeek, *De Ingenieur*, Vol. 36, 1921, p. 501.

(9) "Vorlesungen über Technische Mechanik," by A. Föppl, 4th Ed., Vol. V, Leipzig, 1922.

(10) "Über den Balken auf Nachgiebiger Unterlage," by K. Wieghardt, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 2, 1922, p. 165.

(11) "Platten Rechteckiger Grundriszteilung auf Elastisch Nachgiebiger Unterlage," by Lewe, *Der Bauingenieur*, 1923, p. 453.

---

[16] "Analysis of Continuous Rigid Frames by Graphical Distribution of Moments," by A. A. Eremin, published privately by the author, 1943.

[17] Research Engr., Raymond Concrete Pile Co., New York, N. Y.

(12) **"Om Beregning af Plader paa Elastisk Underlag,"** by H. Westergaard, *Ingeniøren*, Vol. 32, 1923, p. 513.

(13) **"Kreisplatten auf Elastischer Unterlage,"** by F. Schleicher, Julius Springer, Berlin, 1926.

(14) **"Strength of Materials,"** by S. Timoshenko, Pt. II, Van Nostrand, 1930, p. 401.

(15) **"Bodenpressung und Bettungsziffer,"** by K. Terzaghi, *Österreichischen Bauzeitung*, June 18, 1932.

(16) **"Die Bemessung von Flachgründungen aus Eisenbeton und die Neuere Baugrundforschung,"** by O. Froelich, *Beton und Eisen*, No. 12, 1935.

(17) **"Stresses and Deflections in Loaded Rectangular Plates on Elastic Foundations,"** by G. Murphy, *Bulletin No. 135*, Iowa Eng. Experiment Station, Ames, Iowa, 1937.

(18) **"Bending of an Infinite Beam on an Elastic Foundation,"** by M. Biot, *Journal of Applied Mechanics*, March, 1937.

(19) **"Über den Träger auf Elastischer Unterlage,"** by K. Marguerre, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 17, 1937, p. 224.

(20) **"Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth,"** by A. Hogg, *Philosophical Magazine*, London, Vol. 25, 1938, p. 576.

(21) **"Thin Plates on Elastic Foundations,"** by D. Holl, *Proceedings*, Fifth International Cong. for Applied Mechanics, Cambridge, Mass., 1938, p. 71.

(22) **"Mathematical Methods in Engineering,"** by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940, p. 304.

(23) **"Theory of Plates and Shells,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., 1940, p. 248.

(24) **"Mathematical Methods in Engineering,"** by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York and London, 1940, p. 35.

(25) **"An Introduction to the Theory of Elasticity for Engineers and Physicists,"** by R. Southwell, Oxford Univ. Press., 2d Ed., 1941, p. 215.

The author's approach to the problem under discussion appears to be more complex than is necessary. In order to find a solution, the author makes use of slope deflections, moment distribution, and elastic theory based on the differential equation of the elastic curve of the deflected slab. Actually, the problem can be solved by the elastic theory alone with considerably less mathematical work than is involved in the author's solution.

The load condition represented by Fig. 7(*a*) was solved years ago. It is a problem involving a beam resting on a yielding foundation and loaded with two concentrated loads symmetrically placed with respect to the center line. This is just a special case of the problem of the railroad tie which was the principal subject of the early investigations of E. Winkler (Reference 1), J. W. Schwedler (Reference 2) and others. The only difference between Fig. 7(*a*) and the railroad tie is the fact that the concentrated loads on the tie are some distance in from the ends of the tie, whereas the loads in Fig. 7(*a*) are at the ends of the beam. However, the problem with the loads at the ends of the beam has been solved and the complete solution was given by K. Hayashi (Reference 7).

Fig. 7(*b*) refers to the more complicated condition when the backfill is in place and there are various forces and moments acting on the base slab. In order to solve this problem as represented by the force diagram of Fig. 2, the author has introduced slope-deflection equations, beam functions, fixed-end functions, and end-rotation functions. None of these is needed for the solution of this problem, as it is possible to solve the entire problem at one stroke by means of the differential equation of the elastic curve of the beam.

The general solution of Eq. 13 is Eq. 17 which contains four arbitrary constants of integration. The author has chosen to set two of these constants equal to zero for reasons of symmetry and he has then determined the other two constants by the boundary conditions in Eqs. 19. It is this procedure that necessitates the introduction of all those other functions into the analysis. Actually, it is possible to solve the entire problem by means of Eq. 17. To do this, it is only necessary to find the four boundary conditions that will determine the four constants A, B, C, and D. For this purpose the following four boundary conditions are available:

(1) By symmetry, the elastic curve of the deflected slab has a horizontal tangent at the center line. Therefore:

$$\left[\frac{dz}{dx}\right]_{x=0} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(49a)}$$

(2) The bending moment at the right-hand end of the slab is $-M_a$. Therefore:

$$\left[\frac{d^2z}{dx^2}\right]_{x=a} = -\frac{M_a}{E\,I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(49b)}$$

(3) The shear at the right-hand end of the slab is $-F$. Therefore:

$$\left[\frac{d^3z}{dx^3}\right]_{x=a} = -\frac{F}{E\,I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(49c)}$$

(4) The summation of vertical forces must be zero. Therefore:

$$-F = \int_0^{0.5L} w\,dx = -k\int^{0.5L} z\,dx \ldots\ldots\ldots\ldots \text{(49d)}$$

When the four integration constants of Eq. 17 have been determined by means of these four boundary conditions, the resulting equation will be a complete solution of the problem. This equation will contain all of the external forces shown in Fig. 2, and it will contain the elastic coefficients of the soil and the slab. The amount of mathematics required to present the solution would be about the same as that involved in the author's Eqs. 12 to 22, inclusive. The slope-deflection equations, beam functions, fixed-end functions, and end-rotation functions are all unnecessary. The problem of superimposing that external bending moment on the general solution of Eq. 13 is quite simple. Hayashi (Reference 7) devotes only a short paragraph to the subject and merely remarks that the external bending moment can be included in the solution without difficulty.

The author has not offered any explanation as to why he solved this problem on the basis of the incomplete force diagram shown in Fig. 2. A structure of the type under discussion, with the channel empty and with the backfill in place outside the walls, is shown in Fig. 16(*a*). The horizontal thrust, *P*, is combined with the vertical force, *F*, into a resultant, *R*. At the middle plane of the slab, the resultant, *R*, is resolved into a vertical force, *F*′, and a horizontal force, *P*′. These forces are equal, respectively, to *F* and *P*. Since any force may be replaced by a force and a couple, the force *F*′ may be replaced by *F* and M_a. The author has done this in the force diagram of Fig. 2, but the question arises as to what disposition has been made of the horizontal force, *P*′.

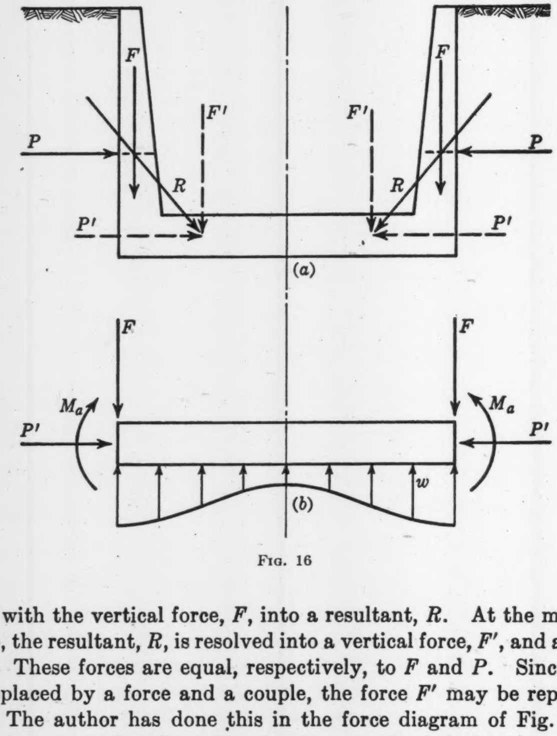


FIG. 16

This horizontal thrust, *P*′, is the force that can cause a retaining wall or a dam to fail by sliding even though the structure may be safe against overturning due to the eccentric force, *F*′. In the design of the side-wall itself, this force,

P', is taken into account because it produces a shearing stress on the horizontal plane where the base of the wall rests on the slab. The design of the bottom of the wall includes an investigation of three kinds of stresses: (*a*) The direct compression produced by F, (*b*) the flexural stresses produced by M_a, and (*c*) the shear produced by P'. Accordingly, when the structure as a whole is being considered, the diagram of external forces should include two equal and opposite horizontal thrusts, P', applied to the edges of the slab as shown in Fig. 16(*b*).

The probable importance of the forces, P', would depend on the dimensions of the structure. In a shallow structure with a thick base slab, the effect of P' would be negligible in comparison with the effects of F and M_a. In a deep structure with a thin base slab, the force, P', might materially increase the bending moments in the slab. In all cases, these forces will exist in any actual structure of this type and they should not be discarded unless it can be demonstrated that their effect is negligible. Furthermore, it should be kept in mind that the slab is bent by transverse loads and bending moments so that these horizontal thrusts are applied to the edges of an already deflected slab.

Eq. 13 is not satisfactory for this problem because it does not include the effect of the horizontal forces, P'. In order to include these forces, it is necessary to start with the differential equation:

$$\frac{d^4z}{dx^4} + \frac{P'}{E\,I}\frac{d^2z}{dx^2} + \frac{k}{E\,I}z = 0 \qquad (50)$$

This is a linear differential equation of the fourth order with constant coefficients. Methods of solving the equation are well known (24) and the general solution may be written in the form

$$z = \sum_{n=1}^{n=4} A_n\, e^{xp_n} \qquad (51)$$

in which A_n represents the four arbitrary constants of integration, p_n represents the four roots in the characteristic equation, and e is the Napierian log base. The four values of p_n are obtained from the equation

$$p^4 + \frac{P'}{E\,I}p^2 + \frac{k}{E\,I} = 0 \qquad (52)$$

After these values of p_n are substituted into Eq. 51, the four values of A_n are determined by means of the boundary conditions given in Eqs. 49. The solution of Eq. 50 is somewhat more laborious than that of Eq. 13, but the use of Eq. 50 with the proper boundary conditions will yield a complete solution without the aid of slope deflections or moment distribution or any other form of structural analysis.

When a mathematical analysis of this kind is applied to an engineering problem it is desirable to keep in mind the fact that the mathematics is only a part of the solution of the problem. All problems of this type involve important practical considerations which are easily overlooked in the maze of mathematics with which the problem is surrounded. Any one contemplating the use of some of these equations as design formulas should understand that this entire analysis is based on certain definite assumptions which may or may not be fulfilled in the actual structure. For example, in connection with Eq. 1, the author has defined the foundation modulus, k, as the constant of proportionality between the upward reaction, w, and the downward displacement, z. Nowhere in the paper is there any mention of the fact that k is supposed to work both ways. In other words, if the bending stresses in the slab tend to lift the slab from the ground, the mathematics require a tensile stress to be generated between the soil and the underside of the slab. This tensile stress is a downward reaction which is proportional to the upward displacement of the slab and the constant of proportionality is the same k that is in Eq. 1.

This idea can be expressed mathematically by the following statement: There is no restriction on the sign of z in Eq. 17. Positive values of z refer to downward displacements, and negative values of z refer to upward displacements. The stress between the soil and the slab may be either tension or compression in direct proportion to the displacement and the constant of proportionality is the foundation modulus, k, in either case. The mathematical analysis therefore requires that k must be able to act in tension or compression.

In contrast with this mathematical requirement, practical considerations lead to the conclusion that the soil cannot exert tension on the underside of the slab in the prescribed manner. If the slab were supported by a pile foundation, the piles would provide anchorage against uplift as well as resistance to downward displacement. However, even under such conditions the elastic behavior of piles subjected to uplift forces would not necessarily be the same as their elastic behavior under downward forces. The analysis of a slab supported by piles which could also act as anchors would require two different values of k. When the slab simply rests on earth or rock, there is no restraint to hold it down if the external forces try to bend it up off the ground.

The author has remarked (see heading, "Design Formulas") that "a practicing engineer instinctively prefers to deal entirely with bending moment values in a structure analysis." In the problem under discussion, it is not safe to trust entirely to instinct and to stop the investigation after the bending moments are determined. The displacements must be investigated also because the entire mathematical analysis is useless if there is any part of the slab for which the value of z is negative. This is a matter of considerable practical importance and it has been discussed thoroughly by other writers, particularly by Hayashi (Reference 7). The author's analysis does not include the final equation for the elastic curve of the deflected slab. However, this equation should be derived and used in the investigation of the structure.

Another practical aspect of this problem is the numerical value of k. The author has suggested load tests to determine the value of k but he recommends a value of 100 lb per cu in. for design purposes in cases where load test results are not available. The writer finds it difficult to accept such a recommendation for several reasons. In the first place, it is quite a problem to get a suitable value for k even when careful load tests are made. The literature in the foregoing list contains numerous demonstrations of this fact. A series of load tests with different sizes of bearing areas on the same soil would give a different value of *k* for each bearing area. The choice of *k* for the full size structure would therefore be a matter of judgment at best. In the second place, the numerical values of *k* that are obtained from load tests on different types of soil will vary over a considerable range. There appears to be no justification for the assumption that 100 lb per cu in. would be a suitable value for all soils merely because no load tests were made. Furthermore, an arbitrary assumption of this kind is hardly compatible with the powerful mathematical analysis the author has applied to the problem.

In conclusion, it seems desirable to make the following observations: Problems of this kind have engaged the attention of engineers for many years and complete mathematical solutions are already available for many of them. In general, these mathematical solutions are much more concise and elegant than the cumbersome procedure used by the author. The real need in connection with these problems is a better understanding of the physical significance of the equations and a considerable number of field measurements on actual structures to find out how accurately the mathematics can predict the behavior of the structure. As the matter stands now, the abstract mathematics has run far ahead of the practical side of the problem.

HARRIS EPSTEIN,[18] ESQ., AND JAMES R. AYERS, JR.,[19] ASSOC. M. AM. SOC. C. E.—The assumption of the proportionality between distributed soil reactions and the deflection of the distributing medium has been used by the writers since 1920.

A particular application of this assumption was made recently, at the suggestion of Ben Moreell, Hon. M. Am. Soc. C. E., to the design of graving docks which consist of two concrete side-walls connected by a floor slab resting either directly on the underlying soil, or on bearing piles, depending on the foundation conditions at a particular site. The side-walls are usually backfilled with earth. A typical cross section appears in Fig. 17, which is quite similar to that of the water channel in Fig. 6.

The side-walls of the water channel, as well as those of a dry dock, are of such height that, in comparison with the floor thickness, they possess almost infinite stiffness in the vertical direction and cannot, therefore, have a curved elastic line. This fact leads to the logical conclusion that the parts of both the force and displacement diagrams, directly under the walls, should be straight lines tangent to the curved part of these diagrams for the floor proper at the intersection of the floor with inner faces of side-walls and extending to the outer faces, as shown in Fig. 17. This contrasts with Fig. 2 in which the curved lines of the force and displacement diagrams extend under the side-walls and terminate at their centers. The omission of the forces on the outer halves of the wall width is certain to influence the magnitude of the moments in the floor slab, the influence increasing with increase in the width of the walls. Therefore the force and displacement diagrams should extend across the entire width of the structure.

---

[18] Prin. Designing Engr., Bureau of Yards and Docks, U. S. Navy Dept., Washington, D. C.

[19] Senior Designing Engr., Bureau of Yards and Docks, U. S. Navy Dept., Washington, D. C.

Let: W_s = weight of the part of the side-wall above the top of the floor slab; and a' = distance from the inner face of the side-wall to the center of gravity of the wall. The effect of the downward distributed load of the side-wall may be incorporated in the solution of the author's fundamental differential equation by applying to the slab, at the inner face of the side-wall, a concentrated load, W_s, and a moment of magnitude $W_s\, a'$. Since the reaction diagram under the wall is a straight line, the shear and moment, as well as the slope of the elastic curve, at the inner face of the side-wall may be expressed in terms of the foundation reactions at the inner and outer faces of the wall. These three expressions, combined with the conditions of symmetry and the expression for foundation reaction at the inner face of the side-wall, are sufficient for evaluating the constants for the general solution of the fundamental differential equation as applied to structures of this shape. Then the expressions for moment, shear, and foundation reaction at any point may be written and used to compute values of these quantities directly, thus eliminating the necessity for balancing the moments by the indirect method suggested by the author.

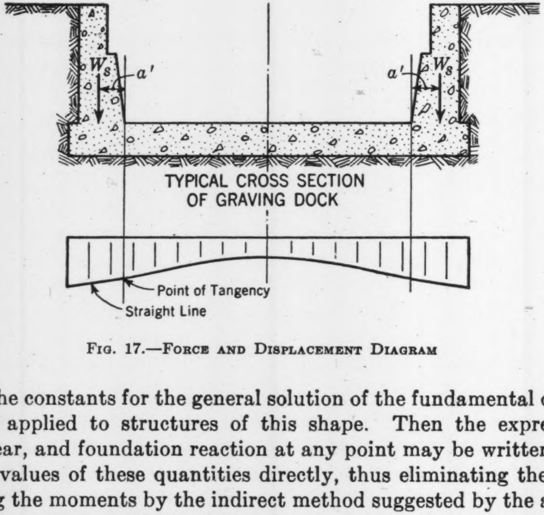


Fig. 17.—Force and Displacement Diagram

In the illustrated example of the water channel, the writers check the author's value of 48 kip-ft for the moment at the center of the slab, assuming a side-wall width of zero; whereas, if the force diagram extends across the entire wall width as a straight line, the moment at the center of the slab reduces to 40 kip-ft, the difference being about 16%.

The maximum value of the negative moment (tension in the top of floor slab) does not always occur at the center of the slab width. In many cases, for large values of α, there are two points of zero shear in the slab in addition to the one occurring at the center. The negative moment, which extends across the entire slab between side-walls, then has a minimum value at the center of the slab, and equal maximum values on either side of the center at the other two points of zero shear. This is due to the fact that the computed distributed reaction acts downward for a part of the slab adjacent to the center of the structure, thus requiring a force to prevent lifting of the slab from the foundation. The dead weight of the slab is usually more than enough to supply this force in the case of a dry dock.

STANLEY U. BENSCOTER,[20] **JUN. AM. SOC. C. E.**—The comments and information in the discussions are appreciated by the writer and will render useful service to the future reader as viewpoints gained from varied experience. The proper assumption as to reaction distribution in the design of a flexible base slab has always been a controversial issue. The writer has seen several types of broken-line diagrams which give a minimum value of base reaction at the center and a maximum value at the edge. Mr. Mensch has illustrated the use of a parabola over part of the half span; and Mr. Eremin, the use of a sine curve. None of these approximate methods of treating the problem contains any rational association with a physical measure of the foundation rigidity. Likewise, none of these approximate methods gives any suggestion as to the increased stiffness of the base slab, which is caused by the foundation and which must be used in an elastic indeterminate structural analysis.

Since the foundation modulus is a readily obtainable measure of the rigidity of the foundation, a method of analysis should be employed which gives results that are directly associated with this modulus. Thus, a rational analysis of the structure is obtained.

Various methods of analysis should be recorded so that calculated results can be compared with experimental field measurements whenever such measurements become available. A stress analyst can only express a plea that future plans for reinforced concrete structures include the installation of strain gages and pressure cells for measuring both reactions and acting earth or water pressures.

Inconsistently, Mr. Mensch objects to using the foundation modulus in a rational analysis of the slab, yet proceeds to use it in calculating the angle of rotation of the side-wall in Fig. 13. The writer wishes to recommend that structural engineers acquire the habit of calculating the angle of rotation of a retaining wall as presented by Mr. Mensch. The acting pressures are closely related to this angle. Although Mr. Mensch neglects the shear through the base slab at the section of the water stop, this shear affects the base reaction and the angle of rotation of the wall. Inclusion of this shearing force permits a reduction in the length of the heel. The writer has never found the loading condition mentioned by Mr. Mensch critical. Stresses are usually at a maximum after the backfill has been placed and before water has been allowed to enter the channel. The critical condition for stability occurs with the channel partly full and must be determined by trial and error. The writer does not extend the base slab beyond the side-wall as illustrated by Mr. Mensch in Fig. 12 when the wall and slab are constructed as a monolithic frame.

The graphs of moment factors presented by Mr. Jameson are a valuable addition to the paper. The writer is inclined to believe that the majority of designers will prefer to calculate internal moments as suggested by Mr. Jameson rather than as presented in the paper. Calculating the simple beam moments and correcting them for the effect of total end moment will resemble more closely the usual analysis of beams with fixed loading. The moment factors for a simple beam would be more instructive if they were plotted as coefficients of the moment $F\,a$ rather than as coefficients of $F\,\lambda$. Thus, the coefficients of $F\,a$ would reflect completely the effect of the foundation rigidity, and the curve for $\alpha = 0$ would be the well-known parabola for uniform loading. A complete set of design information would consist of three families of graphs for z, V, and M for a simple beam and a similar set of three graphs for the effect of a unit end moment.

---

[20] Associate Engr., Bureau of Aeronautics, Navy Bldg., Washington, D. C.

Mr. Parmé has recalled the close relationship between the floating slab theory of an elastic slab on an elastic foundation and an element of an elastic cylinder. The use of moment distribution in cylindrical tank analysis is illustrated in the reference[13] given by Mr. Parmé. The writer has used moment distribution for the analysis of cylindrical tanks and elastic base slabs for several years. In cylindrical tanks, bottom moments and shears generally produce little effect at the top of the cylinder. However, many thick-walled water tanks, built in army camps, designed to conserve steel, showed considerable effect at the top due to bottom moments. It is rather uncommon to find a base slab in which the edge moments and shears produce only a negligible effect at the center.

Mr. Eremin's analysis shows the error of neglecting the effect of foundation rigidity on the stiffness of the base slab. The sine curve gives a reasonable shape for the reaction distribution. The minimum ordinate of the diagram should be associated in some rational manner with the foundation rigidity.

The purpose of the paper is to demonstrate that an indeterminate structure resting on an elastic foundation can be analyzed by moment distribution. The mathematical development was planned and completed with this objective in view. An illustration of the analysis of an indeterminate structure was given in Fig. 8. As a matter of secondary importance, since the solution is well known (as explained by Mr. Cummings), it was shown in Fig. 7 that a determinate structure can also be analyzed by using the same graphs and formulas that are required for the indeterminate case. This inclusion of a determinate case appears to have sidetracked the attention of Mr. Cummings from the primary accomplishment of the paper. The methods of analysis which he presents as being preferable deal only with the determinate case.

Mr. Cummings opens and closes his discussion with statements that "The author's approach to the problem under discussion appears to be more complex than is necessary" and "these mathematical solutions [existing solutions] are much more concise and elegant than the cumbersome procedure used by the author." His entire discussion, which lies between these two criticisms, deals only with a determinate structure and offers the designer no suggestion as to how an indeterminate structure might be analyzed. This fact is particularly significant for the reader who may wish to evaluate the validity and applicability of the criticisms. A more concise method of analyzing the indeterminate case is not to be found in so far as the writer is aware.

Mr. Cummings explains that the complicated functions of the paper arise because the author "has chosen to set" two of the integration constants, C and D, equal to zero. Actually, the solution is unique and the writer had no personal choice in the matter. Eqs. 49, offered by Mr. Cummings in defense of his claim actually disprove it because they show immediately that C and D must be zero. To prove this statement, it is desirable to simplify Eq. 49*d*. With the assistance of Eq. 49*c*, Eq. 49*d* may be reduced to:

$$\left[\frac{d^3z}{dx^3}\right]_{x=0} = 0 \qquad (53)$$

Eq. 53 is recognizable as a condition of symmetry which should be used in place of Eq. 49*d* since it involves considerably more mathematical labor to integrate than to differentiate a sum of products of transcendental functions. Also the third derivative must be computed for use in Eq. 49*c* and thus is available for use in Eq. 49*e*. Substitution of the general formula for z, as given by Eq. 17, into Eqs. 49*a* and 53, which are equations of symmetry, or into Eqs. 49*a* and 49*d* gives,

$$C + D = 0 \qquad (54a)$$

and

$$C - D = 0 \qquad (54b)$$

Eqs. 54 show that C and D are zero. The remainder of the solution for the determinate case consists of solving Eqs. 49*b* and 49*c* simultaneously for A and B, corresponding to the procedure used by the writer for solving the indeterminate case. For that particular determinate case in which V_a and M_a are known, Eqs. 49*b* and 49*c* are correct. For the indeterminate case which should be solved by moment distribution, Eqs. 19*a* and 19*b* give a correct statement of the boundary conditions. There are six possible ways of stating the boundary problem, since there are six combinations of four things taken two at a time. In all six cases C and D must be zero because of symmetry.

The effect of thrust on bending was purposely omitted to conform to standard practice in the design of rigid frames of reinforced concrete. Since the eccentricity of the thrust is several hundred times the maximum relative deflections, the effect of thrust on bending is commonly regarded as negligible in the design of concrete frames. This matter becomes of considerable importance in thin, flexible structures such as those used in the monocoque construction of airplanes.

Mr. Cummings expresses concern over the tensile reactions which may occur at the center of the slab. He states that "the entire mathematical analysis is useless if there is any part of the slab for which the value of z is negative." The writer considers this to be a serious exaggeration of a trivial matter. Tensile reactions occur with flexible beams on rigid foundations, that is, with $\alpha > 2$. Such a case is a concrete conduit on a limestone foundation when the rock has a modulus of elasticity of several million pounds per square inch. The negative calculated reactions must exceed the weight of the slab before a theoretical error occurs. Actually, on such rigid foundations, floating slab theory can only give an approximate value for maximum moments. The error is not due to the slight tensile reactions at the center of the span but, rather, to the need for a more accurate influence relationship between deflections and reactions than that given by floating slab theory.

This paper should be regarded as an introduction to the use of floating slab theory in indeterminate structures. Extension of the concepts to more complicated cases remains for future study.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2221

ILLINOIS WATER LITIGATION, 1940-1941

BY LANGDON PEARSE,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. WILLEM RUDOLFS, L. R. HOWSON, A. M. BUSWELL, LLOYD M. JOHNSON, H. P. RAMEY, AND LANGDON PEARSE.

SYNOPSIS

In 1940–1941, hearings followed a petition to the U. S. Supreme Court by the State of Illinois for a temporary increase in diversion from Lake Michigan. The inquiry was directed to ascertain the effect of odors from a large pool in the heart of Joliet, Ill., on health, the remedial measures available, and the condition of the sewage treatment works of The Sanitary District of Chicago. The Special Master held that no menace to health was proved and that no remedial measures were available as effective or as promptly as the proposed completion of the sewage treatment works of the Sanitary District. The U. S. Supreme Court denied the petition.

GENERAL

Considerable interest was aroused by the petition of the State of Illinois, dated January 11, 1940, for a temporary relaxation of the diversion order restricting the flow of water from Lake Michigan to relieve the condition existing in the upper Illinois Waterway, particularly at Joliet (see Fig. 1). The litigation was somewhat unique in that it involved the question: "When is pollution a menace to health" rather than the well-defined problem of the abatement of a nuisance. Curiously the attorneys on both sides admitted that no cases could be cited as legal precedent.

Prior to July 1, 1930, the War Department Permit of March 3, 1925, limited the diversion to an annual average of 8,500 cu ft per sec, with an instantaneous maximum of 11,000 cu ft per sec. This permit expired on December 31, 1929. A new permit was then issued authorizing an annual average diversion of 7,250 cu ft per sec, or such lesser annual average diversion as would restrict the average annual flow measured at Lockport, Ill., to 8,500 cu ft per sec, until July 1, 1930. The actual diversion in 1929 was 7,699 cu ft per sec, and in 1930, 6,658 cu ft per sec.

NOTE.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

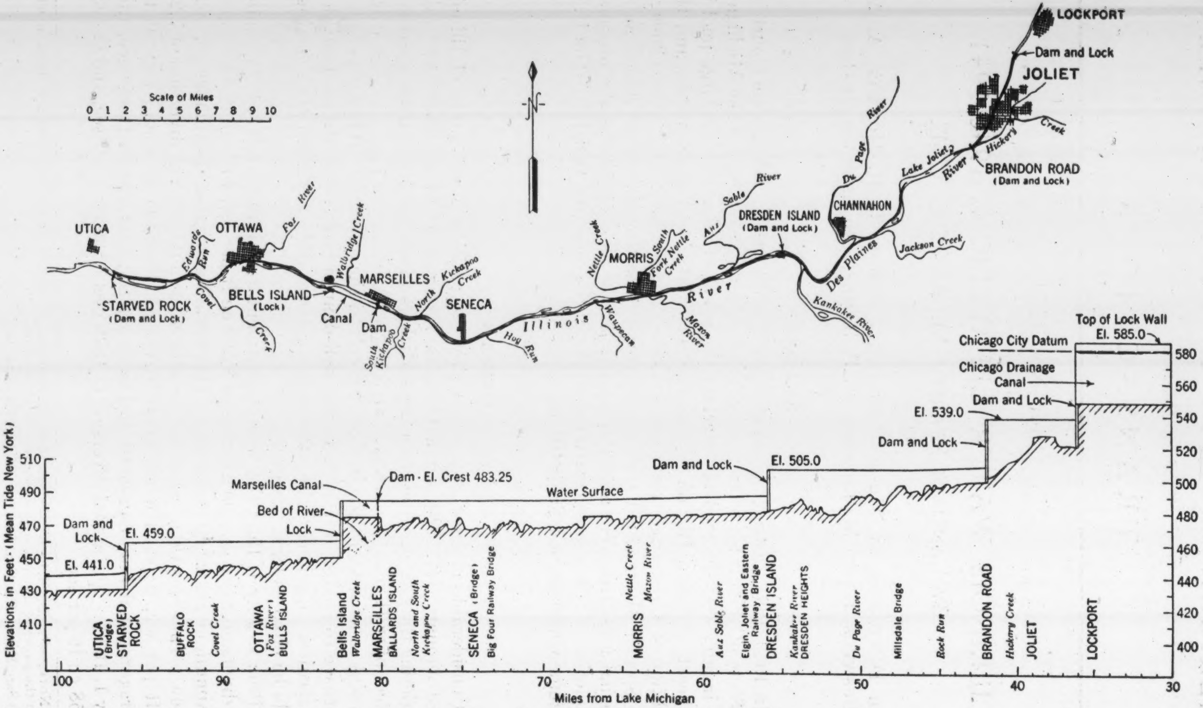

LOCKPORT
Dam and Lock
JOLIET
Hickory
Creek
BRANDON ROAD
(Dam and Lock)
Lake Joliet
River
Du Page River
CHANNAHON
Jackson Creek
Des Plaines
Kankakee River
DRESDEN ISLAND
(Dam and Lock)
Aux Sable River
MORRIS
South Fork Nettle Creek
Nettle Creek
Mazon River
Waupecan
River
Illinois
Hog Run
SENECA
North Kickapoo Creek
South Kickapoo Creek
MARSEILLES
Dam
Canal
Walbridge Creek
BELLS ISLAND
(Lock)
Fox River
OTTAWA
Edwards Run
Covel Creek
STARVED ROCK
(Dam and Lock)
UTICA
Scale of Miles
0 1 2 3 4 5 6 7 8 9 10
N
Top of Lock Wall
El. 585.0
Chicago City Datum
Chicago Drainage Canal
Dam and Lock
El. 539.0
Dam and Lock
Dam and Lock
El. 505.0
Dam - El. Crest 483.25
Water Surface
Marseilles Canal
Bed of River
Lock
Dam and Lock
El. 459.0
El. 441.0
Elevations in Feet - (Mean Tide New York)
510
490
470
450
430
410
580
560
540
520
500
480
460
440
420
400
UTICA (Bridge)
STARVED ROCK
BUFFALO ROCK
Covel Creek
OTTAWA (Fox River)
BULLS ISLAND
Bells Island
Walbridge Creek
MARSEILLES
BALLARDS ISLAND
North and South Kickapoo Creek
SENECA (Bridge)
Big Four Railway Bridge
MORRIS
Nettle Creek
Mazon River
Aux Sable River
Elgin, Joliet and Eastern Railway Bridge
DRESDEN ISLAND
Kankakee River
DRESDEN HEIGHTS
Du Page River
Millsdale Bridge
Rock Run
BRANDON ROAD
Hickory Creek
JOLIET
LOCKPORT
100
90
80
70
60
50
40
30
Miles from Lake Michigan

FIG. 1.—GENERAL PLAN AND PROFILE, ILLINOIS WATERWAY

This litigation was a result of the decree of the Supreme Court of the United States issued April 21, 1930, which provided that the annual average diversion at Chicago, Ill., from Lake Michigan should be reduced in three steps:

Step	Date	Total diversion (cu ft per sec)
1	July 1, 1930	6,500
2	December 31, 1935	5,000
3	December 31, 1938	1,500

These allowances are in addition to the domestic pumpage of Chicago, which amounts to approximately 1,700 cu ft per sec. A controlling works was also to be built at the mouth of the Chicago River on or before December 31, 1935. The diversions were made on schedule and the controlling works were built.

The 1930 decree simply limits the flow of water. It does not specify the type of sewage treatment works to be built or when they shall be completed, although this was discussed by the Special Master, the Hon. Charles E. Hughes, in his report on Re-Reference.

The Court did not foresee the depression which retarded tax collections in 1931–2–3 and thereby stopped The Sanitary District of Chicago from selling bonds because of the technical default of its then outstanding issues. Consequently the timing of the completion of the sewage treatment works gradually fell out of step with the reduced diversion after 1935, with resulting difficulty in 1939, after the final reduction was made. The Sanitary District took the position that it should follow the decree to the letter. After the first two steps the sewage treatment works still provided enough cushion to show some improvement in the condition of the upper Illinois Waterway until 1938, when the third step reduced the flow so greatly that conditions were created that were worse than at any time in the preceding 30 years.

The late Chief Justice William Howard Taft had instructed the Special Master to determine the diversion required for the Port of Chicago. This was interpreted to include the Main Channel at Lockport. Thus, by the Taft instructions, conditions below Lockport were ignored in preparing the Report on Re-Reference and the 1930 decree, because, when Mr. Taft was Chief Justice, the Des Plaines River from Lockport to its junction with the Kankakee River, and thence the Illinois River to Utica, Ill., were not navigable waters of the United States. By the River and Harbor Act of July 3, 1930, the entire waterway from the Chicago River to Utica became a navigable waterway of the United States, more than two months after the decree.

The "Illinois Waterway" is the term now applied to the waterway from Lake Michigan to the Mississippi River. In July, 1930, the United States took over the upper waterway from Utica to Lockport, then under construction by the State of Illinois, and completed the work (7).[2] In the spring of 1933, the waterway was opened for through navigation. In this plan of slack-water navigation a number of pools were created by the dams at Brandon Road, Dresden Island, Marseilles, Ottawa, Peoria, and La Grange, all in Illinois. This paper is concerned principally with the happenings in the upper pool, known as the Brandon Pool (Fig. 1), which extended from Lockport to the Brandon Road dam, a distance of about 4.84 miles, through the heart of Joliet, a city of 42,365 population (1940 census). The pool varies in width from 200 ft to 1,400 ft, with a maximum depth of 27 ft. Prior to the closing of the Brandon Road dam in 1933, the flow passed through Joliet down river with considerable rapidity, and consequently with a high rate of re-aeration. To the end of 1938, however, little was heard of conditions in the pool, although sludge was accumulating in it.

---

[2] Numerals in parentheses, thus: (7), refer to corresponding items in the Bibliography, in the Appendix.

With the reduced diversion, in 1939, conditions changed rapidly for the worse. The 5-day biochemical oxygen demand (B.O.D.) at Lockport rose rapidly. The dissolved oxygen disappeared not only at Lockport (Table 1), but upstream for 30 miles into the South Branch of the Chicago River at Damen Avenue (Table 2). The conditions along the Main Channel were the worst known for more than 30 years.

TABLE 1.—DISSOLVED OXYGEN AND BIOCHEMICAL OXYGEN DEMAND AND DISCHARGE AT LOCKPORT, ILL.

	1938				1939				1940			
			PPM				PPM				PPM	
Month	Discharge (cu ft per sec)	Temperature (degrees C)	Dissolved oxygen	5-Day B.O.D.	Discharge (cu ft per sec)	Temperature (degrees C)	Dissolved oxygen	5-Day B.O.D.	Discharge (cu ft per sec)	Temperature (degrees C)	Dissolved oxygen	5-Day B.O.D.
Jan.	6,408	1.5	7.4	22.6	2,911	5.0	0.5	28.5	2,949	1.5	1.8	14.8
Feb.	7,422	3.5	7.0	21.0	3,989	3.5	2.2	26 6	2,802	5.0	0.4	23.0
Mar.	7,607	6.0	5.6	19.4	3,200	6.5	0.8	25.0	3,167	5.5	1.0	23.2
Apr.	7,686	10.5	3.2	16.3	2,718	13.5	0.1	31 4	3,026	10.0	0.2	26.6
May	6,311	15.0	0.5	17.6	2,623	17.5	0.0	42.3	3,344	14.5	0.1	25.3
June	6,701	19.0	0.3	18.8	4,227	21.5	0.0	37.8	2,876	21.0	0.0	27.9
July	6,814	23.0	0.1	18.5	2,880	24.5	0.0	26.5	3,609	23.5	0.0	18.7
Aug.	6.740	23.0	0.1	14.6	2,902	25.0	0.0	26.2	3,923	23.0	0.1	15.2
Sept.	7,247	20.5	0.3	13.4	2,826	24.0	0.0	23.0	3,125	22.0	0.1	14.3
Oct.	5,512	17.0	0.2	18.6	3,022	18.0	0.0	18.1	3,186	18.5	0.2	14.1
Nov.	5,861	10.5	1.2	18.5	2,820	11.5	0.4	13.8	2,824	11.0	0.8	13.6
Dec.	5,469	4.5	4.3	21.8	3,469	8.0	1.3	13.2	4,996	4.0	4.9	24.7

TABLE 2.—COMPARISON OF CONDITIONS IN AUGUST; MAIN CHANNEL AND SOUTH BRANCH OF THE CHICAGO RIVER

(Units Are Parts per Million)

Description	Chicago Avenue		Damen Avenue		Cicero Avenue		Summit	
	1938	1939	1938	1939	1938	1939	1938	1939
Dissolved oxygen	2.2	0.6	5.5	0.0	2.7	0.0	1.1	0.0
Five-day B.O.D.	6.3	8.2	6.8	32.7	8.3	33.0	13.5	34.2

In the summer of 1939, the outcry from Joliet became so great that on January 11, 1940, the State of Illinois applied for a modification of the 1930 decree to permit a temporary increase of the diversion to 5,000 cu ft per sec in addition to domestic pumpage, until December 31, 1942. After a hearing on this application, the Court, on April 30, 1940, ordered that the petition of the State of Illinois and the return of the Complainant States be referred to Monte M. Lemann as a Special Master

> "* * * with directions and authority to make summary inquiry and to report to this Court with all convenient speed with respect to the actual condition of the Illinois Waterway by reason of the introduction of untreated sewage, and whether, and to what extent, if any, that condition constitutes an actual menace to the health of the inhabitants of the Complaining Communities, and also with respect to the feasibility of remedial or ameliorating measures available to the State of Illinois without an increase in the diversion of water from Lake Michigan."

(In this paper the petitioner, the State of Illinois, is referred to as "Illinois"; the Complainant States, Michigan, Minnesota, New York, Ohio, Pennsylvania, and Wisconsin, are referred to as "Opposing States"; and the Special Master, Mr. Lemann, is referred to as the "Master."

The hearings were held in Chicago over a period of months. The case was argued in New Orleans, La., in February, 1941, before the Master, who rendered a report to the Court on March 31, 1941. On May 26, 1941, the Court denied the petition.

The original petition (January, 1940) requested a temporary increase in diversion to 5,000 cu ft per sec until December 31, 1942. After the completion of the hearings the petition was modified. In its revised petition, Illinois requested a temporary increased diversion during the warmer months of 1941 and 1942 sufficient to maintain a minimum of 1 ppm of dissolved oxygen in the Brandon Pool. The expected need for increased diversion was set up as follows:

Month	1941	1942
April	1,000	....
May	3,000	2,000
June	5,000	4,000
July	6,000	5,000
August	5,000	3,000
September	3,000	1,000
October	1,000	....
Annual average increase	2,000	1,250

The annual average increase in diversion thus requested was 2,000 cu ft per sec in 1941 and 1,250 cu ft per sec in 1942. The earlier records in the case and the findings of Special Master Hughes indicate that the lowering effect of such temporary diversions would not be more than ½ in. in 1941 and ⅓ in. in 1942 in Lake Michigan, Lake Huron, and Lake Erie.

Minimum Dissolved Oxygen

F. W. Mohlman and the writer testified that in the absence of sludge deposits at least 1 ppm of dissolved oxygen is necessary to prevent nuisance. Referring to New York vs. New Jersey, 256 U. S. 296, 311, decided in 1921, the Master expressed his belief that the weight of opinion is that the presence of any dissolved oxygen is sufficient. In that case, however, the Court held that

25% to 50% of saturation is the dissolved oxygen necessary to prevent offensive odors from decomposition of organic matter deposited in the body of water under consideration. As Lake Michigan contains about 8 ppm dissolved oxygen in the summer months, this yardstick would require 2 to 4 ppm of dissolved oxygen in the warmer months in Brandon Pool.

The evidence shows that the most important factor responsible for the foul conditions of the Main Channel and Brandon Pool is undoubtedly the deposits of putrefactive sludge that formed since the diversion was reduced on January 1, 1939. In Brandon Pool, this deposit has accumulated since 1933, from residual solids in the effluents of the various works of the Sanitary District, from raw solids, the discharge of sludge at Stickney (the site of the West and Southwest works of the Sanitary District), and from the storm-water solids.

The accretions of prior years and the preceding winter, which lay dormant through cold weather, exerted a potent effect with the onset of warmer weather and a water temperature of 62° F or higher.

The engineering witnesses on both sides of the case agreed that such sludge deposits are potent in their avid demand for oxygen, and are the most important factor in the production of the foul condition in the Main Channel and Brandon Pool. All agreed that the discharge of settling suspended matter, whether in sewage partly treated or as sludge, should be stopped as soon as possible. This is what the Sanitary District program contemplates doing. Thus the Opposing States and Illinois were agreed that the solids must be kept out of the channels of the Sanitary District and that the sewage must be treated to as high a degree as practicable, as quickly as possible.

The conditions in the Main Channel and Brandon Pool during 1939 and 1940 continued through 1941 and 1942 and will continue thereafter although on a gradually diminishing scale, until the sewage treatment works of the Sanitary District are completed.

The sludge deposits require a long time to lose their putrefactive character or potency. Mr. Mohlman testified that a test of the oxygen demand of sewage sludge overlaid by Illinois River water showed consumption of oxygen for more than 1,100 days. For the first 400 days, this was confirmed by W. Rudolfs, M. Am. Soc. C. E. (1). The results show that during the first summer, for a period of 120 days, the loss of oxygen demand may range from 50% to 55%. During the winter the sludge lies dormant. The following spring possibly 20% to 25% more of the B.O.D. is destroyed. Reference was made to studies of Karl Imhoff, M. Am. Soc. C. E. (2), and the report of the Committee on Sewage Disposal of the Engineering Board of Review (3) which found that the sludge deposits in the Upper Illinois River practically doubled the effect of pollution during the summer.

The condition of the waterway, from Damen Avenue in Chicago to Brandon Road Dam below Joliet, was bad in 1939 and somewhat better in 1940, because of the increased treatment due to the starting of the Southwest works in the summer of 1939. By April, 1940, all the sewage in the Sanitary District was picked up for treatment in its plants except for about 70 mgd and by October, 1940, for about 35 mgd.

From testimony of laymen and experts, there is no question but that hydrogen sulfide was set free. There was controversy over the concentration in the atmosphere, the possible zone of effect, and the presence of other ingredients such as mercaptan, indol, skatole, and cadaverin. One expert witness, C. W. Muehlberger, indicated that persons might become accustomed to hydrogen sulfide but not to fecal odors.

The Health Problem

The proceedings before the Master were clearly divided into three parts, considering—

1. Complaints and the health conditions along the Waterway;
2. Status of sewage treatment by the Sanitary District and the outlook for the summers of 1941 and 1942; and
3. Available remedial or ameliorating measures.

Illinois contended that the present condition of the Illinois Waterway in the Joliet-Lockport area constitutes not only an actual menace to health but also a definite injury to health. Thus, whether or not the Waterway caused the effects alleged to the health of the inhabitants is the initial question in determining whether the conditions, as disclosed by the proof, constitute an actual menace within the meaning of the Court. The evidence shows a two-fold health problem:

(*a*) The actual detriment to health caused by the inhalation of gaseous odors emanating from the Waterway; and
(*b*) The potential hazard, danger, or menace to health occasioned by the present highly polluted and foul condition of the Waterway.

The Opposing States contended

> "* * * that the phrase 'actual menace to health' as used in the order of the United States Supreme Court means that the effects on the health of the people residing near the Waterway must be real and exist in fact, as opposed to any potential, speculative, hypothetical views on what might happen, unsupported by any real cases or actual happenings."

Thus the Opposing States maintained that the evidence falls far short of establishing a menace to health and at most shows only annoyance, discomfort, and inconvenience suffered by some people.

Complaints of Lay Witnesses at Joliet.—The complaints as to the Waterway emanated entirely from residents of Joliet and Lockport, and chiefly from those in Joliet residing alongside the pool and near it. The population of Joliet was reported to be 42,365 in 1940. The Health Commissioner of Joliet estimated 50,000 people were affected, living in and around Joliet. Through Joliet, the Waterway is crossed by eight bridges.

No water from the Waterway is used for drinking purposes. There is no swimming in it. All complaints were based on the offensive odors from the pool, and the effects ascribed to them.

A total of 127 witnesses testified at Joliet, made up as follows:

Lay residents of Joliet	81
Physicians residing in Joliet	15
Nurses and employees of St. Joseph's Hospital	24
Lay residents of Lockport	7
Total	127

It was stipulated that if 200 additional witnesses were called, their testimony would be substantially the same. Three fourths of the witnesses resided within four blocks of the canal, and more than half within two blocks.

The complaints generally were that the pool in the hot summer months of 1939 gave off extremely offensive odors, which in many cases caused nausea, lack of appetite, or inability to sleep. The character of the odors was described variously as the odor of sewers or outhouses or of rotten eggs. Some called it a "stink," others a "terrible" or "unbearable" odor.

All the witnesses agreed that the pool had always given off objectionable odors, but in 1939 the odors were much more offensive than theretofore. The weight of the evidence was that in 1940 the conditions were somewhat better.

The nurses at the hospital (one and one-half to two blocks from the pool) complained of the odors. Many of the patients suffered nervousness, nausea, vomiting, sleeplessness, and headache. Some of the patients were anxious to go home early. However, the average period of stay during 1939 was about one day less than the average for general hospitals in the United States.

The superintendent of grade schools at Lockport (eight blocks from the Waterway) and the principal of an elementary school at Joliet (600 ft from the Waterway) testified as to the type of odor and its nauseating effect on the children in 1939, and to a lesser extent in 1940.

The percentage of sick leaves of the employees in the U. S. Engineer office from Lockport to Starved Rock was somewhat higher in 1939 than in 1938. However, it was but little less than in the Chicago area. In 1940, the percentage in the Joliet area was less than in 1939.

The Health Commissioner of Joliet received hundreds of complaints in June, July, and August, 1939. He stated that there was danger of an epidemic from flies and mosquitoes alighting on the scum; that the stench was nerve wracking; and that the odorous gases irritated the mucous membrane of the nose, throat, and sinus. However, the health conditions in Joliet were good. The only difference in 1939 was the odor.

The other fourteen Joliet physicians considered the pool a menace to health for similar reasons. One was a government surgeon who noted skin diseases attributable to the foul water. Another noted an increase in impetigo along the Waterway from year to year.

The Master concluded that no cases of specific illness clearly attributable to the pool were cited by the physicians.

The statistics on reportable diseases in Illinois show that the case rate per 100,000 inhabitants in 1939 was less in Will County (in which Joliet lies) than the average for the state except for poliomyelitis, malaria, and influenza. In

1939, the death rate was lower in Will County for all reportable diseases than the average for the state. The typhoid rate in Will County in 1939 was 0.9 per 100,000 as compared with an average of 1.8 for the years 1930 to 1938, and a rate of 1.4 for the state. The statistics for the cases and deaths from typhoid show lower rates for counties adjoining the Waterway than for those that do not.

Testimony by Opposing States on Health Conditions.—Casual tests on August 3, 1940, disclosed no trace of hydrogen sulfide in the air over the pool. The bacterial tests on July 16, 1940, disclose only the usual bacteria found in air. Various experts testified that there was no possibility of an epidemic in the adjoining community due to the transmission of germs or bacteria by air from the Waterway. On September 17, 1940, the number of bacteria found over the pool was about one tenth that of other stations. On September 28, 1940, samples of water from the pool showed hydrogen sulfide from 0.1 to 0.2 ppm with an air temperature ranging from 34° to 74° F.

W. H. McNally, M. D., a toxicologist, testified that hydrogen sulfide in the air, concentrated as low as 50 ppm, may produce nausea. A concentration of 100 ppm may cause local irritations and depression of the nervous system. He doubted if such a concentration could occur in the atmosphere over the pool. A concentration of 25 ppm might discolor paint. He quoted Alice Hamilton and Philip Drinker as stating that not more than 20 ppm should be tolerated in industry. The sensitivity of individuals varies. He knew of no recorded instance of damage to health from hydrogen sulfide in a waterway like the Illinois Waterway.

All the physicians, health authorities, and sanitary experts (nine in all) who appeared as witnesses for the Opponents agreed that the Waterway did not constitute a menace to the health of persons living along it.

Testimony for Illinois on Health Conditions.—Mr. Muehlberger, an expert toxicologist, testified that the complaints of nausea, headaches, insomnia, and loss of appetite "may have been caused by hydrogen sulfide." He could not refer to any record in medical literature where an individual was overcome in the open air from a polluted waterway. Upon exposure to more than 20 ppm but less than 50 ppm, the average person would complain of irritation of the mucous surfaces of the eyes, nose, and throat. As stated, decomposing sewage produces organic sulfide compounds, including indol, skatole, mercaptan, and cadavrin, all having offensive odors. Mr. Muehlberger considered the conditions of the Waterway a very definite menace to the health of the inhabitants in the region of Joliet. People may not have died or required hospitalization, but they may have been uncomfortable and unhappy. They are entitled to a decent living condition with good, healthy air. Any contamination of the air is a menace, if a sound state of health is upset. Mr. Muehlberger testified that the odor of the pool in 1939 irritated his throat and nasal passages. However, he had noticed odors in prior years.

A. J. Carlson, M.D., a physiologist, testified that on October 13, 1940, "the present condition is more than a menace to health, it is inimical to health." His conception of a menace to health was typified by conditions which, through accidents or any ordinary course of human events, can lead to injury to health. He regarded such a polluted pool as a menace even if the water is not used for drinking or cooking or bathing. In his opinion, the fact that the communicable disease rate in Joliet is no higher than in other places is no factor whatever in determining whether the pool is a menace to health. Either the inhabitants of Joliet are lucky or the medical statistics are not what they are supposed to be.

Dr. Carlson stated that the Waterway has been a menace since 1934, but that now the menace has been increased. He used the words "menace to health" in a scientific sense as a theoretical danger which cannot be measured in any statistics or in any outward observation by a physician. The restoration of the diversion to 5,000 cu ft per sec was a lessened menace. A. C. Ivy, M. D., a physiologist, corroborated Dr. Carlson, holding that the disturbances reported due to the odor showed an unhygienic condition working as a detriment to the health of the people so affected. He knew of no recorded case of an individual being overcome by hydrogen sulfide from a waterway. H. A. McGuigan, M. D., a pharmacologist, agreed with Drs. Carlson and Ivy, pointing out that hydrogen sulfide was not the only odor in the air from the Waterway.

General Authorities on Sewage Odors

Reference was made to a report of the American Public Health Association which stated (4):

> "A long prevalent theory that sewage odors are directly responsible for disease has been definitely refuted. It is now realized that the physiological effect, if any, is indirect. Odors, by causing worry, loss of sleep, loss of appetite, etc., may be a contributory cause of ill health, and certainly cause discomfort. The courts, in general, have held that the creators of an odor nuisance are responsible therefor, and as far as municipal sewage disposal is concerned, the odor hazard appears to have been shifted from the public health to the public pocketbook."

M. J. Rosenau, M. D., was quoted (5):

> "While odors may be unpleasant, they are not known to seriously influence health. Contrary to common opinion, they are not by any means a reliable sign of danger. * * *
>
> "The effect of odors upon health is not well understood. * * * Odors influence the nervous system in various ways; some stimulate, others depress."

Condition of Waterway in Earlier Years

In weighing the claims of Illinois that the situation in Joliet and Lockport presents a menace to health which makes imperative an increase in diversion, the Master assembled all the available references to the earlier conditions in the Canal and Des Plaines River, which he cited as evidence of gross pollution. This analysis, however, failed to emphasize the changes brought about by the treatment works of the Sanitary District, and the marked improvement in the upper Waterway to the end of 1938. The conditions he cited as of 1872, 1891, 1911, 1919, 1921, 1922, and 1927 had been radically changed for the better.

CONCLUSIONS AS TO HEALTH CONDITIONS

For many years there have been odors from the Des Plaines River at Lockport and through Joliet. In 1925 when the river flowed over rapids at Joliet, the odors were offensive. From 1930 to 1938 conditions at Joliet improved, notwithstanding the reductions in the flow from Lake Michigan because of the added treatment which the Chicago sewage gradually received. The people at Joliet had become accustomed to this improvement. When the flow from Lake Michigan was reduced on December 30, 1938, conditions retrogressed and complaints ensued from the residents, who compared the 1939 conditions with those prevailing in 1938 and immediately prior to 1938. They had either forgotten the conditions prevailing prior to 1930, or, not unnaturally, felt that the improvement should continue. The nuisance conditions in 1939 were much worse than in 1938. They were not as bad in 1940 as in 1939.

The Master stated that this summary disclosed the facts as he found them. In deciding whether these facts constitute a "menace to health" within the meaning of the instructions of the Court, he indicated he lacked a certain guide. The dictionary definitions of menace he considered of no avail, because the Court itself will determine what it meant.

The Master concluded that the Court did not use the expression "menace to health" in the sense which Drs. Carlson and Ivy ascribed to it. If the Court had so intended, it would not have been necessary to refer the case to a Master, because the facts were clear in the papers in the case. If the test be that urged by Drs. Carlson and Ivy, it is clear that a menace would be found to exist. Accepting this definition, the increase of the flow to 5,000 cu ft per sec would not eliminate the menace. Within the suggested definition the stream would continue a menace as long as any untreated sewage is discharged into it.

The Master then stated that

> "It is obvious that the presence of untreated sewage in an open stream is not in accordance with proper standards of sanitation and should be abated. * * *
>
> "The record leaves no reasonable doubt as to the safety of the water supply of Joliet and Lockport. Nor can the effect of odors upon invalids and persons of less than average health be accepted as any test of health menace; even in cases of private nuisance the test is the effect of the acts complained of upon persons of ordinary sensibilities and in normal health. In the present case it is Illinois itself which is creating the nuisance of which it complains and of which it seeks to be relieved by water which has in effect been adjudged by the Court to belong to the opposing States. * * *
>
> "My conclusion is that the facts proven do not establish any menace to the health of the inhabitants of Joliet and Lockport or elsewhere along the Waterway requiring an increase in diversion in water from Lake Michigan."

PRESENT STATUS OF SEWAGE TREATMENT BY SANITARY DISTRICT

The Sanitary District comprises 442 sq miles in area, including Chicago and 59 other municipalities. H. P. Ramey, M. Am. Soc. C. E., testified that this is divided into four main projects, with sewage treatment works as specified:

1. North Side Works.—This is an activated sludge plant, with a present average capacity of 250 mgd. It went into service on October 3, 1928. The effluent goes to the North Shore Channel. The excess sludge and the preliminary settling sludge are pumped 17.5 miles to the Southwest works for ultimate disposal.

TABLE 3.—PROGRESS IN THE TREATMENT OF SEWAGE AT

Year	(a) Flow (Yearly Averages, in Million Gallons Daily)						(b) Record of Sludge and Grit,			
	Des Plaines River (1)	Calumet[b] (2)	North Side[c] (3)	Southwest (4)	West Side (5)	Grand total (6)	Des Plaines River (7)	Calumet[b] (8)	North Side (9)	Southwest (10)
1928	7.5	32.4	28.0[c]	....	....	67.9	..	....	....	....
1929	4.3	58.8	58.8	....	....	121.9	413	5,587	13,764[c]	....
1930	3.1	53.8	139.4[c]	....	22.8	219.1	400	4,984	28,504[c]	....
1931	1.5	55.7	196.0	....	85.1	338.3	198	3,984	40,726	....
1932	Plant shut down October 15, 1931	54.7	200.3	....	125.7	380.7	Plant shut down October 15, 1931	4,086	34,658	....
1933		60.8	199.4	....	135.2	395.4		5,788	35,616	....
1934		52.1	193.5	....	128.4	374.0		5,849	39,264	....
1935		37.8[b]	199.4	....	135.2	372.4		4,175[b]	33,673	....
1936		62.7	204.6	....	197.6	464.9		14,387	36,427	....
1937		65.4	204.8	....	275.6	545.8		13,539	37,145	....
1938		61.8	200.1	....	305.7	567.6		11,055	40,387	....
1939		71.6	202.5	158.0[d]	237.8	669.9		10,277	40,288	28,373[d]
1940		67.2	188.2	313.0	361.5	929.9		11,178	37,658	70,020[e]
1941		65.5	208.3	343.0	472.5	1,089.3		10,785	37,361	61,909

[a] Annual average percentages. [b] The old Calumet Plant was shut down from May 9 to September 13,
Calumet plant. [c] The North Side plant was placed in operation October 3, 1928, and the North Branch
The remaining months of the year were averaged to extend the total to 12 months. In a six-month period
B.O.D. record for Calumet is for the settling in the old Imhoff tanks. After 1935 the record is for activated

2. Calumet Works.—This is an activated sludge plant, with a present average capacity of 136 mgd and a maximum of 200 mgd. Sewage was pumped first on December 3, 1935. Treatment with activated sludge began on December 16, 1935. The excess sludge is conditioned, de-watered, dried for burning, or sold for fertilizer.

3. West Side Works.—This is an Imhoff tank plant, designed for an average flow of 472 mgd and a maximum of 700 mgd. The sludge is pumped to open-air drying beds.

4. Southwest Side Works.—This is an activated sludge plant, nominally designed for an average flow of 400 mgd and a maximum of 600 mgd. The first battery of aeration tanks went into service on June 27, 1939. Sludge disposal apparatus has been in operation since August 28, 1939. On March 20, 1940, the Racine Avenue Pumping Station went into service, thus completing the pickup of sewage except for some minor sewers with a flow of about 70 mgd. By September 26, 1940, all but 35 mgd was connected.

The equivalent population of the Sanitary District is as follows (exclusive of the Corn Products Refining Company):

Population	1930	1940	1941
Human (U. S. Census)	3,901,569	3,962,514	3,962,514
Industrial equivalent	1,486,000	2,237,000	2,549,300
Total	5,387,569	6,199,514	6,511,814

The industrial equivalent population for 1930 was estimated. That for 1940 is the actual value for the period from April to December, inclusive. The peak month in 1940 was November, with a population equivalent of 3,065,000.

Expenditures.—To December 31, 1941, about $166,300,000 had been spent by the Sanitary District for the intercepting sewers, pumping stations, and treatment works. It was estimated that $11,756,900 was required to complete the program. The principal items to be built were at the West-Southwest works—namely, the aeration and final settling tanks and completion of the blower house and sludge handling plant.

THE MAJOR WORKS, THE SANITARY DISTRICT OF CHICAGO

IN TONS PER YEAR			(c) REMOVAL OF 5-DAY B.O.D.[a]				(d) REMOVAL OF SUSPENDED SOLIDS[a]				
West Side (11)	Grand total (12)	To channel (13)	North Side (14)	Calumet[b] (15)	West Side (16)	Southwest (17)	North Side[e] (18)	Calumet[b] (19)	West Side (20)	Southwest (21)	Year
....	....	....	....	....	....	....	....	....	....	....	1928
....	19,764	13,359	....	....	....	....	....	....	....	....	1929
2,805	36,693	25,072	92.0	....	....	....	89.1	....	....	....	1930
9,881	54,789	29,092	92.6	....	56.9	....	91.0	....	58.4	....	1931
15,115	53,859	28,203	91.7	31.3	42.0	....	91.3	56.8	51.2	....	1932
19,655	61,059	32,349	88.3	33.4	42.0	....	89.2	55.8	57.5	....	1933
15,796	60,909	32,764	89.4	38.7	47.5	....	90.5	54.7	61.4	....	1934
11,321	49,169	27,395	91.0	42.9[f]	54.5	....	90.2	54.0[f]	61.1	....	1935
18,993	69,807	36,644	92.2	83.5[f]	49.2	....	89.8	86.9[f]	52.9	....	1936
19,274	69,958	42,495	91.5	86.3	40.7	....	91.5	90.1	40.2	....	1937
29,976	81,418	44,198	92.2	83.1	47.4	....	93.0	90.2	54.5	....	1938
29,985	109,133	53,970	93.5	87.0	53.7	85.8[g]	92.4	90.6	63.0	88.5[g]	1939
33,077	151,933	20,224	94.6	90.7	50.0	67.3	92.4	90.1	58.0	73.4	1940
36,019	146,073	0	94.9	90.5	44.3	56.1	93.8	90.8	54.5	64.9	1941

in 1928. It suspended operations September 15, 1935, and beginning with 1936, records are for the new pumping station began operating June 28, 1930.[c] [d] The Southwest plant began pumping May 23, 1939. 28 mgd were pumped. [e] Complete treatment on approximately 60% of the total flow. [f] Prior to 1936 the sludge. [g] Six months.

PROGRESS IN TREATMENT

Table 3(*a*) shows that the flow of sewage treated increased from 67.9 mgd in 1928 to 929.9 mgd in 1940. Of the latter, 123 mgd was reported untreated in 1940. The total sludge and grit discharged to the Main Channel aggregated 53,970 tons in 1939 (see Table 3(*b*)). In 1938 it was 44,198. On and after September 16, 1940, no sludge and grit was so discharged, because of the provision of some temporary lagoons in addition to the permanent sludge handling

TABLE 4.—DRY VOLATILE SOLIDS DISCHARGED INTO THE CANAL
(Tons per Year)

Solids	1936	1937	1938	1939	1940
Sludge	21,432	24,613	22,450	33,865	14,937
Raw sewage	111,960	90,450	81,490	63,150	20,290
West Side effluent	12,180	20,070	19,160	14,969	28,305
Southwest effluent	0	0	0	3,674	12,940
Total	145,572	135,133	123,100	115,658	76,472

equipment. The efficiencies of the various plants are shown in Tables 3(*c*) and 3(*d*). The 1941 data are from actual records and were not before the Master.

The volatile solids entering the Main Channel decreased from 145,572 in 1936 to 76,472 in 1940 (see Table 4). Briefly for 1938, 1939, and 1940, the percentage reduction in B.O.D. was as shown in Table 5. The Southwest works lacks blowers and sludge handling equipment. Activated sludge treatment was given only to sufficient sewage to produce solids within the capacity of the dewatering and drying equipment. The remainder received preliminary settling treatment only.

TABLE 5.—PERCENTAGE REDUCTION IN B.O.D.

Plant	1938	1939	1940	1941
North Side	92.2	93.5	94.6	94.9
Calumet	83.1	87.0	90.7	90.5
West Side[a]	47.4	53.7	50.0	44.3
Southwest	....	....	67.3	56.1
Weighted average			67.8	60.6

[a] This is only an Imhoff tank plant.

Periods of Flow.—For a clearer understanding of the problem in the Main Channel and Brandon Pool, the time factors of flow must be visualized. The actual periods of flow (in hours) from Damen Avenue to Brandon Road dam, with the average velocities in the Main Channel, are shown in Table 6.

TABLE 6.—FLOW DATA, MAIN CHANNEL AND BRANDON POOL

Month (1939)	Flow (cu ft per sec)	DAMEN AVE. TO SUMMIT		SUMMIT TO LOCKPORT		BETWEEN BRANDON ROAD DAM AND:	
		Hours of flow	Miles per hr	Hours of flow	Miles per hr	Lockport (hr)	Damen Ave. (hr)
Jan.	2,910	30.4	0.26	62.5	0.35	19.0	112.5
June	4,227	24.0	0.33	44.0	0.54	13.0	81.0
June	7,891	11.0	0.72	22.0	1.01	7.0	40.0
June	8,311	11.0	0.72	20.5	1.08	7.0	38.5
....	2,200	62.4	0.13	78.8	0.29	24.8	166.0

PROBABLE CONDITIONS AT LOCKPORT AND JOLIET IN THE SUMMERS OF 1941 AND 1942

At the request of the Master, the experts on both sides submitted estimates on the probable conditions at Lockport and Joliet in the summers of 1941 and 1942. A principal factor was the oxygen demand of sludge deposits in the Main Channel and Brandon Pool. This was difficult to estimate.

The difference between the two sides was enlarged by an over-optimistic viewpoint of the Opposing States as to the date of completion of the remaining Sanitary District works. This viewpoint went so far as to assume that equipment could be delivered long prior to the dates set under contract and the works completed in 1941. As a matter of fact, under the present conditions (November, 1942) of National Defense, it is clear that the works cannot be completed in 1942, and unless priorities are granted, not until after the war. At least two years will then be required, after construction is actively resumed.

The Master concluded that the conditions in Brandon Pool would be better in 1941 and 1942 than in 1939. He noted that in the summer of 1938 there was only 0.2 ppm of dissolved oxygen, and that no special complaints were heard. However, he believed that there would be no dissolved oxygen in the pool in 1941 and that offensive odors were likely to recur.

AVAILABLE REMEDIAL MEASURES

The contention of Illinois was that the only immediate, feasible, remedial or ameliorating measure available for relief of the critical situation at Brandon Pool is an increased diversion of water from Lake Michigan.

The writer testified that a diversion of 5,000 cu ft per sec would improve the conditions over those existing prior to 1939, and that no temporary measures are available which can be applied quickly to obviate the conditions in Brandon Pool other than an increased diversion. Messrs. Calvert and Mohlman concurred. Messrs. A. M. Buswell and Enslow were confident that chlorination would provide more certain relief at Brandon Pool than the diversion of 3,500 cu ft per sec additional; yet Mr. Buswell admitted that dilution reduces the B.O.D. more certainly than chlorine.

Clarence R. Andrew, Assoc. M. Am. Soc. C. E., stated that an additional diversion would reduce the odors. The only two feasible methods known to him were the removal of the pollution at its source or the addition of sufficient water to prevent the odors. Dr. Carlson noted that an increased diversion would lessen both the odors and the menace to health.

As the witnesses for both sides agreed that the critical period of nuisance and health hazard occurs during the warmer months (from early April through October), and as no complaint or health hazard arises with the 1,500 cu ft per sec diversion in the colder months from November through March, the budgeting of the diversion appeared logical on a graduated scale, with a maximum in midsummer and tapering off into spring and fall, with a minimum in the winter.

Mr. Enslow stated the critical temperature of flow to be 62° F. As the temperature rises above this, bacterial action is stimulated, thus increasing gasification from decomposing organic matter. The temperature record of the water at Lockport, in degrees Fahrenheit, is as follows:

Month	1939	1940	Month	1939	1940
January.....	41°	35°	July........	76°	74°
February....	38°	41°	August......	77°	73°
March......	45°	42°	September...	75°	72°
April.......	56°	50°	October.....	64°	65°
May........	64°	58°	November...	53°	52°
June........	71°	70°	December...	46°	39°

Mr Ramey stated that since December 31, 1938, the Sanitary District has budgeted the water diverted from Lake Michigan in a manner to best meet the needs of sanitation, with a diversion limited to an annual average of 1,500 cu ft per sec. This includes the rainfall runoff from the Chicago and Calumet rivers, amounting to 550 cu ft per sec. There remains a dependable dry-weather diversion of approximately 1,000 cu ft per sec. In 1939, from May to October, the dry-weather diversion averaged 1,090 cu ft per sec.

For the protection of the water supply of Chicago, Mr. Ramey did not consider it safe to maintain a flow of less than 2,400 cu ft per sec in the Main Channel at Lockport. Domestic pumpage amounts to 1,650 cu ft per sec and the storm-water runoff equals 550 cu ft per sec, making a total of 2,100 cu ft per sec. This leaves only 300 cu ft per sec to come from Lake Michigan.

To provide greater flow during warmer months, dry-weather flows from 2,400 to 2,600 cu ft per sec were budgeted and attained in the cooler months of 1939 and 1940. Operating results can be improved but little with the present limit on the diversion.

Various remedial or ameliorating measures were proposed by the Opposing States. Illinois contended that these were unsound in principle, effective only to a negligible degree, and available too late to serve the emergency in 1941 and 1942. Illinois further believed that the practicability of the suggestions should be tested from the four standpoints of (*a*) proven reliability and soundness, (*b*) degree of relief effected, (*c*) the time required for installation, and (*d*) the cost.

The suggestions may be divided into temporary and permanent classes, as indicated in Table 7.

TABLE 7.—SCHEDULE OF REMEDIAL OR AMELIORATING MEASURES

No.	Program	Opposing States (temporary)	Illinois (permanent)
1	Chemical treatment at West Side works	✓	....
2	Activated sludge treatment for West Side Imhoff tank effluent	....	✓
3	Eliminate by-passing of Southwest preliminary settled effluent	✓	✓
4	Keep all sludge out of Main Channel	✓	✓
5	Chlorinate the flow in Main Channel	✓	....
6	Chlorinate the effluent of the West Side works	✓	....
7	Double the air used at the North Side works; increase the aeration period from 5 hr to 7 hr to produce 5 ppm of nitrogen as nitrate	✓	....
8	Build cascades at the controlling works at Lockport to aerate the flow above Brandon Pool	✓	....
9	Increase the diversion in summer, in accordance with the results of analyses, to provide a minimum of 1.0 ppm of dissolved oxygen in Brandon Pool	....	✓ (temporary)

The Opposing States urged that the first step to improve the Waterway would be to discontinue the discharge of sludge into it. This was actually done at the end of September, 1940. However, considerable putrescible material is carried over in partly treated effluents. Other suggested remedial measures follow.

Dredging, Draining, or Flushing Brandon Road Pool.—The pool is about 4.84 miles long, with a maximum width of 1,400 ft. Its surface area totals about 12,409,500 sq ft. The water depth varies from 10 to 20 ft. The original volume was approximately 151,000,000 cu ft, which was reduced by 1939 to 120,000,000 cu ft. Mr. Andrew of the U. S. Engineer office estimated about 1,250,000 to 1,500,000 cu yd of deposited material, from 2 to 10 ft deep. He stated that increasing the flow would not remove this deposit, but hydraulic dredging could do so, at a cost of between $500,000 and $700,000, in a period of from 6 to 9 months. Louis R. Howson, M. Am. Soc. C. E., thought the cost might be $400,000 and that any nuisance in lagooning the material could be controlled by chlorine.

Experts on both sides agreed that the denser material has largely lost its potency to cause nuisance, and that hydraulic dredging would remove chiefly material no longer the cause of offense. Mr. Howson doubted the practicability of such dredging as compared to chlorination and other methods. Mr. Mohlman and the writer regarded dredging as an impractical proposition.

Draining the Pool.—Draining the pool was regarded as impracticable by the U. S. Engineer office for a variety of reasons. The Chief of Engineers declined to allow it. From the discussion finally emerged a plan for permitting a flow of 10,000 cu ft per sec in addition to domestic pumpage for 10 days beginning December 2, 1940. To this both sides and the Court agreed.

Experimental 10-Day Flushing.—During the 10-day test beginning December 2, 1940, the total flow at Lockport averaged 9,973 cu ft per sec. The net diversion was 8,430 cu ft per sec. The level of the Brandon Pool was lowered 0.5 ft at first and later an additional foot. The flushing scoured out the Main Channel and deposited in the Brandon Pool more solids than were discharged (see Table 8). However, the solids washed out of the Pool were slightly higher in percentage organic matter than those which entered, as the incoming total solids contained 32.4% volatile matter and the outgoing solids contained 35.6%.

TABLE 8.—RESULTS OF EXPERIMENTAL 10-DAY FLUSHING OF BRANDON POOL

Description	TOTAL SUSPENDED MATTER		VOLATILE SUSPENDED MATTER	
	Tons	%	Tons	%
Delivered to pool	84,010	....	27,067	....
Discharged from pool	21,123	25.2	7,512	27.7
Remainder deposited in pool	62,887	74.8	19,555	72.3

During the test the velocities in the pool averaged from 0.47 to 0.55 ft per sec. The maximum velocities were from 1.33 to 1.58 ft per sec, and the minimum near the banks from 0 to 0.07 ft per sec. Mr. Mohlman estimated about 524,000 cu yd of material were swept in. All the witnesses agreed that in the Main Channel and Brandon Pool a considerable benefit was accomplished, as a whole, by removing sludge from the Main Channel. Mr. Mohlman doubted if this effect would offset the effect of the additional sludge deposited in the pool.

Chlorination of Waterway.—The Opposing States urged the use of chlorine to eliminate odors at Lockport and Joliet.

Mr. Enslow considered the sludge at Brandon Pool to create a bad situation, which would require from three to four times the chlorine necessary if the sludge deposits were removed. In his opinion there is no parallel case in existence for the use of chlorine to prevent septicity for such long periods of flow in a deep river underlaid by sludge, and chlorination at present would be a failure at

Brandon Pool. However, he suggested chlorination at the Southwest plant and at Lockport. Assuming that no new sludge came into the pool his opinion was that 3 ppm of chlorine was required at the Southwest works and 6 ppm at Lockport, although 3 ppm might suffice. If sludge continued to enter, 12 ppm or more, even 15 or 25 ppm, might be required. He considered the attempt speculative, and estimated the cost roughly at $700 to $3,000 per day, based upon chlorine at $43.20 a ton. The equipment might cost $115,000.

Mr. Buswell deprecated the effect of the sludge deposits. He believed 2 to 3 ppm of chlorine at Lockport would be sufficient, with a chloro-boat in the pool to apply 1 ppm if sludge were no longer discharged, and if sludge discharge was continued maybe 5 ppm would be needed. To obtain an authoritative opinion, he admitted, would require tests for at least three months. Mr. Enslow stated that Mr. Buswell's estimate of 1 ppm was too optimistic, but perhaps 3 ppm would be safe, if applied at two points.

Joseph W. Ellms, M. Am. Soc. C. E., estimated that the cost of a 150-ton chlorination plant would not exceed $300,000 and would require six or seven months to build. He felt that chlorine should be applied before septic conditions developed, and at several points. He stated that chlorine could reduce, and perhaps eliminate, odors from hydrogen sulfide, but that hydrogen sulfide would persist as long as organic matter is fed into the pool.

L. F. Warrick testified that chlorine would have a decided beneficial effect upon the Waterway. As a rough estimate he thought the effect of chlorine applied at the Southwest plant might last from 30 min to 2 hr or longer. (The time of flow in June, 1939, actually varied between 24 hr and 76 hr from the Southwest works to Lockport.)

Mr. Howson testified generally that chlorination "is a very effective and positive method of controlling odors." He considered it practicable and effective to relieve the conditions at Joliet and Lockport, and estimated that 6 ppm of chlorine would suffice. With a total flow of 3,130 cu ft per sec, this would amount to 50 tons of chlorine per day. His testimony on the subject of chlorination was based chiefly on Mr. Enslow's testimony.

The Illinois experts conceded that, in many situations, chlorine is used effectively to prevent odors; but, in view of the sludge deposits in the Main Channel and Brandon Pool, they believed that chlorine could not be used effectively to eliminate the odor conditions, even if large quantities were used at great expense.

Mr. Mohlman indicated that at least three points of application would be needed—Damen Avenue, Summit, and Lockport. This might require about 150 tons per day or about $6,000 to $7,000 per day during the summer months. The writer called attention to the fact that chlorine does not dispose of the organic matter that causes the nuisance at Joliet.

Cecil Kirk Calvert, Affiilate Am. Soc. C. E., presented a disappointing experience in 1930, at Indianapolis, Ind., in which an average of 10 ppm of chlorine was applied to the plant effluent. The reduction in B.O.D. was about 10% or 2 ppm of B.O.D. per part per million of chlorine. Usually the chlorine disappeared as soon as the effluent mixed with the river water. On only one occasion was any residual chlorine found farther away than 2,000 ft. He believed that the estimate of 150 tons per day for the Chicago-Joliet situation was very moderate and the application would be required for five or six months. He did not consider that the expenditure was justified, because the nuisance would not be measurably reduced.

Thus the amounts of chlorine required per day were variously estimated between 50 and 150 tons, with very doubtful assurance that the money would be well spent even as an emergency measure. Witnesses on both sides agreed that there is no water or sewage plant in the United States with a capacity for applying chlorine greater than the Detroit (Mich.) Sewage Works with 27 tons per day, and even this plant has not actually applied as much as 10 tons per day.

Cascades at Lockport.—The flow of the Main Channel passes through the turbines at the power house below Lockport and generates electric power which is worth about $1,500 per day to the Sanitary District.

Mr. Howson proposed that this flow be run over a cascade instead of generating power, thereby picking up 6.75 ppm of dissolved oxygen or about 113,000 lb of dissolved oxygen, equivalent to that contained in 2,650 cu ft per sec of lake water. He estimated the cost at between $50,000 and $100,000 for a wooden structure and a period of three to six months for construction. He based the pickup on the experience at the Hastings Dam in the Mississippi River and in the North Side effluent at Chicago.

Mr. Warrick believed that such cascades would have a beneficial effect, inasmuch as at Hastings Dam, in 1933 and 1934, about 7.8 ppm of dissolved oxygen was obtained. He stated that the B.O.D. in the water at the Hastings Dam was only 7 ppm as compared with 30 or 40 ppm in the flow at Lockport. He declined to express any opinion as to how long such pickup could be retained and doubted whether any dissolved oxygen would be carried through.

Mr. Calvert testified that such cascades would improve conditions but slightly, and that they would create a local odor nuisance.

Mr. Mohlman and the writer conceded that some dissolved oxygen would be picked up, not to exceed 3 ppm, but that the effect would be lost before the flow reached the pool because of its anaerobic condition. (As a matter of fact, actual tests made in October, 1924, showed a pickup over the old Bear Trap Dam of 4.1 ppm, which practically disappeared in the flow of 5 miles to the upper end of Brandon Pool.) The writer admitted that the pickup of dissolved oxygen at the North Side works averaged about 6.7 ppm from 1934 to 1939, but he doubted if all the dissolved oxygen so entrained was effective.

Mr. Howson contended that such dissolved oxygen would not be lost because of the 15-ft fall in the flow from the cascade to the pool. Mr. Enslow agreed with Mr. Mohlman that the re-aeration obtained by cascading would not last long under summer conditions and would not justify the cost. If chlorine were added to the cascade, he felt that some benefit would accrue. Otherwise, it would be just a local odor nuisance.

The writer conceded that, if the suggestion of cascading had merit, the effect could be accomplished over the existing control dams at the power house with very small expenditure. He did not consider that enough was gained to justify the loss of $1,500 per day to the Sanitary District. The Opposing States argued that Illinois should be required to try every measure that might possibly ameliorate conditions even at considerable expense, before diverting any additional water from the Lake.

Supply of Additional Oxygen through the Production of Nitrate by Increased Use of Air at North Side and Calumet Works.—Mr. Howson suggested that 40,500 lb of additional oxygen might be developed in the effluents of the North Side and Calumet works by using more air in the activated sludge process, at an operating cost of $89,000 for a 5-month period, or $216,000 for a 12-month period. Messrs. Warrick, Ellms, and Buswell agreed in general. Mr. Buswell stated that he knew of no streams with nitrites and nitrates present in the flow where septic conditions occurred, as nitrates tended to oxidize hydrogen sulfide and prevent its formation. Under existing conditions Mr. Buswell admitted there would be odors even if nitrates were present.

Messrs. Calvert and Mohlman agreed with the writer that such nitrate production would be of little ultimate value. The production of nitrates would require an increase in detention period in the aeration tanks from the present basis of 4.5 or 5 to 7 hr. Such capacity does not exist at the North Side works. Mr. Mohlman indicated that algae and green plants would form a secondary growth in the North Shore Channel which would create a secondary source of B.O.D. He testified that, if an attempt were made to control the odor situation by purchasing sodium nitrate, the cost might reach $17,000 per day.

Chemical Treatment at West Side Works.—Messrs. Buswell and Howson proposed the installation of chemical treatment on the effluent of the Imhoff tanks at the West Side works. This was intended to supplant the treatment by activated sludge which is a part of the Sanitary District permanent program and approved by Messrs. Ellms, Enslow, Howson, and Warrick. Such chemical treatment would reduce the B.O.D. by 21 ppm or, on a flow of 437 mgd, a reduction of 76,000 lb of B.O.D. Mr. Howson estimated that the construction would cost $1,300,000, with an operating expense of from $200,000 for five months to $481,700 for twelve months. (This was substantially the Sanitary District estimate of $1,300,500 (6).) Mr. Howson claimed in September, 1940, that this could be completed by the summer of 1941. According to the Sanitary District report of April, 1940 (6), two and one-half years would be required. Mr. Mohlman and the writer both stated that chemical treatment would not be satisfactory, since it would reduce the B.O.D. at the West Side from 57 to 36 ppm, whereas activated sludge treatment should reduce it to 10 ppm. The Sanitary District estimated that the chemical plant would cost $555,800 per year to operate at the West Side, whereas the activated sludge plant for equivalent purification would only cost $320,600 per year, with a first cost of $3,662,000.

Inasmuch as the Sanitary District has adopted the activated sludge treatment, the Master saw no reason to accept the suggestion that chemical treatment be adopted as a temporary or emergency measure to care for the summer of 1942, at the cost involved, when it would be almost immediately superseded by the superior activated sludge plant.

Chlorination of West Side Imhoff Tank Effluents.—Mr. Howson also described the application of chlorine to the West Side effluent as an alternative to chemical treatment, as suggested by Mr. Enslow. He estimated that this would reduce the B.O.D. by 50,000 lb instead of 76,000 lb by chemical treatment. The cost of the equipment was estimated at $100,000 and the time for construction at six to nine months. The operating expense was estimated at $105,000 for a period of 150 days. Mr. Mohlman and the writer did not consider chlorination of the West Side effluent a practical measure to ameliorate conditions in the Brandon Pool.

The Master concluded that it was obvious that the suggested work could not be done before 1942 and that the extent of amelioration was limited.

Planned Use of Water.—The decree of April 21, 1930, permits a variation in the diversion, which the Sanitary District has utilized to endeavor to provide more flow in the summer. Mr. Howson suggested that the present arrangement be adjusted to provide 4,200 cu ft per sec flow at Lockport during four summer months.

Illinois conceded the merit of this suggestion but indicated that it was limited by the necessity of providing for storm runoffs to prevent reversal in the Calumet River and flooding of basements in the Chicago Loop area. Hence the Sanitary District carries a reserve for use in the event of storm.

In its revised petition, Illinois proposed to take only 1,150 cu ft per sec of water for January, February, March, November, and December. Assuming that this is adequate, the aggregate difference between 1,500 and 1,150 cu ft per sec for five months is 1,750 cu ft per sec. If distributed over four summer months an additional flow of approximately 437 cu ft per sec is provided for each month, or 1,935 cu ft per sec. If this is added to the domestic pumpage of 1,700 cu ft per sec, the total is 3,635 cu ft per sec, as compared with 4,200 cu ft per sec suggested by Mr. Howson.

The Master indicated that this suggestion would only contribute about 22,000 lb of oxygen to meet the B.O.D. situation at Lockport. The lesser amount possible upon the admission of the Sanitary District is not substantially ameliorating. To quote: "In view of the paramount importance to the health of Chicago of avoiding any reversals of the river" the Master hesitated "to make a finding that the Sanitary District should be required to go further in budgeting than in its modified petition it admits to be possible."

Metering of Chicago Domestic Water Supply.—The Opposing States offered in evidence a table showing that the Chicago per capita consumption was 214% more than the average of nineteen other American cities. In Chicago, of 412,228 water services in 1940, only 115,025 were metered.

Illinois offered evidence through Loran D. Gayton, M. Am. Soc. C. E., then city engineer of Chicago, showing that to provide universal metering would require 300,000 meters, costing in excess of $10,000,000 and requiring more than six years to install, at the rate of 50,000 meters per year. In the 10-yr period from 1930 through 1939, the city installed an average of 4,165 meters per year. The greatest number ever placed in one year in Chicago was 16,864 in 1931. No money had been appropriated for their purchase. Under the circumstances, Mr. Gayton stated that he did not believe that any meters installed in 1941 and 1942 on the South Side area (in connection with the new filtration plant) would appreciably reduce the use of water therein.

Should the domestic pumpage be reduced, the Master found no evidence to enable him to appraise the extent of the resulting benefit.

Activated Sludge Treatment at Southwest Works for West Side Imhoff Effluent. —Mr. Howson suggested that a large part of the Sanitary District program for the activated sludge treatment at the Southwest works of the West Side Imhoff effluent could be speeded up, covering an estimated cost of $3,267,200. The writer testified that this sum was inadequate without the addition of certain essential items, totaling approximately $700,000, thus raising the total to $4,048,000. The Sanitary District estimate for the entire extension of the activated sludge plant was $4,878,500. Furthermore, of the program advocated by Mr. Howson, certain conduits and appurtenances costing about $600,000 would be of no value when the plant was completed.

The Comments of the Master

The Master indicated that the contention of the Opposing States is that the Sanitary District might complete its program before the summer of 1942 by proper effort. He commented that

> "The record indicates that the Sanitary District has been influenced, not so much by the desire to make speed, as by the purpose ultimately to complete an efficient system at as little expense as possible to its taxpayers. These are praiseworthy motives when considered from the standpoint of the Sanitary District alone, but they may not place the emphasis upon expedition to which the legitimate protection of the interests of the opposing States entitles them."

The Master also noted that "the record contains numerous references by representatives of the District to the importance of saving the taxpayers' money as a controlling consideration."

Illinois tried to impress on the Master that with the funds available to the Sanitary District for construction and operation, in accordance with its legal limitations, the engineers of the Sanitary District had endeavored to spend such funds to best advantage to produce permanent results, instead of frittering the money away on expensive makeshifts and thereby injuring the operation of the existing works and delaying the construction of the program which the experts on both sides agreed was the only real solution of the problem, the treatment to a high degree of all the sewage and the handling of all the solids so removed, so that a minimum of residual pollution in solution and in settling solids entered the Illinois Waterway.

Recommendations of the Master

On the basis of his conclusions (see Appendix) the Master recommended that a decree be entered dismissing the petition and the modified petition of the State of Illinois for a modification of the decree of April 21, 1930, and taxing the costs of the litigation against the State of Illinois. The Court then dismissed the petition without comment on May 26, 1941.

Acknowledgment

The writer wishes to acknowledge herewith the free use of the report of the Special Master as well as the record and exhibits in the case, from which liberal quotations have been made.

APPENDIX

Conclusions of the Special Master

After hearing all the testimony and inspecting the upper Illinois Waterway from Brandon Road to the Power House, the Master presented the following conclusions to the Court:

> "(1) The actual condition of the Illinois Waterway by reason of the introduction of untreated sewage creates in the summer months a nuisance through offensive odors at Joliet and Lockport, but does not present a menace to health. No nuisance conditions were proven to exist along the Waterway at any other points.
>
> "(2) With respect to remedial or ameliorating measures available to the State of Illinois without an increase in the diversion of water from Lake Michigan, my findings are as follows:
>
> "(*a*) The dredging of Brandon Road Pool would remove chiefly old accumulations of sludge which have completely or largely lost their potency as causes of nuisance and would therefore be of extremely doubtful efficacy. It would cost between $400,000 and $750,000, plus the cost of providing spoil banks and lagoons. It would present problems as to possible nuisance from such spoil banks and lagoons and require further expense for chlorination. I do not think this is a feasible ameliorating measure.
>
> "(*b*) The draining of Brandon Road Pool cannot be accomplished without some interference with navigation, to which the War Department, which has sole jurisdiction over navigation problems, will not consent. For this reason I do not think this suggestion feasible. There would also be presented problems with respect to the water intake pipes of several industries which take water for industrial purposes from the Pool.
>
> "(*c*) Chlorine is an effective measure to reduce and eliminate odors, but owing to the size of the Brandon Road Pool and the large sludge deposits therein and the continuing discharge into the Waterway of incompletely treated sewage, it is impossible to make a reasonably certain estimate of the amount of chlorine which would have to be applied to produce a substantial result.
>
> "In order to have a reasonable prospect of substantially controlling offensive odors, it would be necessary to spend from $3,000 to $4,000 a day for chlorine, plus several hundred thousand dollars for chlorinating equipment.
>
> "(*d*) Cascading the water at Lockport or sending it over the dam there would be remedial to the extent of producing some oxygen at Lockport. How much oxygen would be produced and how much of it would reach Joliet and the Brandon Road Pool is uncertain and could only be determined by actual trial. The use of the water in this way would cost the Sanitary District $1,500 a day in the loss of power and it would be necessary to use an undetermined amount of chlorine to prevent an odor at the point of cascading.
>
> "(*e*) The supply of additional oxygen through production of nitrates by increase of air on the North Side and Calumet plants is not a feasible ameliorating suggestion for the summer of 1941. The evidence before me is not sufficient to prove that it is feasible for 1942, in view of the testimony of the Sanitary District experts that it would require an increase in aeration tank capacity. The proof before me is insufficient to support a conclusion that this suggestion, if it could be put into effect for the summer of 1942, would be substantially ameliorating.
>
> "(*f*) Chemical treatment at the West Side plant would involve a very large permanent expenditure, which could not be made effective in 1941

and if it could be installed by 1942 would be almost immediately superseded by the activated sludge treatment which is provided for by the District's permanent program. This does not seem to me a feasible ameliorating measure.

"(*g*) The estimates as to cost of chlorinating West Side Imhoff tank effluents are too uncertain and the opinions of the experts too conflicting as to the extent of amelioration which it would afford, to enable me to make any finding that such chlorination is a feasible ameliorating measure.

"(*h*) It is feasible for the Sanitary District to budget the 1,500 c.f.s. of water now permitted to be diverted in addition to domestic pumpage so as to divert only 1,150 c.f.s. in the months of January, February, March, November and December, and to allocate the aggregate saving in diversion in those months to the summer months. This ameliorating measure will not, however, materially reduce the BOD at Lockport, and will not, therefore, substantially relieve the odor nuisance.

"(*i*) The adoption of compulsory water metering by Chicago is an ameliorating measure, but the evidence before me is not sufficient to enable me to make any finding as to the extent of amelioration which it would afford or the time within which it could be made available.

"(*j*) The provision of activated sludge treatment at the Southwest plant for the West Side Imhoff tank effluent is a very important and feasible ameliorating measure to which the Sanitary District is committed, and toward which it has made some progress. The extent of the progress will depend upon the industry and enterprise of the Sanitary District. There is no prospect that this ameliorating measure will be operative in the summer of 1941. It is possible, but doubtful, that by special diligence it might be made operative in the summer of 1942 instead of only by the end of 1942, as claimed by the District.

"(*k*) As additional equipment is installed, it will become possible to give complete treatment during the course of 1941 and 1942 to increased quantities of sewage at the Southwest plant. The progress which will be made during these years and the gradually declining influence of sludge deposits from past years will be operative in the summer of 1941 to improve conditions at Lockport and Joliet as compared with 1939 and 1940, and will be operative to a further extent in 1942. The extent of relief from offensive odors which will be afforded at Lockport and Joliet in the summer months of 1941 is very doubtful, but there is a better outlook for the summer months of 1942. Weather conditions will have an important influence.

"Notwithstanding every effort on my part to press the hearings and consideration of this case, one of the three years for which Illinois originally petitioned for relief (1940) has already passed. The people in Joliet and Lockport have submitted to the conditions which prevailed in the summers of 1939 and 1940 without serious consequences to health. If relief is denied for the years 1941 and 1942, the authorities of the Sanitary District may be spurred to increased efforts in the treatment of Chicago sewage. The hearings before me have already resulted in the putting into effect of provisions as to lagooning of incompletely treated sewage which had not previously been adopted."

BIBLIOGRAPHY

(1) **"Stabilization of Sewage Sludge Banks,"** by W. Rudolfs, *Indiana Engineering Chemistry*, Vol. 30, 1938, pp. 337–340; abstracted in *Sewage Works Journal*, Vol. 10, 1938, pp. 636–640.

(2) **"Taschenbuch der Stadtentwasserung,"** by Karl Imhoff, Eighth Ed., 1939, pp. 229–231.

(3) **Report of the Committee on Sewage Disposal,** by H. P. Eddy, et al., Engineering Board of Review, The Sanitary District of Chicago, 1925.

(4) **"Chlorination in Sewage Treatment,"** by L. Pearse, et al., Rept. of the Committee on Sewage Disposal, Eng. Section, Am. Public Health Assn., 1933.

(5) **"Preventive Medicine and Hygiene,"** by M. J. Rosenau, Sixth Ed., 1935, pp. 898–900, 928.

(6) **"Report on Additions to Southwest Sewage Treatment Works and Supplemental Treatment for West Side Treatment Works,"** by F. W. Mohlman, L. M. Johnson, W. A. Dundas, L. C. Whittemore, L. Pearse, and H. P. Ramey, made to Chief Engineer, April 16, 1940.

(7) **"Engineering Features of the Illinois Waterway,"** by Walter M. Smith, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 309.

DISCUSSION

Willem Rudolfs,[3] **M. Am. Soc. C. E.**—Experience revealed by the Illinois Water Litigation in 1940 and 1941 touches three questions of general interest on which more information is needed:

(1) What is the minimum dissolved oxygen required to prevent odor nuisances?

(2) What is the B.O.D. of sludge on prolonged storage?

(3) How long does deposited sludge exert an influence on the overlying water?

Odor nuisances in polluted streams are primarily of two types: Those musty odors related to sewage and sewage effluents, frequently modified by industrial wastes; and the putrid odors (mainly hydrogen sulfide) associated with actively decomposing organic matter. The latter odors are the most objectionable and cause nuisances most frequently, although the former may be disagreeable. Active decomposition of deposited sludge proceeds whether or not the overlying water contains dissolved oxygen. Anaerobic decomposition—and hence gas formation—increases with higher temperatures, resulting in greater activity during summer than winter. The increased biological activity causes the deposited sludge to become gas laden and the sludge rises to the surface in the form of "islands" or is distributed through the overlying water. Under these conditions the available oxygen in the water is used up quickly.

Water containing considerable quantities of oxygen but flowing rapidly over actively digesting sludge banks may be deprived only partly of the available oxygen while gas bubbles escape from the surface. Results obtained have shown that the water may have a dissolved oxygen saturation of 25% or more with appreciable H_2S odors present. In sluggish streams the oxygen nearly always is exhausted when odors are present. It is clear, therefore, that the minimum of dissolved oxygen necessary to prevent odors in the presence of sludge deposits will vary with the character of the stream and the type and quantity of sludge present.

In the absence of sludge deposits, the odors produced come from organic materials in solution or suspension during the course of their decomposition. The gases formed may go into solution or may be released in tiny bubbles. As long as a minimum of dissolved oxygen is present in the water, gross odor nuisances will be prevented. The minimum amount of oxygen required will vary, but the minimum of 1 ppm given by Mr. Mohlman in his testimony probably will suffice in most cases when no sludge deposits are present.

The B.O.D. of sludge is reduced comparatively rapidly during the first nine months of storage. Thereafter the rate of reduction decreases, and finally the material becomes stabilized. Some results obtained from long-time experiments on three types of sewage will illustrate the changes:

---

[3] Chf., Dept. of Water and Sewage Research, New Jersey Agri. Experiment Station, New Brunswick, N. J.

1. Fresh solids, material subjected to anaerobic digestion, not disturbed except when samples were taken for analysis;

2. Fresh solids, material subjected to anaerobic digestion, and the supernatant removed at intervals; and

3. Digested sludge, material previously digested anaerobically for a period of ten years.

The sludges used in the experiments were under observation for a period of from six to sixteen years.

The concentration of the sludges at the beginning and after 2,195 days were as listed in Table 9, Cols. 1 and 2. Values in Cols. 3 and 4 indicate the relatively rapid rate of B.O.D. reduction and the demand on the stream which may be expected over a long period.

TABLE 9.—LONG-TIME EXPERIMENTS WITH SEWAGE SLUDGE

Days	Fresh Solids, Subjected to Anaerobic Digestion								Digested Sludge			
	(a) Undisturbed				(b) Supernatant Removed				(c) Previously Digested for Ten Years			
			B.O.D.				B.O.D.				B.O.D.	
	Total solids (%) (1)	Volatile matter (%) (2)	Parts per million (3)	Milligrams per gram of volatile matter (4)	Total solids (%) (1)	Volatile matter (%) (2)	Parts per million (3)	Milligrams per gram of volatile matter (4)	Total solids (%) (1)	Volatile matter (%) (2)	Parts per million (3)	Milligrams per gram of volatile matter (4)
0	1.80	1.31	7,060	537	1.82	1.33	7,000	526	3.54	3.52	2,400	171
1,050	....	....	295	90	....	....	235	94	....	....	1,290	106
2,195	0.65	0.24	210	89	0.54	0.21	50	24	1.41	1.27	1,080	84

After sixteen years of digestion the unit B.O.D. of the volatile matter remaining is the same as after six years. The sludge residue has become stabilized, but it still exerts an influence on the overlying water as indicated by the further reduction of B.O.D. when the supernatant liquor is removed periodically (see Cols. 4, Table 9). The washing-out effect may be considered similar to the condition in a stream. The quantity of volatile matter remaining in the washed material is only slightly less, but the degree of stabilization is greater, than when the sludge is stored; hence, there is a continuous effect of the sludge on the overlying water.

Summarizing, it appears that:

1. Odor nuisances may be caused when actively decomposing sludge is present, even when several parts per million of oxygen are available in the stream;

2. In the absence of actively decomposing sludge, a minimum of oxygen must be present to prevent odor nuisances;

3. Stabilization of sludge requires a number of years; and

4. Even so-called "stabilized" sludge has a B.O.D. and exerts its influence on a stream. The magnitude of the effect is reduced steadily.

L. R. Howson,[4] M. Am. Soc. C. E.—In all of the litigation regarding diversion at Chicago, extending over a period of fifteen years, the experts for the Opposing States have never been critical of the engineering or construction of the Chicago Sanitary District's works. They have testified repeatedly of the high character of both the design and execution of that work. It is also only fair to state that some of the procedures eventually incorporated into the Sanitary District's program were first urged by the experts for the Opposing States. Such steps include:

(*a*) The recommendation that the Southwest, West Side and Calumet plants be of activated sludge rather than trickling filter type;
(*b*) That the Southwest and West Side projects be consolidated on the West Side site; and
(*c*) That provisions be made for lagooning sludge at the Southwest Side plant in case of breakdown in the sludge drying plant.

To secure a proper perspective of the contentions of the parties in the recent litigation covered by Mr. Pearse's paper, reference is made to the opinion of the U. S. Supreme Court delivered by Chief Justice Taft on January 14, 1929:

> "The Sanitary District authorities relying on the argument with reference to the health of its people, have much too long delayed the needed substitution of suitable sewage plants as a means of avoiding the diversion in the future. Therefore, they cannot now complain if an immediately heavy burden is placed upon the District because of their attitude and course. The situation requires the District to devise proper methods for providing sufficient money and to construct and put in operation, with all reasonable expedition, adequate plants for the disposition of the sewage through other means than lake diversion."

The Supreme Court then instructed the Special Master, the Hon. Charles Evans Hughes, "to determine the practical measures needed to effect the object just stated and the period required for their completion." After an exhaustive series of hearings in which the experts for the Sanitary District estimated that the sewage disposal program would require the expenditure of $176,000,000 and fifteen years in which to build it, and the engineers for the lake states estimated $76,000,000 to $82,250,000 and four to eight years for construction, the Master adopted a construction period of nine years from the date of the order. The project is now (1943) substantially completed. Approximately $80,000,000 have been expended for the sewage disposal program set up in the Master's report. Due to the economic depression delaying the start of construction, most of the program was condensed into a six-year construction period.

Attention is called to Table 3 which discloses that, during 1939, the Sanitary District discharged 53,970 tons of sludge and grit (on a dry basis) into the drainage canal, and that the total volatile solids discharged into the canal in the same year (Table 4) was 115,658 tons. Expressed in another way, from two to three million tons of liquid sludge which had been removed in the treatment plants were dumped back into the channel during 1939 in addition to the solids necessarily carried by the plant effluents. The surprising thing is not that the channel did smell but that it did not smell worse.

---

[4] Cons. Engr. (Alvord, Burdick & Howson), Chicago, Ill.

Some of the hearings were held in the city hall in Joliet. This building is near the drainage channel. The hearings were conducted during the hottest week that has ever been recorded in the Joliet weather records so that the setting was perfect for severe odors. During the period when witnesses were testifying as to the intensity of the odors, the wind was frequently blowing into the court room directly from across the channel. There is such a thing as "judicial notice."

The Special Master records his impressions of the odor situation as he observed it on a boat trip over the Brandon Pool one evening when the hotel thermometer after his return at 9:00 p.m. registered 92°. The following excerpt is from the Master's report:

> "After returning from this inspection trip I noted at 11:30 P.M. in my room at the hotel (which was about 4 blocks from the Waterway) an odor which lasted a minute or two; a few minutes later I got another odor for a minute or two, and then detected no more odors. I slept four nights at Joliet and was not troubled by odors. The only trouble I had in sleeping was due to the heat, which was so great that I kept an electric fan constantly going all night. The City Hall, in which the hearings were held, was about three blocks from the Canal on the east side; I detected an odor only on one day and then only for a few minutes. The four days I spent in Joliet were the hottest of the summer, the temperature reaching 103 degrees on July 25th."

In referring the matter to Monte M. Lemann as Special Master, the Supreme Court in its per curiam opinion (309 U. S. 569) prefaced the quotation referred to by Mr. Pearse with this statement:

> "The State of Illinois has failed to show that it has provided all possible means at its command for the completion of the sewage treatment system as required by the decree as specifically enlarged in 1933 (289 U. S. 395, 710). No adequate excuse has been presented for the delay. Nor has the state submitted appropriate proof that the conditions complained of constitute a menace to the health of the inhabitants of the complaining communities or that the state is not able to provide suitable measures to remedy or ameliorate the alleged conditions without an increase in the diversion of the rights of the complainant states as adjudged by this court."

Accordingly, the position of the Opposing States in this litigation was that their function was simply to suggest various "remedial or ameliorating measures" and that in view of the aforementioned earlier Supreme Court opinions, the cost of such measures was not a consideration. The Opposing States were not concerned as to what steps Chicago or the State of Illinois might take so long as they did not involve increased diversion from Lake Michigan. All engineers were agreed that the Sanitary District should complete its construction program at the earliest possible date. If the State of Illinois considered the Joliet nuisance sufficiently acute or inimical to health, the Opposing States contended that it was obligatory upon Illinois to adopt any or all "remedial or ameliorating measures" at its expense and without detriment to the Opposing States. It should be recalled that the Opposing States did not initiate the 1941 hearings. The State of Illinois was plaintiff.

It is noticed, in Table 7, that Illinois takes credit for No. 4; namely, keeping all sludge out of the Main Channel. At the time of the hearing approximately half of the sludge that had been separated out in the treatment processes was being discharged back into the drainage channel. The experts for the Opposing States, in their suggestion of remedial measures, recommended the immediate cessation of that discharge and that the sludge be deposited in lagoons until the dryers were adequate. Initially the Illinois experts testified that lagooning of sludge was impracticable and could not be adapted to Chicago conditions. Subsequently, however, that position was changed and lagooning, in fact, was provided by the Sanitary District before the Master's report was completed.

Table 7 does not include No. 2, "Activated sludge treatment for West Side Imhoff tank effluent," as a remedial measure in the "Opposing States" column. However, this was one of the measures urged by the Opposing States experts for immediate undertaking and opposed by Illinois as requiring more time than estimated by the Opposing States. In commenting on these opposing views the Master quoted from Special Master Hughes,

> "Much time can be saved or lost in large building operations according to the attitude which is taken as to the importance of early completion. In the present case the Court has already laid down the requirement that the work shall proceed 'with all reasonable expedition.' "

Remedial Measure No. 9 "budgeting of the diversion" was also the initial suggestion of the experts for the lake states. This, too, was a matter on which all parties came to agreement during the progress of the case.

As to most of the other "remedial or ameliorating measures" suggested as available the Master considered them as not "feasible," worth while, or necessary. His report stated:

> "The people in Joliet and Lockport have submitted to the conditions which prevailed in the summers of 1939 and 1940 without serious consequences to health. If relief is denied for the years 1941 and 1942, the authorities of the Sanitary District may be spurred to increased efforts in the treatment of Chicago sewage. The hearings before me have already resulted in the putting into effect of provisions as to lagooning of incompletely treated sewage which had not previously been adopted."

From the fact that there have been four Supreme Court hearings on the diversion of water from Lake Michigan for sewage disposal purposes, it is evident that there have been some divergent views. However, in all of these cases the argument of Illinois has been for diversion. Previous cases have seen much testimony relative to diversion for power, diversion for navigation in the Illinois Waterway, and even the Mississippi River, but in this case the diversion was asked as a direct health measure. The Master recommended against, and the Supreme Court denied, the petition for increased diversion. The Master stated "Every increased diversion from the lakes removes a stimulus to Illinois to speed the work." The Master reported a nuisance but no menace to health.

A. M. **Buswell,**[5] **Esq.**—It is fortunate that Mr. Pearse has prepared for record a summary of the rather unique litigation which, as he states, involved the question, "When is pollution a menace to health" rather than a question of nuisance abatement.

---

[5] Chf., State Water Survey Div., Urbana, Ill.

Although much of the testimony related to evidences of nuisance and the relative effectiveness of various means of relieving odors, the plea was that the situation was so extreme as to have an actual effect on health. On this, the main point of the controversy, the Court stated that (see "Conclusions as to Health Conditions"):

> "In the present case it is Illinois itself which is creating the nuisance of which it complains and of which it seeks to be relieved by water which has in effect been adjudged by the Court to belong to the opposing States.* * *
>
> "My conclusion is that the facts proven do not establish any menace to the health of the inhabitants of Joliet and Lockport or elsewhere along the Waterway requiring an increase in diversion in water from Lake Michigan."

Since one of the principal reasons for increased diversion was to flush out sludge deposits which it was contended had accumulated as a result of decreased diversion, it is of importance to note (Table 3) that the discharge of sludge to the channel increased from some 30,000 tons in 1934 to some 50,000 tons in 1939. As a result of the hearings, this was reduced to 20,000 tons in 1940 and to 0 in 1941. This elimination of the discharge of sludge to the channel was urged by the opponents of diversion as the most effective immediate remedial measure, and it is gratifying to note that the suggestion was acted upon so promptly.

The presentation of these and other suggested ameliorating procedures by Mr. Pearse may be misleading to some readers. The Opposing States did not contend that any one item in their program (Table 7) would completely relieve the situation. They did submit that the effect of combining the permanent cessation of sludge discharge with such temporary measures as chlorination and chemical precipitation at certain plants to improve efficiency, and chlorination where dissolved oxygen deficiencies were imminent, would give greater relief than increased diversion. This contention was supported amply by the results of the 10-day flushing test (see discussion of Table 8) that swept some 500,000 cu yd of sludge into Brandon Pool, increasing the pollution at that critical point.

The page and a half of testimony on chlorination may confuse persons not familiar with the subject. The hesitancy of those offering testimony to state definite requirements and predict quantitative results was due to their realization of the complicated nature of the chlorination reaction. Complete or "break point" chlorination requires relatively large amounts of chlorine but hydrogen sulfide and other odorous compounds are chlorinated far below the break point. It is important to note that all agreed that chlorination was an effective means for odor control. Under the heading, "Available Remedial Measures," the writer is quoted as admitting that "dilution reduces the B.O.D. more certainly than chlorine." Actually the writer's testimony was that the reduction of B.O.D. by dilution can be calculated more accurately in the light of present knowledge than the reduction by chlorine can be calculated. The writer is further quoted (heading, "Supply of Additional Oxygen through the Production of Nitrate by Increased Use of Air at North Side and Calumet Works") as admitting that "there would be odors even if nitrates were present." He wishes here to state that the presence of nitrates is incompatible with putrefactive odors. If the nitrates are not present in sufficient quantity they would be reduced, after which odors would appear.

The summary of testimony on cascades fails to point out that the increase in dissolved oxygen after aeration is not a true measure of the results accomplished. Septic compounds are largely volatilized or oxidized by aeration, thus reducing the B.O.D. by several times the amount of actual oxygen increase. There is no question that aeration is effective, although it may not be worth "$1,500 per day to the Sanitary District."

Emphasis should be placed on the fact that there was no criticism of the final program of the Sanitary District of Chicago for relieving the situation permanently. Both parties to the litigation urged a speeding up of this program as the best solution of the problem. It is apparent that the Special Master came to the same decision. This is brought out forcefully in the final paragraph of his conclusions quoted in the last paragraph of the Appendix to Mr. Pearse's paper.

LLOYD M. JOHNSON,[6] ESQ.—Mr. Pearse's paper records clearly the proceedings in this case. It seems unfortunate that it was necessary for Illinois to prove a menace to health in the conditions existing in the Waterway, in order to obtain a temporary increased flow. To the layman the existence of nuisance would warrant such increase to relieve the conditions.

In view of the accumulations of sludge in the Waterway, and the persistence in oxygen demand of these deposits over several years, a temporarily increased diversion greater than 1,500 cu ft per sec appears extremely desirable during the summer months until the Sanitary District sewage treatment construction program can be completed. This increase would be most effective if taken during the summer months on a schedule similar to that outlined in the modified petition.

A plant of the unprecedented size and complications of the Southwest works of the Sanitary District requires a lengthy "tuning up" period. Scarcity of materials and the priority situation have worked against keeping the plant at maximum operating capacity. Due to limitations in sludge handling capacity the plant has operated so that the reduction of B.O.D. has been approximately 60%. The war has stopped work leading to construction of additional facilities for the complete treatment of all sewage from the Southwest area, and the activated sludge treatment of the West Side Imhoff tank effluent.

Under conditions in 1940–1941 the over-all B.O.D. reduction was approximately 60% for the sewage flow of the four major sewage treatment works of the Sanitary District. In 1938 (the last year of the diversion of 5,000 cu ft per sec and before the Southwest works was placed in operation) the over-all reduction was roughly 35%. With activated sludge treatment for 400 mgd at the Southwest works the reduction would average about 80%, and with such treatment for the West Side flow also, which is now only settled in Imhoff tanks, the value would be 90%. These figures do not take account of the effect of storm flows.

---

[6] Commissioner of Streets and Electricity, City of Chicago, Chicago, Ill. (formerly, Engr. of Maintenance and Operation, The Sanitary District of Chicago).

Table 10 shows a comparison of the residual sewage untreated, on a population basis, and the dilution afforded before and after the reduction in diversion, based on a total population equivalent (human plus industrial equivalent) of 6,512,000, which actually occurred in 1941. The value was probably less in preceding years, but 1941 was the first full year for which data were available.

TABLE 10.—COMPARISON OF RESIDUAL SEWAGE UNTREATED, ON A POPULATION BASIS

Year or condition	Percentage treatment	Residual population untreated	DIVERSION, IN CU FT PER SEC	
			Total population	1,000 population
1938	35	4,220,000	5,000	1.18
1941	60	2,605,000	1,500	0.58
Treatment[a]	80	1,302,000	1,500	1.15
Treatment[b]	90	651,000	1,500	2.30

[a] Activated sludge treatment for 400 mgd at Southwest works and Imhoff tank treatment at West Side works.

[b] Activated sludge treatment for the flow of the entire Sanitary District.

Thus, it is seen that in 1940–1941 the dilution available was about one half that in 1938, before the reduction in diversion. The fact that the lower diversion contained a much higher proportion of direct runoff is disregarded.

Regarding the suggested remedial measures it is fortunate that funds were not used to install chlorinating apparatus or chemical treatment at the West Side works. Any other funds expended for temporary measures would have been largely wasted. In the light of developments since the hearings it is fortunate that the use of chlorine was not adopted, as the war conditions would probably have made it impossible to obtain chlorine for such purpose.

In closing the comments on Mr. Pearse's paper, it is now clear that the most prompt and effective method to ameliorate the condition would have been increased diversion. Unless an emergency order is issued granting increased diversion, the condition of the Illinois Waterway must remain as it is for the duration of the war and for some time thereafter.

H. P. RAMEY,[7] **M. AM. SOC. C. E.**—In Table 7, Mr. Pearse indicates that remedial measure No. 9 was the initial suggestion of the State of Illinois. There was some agreement as to the desirability of budgeting the diversion, but the initial suggestion for it did come from the experts for the State of Illinois. Budgeting of the diversion was started long before the 1940 hearings. As assistant chief engineer of The Sanitary District of Chicago, the writer testified on September 9, 1940, that the flow began to be budgeted very carefully in January, 1939, when the diversion was reduced to 1,500 cu ft per sec. At the same time a tabulation was introduced as Illinois exhibit No. 40, which showed the budgeting, by months, from January, 1939, through 1940, and actual results through July, 1940.

---

[7] Asst. Chf. Engr., The San. Dist. of Chicago, Chicago, Ill.

The representatives of the lake states presented their first testimony regarding remedial measures on September 12, 1940.

Langdon Pearse,[8] M. Am. Soc. C. E.—Following the denial of the petition, work on the additional final settling tanks at the West-Southwest works progressed to about 85% completion, before coming to a stop. Of the three turbo-blowers under contract on August 29, 1940, only one unit was delivered late in 1941 and erected in 1942. However, it was not operated until April, 1943, because of the impossibility of securing necessary gate valves. Twelve contracts were let in 1940–1941 on the construction program to enlarge the Southwest works and provide facilities for treating the effluent of the West Side works. This group of contracts as a whole was only about 22% completed in the fall of 1941, when World War II began. At the end of 1943, construction practically ceased. On some contracts, nothing had been done. As yet (1944) there is no sign of relaxation in priorities, allowing work to be resumed. The completion of the necessary permanent facilities to dispose of the entire production of sludge appears to be postponed indefinitely, until peace comes.

To provide temporarily for the handling of the excess sludge from the West-Southwest plants, plans were drawn for a 16-in. force main and seventy-eight acres of lagoons, about 16 ft deep, divided into sixteen units. The lagoons are located about 4.75 miles down channel from the Southwest works, between the Des Plaines River and the Main Channel. On January 22, 1942, a contract was let for 1,300 tons of cast-iron pipe. This was delivered in May, 1943.

TABLE 11.—Percentage Removal from Four Major Sewage Treatment Works

Period	Yearly Average, Percentage Removal			Average Percentage Removal in June, July, and August		
	Population equivalent (1)	5-day B.O.D. (2)	Suspended solids (3)	Population equivalent (4)	5-day B.O.D. (5)	Suspended solids (6)
1939	....	78.2[a]	82.0[a]	....	76.9[c]	82.0[c]
1940	6,200,000[b]	67.8	72.5	5,735,000	66.6	72.7
1941	6,512,000	60.6	67.5	6,540,000	61.3	69.9
1942	6,693,000	61.5	65.7	6,336,000	65.7	69.5
1943	6,594,000	57.5	62.1	6,417,900	56.0	60.1

[a] Includes Southwest works, July to December, inclusive. [b] April to December, inclusive. [c] Includes Southwest works, July and August only.

Other contracts were let in 1942 for the valves, sludge pumps, and lagoon construction; and, in 1943, for the pipe laying. The pipe line and lagoons were ready for service on November 1, 1943.

Until additional equipment and the requisite structures can be built to complete the Southwest works and supplement the West Side works, there is little chance of meeting the desired goal of a high-grade effluent from all the sewage treatment works of the Sanitary District.

---

[8] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

The load reaching the four major treatment works of the Sanitary District is shown in Cols. 1 and 4, Table 11, based on population equivalents derived from the B.O.D. actually reaching the works. The 1940 census population of the Sanitary District was 3,962,514. From a total of 4,581,111 in 1940 (Cook County representing 4,063,342) the U. S. Census Bureau estimates a decrease of 32,113 (approximately 0.7%) in the population of Cook, Du Page, and Lake counties in Illinois and Lake County, Indiana, between April 1, 1940, and March, 1943.

The results of this population decrease on the Southwest works (Table 3) and the effect on the over-all efficiency of the four major works are explained in Table 12 which shows the apportionment of incoming flow to sewage receiving preliminary settling only and to sewage receiving complete treatment. Under present working conditions, the sewage receiving activated sludge treatment at the Southwest works is limited to that producing the required sludge for dewatering and drying for fertilizer, when added to the North Side sludge.

The over-all efficiency of the four major sewage treatment works of the Sanitary District is illustrated by the data in Cols. 2 and 3, Table 11. The averages for the three months of June, July, and August in Cols. 5 and 6, Table 11, are but slightly different.

TABLE 12.—APPORTIONMENT OF FLOW (IN MILLION GAL PER 24 HR), SOUTHWEST WORKS

Year or period	Preliminary only	Activated sludge	Total
1939[a]	0	282	282
1940	121	192	313
1941	184	154	338
1942	188	143	331
1943	209	143	352

[a] July to December, inclusive.

The condition of the flow from the Main Channel as it enters the Brandon Pool is reflected by the analyses at Lockport. The yearly averages are shown in Table 13(*a*) and the averages for June, July, and August are given in Table 13(*b*). Samples are not collected at the outlet end of the pool.

From September 15, 1940, to November, 1942, no sludge was discharged from the treatment works into the channels of the Sanitary District or into the tributary rivers. Because of the 10-day test in 1940, a large amount of previously deposited sludge was swept down into the Brandon Pool. Apparently conditions in the pool were somewhat worse in the summer of 1941 than in 1940; 1940 was better than 1939 or 1941. In 1942, the condition was somewhat improved over that in 1941, except that an early spring in 1942 caused nuisance conditions at Lockport in April and May, whereas in 1941, the worst month was June. In 1942, from April through November, there was no oxygen (or only a trace) present in the flow at Lockport. In 1943, the dissolved oxygen disappeared in May, and reappeared in October.

Because of the cessation of all construction, an allowed diversion of 3,500 to 5,000 cu ft per sec for the duration of the war is the only measure that is immediately available to alleviate the existing conditions in the Main Channel and Brandon Pool. Such a flow would also be helpful in the lower Illinois River below Peoria, Ill., and Pekin, Ill., where greater activity in the manufacture of alcohol from grain has increased the pollution in the river by an equivalent population of about 750,000 to 1,000,000. As a result, during the summer of 1943, the river was reported almost devoid of dissolved oxygen for a distance of 46 miles below Peoria.

TABLE 13.—Diversion from Lake Michigan and Discharge and Analyses at Lockport, Ill., 1936 to 1943, Inclusive

Year	(a) Yearly Average				(b) Average, June, July, and August			
	Cubic Feet Per Second		Parts Per Million		Cubic Feet Per Second		Parts Per Million	
	Diversion	Discharge	Dissolved oxygen	5-day B.O.D.	Diversion	Discharge	Dissolved oxygen	5-day B.O.D
1936	4,862	6,607	2.9	19.7	4,999	6,838	0.3	20.1
1937	4,989	6,677	3.1	19.0	5,448	7,247	0.1	19.5
1938	4,999	6,648	2.5	18.4	4,891	6,752	0.2	17.3
1939	1,499	3,132	0.4	26.0	1,545	3,336	0.0	30.2
1940	1,681	3,319	0.5	20.1	1,666	3,469	0.0	20.6
1941	1,496	3,341	0.2	22.8	1,929	3,795	0.0	27.7
1942	1,528	3,269	0.4	23.6	1,621	3,429	0.0	21.5
1943	1,500	3,472	0.4	24.4	2,014	3,945	0.0	24.0

Incidentally, it is of interest to note that the level of Lake Michigan has been rising since 1934 and, in August, 1943, averaged 581.47 above mean tide at New York, N. Y., as compared with a previous high monthly average of 583.49 in July, 1876.

In March, 1944, Congress had before it a resolution introduced by the Hon. A. J. Sabath, Representative from Illinois, to allow a diversion of 5,000 cu ft per sec. Hearings have been held before the Rivers and Harbors Committee of the House of Representatives.

From an entirely different angle, the Attorney General of Illinois, George F. Barrett, recently petitioned the United States Supreme Court, on behalf of Illinois, to enjoin the State of Indiana and the cities of East Chicago, Gary, Hammond, and Whiting from polluting Lake Michigan, the source of the Chicago water supply. The Court refused to dismiss the petition and on March 7, 1944, appointed a Special Master, Luther E. Smith. The polluted condition of the lake and the Grand Calumet River in Indiana has been a source of worry to sanitarians since 1909.

The moral of this story has been pointed by the poet, Robert Burns, in his saying:

> "The best laid plans o' mice and men
> Gang aft a-gley."

Certainly, periods of panic and depression and war stress have wrecked the prophecies of optimists and upset the plans for a rational program of construction and operation.

In his testimony, Mr. Buswell stated definitely that there could be odors even if nitrates were present (Record, page 1894). He also stated that "sludge banks, therefore, may prove offensive even though overlaid by a liquid containing some dissolved oxygen" and that "the Drainage Canal would become deoxygenated just as quickly if nitrates were present as if they were absent." His statement, in the discussion of this paper, that "It is important to note that all agreed that chlorination was an effective means for odor control" should be qualified since the witnesses for Illinois limited their opinion on the value of chlorine for odor control to sewage treatment works.

Mr. Howson states that "some of the procedures eventually incorporated into the Sanitary District's program were first urged by the experts for the Opposing States." The engineering staff of the Sanitary District, both regular and consulting, was well aware of the various alternatives available, but did not attempt to decide which was best, under the stress of litigation. Naturally, the staff was gratified to know that the experts for the Opposing States would not oppose certain procedures. The late Harrison P. Eddy, Past-President, Am. Soc. C. E., the late George W. Fuller and the late T. Chalkley Hatton, Members, Am. Soc. C. E., had recommended lagooning of excess sludge in 1923, antedating Mr. Howson's expression of the same idea as of 1941.

Mr. Howson compares the 1927–1928 estimates for a sewage treatment program on the basis of $176,000,000 for the Sanitary District and $76,000,000 to $82,250,000 for the lake states. The record clearly shows that such estimates are not comparable since the lake states omitted more than $46,800,000 of work, including intercepting sewers and pumping stations essential to operation. This is manifestly an incomplete program.

The intercepting sewer system is largely completed (except for the South Side and miscellaneous sewers), the North Side works are practically completed, and the Calumet works nearly completed. However, the Southwest and West Side works are not completed in the sense inferred by Mr. Howson. Lack of funds postponed vital work, so that the efficiency in removal of B.O.D. of the four major plants has averaged about 60% for 1941–1943, instead of 90% or better.

The expenditures for sewage treatment have been as follows:

Period	Actual expenditure
Prior to December 31, 1928 (as reported to U. S. Supreme Court, First Report)	$ 81,917,602.83
December 31, 1928, to December 31, 1938 (as reported to U. S. Supreme Court, Final Report, December 1, 1938)	77,877,728.87
December 31, 1938, to December 31, 1943	11,705,000.00±
Total to December 31, 1943	$171,500,000.00±

The estimated cost of the remaining work is as follows:

Types	Estimated cost
Essential (including completion of Southwest works and addition of activated sludge plant to West Side)	$ 10,500,000.00
Postponed (including South Side intercepting sewers and miscellaneous sewers)	22,940,000.00
Total	$ 33,440,000.00

Contrary to Mr. Howson who states that "Remedial Measure No. 9 'budgeting of the diversion' was also the initial suggestion of the experts for the lake states," budgeting had already been practiced by the Sanitary District as shown by Mr. Ramey's discussion.

The outcome of the litigation was happy in that certain very experimental temporary measures were vetoed by the Master, who favored the permanent work. The writer agrees heartily with the two closing paragraphs of the discussion by Mr. Johnson. To those who have contributed to the discussion the writer is grateful.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2222

CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA

By Luna B. Leopold,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Lawrence Pratt, L. L. Harrold and A. J. Dickson, James Girand, Clarence S. Jarvis, Paul V. Hodges, Edgar E. Foster, R. W. Davenport, A. K. Showalter, Walter B. Langbein Emil P. Schuleen, and Luna B. Leopold.

Synopsis

Seasonal differences in storm types are typical of New Mexico and Arizona. Summer storms ordinarily are of small areal extent and are characterized by high-intensity rainfall in a well-defined pattern with relation to time. Winter storms cover large areas and usually are of low intensity. In this paper, results of frequency analyses of one-day rainfalls are presented for all stations in New Mexico and Arizona, the records of which exceed 15 years in length. The relation of rainfall frequency to geographic position and topographic relief are discussed, and recommendations are made for the use of these data for design purposes.

Introduction

In 1935 when the late David L. Yarnell, M. Am. Soc. C. E., published[2] an analysis of rainfall intensity-frequency data, there were only five rainfall-intensity gages in and near the states of New Mexico and Arizona, the records of which were sufficiently long to be used in the analysis. That number represented one intensity gage per 47,000 sq miles. Since then the U. S. Weather Bureau has launched an extensive program for the collection of intensity records in these, as well as in other, states, which eventually will furnish data needed for engineering designs. The data from these newly installed gages have been published consecutively since January, 1940, and have supplied basic information for studies of intensity patterns, but the records are too short for frequency studies.

To help fill the urgent need for detailed recurrence-interval data on rainfall in New Mexico and Arizona, the present study was made of the records of rainfall collected by standard nonrecording gages of the Weather Bureau.

Note.—Published in February, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Aviation Cadet, U. S. Army Air Forces, Los Angeles, Calif.

[2] "Rainfall Intensity—Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U. S. D. A. August, 1935.

In addition to data on frequency of heavy rainfalls, it usually is necessary, in the design of certain engineering structures, to assume values of area covered by the design storm and to determine a pattern of rainfall intensity before a hydrograph of runoff can be constructed. With these requisites in view, some data on areal patterns of summer storms and rainfall-intensity records were included to add to already published data pertinent to the adoption of a design storm, with particular reference to a small watershed. Due to the limited areal extent of summer-type rainfall in New Mexico and Arizona, large watersheds usually are capable of producing the largest floods as a result of prolonged rainfall covering large areas, or in these states, a typically winter-type storm.

STORM TYPES

The normal rainy seasons in the Southwestern States are July to September, inclusive, and December to March. Valley areas usually receive the highest monthly precipitation in July, August, and September, whereas in the high mountain areas the greatest monthly precipitation occurs from December through March, mostly in the form of snow. These seasonal variations in rainfall amounts are related to storm characteristics and sources of moisture.

Winter precipitation generally is the result of frontal activity associated with influx of cold air masses, modified Polar-Pacific or Polar-Continental. Polar-Pacific air picks up moisture from the ocean and moves into the Southwestern States usually as a relatively moist and cool mass that tends to underrun the warmer dry air. If the moist air mass is forced upward over the mountain ranges, continuous showers or nearly steady precipitation in the form of rain or snow is recorded over a period of 48 hr or more.

Modified Polar-Pacific air may move into Arizona and New Mexico from the south, west, or northwest. Invasion from the south or southwest often results in a relatively stationary east-west front which causes widespread, low-intensity rainfall in these states. If the moist air comes from the northwest, heavy snowfall may be recorded on the north rim of the Grand Canyon and on San Francisco peaks, followed by squalls in southeastern Arizona and southwestern New Mexico.

Polar-Continental air usually comes from the northwest and it tends to lose its moisture in northern Utah or Colorado. Such air masses are relatively dry and cold when they reach New Mexico and Arizona, and they do not constitute an important source of moisture in these states.

The Mogollon Rim, a southwest-facing escarpment extending diagonally across central Arizona, receives most of its winter precipitation from the modified Polar-Pacific air masses. This conclusion is borne out by the flood history of the Gila River which drains the Mogollon escarpment and the mountainous country of west-central Arizona and southwestern New Mexico. Generally, high winter rainfalls in the headwaters of the Gila River are preceded by rain along the southern coast of California.

The source of winter precipitation at least partly explains the fact that winter storms generally affect large areas which receive rainfall at low intensities for periods of two or three days or even a week. Although, in a general way, the total winter precipitation increases with elevation, it is influenced to a great extent by the topographic relief.

These characteristics of winter rainfall contrast with those of summer rainfall, which ordinarily is not the direct result of frontal activity. Summer storms usually are of the thunderstorm type with high intensities covering relatively small areas. Ordinarily the moisture is derived originally from the Gulf of Mexico and is carried to Mexico by subtropical easterlies where convection causes the warmth and moisture to rise to high levels over the Mexican Plateau. This air mass is deflected northward and enters the Southwestern States as a moist tongue at high levels, crossing the Mexican border usually west of El Paso, Tex., and east of Yuma, Ariz. Analyses by H. Wexler and J. Namias[3] show that the path of this moist tongue varies from one season to another, and its position is reflected in the summer rainfall in New Mexico and Arizona.

The mechanism by which this vapor is precipitated usually is a combination of convection caused by local heating of the lower layers of air, and orographic influences of topographic relief. Surface heating of the ground and adjacent layers of air during a summer day tends to increase the steepness of the lapse rate (vertical temperature gradient), and convective instability may result in local precipitation. This process usually results in instability above a mountain slope more quickly than above a valley plain, because the temperature of the ground at the two locations increases at about the same rate. Owing to the higher elevation of the point on the mountain, the vertical temperature gradient above that point is greater than that above the valley floor, and it is more probable that rain will fall on the mountain slope than over the valley.

Infrequently, sections of New Mexico and Arizona experience a general summer storm characterized by moderate rainfall over large areas and scattered local areas of heavy precipitation. Whereas the local summer storms are mostly of the convective orographic type, the general storm usually is associated with frontal activity. The effect of topographic relief is relatively small, although, together with convective lifting, it may tend to promote local high intensities in the area of general rain. Data on intensities and short-period isohyetal maps of storms of this type are meager. The most adequate discussion of such a storm is the report of the U. S. Engineer Office on the storm of August 13–14, 1940.[4]

Storms of the general type caused the floods of the Pecos and Gila rivers in September, 1941, the Bill Williams River flood of September, 1939, and the San Pedro River flood of September, 1926. In general, the available records indicate that the average intensities for durations of 1 hr or less are considerably less in the general storm than in the local summer storms. In the former type, however, the area covered by intense rainfall tends to be somewhat greater, and the rainfall extends over a longer time period. One of the main points of difference apparently lies in the importance of frontal activity as a causal factor.

---

[3] "Mean Monthly Isentropic Charts and Their Relation to Departures of Summer Rainfall," by H. Wexler and J. Namias, *Transactions*, Am. Geophysical Union, Pt. I, 1938, pp. 164–170.

[4] "Hydrologic Data—Storm of August 13–14, 1940, Arizona and New Mexico," mimeographed September, 1941, Los Angeles Dist., U. S. Engr. Office, Los Angeles, Calif.

Because of the relative infrequence of important storms of the general type, most of the high rainfalls experienced in 24-hr periods in summer and comprising the point-rainfall data analyzed in the present study may be assumed to be of the local storm type. Because of the lack of data, adequate analyses of the area covered by the different storm types are not included in this paper, and thus the recurrence-interval data presented subsequently in Tables 2 and 3 are primarily applicable for design purposes to small watersheds.

AREAL PATTERN OF SUMMER STORMS

The relatively great instability in mountainous country at least partly explains the areal distribution of summer rains in the basin and range topography typical of many parts of New Mexico and Arizona. Rainfall records show that summer rainfall in the valley areas and at the base of the mountains is erratic and spotty, whereas at the crest of the higher mountains the total summer rainfall tends to be similar at all points of a given elevation and tends to increase with elevation. Field observations indicate that in any one summer storm the center of greatest precipitation usually is near the base of the mountain slope.

The areal extent of ordinary summer thunderstorms in New Mexico and Arizona is extremely variable, but the center of high intensity is usually about 5,000 acres. It is important to note, however, that many storms of relatively small area occur nearly simultaneously at widely separated points within an area as large as the State of New Mexico. Storms at any particular location evidently are associated with relatively widespread instability, or the influx of moisture into the entire region. Inspection of any monthly summary of precipitation published in *Climatological Data*[5] shows the regional coincidence of rainfall, but studies of the storms at any given station demonstrate that the storms were separate entities of small areal extent.

There is some evidence that the area covered by rainfall in a given summer storm increases with the total precipitation in the storm center, but only a few good isohyetal maps of individual storms are available. It appears certain that the storms occurring in east-central and southeastern New Mexico in the Pecos River basin cover larger areas than storms in any other section of the two states, and also any given amount of storm rainfall occurs there more frequently than in other sections of these states. This presumably may be attributed to the proximity of the Pecos River basin to the source of summer moisture, the Gulf of Mexico, and to the fact that the Pecos area sometimes is subject to the marginal effects of tropical hurricane disturbances that move into Texas from the Gulf.

Samples of the depth-area data available for local summer storms are presented in Table 1. Storm movement and lack of intensity data for various points within the storm area render this type of depth-area data inadequate for making assumptions of the depth-area-time pattern for a design storm, but in most sections of the Southwestern States no better data than the samples

---

[5] *Climatological Data*, published monthly by the U. S. Weather Bureau.

TABLE 1.—Area, in Square Miles, Covered by Various Quantities of Rainfall (Summer Storms)

No.	Location	Date	Rain, in Inches							
			$\geqq 1$	$\geqq 2$	$\geqq 3$	$\geqq 4$	$\geqq 5$	$\geqq 6$	$\geqq 7$	$\geqq 8$
71	Pima, Ariz.	September 16, 1939	132	39	8	1.5	...	...	...	...
3	Las Cruces, N. Mex.	August 29, 1935	...	...	...	...	...	38	7.2	2.6
12	Sierra Ancha, Ariz.	August 5, 1939	...	5.2	1.4	0.6	0.2	...	...	...
72	Rodeo, N. Mex.	August 13, 1940	113	11	4	1	...	...	...	...

in Table 1 are available. A complete report of depth-area-time relations of storms of these states must await the collection of more records.

RAINFALL INTENSITY

Winter-type storms in New Mexico and Arizona usually produce rainfall at relatively low intensities. Summer rainfalls of the thunderstorm type characteristically are intense and of short duration, with the period of highest intensity at, or soon after, the beginning of rainfall. Except in unusual cases, the periods of high intensity are separated by lulls in the downpour, the period of uninterrupted intense rain lasting less than one-half hour. In practically all cases the intensity decreases more or less gradually to the end of the rain. Mass curves of the most intense rainfalls on record at recording gages in New Mexico and Arizona are plotted in Fig. 1(*a*). Many summer rains have some low intensity fall prior to the most intense portion, but in general the curves in Fig. 1(*a*) are representative of the heaviest rains recorded in the Southwest. It is important to note the general similarity of pattern of mass curves of storms from widely separated localities.

From the records of beginning and ending of rainfall observed at Weather Bureau stations where standard rain gages are used, it is possible to collect additional records of heavy rainfalls of short duration. Fig. 1(*b*) presents the results of a systematic search through the original records of 162 stations in New Mexico, comprising 4,656 station-years and certain additional Arizona records, in an effort to determine the minimum duration of heavy rainfalls. The key to numbered points is given in Table 2. No rain less than 2.00 in. or of durations greater than 14 hr is platted, and many 4-in. rains of duration greater than 6 hr are omitted. The envelope curve drawn in Fig. 1(*b*) indicates the maximum rainfall of a given duration that might reasonably be expected in the Southwestern States.

The mass curves of Fig. 1(*a*) indicate the type of intensity pattern that probably comprised the rainfall totals shown in Fig. 1(*b*), and from these two graphs an approximate rainfall histogram can readily be taken for purposes of hydraulic design of engineering structures. It is important to note that none of the rains platted in Fig. 1(*b*) occurred in midwinter, and relatively few occurred outside the season of summer rains, July to September. It may be surmised from this fact that most of these intense rains covered relatively small areas, typical of summer-type storms.

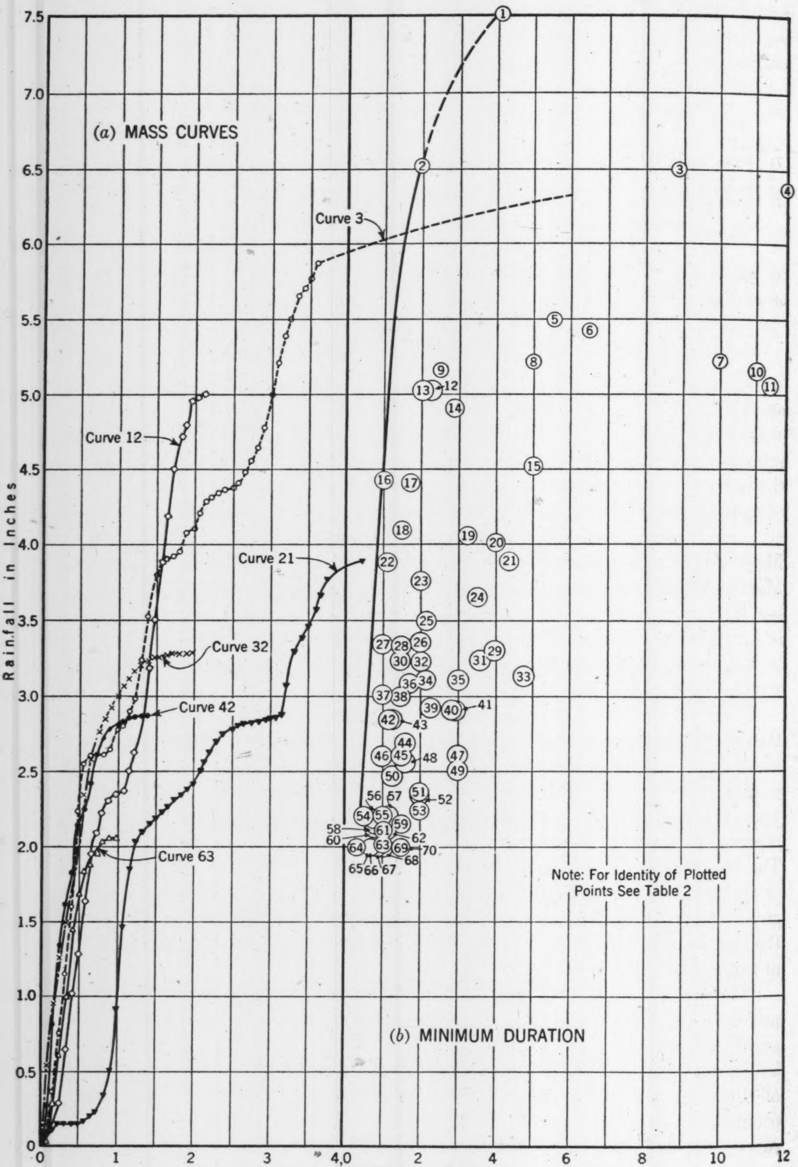


FIG. 1.—RECORDS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA (FROM ORIGINAL NOTES OF WEATHER BUREAU OBSERVERS)

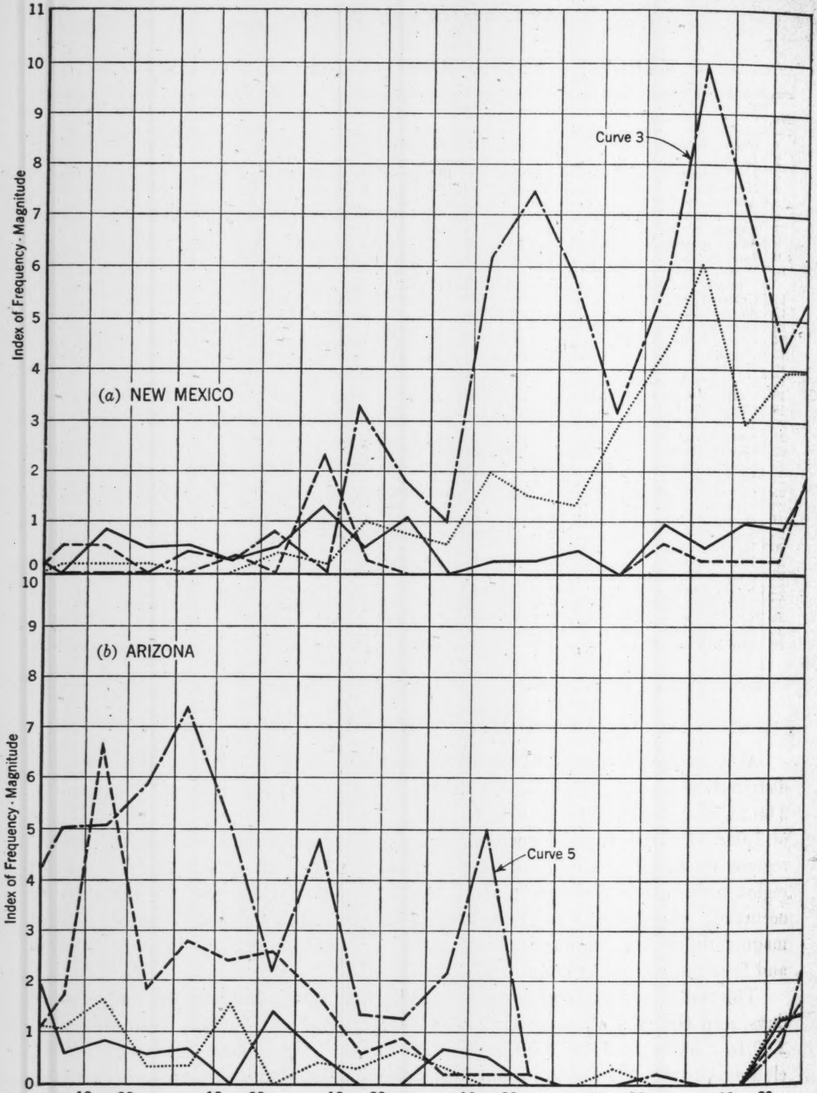
TABLE 2.—KEY TO RAINFALL DATA IN FIG. 1(*b*)

No.	Location	Date	No.	Location	Date
1	Cerro, N. Mex.	August 17, 1922	37	Hope, N. Mex.	June 5, 1921
2	El Paso, Tex.	July 9, 1881	38	Hope, N. Mex.	May 5, 1919
3	Las Cruces, N. Mex.	August 29, 1935	39	Carlsbad, N. Mex.	October 12, 1916
4	Hermosa, N. Mex.	August 30, 1925	40	Torrance, N. Mex.	October 1, 1923
5	Artesia, N. Mex.	July 24, 1911	41	Hachita, N. Mex.	August 6, 1931
6	Casa Grande Ruin, Ariz.	August 1, 1906	42	Pima, Ariz. (near)	August 2, 1939
7	Solano, N. Mex.	July 9, 1929	43	Three Rivers, N. Mex.	August 7, 1919
8	Albuquerque, N. Mex.	October 9, 1865	44	Roswell, N. Mex.	September 16, 1923
9	Ione, N. Mex.	August 21, 1923	45	Valmora, N. Mex.	August 6, 1929
10	White Sands Nat'l Monument, N. Mex.	September 13, 1941	46	Duran, N. Mex.	September 16, 1915
11	Crown King, Ariz.	August 6, 1918	47	Bell Ranch, N. Mex.	June 19, 1926
12	Sierra Ancha, Ariz.	August 5, 1939	48	Levy, N. Mex.	August 15, 1917
13	Carnegie Desert Lab., Ariz.	July 2, 1910	49	Trementina, N. Mex.	August 29, 1916
14	Crown King, Ariz.	August 11, 1927	50	Carlsbad, N. Mex.	July 30, 1916
15	Tatum, N. Mex.	September 14, 1927	51	Gallinas R.S., N. Mex.	May 29, 1930
16	Hobbs, N. Mex.	June 7, 1918	52	Jemez Springs, N. Mex.	July 7, 1923
17	Elephant Butte, N. Mex.	August 7, 1898	53	Jemez Springs, N. Mex.	August 3, 1933
18	Santa Marguerita, Ariz.	August 22, 1935	54	Lake Alice, N. Mex.	August 30, 1936
19	Clovis, N. Mex.	August 4, 1912	55	Roswell, N. Mex.	September 14, 1923
20	Diener, N. Mex.	July 3, 1924	56	Pastura, N. Mex.	August 11, 1933
21	Las Cruces, N. Mex.	September 21, 1941	57	Vermejo Park, N. Mex.	September 9, 1911
22	Clovis, N. Mex.	July 22, 1930	58	Camp 23A, Whitlock Valley, Ariz.	September 6, 1939
23	Obar, N. Mex.	July 2, 1906	59	Harvey's Upper Ranch, N. Mex.	July 27, 1919
24	Ft. Sumner, N. Mex.	August 13, 1908	60	Peoria, Ariz.	August 6, 1939
25	Corona, N. Mex.	August 14, 1934	61	Silver City, N. Mex.	July 20, 1929
26	Mosquero, N. Mex.	May 30, 1938	62	Mescalero, N. Mex.	August 28, 1923
27	Hope, N. Mex.	August 22, 1908	63	Tucson, Ariz.	July 31, 1935
28	Santa Rosa, N. Mex.	May 30, 1930	64	Cuervo, N. Mex.	July 21, 1926
29	Roy, N. Mex.	May 30, 1938	65	Deming, N. Mex.	July 18, 1896
30	Elk, N. Mex.	September 25, 1906	66	Newman, N. Mex.	September 29, 1915
31	Ancho, N. Mex.	June 26, 1937	67	Jemez Springs, N. Mex.	July 12, 1910
32	Sierra Ancha, Ariz.	September 10, 1933	68	Mineral Hill, N. Mex.	April 23, 1919
33	Marinette, Ariz.	August 6, 1939	69	Laguna, N. Mex.	July 22, 1912
34	Lake Alice, N. Mex.	August 4, 1935	70	Gallinas R.S., N. Mex.	August 16, 1919
35	San Marcial, N. Mex.	August 29, 1895	71	Pima, Ariz.	September 16, 1939
36	Lindrith, N. Mex.	July 30, 1931	72	Rodeo, N. Mex.	August 13, 1940

SEASONAL DISTRIBUTION OF HIGH RAINFALLS

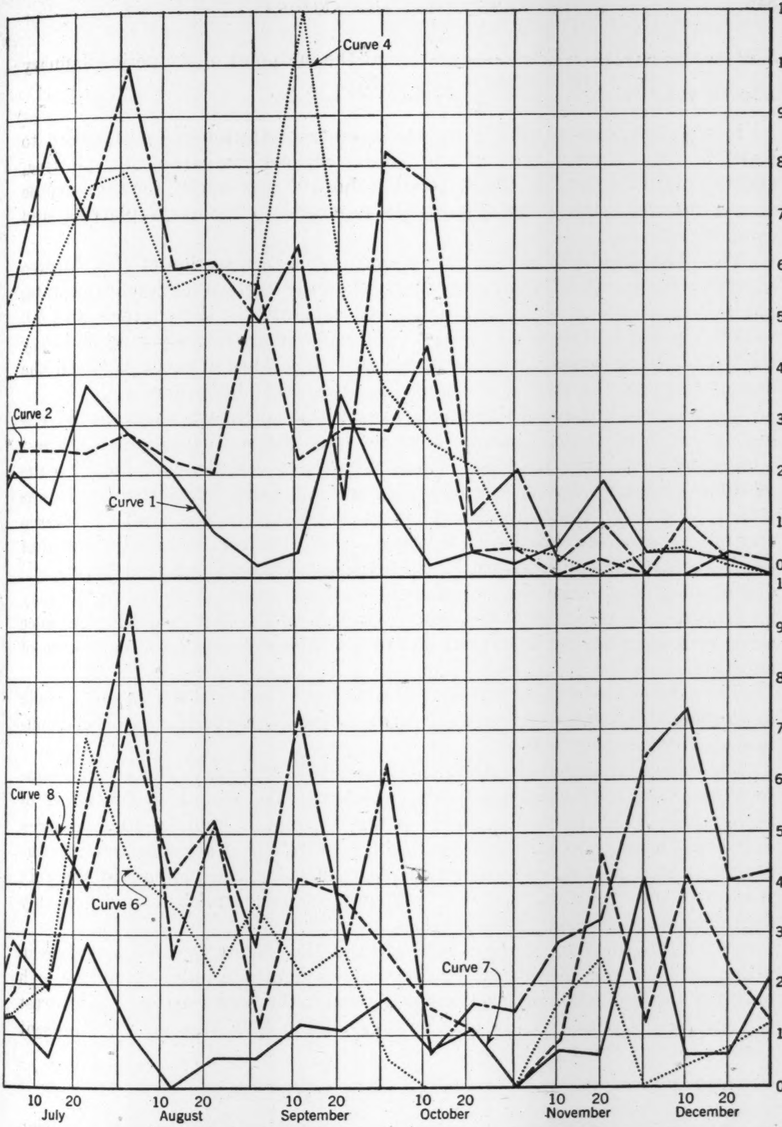
A separate analysis furnishes additional information on the seasonal distribution of high rainfalls. A tabulation was made of all rainfalls exceeding 2.00 in. in a 1-day period for 268 stations in the two states, comprising a total of 7,048 station-years of record. Stations were grouped into eight general regions by dividing each state approximately into four quadrants. For each region, rainfall depths were platted against the day of the year on which each occurred. An index was thus derived that weights both the frequency and magnitude of high rains, and is platted for the four regions in New Mexico and four regions in Arizona in Fig. 2.

The index was derived by dividing the year into periods of ten consecutive days, and the rainfall magnitude into categories of half inches, beginning with 2.00 to 2.50 in., 2.50 to 3.00 in., and so forth. For a given 10-day period including all stations, the number of rainfalls that fell in each magnitude category was multiplied by the average value of the magnitude category, and the sum of the products times 100, divided by the station-years of record for the region, was used as the index of magnitude-frequency. For example, in the upper Gila River and southeastern Arizona area, 35 stations were used, comprising 976 station-years of record. Between January 1 and January 10, there were five rains between 2.00 in. and 2.50 in. in magnitude, and one rain between



	(a) New Mexico; Number of:	Stations	Station-years
1	Upper Rio Grande watershed and northwestern New Mexico	37	925
2	Lower Rio Grande watershed and southwestern New Mexico	25	802
3	Canadian River watershed and northeastern New Mexico	39	1,068
4	Pecos River watershed and southeastern New Mexico	47	1,227

Fig. 2.—Seasonal Variation of Rainfall Frequency (Magnitude



	(b) Arizona; Number of:	Stations	Station-years
5	Verde River watershed and northwestern Arizona	43	988
6	Lower Gila River watershed and southwestern Arizona	27	671
7	Little Colorado River watershed and northeastern Arizona	15	391
8	Upper Gila River watershed and southeastern Arizona	35	976

Index Derived from Rains Exceeding 2.00 In. in One Day)

5.50 in. and 6.00 in. The index platted at the midpoint of the period January 1 to 10 was $100 \times \frac{(5 \times 2.25) + (1 \times 5.75)}{976} = 1.74$.

It was necessary to divide by the number of station-years of record to make the computed indexes of the various regions comparable for platting, and the multiplication by the arbitrary value 100 was simply for the purpose of making the platted indexes whole numbers to facilitate platting and comparison.

The platted indexes in Fig. 2 show that the eastern section of New Mexico experiences high rainfalls more frequently than the western section of the state; but all sections receive high rainfalls predominantly in the summer and fall seasons, whereas November through March is comparatively free of heavy rainfalls. The heaviest rainfalls in western New Mexico occur between the first of July and the middle of August, and again in mid-September.

In Arizona, however, all regions except the northeast quadrant have a high index in summer, whereas in winter only the southeast and northwest quadrants have high indexes. These differences can be explained by the relation of topography, geographic position, and source of moisture. It was stated previously that summer moisture originates in the Gulf of Mexico and moves into New Mexico and Arizona primarily from the south, and that the Pecos Area in southeastern New Mexico sometimes is affected by tropical disturbances that cause great rainfalls in southern and central Texas. It may be expected, therefore, that the eastern section of the state would experience more high rains than the western because of the proximity to the source of moisture.

The southwestern section of New Mexico has a higher frequency-magnitude index than the northwest, since the latter is in the rain shadow of the mountainous southwestern section.

The index for regions in Arizona in summer is highest in the mountainous southeast and northwest quadrants, which can be related to the effect of topography. The index for southwestern Arizona is high in midsummer—July and August—because of the convective storms that are common over the desert at that season. Northeastern Arizona is on the plateau in the rain shadow of the Mogollon Rim and the mountainous central belt, and probably for this reason the index is low.

In winter, when moisture is derived from the Pacific Ocean and moves in primarily from the Southwest, the frequency-magnitude index should be high only for the mountainous sections of Arizona, the southeast and northwest quadrants, and relatively low in other areas and in New Mexico. The present study supports this hypothesis.

Frequency of High Rainfalls

Recurrence-interval curves were drawn for each standard rain-gage station in New Mexico and Arizona whose records exceed 15 years, and for certain stations of shorter record, in order to derive values of 24-hr rainfall that may be expected to be equaled or exceeded in a given period of years. These data are useful in the design of engineering structures, especially where it is necessary to design for a rain greater than has been experienced in the past. The data also indicate the recurrence-interval of rainfalls that have occurred in the past.

In this analysis the highest one-day rainfall in each year of record at the station was tabulated, the data arranged in order of magnitude, and the curve platted by the "exceedance interval," or "California," method. From the platted curves, the 24-hr rainfall that probably will be equaled or exceeded in 5, 10, 15, 20, 35, 50, 75, and 100 years, was tabulated and the data are presented in Table 3(*a*) for New Mexico stations and Table 3(*b*) for Arizona stations.

The rainfall data used were recorded by standard nonrecording rain gages maintained by the Weather Bureau. The rainfall depths were taken from the original records on which, in about half the cases, the observer had noted the time of beginning and time of ending of the precipitation. By use of these notations of times it was possible to ascertain the greatest rainfall in a 24-hr period, which was the value used in the frequency array wherever possible. Particularly in the case of summer rain, the precipitation often starts in mid-afternoon and ends during the night, which is a period of less than 24 hr. Many observers read the gage at sunset; so, in the foregoing example, if no rain fell on the day following, the sum of the two sunset readings equaled the amount that fell in a 24-hr period. Rains continuing through two or more days could not be broken down accurately to determine the largest amount in a 24-hr period and, therefore, the largest amount recorded for one day was used in the recurrence-interval array. The method used approximates, closely, the actually greatest fall in a 24-hr period, since a large percentage of the high rains fell in summer and lasted less than 24 hr.

The characteristics of summer and non-summer precipitation were considered sufficiently distinct and important to warrant separation in the recurrence-interval determinations. Arrays were made separately for the highest rainfall recorded each year between June 1 and September 30 (the summer period), and the highest rain recorded in each complete year. Certain stations may be expected to experience the same 24-hr rainfall at a given recurrence-interval in summer as they will experience in the entire year as can be seen in the tabulations of 50, 75, and 100-yr frequencies. This simply indicates that that station characteristically received the largest 24-hr rainfalls in the summer period and, therefore, the recurrence-interval curves for summer and all year would be practically identical.

Winter values of rainfall amounts probably are somewhat less accurate than summer values owing to the effect of snow. One source of error was the failure of a gage to catch the full snowfall, but probably a more important error was introduced by the general policy of assuming that 1 in. of snow equaled 0.10 in. of precipitation in the form of water. Investigations by the Weather Bureau indicate that, in particular instances, much larger amounts of precipitation were assumed by the use of this policy than actually fell.

The compilation of a recurrence-interval array necessarily involves some exercise of judgment. In many cases, particular months were missing from a

TABLE 3.—FREQUENCY-
(Data from Records Pub-

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
								(a) NEW
1	Dulce	6,767	18.13	34	1906–1915; 1917–1939	33	1.16	1.34
2	Aztec	5,590	9.06	38	1895–1901; 1911–1916; 1918–1923; 1925–1939	34	.91	1.01
3	Fruitland	5,165	6.68	35	1893; 1895; 1897; 1904–1915; 1917–1922; 1939	22	.79	.95
4	Farmington	5,220	8.35	25	1914–1930; 1933–1939	24	.87	.94
5	Bloomfield	5,500	8.79	41	1905–1939	35	.90	.97
6	Haynes	6,600	11.80	16	1910–1917; 1920–1926	15	1.16	1.20
7	Tohatchi	6,800	11.07	24	1915; 1918; 1923; 1926–1927; 1929–1939	16	1.13	1.19
8	Crownpoint	6,800	11.15	26	1914–1919; 1921–1939	25	1.05	1.11
9	Gamerco	6,785	11.94	22	1922–1939	18	.91	1.12
10	Ft. Wingate	6,997	14.45	48	1897–1910	14	1.23	1.44
11	McGaffey Ranger Station	8,000	17.77	27	1916–1939	24	1.10	1.41
12	Bluewater	6,732	10.28	34	1897–1902; 1909–1917; 1926–1939	29	1.06	1.20
13	Ramah	7,200	13.68	15	1922; 1924; 1927–1939	15	1.36	1.68
14	San Fidel	6,100	10.37	20	1920–1939	20	1.20	1.22
15	Black Rock	6,455	12.29	31	1909–1910; 1912; 1914–1939	29	1.01	1.10
16	Chama	7,851	22.10	44	1893–1896; 1905–1918; 1921–1939	37	1.07	1.40
17	Cerro	7,668	13.98	29	1910–1916; 1918–1930; 1932–1939	28	1.20	1.33
18	Red River Canyon	8,956	21.65	34	1906–1907; 1910–1939	32	1.02	1.19
19	Tres Piedras	8,076	16.05	40	1905–1910; 1912–1914; 1916–1939	33	1.11	1.28
20	Aspen Grove Ranch	9,000	23.38	31	1897–1898; 1910–1939	32	1.10	1.40
21	Eagle Rock Ranger Station	8,000	15.70	17	1914–1930	17	1.07	1.22
22	Bateman's Ranch	8,900	23.48	31	1910–1939	30	1.24	1.46
23	Taos Canyon	8,959	19.52	31	1909–1939	31	1.04	1.22
24	Taos	6,983	12.61	47	1893–1895; 1902–1939	41	.93	1.08
25	Truchas	7,935	15.55	31	1905; 1909; 1910–1916; 1918–1939	31	1.14	1.29
26	Espanola	5,590	9.96	35	1895–1900; 1902; 1905; 1907–1909; 1911–1918; 1920–1929; 1938–1939	31	1.04	1.14
27	Selsor Ranch	8,500	22.19	28	1894; 1912–1926; 1928–1939	28	1.30	1.46
28	Alamos Ranch	7,328	18.13	29	1911–1916; 1918–1920; 1922–1939	27	1.34	1.57
29	Frijoles Canyon	6,200	15.38	16	1924–1939	16	1.54	1.66
30	Jemez Springs	6,100	18.07	30	1910–1939	30	1.49	1.63
31	Lee's Ranch	8,691	22.13	16	1924–1939	16	1.89	2.08
32	Santa Fe Canyon	8,100	17.69	13	1910–1916; 1923–1928	13	1.14	1.26
33	Santa Fe	6,994	14.39	90	1850; 1853–1861; 1863–1866; 1868–1882; 1885–1936; 1938–1939	83	1.15	1.29
34	Stanley	6,317	12.20	29	1909–1913; 1916–1926; 1929–1939	27	1.32	1.39
35	Albuquerque	5,314	8.18	69	1850–1854; 1856–1861; 1864–1865; 1879; 1889–1890; 1892–1908; 1910–1939	63	.95	1.13
36	Laguna	5,840	11.14	30	1905–1914; 1919–1921; 1927–1934; 1936–1939	25	1.27	1.48
37	Estancia	6,300	15.14	26	1905–1907; 1909–1911; 1913–1917; 1923–1927; 1937–1939	19	1.21	1.35
38	McIntosh	6,355	13.04	12	1928–1939	12	1.26	1.53
39	Los Lunas	4,900	8.46	51	1893–1897; 1899–1901; 1903–1909; 1911–1939	44	1.16	1.31
40	Tajique	7,100	19.68	20	1910–1918; 1920–1939	29	1.47	1.73
41	Mountainair	6,475	16.03	33	1902–1915; 1917; 1919–1921; 1923; 1928–1939	31	1.45	1.70
42	Datil	7,850	13.22	28	1906–1908; 1916–1924; 1926–1939	26	1.19	1.27
43	Magdalena	6,556	12.68	37	1905–1915; 1918–1925; 1927–1931; 1934–1939	30	1.14	1.31
44	Socorro	4,600	10.19	53	1893–1894; 1896–1897; 1899–1901; 1904–1939	43	1.19	1.38
45	San Marcial	4,447	9.11	83	1895–1899; 1901; 1904–1909; 1912–1919; 1921–1939	39	1.17	1.38
46	Glorieta Ranch	6,300	11.88	19	1910–1928	18	1.10	1.40
47	Chloride Ranger Station	6,500	14.84	23	1922–1939	18	1.26	1.49

---

[1] Average of the maximum 1-day rain recorded for the
[2] Average of the maximum 1-day rain recorded in each

MAGNITUDE OF DEPRECIATION
lished by U. S. Weather Bureau)

ONE-DAY RAINFALL WHICH PROBABLY WILL BE EQUALED OR EXCEEDED ONCE IN:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
MEXICO																
1.33	1.65	1.75	2.20	2.06	2.60	2.30	2.90	2.91	3.65	3.40	4.25	4.00	5.00	4.50	5.60	1
1.25	1.35	1.48	1.59	1.60	1.70	1.69	1.80	1.83	1.98	1.92	2.09	2.02	2.19	2.10	2.28	2
1.13	1.24	1.40	1.51	1.58	1.66	1.70	1.78	1.96	2.00	2.12	2.18	2.34	2.35	2.50	2.50	3
1.21	1.29	1.51	1.58	1.69	1.71	1.80	1.82	2.02	2.05	2.20	2.20	2.38	2.38	2.50	2.50	4
1.30	1.30	1.49	1.50	1.57	1.58	1.60	1.61	1.65	1.70	1.68	1.71	1.68	1.75	1.70	1.78	5
1.73	1.73	2.05	2.05	2.20	2.20	2.31	2.31	2.50	2.50	2.65	2.65	2.77	2.77	2.86	2.86	6
1.70	1.70	2.31	2.31	2.70	2.70	3.05	3.05	3.78	3.78	4.30	4.30	4.95	4.95	5.50	5.50	7
1.48	1.50	1.71	1.72	1.82	1.85	1.91	1.92	2.05	2.08	2.16	2.16	2.23	2.23	2.30	2.30	8
1.38	1.70	1.60	2.01	1.70	2.16	1.78	2.22	1.89	2.35	1.92	2.50	1.98	2.47	2.00	2.50	9
1.50	1.86	1.78	2.28	1.93	2.53	2.04	2.72	2.28	3.14	2.43	3.42	2.60	3.77	2.70	4.00	10
1.58	1.88	1.90	2.25	2.05	2.46	2.17	2.60	2.37	2.92	2.50	3.11	2.63	3.36	2.71	3.50	11
1.40	1.64	1.68	2.01	1.81	2.25	1.92	2.41	2.12	2.77	2.28	3.00	2.40	3.30	2.50	3.50	12
2.10	2.40	2.55	2.75	2.73	2.91	2.85	3.05	3.10	3.22	3.25	3.35	3.41	3.48	3.50	3.50	13
1.54	1.70	1.68	1.92	1.72	2.06	1.78	2.15	1.81	2.30	1.85	2.40	1.88	2.50	1.89	2.55	14
1.40	1.51	1.78	1.89	1.98	2.08	2.11	2.21	2.42	2.50	2.67	2.69	2.90	2.90	3.10	3.10	15
1.46	1.70	1.74	2.08	1.89	2.33	1.99	2.54	2.17	3.00	2.28	3.32	2.40	3.75	2.50	4.10	16
1.42	1.62	2.00	2.22	2.42	2.65	2.79	3.00	3.58	3.74	4.20	4.30	4.95	5.00	5.60	5.60	17
1.31	1.51	1.60	1.78	1.75	1.90	1.88	2.00	2.10	2.20	2.29	2.31	2.48	2.48	2.60	2.60	18
1.50	1.60	1.81	1.89	2.00	2.06	2.12	2.20	2.41	2.46	2.60	2.62	2.83	2.85	3.00	3.00	19
1.48	1.96	1.72	2.42	1.82	2.70	1.91	2.90	2.05	3.31	2.16	3.61	2.26	3.90	2.31	4.12	20
1.40	1.49	1.57	1.61	1.63	1.69	1.70	1.72	1.80	1.81	1.87	1.89	1.92	1.92	1.99	1.99	21
1.62	1.85	1.88	2.14	1.99	2.30	2.08	2.40	2.21	2.62	2.31	2.78	2.40	2.90	2.50	3.00	22
1.33	1.55	1.55	1.69	1.65	1.78	1.71	1.82	1.85	1.92	1.92	2.00	2.01	2.05	2.10	2.10	23
1.30	1.39	1.48	1.55	1.53	1.61	1.59	1.65	1.66	1.70	1.70	1.74	1.74	1.78	1.78	1.80	24
1.66	1.91	2.05	2.24	2.22	2.37	2.32	2.46	2.51	2.59	2.62	2.69	2.75	2.75	2.82	2.82	25
1.44	1.50	1.65	1.71	1.72	1.83	1.79	1.91	1.88	2.02	1.91	2.11	1.98	2.20	2.00	2.28	26
1.70	1.95	2.00	2.22	2.18	2.36	2.30	2.48	2.52	2.68	2.70	2.80	2.89	2.93	3.00	3.05	27
1.80	2.03	2.18	2.59	2.40	2.90	2.58	3.18	2.91	3.70	3.19	4.11	3.46	4.60	3.68	4.95	28
2.18	2.18	2.62	2.62	2.80	2.80	3.00	3.00	3.25	3.35	3.46	3.60	3.65	3.89	3.80	4.10	29
2.01	2.19	2.38	2.61	2.51	2.80	2.64	2.97	2.83	3.25	2.99	3.41	3.10	3.63	3.20	3.80	30
2.72	2.85	3.04	3.21	3.20	3.40	3.30	3.51	3.43	3.79	3.52	3.90	3.60	4.10	3.70	4.20	31
1.58	1.63	1.80	1.86	1.91	1.98	2.00	2.07	2.19	2.21	2.30	2.30	2.40	2.40	2.50	2.50	32
1.52	1.60	1.83	1.97	2.05	2.22	2.21	2.48	2.58	2.98	2.81	3.39	3.13	3.95	3.40	4.50	33
1.79	1.90	2.17	2.23	2.40	2.41	2.57	2.59	2.85	2.85	3.09	3.09	3.33	3.33	3.50	3.50	34
1.30	1.50	1.89	2.14	2.28	2.63	2.60	3.02	3.35	3.95	3.89	4.69	4.50	5.45	5.00	6.00	35
1.68	1.90	1.88	2.29	1.94	2.50	1.99	2.69	2.06	3.00	2.10	3.28	2.13	3.53	2.17	3.75	36
1.80	1.84	2.15	2.15	2.35	2.35	2.45	2.45	2.71	2.71	2.90	2.90	3.10	3.10	3.21	3.21	37
1.62	1.93	2.02	2.31	2.30	2.58	2.50	2.78	2.93	3.12	3.25	3.40	3.60	3.70	3.89	3.90	38
1.61	1.78	1.91	2.04	2.01	2.15	2.10	2.21	2.23	2.35	2.31	2.41	2.40	2.50	2.48	2.57	39
1.98	2.18	2.38	2.56	2.60	2.79	2.80	2.97	3.18	3.30	3.45	3.52	3.71	3.79	4.00	4.00	40
1.95	2.18	2.29	2.49	2.46	2.65	2.61	2.78	2.90	2.99	3.11	3.15	3.30	3.35	3.43	3.49	41
1.51	1.58	1.71	1.73	1.81	1.82	1.90	1.90	2.02	2.02	2.11	2.11	2.21	2.21	2.30	2.30	42
1.50	1.78	1.70	2.07	1.78	2.19	1.82	2.29	1.90	2.44	1.95	2.57	1.98	2.68	2.00	2.76	43
1.53	1.87	2.05	2.59	2.41	3.10	2.71	3.51	3.37	4.50	3.87	5.12	4.50	6.20	5.00	7.00	44
1.70	1.90	2.40	2.62	2.90	3.11	3.35	3.50	4.30	4.44	5.00	5.10	6.00	6.00	6.70	6.70	45
1.43	1.70	1.69	1.90	1.83	2.02	1.95	2.11	2.15	2.30	2.30	2.40	2.45	2.53	2.58	2.60	46
1.65	1.83	1.81	2.21	1.90	2.48	1.98	2.69	2.07	3.03	2.12	3.30	2.20	3.60	2.24	3.80	47

period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	Record Used for Frequency Array		Average of Maximum 1-Day Rain Each:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(a) New Mexico—	
48	Hermosa	7,200	17.79	15	1914–1926	13	1.75	2.18
49	Elephant Butte	4,265	9.91	55	1895–1904; 1908–1916; 1918–1921; 1923–1939	40	1.38	1.42
50	Kingston	6,500	18.18	25	1916–1939	24	1.59	1.72
51	Hillsboro	5,236	12.61	34	1896–1900; 1904–1909; 1911; 1925–1933; 1935–1939	26	1.48	1.54
52	Garfield	4,100	10.09	33	1894–1899; 1905–1917; 1920–1923; 1925; 1927	25	1.24	1.27
53	Lake Valley	5,412	13.41	33	1905–1923; 1927–1939	32	1.52	1.59
54	Jornada Experimental Range	4,150	9.28	26	1914–1939	26	1.05	1.14
55	Deming	4,331	9.66	64	1893–1939	47	1.15	1.23
56	Las Cruces (Agricultural College)	3,909	8.66	83	1892–1939	48	1.11	1.17
57	Cambray	4,225	8.92	39	1899–1933; 1936–1939	39	1.16	1.28
58	Newman	3,989	9.05	24	1909–1932	24	1.29	1.35
59	Lanark	4,156	8.20	25	1899–1918; 1921	21	1.14	1.18
60	Columbus	4,054	9.58	31	1909–1939	31	1.25	1.32
61	Horse Springs	7,070	13.79	31	1900–1902; 1906–1907; 1909–1921; 1923–1927; 1930–1938	32	1.03	1.20
62	Luna Ranger Station	7,300	16.18	40	1903–1906; 1908–1918; 1920–1939	35	1.09	1.18
63	Hood Ranger Station	5,838	14.19	23	1916 1939	24	1.01	1.24
64	Alma	4,800	14.89	26	1894–1901; 1903–1905; 1910–1925	27	1.12	1.34
65	Cliff (Gila)	4,470	14.22	32	1895–1900; 1905–1910; 1912–1914; 1916; 1931–1939	25	1.44	1.61
66	Pinos Altos	7,000	22.89	29	1911–1932; 1934–1939	28	1.48	1.65
67	Mimbres	6,250	18.33	26	1905–1919; 1930–1939	25	1.35	1.54
68	Ft. Bayard	6,152	15.70	71	1897–1919; 1921–1939	42	1.38	1.54
69	Silver City	5,937	17.38	37	1901; 1912–1939	29	1.46	1.52
70	Redrock	4,150	12.30	35	1905–1939	35	1.32	1.53
71	Lordsburg	4,245	9.57	59	1893–1939	47	1.10	1.26
72	Hachita	4,504	10.76	30	1909–1939	31	1.34	1.44
73	Rodeo	4,118	11.04	31	1909–1939	31	1.29	1.48
74	Lake Alice	6,950	19.85	31	1909–1911; 1913–1939	30	1.48	1.86
75	Vermejo Park	7,600	17.13	19	1904–1920; 1924–1925	19	1.25	1.38
76	Raton	6.660	15.61	44	1895–1905; 1909; 1912–1939	40	1.75	1.98
77	Des Moines	6,622	17.23	24	1916–1939	24	1.64	1.79
78	Dawson	6,396	15.54	31	1909–1939	31	1.44	1.76
79	Maxwell	5,894	14.53	27	1905; 1908–1931	25	1.40	1.60
80	Cimarron	6,427	15.56	36	1904–1939	36	1.30	1.74
81	Clayton	5,049	15.62	35	1896–1899; 1909–1911; 1913–1939	34	1.49	1.78
82	Springer	5,857	14.18	51	1893–1894; 1896–1897; 1900–1901; 1904–1939	42	1.44	1.59
83	Miami	6,000	15.73	32	1908–1917; 1919–1926; 1928–1939	30	1.47	1.68
84	Taylor	5,661	15.57	24	1909–1932	24	1.20	1.35
85	Pasamonte	5,650	15.11	30	1910–1939	30	1.57	1.71
86	Black Lake	8,348	16.21	31	1909–1939	31	1.33	1.46
87	Aurora	8,849	20.45	31	1909–1939	31	1.45	1.77
88	Vance (Thomas)	4,825	15.28	29	1911–1939	29	1.57	1.82
89	Abbott	5,771	14.98	32	1909–1939	31	1.59	1.82
90	Chacon	8,510	20.38	31	1909–1939	31	1.20	1.39
91	Levy	6,252	17.24	30	1909–1918; 1922–1939	28	1.68	1.86
92	Palo Verde	5,880	15.97	29	1912–1939	28	1.39	1.86
93	Roy	5,884	15.18	31	1909-1939	31	1.55	1.92
94	Ft. Union	6,885	18.01	62	1895; 1897–1918	23	1.71	1.87
95	Solano	5,622	16.71	31	1909–1939	31	1.89	2.14
96	Hoosier Ranch	5,680	15.40	29	1911–1939	29	1.80	2.29
97	Valmora	6,200	16.51	22	1917–1923; 1926–1939	21	1.52	1.67
98	Mosquero	5,579	16.02	25	1915–1939	25	1.78	1.96
99	Ione	4,850	16.57	30	1916–1939	24	2.19	2.40
100	Bell Ranch	5,400	15.24	41	1899–1939	41	1.62	1.91
101	Obar	4,225	16.36	34	1906–1911; 1914–1923; 1926–1939	30	1.80	2.13
102	Trementina	5,000	15.68	22	1908–1918; 1920–1928	20	1.52	1.91
103	Logan	3,851	16.29	34	1906-1920; 1922–1937; 1939	32	1.85	1.91
104	Tucumcari No. 1	4,100	16.35	35	1905–1939	35	1.74	2.05

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		No.
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
2.25	3.10	2.95	3.85	3.41	4.28	3.80	4.55	4.55	5.15	5.10	5.50	5.80	5.95	6.30	6.30	48
1.75	1.81	2.29	2.38	2.67	2.70	2.95	3.00	3.60	3.60	4.06	4.06	4.60	4.60	5.00	5.00	49
2.10	2.30	2.52	2.73	2.80	2.97	2.99	3.13	3.35	3.50	3.64	3.78	3.95	4.00	4.20	4.20	50
1.98	2.00	2.32	2.34	2.55	2.55	2.68	2.68	2.97	2.97	3.12	3.12	3.34	3.34	3.50	3.50	51
1.70	1.71	1.91	1.91	2.00	2.00	2.05	2.05	2.15	2.15	2.20	2.20	2.26	2.26	2.30	2.30	52
1.90	1.98	2.49	2.51	2.90	2.95	3.22	3.29	4.00	4.03	4.60	4.60	5.40	5.40	6.00	6.00	53
1.40	1.53	1.76	1.81	1.96	2.00	2.12	2.17	2.45	2.46	2.68	2.71	2.95	2.97	3.20	3.20	54
1.65	1.70	1.84	1.91	1.92	1.99	1.98	2.01	2.02	2.08	2.05	2.10	2.06	2.13	2.08	2.12	55
1.45	1.55	1.99	2.09	2.36	2.46	2.68	2.76	3.35	3.40	3.88	3.98	4.50	4.58	5.00	5.00	56
1.59	1.74	1.89	2.01	2.04	2.11	2.13	2.21	2.35	2.39	2.49	2.51	2.60	2.62	2.70	2.70	57
1.63	1.66	1.88	1.90	2.00	2.03	2.12	2.15	2.33	2.35	2.47	2.50	2.60	2.63	2.70	2.70	58
1.65	1.65	2.02	2.02	2.30	2.30	2.44	2.44	2.80	2.80	3.05	3.05	3.35	3.35	3.58	3.58	59
1.75	1.80	2.20	2.21	2.48	2.50	2.68	2.69	3.10	3.10	3.40	3.40	3.75	3.75	4.00	4.00	60
1.33	1.55	1.50	1.76	1.60	1.88	1.66	1.98	1.80	2.10	1.89	2.20	1.97	2.30	2.01	2.38	61
1.41	1.68	1.68	1.92	1.82	2.06	1.94	2.19	2.18	2.35	2.32	2.50	2.50	2.64	2.62	2.76	62
1.35	1.52	1.53	1.70	1.62	1.80	1.68	1.89	1.78	2.03	1.84	2.11	1.90	2.20	1.92	2.29	63
1.40	1.68	1.59	1.92	1.69	2.08	1.78	2.18	1.90	2.37	1.99	2.50	2.08	2.65	2.12	2.78	64
1.81	2.10	2.19	2.38	2.31	2.50	2.41	2.59	2.64	2.75	2.79	2.85	2.90	2.95	3.00	3.01	65
1.90	2.05	2.15	2.30	2.30	2.40	2.41	2.50	2.60	2.69	2.78	2.80	2.90	2.90	3.00	3.00	66
1.78	2.00	2.08	2.29	2.25	2.41	2.38	2.52	2.60	2.70	2.75	2.82	2.90	2.98	3.00	3.09	67
1.80	1.98	2.21	2.39	2.49	2.62	2.70	2.81	3.10	3.20	3.41	3.50	3.80	3.80	4.00	4.00	68
1.90	1.92	2.11	2.13	2.22	2.25	2.30	2.31	2.38	2.45	2.42	2.51	2.48	2.60	2.50	2.68	69
1.70	2.00	1.92	2.40	2.05	2.60	2.13	2.79	2.29	3.09	2.39	3.30	2.40	3.55	2.58	3.70	70
1.43	1.61	1.77	1.98	1.98	2.18	2.11	2.34	2.43	2.68	2.63	2.90	2.88	3.40	3.05	3.40	71
1.94	2.07	2.39	2.50	2.60	2.75	2.79	2.93	3.10	3.28	3.32	3.50	3.60	3.78	3.80	4.00	72
1.80	2.03	2.13	2.32	2.30	2.45	2.41	2.52	2.60	2.68	2.70	2.79	2.83	2.85	2.90	2.90	73
1.90	2.40	2.28	2.97	2.50	3.30	2.66	3.53	3.00	4.10	3.21	4.43	3.50	4.85	3.70	5.10	74
1.73	1.90	2.01	2.17	2.20	2.30	2.31	2.40	2.55	2.60	2.70	2.72	2.88	2.88	3.00	3.00	75
2.40	2.68	3.19	3.40	3.65	3.85	4.01	4.20	4.85	4.95	5.48	5.59	6.20	6.30	6.80	6.80	76
2.45	2.65	3.00	3.10	3.25	3.30	3.40	3.45	3.63	3.65	3.78	3.80	3.90	3.95	4.00	4.00	77
1.88	2.26	2.38	2.71	2.68	3.00	2.91	3.21	3.43	3.65	3.80	4.00	4.25	4.35	4.60	4.60	78
1.90	2.24	2.37	2.78	2.62	3.02	2.83	3.20	3.25	3.50	3.51	3.70	3.85	3.90	4.10	4.10	79
1.75	2.32	2.31	2.95	2.70	3.35	3.00	3.60	3.60	4.12	4.08	4.49	4.60	4.85	5.00	5.09	80
1.95	2.31	2.45	2.90	2.73	3.30	2.99	3.59	3.45	4.20	3.80	4.70	4.20	5.20	4.50	5.60	81
1.82	2.02	2.28	2.43	2.50	2.70	2.71	2.89	3.15	3.25	3.48	3.50	3.80	3.80	4.00	4.00	82
1.90	2.18	2.20	2.60	2.35	2.88	2.47	3.09	2.68	3.50	2.81	3.80	2.93	4.13	3.00	4.30	83
1.65	1.75	1.88	1.93	1.95	2.00	2.00	2.05	2.09	2.11	2.12	2.17	2.18	2.18	2.20	2.20	84
2.21	2.36	2.61	3.00	2.80	3.35	2.93	3.60	3.18	4.18	3.31	4.59	3.48	5.00	3.60	5.38	85
1.80	1.92	2.03	2.10	2.10	2.16	2.15	2.20	2.22	2.25	2.27	2.29	2.29	2.29	2.30	2.30	86
1.90	2.48	2.49	3.04	2.85	3.40	3.16	3.61	3.80	4.15	4.27	4.50	4.80	4.90	5.20	5.20	87
2.21	2.58	2.69	3.00	2.92	3.16	3.10	3.30	3.40	3.55	3.59	3.70	3.79	3.88	3.90	4.00	88
2.26	2.50	2.82	3.09	3.18	3.45	3.40	3.71	3.90	4.30	4.28	4.69	4.60	5.10	5.00	5.40	89
1.51	1.82	1.80	2.13	1.95	2.32	2.09	2.48	2.33	2.70	2.50	2.89	2.70	3.08	2.84	3.20	90
2.05	2.35	2.39	2.70	2.58	2.82	2.70	2.97	2.98	3.15	3.16	3.29	3.35	3.40	3.50	3.50	91
1.87	2.55	2.20	3.09	2.40	3.35	2.57	2.59	2.88	4.05	3.09	4.40	3.30	4.75	3.50	5.00	92
2.10	2.60	2.65	3.38	3.00	3.90	3.27	4.31	3.80	5.20	4.19	5.84	4.70	6.70	5.00	7.20	93
2.34	2.42	2.68	2.70	2.80	2.82	2.90	2.91	3.05	3.06	3.12	3.12	3.22	3.22	3.30	3.30	94
2.40	2.65	3.00	3.30	3.40	3.75	3.71	4.10	4.40	4.75	4.99	5.30	5.60	5.85	6.20	6.20	95
2.68	3.45	3.30	4.41	3.65	4.95	3.90	5.39	4.40	6.20	4.78	6.80	5.15	7.50	5.40	8.00	96
1.95	2.20	2.27	2.53	2.46	2.71	2.61	2.86	2.90	3.10	3.11	3.30	3.30	3.45	3.50	3.60	97
2.20	2.46	2.42	2.87	2.55	3.08	2.65	3.26	2.80	3.55	2.91	3.77	3.05	3.97	3.10	4.10	98
3.35	3.50	4.30	4.50	4.95	5.10	5.40	5.58	6.30	6.40	7.00	7.10	7.70	7.90	8.30	8.40	99
2.03	2.45	2.40	2.99	2.60	3.25	2.78	3.50	3.10	3.95	3.32	4.28	3.60	4.65	3.80	5.00	100
2.40	2.95	3.10	3.69	3.52	4.10	3.89	4.45	4.65	5.10	5.22	5.62	5.95	6.30	6.50	6.80	101
2.15	2.52	2.98	3.10	3.45	3.50	3.80	3.80	4.40	4.40	4.90	4.90	5.40	5.40	5.78	5.78	102
2.72	2.82	3.17	3.25	3.35	3.42	3.49	3.53	3.70	3.75	3.80	3.85	3.95	3.95	4.00	4.00	103
2.42	2.70	2.85	3.15	3.08	3.40	3.20	3.60	3.40	3.98	3.58	4.23	3.75	4.55	3.82	4.80	104

period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1939 (inches)	Length of record (years)	Record Used for Frequency Array		Average of Maximum 1-Day Rain Each:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
						(a) New Mexico—		
105	Montoya	4,335	14.06	24	1910–1932	23	1.54	1.77
106	San Jon	3,982	15.69	33	1907–1939	33	1.86	2.18
107	Winsor's Ranch	8,800	24.49	45	1897–1939	43	1.32	1.49
108	Harvey's Upper Ranch	9,400	33.43	15	1909–1923	15	1.80	2.05
109	Onava	6,700	15.78	32	1909–1935; 1937–1939	30	1.47	1.69
110	Gallinas Plant. Station	7,500	25.64	25	1907–1930	24	1.74	1.84
111	Mineral Hill	7,050	21.06	20	1905–1921; 1923	18	1.72	1.84
112	Las Vegas	6,400	17.77	56	1893–1939	47	1.63	1.75
113	Pecos Ranger Station	6,900	15.99	22	1916; 1919–1920; 1923–1939	20	1.29	1.37
114	Doretta	6,381	15.79	21	1915–1920; 1925–1938	20	1.46	1.57
115	Cuervo	4,849	13.26	31	1909–1939	31	1.46	1.75
116	Palma	7,000	11.78	26	1906; 1916–1939	25	1.28	1.42
117	Santa Rosa	4,624	14.28	40	1908–1916; 1919–1939	30	1.38	1.68
118	Pastura	5,285	12.69	31	1909–1928; 1930–1939	30	1.53	1.61
119	Vaughn	5,930	13.01	31	1909–1939	31	1.32	1.58
120	Lagunita	4,500	14.34	22	1905–1913; 1917–1926	19	1.52	1.72
121	Ft. Sumner	4,028	15.55	38	1908–1939	32	1.63	1.96
122	Duran	6,272	14.23	25	1908–1932	25	1.29	1.35
123	Melrose	4,400	16.28	32	1908–1926; 1933–1939	26	1.78	1.99
124	St. Vrain	4,250	16.57	28	1913–1939	27	1.68	1.97
125	Clovis	4,262	18.30	29	1911–1939	29	2.32	2.47
126	Torrance	6,433	12.89	19	1909–1925	17	1.53	1.70
127	Corona	6,666	14.77	31	1909–1939	31	1.29	1.36
128	Portales	4,004	17.55	33	1905–1906; 1912–1939	30	1.92	2.25
129	Gallinas Ranger Station	6,636	14.26	31	1909–1939	31	1.26	1.38
130	Elida	4,345	15.04	27	1913–1939	27	1.75	2.00
131	Ancho	6,112	12.47	30	1909–1939	31	1.35	1.42
132	Richland	4,000	14.51	27	1913–1939	27	1.95	2.04
133	Boaz	4,154	13.16	31	1909–1939	31	1.89	2.07
134	Carrizozo	5,438	13.42	32	1908–1935; 1937–1939	31	1.17	1.25
135	Arabela	6,700	19.54	17	1902–1905; 1916–1921; 1930–1939	20	2.28	2.58
136	Capitan	6,348	17.11	31	1909–1939	31	1.49	1.55
137	Ft. Stanton	6,231	15.96	66	1893; 1897; 1900–1939	42	1.37	1.45
138	Oscura No. 2	5,016	9.95	19	1909–1927	19	.95	1.05
139	Roswell	3,602	14.38	62	1894–1939	46	1.60	1.88
140	Three Rivers	4,436	9.56	24	1910–1932	23	1.01	1.16
141	White Tail	7,000	22.88	56	1915–1918; 1920–1939	24	1.65	1.70
142	Tatum	3,950	15.75	21	1919–1925; 1927; 1929–1939	19	2.07	2.23
143	Mescalero	6,627	19.46	28	1911–1916; 1918–1939	28	1.58	1.65
144	Prairieview	3,825	15.11	28	1911–1916; 1918–1939	28	1.66	1.79
145	Tularosa	4,436	9.32	32	1908–1910; 1914–1939	29	1.40	1.45
146	Cloudcroft	8,650	24.56	38	1903–1939	37	1.61	1.76
147	Elk	5,350	17.42	33	1895–1901; 1904–1909; 1922–1939	31	1.96	2.07
148	Lovington	3,900	14.37	32	1910–1915; 1917–1925; 1927–1928; 1932–1939	25	1.81	1.98
149	Mountain Park	6,720	18.42	25	1912–1924; 1928–1929; 1931–1939	24	1.58	1.79
150	Mayhill Ranger Station	6,400	19.63	23	1917–1939	23	1.89	1.92
151	Alamogordo	4,338	11.18	45	1901–1925; 1927–1939	38	1.23	1.39
152	Artesia	3,350	12.29	33	1905–1906; 1910–1939	32	1.60	1.88
153	Weed Ranger Station	7,200	21.27	15	1905–1906; 1916–1929	16	1.83	1.93
154	Hope	4,000	13.63	21	1906–1908; 1916; 1919–1938	24	1.64	1.86
155	Hobbs	3,600	15.45	22	1913–1929; 1932–1933; 1938–1939	21	2.20	2.24
156	Pearl	3,700	13.02	24	1915–1925; 1927–1928; 1930; 1932–1939	22	1.87	2.06
157	Lakewood	3,325	10.95	17	1912–1925; 1927	15	1.71	2.11
158	Lake Avalon	3,200	11.74	26	1915–1939	25	1.62	1.83
159	Carlsbad	3,120	12.83	48	1894–1899; 1901–1939	45	1.72	1.84
160	Oro Grande	4,171	9.43	33	1905; 1910–1939	31	1.46	1.53
161	Loving	3,045	10.94	22	1918–1939	22	1.27	1.51
162	Carson Seep	6,500	19.29	26	1914–1939	26	2.24	2.42

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		No.
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
2.50	2.70	3.30	3.60	3.80	4.20	4.20	4.70	5.00	5.70	5.60	6.40	6.35	7.30	7.00	8.00	105
2.75	3.25	3.31	4.29	3.65	4.95	3.85	5.40	4.25	6.50	4.55	7.22	4.80	8.20	5.00	9.00	106
1.70	1.90	2.00	2.13	2.16	2.26	2.28	2.35	2.45	2.51	2.59	2.60	2.70	2.70	2.79	2.79	107
2.31	2.66	2.67	3.18	2.85	3.45	3.10	3.69	3.26	4.15	3.44	4.45	3.62	4.80	3.76	5.00	108
2.07	2.36	2.50	2.75	2.75	2.91	2.90	3.01	3.16	3.22	3.30	3.35	3.43	3.46	3.50	3.52	109
2.35	2.45	2.82	2.91	3.12	3.21	3.40	3.43	3.90	3.95	4.30	4.30	4.70	4.70	5.00	5.00	110
2.25	2.33	2.66	2.79	2.91	3.05	3.10	3.25	3.50	3.65	3.72	3.93	4.00	4.25	4.20	4.45	111
2.25	2.25	2.55	2.55	2.75	2.75	2.82	2.83	2.98	3.08	3.05	3.23	3.15	3.40	3.20	3.50	112
1.70	2.10	2.41	2.61	2.74	2.90	2.98	3.10	3.45	3.55	3.80	3.85	4.20	4.20	4.50	4.50	113
2.06	2.18	2.39	2.45	2.50	2.54	2.56	2.60	2.65	2.70	2.70	2.74	2.75	2.79	2.79	2.80	114
2.08	2.50	2.42	3.00	2.60	3.28	2.70	3.45	2.92	3.80	3.06	4.05	3.20	4.30	3.30	4.46	115
1.86	2.06	2.62	2.82	3.15	3.37	3.60	3.80	4.60	4.80	5.40	5.50	6.40	6.40	7.20	7.20	116
1.74	2.18	1.91	2.50	2.03	2.68	2.11	2.80	2.27	3.03	2.35	3.20	2.45	3.35	2.50	3.48	117
2.08	2.12	2.40	2.40	2.56	2.56	2.70	2.70	2.88	2.88	3.00	3.00	3.11	3.11	3.20	3.20	118
1.75	2.10	2.19	2.50	2.45	2.75	2.65	2.95	3.10	3.31	3.40	3.59	3.75	3.86	4.00	4.04	119
2.30	2.30	2.90	2.90	3.22	3.22	3.50	3.50	4.00	4.05	4.30	4.45	4.70	4.95	5.00	5.30	120
2.42	2.80	2.91	3.20	3.16	3.40	3.31	3.52	3.65	3.75	3.85	3.92	4.08	4.10	4.20	4.20	121
1.80	1.81	2.25	2.25	2.55	2.55	2.80	2.80	3.20	3.20	3.50	3.50	3.90	3.90	4.20	4.20	122
2.31	2.58	2.81	3.12	3.13	3.49	3.38	3.80	3.90	4.21	4.25	4.60	4.68	5.00	5.00	5.30	123
2.25	2.40	2.60	2.90	2.80	3.20	2.97	3.41	3.28	3.92	3.50	4.30	3.73	4.65	3.90	4.90	124
3.38	3.50	4.00	4.15	4.30	4.40	4.30	4.60	4.83	4.90	5.01	5.04	5.25	5.30	5.40	5.40	125
2.20	2.20	2.90	2.90	3.35	3.35	3.70	3.70	4.50	4.50	5.10	5.10	5.85	5.85	6.40	6.40	126
1.80	1.90	2.29	2.40	2.60	2.72	2.87	2.99	3.40	3.50	3.80	3.82	4.30	4.30	4.60	4.60	127
2.62	3.00	3.49	3.85	4.05	4.40	4.52	4.88	5.60	5.80	6.35	6.58	7.25	7.40	8.00	8.00	128
1.73	1.83	2.04	2.14	2.21	2.30	2.32	2.40	2.58	2.60	2.71	2.73	2.89	2.89	3.00	3.00	129
2.43	2.75	3.30	3.61	3.88	4.20	4.35	4.67	5.50	5.65	6.20	6.40	7.20	7.30	8.00	8.00	130
1.92	1.92	2.51	2.59	3.04	3.04	3.31	3.38	4.18	4.18	4.70	4.70	5.42	5.42	6.00	6.00	131
2.79	2.79	3.40	3.40	3.60	3.60	3.89	3.89	4.20	4.20	4.50	4.50	4.78	4.78	5.00	5.00	132
2.80	2.80	3.60	3.60	3.81	3.81	4.20	4.20	4.60	4.60	5.00	5.00	5.35	5.35	5.60	5.60	133
1.58	1.65	1.88	1.91	2.05	2.10	2.20	2.21	2.45	2.49	2.65	2.65	2.85	2.85	3.00	3.00	134
3.40	3.65	4.50	4.70	5.20	5.43	5.80	6.00	7.10	7.20	8.00	8.01	9.10	9.10	10.00	10.00	135
2.00	2.00	2.25	2.25	2.32	2.32	2.40	2.40	2.47	2.47	2.51	2.51	2.58	2.58	2.59	2.60	136
1.93	2.05	2.38	2.49	2.62	2.73	2.80	2.90	3.20	3.28	3.48	3.50	3.78	3.80	4.00	4.00	137
1.28	1.37	1.52	1.60	1.68	1.73	1.80	1.81	2.00	2.01	2.13	2.13	2.28	2.28	2.40	2.40	138
2.22	2.63	3.00	3.50	3.55	4.05	4.00	4.50	4.90	5.50	5.60	6.20	6.50	7.10	7.10	7.90	139
1.43	1.62	1.89	2.09	2.18	2.38	2.40	2.60	2.90	3.04	3.25	3.35	3.68	3.72	4.00	4.00	140
2.10	2.15	2.48	2.49	2.69	2.69	2.82	2.82	3.13	3.13	3.34	3.34	3.55	3.55	3.70	3.70	141
2.85	3.23	3.90	4.30	4.67	5.10	5.40	5.70	6.70	6.98	7.70	8.00	9.00	9.10	10.00	10.00	142
2.05	2.08	2.37	2.40	2.56	2.60	2.70	2.72	3.00	3.00	3.18	3.20	3.38	3.39	3.50	3.50	143
2.42	2.42	3.00	3.00	3.16	3.16	3.45	3.45	3.78	3.78	4.08	4.08	4.35	4.35	4.60	4.60	144
2.03	2.04	2.65	2.65	3.05	3.05	3.40	3.40	4.13	4.13	4.65	4.65	5.30	5.30	5.80	5.80	145
2.10	2.32	2.45	2.65	2.65	2.82	2.80	2.95	3.05	3.15	3.20	3.25	3.40	3.40	3.50	3.50	146
2.75	2.90	3.50	3.60	3.95	4.05	4.30	4.40	5.05	5.10	5.55	5.60	6.20	6.20	6.60	6.60	147
2.60	2.85	3.25	3.50	3.68	3.90	4.00	4.20	4.62	4.78	5.10	5.20	5.60	5.63	6.00	6.00	148
2.28	2.45	2.88	2.98	3.20	3.28	3.45	3.50	3.95	4.00	4.30	4.30	4.70	4.70	5.00	5.00	149
2.50	2.55	2.90	2.90	3.08	3.10	3.20	3.20	3.42	3.42	3.55	3.55	3.69	3.69	3.80	3.80	150
1.64	1.78	2.12	2.25	2.45	2.58	2.70	2.81	3.25	3.35	3.65	3.74	4.15	4.20	4.50	4.50	151
2.10	2.50	2.71	3.18	3.15	3.64	3.50	4.00	4.32	4.75	4.98	5.30	5.78	5.95	6.40	6.40	152
2.43	2.49	2.75	2.75	2.89	2.90	3.00	3.00	3.18	3.20	3.30	3.30	3.45	3.45	3.50	3.50	153
2.35	2.42	3.05	3.05	3.47	3.47	3.80	3.80	4.50	4.50	5.00	5.00	5.55	5.55	6.00	6.00	154
3.38	3.48	4.20	4.30	4.65	4.75	5.00	5.05	5.70	5.75	6.10	6.20	6.60	6.70	7.00	7.00	155
2.80	2.80	3.40	3.40	3.70	3.70	4.00	4.00	4.45	4.45	4.80	4.80	5.10	5.10	5.40	5.40	156
2.62	3.23	3.50	4.10	4.05	4.64	4.50	5.00	5.40	5.80	6.05	6.40	6.80	7.00	7.50	7.50	157
2.40	2.50	2.81	2.87	3.05	3.10	3.21	3.25	3.54	3.55	3.71	3.71	3.95	3.95	4.10	4.10	158
2.46	2.67	3.15	3.40	3.60	3.82	3.90	4.15	4.60	4.81	5.10	5.30	5.70	5.80	6.20	6.20	159
2.10	2.10	2.61	2.61	2.82	2.82	3.15	3.15	3.50	3.50	3.83	3.83	4.15	4.15	4.40	4.40	160
1.83	2.03	2.30	2.40	2.55	2.69	2.80	2.90	3.18	3.28	3.50	3.50	3.85	3.85	4.10	4.10	161
3.30	3.45	4.30	4.40	4.85	4.93	5.30	5.38	6.15	6.20	6.79	6.80	7.45	7.45	8.00	8.00	162

period June 1–September 30 in each year of record.
year of record.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	Record Used for Frequency Array		Average of Maximum 1-Day Rain Each:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
								(b) Ari-
163	San Carlos Reservoir...	2,532	12.12	50[3]	1885–1887; 1898–1901; 1903–1911; 1917–1918; 1921–1923	26	1.20	1.47
164	Thatcher............	2,800	9.55	39	1904–1937	34	1.19	1.32
165	Clifton..............	3,465	13.43	50	1906–1907; 1909–1940	34	1.31	1.43
166	Benson..............	3,523	11.07	58	1898–1940	43	1.18	1.24
167	San Simon...........	3,609	7.68	48[3]	1899–1900; 1903–1910; 1913–1914; 1922–1923; 1925–1940	30	1.07	1.24
168	Bowie...............	3,756	10.77	66	1881–1887; 1899–1901; 1903–1916; 1919–1939	45	1.17	1.36
169	Fairbank............	3,862	12.00	17	1910–1918; 1924–1940	26	1.41	1.48
170	Douglas.............	3,939	12.78	35	1904–1940	37	1.30	1.43
171	Lewis Springs........	4,029	11.17	22	1910–1918; 1920–1933	23	1.45	1.49
172	Allaires Ranch.......	4,184	11.55	21	1895–1913	19	1.50	1.61
173	Willcox..............	4,200	11.99	57[3]	1888–1890; 1898–1921; 1923–1925; 1927–1938; 1940	43	1.30	1.46
174	Cochise..............	4,250	11.39	27[3]	1899; 1901–1923	24	1.22	1.44
175	Leslie Canyon........	4,461	13.44	23	1916–1918; 1920–1940	24	1.44	1.48
176	Naco................	4,579	14.44	25	1914–1918; 1925–1934	15	1.62	1.79
177	Tombstone...........	4,540	14.85	41	1897–1899; 1901–1940	43	1.51	1.60
178	Ft. Grant............	4,833	13.32	65	1878–1891; 1898–1905; 1918–1919; 1921–1923; 1926–1928; 1931; 1933–1940	39	1.36	1.58
179	Ft. Huachuca.........	5,100	16.96	35	1900–1918	19	1.44	1.75
180	Ft. Apache...........	5,300	18.45	62	1879–1888; 1897–1918; 1921–1933	45	1.36	1.67
181	Bisbee...............	5,425	19.98	50	1893–1929; 1931–1940	47	1.79	1.99
182	Paradise.............	5,436	18.73	32	1914–1937	24	1.43	1.87
183	Rucker Canyon........	5,634	21.38	28	1917; 1924–1940	18	1.73	1.89
184	Snowflake............	5,644	12.98	39	1897–1900; 1911–1938; 1940	33	1.21	1.27
185	Portal...............	6,000	18.53	25	1914–1917; 1921–1923; 1926–1937; 1940	20	1.69	1.86
186	Show Low............	6,300	17.64	30[3]	1893–1898; 1909; 1916–1925	17	1.14	1.63
187	Heber...............	6,484	18.94	15[3]	1915–1921; 1926–1930; 1935	13	1.35	1.98
188	Blue................	6,500	18.89	27	1914–1930	15	1.30	1.59
189	Pinedale............	6,500	19.07	26	1912–1940	29	1.38	1.68
190	Springerville.........	6,862	11.79	40	1911–1940	30	1.12	1.17
191	Henrys Camp.........	6,925	23.11	23	1916–1938	23	1.28	1.67
192	Lakeside............	7,054	22.91	20[3]	1911–1926; 1929–1933	21	1.43	1.93
193	Alpine..............	8,500	23.28	29[3]	1905; 1912–1917; 1920–1922; 1925–1933; 1935–1940	25	1.46	1.76
194	Greer...............	8,500	22.18	22[3]	1904–1910; 1913–1916; 1918; 1920–1923; 1925–1926	18	1.23	1.42
195	Buckeye.............	980	7.80	50	1893–1940	48	1.03	1.37
196	Phoenix W. B.........	1,108	7.78	62	1897–1940	44	1.10	1.27
197	Marinette............	1,150	7.82	25[3]	1913–1915; 1917–1918; 1921–1923; 1925–1940	24	.97	1.40
198	Tempe Date Orchard...	1,165	8.74	38	1905–1940	36	1.04	1.33
199	Litchfield Park........	1,180	8.22	21	1918–1937; 1939–1940	22	1.17	1.45
200	Maricopa............	1,186	8.12	64	1898–1940	43	1.26	1.41
201	Goulds Ranch........	1,195	7.41	24	1915–1940	26	.97	1.35
202	Goodyear............	1,203	7.78	14	1919–1932	15	1.01	1.24
203	Chandler............	1,213	8.16	22	1913–1933	21	1.10	1.38
204	Mesa Experimental Farm	1,245	8.69	44	1896–1912; 1915–1940	43	1.02	1.36
205	Sacaton.............	1,280	9.60	31	1908–1940	33	1.28	1.44
206	Granite Reef Dam.....	1,325	9.52	46	1893–1894; 1896–1923; 1925–1940	46	.97	1.31
207	Casa Grande.........	1,400	8.30	50[3]	1898–1908; 1910–1914; 1917–1919; 1921–1922; 1925–1938; 1940	36	1.14	1.36
208	Casa Grande Ruin.....	1,422	9.73	23[3]	1906; 1909–1915; 1932–1937; 1940	15	1.25	1.62
209	Florence............	1,500	9.88	38[3]	1893; 1907; 1910; 1912–1917; 1921; 1923–1940	28	1.18	1.37
210	Morman Flats.........	1,815	13.54	14[3]	1923–1937	14	1.64	1.87
211	Redrock.............	1,856	9.61	35[3]	1893–1894; 1908–1933; 1935	29	1.17	1.38
212	Canon..............	1,990	17.92	15[3]	1916–1929	12	1.57	2.34
213	Dudleyville..........	2,204	14.60	37	1893–1909; 1911–1923	30	1.34	1.56
214	Intake..............	2,230	13.41	33	1907–1940	34	1.29	1.55
215	Roosevelt...........	2,275	17.86	34	1905–1910; 1912–1940	35	1.09	1.60
216	Gisela..............	2,300	18.78	27[3]	1916–1923; 1929–1940	20	1.19	1.57

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

(Continued)

ONE-DAY RAINFALL WHICH PROBABLY WILL BE EQUALED OR EXCEEDED ONCE IN:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
ZONA																
1.75	1.88	2.21	2.40	2.49	2.65	2.70	2.81	3.09	3.19	3.37	3.41	3.70	3.72	3.91	3.91	163
1.65	1.75	1.98	2.00	2.15	2.15	2.25	2.26	2.45	2.45	2.55	2.57	2.70	2.70	2.79	2.80	164
1.78	1.81	1.92	1.93	1.98	1.98	2.00	2.01	2.05	2.05	2.08	2.08	2.09	2.09	2.09	2.10	165
1.54	1.60	1.82	1.88	1.98	2.03	2.09	2.12	2.30	2.33	2.42	2.47	2.58	2.60	2.70	2.70	166
1.42	1.58	1.71	2.08	1.88	2.38	2.00	2.63	2.22	3.16	2.40	3.55	2.61	4.00	2.75	4.35	167
1.59	1.76	1.79	2.00	1.90	2.07	1.92	2.18	2.05	2.24	2.09	2.30	2.16	2.36	2.17	2.40	168
2.09	2.10	2.48	2.49	2.62	2.62	2.70	2.70	2.82	2.82	2.90	2.90	2.95	2.95	3.00	3.00	169
1.65	1.84	2.09	2.28	2.35	2.54	2.58	2.72	3.00	3.15	3.30	3.40	3.69	3.75	4.00	4.00	170
1.79	1.84	2.20	2.25	2.46	2.51	2.68	2.71	3.08	3.12	3.36	3.41	3.70	3.78	3.98	4.01	171
2.06	2.18	2.42	2.49	2.60	2.63	2.70	2.72	2.90	2.91	3.01	3.01	3.15	3.15	3.22	3.22	172
1.70	2.00	2.10	2.44	2.40	2.69	2.60	2.89	3.04	3.25	3.35	3.50	3.72	3.79	4.00	4.00	173
1.78	1.93	2.18	2.29	2.35	2.45	2.50	2.58	2.73	2.80	2.90	2.95	3.08	3.10	3.20	3.20	174
1.97	2.00	2.40	2.40	2.62	2.62	2.80	2.80	3.12	3.12	3.39	3.39	3.59	3.59	3.79	3.79	175
2.32	2.49	2.91	3.02	3.25	3.34	3.49	3.58	3.97	4.01	4.30	4.30	4.68	4.68	5.00	5.00	176
1.96	2.15	2.31	2.50	2.50	2.68	2.61	2.79	2.85	2.98	3.00	3.10	3.18	3.21	3.30	3.30	177
1.96	2.13	2.40	2.45	2.60	2.63	2.75	2.79	3.00	3.00	3.20	3.20	3.40	3.40	3.55	3.55	178
1.76	2.16	1.92	2.39	2.00	2.50	2.02	2.60	2.10	2.72	2.18	2.81	2.18	2.90	2.20	2.98	179
1.79	2.15	2.21	2.52	2.49	2.75	2.70	2.94	3.11	3.29	3.40	3.50	3.77	3.81	4.00	4.00	180
2.32	2.55	2.82	3.10	3.16	3.40	3.40	3.67	3.91	4.15	4.30	4.50	4.70	4.90	5.00	5.20	181
1.99	2.40	2.32	3.00	2.48	3.30	2.58	3.54	2.75	4.00	2.87	4.33	2.98	4.74	3.05	5.00	182
2.10	2.27	2.35	2.50	2.48	2.65	2.58	2.75	2.71	2.91	2.81	3.00	2.92	3.14	3.00	3.20	183
1.60	1.60	1.95	1.95	2.15	2.15	2.30	2.31	2.62	2.65	2.85	2.89	3.10	3.15	3.30	3.30	184
2.02	2.26	2.20	2.48	2.28	2.69	2.34	2.64	2.45	2.79	2.50	2.85	2.60	2.95	2.65	3.00	185
1.33	2.06	1.52	2.30	1.65	2.43	1.72	2.51	1.90	2.70	2.00	2.80	2.11	2.90	2.20	3.00	186
1.83	2.60	2.18	3.18	2.34	3.36	2.48	3.50	2.70	3.69	2.89	3.80	3.06	3.94	3.20	4.00	187
1.83	2.14	2.06	2.39	2.18	2.50	2.25	2.59	2.38	2.70	2.48	2.79	2.52	2.83	2.60	2.90	188
1.65	2.04	1.86	2.35	1.97	2.50	2.06	2.59	2.22	2.73	2.30	2.81	2.43	2.92	2.50	3.00	189
1.42	1.46	1.69	1.70	1.82	1.84	1.92	1.92	2.12	2.12	2.28	2.28	2.40	2.40	2.50	2.50	190
1.65	2.18	2.02	2.71	2.28	3.08	2.48	3.35	2.85	3.95	3.12	4.35	3.43	4.85	3.65	5.20	191
1.76	2.38	2.20	2.80	2.45	3.04	2.65	3.19	3.07	3.46	3.37	3.65	3.70	3.85	4.00	4.00	192
1.81	2.18	2.18	2.45	2.40	2.58	2.52	2.64	2.75	2.79	2.85	2.89	2.95	2.95	3.00	3.00	193
1.60	1.84	1.87	2.10	1.97	2.25	2.05	2.32	2.19	2.49	2.28	2.59	2.35	2.69	2.40	2.74	194
1.57	1.79	2.15	2.20	2.48	2.49	2.70	2.70	3.09	3.09	3.40	3.40	3.75	3.75	4.00	4.00	195
1.59	1.78	2.16	2.16	2.40	2.40	2.50	2.50	2.80	2.80	3.00	3.00	3.08	3.08	3.20	3.20	196
1.50	1.95	2.10	2.50	2.46	2.84	2.76	3.10	3.40	3.69	3.89	4.10	4.45	4.60	4.90	5.00	197
1.50	1.66	1.91	2.01	2.17	2.25	2.35	2.40	2.72	2.76	3.00	3.00	3.29	3.29	3.50	3.50	198
1.96	2.22	2.39	2.61	2.56	2.75	2.66	2.82	2.81	2.91	2.90	2.95	2.98	2.98	3.00	3.00	199
1.98	2.06	2.62	2.69	2.95	2.98	3.10	3.14	3.39	3.40	3.50	3.52	3.62	3.65	3.70	3.70	200
1.55	1.90	2.02	2.15	2.21	2.25	2.30	2.30	2.40	2.40	2.45	2.45	2.48	2.48	2.50	2.50	201
1.62	1.77	1.99	2.03	2.17	2.20	2.31	2.31	2.54	2.54	2.70	2.70	2.85	2.85	3.00	3.00	202
1.69	1.92	2.15	2.29	2.39	2.49	2.55	2.63	2.88	2.93	3.09	3.11	3.31	3.33	3.50	3.50	203
1.43	1.62	1.85	2.00	2.08	2.22	2.30	2.39	2.67	2.74	2.91	2.99	3.25	3.28	3.50	3.50	204
1.83	2.01	2.18	2.27	2.33	2.39	2.44	2.50	2.65	2.68	2.79	2.79	2.90	2.90	3.00	3.00	205
1.64	1.87	1.90	2.04	1.95	2.13	2.01	2.20	2.05	2.31	2.08	2.39	2.09	2.48	2.10	2.50	206
1.63	1.87	2.30	2.51	2.73	2.96	3.11	3.30	3.95	4.10	4.58	4.70	5.40	5.40	6.00	6.00	207
1.71	1.90	2.28	2.38	2.60	2.68	2.81	2.90	3.35	3.40	3.70	3.80	4.13	4.20	4.50	4.50	208
1.72	1.76	1.93	1.93	1.98	1.98	2.00	2.00	2.05	2.05	2.08	2.08	2.08	2.08	2.09	2.09	209
2.28	2.61	2.81	3.20	3.18	3.50	3.41	3.70	3.91	4.15	4.29	4.45	4.68	4.75	5.00	5.00	210
1.76	1.89	1.96	2.06	2.03	2.12	2.08	2.16	2.18	2.21	2.21	2.23	2.28	2.28	2.30	2.30	211
2.44	3.40	2.90	4.00	3.10	4.27	3.22	4.50	3.45	4.79	3.60	5.00	3.71	5.20	3.80	5.40	212
1.89	2.02	2.22	2.30	2.38	2.44	2.49	2.52	2.68	2.70	2.79	2.80	2.90	2.90	3.00	3.00	213
1.86	2.13	2.21	2.52	2.38	2.72	2.49	2.87	2.68	3.10	2.80	3.21	2.92	3.40	3.00	3.50	214
1.47	2.04	1.70	2.30	1.83	2.44	1.91	2.53	2.06	2.70	2.18	2.80	2.28	2.92	2.32	3.00	215
1.62	1.88	1.85	2.04	1.98	2.13	2.05	2.20	2.20	2.31	2.30	2.38	2.41	2.45	2.49	2.50	216

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(b) ARIZONA—	
217	Tucson, Univ. of Ariz.	2,423	11.28	72	1915–1940	26	1.32	1.46
218	Childs (Verde Hot Springs)	2,650	16.93	24	1915–1940	26	1.38	2.17
219	Superior	2,990	18.63	19	1920–1940	21	1.52	1.81
220	Alamo Ranger Station	3,155	13.88	24[3]	1916–1940	25	1.34	1.48
221	Vail	3,241	9.07	14[3]	1899–1910; 1912–1915	16	1.08	1.26
222	Globe	3,440	16.17	20	1894; 1901–1902; 1905; 1907–1928; 1930–1940	37	1.29	1.50
223	Clemenceau	3,465	11.31	17	1920–1936	17	1.21	1.38
224	Walnut Grove	3,469	17.79	49	1897–1940	45	1.51	2.28
225	Miami	3,603	19.82	25	1914–1940	27	1.47	1.83
226	Silverbell	3,604	13.19	15	1906–1915; 1917–1920	14	1.32	1.57
227	Cordes	3,773	12.89	14	1926–1940	15	1.04	1.55
228	Nogales	3,839	17.19	33	1900–1903; 1909; 1914–1940	32	1.51	1.66
229	Fresnal Ranch	4,000	18.40	15[3]	1923–1929; 1931–1937	14	1.43	1.77
230	Santa Marguerita	4,000	19.28	22	1917–1940	24	1.82	1.98
231	Sycamore Ranger Station	4,000	21.89	20[3]	1919–1921; 1924–1940	20	1.62	2.01
232	Patagonia	4,044	17.85	18	1922–1940	19	1.46	1.52
233	Helvetia	4,300	19.70	24[3]	1916–1919; 1921–1940	24	1.69	1.81
234	Young	4,400	21.75	29	1918; 1920–1923; 1925–1927; 1929–1935; 1938–1940	18	1.46	1.97
235	Pinal Ranch	4,520	25.57	46	1895–1940	46	1.65	2.44
236	Oracle	4,522	19.72	46[3]	1894–1910; 1913–1921; 1923–1927; 1929–1940	43	1.44	1.93
237	Tonto Ranger Station	4,732	16.36	24	1915–1928; 1930–1937; 1940	23	1.53	2.04
238	Rosemont	4,800	17.73	17	1914–1918; 1923–1930	13	1.42	1.87
239	Yeager Canyon	4,850	18.10	22	1918–1940	23	1.45	2.14
240	Elgin	4,900	15.85	27	1913–1940	28	1.62	1.73
241	Payson	4,906	21.04	40[3]	1893; 1895–1897; 1909–1916; 1918–1940	35	1.56	2.17
242	Natural Bridge	4,607	25.50	49	1893–1940	48	1.62	2.25
243	Canille	5,225	19.82	28	1910–1921; 1923–1940	30	1.89	1.98
244	Jerome	5,250	19.12	42	1898–1940	43	1.32	1.83
245	Prescott	5,354	20.52	73	1916–1940	25	1.31	2.17
246	Crown King	6,000	28.09	24	1915–1918; 1920–1940	25	1.96	3.39
247	Yuma	141	3.47	69	1898–1940	43	.90	1.10
248	Parker	350	5.16	46	1894–1898; 1900–1905; 1907–1940	45	.80	1.24
249	Aztec	492	4.81	23	1899–1921	23	.68	1.12
250	Mohawk	538	4.12	56	1900–1940	41	.83	1.10
251	Sentinel	685	4.57	21[3]	1899–1918	20	.90	1.13
252	Gila Bend	737	5.88	45[3]	1900–1925; 1928–1940	39	.81	1.05
253	Quartzsite	871	6.19	24	1908–1910; 1912–1914; 1920–1940	27	.95	1.24
254	Signal	1,652	7.33	18[3]	1893–1906	13	.83	1.26
255	Ajo	1,770	9.16	26	1914–1940	27	1.12	1.44
256	Salome	1,775	8.22	30	1908–1912; 1914–1940	32	1.01	1.44
257	Wickenburg	2,072	9.55	38[3]	1908; 1910–1940	32	1.25	1.64
258	Congress	3,688	13.55	14	1897–1910	14	1.22	1.76
259	Yarnell	4,782	19.10	25[3]	1899–1910; 1925–1926; 1936–1939	18	1.56	2.13
260	Leupp	4,150	7.07	12	1915–1921; 1925; 1934–1940	15	.69	.76
261	Tuba City	4,900	6.29	44	1897; 1899–1902; 1904; 1906; 1908–1917; 1920–1922; 1924–1927; 1930–1940	35	.75	.89
262	Winslow	4,848	8.60	40	1899–1900; 1909–1910; 1915–1940	30	1.00	1.08
263	Holbrook	5,069	9.25	42	1893–1899; 1904–1940	44	.97	1.11
264	St. Johns	5,650	12.17	39[3]	1902; 1904; 1906–1907; 1909–1918; 1920–1940	35	1.06	1.11
265	Pinto	5,660	11.83	19	1905–1922	18	1.06	1.20
266	Kayenta	5,800	8.80	26	1915–1917; 1919–1938; 1940	24	.82	1.09
267	Chin Lee	6,090	8.75	25	1909–1924; 1927; 1936–1940	22	.92	.96
268	Ft. Defiance	6,950	13.68	30	1897–1905; 1930–1940	20	.94	1.16
269	St. Michaels	6,950	13.15	22	1906–1925	20	1.12	1.24
270	Ft. Mohave	540	5.09	58	1893; 1896–1910; 1912–1913; 1926–1928; 1930–1934; 1916–1919; 1922–1937	26	.81	1.04
271	Lee's Ferry	3,142	6.20	22	1916–1919; 1922–1937	20	.73	.82

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		No.
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
1.85	2.12	2.30	2.51	2.50	2.72	2.68	2.85	3.00	3.10	3.20	3.25	3.43	3.43	3.60	3.60	217
1.73	2.57	2.38	3.30	2.80	3.76	3.19	4.20	4.00	5.00	4.64	5.60	5.42	6.40	6.00	7.00	218
2.19	2.38	2.61	2.72	2.85	2.95	3.00	3.10	3.35	3.40	3.56	3.60	3.80	3.82	4.00	4.00	219
1.92	2.03	2.30	2.37	2.50	2.55	2.61	2.66	2.87	2.90	3.00	3.05	3.18	3.20	3.30	3.30	220
1.75	1.97	2.10	2.31	2.28	2.48	2.38	2.52	2.49	2.61	2.57	2.65	2.65	2.68	2.70	2.70	221
1.61	1.98	1.90	2.28	2.08	2.41	2.21	2.51	2.49	2.70	2.66	2.80	2.86	2.91	3.00	3.00	222
1.78	1.93	2.68	2.81	3.35	3.50	3.90	4.08	5.30	5.40	6.40	6.50	7.85	7.90	9.00	9.00	223
2.02	2.96	2.58	3.70	2.95	4.23	3.20	4.60	3.75	5.40	4.15	6.00	4.61	6.70	5.00	7.10	224
1.91	2.25	2.28	2.60	2.48	2.81	2.61	2.99	2.92	3.25	3.11	3.45	3.35	3.65	3.50	3.80	225
2.00	2.05	2.19	2.20	2.25	2.26	2.30	2.30	2.38	2.38	2.40	2.40	2.45	2.45	2.50	2.50	226
1.61	2.35	2.28	3.00	2.72	3.44	3.10	3.79	3.94	4.45	4.57	4.90	5.40	5.50	6.00	6.00	227
1.91	2.03	2.30	2.50	2.55	2.79	2.71	3.00	3.09	3.48	3.32	3.80	3.59	4.19	3.80	4.45	228
1.88	2.10	1.99	2.26	2.03	2.31	2.07	2.36	2.12	2.41	2.16	2.45	2.19	2.47	2.20	2.50	229
2.48	2.60	3.20	3.30	3.63	3.75	4.00	4.10	4.74	4.85	5.30	5.40	5.95	6.00	6.40	6.50	230
2.55	2.75	3.08	3.15	3.30	3.34	3.42	3.45	3.66	3.66	3.80	3.80	3.90	3.90	4.00	4.00	231
1.88	2.03	2.29	2.45	2.55	2.70	2.71	2.89	3.12	3.25	3.40	3.50	3.75	3.77	4.00	4.00	232
2.28	2.40	2.60	2.63	2.75	2.75	2.81	2.82	2.95	2.98	3.08	3.08	3.15	3.15	3.20	3.20	233
2.11	2.71	2.51	3.05	2.70	3.19	2.82	3.25	3.06	3.36	3.20	3.40	3.38	3.45	3.50	3.50	234
2.28	3.10	2.69	3.80	2.90	4.27	3.04	4.60	3.30	5.30	3.49	5.79	3.65	6.35	3.80	6.80	235
1.98	2.50	2.30	3.00	2.43	3.31	2.51	3.55	2.70	4.00	2.80	4.30	2.91	4.70	3.00	5.00	236
2.11	2.76	2.63	3.18	2.93	3.40	3.15	3.57	3.59	3.88	3.89	4.10	4.23	4.31	4.50	4.50	237
1.89	2.40	2.22	2.95	2.43	3.28	2.60	3.50	2.90	4.00	3.10	4.30	3.32	4.58	3.50	5.00	238
1.96	2.55	2.35	3.00	2.55	3.23	2.70	3.40	3.00	3.80	3.20	4.00	3.40	4.30	3.50	4.50	239
2.11	2.11	2.50	2.55	2.70	2.78	2.82	2.96	3.08	3.30	3.21	3.55	3.40	3.81	3.50	4.00	240
2.18	2.70	2.58	3.41	2.80	3.88	2.99	4.22	3.32	4.95	3.55	5.50	3.80	6.15	4.00	6.60	241
2.06	2.90	2.72	3.51	3.19	3.90	3.56	4.20	4.33	4.80	4.91	5.22	5.62	5.75	6.20	6.20	242
2.50	2.60	3.12	3.23	3.50	3.60	3.80	3.91	4.40	4.48	4.80	4.90	5.30	5.35	5.60	5.70	243
1.83	2.41	2.11	2.85	2.25	3.08	2.31	3.20	2.46	3.50	2.53	3.70	2.62	3.89	2.68	4.00	244
1.70	2.87	1.88	3.48	1.96	3.82	2.01	4.10	2.12	4.68	2.20	5.01	2.25	5.50	2.30	5.80	245
2.60	4.70	3.49	5.98	4.05	6.65	5.54	7.20	5.60	8.40	6.40	9.20	7.35	10.01	8.00	10.80	246
1.30	1.57	1.92	2.18	2.32	2.52	2.62	2.82	3.32	3.46	3.83	3.95	4.49	4.51	5.00	5.00	247
1.05	1.60	1.46	1.98	1.72	2.20	1.91	2.39	2.38	2.75	2.70	3.00	3.12	3.30	3.45	3.51	248
.96	1.62	1.12	1.88	1.20	1.98	1.25	2.03	1.31	2.14	1.34	2.20	1.37	2.26	1.40	2.30	249
1.27	1.56	1.68	1.91	1.86	2.10	1.99	2.22	2.18	2.50	2.30	2.68	2.42	2.87	2.50	3.00	250
1.43	1.53	1.90	1.91	2.15	2.15	2.31	2.31	2.72	2.72	3.00	3.00	3.28	3.28	3.50	3.50	251
1.17	1.47	1.37	1.71	1.45	1.82	1.50	1.90	1.58	2.05	1.61	2.14	1.67	2.24	1.70	2.30	252
1.35	1.69	1.90	2.21	2.28	2.59	2.60	2.89	3.30	3.48	3.81	3.95	4.50	4.50	5.00	5.00	253
1.06	1.64	1.28	1.81	1.38	1.90	1.45	1.92	1.58	2.02	1.67	2.08	1.73	2.09	1.80	2.10	254
1.53	2.10	1.71	2.52	2.14	2.75	2.32	2.91	2.69	3.22	2.95	3.40	3.26	3.65	3.50	3.80	255
1.47	2.05	1.91	2.41	2.18	2.60	2.33	2.70	2.65	2.90	2.85	3.01	3.05	3.16	3.20	3.25	256
1.87	2.03	2.30	2.39	2.53	2.58	2.70	2.72	2.99	3.02	3.19	3.20	3.40	3.43	3.55	3.60	257
2.02	2.64	2.60	2.85	2.75	2.91	2.80	3.00	2.90	3.04	2.95	3.09	2.99	3.09	3.00	3.10	258
2.00	2.81	2.29	3.21	2.41	3.40	2.51	3.50	2.70	3.71	2.80	3.81	2.91	3.95	3.00	4.00	259
.98	1.03	1.15	1.18	1.20	1.22	1.23	1.27	1.30	1.31	1.34	1.36	1.38	1.38	1.40	1.41	260
1.05	1.19	1.32	1.48	1.49	1.65	1.60	1.78	1.84	2.03	2.00	2.20	2.19	2.40	2.31	2.54	261
1.38	1.48	1.73	1.81	1.95	2.01	2.09	2.15	2.40	2.43	2.60	2.61	2.85	2.85	3.00	3.00	262
1.31	1.40	1.65	1.71	1.83	1.90	1.99	2.05	2.30	2.35	2.51	2.58	2.78	2.82	2.98	3.00	263
1.33	1.42	1.63	1.70	1.80	1.88	1.95	2.00	2.23	2.26	2.42	2.42	2.65	2.65	2.80	2.80	264
1.34	1.52	1.61	1.81	1.80	2.01	1.94	2.15	2.26	2.42	2.49	2.61	2.75	2.85	2.97	3.00	265
1.30	1.50	1.51	1.93	1.55	2.22	1.58	2.47	1.60	2.93	1.61	3.35	1.65	3.65	1.66	4.00	266
1.29	1.29	1.52	1.55	1.67	1.67	1.70	1.72	1.83	1.83	1.89	1.90	1.96	1.96	2.00	2.00	267
1.16	1.46	1.26	1.62	1.31	1.74	1.34	1.80	1.40	1.95	1.42	2.03	1.47	2.12	1.50	2.20	268
1.45	1.60	1.55	1.92	1.59	2.10	1.61	2.24	1.66	2.49	1.69	2.66	1.71	2.85	1.71	3.00	269
1.32	1.65	1.90	1.98	2.05	2.10	2.10	2.20	2.19	2.31	2.22	2.40	2.26	2.48	2.29	2.51	270
1.03	1.25	1.39	1.60	1.60	1.80	1.78	1.98	2.15	2.30	2.42	2.53	2.77	2.81	3.00	3.00	271

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

TABLE 3.—

No.	Station	Elevation (feet)	Average annual precipitation through 1938 (inches)	Length of record (years)	RECORD USED FOR FREQUENCY ARRAY		AVERAGE OF MAXIMUM 1-DAY RAIN EACH:	
					Dates (inclusive)	Number years	Summer[1]	Year[2]
							(*b*) ARIZONA—	
272	Supai	3,200	8.89	25	1914–1915; 1917–1918; 1921–1922; 1924–1927; 1930–1932	13	.89	1.17
273	Kingman	3,326	10.92	40	1901–1902; 1904–1940	39	1.13	1.74
274	Truxton	3,997	9.91	21	1910–1917; 1921–1922; 1924–1925; 1927–1940	26	1.24	1.46
275	Cedar Glade	4,610	14.30	24	1915–1940	26	1.28	1.68
276	Prescott Dry Farm	5,008	14.34	18	1913–1928	16	1.47	1.59
277	Walnut Creek Ranger Station	5,138	19.15	24	1916–1940	25	1.42	1.95
278	Ashfork	5,160	12.60	30	1909–1910; 1912–1913; 1915–1928; 1930–1940	29	1.20	1.44
279	Seligman	5,219	11.04	34[3]	1905–1909; 1912–1935; 1937–1940	33	1.14	1.37
280	Williams	6,750	22.32	43	1904–1905; 1908–1940	35	1.39	1.97
281	Grand Canyon	6,866	17.22	38	1904; 1906–1940	36	1.10	1.38
282	Flagstaff	6,907	22.80	51	1894; 1897–1940	45	1.20	1.62
283	Ft. Valley	7,500	24.33	30	1914–1924; 1935–1940	17	1.25	1.73

[1] Average of the maximum 1-day rain recorded for the

[2] Average of the maximum 1-day rain recorded in each

[3] Some missing months interpolated from surrounding

given year, in which case it was necessary to decide whether to include the year. The decision was made after inspection of the complete record to determine what quantity constituted a high rainfall at that station. Depending on the number and season of the missing months, the year was included in the array if it appeared probable that the available record included the day of highest rainfall for the year.

If the data do not plat as a straight line in the recurrence-interval curve, the relatively few widely-spaced points in the upper limits of the series determine how the curve will be drawn and will control the extrapolation. In this study two curves (summer and all-year) were drawn for each of 283 stations. They represent stations in all kinds of topography in regions of average annual precipitation varying from 8 to 30 in. By far the most important factor affecting the extrapolation of the curves was neither the manner of choosing the series, nor the length of record, but the occurrence of aberrant points in the upper limits of the series. When such an aberrancy occurred the position of the curve was mostly a matter of judgment.

In the Southwestern States there is great variability in both the average annual precipitation and in the magnitude of the highest one-day rainfalls that occur in successive years. One year may include a number of heavy rainfalls and may have a high total precipitation, whereas the following year may be dry, with no high rains. To demonstrate this variability the heaviest rain of record each year was used in the present analysis. The recurrence-interval array, therefore, is comparable to the method used by the late Allen Hazen,[6] M. Am. Soc. C. E., in which he computed from stream-flow records

---

[6] "Flood Flows," by Allen Hazen, John Wiley and Sons, New York, N. Y., 1930.

(Continued)

One-Day Rainfall Which Probably Will Be Equaled or Exceeded Once In:																No.
5 years		10 years		15 years		20 years		35 years		50 years		75 years		100 years		
Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	Summer	All year	
(Continued)																
1.28	1.95	1.65	2.60	1.82	2.90	1.93	3.11	2.14	3.47	2.29	3.68	2.40	3.86	2.50	4.00	272
1.83	2.52	2.31	3.10	2.61	3.40	2.71	3.61	2.89	4.05	2.95	4.31	2.99	4.65	3.00	4.90	273
1.83	2.24	2.51	2.82	2.90	3.18	3.20	3.41	3.75	3.95	4.20	4.30	4.65	4.72	5.00	5.00	274
1.83	2.21	2.09	2.51	2.16	2.68	2.20	2.78	2.28	2.93	2.30	3.00	2.32	3.10	2.32	3.20	275
1.96	2.00	2.23	2.23	2.33	2.35	2.41	2.41	2.55	2.55	2.61	2.63	2.71	2.71	2.75	2.75	276
1.95	2.41	2.60	3.05	3.05	3.46	3.40	3.79	4.20	4.46	4.75	5.00	5.50	5.55	6.00	6.00	277
1.68	1.77	1.90	1.98	2.00	2.07	2.04	2.14	2.12	2.28	2.18	2.35	2.21	2.42	2.22	2.50	278
1.64	1.84	1.89	2.17	2.00	2.30	2.09	2.41	2.24	2.62	2.31	2.75	2.41	2.90	2.48	3.00	279
1.79	2.65	1.96	3.10	2.04	3.33	2.10	3.51	2.20	3.86	2.27	4.10	2.31	4.33	2.35	4.50	280
1.50	1.70	1.81	2.00	2.00	2.17	2.15	2.30	2.42	2.53	2.60	2.70	2.83	2.88	3.00	3.00	281
1.54	1.97	1.73	2.24	1.85	2.40	1.91	2.51	2.06	2.75	2.18	2.90	2.27	3.08	2.32	3.19	282
1.55	2.05	1.79	2.39	1.90	2.57	2.00	2.70	2.16	3.00	2.28	3.18	2.39	3.38	2.48	3.50	283

period June 1–September 30 in each year of record.
year of record.
stations for computation of average annual precipitation.

the highest peak discharge each year of record, the average of which he termed "average annual flood." In Table 3, the data on the average of highest rainfall each summer and of highest rainfall each year are comparable to Mr. Hazen's "average annual flood."

ISOPLUVIAL MAPS

The isopluvial maps published by Mr. Yarnell[2] were based on five stations in the Southwestern States. In spite of the fact that the present analysis includes frequency curves for 283 stations in the same region, due to marked variation in frequency curves for closely adjacent stations of comparable records, isopluvial maps are difficult to construct. This difficulty can be attributed to a number of facts. Owing to the great divergence in topographic relief between closely adjacent points, two stations within a very short distance of one another may be in very different topographic settings. One may be in the middle of a broad, flat plain, the other in a deep, narrow mountain canyon. It appears from the present analysis that stations above El. 7500 experience relatively few high rainfalls. This indicates that elevation above sea level has some effect on the frequency characteristics, although ruggedness of relief and position of the station in relation to the source of moisture and the surrounding mountainous areas probably are of more direct importance.

The small areal extent of high rainfalls occurring in summer-type storms is a factor leading to differences between stations since any high-intensity, individual storm rarely is recorded at more than one station.

The 24-hr rainfall quantity expected to be equaled or exceeded once in 50 years at various stations in relation to topography is shown by the relief maps

GRAND CANYON 2.70
TUBA CITY
CHIN LEE
FT. DEFIANCE
ST. MICHAELS
TRUXTON
SELIGMAN 2.70
KINGMAN
ASHFORK
WILLIAMS
FT. VALLEY
LEUPP
FLAGSTAFF
CEDAR GLADE
WALNUT CREEK RANGER STATION
PRESCOTT DRY FARM
JEROME 2.70
CLEMENCEAU
PRESCOTT
WALNUT GROVE
SYCAMORE RANGER STATION
CONGRESS
CROWN KING
WINSLOW
HOLBROOK
PINTO
SNOWFLAKE
ST. JOHNS
HEBER
PINEDALE
SHOW LOW
LAKESIDE

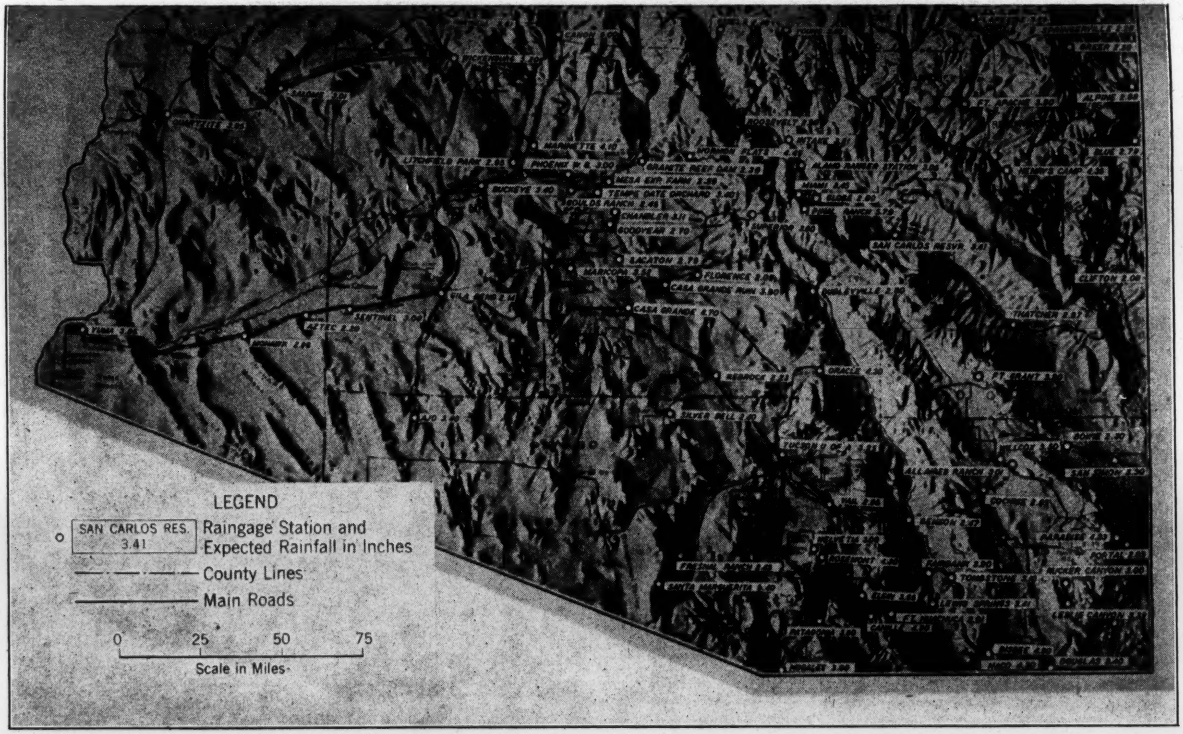
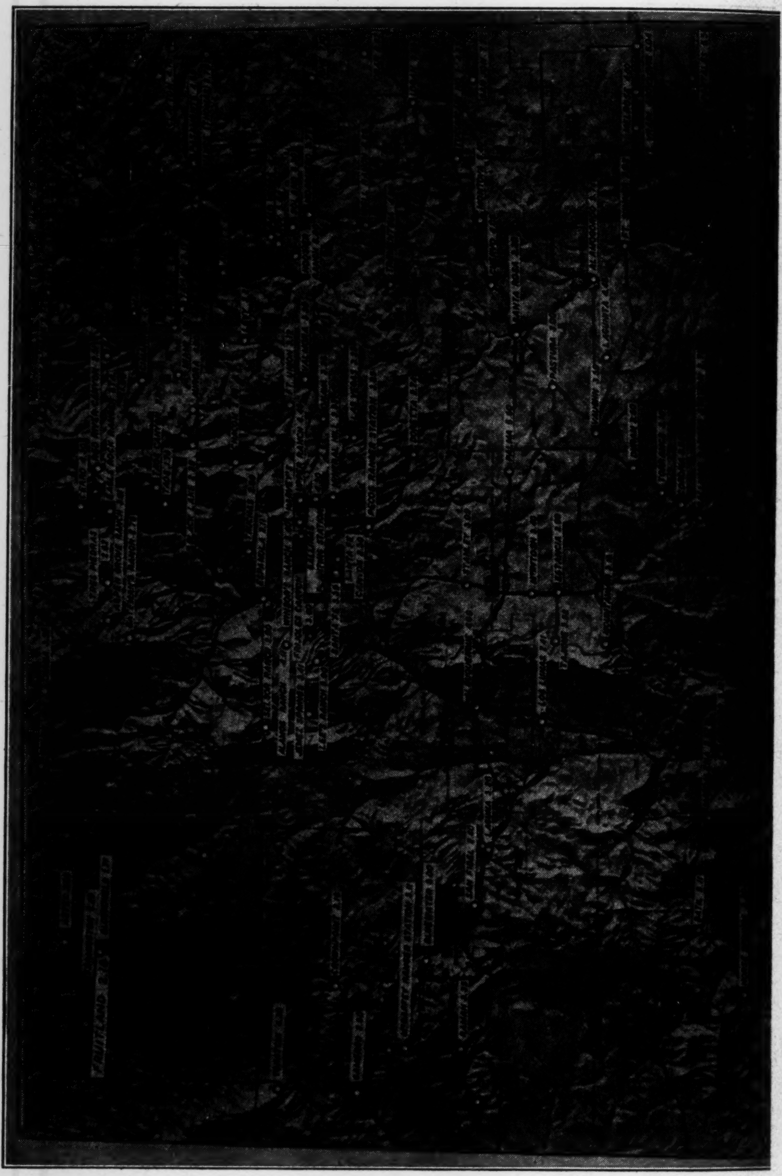

QUARTZSITE 2.95
SALOME 3.01
YUMA 3.06
MOHAWK 2.96
AZTEC 2.30
SENTINEL 3.00
GILA BEND 2.14
AJO 3.40
CANON 3.00
LITCHFIELD PARK 2.85
PHOENIX W. B. 3.00
MARINETTE 4.10
BUCKEYE 3.40
GRANITE REEF DAM 2.38
MESA EXP. FARM 2.59
TEMPE DATE ORCHARD 3.40
BOULDS RANCH 2.46
CHANDLER 3.11
GOODYEAR 2.70
SACATON 2.78
MARICOPA 3.51
FLORENCE 2.09
CASA GRANDE RUIN 3.80
CASA GRANDE 4.70
REDROCK 2.23
ROOSEVELT
MORMON FLATS 4.41
INTAKE 3.81
ALAMO RANGER STATION 3.06
MIAMI 3.45
GLOBE 2.60
DUNN RANCH 3.78
SUPERIOR 3.60
SAN CARLOS RESVR. 3.41
DUDLEYVILLE 2.90
SPRINGERVILLE 2.38
GREER 2.38
FT. APACHE 3.80
ALPINE 2.88
BLUE 2.79
HEMMI'S CAMP 4.30
CLIFTON 2.08
THATCHER 2.57
ORACLE 4.30
FT. GRANT 3.80
SILVER BELL 2.40
TUCSON U OF A 3.25
BOWIE 2.30
WILLCOX 3.50
SAN SIMON 2.30
VAIL 2.95
COCHISE 2.65
BENSON 2.47
PARADISE 4.33
HELVETIA 3.60
ROSEMONT 4.00
FRESNAL RANCH 2.43
FAIRBANK 2.90
PORTAL 2.83
RUCKER CANYON 3.00
TOMBSTONE 3.10
SANTA MARGARITA 3.40
ELGIN 3.55
LEWIS SPRINGS 2.41
FT. HUACHUCA 2.91
PATAGONIA 3.50
LESLIE CANYON
BISBEE 4.30
NACO 4.30
DOUGLAS 3.40
NOGALES 3.90
LEGEND
○ SAN CARLOS RES. 3.41 Raingage Station and Expected Rainfall in Inches
County Lines
Main Roads
0
25
50
75
Scale in Miles

Fig. 3.—Arizona: 24-Hr Rainfall Expected to Be Equaled or Exceeded Once in 50 Years



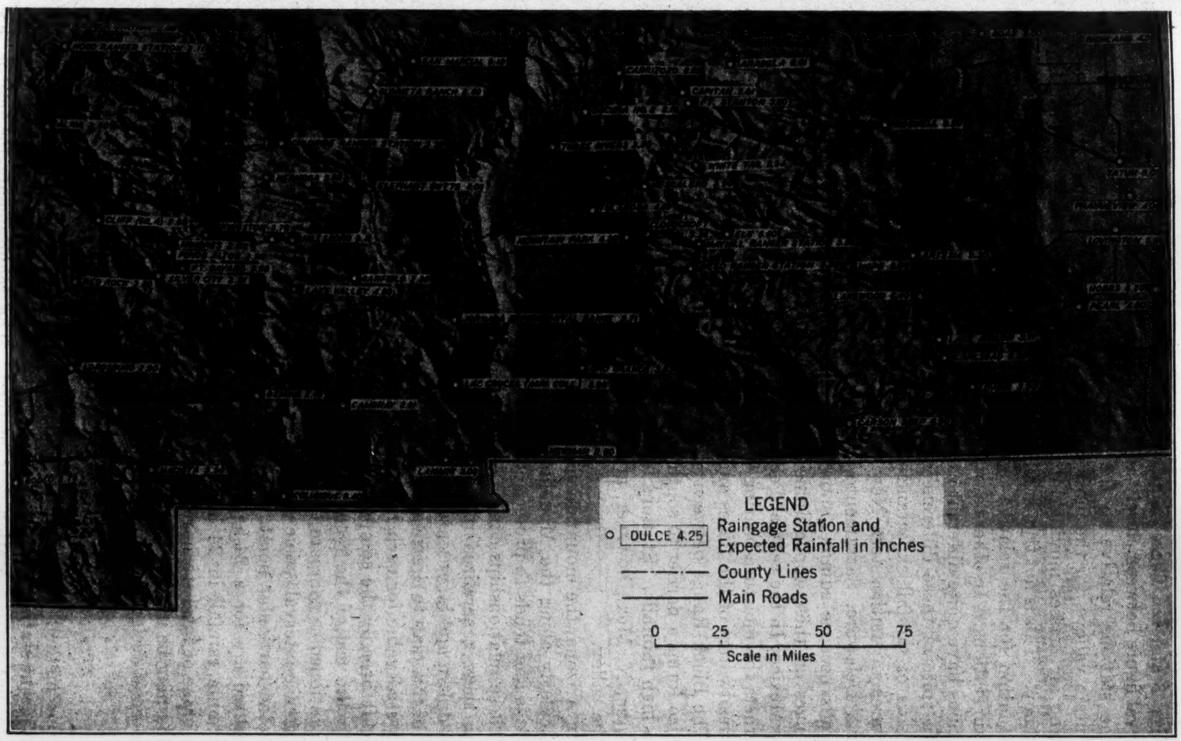

LEGEND
DULCE 4.25
Raingage Station and Expected Rainfall in Inches
County Lines
Main Roads
0
25
50
75
Scale in Miles

FIG. 4.—NEW MEXICO: 24-HR RAINFALL EXPECTED TO BE EQUALED OR EXCEEDED ONCE IN 50 YEARS

of Figs. 3 and 4. These maps demonstrate the variability of the recurrence-interval figure between closely adjacent stations, and they indicate the position of the stations relative to the major mountain masses. Stations in deep mountain valleys, for the most part, are shielded from the effect of moisture-bearing air masses moving into the area and tend to have low rainfall expectancy. This is not true in all cases, but the relief apparently offers sufficient explanation for the relative magnitude of the recurrence-interval figure for certain stations. In other cases, two stations in apparently similar topographic positions have quite different expectancies.

In spite of these differences between individual stations, certain general patterns of rainfall expectancies are apparent in New Mexico and Arizona. The area in southeastern New Mexico, particularly along the eastern flanks of the Delaware, Sacramento, and Capitan mountains, is subject to high rainfalls because summer moisture moving in from the south or southwest impinges on these topographic barriers. The same is true for northeastern New Mexico: In the central Pecos watershed no important topographic barrier obstructs the moisture-bearing winds south of the relatively rugged country in the headwaters of the Pecos and Canadian rivers.

The Black Range between Silver City and Hillsboro in southwestern New Mexico forms a barrier to south winds and protects northwestern New Mexico from high rainfalls. Stations in the Rio Grande Valley as far north as Albuquerque, N. Mex., however, are subject to high rains from moisture moving up the valley.

In Arizona, the mountainous area of the central and northwestern portion, particularly along the Mogollon and Yarnell escarpments, is subject to high rainfalls and tends to protect or shadow the northeastern plateau country.

These relationships can be easily visualized by inspection of Figs. 3 and 4. From these generalized relations an isopluvial map for 50-yr frequency has been drawn and is presented in Fig. 5. No attempt was made in the preparation of this map to draw each "isopluve" in the exact position indicated by the computed value for each individual station, but the lines were generalized to indicate broadly the comparative expectancies of various regions.

In the use of the recurrence-interval data presented herewith, engineers are cautioned to compare the topography and position of the point for which a recurrence-interval value is desired with those factors affecting all stations near the particular point. Moreover, it must be remembered that the data presented are for a 24-hr period, but in the summer type of rainfalls, the maximum rainfall in 24 hr probably fell in a much shorter period of time. The value of total storm rainfall for a given recurrence-interval in the summer period may be used in connection with average intensity-time patterns for the purpose of designing engineering structures.

Applicability of Station-Year Analysis

One method of ironing out differences between recurrence-interval curves of adjacent stations and improving the value of short records is a combination of stations by the station-year method. The requirements for use of this method discussed by H. C. S. Thom[7] and Katharine Clarke-Hafstad[8] may be summarized as follows:

(1) Stations grouped must be mutually independent—that is, far enough apart so that no single storm covers more than one of the stations; and

(2) Stations grouped must be included in a meterologically homogeneous region.

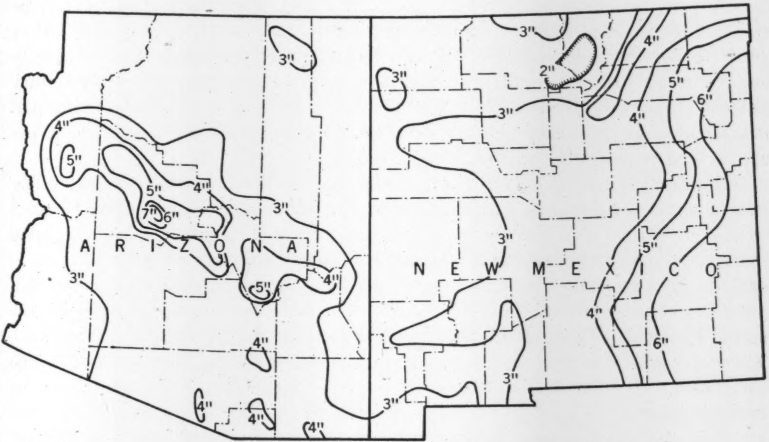


FIG. 5.—ISOPLUVIAL MAP OF ARIZONA AND NEW MEXICO (24-HR RAIN EQUALED OR EXCEEDED ONCE IN 50 YEARS)

Mr. Thom defines a meteorologically homogeneous region as an area each point of which would have identical frequency curves of rainfall. He states that delineation of such a region by this criterion is impractical since "assignable causes," such as topographic differences, operate to produce dissimilarity of rainfall experience at various points within the region; and he suggests, therefore, that homogeneity must be established by meteorological studies. Assignable causes operate in the following way: A storm situation, one favorable for rain, prevailing over a region may be so weak that rainfall occurs only at points that are subject to significant orographic effects, and these stations, over a long period, would therefore experience more rainy days than do stations with assignable causes of lesser magnitude.

Although it is possible, in New Mexico and Arizona, to delineate meteorologically homogeneous areas defined by meteorologic studies, recurrence-interval curves at various stations within such a region may be so different that assignable causes appear to operate in these states to such an extent that there may be more difference between curves of stations within the homogeneous area than between two stations in dissimilar meteorologic regions.

---

[7] "On the Statistical Analysis of Rainfall Data," by Herbert C. S. Thom, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 490–499.

[8] "Reliability of the Station-Year Rainfall-Frequency Determinations," by Katharine Clarke-Hafstad, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 633.

Examples of station-year groupings were platted in connection with the present study. It did not appear satisfactory to group stations by the station-year method, although the stations were in meteorologically similar regions, inasmuch as the frequency curves were so distinctly different.

Comparison of Magnitude of 1-Day, 2-Day, and 3-Day Rains

In the use of frequency-magnitude data for design, it usually is necessary to apply an intensity pattern to a given rainfall total. In general, it may be stated that in New Mexico and Arizona summer rainfall depths published as 24-hr totals characteristically fall in a period much less than 24 hr—usually less than 10 hr. Since design storms ordinarily are assigned an intensity pattern that would result in relatively high stream discharge, the rainfall magnitudes for summer listed in Table 3 logically may be used with the characteristic intensity patterns presented in Fig. 1(*a*).

The great winter floods in these states are the result of rainfalls extending over a period of days, as was the case in the flood of the upper Gila River in January, 1916. For the determination of the probability of long-duration storms, the recurrence-interval of various total amounts of precipitation in periods greater than 24 hr was derived by tabulating the highest rainfall amounts each year that fell on two or three consecutive days, arranging the terms in order of magnitude, and platting as in the studies of 24-hr rains. These arrays were prepared for a limited number of stations, each of which was chosen to represent a certain area with characteristic topographic relief and elevation.

This analysis showed that the ratio of the total rainfall in a 48-hr period to that falling in a 24-hr period is essentially constant regardless of the recurrence-interval of the rain, and similarly, the ratio of 72-hr total to 24-hr is approximately constant. At any given recurrence-interval the amount of rain in any consecutive 48-hr period is about 10% to 20% greater than the 24-hr amount expected, and the total for any 72 consecutive-hr period is 20% to 30% greater than the amount in 24 hr.

These ratios are only an indication of the relation between 1-day, 2-day, and 3-day rains, since the present analysis did not include sufficiently detailed studies of long-duration rains and winter intensity patterns to form the basis for positive statements on this subject. The rains experienced in the Pecos River watershed of eastern New Mexico in September, 1941, indicate that, at least in that section of the state, summer-type rainfalls of very high amounts may fall on two consecutive days.

Acknowledgments

The writer gratefully acknowledges the assistance of Thomas Maddock, Jr., for suggestions during the progress of the study, B. T. Mitchell in checking original records, and a number of workers of the Work Projects Administration, principally the late E. F. Blanchard, who copied data from the original observers' records and assisted in the frequency tabulations. The cooperation of E. L. Hardy and G. K. Greening, U. S. Weather Bureau, in allowing access to the original records of cooperative observers is appreciated.

DISCUSSION

Lawrence Pratt,[9] Assoc. M. Am. Soc. C. E.—Engineers of the Southwest will find this paper a valuable compilation of 1-day rainfall records, supplementing the previous work of Mr. Yarnell.[2]

It has been observed at Tucson, Ariz., that, for a period of six to eight weeks during July and August, thunderstorms are of almost daily occurrence. A notable feature of these storms is the punctuality of their appearance. About 10:00 a.m. high cumulus "thunderheads" appear in the northwest and mount rapidly to the zenith. The first drops of rain may be expected about 2:00 p.m. but within another 2 hr the storm will have passed and the sun will be shining again. The first raindrops are very large, making splashes on the ground the size of silver dollars, but quickly becoming smaller. Probably the heaviest intensity during the storm occurs within the first 15 min. Each storm is accompanied by a sharp temperature drop of 20° to 30°, and there may or may not be a strong northwest wind.

Another type of storm which may be peculiar to southern Arizona and southern New Mexico is the "duster" which occurs in early and late summer. This is characterized by a persistent wind which at times reaches a high velocity and which carries great quantities of dust. The usual duration of this storm is 3 days. The wind increases in velocity from day to day and the storm culminates in a thundershower which may last no longer than 30 min. It is hoped that the author will explain the meteorological aspects of this type of storm.

From the summit of Bill Williams Mountain, 4 miles south, and 2,500 ft above Williams, Ariz., the writer had an opportunity to observe weather phenomena throughout one summer. On certain days as many as four separate and apparently independent thunderstorms were seen at the same time, moving from west to east over the plateau which reaches north to the Grand Canyon. These storms were of small extent, with sharply defined boundaries, covering from 1 to perhaps 4 or 5 sq miles.

Because of the limited area and short duration of high-intensity summer storms, the problem of deducing runoff from rainfall is complicated. Only the smaller drainage basins have times of concentration sufficiently short to coincide with the times of precipitation of the storms, although local flash floods may be produced occasionally even in streams of considerable drainage area. However, notable floods on the larger rivers, the Colorado, Rio Grande, Pecos, and Gila, are generally associated with winter storms of low intensity, wide extent, and long duration.

The danger of using 24-hr records from nonrecording rain gages should be emphasized. Unless the times of beginning and ending of precipitation are noted carefully at the time of observation, the total rainfall is of little value in any intensity study.

---

[9] Los Angeles, Calif.

L. L. HARROLD,[10] ASSOC. M. AM. SOC. C. E., AND A. J. DICKSON,[11] ESQ. —The data presented by the author will be very useful to engineers engaged in hydraulic designs for the arid Southwest region. The writers have had occasion to make a concentrated tabulation and study of the type of rainfall reported in this paper.

The statement under the heading, "Storm Types," that "Valley areas usually receive the highest monthly precipitation in July, August, and September * * *" is assumed to mean that the precipitation in these months is usually higher than values for other months of the year. The records at the Navajo Experiment Station, Mexican Springs, N. Mex., verify this conclusion. Furthermore, the mountain gages at this research station usually record more monthly rainfall for these three months than for the other months of the year. This is contrary to the author's findings which probably are based on records from areas in Arizona and New Mexico where the winter snowfall is greater than at this research station. It is true, however, that the winter precipitation, December through March, at the higher elevations is significantly greater than that for the valley areas.

The data on areal extent of various quantities of summer rainfall (Table 1) are of great value even though rain gages were spaced rather far apart to delineate the limits of these areas adequately. The areal extent of summer storms over the watersheds of the Navajo Experiment Station (Fig. 6), 25 miles north of Gallup, N. Mex., cannot be well defined at all times although each of 83 rain gages represents an area of only 0.8 sq mile. Fig. 7 shows isohyetal lines for the total rainfall of the storm of August 6, 1941, a typical summer storm. It can be seen that the position of these isohyets would be entirely different if there were fewer gages so that each rain gage represented 10 sq miles. In areas where the gages are spaced 10 to 100 miles or more apart, it is no wonder that so little is known of the occurrence and distribution of summer storms. It is believed that on the Navajo Experiment Station there are more recording rain gages per unit than on any other area of the same size or larger in this region. These gages are operated in connection with runoff stations for the purpose of studying rainfall-runoff relations, land use, and water spreading. A description of some summer storms for 1941 on this experimental area is given elsewhere.[12,13]

The records at this station confirm the author's statement (see heading, "Storm Types") that "The effect of topographic relief is relatively small * * *." They do not, however, tend to confirm his statement under the heading, "Areal Pattern of Summer Storms," that "* * * in any one summer storm the center of greatest precipitation usually is near the base of the mountain slope." On the Navajo Experiment Station area, elevations range from 6,200 to 8,800 ft, and, during 1941, six storms had thirteen different centers of greatest pre-

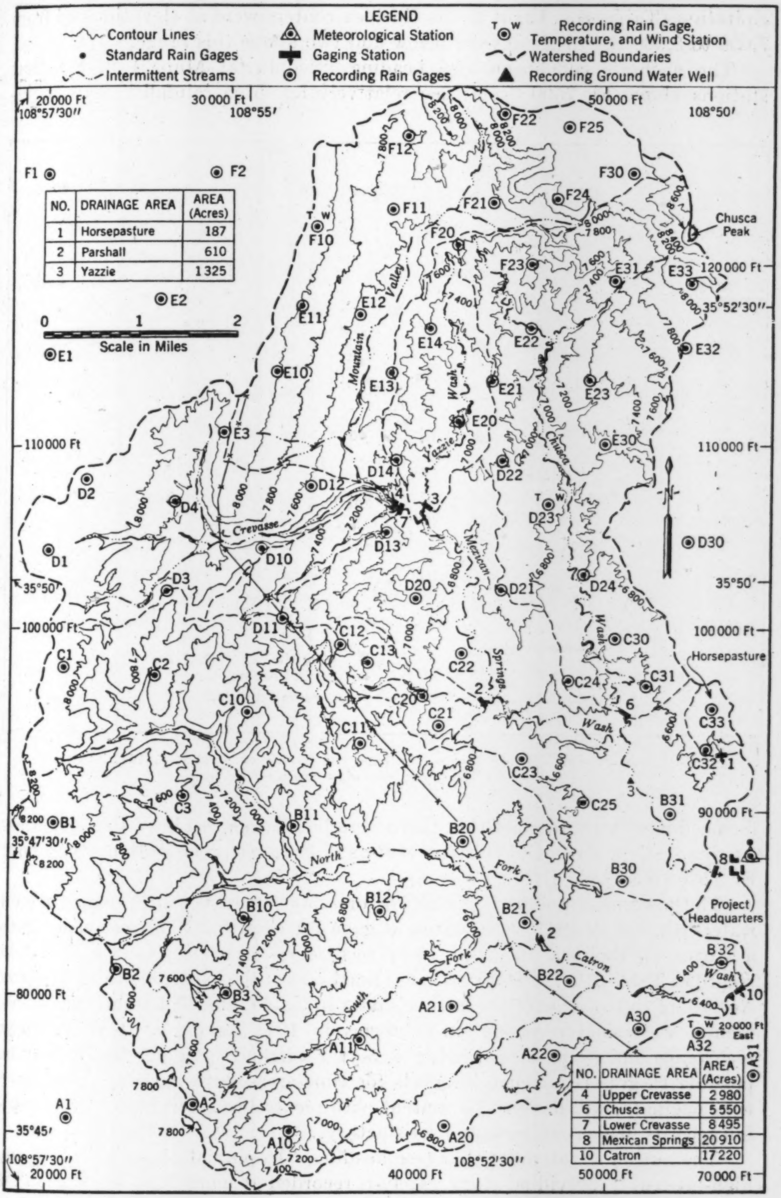
---

[10] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[11] Asst. Hydr. Engr., SCS, U. S. Dept. of Agriculture, Mexican Springs, N. Mex.

[12] "Agriculture, Soils, Geology, Topography, and 1941 Data, Navajo Conservation Experiment Station, Mexican Springs, N. Mex.," *Hydrologic Bulletin No. 6*, U.S.D.A. (publication pending).

[13] "Thunder-storms and Runoff at High Elevations in Northwestern New Mexico," by L. L. Harrold, *Transactions*, Am. Geophysical Union, Pt. II, 1943, p. 425.


LEGEND
Contour Lines
Standard Rain Gages
Intermittent Streams
Meteorological Station
Gaging Station
Recording Rain Gages
T W Recording Rain Gage, Temperature, and Wind Station
Watershed Boundaries
Recording Ground Water Well
20000 Ft
108°57'30"
30000 Ft
108°55'
50000 Ft
108°50'
120000 Ft
35°52'30"
110000 Ft
35°50'
100000 Ft
90000 Ft
35°47'30"
80000 Ft
35°45'
40000 Ft
108°52'30"
70000 Ft
20000 Ft East
0 1 2
Scale in Miles
Chusca Peak
Horsepasture
Project Headquarters
Crevasse
Mountain Valley
Yazzie Wash
Chusca Wash
Mexican Springs
North Fork
South Fork
Catron Wash
F1 F2 F10 F11 F12 F20 F21 F22 F23 F24 F25 F30
E1 E2 E3 E10 E11 E12 E13 E14 E20 E21 E22 E23 E30 E31 E32 E33
D1 D2 D3 D4 D10 D11 D12 D13 D14 D20 D21 D22 D23 D24 D30
C1 C2 C3 C10 C11 C12 C13 C20 C21 C22 C23 C24 C25 C30 C31 C32 C33
B1 B2 B3 B10 B11 B12 B20 B21 B22 B30 B31 B32
A1 A2 A10 A11 A20 A21 A22 A30 A31 A32
1 2 3 4 6 7 8 10
8600 8200 8000 7800 7600 7400 7200 7000 6800 6600 6400

NO.	DRAINAGE AREA	AREA (Acres)
1	Horsepasture	187
2	Parshall	610
3	Yazzie	1 325

NO.	DRAINAGE AREA	AREA (Acres)
4	Upper Crevasse	2 980
6	Chusca	5 550
7	Lower Crevasse	8 495
8	Mexican Springs	20 910
10	Catron	17 220

FIG. 6

cipitation (Table 4). Eight of the thirteen centers were at elevations of from 7,000 to 8,000 ft and three were below and two above this range.

The author's conclusion (see heading, "Isopluvial Maps") "* * * that stations above El. 7500 experience relatively few high rainfalls" may be influenced largely by the fact that there were but few gages above this elevation. At this station, six out of thirteen centers of greatest precipitation were above El. 7500 (see Table 4).

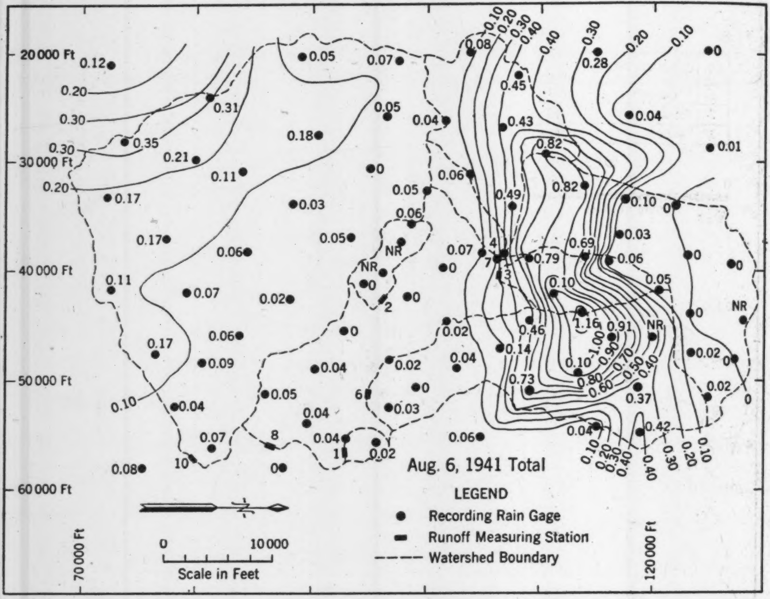

20000 Ft
30000 Ft
40000 Ft
50000 Ft
60000 Ft
70000 Ft
120000 Ft
0.12
0.20
0.30
0.31
0.35
0.21
0.17
0.11
0.05
0.07
0.18
0.03
0.06
0.02
0.09
0.04
0.08
0.10
0.28
0.40
0.45
0.43
0.82
0.49
0.79
0.69
0.46
0.14
0.73
1.16
0.91
1.00
0.90
0.80
0.70
0.60
0.50
0.37
0.42
0.01
NR
Aug. 6, 1941 Total
LEGEND
Recording Rain Gage
Runoff Measuring Station
Watershed Boundary
0
10000
Scale in Feet

FIG. 7.—ISOHYETAL MAP OF TOTAL RAINFALL FOR STORM OF AUGUST 6, 1941, NAVAJO EXPERIMENT STATION, MEXICAN SPRINGS, N. MEX.

The maximum point of rainfall for one day recorded on the experimental watershed during 1941 was 2.40 in. at gage D1 (Fig. 6). This corresponds to a storm of about a 10-yr to 15-yr recurrence according to the Tohatchi record in Table 3. Tohatchi is 7 miles northeast of the experimental watershed. According to the scale of the author's data in Table 3 for Tohatchi, the experimental watershed recorded three storms in 1941 having a 10-yr to 15-yr recurrence and two storms having a 5-yr recurrence. Some day it will be possible to give recurrence intervals for time-area-depth curves. Until such time engineers will have to be content with recurrence intervals for point rainfall records, which are none too satisfactory.

The author's statement that (see heading, "Isopluvial Maps") "* * * any high-intensity, individual storm rarely is recorded at more than one station" is obviously dependent on the spacing of rain gages. At this experiment station, where the gages are spaced about 1 mile apart, the following amounts, in inches, for the storm of July 23, 1941, were recorded at different gages: (2.40), (2.05), (2.02), (1.82), 1.73, 1.50, 1.39, and 1.07, in which values in parentheses belong to the same storm center.

TABLE 4.—CENTERS OF GREATEST PRECIPITATION FOR STORMS ON NAVAJO EXPERIMENT STATION, 1941

Date (1)	Rainfall station[a] (2)	Rainfall amount (in.) (3)	Elevation (ft) (4)	Date (1)	Rainfall station[a] (2)	Rainfall amount (in.) (3)	Elevation (ft) (4)
July 19	C32	0.87	6,620	August 8	B32	1.57	6,400
	D4	0.56	7,730		A22	1.39	6,710
	D13	0.56	7,300		D10	1.12	7,740
July 23	D1	2.40	8,080		E33	0.96	8,150
	E13	1.73	7,540		F21	0.80	7,740
August 6	E21	1.16	7,200	August 15	D22	1.05	7,020
				October 13	E22	1.44	7,060

[a] See Fig. 6.

This discussion is presented to emphasize the need for additional information on the behavior of summer storms of high intensity. As these storms are extremely erratic in time and areal distribution, the need can only be met by a rather concentrated network of recording rain gages operated over a long period. In order to give adequate areal coverage for deriving time-area-depth curves, it is estimated that the gages should be spaced 1 or 2 miles apart and the network should extend over roughly 250 sq miles.

JAMES GIRAND,[14] ASSOC. M. AM. SOC. C. E.—The utilization of meteorologic data in the design of engineering projects always has been surrounded with uncertainties. The engineer approaches his problem with the conviction that some definite correlation between the rainfall records and the runoff data does exist. To establish a proof that will satisfy design requirements is another matter. It is apparent that any relations that do exist are highly complex, requiring the greatest perseverance to discover.

The author was fortunate in having available to him a relief project furnishing the large number of man-hours necessary to make a detailed study of a vast amount of rainfall data. This man power is seldom available to the designing engineer. The publication of the results of these studies is doubly valuable in that it opens a vista of the possibilities of further study and also develops definite information as to what continuing data should be collected. As the science of analyzing climatological data progresses, it should be possible to induce the Weather Bureau to collect its records in the most useful form. Eventually a card system might be devised so that these studies could be made with tabulating machines instead of such great expenditures in man-hours.

The interpretation of weather data is essentially a laborious process requiring a great amount of time and energy; but before this process can be set in motion, it is necessary to determine the basis upon which the analysis is to be made, the result to be sought, and the method of assembling and analyzing the recorded data. The author has not given a sufficiently complete exposition of the various assumptions and methods involved for others to apply his methods.

---

[14] Cons. Engr., Phoenix, Ariz.

Weather data are basically erratic and unreliable. The use of short-term or broken weather records is extremely hazardous. However, meteorologic observations can be valuable when carefully analyzed so that the weight of numbers will offset the individual errors. This effect of erratic records is quite apparent in Table 3 under stations 206, 207, and 209, for Granite Reef Dam, Casa Grande, and Florence, respectively. These stations are about 25 miles apart and are similarly situated in regard to topographic features. However, Table 3 indicates that the one-day rainfall that probably will be reached or exceeded once in a hundred years is 2.50, 6.00, and 2.09 in., respectively. Obviously such wide variation could not be expected at three

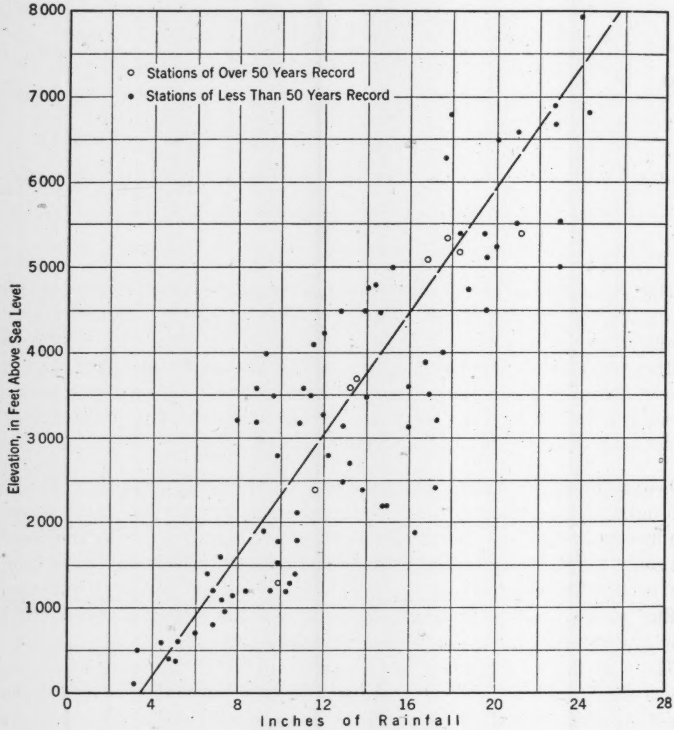


FIG. 8.—RAINFALL IN ARIZONA

similar stations so closely grouped. This divergence is accounted for possibly by erratic records. Further examination shows that the author predicted 5-yr storms for the same stations to be 1.64, 1.63, and 1.72 in., respectively. These latter values appear reasonable.

The effect of elevation on rainfall in the Southwest is a field that deserves further study. The author touches on this by stating "rainfall * * * tends to increase with elevation," but he fails to investigate further. The writer's conclusion, based on numerous studies in Arizona, is that the rainfall is almost exactly proportional to the elevation, as shown in Fig. 8. It is surprising how close a relation is shown, particularly in the long-record stations. These studies seem to indicate that topographic influences are not as important as might be concluded from limited observations. If the relation between elevation and rainfall were introduced into the author's studies, more consistent and satisfactory results might be obtained.

The correlation of observation stations and elimination of erratic data should be the first steps in any rainfall-runoff study. Short-term or broken records should be correlated to that of the longest-term record available. When such a correlation is made, the probability curves of all similar stations will have the same shape and slope. The final results will then be consistent and will represent the nearest approach to an accurate solution.

In many cases it may be desirable to go beyond the terms of available records by some other method, such as the tree-ring technique of A. E. Douglass,[15] as applied by John Girand,[16] Assoc. M. Am. Soc. C. E. By such devices it is possible to ascertain whether the available records represent normal conditions, or periods of excessive drought or flood. Accurate predictions must be based on a complete knowledge of whether the data extrapolated represent normal, wet, or dry years. The tree-ring technique has demonstrated the periodic recurrence of an excessive drought in Arizona at 300-yr intervals. This is highly important to the irrigation or water supply engineer. The presence of long-term periodic floods has not been proved adequately. For flood studies it is possible that existing rainfall records, some of which exceed 70 years, may be adequate as a base from which to predict future floods. The use of any shorter base is certain to lead to erroneous results.

Another valuable field of research would be to amplify the results shown in Table 1. The author's work and results, as set forth in Fig. 5, are for small watersheds. For the design of structures other than the smallest, it is essential that some relationship be found in terms of watershed area. The use of these figures for watersheds of even limited extent would lead to astronomical runoffs.

The author has made a substantial contribution to rainfall studies and it is hoped that he will be able to continue this work after he returns from duty with the armed forces. It is also to be hoped that, in the future, through various governmental agencies, the man power will be made available to the engineering profession for detailed studies such as this.

---

[15] "Climatic Cycles and Tree Growth," by A. E. Douglass, Carnegie Institution, Washington, D. C., 1928.

[16] "Water Supply on Upper Salt River, Arizona," by John Girand, *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 398.

Clarence S. Jarvis,[17] M. Am. Soc. C. E.—No matter how many hours were required to assemble and analyze those hundreds of station records for the two states, comprising more than 7,000 station-years, full recompense is realized in the third from the last paragraph of the noteworthy paper under discussion. To be able to state with definite assurance that two-day rainfalls for any given recurrence interval may be taken as 10% to 20% greater than for the 24-hr record depth, or 20% to 30% greater for a three-day period, is proof of a notable achievement by the author.

The impressive array of authoritative information shown on both the relief maps and the isopluvial maps of the two states marks additional steps forward toward a firm grasp and understanding of a subject hitherto regarded as too vague and irregular for actual analysis and classification.

The writer recalls his personal observations at the Grand Canyon of the Colorado River one summer afternoon, when six or more distinct showers were in progress, moving like gray curtains against the backgrounds of intervening blue sky or red sandstone formation. Although there was no way of determining the actual intensity of each disturbance, it was noted that some of the curtains were like gauze, whereas others were nearly opaque. Presumably, other than orographic influences determined which storm centers should yield with high intensity; for all of the cloud forms were high above the canyon rims, and still higher above the areas of precipitation, which were almost invariably within the confines of the chasm.

Having been permitted to review the early draft of the paper under discussion, and having noted the distinct improvements accomplished by the author, it is the writer's pleasure to enumerate some of the distinctive modifications:

(1) Reduction of the number of objectives;

(2) Unification of effort toward those tangible, simple objectives;

(3) Elimination of such features as might have afforded something of a diversion without particularly strengthening the main thesis;

(4) Employment of graphical presentation at its best for the immediate purposes;

(5) According as much prominence to the deficiencies in basic hydrologic data as to the wealth of records discussed in the textual paragraphs; and

(6) Securing the most effective teamwork from both those in charge of the precipitation records, and those making the necessary transcriptions and orderly tabulations. This should amount to a guarantee as to the correctness of data used and summarized.

It is the writer's belief that the author will be able to extend the generalizations in his closure to great advantage. More of that kind of scientific contribution and discussion is needed, especially where the field has yielded so readily to a patiently conducted, yet masterly, approach.

Paul V. Hodges,[18] M. Am. Soc. C. E.—The assembly and analysis of rainfall records from the many stations, together with the characteristics of certain heavy storms, add a great amount of valuable data to the somewhat meager information available for this area. In Table 3, the summer rains, as the author states, usually occur in periods of less than 10 hr, and therefore this fact should be considered in the use of the table for flood studies.

---

[17] Hydr. Engr., With Office, Chf. of Engrs., U. S. Army, Washington, D. C.

[18] Care, U. S. Engrs. Office, Omaha, Nebr.

The shorter the period of rainfall, the greater the number of rains, probably, in inverse proportion to the rainfall period; whereas summer rains of one or two days would be relatively infrequent.

An example of a rainfall of long duration is the storm producing the San Pedro River flood of September 28, 1926, which had a momentary peak discharge of 98,000 cu ft per sec and a calendar-day discharge of 28,800 cu ft per sec at Fairbank station. The average precipitation over the basin for a 7-day period (September 24 to 30) was 5.04 in., producing a runoff at the Fairbank station of 110,000 acre-ft or 1.38-in. depth over the area during the same period—that is, 27.4% of the precipitation.

From a study of the record flood flows of the upper Gila Basin, a formula representing the relation of momentary peak discharge in cubic feet per second to drainage area in square miles was derived.

The formula $Q = 2{,}070\,A^{0.44}$ represents the maximum recorded floods of the larger rivers, and about the average of flood estimates of many small tributaries. This formula gives the following peak discharge:

A, in square miles	Discharge, in cubic feet per second	Discharge, in inches per hour
1	2,070	3.20
5	4,200	1.30
10	5,700	0.88
20	7,700	0.60
30	9,250	0.48
50	11,600	0.36
100	15,700	0.24

Summer rains are very erratic in size and location, and records at any one station would include but a small percentage of the total number of storms, and those of the highest intensity would very rarely be recorded or measured at any particular station. For this reason the data, for most of the stations, are considered insufficient for determining the maximum rainfall to be expected over long periods of time.

The isopluvial map of New Mexico as developed in Fig. 5 shows higher 24-hr rain than that indicated for western Texas by the probability chart for 50-yr frequency, shown in "Excessive Rainfall in Texas."[19] This is due, no doubt, to the longer record available to the author.

The writer made a study of the rainfall and runoff of the upper Gila River Basin in Arizona and New Mexico (unpublished report of August 14, 1936), in which he determined the average precipitation for different periods of the year, for the precipitation stations having 19 to 38 years of record. It was determined that the average precipitation for the month of July was more than three times the monthly winter precipitation on an average, and that the average precipitation during July, August, and September was greater than that for any other period, being 48% of the yearly total for all stations during periods of record.

---

[19] *Bulletin No. 25*, State of Texas, Reclamation Dept., November, 1934, p. 110.

The average number of days with a precipitation of 0.01 in. or more was computed for the 15 upper basin stations having a station-year record of 315 years. The distribution and average number of days, for different periods of the year, were found to be:

Period	Average number of days	Percentages
April to June	7.48	13.7
July to September	24.22	44.5
October to March	22.80	41.8
Twelve months	54.50	100.0

From this study it seems that the summer storms have a greater daily precipitation than other storms; that is, the summer storms, representing 44.5% of occurrences, produce 48% of the annual precipitation.

Mr. Leopold has presented much information of great value on intensity and the occurrence of rains at the various stations. Although the data for some of the stations may give reliable indications, the great variation in rainfall between near-by stations would tend to discount the value of many predictions of maximum rainfall in Table 3. With longer records the maximum rainfall would tend, no doubt, to be more uniformly distributed.

Edgar E. Foster,[20] **Assoc. M. Am. Soc. C. E.**—A fundamentally sound analysis of the precipitation in the two southwestern states, New Mexico and Arizona, has been presented in this interesting paper. The author has extended the analysis to the basic causes, such as type and movements of air masses, type of storms, and effect of topography. These and related climatic factors should be the first items considered in an investigation of the hydrology of a given region, since they are the underlying causes of the occurrence and distribution of precipitation.

The region under consideration is somewhat off the usual paths of air masses affecting the United States other than the adjacent areas of the Great Basin and Colorado Plateau, and it is protected furthermore from invasion of outside air by the border of high mountains. It is isolated meteorologically to the extent that H. C. Willett[21] designates it as a source region of tropical continental air masses. Since air must remain quiescent to acquire the characteristic properties of a source region, the applicability of such a designation indicates that there could be relatively little interchange of air with outside areas.

---

[20] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[21] "American Air Mass Properties," by H. C. Willett, Papers in Physical Oceanography and Meteorology, Mass. Inst. of Tech. and Woods Hole Oceanographic Inst., Vol. 2, No. 2, 1933.

This meteorological isolation probably promotes the spotted, irregular summer precipitation. This irregularity, added to the effect of the rugged, mountainous topography, would make difficult the construction of isohyetal maps of storms, as the author has indicated by the omission of such maps from his paper. Furthermore, the data of precipitation at one station should be used at other points only after a careful consideration of all pertinent factors that may vitiate the results. For this reason the station-year method of constructing frequency curves would be open to some question because topographical features may cause sufficiently discordant data to offset climatic homogeneity. The author wisely has advised caution in the use of the recurrence-interval data, even through curves were drawn for each of the stations listed.

The writer questions the use of the term "recurrence-interval" when referring to data or curves that show frequency. These are in reality "frequency data" or "frequency curves." It appears desirable to adhere to commonly and widely accepted terms rather than those relatively unknown. The data of 1-day rainfall which probably will be equaled or exceeded once in the specified period of years may be obtained from frequency curves that have been expressed in the integral form as $P = \int_{-\infty}^{+\infty} e^{-u^2} du$. The fact that the data were taken from empirical curves would not alter this principle.

Where rainfall frequencies are important factors in an investigation, the writer suggests that they be computed by methods based on the mathematical theory of statistics or probability. For this purpose the unbounded function proposed by J. J. Slade[22] is recommended. The use of this function involves considerably more work, but it takes into proper consideration the "widely-spaced points in the upper limits" which are the values of rare intense rainfall.

R. W. DAVENPORT,[23] M. AM. SOC. C. E.—The derivation of Fig. 5 showing, for Arizona and New Mexico, the 24-hr rains equaled or exceeded once in 50 years may well be considered in the light of inherent statistical characteristics of the basic data. These data are summarized in Table 3.

Out of the 283 station records that are used, two records have frequency arrays of 50 years or more. The average length of record is approximately 27 years. Substantially, therefore, the records may be considered as offering 283 samples of the highest 24-hr rain in periods of about 27 years. The extrapolation of the frequency arrays to show rains equaled or exceeded once in 50 years produces results that fall short of being equivalent to actual samples of the highest 24-hr rains experienced in periods of 50 years. For purposes of this discussion, however, the data in the "50 years—all year" column of Table 3 will be considered as representing maximum 24-hr rains in 50-yr records at the respective stations.

If long-time frequency relations for 24-hr rainfalls at the respective stations were available, the 50-yr maxima could be assigned frequencies according to their positions on the magnitude-frequency scales. One maximum might be such as the long-time relation showed would be equaled or exceeded once in 40 years, another once in 75 years, another once in 150 years, etc.

---

[22] "An Asymmetric Probability Function," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 35.

[23] Chf., Div. of Water Utilization, Dept. of the Interior, U. S. Geological Survey, Washington, D. C.

The probability that the value to be equaled or exceeded once in 50 years would be equaled or exceeded in a given year is $\frac{1}{50}$. The probability that such value would be equaled or fallen short of in a given year is $\frac{49}{50}$ and in a 50-yr period $(\frac{49}{50})^{50}$, or 0.364. This signifies that, in a large group of 50-yr samples, the maxima in 36.4% would equal or fall short of the values to be equaled or exceeded once in 50 years.

TABLE 5.—DISTRIBUTION OF 283 MAXIMA ACCORDING TO PERCENTAGES

Frequency scale[a] (1)	% (2)	Distribution (3)
50 years or less.......	36.4	103
50 to 75 years........	14.7	41
75 to 100 years.......	9.4	27
100 to 500 years.......	30.0	85
500 years and more....	9.5	27

[a] Equaled or exceeded once in the number of years indicated.

Similarly the value to be equaled or exceeded once in 75 years would be equaled or fallen short of $(\frac{74}{75})^{50}$, or 51.1%, of the samples. The corresponding results for values to be equaled or exceeded once in 100 years and once in 500 years are 60.5% and 90.5%, respectively.

The foregoing shows that for a large number of 50-yr samples the maxima would be distributed on the frequency scale according to the percentages shown in the following tabulation. If the distribution of the 283 assumed maxima of Table 3 conformed to these percentages, the classification of the station records according to frequencies would be as shown in Col. 3, Table 5. Of course, it would be possible to have a great variety of other distributions in samples limited in number to 283, but that shown is reasonably typical.

If frequencies were assigned to the values shown in Figs. 3 and 4, they would be distributed somewhat as shown in Col. 3, Table 5. The median frequency would be that equaled or exceeded once in slightly less than 75 years.

It would seem that, in the definition of isopluvial lines by such data, considerable uncertainty would be involved as to the extent to which they represent variations of magnitude incident to topography and other local causes or variations incident to the use of samples of limited size. The latter variations might tend to obscure the former. Moreover, the isopluvials developed might correspond most closely to the median of the frequencies represented; that is, 24-hr rains equaled or exceeded once in 75 years. Viewed in the light of the uncertainties discussed herein, the significance of Fig. 5 is not clear.

The author is to be commended for his extensive compilation of rainfall data and the presentation of the results in compressed form convenient for reference and use.

A. K. SHOWALTER,[24] ESQ.—This paper is a contribution to the hydroclimatic history of New Mexico and Arizona. The results are representative of the available data, but it is doubtful whether point-rainfall frequencies can indicate directly the frequencies of selected average depths over specific drainage areas. The author demonstrates, for example, that rainfall is more frequent at higher altitudes but that, nevertheless, in a single storm the heaviest rainfall usually occurs near the base of the mountain. If data on frequency of average depths of rainfall over a small, steep drainage area were desired, the results would differ radically if in one case the key station were leeward of a high mountain and in another case at a low elevation directly exposed to the prevailing storm wind.

---

[24] Meteorologist in Charge, Hydrometeorological Section, Weather Bureau, U. S. Dept. of Commerce, Washington, D. C.

If Mr. Leopold's data are to be used in design, it is recommended that pilot studies be conducted to determine the typical relationship between the depth of rainfall at the key station and the average depth over the project basin. Such studies would seem to be especially important in regions dominated by pronounced orographic influences.

More complete data are needed on typical depth-area-time relationships in this region, and it is probable that adequate data will never be obtained without the use of very dense networks of electrically synchronized automatic rain gages. Sampling networks should be located in the relatively flat areas and also in regions of marked topographic variation.

The author neglected to mention that tropical Pacific hurricanes sometimes move up the west coast of Mexico or through the Gulf of California bringing with them deep currents of moist air and subsequently heavy widespread rains over parts of Arizona or New Mexico long after surface barometric indications of the tropical cyclone have disappeared.

Another storm type not mentioned is a deep, cold, cyclonic circulation which may move slowly across the region and cause the northward-flowing, moist, tropical, maritime air to the east to curve cyclonically as it advances up slope through Texas and northern Mexico into New Mexico and Arizona. Such a circulation pattern in a surface air mass must necessarily be accompanied by horizontal convergence and vertical stretching which tend to accelerate the release of potential instability and convective overturning. Such patterns are typical of eastern Colorado and New Mexico from late April through early June.

Mr. Leopold's data and conclusion regarding the relationship between rainfall intensity and height seem more consistent with theoretical deductions than much of the previous literature on this subject. The opportunity for rainfall is increased at high-level stations by the increased lift, and therefore there is a higher frequency at the higher elevations if only average rains or rains of low magnitude are considered. However, as the height increases, the depth of available precipitable water in the air above decreases (neglecting localized chimney effects which carry suspended condensation particles), and also the zones of horizontal convergence are not so well defined. The result is lesser frequency of excessive rains of a given magnitude at the higher elevations.

WALTER B. LANGBEIN,[25] ASSOC. M. AM. SOC. C. E.—In general, rainfall regimes for engineering purposes are completely defined only by evaluation of the depth-area-intensity-frequency characteristics—a four-dimensional problem that has nowhere been solved. Merrill Bernard[26] and Gail A. Hathaway,[27]

---

[25] Associate Engr., U. S. Geological Survey, Washington, D. C.

[26] "Primary Rôle of Meteorology in Flood Flow Estimating," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 311.

[27] *Ibid.*, p. 370.

Members, Am. Soc. C. E., have described the latest methods of deriving depth-area-intensity relations, frequency being omitted. Mr. Yarnell's useful publication[2] treats the depth-intensity-frequency relations of "point" rainfall at first-order Weather Bureau stations exclusively, omitting area of storm rainfall. In the western part of the United States the sparsity of first-order stations imposes a serious shortcoming. The author contributes much toward filling this need by incorporating information from all stations, although his compilation is limited in turn because of this broader coverage to 24-hr periods. Thus, he reveals an expectancy of 24-hr rainfall of 6 in. or more once in 50 years on the average in Arizona, whereas Mr. Yarnell showed nothing more than 3.5 in. The deviation from Mr. Yarnell's results in New Mexico, although appreciable, is not so marked.

Isopluvial maps such as Fig. 5 are open to many kinds of errors, including errors of measurement, errors of sampling (the next 30 years might produce materially different frequency curves), and also errors arising from necessary generalization. Such errors are inherent in the statistical methods available to the engineer and are not peculiar to Mr. Leopold's expert handling.

Statistical theory is not at all clear even in the matter of fitting a frequency time scale to the annual data. Mr. Leopold has computed his plotting positions by the California method, in which recurrence-intervals are computed by the formula $\frac{y}{m}$ (y being the number of years in the series, and m, the relative rank of an annual rainfall, listed in decreasing order of magnitude). Equally legitimate is the method of listing in the reverse order, in which case the interval in years is $\frac{y}{y - n}$ (n being the relative rank beginning with the lowest as 1). The values obtained by the second formula are generally called exceedance intervals. A graph plotted to conform with the recurrence-interval plotting positions will be higher than the exceedance-interval graph (that is, for a given interval in years the former will give the higher rainfall). There are other methods of plotting;[28, 29] all produce graphs that lie between the recurrence-interval graph and the exceedance-interval graph. A common method is the Hazen compromise, $\frac{y}{m - 0.5}$ or $\frac{y}{y - n + 0.5}$, these two ratios becoming numerically equal. However, it is desirable to emphasize that Mr. Leopold's results, particularly the extrapolated 50-yr and 100-yr rainfalls, are higher than would be obtained by any other method of plotting now in use, applying the same data.

Assuming that the frequency data in Table 3 and Fig. 5 are accepted, this paper presents a wealth of material for study and analysis. For example, a search might profitably be made to find the effects of altitude or other mea-

---

[28] "Statistical Analysis in Hydrology," by L. R. Beard, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1110.

[29] "On Serial Numbers," by E. J. Gumbel, *Annals of Mathematical Statistics*, Vol. 14, No. 2, June, 1943, pp. 163–178.

surable factors on rainfall intensity. The following statement by G. N. Brancato[30] seems pertinent to such an inquiry:

> "However, contrary to published and popular accounts, the thunderstorms produced the greatest amount of precipitation at the lower elevations and not on the mountain slopes. The most favorable condition for the production of heavy rain is the presence of an air mass with a sufficient amount of available energy and the greatest possible amount of moisture. Orographic lifting is very effective as a mechanism to release the latent energy in an air mass but as the air is lifted over progressively higher terrain the total amount of available precipitable water above any given area becomes progressively smaller. Over the lower level areas therefore there is always more potential rain waiting to be released, but the opportunity for realization occurs less frequently than at the higher elevations which are favored by orographic lifting."

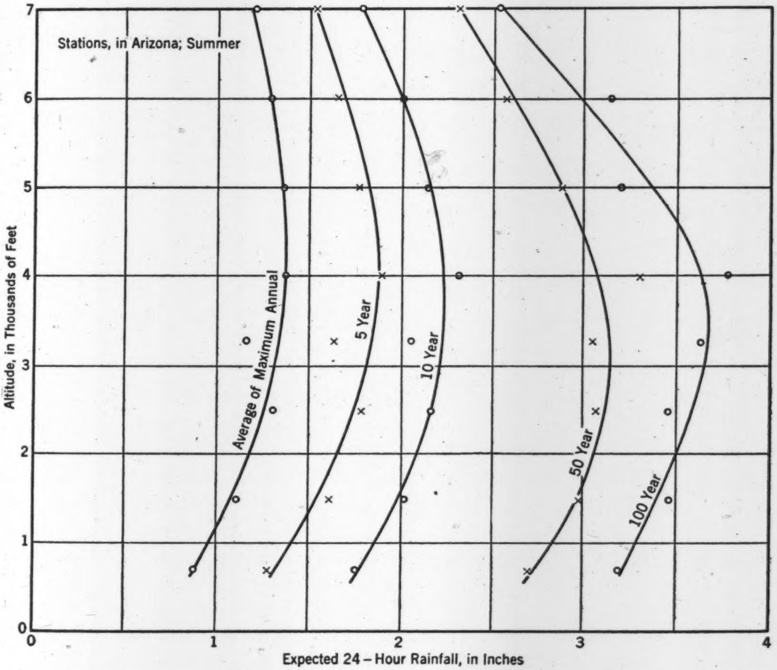


FIG. 9.—STUDY OF RELATION BETWEEN ALTITUDE AND EXPECTED 24-HR RAINFALL OF INDICATED RECURRENCE INTERVALS

Fig. 9 shows results of averaging groups of stations of like altitude for the summer in Arizona. The graphs show that, on the average, rainfall intensities are greatest in the range of altitudes between 3,000 ft and 4,000 ft. The decline in expected intensities at high altitudes is especially noteworthy.

---

[30] "The Meteorological Behavior and Characteristics of Thunderstorms," by G. N. Brancato, Hydrometeorologic Section, U. S. Weather Bureau, April, 1942 (processed).

Furthermore, the spread between the 5-yr and 50-yr recurrence-interval graphs is greater at low altitudes than at high altitudes. Indeed, the indicated 50-yr rainfall is greater on the average at altitudes below 1,000 ft than at the 7,000-ft level. However, 5-yr rainfall is greater at the 7,000-ft level; and 10-yr rainfall is nearly the same at both levels. These relationships probably result from greater thunderstorm frequency at high levels due to instability created by lifting of the air. Seemingly, when, at rare intervals, conditions favor the development of critical storms at low levels, the resulting intensities are apt to be greater because of the greater amount of precipitable moisture in the air column.

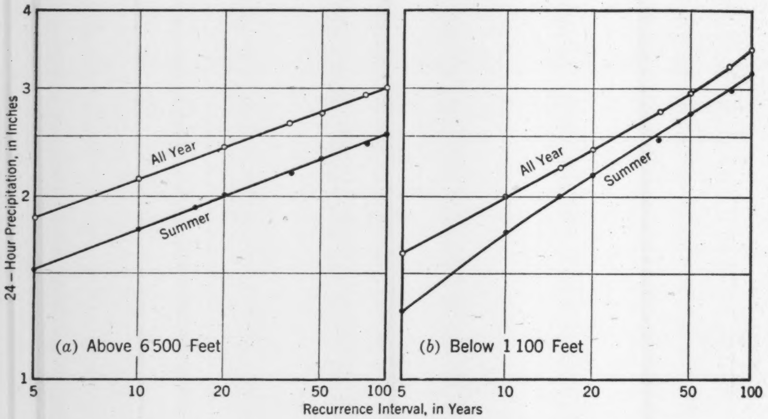


FIG. 10.—COMPARISON OF AVERAGE FREQUENCY CURVES FOR SUMMER AND ALL YEAR, AT ALTITUDES BELOW 1,100 FT AND ABOVE 6,500 FT, IN ARIZONA

These ideas are further explored in Fig. 10. There is a 20% difference between the "summer" and the "all-year" curve at altitudes higher than 6,500 ft (see Fig. 10(*a*)), indicating that non-summer rainfalls are important contributors to storm intensities at high altitudes. At stations below 1,100 ft (Fig. 10(*b*)) the spread is much less, especially at the upper end. At low altitudes in Arizona, the characteristics of rainfall intensities are seemingly dominated by those of summer. The difference in slope of the high-altitude and low-altitude graphs indicates a greater range in expected intensities at low altitudes than at high altitudes, as already has been mentioned.

Mr. Brancato outlines two influences on the intensity of rainfall produced by thunderstorms: (1) Lifting forces, and (2) amount of precipitable moisture. Altitude is a measure of the lifting indigenous to a station. It would seem that, for stations at the same altitude, mean seasonal or annual precipitation might furnish an approximate measure of such meteorologic and local effects as are indicative of precipitable moisture. Furthermore, comparisons of intensities at stations having the same mean seasonal or annual rainfall would provide a basis for study of the effect of altitude separately. These studies are made most simply by means of a multiple-correlation chart.

Table 3 does not list mean summer precipitation, and therefore "all-year" data have been used in this correlation. Altitude and mean annual precipitation have been correlated with "average of maximum 1-day rainfall" for "all year." Separate correlations would need to be made for each recurrence-interval as well. For the longer recurrence-intervals the correlations become less accurate.

Diagrams were prepared on which were plotted altitude as ordinate and "average maximum 24-hr rainfall" as abscissa; and mean annual precipitation was written along each plotted point. On this background of points, contours

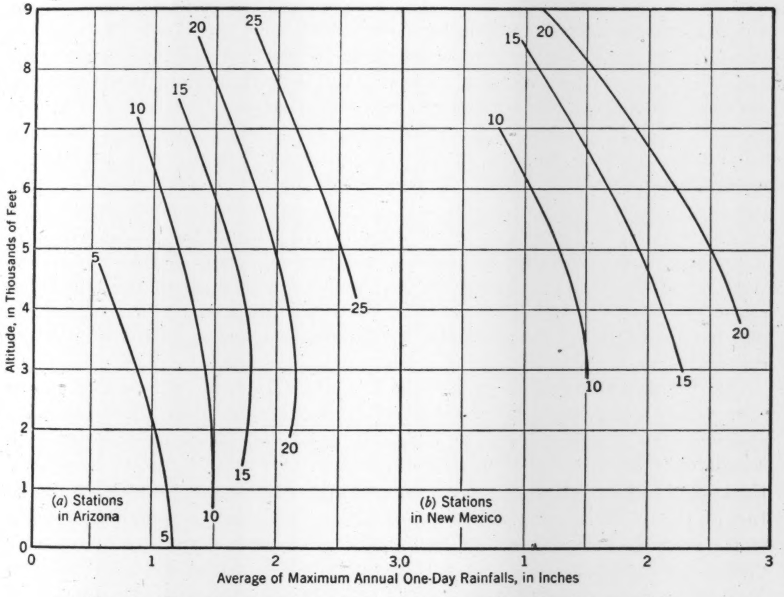


FIG. 11.—RELATION OF ALTITUDE AND AVERAGE OF MAXIMUM ANNUAL 1-DAY RAINFALLS IN TERMS OF MEAN ANNUAL PRECIPITATION

of equal mean annual precipitation were drawn. The resultant contours are shown in Fig. 11 for Arizona and New Mexico, respectively. The mean error of estimate in Fig. 11(*a*) is 0.15 in., and in Fig. 11(*b*), 0.20 in. The decrease in rainfall intensity with altitude, mean annual precipitation constant, is to be noted. Also, for a given altitude there is a marked increase in rainfall intensity with increase in mean annual precipitation. Diagrams similar to Fig. 11 should provide means for estimating rainfall intensity at places in Arizona and New Mexico when the altitude and mean annual precipitation are known.

Mean annual precipitation can be interpolated from isohyetal maps modified if necessary on evidence given by native vegetation.

Any generalization, whether by graph or map, however, would be subject to erratic variation because of the great divergence in relief in the mountainous and intermountainous parts of the West. Measurements of rainfall provide the best and only reliable guide to storm frequency and intensity. As Mr. Leopold states, caution is imperative in transferring the results to locations distant from the place of observation, especially if the topographic setting or orientation with respect to the moisture-carrying winds is markedly different.

Emil P. Schuleen,[31] Assoc. M. Am. Soc. C. E.—In making available to the engineering profession the results of his extensive study, the author is to be commended. A great amount of the detailed work involved in his analyses apparently was done by a number of workers from the Work Projects Administration. The author was fortunate in having had such assistance at his disposal. The writer wonders, however, if (as is sometimes the case when assistance is available in abundance) the work has not been reduced largely to mechanical procedures with efforts directed toward volume rather than reliability. In this respect he has reference particularly to the extensive results summarized in Table 3, which will be discussed further in later paragraphs.

The section on "Seasonal Distribution of High Rainfalls" might well have followed immediately after the section on "Storm Types," for these two sections, together with Fig. 2, provide an excellent treatment of the relation of storm types and topographic relief to the intensity, geographic distribution, and seasonal occurrence of rainfall in the various parts of the area under consideration.

In the last paragraph under the heading, "Aerial Pattern of Summer Storms," it is implied that the depth-area data given in Table 1 are but a few samples, and that other similar data are available, although considered inadequate to indicate depth-area-time patterns for design purposes because of the lack of definite information concerning intensities. Apparently the limitation on the use of the available data for making such analyses applies primarily to the case of small areas where rainfalls of a few hours duration would govern design requirements. Would not the available data, however, permit analyses for larger areas where rainfalls of 1-day or 24-hr duration would be important? Data of that nature, separated in accordance with the summer-type and winter-type storms, would provide a valuable addition to the author's paper in the interest of design storms and floods for larger drainage areas.

In the first paragraph under the heading, "Rainfall Intensity," it is indicated that the mass curves in Fig. 1(*a*) represent the most intense rainfalls on record at recording gage stations in New Mexico and Arizona. In the second paragraph of this section it is indicated that the points in Fig. 1(*b*) were obtained from the records of standard nonrecording rain gages where the times of beginning and ending of rainfall had been noted by the observer. The writer is not familiar with the available data, but suggests that it may have been possible to develop reasonably accurate mass curves for some or many of the 72 storms represented, by comparing the records from the nonrecording stations with those from recording stations in the vicinity and by comparing the times of beginning and ending of rainfall at the respective stations. This analysis would be facilitated further if the regular time of observation differed materially among the several stations, as is frequently the case. If such mass curves were available, presumably their envelope would approximate the envelope curve shown in Fig. 1(*b*). This envelope curve, therefore, would serve as the mass curve for the design storm, if the project were to be designed for the maximum storm possibility.

---

[31] Senior Engr., U. S. Engr. Office, Pittsburgh, Pa.

The author states that the frequency data given in Table 3 are based on records for 15 years or more from standard nonrecording rain gages, and that in a few instances shorter record periods were used. There is reason to question the reliability of frequency determinations for intervals of as many as 100 years when the studies are based on records for as few as 15 years. Most extensive rainfall records indicate tendencies for cyclic variation with alternate periods of heavy and light rainfall. Frequency studies based on short periods of record, therefore, may not reflect truly average conditions. There is also the possibility in the use of short records for one or two abnormal or subnormal values to warp the true tendency of the remaining data. The author has recognized this possibility by his reference (see heading, "Frequency of High Rainfalls") to "aberrant points in the upper limits of the series." It is noted also that in some cases the rainfall values were "24-hr values" based on the observer's notations with respect to the time of beginning and ending of rainfall, which in some instances indicated a total lapsed time of considerably less than 24 hr, whereas in other cases the values were "1-day values" as obtained from consecutive regular daily observations. It is noted further that, where rains continued through two or more days, the largest amount recorded for one day was used. All these variations in the manner of applying the basic data, while difficult to avoid, affect the reliability of the results.

Illustrations of apparent inconsistencies in results may be had from a study of the rainfall values for various frequencies given in Table 3, in conjunction with reference to the relief maps of Figs. 3 and 4. The writer has selected two instances as examples. The first of these involves a comparison of the stations at Tatum and Prairieview, N. Mex. These two stations differ but 125 ft in elevation and are only 10 or 12 miles apart in an area where the topographic relief apparently is uniform and similar, yet the 50-yr value for the first is almost twice that for the second. The second instance involves a comparison of the stations at Palo Verde, Roy, Solano, Hoosier Ranch, and Mosquero, in New Mexico. These stations range from El. 5579 to El. 5884, an over-all difference of 305 ft. All five stations are located within a radius of 6 or 7 miles in an area which, from Fig. 4, appears to be relatively uniform and similar in topographic relief; yet the 50-yr rainfall values range from 3.77 to 6.80, an over-all difference of about 80%. Two of these stations, Palo Verde and Roy, are indicated by Fig. 4 to be only 3 or 4 miles apart and by Table 3 to differ in elevation by only 4 ft, yet the 50-yr rainfall value for the second is more than 30% greater than that for the first. One might expect that rainfall frequencies should be similar at the various stations in these respective areas, unless there are topographic features, not apparent in Fig. 4, that are of sufficient extent to explain the differences in the rainfall frequency values given for those stations in Table 3. If no such explanation can be found, these differences must be attributed to inconsistencies in the basic data or to the method in which the data have been applied to obtain the results.

In commenting on the fact that Table 3 indicates certain stations to have the same 24-hr rainfall expectancy at a given recurrence-interval for summer as for the entire year, the author makes the statement that (see heading, "Frequency of High Rainfalls") "This simply indicates that that station characteristically received the largest 24-hr rainfalls in the summer period and, therefore, the recurrence-interval curves for summer and all year would be practically identical." It is recognized that this might be true in some cases, but in others it is difficult to understand why it should be true with respect to one station and not with respect to another in the immediate vicinity and in the same general topographic setting. Referring again to the stations at Tatum and Prairieview, Table 3 indicates the 50-yr values for the former to be 7.70 and 8.00 for summer and all year, respectively, whereas in the case of the latter both values are given as 4.08. Unless (as mentioned in the first comparison of these two stations) there are topographic features, not apparent in the relief map, that affect one station and not the other, it does not appear reasonable that the greatest rainfalls characteristically should occur in the summer at one station and during the remaining seasons at the other.

Under the heading, "Isopluvial Maps," it is stated that "The small areal extent of high rainfalls occurring in summer-type storms is a factor leading to differences between stations since any high-intensity, individual storm rarely is recorded at more than one station." By this statement the author indicates that the records in some instances are erratic. It is probable also that, in many instances, the greatest rainfall during a high-intensity storm was not recorded at any standard station. Where such spotty rainfall occurs in a region of relatively uniform topographic relief, and heavy rainfall is recorded at one station and not at another near-by station, the location of the point of heaviest rainfall must be attributed to mere chance. Presumably, if both stations had long record periods, the occurrence of high rainfall values would be about the same at each and their frequency curves would approach one another.

The writer's comments, particularly with respect to the variable nature of the results in Table 3, are made with the intent of calling attention to possible sources of inconsistencies in the basic data and in their application leading to the tabulated results. He is not familiar with the topographic details of New Mexico and Arizona and his comments regarding the effects of such details on rainfall intensity and distribution are based solely on the apparent variations in relief indicated by Figs. 3 and 4.

The paper as a whole is very well prepared and presents much valuable information to the engineering profession. It is suggested, however, that the value would be enhanced if the results in Table 3 were classified as "excellent," "good," "fair," etc., in accordance with the author's knowledge of the relative merits of the basic data and his results.

LUNA B. LEOPOLD,[32] JUN. AM. SOC. C. E.—When the writer started the preparation of the tables of frequency-magnitude of precipitation (Table 3) for nonrecording rain gage stations in New Mexico and Arizona, he was confronted with the task of making estimates of design floods for numerous flood control structures on small watersheds scattered throughout the two states. Each time such a design problem arose the rainfall records available for stations near the watershed in question had to be analyzed. The watersheds were scattered through two states where topographic relief, elevation, soils, and rainfall vary tremendously within short horizontal distances, and the flood control structures were often to be located in inaccessible country where rainfall stations were scarce. It was necessary, therefore, not only to analyze rainfall records for stations adjacent to the watershed in question but to attempt to find other watersheds where the factors affecting runoff would be comparable. Thus, tabulations of rainfall records soon were accumulated for rainfall stations located all over the two states. In practically all cases, intensity records were lacking for stations where rain analyses were necessary.

As a result of these circumstances and limitations of time, the writer was not able to make detailed studies of storms of long duration which would be applicable to the design of major structures on large watersheds. However, even the problem of intensities and frequency of storms of small areal extent in the Southwest has always been one of considerable importance owing to the scarcity of rain gages, particularly intensity gages, in such a large area of diverse topography. The writer wishes to emphasize the comments made by Mr. Girand and Mr. Hodges that, as stated in the paper, the frequency data were prepared primarily to study the storm rainfall on small watersheds, and have limited applicability to long duration storms. Mr. Schuleen suggested that analyses of the rainfall for long duration storms would add to the value of the paper. The writer is in entire agreement and regrets that the study could not, at that time, have been so extended.

The data presented under the heading, "Comparison of Magnitude of 1-Day, 2-Day, and 3-Day Rains," and discussed by Mr. Jarvis represent a preliminary approach to the problem of long duration storms, but, as stated in that section of the paper, the results are indicative only. Even since the installation of additional intensity gages by the Weather Bureau in 1940, numerous storms of long duration and considerable magnitude have occurred, the records of which should certainly be considered in any detailed study of extended storms in the area.

The variability of the frequency-magnitude values for closely adjacent stations was noted by Messrs. Girand, Hodges, and Schuleen. At first glance, these variations seem to invalidate the final results, and they lead Mr. Schuleen to conclude that either the basic data are inconsistent or the method of analysis is at fault. The writer called attention to these variations under the heading,

---

[32] 2d Lt., Air Force, U. S. Army, Los Angeles, Calif.

"Isopluvial Maps," and, to give added prominence to the variations, the 50-yr values were plotted on topographic maps in Figs. 3 and 4.

This variation between stations is a reflection of an important characteristic of the summer-type rainfall. Because of the physical nature of the short duration storms, the area of most intense rainfall is small, and one station may receive a large amount of rainfall while adjacent stations receive none. An example is the storm at Las Cruces, N. Mex., on August 29, 1935. At that station 6.49 in. of rain fell in 7 hr while other stations in the vicinity recorded "none" or "trace" up to 1.83 in.[33]

Both from the standpoint of the actual frequency figures computed and from a consideration of the physical character of the short duration storms, the element of chance seems to play an important part in determining the rainfall values actually recorded at a given station. The available records for the Southwest generally cover too short a period to give either a true picture of the frequency of the infrequent storms, or consistent frequency values at any pair of adjacent stations.

W. V. Turnage and T. D. Mallery[34] analyzed one year of rainfall records from sixteen standard rain gages in a 300-m square plot and concluded that, although one rainstorm gave considerable variation of precipitation between gages, at the end of the season, the total rainfall for each gage compared closely with the average seasonal rainfall for all gages. They noted that the variation in total seasonal rainfall between near-by gages was smaller for stations in Arizona near Tucson than for stations near Yuma, and concluded that spatial variation in seasonal precipitation is greater in areas of low rainfall than in areas of high rainfall.

The variation between adjacent stations does tend to iron out as the length of record increases, but, when a station happens to experience an exceptionally great rainfall, a long time might elapse before other rains of that magnitude are recorded again at the station (as, for example, at Las Cruces). In the 48-yr record at Las Cruces the greatest rain recorded in 24 hr was 6.49 in., which actually fell during a 7-hr period. The second largest rain at that station was 2.43 in.

The topographic maps of Figs. 3 and 4 show that the frequency-magnitude values in a given broad area differ considerably from the data for other areas in a different geographic setting, in spite of the tendency for variations between adjacent stations to mask the broad differences. As an example, compare the data for a 50-yr expectancy (summer) for stations in a west-east strip across east central New Mexico. The stations chosen for Table 6 were divided arbitrarily into three groups separating stations into geographic areas more or less centered around Torrance, Guadalupe, and Quay counties. In this strip of territory there are no important mountain ranges, and all stations are approximately at the same elevation. However, the stations in the Estancia Valley area (Torrance County) are more or less protected by the Sangre de Cristo Mountains to the north, the Manzanos on the west, and the Gallina-Capitan ranges on the south. This relative isolation differentiates the Estancia Valley from the high plains along the east edge of the state where the effect of the frontal weather of west central Texas is often felt. No important topographic barriers protect the high plains stations from southerly wind currents.

---

[33] "Areal Extent of Intense Rainfall, New Mexico and Arizona," by Luna Leopold, *Transactions*, Am. Geophysical Union, Pt. II, November, 1942, pp. 558–563.

[34] "An Analysis of Rainfall in the Sonoran Desert and Adjacent Territory," by W. V. Turnage and T. D. Mallery, *Publication No. 529*, Carnegie Inst. of Washington, 1941.

TABLE 6.—COMPARISON OF 50-YR (SUMMER) FREQUENCY-MAGNITUDE VALUES FOR STATIONS IN EAST CENTRAL NEW MEXICO

WEST			CENTRAL			EAST		
Station	El. (ft)	50-yr frequency (in.)	Station	El. (ft)	50-yr frequency (in.)	Station	El. (ft)	50-yr frequency (in.)
Stanley	6,317	3.09	Palma	7,000	5.40	Logan	3,851	3.80
McIntosh	6,355	3.25	Cuervo	4,849	3.06	Tucumcari	4,100	3.58
Estancia	6,300	2.90	Santa Rosa	4,624	2.35	San Jon	3,982	4.55
Tajique	7,100	3.45	Pastura	5,285	3.00	Montoya	4,335	5.60
Mountainair	6,475	3.11	Vaughn	5,930	3.40	Melrose	4,400	4.25
....	....	....	Lagunita	4,500	4.30	St. Vrain	4,250	3.50
....	....	....	Alamogordo	4,338	3.65	Clovis	4,262	5.01
....	....	....	Ft. Sumner	4,028	3.85	....	....	....

Table 6 indicates an increase in the magnitude of 24-hr summer-type rain of 50-yr frequency from Torrance County on the west to Quay County on the east, although closely adjacent stations vary considerably between themselves.

The manner of grouping stations or records that do so differ is a question on which probably no two persons would agree. One method might be by station-year grouping. The requirements for the use of such a grouping were discussed under the section, "Applicability of Station-Year Analysis." As stated there, frequency curves differ so much that it is difficult to decide whether the rainfall experience at certain stations differs mostly due to chance, or whether assignable causes, such as topography, were operative. The assignable causes may operate in subtle ways which are not immediately apparent. Thus, different persons may not agree on which stations should be grouped together.

In order not to generalize the basic data by grouping stations, the writer presented the frequency data separately for each station, and then generalized in the construction of the isopluvial map. Since the number of rain gages is small per unit area, the short records are included even though their value is less than that of the long records. For comparability the short records were analyzed in the same manner as longer records, and the engineer who uses the data must consider the length of record in weighing its worth. To facilitate this, the writer included in Table 3 the exact years of record used in making the frequency arrays. The writer agrees with Mr. Schuleen that the 100-yr frequency value based on a 15-yr record is of questionable reliability, but the short records should be considered rather than disregarded. Since the grouping of stations appeared to be open to many questions, particularly concerning which stations should be thrown together, the writer considered it justifiable to present the analysis of the short records as well as that of the long records, with the caution that reliability of the computed frequencies varies.

The use of the highest rain on record each year for the construction of frequency curves is comparable to the annual flood method.[35] The recurrence-interval or California Method of plotting[36] has had considerable popularity, but it represents only one of many procedures which might have been employed in this analysis and, as Mr. Langbein stated, gives values somewhat higher than other methods.

Comparison of the results obtained by various methods of plotting indicated that the magnitude of rainfall values for frequencies of 50 to 100 years was determined mostly by the four or five highest rainfall values recorded at the station. If these highest rainfall values fall on a smooth curve, the method of computing the plotting position is of importance. However, for many stations whose records were short, the highest values did not plot in a smooth curve and the fitting of a curve through the points was a matter of individual judgment. Consequently, the writer concluded that the different methods of plotting in such a case give less variation in final values than would the positions of a smooth curve drawn through the plotted points by different workers.

The construction of an isopluvial map such as Fig. 5 is open to many kinds of errors, as Messrs. Langbein and Davenport have explained. It is presented, however, as one way the frequency data may be generalized to give a rough over-all picture of the geographic distribution of the rainfall expectancy. As shown by Mr. Davenport, with a large number of 50-yr samples of maximum 24-hr rainfalls, these maxima would be scattered around their mean in a probability curve, or with random distribution. This would be true if there were no assignable causes such as topography to make the rainfall differ between stations. In such a case, the geographic distribution of maxima would also be random.

However, when the data are plotted on a map, the geographic distribution of values is not random but, as demonstrated by Fig. 5, tends to show a distribution which can logically be explained by topographic differences and relation of the area to sources of moisture. Thus, if an isopluvial map were properly analyzed, it should provide a key to the geographic distribution of causes which tend to vary the record between stations—all causes being integrated into the final map. In practice, however, only general principles can be cited to explain why isopluves are distributed as they are. More detailed studies of macro-climate and micro-climate are necessary before all the obscure effects of assignable causes can be evaluated.

The rainfall frequency for a given small area should be derived from the applicable station frequency-magnitude curves rather than from a general isopluvial map as presented in Fig. 5. As a generalized picture of the distribution of frequency data over a large area, the isopluvial map is a convenient picture but should not be trusted alone to define the frequency-magnitude values for a given spot.

Mr. Langbein points out that the highest 24-hr isopluve for 50 years shown by Mr. Yarnell[2] in Arizona is 3.5 in., whereas the writer placed a 6-in. isopluve over the mountain area surrounding Crown King, Ariz. This difference might be expected inasmuch as the maps constructed by Mr. Yarnell were based on only two intensity records in Arizona—at Phoenix and Flagstaff. In the record analyzed by Mr. Yarnell, the greatest rain at Phoenix was 1.15 in. in a 30-min period, whereas the greatest 24-hr rain (during the summer season) in the writer's analysis for Phoenix was 2.91 in. In a 25-yr record, on the other hand, Crown King experienced three rains in the summer season which exceeded 3 in. in 24 hr, the largest of which was 5.05 in.

---

[35] "Floods in the United States," by C. S. Jarvis and others, *Water-Supply Paper No. 771*, U. S. Geological Survey 1936, p. 54.

[36] *Ibid.*, p. 58.

Thus, the importance of adding to the very valuable material published by Mr. Yarnell is apparent, even if the data available for analysis are subject to the limitation that they are 24-hr totals and not intensity records.

Although the isopluvial values presented by the writer in Fig. 5 are in closer agreement with those of Mr. Yarnell for New Mexico than for Arizona, they are higher for both states. The isopluves of Mr. Yarnell for New Mexico agree very satisfactorily with those derived by Robert L. Lowry, Jr.,[37] M. Am. Soc. C. E., for the western part of Texas. A reconsideration of the position of the isopluves in Fig. 5 for eastern New Mexico would probably be profitable in order that they might be drawn in a more easterly position for better agreement with those of Messrs. Yarnell and Lowry.

Mr. Langbein's valuable discussion presents more detailed studies of the relation of intensity to elevation. The writer indicated that the center of greatest precipitation occurred near the base of the mountain slope. Mr. Langbein's further analysis of the same data showed a decrease of intensity with elevation. The same result was reached by Mr. Brancato[30] whose conclusions were supported by Mr. Showalter at least from theoretical considerations; but Messrs. Harrold and Dickson show that the data from Navajo Experiment Station are not in agreement with this conclusion. As Mr. Harrold stated, there is need for additional information before the details are understood.

In summary, the lack of long records and the scarcity of rain gages make it necessary to utilize records that are really too short to be considered reliable for frequency determinations The physical character of summer-type storms leads to differences between records of adjacent stations. These differences result from a combination of chance and assignable causes, which cannot be satisfactorily separated. Workers agree that most of the high rainfalls which fall in summer, and are recorded as 24-hr totals, actually fall in periods of less than 10 hr, and a study of intensity records and times of beginning and ending of rain shows that many of these rains fall during periods of 2 to 3 hr.

The intensity-time curves presented in Fig. 1 probably represent the intensity patterns for most of the summer-type, 24-hr rains which were analyzed for frequency, but, at best, the combination of an average intensity histogram with a total rainfall amount for a given frequency only provides additional data for depth-intensity-frequency relations of "point" rainfall. The relation of these factors to areal distribution of a single storm still remains to be demonstrated by research such as that being conducted at Navajo Experiment Station and Southwestern Forest and Range Experiment Station.

---

[37] "A Study of Rainfall in Texas," by Robert L. Lowry, *Bulletin No. 18*, State of Texas Reclamation Dept., August, 1929.

Engineers who use the data presented in Table 3 should do so with caution, because in many cases the variation of frequency values between adjacent stations cannot be explained satisfactorily. Only by detailed studies of assignable causes can the effect of chance be properly evaluated. Consideration of the frequency-magnitude curves for individual stations will lead to a better choice of a composite frequency value than will the use of a generalized isopluvial map.

Because of the inherent differences between long duration storms typical of winter and the intense thunderstorm type of summer rainfall, the frequency-magnitude values are applicable primarily to small watersheds, whose maximum flood flows result from the cloudburst type of storm.

The writer wishes to express his appreciation for the contributions made by the discussers. They are pertinent and valuable additions to the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2223

FLOW CHARACTERISTICS AT RECTANGULAR OPEN-CHANNEL JUNCTIONS

By Edward H. Taylor,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Harold K. Palmer, G. H. Hickox, J. C. Stevens, C. J. Posey, I. B. Hosig, L. Standish Hall, Brooks T. Morris, and Edward H. Taylor.

Synopsis

The research of which this paper is a part comprised a study of the phenomena associated with the combining and dividing flow of water in open and closed conduits. The paper is concerned only with the open-channel problem, in which the variables involved are so numerous that only an empirical solution seems possible. The conformity of actual behavior to theory is discussed, and an empirical solution is included which may be applied to rectangular channels. For other prismatic channels or for natural streams, the flow characteristics at the junction are best determined by models controlled in an hydraulic laboratory.

Notation

The letter symbols introduced in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

Combining Flow

The principal problem of combining flow may be stated as follows: When two streams combine in a single channel, the depth just below the junction will be fixed by the backwater characteristics of that channel and the magnitudes of the combined rates of flow. The problem is to predict the depth in each tributary channel just upstream from the junction. One of the factors involved is the ratio in which the incoming flows are divided. In studying the combining streams, this ratio was made one of the independent variables.

Note.—Published in November, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Naval Ordnance Laboratory, Washington, D.C.; formerly Instr., Mech. Eng., Univ. of California, Berkeley, Calif.

It is immediately apparent that an unlimited number of different types of stream intersections could be devised. The angles of intersection, the breadths of the channels, the directions of flow, etc., could be combined in such a multiplicity of ways as to make a completely exhaustive investigation a matter of many years' work. The studies have indicated that generalization of the results presented is not possible, or even desirable, so that no attempt has been made to present a mathematical statement applicable to every type of stream intersection. Rather, the attempt has been to investigate the importance of the problem, to point out limitations in the theory, and to demonstrate how the results may be applied in practice.

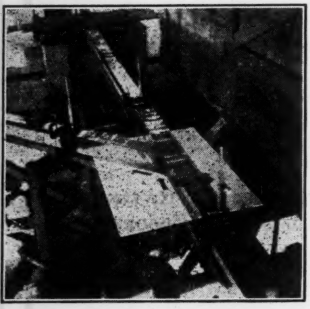


FIG. 1

Experimental Equipment.—The tests were conducted in small, horizontal, rectangular channels, all 4 in. wide and with a maximum depth of 4 in. The intersections were constructed of a transparent plastic, and the remainder of the channels were built of galvanized sheet metal. Fig. 1 gives a general view of the experimental setup. Water-surface elevations were measured with point gages, and rates of flow by calibrated diaphragm orifices in the supply pipe. The elevations of the water surfaces in both channels could be varied by means of adjustable gates at the lower ends.

Theory and Assumptions.—The scope of discussion will be limited to the case shown in Fig. 2. The restrictions are: (*a*) The channels are of equal width, (*b*) the bottom slopes are all zero, (*c*) the flow is from channels 1 and 2 into channel 3, and (*d*) channels 1 and 3 lie in a straight line. The assumptions are: (1) The flow is parallel to the channel walls immediately above and below the junction, (2) ordinary wall friction is negligible in comparison with other forces involved, and (3) the depths in channels 1 and 2 are equal immediately above the junction.

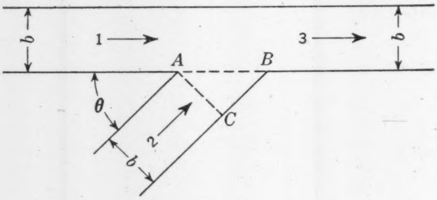


FIG. 2

From the principle that the net force acting upon a fluid system is equal to the rate at which the momentum of the system is changed, the following equations are obtained:

The net force, F, acting in the direction 1 to 3 is—

$$F_{1-3} = \frac{\gamma\, b\, (y_a)^2}{2} + \frac{\gamma\, b\, (y_a)^2}{2} \cos\theta - \frac{\gamma\, b\, (y_b)^2}{2} + U \ldots\ldots\ldots (1)$$

in which: γ = specific weight of water; b = width of a rectangular channel section; y_a = depth of flow above the junction; y_b = depth of flow below the junction; θ = angle between the diverging channels; and U = an unknown reaction. The rate, ΔM_{1-3}, at which momentum is changed in the direction 1 to 3 is—

$$\Delta M_{1-3} = \frac{Q_3 \gamma V_3}{g} - \frac{Q_1 \gamma V_1}{g} - \frac{Q_2 \gamma V_2}{g} \cos\theta \ldots\ldots\ldots\ldots (2)$$

in which: Q = rate of flow, the subscript referring to the appropriate channel; V = average velocity in the channel denoted by an appropriate subscript; and g = acceleration of gravity.

Upon examining Eq. 1 more closely, it is apparent that U must be the component of the pressure force upon the part of the wall marked BC in Fig. 2. If a further simplification is made so that the depth in the triangle ABC is everywhere equal to y_a, then U evidently must be equal and opposite to the term $\frac{\gamma\, b\, (y_a)^2}{2} \cos\theta$. Eq. 1 is now simply:

$$F_{1-3} = \frac{\gamma\, b\, (y_a)^2}{2} - \frac{\gamma\, b\, (y_b)^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equating Eqs. 1 and 2:

$$\frac{Q_1 \gamma}{g} V_1 + \frac{\gamma\, b\, (y_a)^2}{2} + \frac{Q_2 \gamma}{g} V_2 \cos\theta = \frac{Q_3 \gamma}{g} V_3 + \frac{\gamma\, b\, (y_b)^2}{2} \ldots\ldots (4)$$

Eq. 4 may be put in dimensionless form by the introduction of the factors n_q, n_y, and k_3. The final form is:

$$(n_q)^2 (1 + \cos\theta) - 2\, n_q + 1 = \left[\frac{1 - (n_y)^2}{4\, k_3} + 1 \right] n_y \ldots\ldots\ldots (5)$$

Experimental Results.—The factor k_3 in Eq. 5 varies inversely as the cube of the depth y_b. As this depth was difficult to measure, owing to the rough water surface below the junction, the numerical value of the factor k_3 as obtained experimentally was of doubtful accuracy. Therefore, Eq. 5 was modified so that the value of k_2, or the ratio of the velocity head to the depth in the branch channel, was made the independent variable. The resulting equation is:

$$k_2 = \frac{(n_q)^2 \left[(n_y)^2 - 1\right]}{4\, (n_y)^2 \left[2\, n_q - (n_q)^2 (1 + \cos\theta) + n_y - 1\right]} \ldots\ldots\ldots (6)$$

Eq. 6 is shown graphically in Fig. 3 for intersection angles of 45° and 135°, and for values of n_q equal to 0.4, 0.6, and 0.8. Plotted thereon are a few of the points obtained in the laboratory. The agreement with the theory is seen to be quite good for $\theta = 45°$ (Fig. 3(*a*)) and rather poor for $\theta = 135°$ (Fig. 3(*b*)).

Summary; Combining Flow.—(1) The agreement between theory and experiment for the 45° junction supports the conclusion that all the aforementioned assumptions are justified in this case of combining flow;

(2) Since the boundary friction was the same in both cases, it may be considered as a negligible influence in comparison with the other forces;

(3) The experimental data clearly showed that the depths in the two channels upstream from the junction had nearly the same value, regardless of the angle of intersection; and

(4) The lack of agreement between theory and experiment for the 135° intersection is due to the distortion of the velocity distribution below the junction and to the fact that the flow does not remain parallel to the channel walls. Item (4) is apparent from a comparison of Figs. 4(*a*) and 4(*b*) with Figs. 4(*c*) and 4(*d*). It is worthy of note that the injection of dye in the junction does not clearly define the line of demarcation between the two streams after they have joined. Notice that the line of separation is different in Figs. 4(*a*) and 4(*b*), whereas the photographs are of identical conditions, except as to the dye. The differences between the two lines must represent the region in which diffusion and turbulent mixing are taking place.

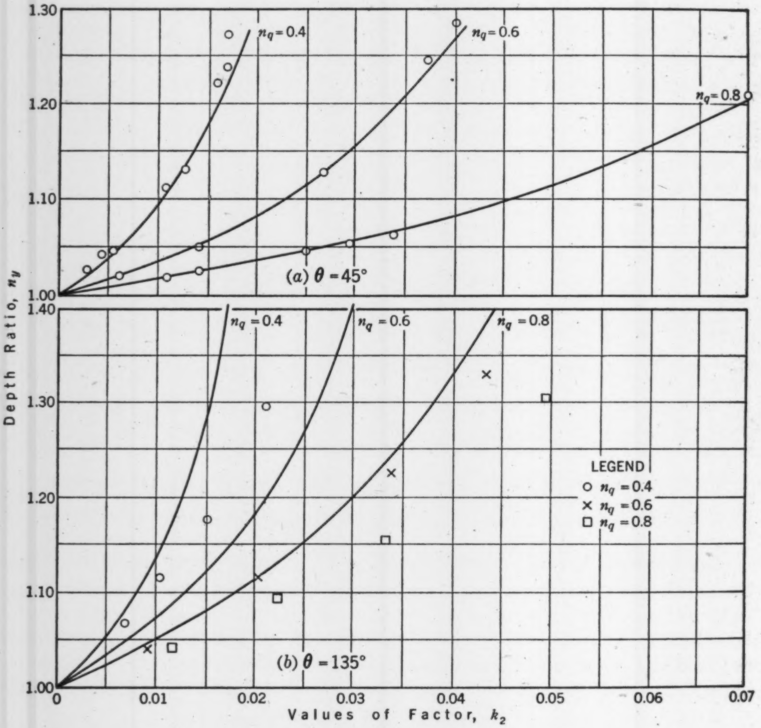
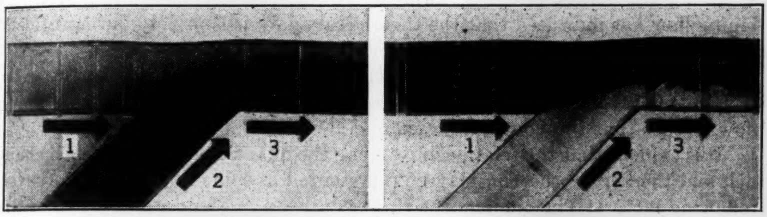
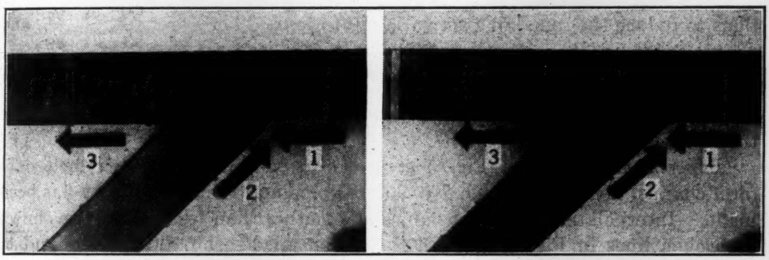


FIG. 3.—INTERSECTION CHARACTERISTICS, COMBINING FLOW

Application of the Results.—These results are not directly applicable to natural streams because of the restrictions placed upon the experimental procedure. A little reflection will show, however, that the problem is of minor importance if deep, slow-moving rivers are considered. The ratio of the velocity head to the depth is such a small number that the resulting ratio of depth above (y_a) to depth below (y_b) the junction will be close to unity. For example, consider a stream entering another at 45° and carrying 60% of the total flow. Suppose this tributary is 20 ft deep and is moving at 3 ft per sec.



(a) θ = 45°; Dye in Channel 2 (b) θ = 45°; Dye in Channel 1



(c) θ = 135°; Dye in Channel 1 (d) θ = 135°; Dye in Channel 2

FIG. 4.—FLOW PATTERNS, COMBINING FLOW

According to Fig. 3(a), the difference in depth between two points above and below the junction would be approximately 2%, or 0.40 ft.

The results show clearly that, owing to the negligible effect of friction, model studies of special stream intersections (if the problem were sufficiently important) would be successful. This procedure is recommended since, as stated, theoretical and experimental investigation of every conceivable type of intersection is impracticable.

DIVIDING FLOW

In dividing flow the problem is somewhat different from the case of combining flow. A logical statement of this problem would be: A given stream branches into two separate channels. What will be the division of the flow? This will depend upon the backwater characteristics of the two branch channels and the dynamic conditions existing at the junction.

Analysis of the problem of dividing flow is considerably more difficult than that of combining flow, for the following reason: In the former case, it was possible to assume that the depths in the tributary channels were equal immediately above the junction. No analogous assumption is permissible in the case of the dividing flow. The momentum equations are then complicated to the extent of containing an additional unknown term. In view of this fact, no attempt at rational analysis in terms of momentum was made, because assuming a relation among the three depths would be almost tantamount to assuming the solution to the problem. Furthermore, experience with combining flow has indicated that the theoretical analysis is in only fair agreement with the observed facts.

This part of the paper will be limited to the case in which the flow of water in Fig. 2 is reversed. The restrictions will be the same as for the combining flow, except that the flow is from channel 3 into channels 1 and 2.

It is evident that the relationship among the quantities flowing in each channel, and the depths therein, may be represented as some function of the seven following factors: The angle θ; the three depths, y_1, y_2, and y_3; any two of the three quantities, say Q_3 and Q_2; and the velocity with which the stream enters the junction. For any given angle θ, it is possible by dimensional considerations to reduce the remaining six variables to four. They are: $\frac{Q_2}{Q_3}$, $\frac{y_3}{y_2}$, $\frac{y_1}{y_2}$, and $\frac{(V_3)^2}{2\,g\,y_3}$ $(= k_3)$. It is proposed to present the experimental data for one type of intersection and then demonstrate how these data may be applied to a case that may occur in practice.

Experimental Results.—Fig. 5 gives the experimentally determined data for a channel division in which $\theta = 90°$. No comparison with theory has been made. Interpolated curves for values of k_3, other than test values, are shown as broken lines in Fig. 5. Fig. 6 shows the relationship that was found to exist among the three depths within the range of the values of k_3 considered. Figs. 7(*a*) and 7(*b*), which are of identical flow conditions, except as to the dye, show qualitatively the manner in which the flow divides.

Application of Results.—It has been stated previously that a practical use of a study in stream division would be to predict the division of flow when a given stream branches into two separate channels. It is assumed, of course, that the inflows to the branches are not controlled by gates but merely by the characteristics of the junction and the branches themselves. The following hypothetical case will be assumed to exist: A rectangular canal, 10 ft wide, divides at a 90° branch. The branch channels are also 10 ft wide and rectangular. They are of such slopes and roughness that their rating curves for sections just below the division point are as shown in Fig. 8. The problem is to predict how much water will be naturally diverted to each branch channel if 500 cu ft per sec are admitted to the main canal.

Determination of the division of flow involves the following steps:

(1) Assume a certain division, say $Q_2 = 250$ cu ft per sec and $Q_1 = 250$ cu ft per sec $\left(\frac{Q_2}{Q_3} = 0.5\right)$. This fixes the depths y_1 and y_2. The ratio of y_3 to y_2

may then be determined by means of Fig. 6. It is possible, by choosing various values of Q_2, to obtain a curve between $\frac{Q_2}{Q_3}$ and $\frac{y_3}{y_2}$. For the conditions assumed herein, this curve appears in Fig. 5 and is marked A.

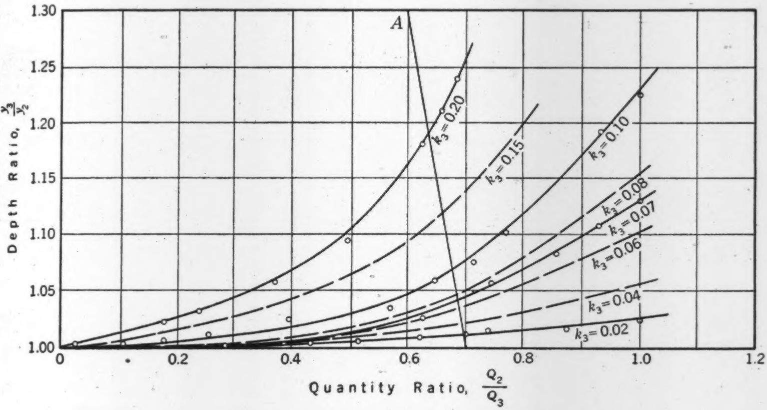


FIG. 5.—CHARACTERISTICS OF STREAM DIVISION IN WHICH $\theta = 90°$

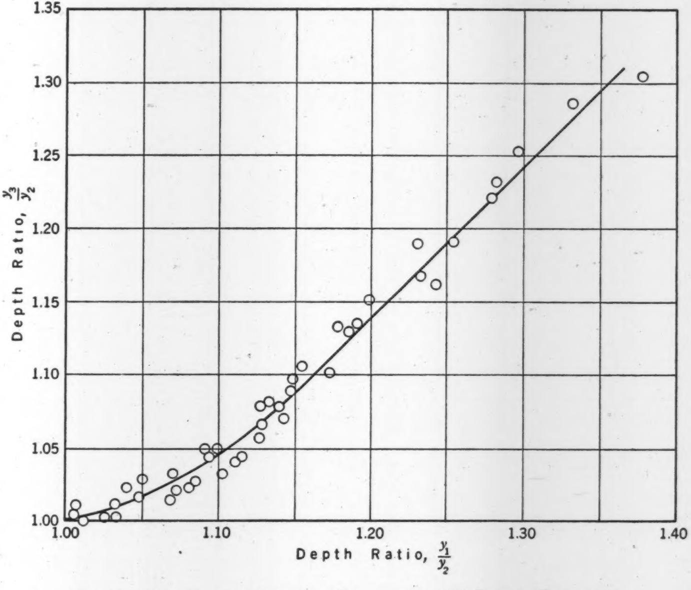
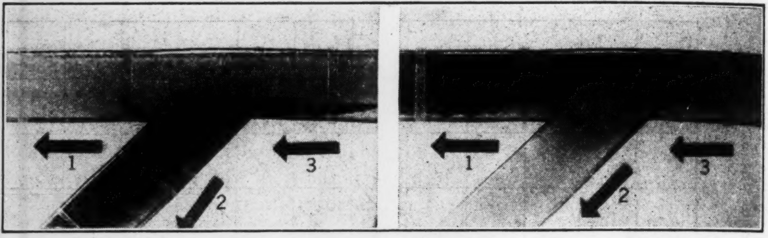


FIG. 6.—RELATIONSHIP BETWEEN DEPTHS IN A 90° STREAM DIVISION

(2) The locus of the points of intersection of curve A and the k_3-curves (Fig. 5) now gives all possible combinations of the variables. Still undetermined is the value of k_3 with which the flow of 500 cu ft per sec enters the junction. It is necessary to replot curve A in terms of the numerical values of the depth y_3 and the factor k_3. This is done as follows: Take the intersection of curve A and $k_3 = 0.10$; $\frac{y_3}{y_2} = 1.067$ and $\frac{Q_2}{Q_3} = 0.677$. Then $Q_2 = 338$ cu ft per sec and, from Fig. 8, $y_2 = 6.0$ ft and $y_3 = 1.067 \times 6 = 6.4$ ft. The point ($k_3 = 0.10$ and $y_3 = 6.4$) is plotted in Fig. 9. Other points are plotted in the same manner. The resulting curve is marked B in Fig. 9.



(a) Dye Entering Channel 1 (b) Dye Entering Channel 2

FIG. 7.—FLOW PATTERNS, DIVIDING FLOW; $\theta = 45°$

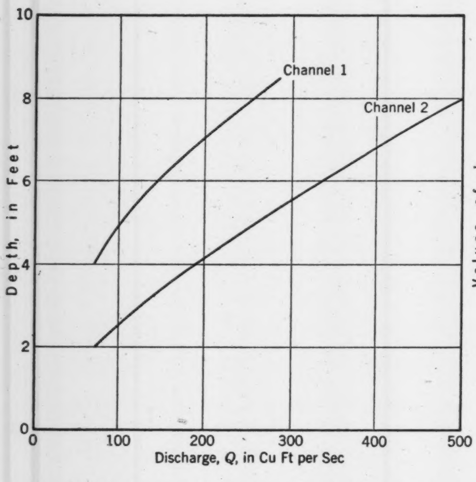


FIG. 8.—RATING CURVES FOR BRANCH CHANNELS

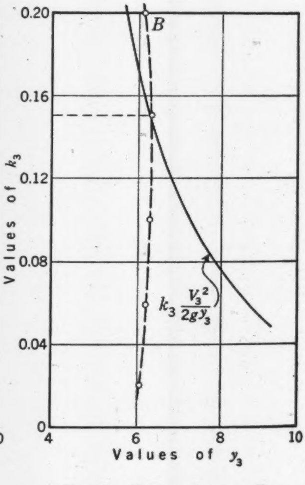


FIG. 9.—FACTOR k_3 AS A FUNCTION OF y_3

(3) The proper value of k_3 must satisfy not only curve B, but also the relationship $k_3 = \frac{(V_3)^2}{2\,g\,y_3}$. This is easily found by plotting $k_3 = \frac{(V_3)^2}{2\,g\,y_3}$ in Fig. 9 and obtaining the intersection with curve B. For these hypothetical conditions, k_3 is found to be 0.15 and $y_3 = 6.4$ ft.

(4) Entering Fig. 5 with $k_3 = 0.15$ at its intersection with curve A, the value of $\frac{Q_2}{Q_3}$ is seen to be 0.66. Then $Q_2 = 330$ cu ft per sec and $Q_1 = 500 - 330 = 170$ cu ft per sec.

Conclusion

The example given is admittedly a very special case, as such an ideal set of conditions could hardly be expected to occur in practice. It has only been included to indicate the data that would be necessary in order to solve a general problem of this type, as it is felt that rational analysis is not practical.

The research described in the foregoing may be considered by some to be incomplete in that no attempt was made to study the details of the mixing process in quantitative terms. Studies have since been undertaken, and are still continuing, at the University of California under somewhat simpler conditions; namely, the mixing of parallel streams, in which velocity and pressure gradients have been measured in the region of mixing.

Acknowledgment

The Committee of the Hydraulic Division on Hydraulic Research has had for one of its projects "The Phenomenon of Intersecting Streams." With funds allotted by The Engineering Foundation, the University of California conducted the investigation, with Morrough P. O'Brien, M. Am. Soc. C. E., as the cooperating member of the Committee.

The quantitative data from which this paper was prepared were taken from two sources: "Combining of Streams in Open Rectangular Channels,"[2] by F. C. Horowitz and John R. Morgan, Juniors, Am Soc. C. E., and "The Characteristics of Dividing Streams in Open Rectangular Channels,"[3] by Lt. Ivan C. Rumsey. The photographs were obtained from a thesis entitled "A Qualitative Study of the Intersections of Converging and Dividing Streams,"[2] by R. M. Bickerstaff, Jun. Am. Soc. C. E. R. L. Stoker was immediate supervisor of these studies.

APPENDIX

Notation

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[4] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942 (numeral subscripts refer to appropriate channel numbers, channel 3 being the main stream and channel 2 the channel entering or leaving the straight axis of flow):

---

[2] Thesis submitted to the University of California in partial fulfilment of the requirements for the degree of Bachelor of Science in Engineering.

[3] Thesis submitted to the University of California in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

[4] ASA-Z10.2—1942.

b = width of rectangular channel section;
F = net force acting on a fluid system: F_{1-3} = net force acting in the direction 1 to 3;
g = acceleration of gravity;
k = a factor $= \frac{V^2}{2\,g\,y}$;
M = momentum of a system: ΔM_{1-3} = the rate at which momentum is changed in the direction 1 to 3;
n = ratio symbol: $n_q = \frac{Q_2}{Q_3}$ and $n_y = \frac{y_a}{y_b}$;
Q = rate of flow: Q_2 is the flow from or to the side tributary or branch channel; and $Q_3 = Q_1 + Q_2$ (see Fig. 2);
U = an unknown reaction in Eq. 1;
V = average velocity across a given cross section;
y = depth of flow: y_a = depth above the junction and y_b = depth below the junction;
γ = force per unit weight; specific weight of water; and
θ = angle between merging or diverging channels.

DISCUSSION

HAROLD K. PALMER,[5] M. AM. SOC. C. E.—Some interesting experiments are offered in this paper in an attempt to find a solution of some important hydraulic problems; but unfortunately some of the author's hypotheses are not confirmed by the experiments, which leads to the suggestion that the plan of attack may require some modification.

An examination of the photographs in Fig. 4 shows clearly that there are three zones at the junction: A zone of slight deflection, and the narrowing of stream 1 along the straight side; a bending and narrowing of stream 2 around the angle; and a zone of turbulence between the two outer zones, where the mixing of the two streams takes place. The deflection of a stream will cause a small loss of energy and in the zone of turbulence there will be a much larger loss, but the two are of such different natures, and affect such different parts of the same stream, that they should be considered as two terms with different coefficients. The shape of the junction would have a marked effect on the loss due to bending stream 2 and might affect somewhat the loss due to turbulence. It appears to the writer, therefore, that the problem should be approached from the standpoint of loss of energy rather than from change of momentum.

To solve the problem by this method it would be necessary to assume y_3 and v_3 below the junction of the converging streams, thus establishing the relation:

$$E_3 = y_3 + H\,v_3 \qquad \text{(7a)}$$

in which E represents the total energy in the part of the stream indicated by the subscript.

Stream 1 would lose some energy in the zone of turbulence and in being confined to a narrow strip, and it would require some experiments to evaluate this combination of losses. Once this loss is found, the energy in this stream would be expressed by the relation

$$E_1 = E_3 + \Delta E_1 \qquad \text{(7b)}$$

In the same way,

$$E_2 = E_3 + \Delta E_2 \qquad \text{(7c)}$$

In the latter case, the loss due to deflection would be much greater than in stream 1, but still would be much less than the turbulence losses.

To find the constants to be used in this method, it would be well to conduct experiments on the deflection of a single stream, avoiding the turbulence losses. The only way to segregate the deflection and turbulence losses is first to find the magnitude of one and then subtract that from the total to find the other. Such a study may determine the importance of rounding corners on the concave side of the entering stream to avoid excessive loss, and it would also show the effect of the width of the side channel.

---

[5] Office Engr., Los Angeles County Sanitation Dists., Los Angeles, Calif.

The same methods could be applied to the case of dividing flows. In this case, the deflection loss is very important, as the momentum of the water tends to make it continue down the straight channel. The width of the side channel is even more important in this case and, if the side channel can be so designed that the stream is always in contact with the convex bank, there will be no backwater, and losses will be minimized.

This proposed method of attack may not be the solution, but the writer believes it accounts more nearly for the forces acting on the streams, and that therefore some constants may be found that would give an approximate idea of what happens in either combining or dividing streams.

If such studies should prove successful, it would not be necessary to construct a model for every case, as suggested by the author. It is to be hoped that the continuing studies at the University of California, which the author mentions in his "Conclusion," may help to solve this general problem by furnishing the necessary constants.

G. H. Hickox,[6] M. Am. Soc. C. E.—By attacking a problem which has received comparatively little attention Mr. Taylor has made a distinct contribution to engineering literature. His attempt at a solution points the way to a better understanding of the general subject of branching flow. A similar problem exists in pipe flow where several attempts have been made to predict the discharge through the orifices of irrigation sprinkler systems. Some of these attempts have been made on the basis of the momentum principle, relating the change in pressure in the feeder pipe to the rate of change of momentum. As far as the writer is aware, however, the pressure differences on the walls of the lateral outlets, parallel to the flow in the feeder pipes, have not been taken into account. The author's method of attack, considering the effect on the momentum equation of the wall pressures in the branch channel, is suggestive of similar treatment of the problem in pipe flow.

With respect to the application of the momentum principle to the problem of combining flow, Eqs. 1 and 2 state the sums of pressures and rates of momentum change correctly. The derivation of Eq. 3 from Eq. 1, however, is of somewhat doubtful validity. It assumes that the depths in the main channel and the branch channel at their junction are the same and are uniform across the channel. Also, it assumes that the pressure on the wall of the branch channel is equal to the hydrostatic pressure of still water at the same depth. This would be the case if the flow in the branch channel was everywhere parallel to the walls. At the junction, however, and particularly in the vicinity of the wall BC, the flow is not parallel to the wall but is curved. This curvature of flow will modify both assumptions. The water surface across the channel no longer will be level, and it is possible that the variation of pressure with depth may not be quite hydrostatic. Also, there may be some modification of pressure on the walls of the branch channel above section AC. From a further application of the momentum principle, the elevation of the water in the branch channel should be lower than that at the junction since there is a component of velocity normal to the main channel which is reduced to zero.

---

[6] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

It would have been interesting if the author had developed his equations without the simplification introduced in Eq. 3 and had evaluated the term U from the experimental data. The results then might have been expressed in terms of a coefficient relating the experimental value of U to the calculated value.

Also it might have been enlightening if the pressures on the walls of the branch channel had been measured by means of piezometers. There seems to be no reason why the momentum principle should not give excellent results if it is applied correctly. Its proper application depends upon a complete knowledge of all the pressures having components in the direction of flow. The assumptions made in this case regarding pressure might well have been verified experimentally.

J. C. STEVENS,[7] M. AM. SOC. C. E.—It would be worth while for the reader of this excellent paper to look back 17 years and review the writer's short article[8] on similar flow characteristics at pipe intersections and particularly the discussions that followed, by Harold A. Thomas[9] and Julian Hinds,[10] Members, Am. Soc. C. E. In that article it was assumed that the pressure in each intersecting conduit would merge into a common value at the intersection. Momentum and energy relations were based on mean velocities. There followed from those assumptions a formula purporting to give the theoretical energy losses when n pipes of varying sizes join at a common junction at varying angles from which the water flowed away in a main.

Another formula gave expression to the theoretical losses resulting from n pipes of varying sizes discharging at right angles into a header of constant diameter. It made no difference in which direction the water was flowing—for example, the water flow could divide from the main to the several pipes and the same theoretical energy losses would result.

Those formulas will give approximate results. The errors are on the side of safety in that the losses indicated are on the excessive side. However, the formulas do illustrate the danger of simplifying assumptions if accuracy is important. Mr. Taylor was able to simplify his theoretical analysis by assuming that the depth in the branch remained constant to the wall of the main and that it was also equal to the depth in the main above the junction. Both of these assumptions are somewhat similar to the writer's assumption of uniform pressure at pipe intersections, and both are incorrect; yet approximate results may be obtained from the use of the author's formula and diagrams.

The sources of error in these analyses are obvious. The water never has a uniform velocity distribution. After the junction a greatly distorted velocity distribution results. Both momentum and energy relationships based on a $\frac{Q}{A}$-mean are therefore considerably in error.

---

[7] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[8] "Theoretical Energy Losses in Intersecting Pipes," by J. C. Stevens, *Engineering News-Record*, July 22, 1926, p. 140.

[9] *Ibid.*, p. 437.

[10] *Ibid.*, pp. 883–884.

The force due to pressure on certain parts of the peripheries of the conduits at their junction, even in the case of uniform size conduits intersecting at right angles, cannot be expressed in known terms. In the paper cited,[8] this force was simply ignored as the easiest solution. Mr. Taylor assumes it to be equal and opposite to the pressure force in the branch, which, although preferable to ignoring it completely, still leaves much to be desired.

The Society's Committee on Hydraulic Research assigned the research project "Phenomenon of Intersecting Streams" to Professor M. P. O'Brien in 1936. Mr. Taylor's paper is the final paper on the open-channel phase of this subject. Research on intersecting pipes is being continued, although temporarily postponed in favor of more urgent war activities.

Flow in open-channel intersections was found to involve too many unknowns. Flow in intersecting pipes also contains too many complexities until more can be learned about how two streams mingle. As stated by the author, that phase is under study—first in its simplest form of mixing parallel streams. The next step will be to study the mixing of two nonparallel streams. Upon completion of these investigations, it is anticipated that studies on the flow characteristics of intersecting pipes can be resumed.

Almost any kind of research progresses slowly. The first steps open new vistas that must be explored before tangible results can be achieved on the original objective. Those engaged in teaching, of course, must make that their first duty, and research in such cases must be satisfied with what can be accomplished as extracurricular activities.

C. J. Posey,[11] Assoc. M. Am. Soc. C. E.—It should be emphasized that the data and conclusions of this paper are for flow deeper than the critical throughout—that is, with a ratio of velocity head to depth of less than one half in both branches of the stream. Although no statement to this effect appears in the paper, it is obviously true from the results, and is implicit in the author's statement that the elevations of the water surface in both channels could be varied by means of adjustable gates at the lower ends.

The data presented by the author justify the assumption customarily made in dealing with problems in combining or dividing flow at greater than critical depth—namely, that depths in the intersecting channels are substantially the same, with the division, in case of dividing flow, depending chiefly upon the backwater characteristics of the two branch channels. If greater refinement were needed, in some particular case, a model study patterned after the experiments reported in this paper should yield the desired information.

I. B. Hosig,[12] Esq.—To the writer this paper is of interest principally because it presents an additional case in which the application of the principles of analysis by forces yields results in an hydrodynamic problem. It is gratifying to find this method of analysis being used by the young men entering hydraulic work and, although, to date, its practical applications have been few, it has elucidated with brilliancy the problem of the hydraulic jump—for instance, where work with the principles of analysis by energy content had enveloped that problem in a fog of empiricism.

---

[11] Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa; and Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[12] Engr., Bureau of Reclamation, Denver, Colo.

The author modestly disclaims a solution of great practical value because the theory is adequately bolstered only by experiments in which the angle between the two intersecting streams is 45°. However, it is probable that in cases in nature the angle will be found to be that or less, and thus the work is practical.

In the irrigation and drainage work with which the writer is familiar, the case of division of water is entirely ruled out of the class of problem herein discussed because division is under control by means of structures. Division is a major work in the control of irrigation water.

The joining of streams in irrigation work consists principally of taking storm water or return flow into supply canals. The water surface of the principal stream is usually much lower than that of the incoming stream and the volume is also much greater. Hence the question of the elevation of the joined waters is of no interest; but the problem of the force of the incoming water is of paramount importance.

In the case of drainage channels the resultant water surface at the junction may be of considerable importance. In the writer's practice the method of treating the problem has been to make the channel below the junction of such size that it will handle the anticipated flow with the water surface at the junction at the same elevation as that of the incoming channels, assuming flow in each branch to follow Chézy's formula where the anticipated flows are of consequence. Otherwise, the bottom grades are all brought to one elevation. In the writer's practice observation of the behavior of drains at junctions has not been very illuminating as discharges are difficult to predict and are highly variable; hence flows observed are usually not those assumed when making the layout but the observations are clear in one respect—that the forces involved govern resultant channel conditions. It is sometimes necessary either to direct the combined channel as a resultant of the two branches or to contemplate additional work in the nature of bank protection which is in effect the introduction of additional force. This leads to the idea that more fruitful results will be obtained if the problem is presented in such a way as to recognize all forces of such magnitude as can appreciably affect the motions of the body involved.

In the case in the paper where two streams meet at an angle of 45°, the transverse (with respect to the straight channel) component of the $\frac{Q\,v}{g}$ force of the stream coming in at an angle was not included in the mathematical treatment. The author recognized the omission but the photographs show that the force does have an effect on flow conditions. The writer believes that it is this force which limits the use of the mathematical treatment presented to angles of an order of 45° or less for the case considered. This is not a criticism which is intended to imply that analysis by forces in this case is a weak approach to the problem. In fact it appears to be the only analytical approach and the proper course is a more comprehensive setup. Model study probably would solve a particular problem, but model study without analysis is a barren plant in the field of engineering science.

L. STANDISH HALL,[13] M. AM. SOC. C. E.—Little research has been done on the phase of hydraulics described in this paper. Many variables are involved in the solution of the problem, one of which has been eliminated in the paper by making the bed horizontal. The flat grade produces low velocities and, consequently, small values of kineticity.

In many of the design problems involving converging channels, the writer has found that the velocities have been great enough to require paving or a lined channel. The velocities often are greater than the critical and the kineticity greater than 2. With kineticity of this magnitude, the depth of flow above the junction is not affected by backwater from the combined flow; hence, the depth of flow in the two converging channels is not necessarily the same. When the depths are different, cross waves will develop that are transmitted downstream, sometimes with undesirable results.

Even with the velocities below critical it would be difficult to project the curves to include higher kineticity on the basis of the data presented in the paper.

For the data plotted in Fig. 3, it should be noted that, for small values of n_q, n_y would be independent of the value of k_2. For this reason, the intersection characteristics should be plotted against k_3.

If the flow from the side channel becomes zero, the ratio $n_q = 0$. Then, from Eq. 5,

$$(n_y - 1)[(n_y)^2 + n_y - 4\,k_3] = 0 \qquad (8)$$

With $n_q = 0$, assuming that wall friction can be neglected, the depth ratio, $n_y = \frac{y_a}{y_b} = 1$ can only occur when $k_3 = \frac{1}{2}$; or, expressed in words, under the condition of no flow in the side channel, the velocity head is always one half of the depth. This may be true, but it is difficult to believe. If Eq. 8 is solved for n_y, then

$$n_y = \frac{-1 \pm \sqrt{1 + 16\,k_3}}{2} \qquad (9)$$

From Eq. 9, it is seen that one root is positive and one negative; and with kineticity less than $\frac{1}{2}$, such as was observed in the author's experiments, the value of n_y should be less than unity. In other words, a jump should occur at very low velocities. This is contrary to accepted hydraulic theory, as a jump can only occur when k_3 is greater than $\frac{1}{2}$.[14] The solution to this dilemma lies in the fact that in Eq. 5, when $n_y = 1$ and $n_q = 0$, the term containing k_3 drops from the equation. Hence, for this special case the kineticity is immaterial, and Eq. 9 can apply only when $n_y \neq 1$.

---

[13] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[14] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 630.

For the other extreme, if all of the flow originates from the side channel, $n_q = 1$. Then, from Eq. 5, if $\theta = 90°$, the relation

$$n_y = \sqrt{1 + 4\,k_3} \qquad (10a)$$

is obtained and, from Eq. 6,

$$k_2 = \frac{k_3}{(1 + 4\,k_3)^{1.5}} \qquad (10b)$$

These relations indicate that depths in channels 1 and 2 (Fig. 2) would be greater than in channel 3, and k_2 would be less than k_3. The lack of agreement between theory and experiment shown in Fig. 3 makes the application of Eqs. 10 questionable. The results of experiments covering this condition would be interesting if available.

For converging channels, the lack of agreement of the results with the momentum equation when $\theta = 135°$ would suggest that a comparison of the energy losses might yield a more satisfactory solution. In Fig. 3, when $n_q = 0.4$, the relation between $\frac{y_a}{y_b}\left(\text{or } \frac{y_1}{y_2}\right)$ and k_2 is identical for both $\theta = 45°$ and $\theta = 135°$. With $n_q = 0.6$ there is very little difference in the relation with either angle. Only when $n_q = 0.8$—that is, with 80% of the flow entering from the side channel—is definite difference in the relation apparent between supplementary angles of convergence. The results would indicate that the momentum of translation is converted into a momentum of rotation, and, hence, with small flows in the side channel, the angle of entrance is immaterial.

An example of dividing flow is given by a side spillway for regulating the diversion into a canal system. The conditions are not parallel with those given by the author since the bottom of the side channel is at a higher elevation than the bottom of the main canal. The momentum principle can be used in solving this problem. In the case of the change of momentum affecting the water surface profile in such a canal, it has been assumed that momentum is lost with the overflowing water.[15]

On the other hand, in a side-channel spillway from a reservoir (an example of combining flow), it has been assumed that the momentum of the water discharging over the spillway lip is entirely lost upon impact with the water in the spillway channel. This assumption has been made since the flow moves over the crest of the weir in a direction approximately at right angles to the axis of the spillway channel.[16] This would correspond with the assumption made by the author in Eq. 2.

The variables affecting combining and dividing flow are so numerous that no general relation applicable to all conditions can be found. In most cases, when a prior knowledge of the flow characteristics at an intersection is important, a model should be constructed as suggested by the author. General relations applicable to specific cases may be obtained where it is possible to control a sufficient number of the variables. The results presented in this paper are a step in the right direction.

---

[15] "Side Spillways for Regulating Diversion Canals," by W. H. R. Nimmo, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1561.

[16] "Side Channel Spillways," by Julian Hinds, *ibid.*, Vol. 89 (1926), p. 881.

BROOKS T. MORRIS,[17] JUN. AM. SOC. C. E.—Experiments on so general a problem as channel junction performance cannot be expected to cover more than a fraction of the field. Within the limits of his apparatus and of the time given to its operation, Mr. Taylor has developed good data. The quantities employed in both combining and dividing streams cover the range of tranquil flow nearly to the limit of critical velocity in the most heavily loaded conduit.

The analyses attempted by Mr. Taylor make the data of considerable use in revealing the weaknesses of certain assumptions which are customarily employed in hydraulic computations but which are inappropriate to problems of combining and dividing flow. The first of these is the assumption of uniform velocity distribution over the channel cross sections. Theoretical analysis and long experience each have shown that this assumption is adequate in ordinary pipe and channel calculations where similar, although not strictly uniform, velocity distributions exist at the various sections for which force-balance equations are written. On the other hand, within individual cross sections, manifold pipes and branched channels contain extreme variations of velocity distribution. Such conditions, like the 135° intersection, contain stream filaments moving in opposite directions in the same cross section. To describe such conditions, formulas like Eq. 2 should contain distribution coefficients corresponding to the ratio of the integrated mean momentum to that calculated from the mean velocity of flow. As long as those coefficients approach unity, as in the 45° junction, the analysis as presented by the author is successful.

The second familiar assumption is the dual one of hydrostatic pressure distribution and of horizontal surface at each cross section. This assumption leaves the author short one equation in studying the dividing flow problem and makes necessary his assumption (3) regarding the depths in combining flow. Except for this, the fact of a gradient of depth, or pressure, gradient across the 1–3 channel might be recognized and the second equation of momentum might be written. The writer recognizes the difficulties inherent in writing this equation, but he believes that it is a necessary step in the rational description of combining or dividing flow. Also, the determination of momentum distribution coefficients suited to varying junction geometry is not easy. The continuing studies on the mixing of parallel streams should be informative in this regard. Excellent investigations of certain types of such mixing have been made already by W. Tollmien and A. M. Kuethe.[18]

The writer felt handicapped by the coefficient k until he recognized it as merely another form of the Froude number or kineticity of flow such that

$$k = \frac{V^2}{2\,g\,y} = \frac{\mathbf{F}^2}{2} = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{y_c}{y}\right)^3 \dots\dots\dots\dots\dots (11)$$

Mr. Bickerstaff's photographs are very helpful in picturing the conditions in the author's experiments. However, the writer is concerned over the effects that departure of the dye interface from the vertical might have on the interpretation of the illustrations. Was the dyed stream only a shallow layer or did it occupy the full depth of the section? If the latter was the case, did the interface remain vertical?

---

[17] Research Engr., C. F. Braun & Co., Alhambra, Calif.

[18] "Modern Developments in Fluid Dynamics," by S. Goldstein, Oxford, 1938, p. 592.

The verity of Mr. Taylor's data is undeniable, and his conclusions have been drawn with care. Still, the writer would be sorry to see either used to discourage further attempts at theoretical analysis of the problem. It is for that reason that he has sought to show the weaknesses of the underlying assumptions.

EDWARD H. TAYLOR,[19] JUN. AM. SOC. C. E.—The interest shown in this rather specialized problem has been an agreeable surprise to the writer. The discussers have uniformly realized that the purpose of the research was to investigate the degree to which elementary physical laws can be applied to the situations devised, rather than to discuss the problem of turbulent mixing in detail. The mixing problem is essentially complicated, and thus should be approached from a consideration of pressure and momentum. If it is possible to write down correct expressions for the pressures and momenta at the boundaries of the process in question, the intermediate details can be disregarded.

Mr. Palmer suggests that the problem should be studied from the standpoint of energy loss rather than from that of change of momentum. This would require knowledge of the mechanism of energy loss, which was not contemplated in the original studies. Thus, in the absence of theoretical considerations relating to these losses, such an approach would seem to require greater recourse to empiricism than had been intended.

In raising the question of evaluation of the pressure force on the walls of the branch channel, both Mr. Hickox and Mr. Stevens have accurately stated the chief source of disparity between the experimental results and the equations. The measurements substantiated the assumption of equal depths above the junction; but this, of course, disregards wall pressures arising due to curvature of flow in the branch between the junction and the point of measurement of depth. The writer now recognizes that failure to measure the static pressures in the region constitutes an important omission.

Mr. Stevens brings up the same question in connection with pipe intersections. In the article he cites,[8] the pressures are assumed uniform over the pipe cross sections immediately upstream from the junction. In measurements[20] made at the University of California on a 90° pipe intersection, this factor was investigated. It was found that an appreciable lateral pressure gradient existed in the branch for some distance upstream. This is due to the fact that the flow in the branch does not change abruptly on reaching the junction, but starts to curve while it is yet in the straight part of the branch. This section of the flow system is included in writing the equations, since at its upstream end the direction of the velocity is known to be parallel to the walls. However, its inclusion introduces a component of pressure force in the direction of summation which cannot be accounted for by elementary methods.

---

[19] Washington, D. C.; formerly Instr., Mech. Eng., Univ. of California, Berkeley, Calif.

[20] "Energy Loss in Mixing Streams in Closed Circular Conduits," by William Reeves Shuler, thesis presented to the Univ. of California in Berkeley in May, 1940, in partial fulfilment of the requirements for the degree of Master of Science (unpublished).

Professor Posey correctly states that no consideration has been given to cases involving flow at less than critical depth. The same fundamental principles would apply to such cases, although the problem of maintaining supercritical velocities throughout the junction might prove troublesome. It is also likely that the factors causing disagreement would be amplified by the higher velocities.

The comments of Mr. Hosig regarding the practical aspects of this research are very welcome, since such considerations were quite beyond the scope of the original studies. It is gratifying to have Mr. Hosig's testimony that these results are not entirely academic.

In his examination of the momentum equation, Mr. Hall has reached certain conclusions which are not correct, and at the risk of appearing pedantic, the writer wishes to point out the errors involved. Mr. Hall finds that "under the condition of no flow in the side channel, the velocity head is always one half of the depth." The algebraic steps leading to this result involved division by zero, which, of course, invalidates any subsequent conclusion.

In Eq. 9, Mr. Hall has derived the hydraulic jump relationship, but appears concerned because of the possibility that the jump might occur in the reverse direction, which he notes, is "contrary to accepted hydraulic theory." Mr. Hall has overlooked the fact that, in so far as momentum changes alone are concerned, the jump is quite a reversible process. A reverse jump would require an increase in total energy in the direction of flow, which presumably accounts for the rarity of its occurrence in nature.

Both Mr. Morris and Mr. Stevens note that the momentum terms are based on average velocities, and, to be strictly correct, such terms should contain a velocity distribution coefficient. Although this is undeniably true, it seems to be of secondary importance in comparison with inability to express all pressure forces correctly. This is particularly true of the experiments described in this paper, in which tranquil flows were maintained in all cases. Mr. Morris draws attention to Mr. Bickerstaff's photographs, which are isolated examples taken from a large collection of very good pictures. In each case the dye had been injected so that, by the time it had reached the junction, it occupied the full depth of the section. If the writer's memory serves correctly, the interface remained essentially vertical in the vicinity of the intersection.

Messrs. Hickox, Stevens, and Morris are in substantial agreement on the uncertainties in the analysis as presented. In the event that a problem similar to this one should be investigated in the future, the experimenter would be well advised to heed their comments.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2224

SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES

By Richard M. Hodges,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. D. Geisler, Arthur G. Hayden, Bernard L. Weiner, Alfred L. Parmé, J. Charles Rathbun, Maurice Barron, Phillips H. Lovering, Jaroslav Polivka, A. A. Eremin, and Richard M. Hodges.

Synopsis

The principal purpose of this paper is to present a simplified general method of analysis for solid-barreled skewed reinforced concrete rigid frames and arches which can be understood readily by capable designers who know something about the theory of elasticity, without necessarily having made a special study of the subject of skewed arches. All unnecessarily complicated terms and expressions have been avoided, particularly those resulting from the solution in general terms of simple simultaneous equations which can be handled more easily by the direct substitution of numerical quantities.

The method presented is based on the assumption ordinarily made in the analysis of rigid frame bridges on earth foundations—that virtual hinges exist along the z-axes at the bottoms of the footings, due to the compressibility of the soil. Methods previously presented also have been based on the additional assumption that complete fixity against rotation exists about the other two axes. In view of the evident inconsistency of these two assumptions, the probability of slight rotations about the x-axes and the y-axes has been taken into account also. It will be found that the necessary adjustments can be made with very little difficulty on account of the simplification effected in the general analytical process.

A practical application is made to the design of a grade separation access structure of small span but heavy skew, constructed by the Westchester (N. Y.) Cross County Parkway Authority in 1942, which is typical of the kind of structure with which this paper is mainly concerned. Suggestions are given

---

Note.—Published in May, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Engr., Tunnel Design, New York City Tunnel Authority, New York, N. Y.

which, it is hoped, will be helpful in the matter of procedure to those not familiar with the subject.

In the Appendix, the original four simultaneous equations for determining the redundant reaction components of a skewed arch according to the method of analysis first published[2] in 1931 by Arthur G. Hayden, M. Am. Soc. C. E., are given, and certain deductions are made as an additional confirmation of the fundamental relationship on which this paper is based. It will be found unnecessary to make further reference, in this connection, to Mr. Hayden's equations—the simplified method, as here presented, is complete in itself.

The term "frame," or "rigid frame," as used in this paper, is defined to include structures of the full-centered arch type, such as are sometimes used in the form of elliptical arches for grade crossing eliminations.

INTRODUCTION

Following the general adoption of rigid frame bridges along the parkways of Westchester County and Long Island, New York, there has been a rapid increase in the construction of this type of bridge, due, in a large measure, to the particular suitability of solid-barreled frames for grade separations. In general, grade separations are seldom at right angles. It is probably an understatement to say that at least 25% of the frames used for grade separation work either are, or should be, skewed to an extent sufficient to affect the design. Such bridges are normally small and inexpensive. It should be made possible for any ordinary highway bridge design force, or small design organization, to handle them correctly, confidently, and with a reasonable amount of speed and economy. It should never be necessary, under any conditions, to resort to a bridge of less suitable type, or to juggle the alinement in order to dodge the problem.

It is unfortunate, therefore, that so little has been done to simplify the available methods of analysis, and to make them accessible, in effect, to the average designer. During the six or seven years immediately following the presentation, in 1924, by J. Charles Rathbun, M. Am. Soc. C. E., of a paper[3] giving, for the first time, a rational basis for the design of skewed arches and frames, several variations and modifications were proposed, all leading by more or less devious channels to identical results (except the distribution of the internal stresses), and all being open, in a greater or less degree, to the same objection. They are all tedious and complicated, and difficult to apply without an unreasonable amount of time and labor.

The method published by Mr. Hayden is perhaps the easiest, or rather the least difficult, for the average designer. In the method referred to, Professor Rathbun's theory, as applied to the determination of the external forces, was adapted by the writer, under Mr. Hayden's direction, to the design methods used for rectangular two-hinged rigid frame bridges in the Design Department, Westchester County Park Commission, the internal stress distribution being handled (in the later edition) in very much the same way as was proposed originally by Professor Rathbun. The process has been simplified in this paper to such extent that more than one half of the work formerly involved in a complete analysis and design has been eliminated.

---

[2] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed., pp. 148–149.

[3] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

DERIVATION OF METHOD FOR DETERMINING REACTIONS

General Basis of Derivation.—The redundant reaction components H acting at the supports of a two-hinged rectangular rigid frame are so nearly identical with the corresponding redundant reaction components R_x acting at the supports of a two-hinged skewed rigid frame of the same right span that, for purposes of practical design, the difference is negligible. This fact has been attested by comparative analyses, regardless of skew, span, or deck curvature. It was verified by comparative analyses of flat-topped and elliptical frames, made under the writer's supervision in 1932, for skews up to 60°, and has been further confirmed as one of the results of a study,[4] by Ernest J. Clark (Supervising Designer for W. Earle Andrews, Consulting Engineer, New York, N. Y.), of a skewed frame of comparatively long span, using entirely different methods.

The identity referred to has been applied for years as an approximate method of analysis,[5] under variously defined limitations, but its implications have not been thoroughly understood. It can be derived mathematically for flat-topped frames, and can be demonstrated as applicable to any type of frame by making an assumption (see Appendix), the practical validity of which, for cases of unrestricted deck curvature, it has been necessary to establish by trial. No comparative analyses have been made, to the writer's knowledge, of arches of the segmental type, and it is not certain, in the absence of such supporting evidence, that the simplified method is valid in such cases. Skewed segmental arches, however, are rather too scarce, in general, to be of much importance in this connection, since, when used, they are usually sufficiently large and expensive to justify more extended methods of analysis.

The general applicability of the foregoing relationship to all kinds of frames is of fundamental importance in any real simplification of skewed frame analysis. The reaction components H and their static equilibrants ϵH (see Fig. 1) which thus have been found to be independent of the torsional elastic deformations in the structure, are of much more importance to the design than any of the remaining redundant quantities, and the advantages to be gained by considering them independently are obvious.

It will be noted that the equilibrants ϵH are the principal parts of the total reaction components R_z. The remaining parts of R_z, which have been designated as R'_z, together with the reaction components M_x and M_y, constitute an independent group, all of which are functions of the torsional elastic deformations, and no one of which is of much more (or less) importance in its effect on the design than either of the others. The reaction component M_{yL} at the left support is equal to the reaction component M_{yR} at the right support for all cases of symmetrical loading, and both can be obtained by statics as soon as

---

[4] Thesis presented in 1942 to the Polytechnic Inst. of Brooklyn (New York), in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[5] "Approximate Design Method for Concrete Skew Rigid Frames," by Edward F. Gifford, *Engineering News-Record*, May 3, 1934.

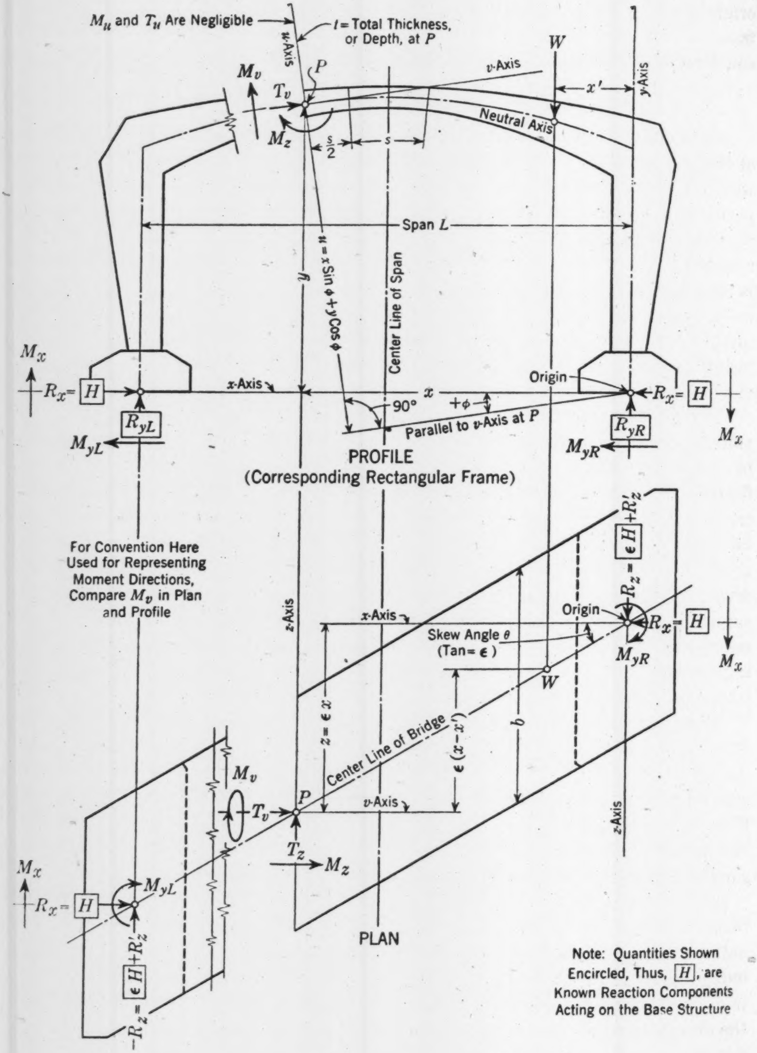

M_u and T_u Are Negligible
t = Total Thickness, or Depth, at P
u-Axis
W
P
M_v
T_v
M_z
v-Axis
x'
y-Axis
Neutral Axis
$\frac{s}{2}$
s
Span L
y
$u = x \sin\phi + y \cos\phi$
Center Line of Span
M_x
$R_x = \boxed{H}$
x-Axis
x
90°
$+\phi$
Origin
Parallel to v-Axis at P
$\boxed{R_{yL}}$
M_{yL}
$\boxed{R_{yR}}$
M_{yR}
PROFILE
(Corresponding Rectangular Frame)
For Convention Here Used for Representing Moment Directions, Compare M_v in Plan and Profile
z-Axis
x-Axis
Skew Angle θ (Tan= ϵ)
Origin
$R_z = \boxed{\epsilon H} + R'_z$
$R_x = \boxed{H}$
M_{yR}
M_x
W
$z = \epsilon x$
b
$\epsilon(x - x')$
Center Line of Bridge
M_v
T_v
P
v-Axis
T_z
M_z
z-Axis
M_x
$R_x = \boxed{H}$
M_{yL}
$-R_z = \boxed{\epsilon H} + R'_z$
PLAN
Note: Quantities Shown Encircled, Thus, $\boxed{H}$, are Known Reaction Components Acting on the Base Structure

FIG. 1.—GEOMETRICAL PROPERTIES AND FORCES ACTING (VERTICAL LOADING)

R'_s has been found. Therefore, the determination of the remaining redundant reaction components requires the solution of only two simultaneous equations, with simple corrections for unsymmetrical loading. Separate solutions for the two independent groups of redundant quantities acting on a skewed frame not only shorten the calculations but also (which is more important) reduce the probability of serious error. It may be added that the extreme precision required for the solution of four simultaneous equations, each involving quantities belonging to two separate elastic systems, is no longer necessary.

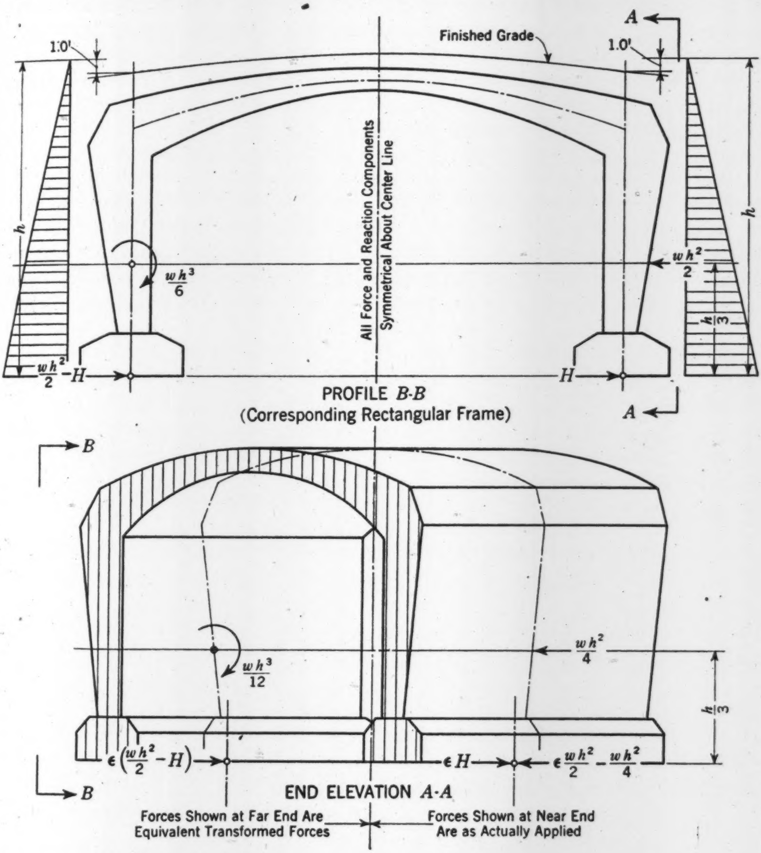

1.0'
Finished Grade
1.0'
A
h
All Force and Reaction Components Symmetrical About Center Line
$\frac{wh^3}{6}$
$\frac{wh^2}{2}$
h
$\frac{h}{3}$
$\frac{wh^2}{2}-H$
H
PROFILE B-B
(Corresponding Rectangular Frame)
A
B
$\frac{wh^2}{4}$
$\frac{wh^3}{12}$
$\frac{h}{3}$
$\epsilon\left(\frac{wh^2}{2}-H\right)$
ϵH
$\epsilon\frac{wh^2}{2}-\frac{wh^2}{4}$
B
END ELEVATION A-A
Forces Shown at Far End Are Equivalent Transformed Forces
Forces Shown at Near End Are as Actually Applied

Fig. 2.—Balanced Earth-Pressure Forces and Reactions Acting on the Base Structure

Although this paper is not primarily concerned with the problem of internal stress distribution, the design method originally proposed by Professor Rathbun has been adopted, in the form presented by Mr. Hayden, and has been simplified further. The method presented by Bernard L. Weiner,[6] M. Am. Soc. C. E., was a valuable contribution to the theory of the subject, but the writer, as well as other designers, has used the very much simpler Rathbun method for many years with satisfactory results.

Details of Derivation.—All of the forces acting on the structure, as well as all of the internal stress components which need to be considered in any ordinary practical problem, are shown in Figs. 1 and 2. Unbalanced earth pressure is so seldom used in practical design that it has not been included.

The redundant reaction components to be determined directly are those acting at the point of origin at the right support. The complete solution for any loading condition is made in two steps:

1. The first step requires very little explanation. It consists of the determination of the horizontal reactions H for an ordinary two-hinged rectangular rigid frame (designated as the "corresponding rectangular frame"), which is outlined in longitudinal section by projecting the skewed frame under consideration on a vertical plane perpendicular to the abutments, or parallel to the xy-plane. The same reactions apply to the skewed frame, without any change except that the H for temperature change must be multiplied by $(1 + \epsilon^2)$.
2. The second step consists of setting up and solving the equations for the redundant reaction components R'_z, M_x, and M_{yR}, using, as the base structure, the skewed frame in equilibrium with the reaction components H and ϵH included. Except for the number of redundants involved, the type of base structure used, and the fact that only one elastic system is used instead of two, the method is very similar to that originally used in deriving the equations given in the Appendix.

In setting up the equations, the internal stresses acting at any point P (Fig. 1) are represented as a system of external moment and force components exerted by the portion of the structure to the left of P on the part of the structure to the right. The applied loading and the reaction components acting on the base structure are shown in Figs. 1 and 2 as applied, of course, in their known directions. The unknown redundant reaction components, and also the components representing the unknown internal stresses, are shown applied in their assumed positive directions. The convention used for indicating moment directions is indicated in Fig. 1, the subscript representing, in each case, the axis about which rotation tends to occur.

The equations state the customary assumption that each of the redundant reaction components will prevent displacement or rotation in its particular direction. Since this assumption is open to question under ordinary soil conditions (see Fig. 6, under "Application of the Method to a Practical Example"), additional equations are set up and solved to find the effect on the design if free rotation is allowed about the x-axes, or about both the x-axes and the y-axes. The structure can be reinforced, at very small extra expense, to meet all of these assumed conditions of rotation or fixity.

---

[6] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1212.

Geometrical Relations and Definitions.—The geometrical relations involved are indicated in Fig. 1. The coordinate axes, applied loads, reaction components, and the components representing the internal stresses at any point P are all applied in directions parallel with, or perpendicular to, a vertical plane perpendicular to the abutments. The profiles shown in Figs. 1 and 2 are projections of the skewed frame on such a vertical plane. The point P is taken on the neutral axis at the center of any subdivision s, the coordinates of which are x, y, and z. The components representing the internal stresses at P are assumed to act along the directions of the u, v, and z-axes passing through point P. These directions (except the direction of z) are variable. The angle ϕ is the angle of slope of the neutral axis (or of the v-axis) at P; it can be represented conveniently as shown. The quantity u is shown to be the component parallel to the u-axis of the distance between point P and the origin. It cannot be emphasized too strongly that a complete understanding of the basic relations shown in Fig. 1 will remove the principal source of difficulty in skewed arch or frame analysis.

The following definitions are supplementary to the information shown in Figs. 1 and 2:

c = coefficient of linear expansion for concrete and reinforcing steel;

$t°$ = temperature rise or fall, in degrees Fahrenheit;

w = equivalent fluid pressure of earth, in pounds per square foot per foot of depth;

M_o = simple span moment on the corresponding rectangular frame, for vertical loading;

$$K = \frac{\text{Modulus of elasticity for concrete under axial stress}}{\text{Modulus of elasticity for concrete in shear}} = \frac{E}{G};$$

F = factor of torsion (a quantity corresponding to the moment of inertia I where ordinary flexure is concerned) $= \dfrac{b\,t^3}{3.58}$.

Basic Equations.—The basic equations are developed under the assumption that no rotation can occur at the footings, except about the z-axes, and that no movement of translation can occur in any direction. There are five reaction components at each footing, therefore; at the right-hand footing they are R_x, R_{yR}, R_z, M_x, and M_{yR}. The components R_x and R_{yR} are known, as well as ϵR_x, which is a part of the total component R_z. The redundant reaction components to be considered are therefore R'_z, M_x, and M_{yR}. The deflections at the right support along the lines of action of the redundants, due to all of the forces acting, are assumed, in each case, to be equal to zero. The usual assumption is made that the effects of all thrusts and shears can be neglected in considering deflections. It is only necessary, therefore, to consider the deflections due to the torsional moments about the v-axis, at point P.

Let:

M_v = moment at P about the v-axis, due to all the forces acting;

$m_{v(z)}$ = moment at P about the v-axis, due to unit force acting along and in the direction of R'_z;

$m_{v(ox)}$ = moment at P about the v-axis, due to unit couple acting along and in the direction of M_x; and

$m_{v(oy)}$ = moment at P about the v-axis, due to unit couple acting along and in the direction of M_y.

Summing up for all points P, the three deflection equations are:

$$\frac{s}{G}\sum\frac{M_v\,m_{v(z)}}{F} = 0\ (1a),\quad \frac{s}{G}\sum\frac{M_v\,m_{v(ox)}}{F} = 0\ (1b),\ \text{and}\ \frac{s}{G}\sum\frac{M_v\,m_{v(oy)}}{F} = 0\ (1c)$$

The fundamental expression for M_v (for vertical loading), in terms of all of the external forces acting on the structure, can be written from a careful inspection of Fig. 1, as follows:

$$\begin{aligned} M_v &= R_x\,\epsilon\,x\sin\phi - R_z\,u + M_x\cos\phi - M_{yR}\sin\phi \\ &\quad + R_{yR}\,\epsilon\,x\cos\phi - W\,\epsilon\,(x - x')\cos\phi \\ &= R_x\,\epsilon\,x\sin\phi - R_z\,u + M_x\cos\phi - M_{yR}\sin\phi + M_o\,\epsilon\cos\phi \quad\ldots\ldots(2) \end{aligned}$$

Also, by inspection:

$$m_{v(z)} = -\,u \quad\ldots\ldots(3a)$$

$$m_{v(ox)} = \cos\phi \quad\ldots\ldots(3b)$$

and

$$m_{v(oy)} = -\sin\phi \quad\ldots\ldots(3c)$$

Substituting Eqs. 2 and 3 in Eqs. 1, making $R_x = H$, and noting that the summation terms $\sum\frac{\sin\phi\cos\phi}{F}$ and $\sum\frac{y\sin\phi\cos\phi}{F}$ are each equal to zero, by symmetry, Eqs. 1 for vertical loading can be written, in preliminary form, as follows:

$$\begin{aligned} R'_z\sum\frac{u^2}{F} - M_x\sum\frac{u\cos\phi}{F} + M_{yR}\sum\frac{x\sin^2\phi}{F} \\ = \epsilon\left(\sum\frac{M_o\,u\cos\phi}{F} - H\sum\frac{u\,y\cos\phi}{F}\right) \quad\ldots\ldots(4a) \end{aligned}$$

$$R'_z\sum\frac{u\cos\phi}{F} - M_x\sum\frac{\cos^2\phi}{F} = \epsilon\left(\sum\frac{M_o\cos^2\phi}{F} - H\sum\frac{y\cos^2\phi}{F}\right) \ldots(4b)$$

and

$$R'_z\sum\frac{x\sin^2\phi}{F} + M_{yR}\sum\frac{\sin^2\phi}{F} = \epsilon\sum\frac{M_o\sin\phi\cos\phi}{F} \quad\ldots\ldots(4c)$$

The expression for M_v in Eq. 2 and the resulting Eqs. 4 can be adjusted to include balanced earth pressure and temperature loading, as follows:

It should be noted that the simple span moment, or M_o, as used in the equations for vertical loading, has no direct component in the direction of M_v. It is simply a convenient abbreviation of the terms $R_{yR}\,x - W\,(x - x')$, which happen to enter into the expression for M_v as $[R_{yR}\,x - W\,(x - x')]\,\epsilon\cos\phi$. No such expedient is possible in the cases of earth pressure and temperature loading. For balanced earth-pressure loading, the corresponding terms may be written: $\left(\frac{w\,h^2}{2}\right)\left(y - \frac{h}{3}\right) - H\,y$, which equals $\left(\frac{w\,h^2}{2} - H\right)y - \frac{w\,h^3}{6}$.

As far as its effect on M_v is concerned, $\frac{w\,h^3}{6}$ can be omitted (see Fig. 2), and the moment $\left(\frac{w\,h^2}{2} - H\right) y$, like the moment M_o, has no direct M_v component. The force $\left(\frac{w\,h^2}{2} - H\right)$, however, cannot be neglected. For balanced earth pressure, therefore, the M_o-terms are to be omitted both from the expression for M_v and from the equations, and H is to be replaced throughout by $\left(\frac{w\,h^2}{2} - H\right)$.

For temperature rise, replace H throughout by $H\,(1 + \epsilon^2)$, which corresponds to R_x in this case (see the Appendix). Omit the M_o-terms in the expression for M_v and in Eqs. 4*b* and 4*c*, and replace the M_o-term in Eq. 4*a* by the quantity $\frac{E\,c\,t^\circ\,L}{K\,s}$. The latter substitution simply represents the substitution of the direct temperature displacement in the direction of R'_x in place of the elastic displacement due to the applied loading. There is no corresponding direct temperature displacement in the direction of M_x or of M_{yR}.

The preliminary equations can be simplified also in the following manner: In Eqs. 4*a* and 4*c*, $\sum \frac{x \sin^2 \phi}{F} = \frac{L}{2} \sum \frac{\sin^2 \phi}{F}$, since the values of x entering the first summation can be taken in pairs, each pair being equal to $\frac{L}{2}$. For any kind of symmetrical loading, the M_o-term in Eq. 4*c* is equal to zero. Eq. 4*c* reduces to the simple expression $M_{yR} = M_{yL} = -\frac{L}{2} R'_x$, and there are only two simultaneous equations left to be solved for R'_x and M_x. For symmetrical loading, R'_x and M_x at the right support are respectively equal to R'_x and M_x at the left support. Similarly, $M_{yR} = M_{yL}$ for symmetrical loading. For unsymmetrical vertical loading (Eq. 4*c*), $M_{yR} = -R'_x \frac{L}{2} + C$, in which C represents the quantity $\sum \frac{M_o \sin \phi \cos \phi}{F} \div \sum \frac{\sin^2 \phi}{F}$. By statics, $M_{yR} + M_{yL} = -R'_x\,L$, and therefore $M_{yL} = -R'_x \frac{L}{2} - C$.

Now assume the unsymmetrical loading balanced by adding equal loading placed symmetrically with respect to the original unsymmetrical loading. Evidently M_{yR} (or M_{yL}) for the balanced loading is equal to $-R'_x \frac{L}{2}$ for the unsymmetrical loading, which leads directly to Eqs. 5*d*. The redundants R'_x and M_x for unsymmetrical loading are each equal to one half, respectively, of the corresponding quantities for balanced loading. It is possible, therefore, to make the two simultaneous equations apply, not only to symmetrical loading, but also to unsymmetrical loading, by balancing, solving, and making the corrections noted.

Performing the substitutions and simplifications indicated, Eqs. 4 reduce to the following final equations for R'_x and M_x and formulas for M_{yR} and M_{yL}:

For vertical loads,

$$R'_z\left(\sum\frac{u^2}{F}-\frac{L^2}{4}\sum\frac{\sin^2\phi}{F}\right)-M_x\sum\frac{u\cos\phi}{F}$$

$$=\epsilon\left(\sum\frac{M_o\,u\cos\phi}{F}-H\sum\frac{u\,y\cos\phi}{F}\right)\ldots\ldots\ldots\ldots$$

for balanced earth pressure,

$$=-\epsilon\left(\frac{w\,h^2}{2}-H\right)\sum\frac{u\,y\cos\phi}{F}\ldots\ldots\ldots\ldots \tag{5a}$$

and, for temperature,

$$=\epsilon\left(\frac{E\,c\,t^\circ\,L}{K\,s}-H\,(1+\epsilon^2)\sum\frac{u\,y\cos\phi}{F}\right)\ldots\ldots\ldots\ldots$$

Also, for vertical loads,

$$R'_z\sum\frac{u\cos\phi}{F}-M_x\sum\frac{\cos^2\phi}{F}=\epsilon\left(\sum\frac{M_o\cos^2\phi}{F}-H\sum\frac{y\cos^2\phi}{F}\right)$$

for balanced earth pressure,

$$=-\epsilon\left(\frac{w\,h^2}{2}-H\right)\sum\frac{y\cos^2\phi}{F}\ldots\ldots\ldots\ldots \tag{5b}$$

and, for temperature,

$$=-\epsilon\,H\,(1+\epsilon^2)\sum\frac{y\cos^2\phi}{F}\ldots\ldots\ldots\ldots$$

For symmetrical loading,

$$M_{yR}=M_{yL}=-R'_z\frac{L}{2}\ldots\ldots\ldots\ldots \tag{5c}$$

and, for unsymmetrical vertical loading,

$$M_{yR}=+\frac{M_y}{2}+C,\qquad\text{and}\qquad M_{yL}=+\frac{M_y}{2}-C\ldots\ldots \tag{5d}$$

in which M_y is equal to either M_{yR} or M_{yL} for balanced vertical loading, and

$$C=\sum\frac{M_o\sin\phi\cos\phi}{F}\div\sum\frac{\sin^2\phi}{F}\ldots\ldots\ldots\ldots \tag{5e}$$

using the values of M_o for unsymmetrical vertical loading. Also R'_z and M_x for unsymmetrical loading are equal to one half, respectively, of R'_z and M_x, as previously noted, for balanced vertical loading.

The solution of Eqs. 5 completes the analysis of the external forces acting on the frame for the basic assumption of complete fixity at the footings, except about the z-axes. (A correction, which is considered subsequently, will be necessary in analyzing the effect on the footings of the component M_x for earth pressure loading.)

The corresponding general expression for M_v, expressed by Eq. 2, must be adjusted similarly for the various loading conditions, as follows: For vertical loading, $R_x = H$; for balanced earth pressure, $R_x = \frac{w\,h^2}{2} - H$; and, for temperature rise, $R_x = (1 + \epsilon^2)\,H$. The simple span moment M_o is to be omitted for all loading conditions except vertical loading.

Equations for Rotations at Footings.—The customary assumption that the footings are fixed rigidly at the footings against rotations about the x-axes and y-axes (especially the x-axes) seems to be unwarranted under ordinary soil conditions and for bridges of the type here considered. The degree of rotation necessary to overcome a condition of complete fixity in any direction is so small that even elaborate and expensive provisions for insuring fixity cannot be relied upon with any degree of certainty. The soil pressure distribution about the x-axes (see Fig. 6 under "Application of the Method to a Practical Example") is such that variable settlement and resulting rotation are to be expected. Theoretically, uniform pressure can be obtained by splaying the footings; but, aside from the fact that temperature effects (which are very important for heavy skews) are reversible, this is objectionable from a practical standpoint. On the other hand, any assumption of unrestricted rotation is also questionable unless special measures are taken to separate the frame and approach wall footings. Since the analysis can be made easily for both of these assumptions, there can be no reason for basing the design exclusively on either one of them. Therefore, in addition to the equations previously derived for the basic assumption, hereinafter called Assumption 1, equations will be derived for Assumption 2—free rotation about the x-axes; and for Assumption 3—free rotation about both the x-axes and y-axes.

Assumption 2.—Assuming free rotation about the x-axes, Eq. 5*b* is omitted and the M_x-term in Eq. 5*a* becomes zero, for obvious reasons. The right-hand terms of Eq. 5*a* remain as before, except that, for balanced earth-pressure loading, there is an additional term $\frac{w\,h^3}{12} \sum \frac{u \cos \phi}{F}$ · (An explanation of this is given subsequently under "Earth-Pressure Loading Considerations.") The M_y formulas and corrections remain unchanged.

In the expression for M_v (Eq. 2) omit the M_x-term for all loading conditions, and, for balanced earth-pressure loading, add the term $\frac{w\,h^3}{12} \cos \phi$ (see "Earth-Pressure Loading Considerations").

Assumption 3.—Assuming free rotation about both the x-axes and the y-axes, it is only necessary to solve Eq. 5*a*, which is exactly the same as for the previous assumption of rotation about the x-axes, except that the coefficient for R'_x becomes simply $\sum \frac{u^2}{F}$ · In the expression for M_v (Eq. 2), omit both the M_x-term and the M_{yR}-term, and make the same addition as before for balanced earth-pressure loading.

Earth-Pressure Loading Considerations.—The effect of earth-pressure loading on the torsional and transverse shearing forces is much greater, in general, than that of dead loading or any of the other loadings to which a skewed frame is subjected. It is more difficult, in many ways, to analyze, and is based on questionable assumptions. It deserves more attention, therefore, than it usually gets.

Fig. 2 represents the forces acting, and the transformations used in analysis. It should be noted that the figure shows only the external forces acting on the base structure; the redundant reaction components and the stress components at P are applied as shown in Fig. 1.

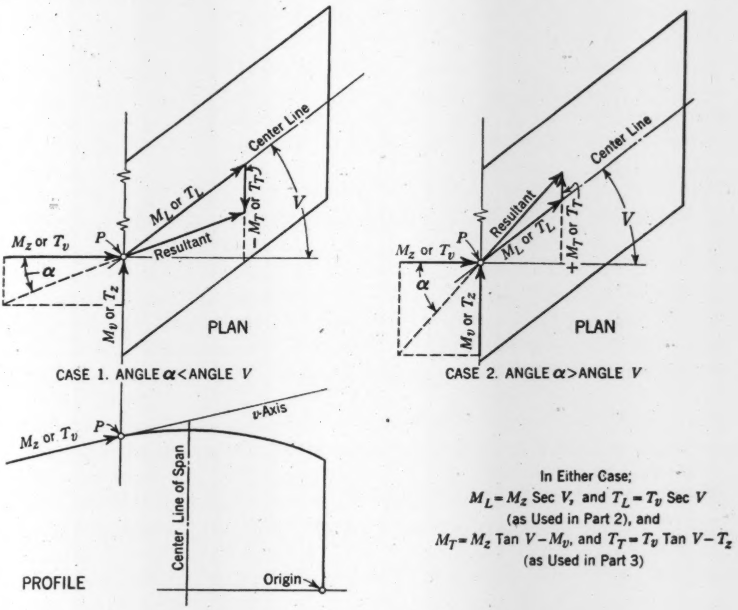
The reactions $\frac{w\,h^2}{4}$ shown acting at the elevation of the resultant total earth pressures are the most that can be taken above the footings, assuming a coefficient of friction of earth on concrete (or mortar waterproofing protection) of 0.5. The remainder, or $\epsilon\left(\frac{w\,h^2}{2}\right) - \frac{w\,h^2}{4}$, is assumed to be applied, in each case, directly against the footing. The customary assumption in practice has been that the entire equilibrating reaction is applied at the elevation of the bases of footings. On the contrary, the writer believes that the tendency is for the abutments to transmit, directly, as much of it as the coefficient of friction will permit. Admitting the many uncertainties affecting the action of earth pressures in general, and especially in this particular case, it is believed that the distribution here proposed is more reasonable.

The transformation of lateral reactions, as shown on the end elevation in Fig. 2, fails to take into account the effects of the forces on the vertical legs of the frame, but, since these effects are negligible, it makes no difference. The couples $\frac{w\,h^3}{12}$ are either directly counterbalanced at each footing, if the footings are assumed fixed against rotation in the direction of M_x, or counteracted across the frame, if the footings are assumed free to rotate. In the first case, these couples can be disregarded, except in so far as they reduce the reaction components M_x; in the second case, they must be taken into account in the equations and in the expressions for M_v. It will be seen that the moment $\frac{w\,h^3}{12}$, at the right end, is in the same direction as M_o for vertical loading, and therefore its M_v component can be obtained by substitution. The necessary corrections have been made accordingly in the equations and expressions for M_v as noted previously in the last two paragraphs under the heading "Equations for Rotations at Footings."

The couples $\frac{w\,h^3}{6}$, as shown on the profile, Fig. 2, are taken into account, in effect, in the analysis of the corresponding rectangular frame, but (since they have no M_v components) can be disregarded here.

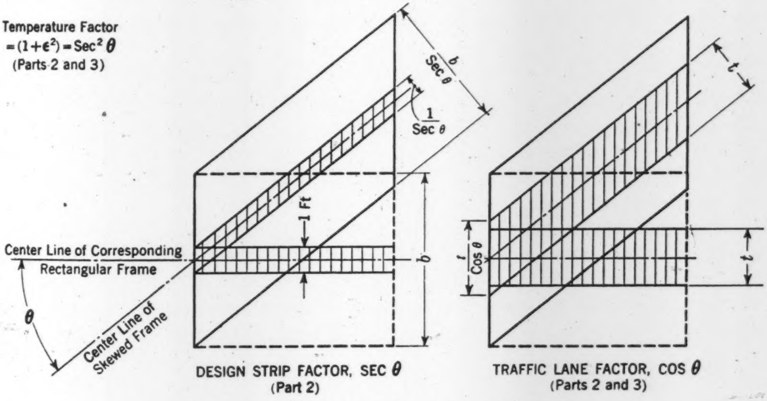
DESIGN

Preliminary.—Having determined all of the redundant reaction components acting on the structure for the assumed conditions at the footings, the remaining problem, of course, is the determination of the effect of all forces acting on the structure, and the proportioning of the structure to withstand them with safety and economy. The stress at any section (point P) of the frame is represented by the moment and force components M_x, T_v, M_v, and T_x, as


Center Line
V
M_L or T_L
$-M_T$ or T_T
M_z or T_v
P
Resultant
α
M_v or T_z
PLAN
CASE 1. ANGLE α < ANGLE V
Center Line
Resultant
M_L or T_L
$+M_T$ or T_T
V
M_z or T_v
P
α
M_v or T_z
PLAN
CASE 2. ANGLE α > ANGLE V
M_z or T_v
P
v-Axis
Center Line of Span
PROFILE
Origin

In Either Case;
$M_L = M_z$ Sec V, and $T_L = T_v$ Sec V
(as Used in Part 2), and
$M_T = M_z$ Tan $V - M_v$, and $T_T = T_v$ Tan $V - T_z$
(as Used in Part 3)

FIG. 3.—TRANSFORMATION OF FORCES


$\frac{b}{\text{Sec }\theta}$
$\frac{1}{\text{Sec }\theta}$
1 Ft
b
Center Line of Corresponding Rectangular Frame
θ
Center Line of Skewed Frame
DESIGN STRIP FACTOR, SEC θ (Part 2)
t
$\frac{t}{\text{Cos }\theta}$
t
TRAFFIC LANE FACTOR, COS θ (Parts 2 and 3)

Temperature Factor
$= (1+\epsilon^2) = \text{Sec}^2\,\theta$
(Parts 2 and 3)

FIG. 4.—SKEW CORRECTIONS

indicated in Fig. 1. The moment and thrust M_z and T_v are obtained from the analysis of the corresponding rectangular frame, which needs no explanation; M_v has already been considered (see Eq. 2) and T_z is equal to R_z in every case.

Transformation of Forces.—The moment and force components referred to under the heading "Design: Preliminary" represent components of the internal stresses at the point P, acting in directions parallel with and perpendicular to the abutments. The next step will be to resolve or transform them into components acting in the directions most convenient for practical design; that is, parallel to the skewed direction of the bridge and parallel to its abutments. These transformations, the formulas for which are given in Fig. 3, result in a direct moment M_L and an axial thrust T_L acting in the longitudinal direction, and in a torsional moment M_T and a shear T_T acting in the transverse direction.

Design for Longitudinal Forces.—The design for the longitudinal moments and thrusts controls the proportioning of the structure, as well as the longitudinal reinforcement, but, since it consists of the same methods as are used generally for rectangular frames, it will not be considered here, except to note the necessary skew corrections, as indicated in Fig. 4. The procedure to be followed in applying the corrections is thoroughly explained and illustrated under the heading "Application of the Method to a Practical Example."

Design for Transverse Forces.—The design for the transverse torsional moments and shears determines the transverse reinforcement required in the structure as previously proportioned from the design for the longitudinal forces. As previously stated, the method used is fundamentally the same as that originally proposed by Professor Rathbun. The writer could do no better than to restate briefly, for the reader's convenience, a detailed outline of the process as presented by Mr. Hayden, with certain modifications.

The direct transverse shear T_T is assumed to be distributed parabolically across the width of the frame, in the same way as vertical shear is distributed along the depth of an ordinary beam. The maximum intensity at any section is constant throughout the depth, and, at the center line of the frame, amounts to three halves of the average unit shear.

The maximum intensity of the torsional shear at any section is given by Merriman's formula for elongated rectangular shafts (see Formula I, Table 11, presented subsequently). This maximum unit stress, which also exists at the center line of the frame, is exerted in opposite directions at the extrados and at the intrados, with a constant variation throughout the depth of the section and zero stress at the center, as shown in Table 11.

The shearing capacity of concrete (assumed in the example as 60 lb per sq. in.) can be deducted from the combination of the total net shears acting across the center line of the frame at the point considered, and the transverse reinforcing steel is designed to take care of the remainder by a method analogous to that followed in the design of vertical stirrups for a concrete beam whose depth is equal to the skew width of the bridge, and whose width is the depth of the section.

Fig. 1 indicates the positive directions of M_v and T_z at any point P. Whenever M_v and T_z are found to be of the same sign, they work together to cause shearing stress at the extrados and against each other to cause shearing stress at the intrados; and conversely, when they are found to be of different sign, they combine to cause shearing stress at the intrados and work against each other to cause shearing stress at the extrados. Since M_T and T_T are positive in the same directions as M_v and T_z, respectively, the same effects hold true for the transformed moments and forces M_T and T_T. This leads to the following rules for combining the unit torsional shears v_t and unit direct shears v_s for maximum effects:

(1) When v_t and v_s are of the same sign, add to obtain the resultant shear at the extrados and subtract to obtain the resultant shear at the intrados.

(2) When v_t and v_s are of different sign, add (regardless of sign) to obtain the resultant shear at the intrados and subtract (regardless of sign) to obtain the resultant shear at the extrados.

Since the differences between the resulting shears at the intrados and at the extrados are proportionately small, except where the amount of transverse reinforcement is negligible, it is only necessary, in practice, to design for an equal amount of steel in both locations, based, in both cases, on the arithmetical sum of v_t and v_s. The areas of transverse steel reinforcement required at the various subdivision points are calculated by a semigraphical process which can be understood readily by referring to Table 11 (under the heading "Application of the Method to a Practical Example"). The calculations have been shortened by representing v_s in a diagonal instead of in a vertical position, and by deducting, for the shearing resistance of concrete, an amount varying uniformly from zero at the neutral axis of the section to the full allowable value at the top and bottom.

APPLICATION OF THE METHOD TO A PRACTICAL EXAMPLE (SEE FIG. 5)

Part 1. Analysis of Corresponding Rectangular Frame.—The complete analysis of the corresponding rectangular frame, to the point of determining the reactions and the maximum moments and thrusts for the loading conditions considered, constitutes Part 1 of the procedure. There are various well-known methods of procedure which, of course, need not be mentioned. The solution appears in Table 1.

TABLE 1.—MOMENTS (M) AND THRUSTS (N) RESULTING FROM VARIOUS LOADS ON THE CORRESPONDING RECTANGULAR FRAME

Loads	POINT 0		POINT 1R		POINT 2R		POINT 3R		POINT 4R[a]		POINT 6R[a]	
	M	N	M	N	M	N	M	N	M	N	M	N
Dead load plus earth pressure ..	+4.8	7.1	−1.3	7.1	−21.5	7.1	−62.0	13.4	−23.8	17.2	+10.5	23.8
Live Load:[b]												
Point 0........	+7.7	1.1	−2.7	1.1	−12.9	1.0	−22.1	2.0	−15.5	1.7	− 3.7	1.7
Point 1R.......	+1.7	0.9	+9.0	0.9	− 4.8	0.9	−18.0	2.2	....	....	− 3.0	2.3
Point 1L.......	+1.7	0.9	−5.5	0.9	−12.9	0.9	−18.0	1.4	....	....	− 3.0	1.1
Temperature:												
Rise Fall	∓3.0	±0.1	∓3.0	±0.1	∓ 2.9	±0.1	∓ 2.7	±0.1	∓ 1.9	0.0	∓ 0.4	0.0

[a] Subdivision $5R$ (Fig. 5) has been omitted since the moments and thrusts at this point are negligible.

[b] See alternate locations of concentrated live loads in Fig. 5.

Part 2. Design of Skewed Frame for Longitudinal Forces.—Part 2 consists, in general, of the complete design of the skewed frame for the longitudinal moments and thrusts found from Part 1, after transforming and correcting them in accordance with the following procedure:

The transformation of forces and the resulting formulas are shown in Fig. 3. The formulas for M_L and T_L applying to Part 2 are:

$$M_L = M_z \sec V \qquad \text{(6a)}$$

and

$$T_L = T_v \sec V \qquad \text{(6b)}$$

The thing to be particularly noted here is that the formulas happen to work out in such a way that M_L and T_L are entirely independent of M_v and T_z, which appear in the corresponding formulas for Part 3. This means that Part 2 of the design, which controls the proportioning of the structure, can be completed without any reference to Part 3; and this is a very decided advantage. With reference to Fig. 3, it should be noted that the moments and forces shown in plan are all acting in the *vz*-plane (tangent to the neutral axis at point *P*). The angle *V* is the projection on the *vz*-plane of the skew angle θ (see calculations in Table 2). In Fig. 3, it is shown projected back on a horizontal plane.

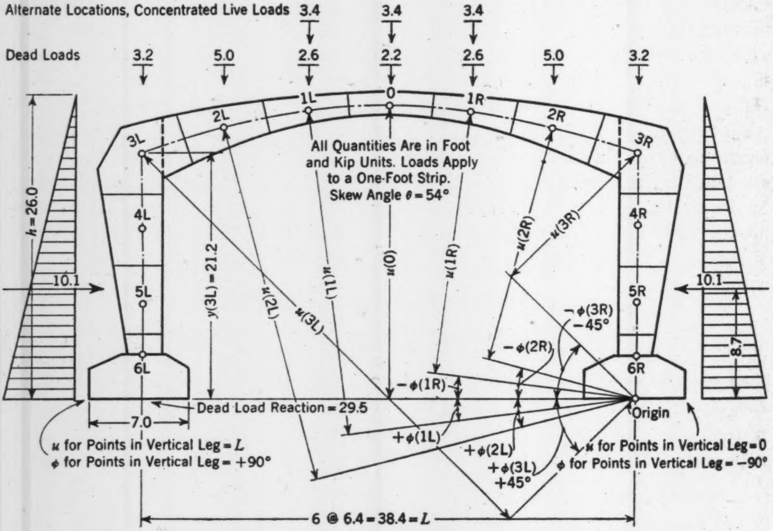

Alternate Locations, Concentrated Live Loads
3.4
3.4
3.4
Dead Loads
3.2
5.0
2.6
2.2
2.6
5.0
3.2
1L
0
1R
2L
2R
3L
3R
All Quantities Are in Foot and Kip Units. Loads Apply to a One-Foot Strip. Skew Angle θ = 54°
h = 26.0
4L
4R
y(3L) = 21.2
u(2L)
u(1L)
u(0)
u(1R)
u(2R)
u(3R)
u(3L)
10.1
10.1
5L
5R
−φ(3R)
−45°
−φ(2R)
6L
6R
8.7
−φ(1R)
Dead Load Reaction = 29.5
Origin
7.0
+φ(1L)
+φ(2L)
+φ(3L)
+45°
u for Points in Vertical Leg = L
φ for Points in Vertical Leg = +90°
u for Points in Vertical Leg = 0
φ for Points in Vertical Leg = −90°
6 @ 6.4 = 38.4 = L

FIG. 5.—DIMENSIONS, LOADINGS, AND PROPERTIES OF CORRESPONDING RECTANGULAR FRAME

The first correction considered is due directly to the fact that *H* for temperature loading, as found for the corresponding rectangular frame, must be multiplied by $(1 + \epsilon^2)$, or $\sec^2 \theta$, in order to make it applicable to a skewed frame (see Fig. 4). The additional skew corrections, which are illustrated in

Fig. 4, consist of multiplying M_L and T_L by sec θ for all conditions of loading, and also multiplying M_L and T_L again, for live loading, by cos θ. Although the two factors evidently cancel for live loading, it is found convenient in tabulation to perform both multiplications. The resulting procedure, as adopted for the present example, is as shown in Table 3.

TABLE 2.—VALUES OF FUNCTIONS OF THE ANGLE V (SEE FIG. 3)

($\theta = 54°$; $\epsilon = 1.376$; and $\epsilon^2 = 1.893$)

Function	Point 0	Point 1R	Point 2R	Point 3R	4R	5R	6R
Cos ϕ	1.00	0.99	0.98	0.95[a]	0	0	0
Tan $V = \epsilon \cos \phi$	1.376	1.362	1.348	1.307	0	0	0
V	54° 00′	53° 45′	53° 25′	53° 35′	0	0	0
Sec V	1.701	1.69	1.68	1.64	1.0	1.0	1.0
Sec θ sec V	2.90	2.88	2.86	2.79	1.7	1.7	1.7

[a] The angle ϕ at point 3R is in a state of transition. The value of cos ϕ, as used for Part 2, is based on a point in the deck portion just to the left of point 3R—which represents the most severe assumption. For Part 3, where it makes very little difference one way or the other, an angle of 45° is assumed (see Table 4).

The remainder of Part 2 consists simply of designing and proportioning the structure from the corrected moments and thrusts in exactly the same manner as for a rectangular rigid frame. The calculated depth at any section is to be measured in a vertical plane perpendicular to the abutments, which is the same as the depth for the corresponding rectangular frame. The calculated steel areas will be the areas required per foot of square width of the structure—that is, perpendicular to the faces of the bridge.

TABLE 3.—QUANTITIES IN TABLE 1 CORRECTED FOR SKEW

Loads	Step[a]	POINT 0		POINT 1R		POINT 2R		POINT 3R		POINT 4R		POINT 6R	
		M_s	T_s	M_s	T_s	M_s	T_s	M_s	T_s	M_s	T_s	M_s	T_s
Dead load plus earth pressure	..	+ 4.8	7.1	− 1.3	7.1	− 21.5	7.1	− 62.0	13.4	−23.8	17.2	+10.5	23.8
Live Load:[b]													
Point 0	1	+ 4.5	0.6	− 1.6	0.6	− 7.6	0.6	− 13.0	1.2	− 9.1	1.0	− 2.2	1.0
Point 1R	1	+ 1.0	0.5	+ 4.0	0.5	− 2.8	0.5	− 10.6	1.4	....	..	− 1.6	0.6
Point 1L	1	+ 1.0	0.5	− 3.2	0.5	− 7.5	0.5	− 10.6	1.4	....	..	− 1.6	0.6
Temperature:													
Rise Fall	2	∓ 8.7	±0.3	∓ 8.7	±0.3	∓ 8.4	±0.3	∓ 7.8	±0.3	∓ 5.5	0.0	∓ 1.2	0.0
Totals:													
Maximum	3	+18.0	7.5	+11.4 −13.2	7.3 7.9	− 37.4	7.9	− 82.8	14.9	−38.4	18.2	+11.7	23.8
Corrected	3	+52.2	21.6	+32.8 −37.9	21.1 22.8	−106.9	22.7	−231.0	41.5	−65.3	30.9	+19.9	40.5

[a] Note that Steps 1 and 2, following, apply also to Part 3:
Step 1.—Live load quantities multiplied by cos θ (0.588).
Step 2.—Temperature quantities multiplied by $\sec^2 \theta$ (2.90).
Step 3.—Maximum moments and thrusts summed up and multiplied by sec θ sec V (Table 2).

[b] See alternate locations of concentrated live loads in Fig. 5.

These matters have been described at some length because frequently they are sources of confusion and error. In the tables, the calculations have been extended only to the point of tabulating the maximum moments and thrusts at the various points, including the necessary skew corrections. It would serve no useful purpose to include in the example details of reinforced concrete design for bending and direct stress. It is worthy of note, however, that in the actual design of the structure used here as an example (Fig. 5), it was necessary to make several preliminary trials before the final dimensions were established, and it was found necessary, in the final design, to reduce the depth of concrete at and adjacent to the crown (the critical section), to a depth considerably smaller than that assumed in the beginning. This is typical of severe skews, where the temperature moments increase very rapidly in proportion to the other moments acting. The preponderant effect of temperature in such cases leads to the anomalous condition that the better the quality of concrete assumed in the design, the weaker the structure tends to become theoretically, until finally a point is reached where it becomes impossible, for any assumed crown thickness, to find room for the amount of tensile and compressive reinforcing steel required.

Part 3. Analysis and Design of Skewed Frame for Transverse Forces.—Part 3 includes (1) the analysis of the frame for the torsional and shearing forces acting in the transverse direction, and (2) the design of the transverse reinforcement needed in the structure as previously proportioned under Part 2.

It is unfortunate that, although Part 3 is of comparatively minor importance, it requires a comparatively larger amount of work. In many cases of moderate skew, the transverse steel will amount to little, if any, more than the spacer bars that would be used in any case, and therefore Part 3 could be omitted entirely. However, it is difficult, if not impossible, to be sure about this in any particular case (unless the skew amounts to less than about 25°) without working it out first.

TABLE 4.—PHYSICAL CONSTANTS, ELASTIC PROPERTIES, AND SUMMATIONS FOR EQUATIONS

Point	(a) QUANTITIES USED IN SUMMATIONS						(b) CONSTANT TERMS					
	t	$\frac{1}{F}$	y	Sin ϕ	Cos ϕ	u	$\frac{u^2}{F}$	$\frac{\sin^2 \phi}{F}$	$\frac{\cos^2 \phi}{F}$	$\frac{u \cos \phi}{F}$	$\frac{u y \cos \phi}{F}$	$\frac{y \cos^2 \phi}{F}$
6R	3.0	0.133	3.5	−1.00	0.00	0.0	0.0	0.1326	0.0000	0.000	0.00	0.00
5R	3.35	0.095	8.4	−1.00	0.00	0.0	0.0	0.0952	0.0000	0.000	0.00	0.00
4R	4.35	0.043	14.8	−1.00	0.00	0.0	0.0	0.0435	0.0000	0.000	0.00	0.00
3R	5.00	0.029	21.2	−0.71	0.71	15.0	6.4	0.0143	0.0143	0.303	6.43	0.30
2R	3.30	0.100	22.8	−0.19	0.98	21.1	44.3	0.0036	0.0956	2.060	47.00	2.18
1R	1.79	0.624	23.6	−0.09	0.99	22.2	308.0	0.0050	0.612	13.71	324.0	14.44
0	1.33	1.522	23.9	0.00	1.00	23.9	869.0	0.0000	1.522	36.4	869.0	36.40
1L	1.79	0.624	23.6	+0.09	0.99	25.6	409.0	0.0050	0.612	15.81	373.0	14.44
2L	3.30	0.100	22.8	+0.19	0.98	28.4	80.3	0.0036	0.0956	2.77	63.2	2.18
3L	5.00	0.029	21.2	+0.71	0.71	42.2	50.9	0.0143	0.0143	0.854	18.10	0.30
4L	4.35	0.043	14.8	+1.00	0.00	38.4	64.2	0.0435	0.0000	0.000	0.00	0.00
5L	3.35	0.095	8.4	+1.00	0.00	38.4	140.4	0.0952	0.0000	0.000	0.00	0.00
6L	3.00	0.133	3.5	+1.00	0.00	38.4	195.5	0.1326	0.0000	0.000	0.00	0.00
Σ	....	....	....	....	....	....	2,168.0	0.588	2.97	71.91	1,701.00	70.25

The example includes, for purposes of illustration, the most extensive analysis necessary under any circumstances. In most cases the work can be shortened. For instance, it will be noted that Assumption 3 (free rotation about both the x-axes and the y-axes) controls the design throughout, as might be expected, except that Assumption 2 (free rotation about the x-axes) controls at points along the vertical legs. It will also be seen that live loading could be omitted entirely without materially affecting the results.

TABLE 5.—LOADING-TERM SUMMATIONS FOR EQUATIONS

Point	DEAD LOAD			LIVE LOAD[a] 0			BALANCED LIVE LOAD[a] 1R			LIVE LOAD[a] 1R		
	M_o	$\frac{M_o\, u \cos \phi}{F}$	$\frac{M_o \cos^2 \phi}{F}$	M_o	$\frac{M_o\, u \cos \phi}{F}$	$\frac{M_o \cos^2 \phi}{F}$	M_o	$\frac{M_o\, u \cos \phi}{F}$	$\frac{M_o \cos^2 \phi}{F}$	M_o	$\frac{\text{Sin}\, \phi \cos \phi}{F}$	$\frac{M_o \sin \phi \cos \phi}{F}$
2R	55.7	115.0	5.33	10.9	22.4	1.04	21.7	44.8	2.08	14.5	−0.0185	−0.268
1R	79.4	1,089.0	48.60	21.8	299.0	13.33	43.5	597.0	26.60	29.0	−0.0556	−1.612
0	86.4	3,140.0	131.50	32.6	1,186.0	49.60	43.5	1,583.0	66.20	21.8	0.0	0.0
1L	79.4	1,256.0	48.60	21.8	345.0	13.33	43.5	688.0	26.60	14.5	+0.0556	+0.806
2L	55.7	154.0	5.33	10.9	30.2	1.04	21.8	60.3	2.08	7.2	+0.0185	+0.134
Σ	....	5,754.0	239.4	....	1,882.6	78.3	....	2,973.1	123.6	....	....	−0.940

[a] See alternate locations of concentrated live loads in Fig. 5.

The procedure for Part 3 is illustrated fully by Tables 4 to 11, and no further explanation is necessary. Experience shows, however, that mistakes occur most frequently in assembling and calculating the proper numerical quantities for substitution in the Eqs. 5. The following two paragraphs, in connection with Table 6, show a systematic arrangement of these quantities.

Quantities for Direct Substitution in Equations, Assumption 1.—The following values of H are obtained from Part 1:

Dead load	Balanced earth pressure	Temperature rise	Live load, Point 0	Live load, Point 1R
3.28	6.3	0.126	1.05	0.85

Substitutions for the left-hand sides of the equations are:

Eq.	Expression	Solution	Substitution
5*a*	$\sum \frac{u^2}{F} - \frac{L^2}{4} \sum \frac{\sin^2 \phi}{F}$	$2{,}168 - 19.2^2 \times 0.588$	1,951
5*a* and 5*b*	$\sum \frac{u \cos \phi}{F}$	....	71.9
5*b*	$\sum \frac{\cos^2 \phi}{F}$	....	2.97

The corresponding substitutions for the right-hand sides of the equations are listed in Table 6.

Quantities for Direct Substitution in Equations, Assumptions 2 and 3.—The balanced earth-pressure substitutions, in the right-hand sides of Eqs. 5*a* and 5*b*, for Assumptions 2 and 3, are as follows: $- \epsilon \left(\frac{w\, h^2}{2} - H \right) \sum \frac{u\, y \cos \phi}{F}$

$+ \frac{w\,h^3}{12} \sum \frac{u \cos \phi}{F} = -\,8{,}890 + 43.4 \times 71.9 = -\,5{,}770$. The substitution for the left-hand side (Assumption 3) is $\sum \frac{u^2}{F} = 2{,}168$. The M_v-correction for Assumption 2 is the same as shown in Table 6 for Assumption 1. With the aforementioned exceptions, all substitutions in the right-hand sides of Eqs. 5*a* for Assumptions 2 and 3 are the same as the corresponding substitutions in Eqs. 5*a*, for Assumption 1, as shown in Table 6.

TABLE 6.—SUBSTITUTIONS FOR THE RIGHT-HAND SIDE OF EQUATIONS ASSUMPTION 1

Loading	Expression	Computation	Substitution
(*a*) Eqs. 5*a*			
Dead load.......	$\epsilon \left(\sum \frac{M_s\, u \cos \phi}{F} - H \sum \frac{u\,y \cos \phi}{F}\right)$	1.376 (5,754 − 3.28 × 1,701)	240
Balanced earth pressure	$-\,\epsilon \left(\frac{w\,h^2}{2} - H\right) \sum \frac{u\,y \cos \phi}{F}$	− 1.376 (10.1 − 6.3) × 1,701	−8,890
Temperature rise	$\epsilon \left[\frac{E\,c\,t^\circ L}{K\,s} - (1 + \epsilon^2)\, H \sum \frac{u\,y \cos \phi}{F}\right]$	(see next line)	
		$1.376\, \frac{288{,}000 \times 0.0000065 \times 50 \times 38.4}{2.67 \times 6.4} - (1 + 1.376^2)\, 0.126 \times 1{,}700$ $= 1.376\,(210 - 620)$	−564
Live Load:			
0............	(Dead-load expression × cos θ)	1.376 (1,883 − 1.05 × 1,701) × 0.588	78.4
1R (balanced).	(Dead-load expression × cos θ)	1.376 (2,973 − 1.7 × 1,701) × 0.588	62.4
(*b*) Eqs. 5*b*			
Dead load......	$\epsilon \left(\sum \frac{M_s \cos^2 \phi}{F} - H \sum \frac{y \cos^2 \phi}{F}\right)$	1.376 (239.4 − 3.28 × 70.25)	12.38
Balanced earth pressure	$-\,\epsilon \left(\frac{w\,h^2}{2} - H\right) \sum \frac{y \cos^2 \phi}{F}$	− 1.376 × 3.8 × 70.25	−367
Temperature rise	$-\,\epsilon(1 + \epsilon^2)\, H \sum \frac{y \cos^2 \phi}{F}$	− (1.376 + 1.376³) 0.126 × 70.25	−35.2
Live Load:			
0............	(Dead-load expression × cos θ)	1.376 (783 − 1.05 × 70.25) × 0.588	3.67
1R (balanced).	(Dead-load expression × cos θ)	1.376 (123.6 − 1.7 × 70.25) × 0.588	3.38
M_v-correction C (Eq. 5*e*)	$\epsilon \sum \frac{M_s \sin \phi \cos \phi}{F} \div \sum \frac{\sin^2 \phi}{F}$	$\frac{1.376 \times (-\,0.94)}{0.588}$	−2.20

Note that the values assumed for the various constants *w*, *c*, *t*°, etc. are plainly indicated in the above substitutions.

The individual reactions, as given in Table 8, are to be used in calculating the values of M_v in Table 9; the summations will apply to the determination of soil pressures at the footings (see Fig. 6), with the following corrections: Deduct $\frac{w\,h^2}{2}$ (or 10.1) from the total for R_x, and $\epsilon \frac{w\,h^2}{2}$ (or 13.9) from the total for R_z,

TABLE 7.—SOLUTION OF EQUATIONS, FOR VARIOUS ASSUMED FOOTING CONDITIONS

Eq.	Left-hand sides	Dead load	Balanced earth pressure	Temperature rise	LIVE LOAD[a] 0	LIVE LOAD[a] 1R (balanced)	LIVE LOAD[a] 1R
	(a) ASSUMPTION 1, ROTATION ABOUT THE z-AXES ONLY						
5a	1,951 R'_z −71.9 M_z =	+240	−8,890	−564	+78.4	+62.4	....
5b	71.9 R'_z −2.97 M_z =	+ 12.38	−367	− 35.2	+ 3.67	+ 3.38	....
	Dividing through by the coefficients of R_z:						
5a	R'_z −0.0369 M_z =	+ 0.1230	− 4.56	− 0.289	+ 0.0402	+ 0.0320	....
5b	R'_z −0.0413 M_z =	+ 0.1722	− 5.10	− 0.490	+ 0.0510	+ 0.0470	....
5a–5b	0.0044 M_z =	− 0.0492	+ 0.54	+ 0.201	− 0.0108	− 0.0150	....
....	M_z =	− 11.18	+122.7	+ 45.7	− 2.45	− 3.41	−1.70
....	+0.0369 M_z =	− 0.413	+ 4.53	+ 1.686	− 0.0904	− 0.1260	....
....	Adding to Eq. 5a, R'_z =	− 0.290	− 0.03	+ 1.397	− 0.0502	− 0.0940	−0.047
5c	$M_y = -\frac{L}{2} R'_z$ $= -19.2 R'_z$ =	+ 5.57	+ 0.58	− 26.80	+ 0.964	+ 1.806	+0.903 (Av.)
	For Live Load[a] at 1R						
5d	M_{yR} =	+0.903 −2.20 = −1.297					
5d	M_{yL} =	+0.903 +2.20 = +3.103					
	(b) ASSUMPTION 2, ROTATION ABOUT x-AXES AND z-AXES						
5a	1,951 R'_z =	+240	−5,770	−564	+78.4	+62.4	....
....	R'_z =	+ 0.123	− 2.96	− 0.289	+ 0.0402	+ 0.032	+0.016
5c	$M_y = -19.2 R'_z$ =	− 2.36	+ 56.9	+ 5.55	− 0.772	− 0.614	−0.307 (Av.)
	For Live Load[a] at 1R:						
5d	M_{yR} =	−0.307 −2.20 = −2.507					
5d	M_{yL} =	−0.307 +2.20 = +1.893					
	(c) ASSUMPTION 3, ROTATION ABOUT x-AXES, y-AXES, AND z-AXES						
5a	2,168 R'_z =	+240	−5,770	−564	+78.4	+62.4	....
....	R'_z =	+ 0.111	− 2.66	− 0.260	+ 0.036	+ 0.029	+0.014

[a] Alternate locations of concentrated live loads shown in Fig. 5.

TABLE 8.—TABULATION OF REACTIONS

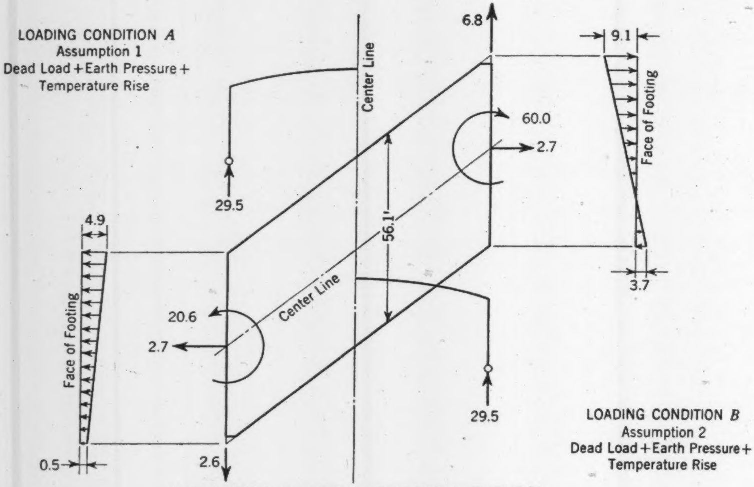
No.	Load	R_x	eR_x	ASSUMPTION 1				ASSUMPTION 2			ASSUMPTION 3	
				R'_z	M_{yR}	M_z	R_z	R'_z	M_{yR}	R_z	R'_z	R_z
1	Dead load.....	3.28	4.51	−0.290	+ 5.57	− 11.2	+ 4.22	+0.123	− 2.36	+4.63	+0.111	+4.62
2	Earth pressure.	3.80	5.23	−0.030	+ 0.58	+122.7	+ 5.20	−2.96	+56.9	+2.27	−2.66	+2.57
3	Total.......	7.08	9.74	−0.320	+ 6.15	111.5	+ 9.42	−2.84	54.5	+6.90	−2.55	+7.19
	Live Load:[a]											
4	0..........	0.62	0.85	−0.050	+ 0.96	− 2.5	+ 0.80	+0.04	− 0.77	+0.89	+0.04	+0.89
5	1R.........	0.50	0.69	−0.050	− 1.30	− 1.7	+ 0.64	+0.02	− 2.51	+0.71	+0.01	+0.70
6	1L.........	0.50	0.69	−0.050	+ 3.10	− 1.7	+ 0.64	+0.02	+ 1.89	+0.71	+0.01	+0.70
	Temperature:											
7	Rise Fall	±0.37	±0.50	±1.40	∓26.80	± 45.7	± 1.90	∓0.29	± 5.55	±0.21	∓0.26	±0.24
	Summations including Temperature:											
8	Rise........	7.45	....	....	−20.6	+157.2	+11.32	....	+60.0	+7.11	....	+7.43
9	Fall........	6.71	....	....	+33.0	+ 65.8	+ 7.52	....	+49.0	+6.69	....	+6.95

[a] Alternate locations of concentrated live loads in Fig. 5. Note that the live loadings (Items 4, 5, and 6) are not included in the summations (Items 8 and 9) since live loading has been neglected in the investigation of soil pressures (see Fig. 6).

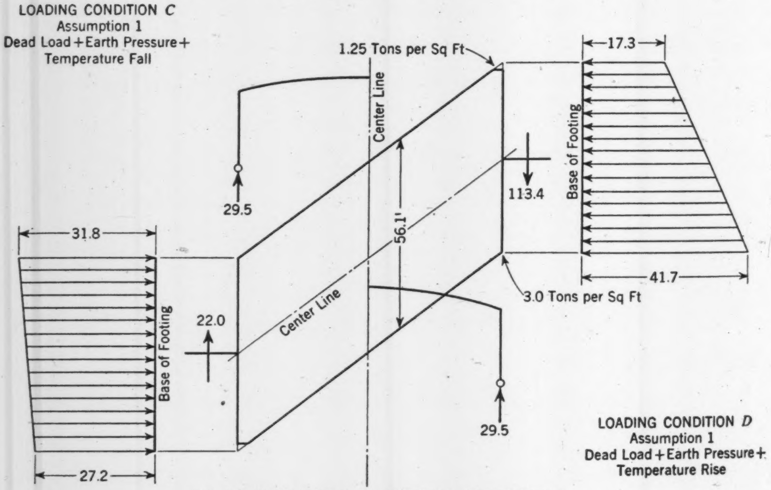
Note: Values of R_x for dead load, earth pressure, live load, and temperature rise are, respectively: H, $0.5\, w\, h^2 - H$, $H \cos \theta$, and $(1 + e^2)\, H$. The values of H are obtained from Part 1.

for each of the three assumptions; and deduct $\frac{w\,h^3}{12}$ (or 43.8) from the total for M_x, under Assumption 1.

The resulting foundation conditions are indicated graphically in Fig. 6, as plotted from information given in Table 8. It is evident from inspection that


LOADING CONDITION *A*
Assumption 1
Dead Load + Earth Pressure + Temperature Rise
6.8
9.1
Center Line
Face of Footing
60.0
2.7
29.5
56.1'
4.9
3.7
Center Line
20.6
2.7
Face of Footing
29.5
LOADING CONDITION *B*
Assumption 2
Dead Load + Earth Pressure + Temperature Rise
0.5
2.6

(*a*) TWISTING EFFECTS; HORIZONTAL THRUSTS


LOADING CONDITION *C*
Assumption 1
Dead Load + Earth Pressure + Temperature Fall
1.25 Tons per Sq Ft
17.3
Center Line
Base of Footing
113.4
29.5
31.8
56.1'
41.7
3.0 Tons per Sq Ft
Center Line
22.0
Base of Footing
29.5
LOADING CONDITION *D*
Assumption 1
Dead Load + Earth Pressure + Temperature Rise
27.2

(*b*) ROCKING EFFECTS; VERTICAL PRESSURES

FIG. 6.—FORCES DEVELOPED AT FOOTINGS (ALL THRUSTS AND PRESSURES ARE KIPS PER LINEAR FOOT OF FOOTING—EXCEPT AS NOTED—AND ALL MOMENTS ARE FOOT-KIPS PER LINEAR FOOT OF FOOTING)

complete fixity against rotation about the x-axis should not be depended on under ordinary conditions.

Attention is called to a departure from general practice in basing the calculations, in the example, on a 1-ft longitudinal design strip instead of the total

TABLE 9.—COMPUTATIONS FOR M_v

Formulas for M_v, under Assumptions 1 and 3, are given in this table. For Assumption 2, the formula is the same as for Assumption 3 except that the quantity $-M_y \sin\phi$ is added. For earth pressure loading, M_{yR} is relatively quite large (see Table 8), but since $\sin\phi$ is very small from point 0 to point 2R, and also since point 3R (because of the increased depth at this point) is not a critical design section, Assumption 2 is confined to points 4R to 6R, inclusive. Values of M_o are given in Table 5, and of R_z, R_s, M_z, and M_{yR}, in Table 8. $\epsilon = 1.376$ (see Table 2).

Load	ϵM_o	ASSUMPTION 1						ASSUMPTION 3				
		$R_z \epsilon x \sin\phi$	− $R_z u$	+ $M_z \cos\phi$	− $M_{yR} \sin\phi$	+ $M_o \epsilon \cos\phi$	= M_v	$R_z \epsilon x \sin\phi$	− $R_s u$	+ $\frac{1}{12} w h^3 \cos\phi$	+ $M_o \epsilon \cos\phi$	= M_v
(*a*) POINT 0: $x = 19.2$; SIN $\phi = 0$; COS $\phi = 1.00$; AND $u = 23.9$												
Dead load[a].....	118.9	....	−225.1	+111.5	....	+118.9	+ 5.3	....	−172.1	+43.8	+118.9	− 9.4
Live Load:[b]												
Point 0.....	26.2	....	− 19.1	− 2.5	....	+ 26.2	+ 4.6	....	− 21.2	....	+ 26.2	+ 5.0
Point 1.....	17.5	....	− 15.3	− 1.7	....	+ 17.5	+ 0.5	....	− 16.7	....	+ 17.5	+ 0.8
Temperature:												
Rise Fall	....	....	∓ 45.5	± 45.7	....	....	± 0.2	....	∓ 6.0	....	....	∓ 6.0
(*b*) POINT 1R: $x = 12.8$; SIN $\phi = -0.09$; COS $\phi = 0.99$; AND $u = 22.2$												
Dead load[a].....	109.3	−11.3	−209.1	+110.5	+ 0.6	+108.2	− 1.1	−11.3	−160.0	+43.4	+108.2	−19.7
Live Load:[b]												
Point 0.....	17.5	− 1.0	− 17.8	− 2.5	+ 0.1	+ 17.4	− 3.8	− 1.0	− 19.8	....	+ 17.4	− 3.4
Point 1R...	23.2	− 0.8	− 14.2	− 1.7	− 0.1	+ 23.0	+ 6.2	− 0.8	− 15.6	....	+ 23.0	+ 6.6
Point 1L...	11.6	− 0.8	− 14.2	− 1.7	+ 0.3	+ 11.5	− 4.9	− 0.8	− 15.6	....	+ 11.5	− 4.9
Temperature:												
Rise Fall	....	∓ 0.6	∓ 42.4	± 45.3	∓ 2.4	....	∓ 0.2	∓ 0.6	∓ 5.5	....	....	∓ 6.1
(*c*) POINT 2R: $x = 6.4$; SIN $\phi = -0.19$; COS $\phi = 0.98$; AND $u = 21.1$												
Dead load[a].....	76.6	−11.9	−199.0	+109.2	+ 1.2	+ 75.0	−25.5	−11.9	−152.0	+42.9	+ 75.0	−46.0
Live Load:[b].....												
Point 0.....	8.7	− 1.0	− 16.9	− 2.4	+ 0.2	+ 8.5	−11.6	− 1.0	− 18.7	....	+ 8.5	−11.2
Point 1R...	11.6	− 0.8	− 13.5	− 1.7	− 0.2	+ 11.4	− 4.8	− 0.8	− 14.8	....	+ 11.4	− 4.2
Point 1L...	5.7	− 0.8	− 13.5	− 1.7	+ 0.6	+ 5.6	− 9.8	− 0.8	− 14.8	....	+ 5.6	−10.0
Temperature:												
Rise Fall	....	∓ 0.6	∓ 40.3	± 44.7	∓ 5.1	....	∓ 1.3	∓ 0.6	∓ 5.1	....	....	∓ 5.7
(*d*) POINT 3R: $x = 0.0$; SIN $\phi = -0.71$; COS $\phi = 0.71$; AND $u = 15.0$												
Dead load[a].....	....	....	−141.3	+ 79.1	+ 4.4	....	−57.8	....	−107.8	+31.1	....	−76.7
Live Load:[b]												
Point 0.....	....	....	− 12.0	− 1.8	+ 0.7	....	−13.1	....	− 13.3	....	....	−13.3
Point 1R...	....	....	− 9.6	− 1.2	− 0.9	....	−11.7	....	− 10.5	....	....	−10.5
Point 1L...	....	....	− 9.6	− 1.2	+ 2.2	....	− 8.6	....	− 10.5	....	....	−10.5
Temperature:												
Rise Fall	....	....	∓ 28.7	± 32.5	∓ 19.1	....	∓ 15.2	....	∓ 3.8	....	....	∓ 3.8

(*e*) ASSUMPTION 2; POINTS 4R TO 6R, INCLUSIVE: $x = 0$; SIN $\phi = 0$; COS $\phi = -1.00$; AND $u = 0$ Evidently $M_v = -M_{yR} \sin\phi = M_{yR}$; and M_v can be taken directly from Table 8.

---

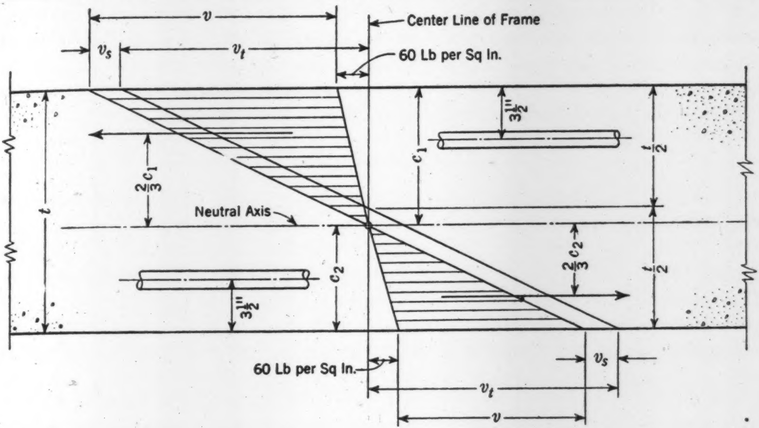
[a] Combined vertical load plus earth pressure. [b] Alternate locations of concentrated live loads in Fig. 5.

TABLE 10.—Transverse Torsional Moments[a] M_T and Shears[a] T_T

Loading	M_s	M_s tan V	T_v	T_v tan V	Assumption 1				Assumption 3			
					M_v	M_T	T_s	T_T	M_v	M_T	T_s	T_T
(*a*) Point 0: Tan V = 1.376												
Dead load plus earth pressure	+ 4.8	+ 6.6	+ 7.1	+ 9.8	+ 5.2	+ 1.4	+9.4	+0.4	− 9.5	+16.1	+7.2	+2.6
Live Load:												
Point 0	+ 4.5	+ 6.2	+ 0.6	+ 0.8	+ 4.6	+ 1.6	+0.8	+0.0	+ 5.0	+ 1.2	+0.9	−0.1
Point 1	+ 1.0	+ 1.4	+ 0.5	+ 0.7	+ 0.5	+ 0.9	+0.6	+0.1	+ 0.8	+ 0.6	+0.7	−0.0
Temperature:												
Rise Fall	∓ 8.7	∓12.0	± 0.3	± 0.4	± 0.3	∓12.3	±1.9	∓1.5	∓ 6.0	∓ 6.0	±0.2	±0.2
Summation	....	....	....	....	....	+15.3	....	+1.9	....	+23.3	....	+2.3
(*b*) Point 1R: Tan V = 1.362												
Dead load plus earth pressure	− 1.3	− 1.8	+ 7.1	+ 9.7	− 1.1	− 0.7	+9.4	+0.3	−19.7	+17.9	+7.2	+2.5
Live Load:												
Point 0	− 1.6	− 2.2	+ 0.6	+ 0.8	− 3.8	+ 1.6	+0.8	0.0	− 3.4	+ 1.2	+0.9	−0.1
Point 1R	+ 4.0	+ 5.4	+ 0.5	+ 0.7	+ 6.2	− 0.8	+0.6	+0.1	+ 6.6	− 1.2	+0.7	0.0
Point 1L	− 3.2	− 4.3	+ 0.5	+ 0.7	− 4.9	+ 0.6	+0.6	+0.1	− 4.9	+ 0.6	+0.7	0.0
Temperature:												
Rise Fall	∓ 8.7	∓11.8	± 0.3	± 0.4	∓ 0.1	∓11.7	±1.9	∓1.5	∓ 6.1	∓ 5.7	±0.2	±0.2
Summation	....	....	....	....	....	−13.2	....	−1.1	....	+24.8	....	+2.2
(*c*) Point 2R: Tan V = 1.348												
Dead load plus earth pressure	−21.5	−29.0	+ 7.1	+ 9.6	−25.5	− 3.5	+9.4	+0.2	−46.0	+17.0	+7.2	+2.4
Live Load:												
Point 0	− 7.6	−10.2	+ 0.6	+ 0.8	−11.6	+ 1.4	+0.8	0.0	−11.2	+ 1.0	+0.9	−0.1
Point 1R	− 2.8	− 3.8	+ 0.5	+ 0.7	− 4.8	+ 1.0	+0.6	+0.1	− 4.2	+ 0.4	+0.7	0.0
Point 1L	− 7.5	−10.1	+ 0.5	+ 0.7	− 9.8	− 0.3	+0.6	+0.1	−10.0	− 0.1	+0.7	0.0
Temperature:												
Rise Fall	∓ 8.4	∓11.3	± 0.3	± 0.4	∓ 1.3	∓10.0	±1.9	∓1.5	∓ 5.9	∓ 5.4	±0.2	±0.2
Summation	....	....	....	....	....	−13.8	....	−1.2	....	+23.4	....	+2.1
(*d*) Point 3R: Tan V = 0.977												
Dead load plus earth pressure	−62.0	−60.6	+13.4	+13.4	−57.8	− 2.8	+9.4	+4.0	−76.7	+16.1	+7.2	+6.2
Live Load:												
Point 0	−13.0	−12.7	+ 1.2	+ 1.2	−13.1	+ 0.4	+0.8	+0.4	−13.3	+ 0.6	+0.9	+0.3
Point 1R	−10.6	−10.3	+ 1.4	+ 1.4	−11.7	+ 1.4	+0.6	+0.8	−10.5	+ 0.2	+0.7	+0.7
Point 1L	−10.6	−10.3	+ 1.4	+ 1.4	− 8.6	− 1.7	+0.6	+0.8	−10.5	+ 0.2	+0.7	+0.7
Temperature:												
Rise Fall	∓ 7.8	∓ 7.6	± 0.3	± 0.3	∓15.2	± 7.6	±1.9	∓1.6	∓ 3.8	∓ 3.8	±0.2	±0.1
Summation	....	....	....	....	....	−12.1	....	+6.4	....	+20.5	....	+6.4
(*e*) Points 4R, 5R, and 6R: Tan V = 0					Assumption 1				Assumption 2			
Dead load plus earth pressure	0	0	0	0	+ 6.2	− 6.2	+9.4	−9.4	+54.5	−54.5	+6.9	−6.9
Live Load:												
Point 0	0	0	0	0	+ 1.0	− 1.0	+0.8	−0.8	− 0.8	+ 0.8	+0.9	−0.9
Point 1R	0	0	0	0	− 1.3	+ 1.3	+0.6	−0.6	− 2.5	+ 2.5	+0.7	−0.7
Point 1L	0	0	0	0	+ 3.1	− 3.1	+0.6	−0.6	+ 1.9	− 1.9	+0.7	−0.7
Temperature:												
Rise Fall	0	0	0	0	∓26.8	±26.8	±1.9	∓1.9	± 5.6	∓ 5.6	±0.2	∓0.2
Summation	....	....	....	....	....	−36.1	....	−8.1	....	−62.0	....	−7.8

[a] M_T and T_T (see Fig. 3) are equal, respectively, to: M_s tan $V - M_v$; and, T_v tan $V - T_s$. Values of M_s and T_v are given in Table 3, M_v in Table 9, and $T_s = R_s$ in Table 8.

TABLE 11.—RESULTANT UNIT SHEARS AND TRANSVERSE REINFORCEMENT



Referring to the diagram—

$$v_t = \frac{4.5\, M_T}{12\, t^2} \quad \text{I}$$

$$v_s = \frac{1.5\, T_T}{12\, t} \quad \text{II}$$

and

$$v = (v_t \pm v_s) - 60 \quad \text{III}$$

Furthermore—

$$c_1 = \frac{v_t + v_s}{2\, v_t} t \quad \text{IV}$$

$$A_s = \frac{v\, c^2_1}{4{,}500\,(c_1 - 3.5)} \quad \text{V}$$

and

$$A_s = \frac{v\, c_1}{3{,}000} \quad \text{VI}$$

The factor b, which should appear in the denominators of Formulas I and II, is not shown since it is equal to unity. c_1 (see Formula IV) locates the neutral axis, and is the greater of the two c distances, whether measured from the intrados or from the extrados. Formula V equates $A_s f_s$ to the total shear, as represented by the larger shear triangle in the diagram. Formula VI equates the moment $A_s f_s$ about the neutral axis to the moment of the larger shear triangle about the neutral axis. Formulas V and VI are based on $f_s = 18{,}000$ lb per sq in.; and the transverse steel is designed from the greater of the two values of A_s.

Point	Torsional Shear					Direct Shear		Total		Steel Reinforcement		
	M_T		t (in.)	t^2 (sq in.)	v_t (lb per sq in.)	T_T (kips per ft)	v_s (lb per sq in.)	v (Lb per Sq In.)		c_1 (in.)	A_s (sq in. per ft)	
	Kip-ft per ft	Kip-in. per ft						Extrados	Intrados			
Equations					I		II	III	III	IV	V	VI
0	+23.3	279.6	16	256	410	+2.3	18.0	368	332	8.4	1.18	1.03
1	+24.8	297.6	21.5	462	241	+2.2	12.8	194	168	11.3	0.71	0.73
2	+23.4	280.8	39.6	1,570	67	+2.1	6.6	14	....	21.9	0.08	0.10
3	+20.5	246.0	60	3,600	26	+6.4	13.3	....	....	....	....	....
4	−62.0	744.0	52.2	2,730	102	−7.8	18.7	61	23	31.0	0.47	0.63
5	−62.0	744.0	40.2	1,620	172	−7.8	24.3	136	88	23.0	0.83	1.04
6	−62.0	744.0	36.0	1,296	215	−7.8	27.1	182	128	20.3	0.99	1.25

width of the bridge. It should be understood, of course, that the bridge is considered as a whole in this part of the analysis and design. It is possible to use a 1-ft strip only because it just happens that the total width "b" enters into the expression for the factor of torsion, F (under the heading "Geometrical Relations and Definitions"), and into the torsion and shear formulas (Table 11), as a linear quantity. In such case, it is a practical waste of time to introduce "b" into the calculations, since it automatically cancels itself.

CONCLUSIONS

(1) There is no justification for avoiding the use of solid-barreled skewed rigid frame bridges on the basis of difficulties, delays, or uncertainties in design. Parts 1 and 2 of this paper (including the analysis and design for the transformed longitudinal forces), taken together, represent very little more work than that necessary for the design of any ordinary rectangular rigid frame. Part 3 (the analysis and design for the transformed transverse forces) has been developed at some length in the example to illustrate the comparative effects of various loading conditions and various foundation assumptions, but the amount of work involved (a large proportion of which, it will be found, can be omitted in practice) is still considerably less than that formerly required and a great deal easier to follow.

In Part 3, it will be noted that Assumption 1 (footings fixed against rotation about the x-axes and y-axes) leads to transverse torsional moments amounting, in the example, to less than two thirds, at any point, of the torsional moments obtained by either one of the other two assumptions. Although there is reason to believe that this will be true for any skewed frame, it is impossible, at present, to make any such general inference. With accumulating experience, however, it should be possible to eliminate Assumption 1, in at least the majority of cases, which will mean a substantial further reduction in the amount of work required. The example also illustrates the practically negligible effects of live loading. In practice, it ordinarily can be omitted safely, except as a small plus-or-minus percentage (say ± 10%) of the combined dead load and earth pressure. This still further reduces the work necessary for Part 3.

Admitting that various uncertainties exist in skewed arch or frame design (notably the uncertainty as to the correctness of any of the various versions of the basic torsion formula underlying all methods of internal stress distribution), it should be remembered that they apply, for the most part, to Part 3, which is after all a minor part of the design. Attention is called again to the fact that Parts 1 and 2, which control the proportioning of the structure, are independent of Part 3, and of any assumption that has been made therein.

The method of design described for Part 3 is based on the practical method of internal stress distribution originally proposed by Professor Rathbun. It is believed to be well within the limits of justifiable precision. Since Part 3 is of secondary importance and the transverse reinforcement amounts to relatively little in any case, and in view of the uncertainties previously referred to, any greater degree of precision would seem to be both useless and misleading for all practical purposes.

(2) The customary assumption that the footings will be fully restrained against rotation about the x-axes and y-axes is unaccompanied usually by any attempt to make certain, either in the design of the structure or in the details of the footings, that such restraint can be accomplished. It is evident from Fig. 6 that rotation about the x-axes (in the case of the bridge in the example) would be difficult to prevent, in any case. The proposed practice of designing skewed frames to meet either or both of the extreme conditions of complete restraint and free rotation about the x-axes and y-axes meets any condition, and furnishes security against any degree of rotation or restraint that can possibly result. The cost of the small amount of additional transverse reinforcement necessary for this purpose is a small price to pay for the added security obtained.

The practice of designing skewed frames on an assumption of free rotation about the z-axes is based on the well-established practice of designing rectangular frames on a similar assumption. There seems to be no reason why the assumption should not be as applicable in one case as in the other.

(3) The maximum degree of skew for which a frame can be designed successfully by the method here adopted is controlled, in any particular case, by the effects of temperature drop on the crown section under Part 2 (longitudinal forces). The temperature effects increase in the ratio of the fourth power of the secant of the skew angle. They also increase with the strength and stiffness of the concrete used in construction.

(4) It should be noted that the width of the structure, b, does not enter into the analysis or design, except indirectly as a factor in the intensity of live loading to be considered. The width-span ratio, therefore, has little effect on the results, and, according to the method of design here employed, has no more effect on the strength and stability of a skewed frame than it has in the case of an ordinary rectangular frame. Consequently (contrary to the popular belief), it makes no difference whether or not any part of one abutment can be projected, perpendicular to itself, upon the other.

(5) Such tests as have been made in connection with skewed arches and frames have dealt mostly with the effects of live loading in various positions. As previously noted, the effects of live loading are subordinate to temperature effects in Part 2, and are almost negligible in comparison with the effects of earth pressure in Part 3. Therefore, it would be more to the point (at least as far as the ordinary type of skewed frame is concerned) to test the effects of temperature fall and the distribution and intensity of earth pressure forces, as well as the amounts of rotation, in the various directions, that can be expected at the footings under ordinary soil conditions.

ACKNOWLEDGMENTS

The writer gratefully acknowledges the encouragement and assistance of Mr. Hayden in making this paper possible, and the helpful suggestions of Phillips H. Lovering, Assoc. M. Am. Soc. C. E., who checked parts of the manuscript. Permission was granted by Gilmore D. Clarke, M. Am. Soc. C. E., to make use of the design calculations for a grade separation access structure constructed by the Westchester Cross County Parkway Authority. The

skewed arch equations given in the Appendix, together with certain excerpts noted in the body of the paper, have been included by permission of Mr. Hayden and of John Wiley and Sons, Inc. Valuable improvements were pointed out by Norman D. Richardson, M. Am. Soc. C. E.

APPENDIX

DEDUCTIONS FROM SKEWED ARCH EQUATIONS AS PUBLISHED BY ARTHUR G. HAYDEN[3]

The equations presented by Mr. Hayden[3] are:

$$R_x\left(\epsilon^2 K \sum \frac{x^2 \sin^2 \phi}{F} + \sum \frac{y^2}{I}\right) - R_z\, \epsilon K \sum \frac{u\, x \sin \phi}{F}$$

$$+ M_x\, \epsilon K \sum \frac{x \sin \phi \cos \phi}{F} - M_y\, \epsilon K \sum \frac{x \sin^2 \phi}{F}$$

$$= -\epsilon^2 K \sum \frac{M_o \sin \phi \cos \phi\, x}{F} + \sum \frac{M_o\, y}{I} \ldots\ldots\ldots\ldots (7a)$$

$$R_x\, \epsilon \sum \frac{u\, x \sin \phi}{F} - R_z \sum \frac{u^2}{F} + M_x \sum \frac{u \cos \phi}{F}$$

$$- M_y \sum \frac{u \sin \phi}{F} = -\epsilon \sum \frac{M_o\, u \cos \phi}{F} \ldots\ldots\ldots\ldots (7b)$$

$$R_x\, \epsilon \sum \frac{x \sin \phi \cos \phi}{F} - R_z \sum \frac{u \cos \phi}{F} + M_x \sum \frac{\cos^2 \phi}{F}$$

$$- M_y \sum \frac{\sin \phi \cos \phi}{F} = -\epsilon \sum \frac{M_o \cos^2 \phi}{F} \ldots\ldots\ldots\ldots (7c)$$

and

$$R_x\, \epsilon \sum \frac{x \sin^2 \phi}{F} - R_z \sum \frac{u \sin \phi}{F} + M_x \sum \frac{\sin \phi \cos \phi}{F}$$

$$- M_y \sum \frac{\sin^2 \phi}{F} = -\epsilon \sum \frac{M_o \sin \phi \cos \phi}{F} \ldots\ldots\ldots\ldots (7d)$$

To use these equations for temperature reactions substitute the following quantities for the right-hand members of Eqs. 7: Eq. 7*a*, $+\frac{E\, c\, t^\circ L}{s}$; Eq. 7*b*, $-\frac{E\, \epsilon\, c\, t^\circ L}{K\, s}$; Eq. 7*c*, 0; and Eq. 7*d*, 0. For the sake of simplicity, the substitutions for horizontal earth-pressure loading have not been included.

Substituting, in Eqs. 7, the value

$$u = x \sin \phi + y \cos \phi \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

the following results are obtained:

$$R_x\left(\epsilon^2 K \sum \frac{x^2 \sin^2 \phi}{F} + \sum \frac{y^2}{I}\right) - R_z\, \epsilon\, K \left(\sum \frac{x^2 \sin^2 \phi}{F}\right.$$

$$\left. + \sum \frac{x\, y \sin \phi \cos \phi}{F}\right) + M_x\, \epsilon\, K \sum \frac{x \sin \phi \cos \phi}{F}$$

$$- M_y\, \epsilon\, K \sum \frac{x \sin^2 \phi}{F} = - \epsilon^2 K \sum \frac{M_o \sin \phi \cos \phi\, x}{F} + \sum \frac{M_o\, y}{I} \ldots (9a)$$

$$R_x\, \epsilon \left(\sum \frac{x^2 \sin^2 \phi}{F} + \sum \frac{x\, y \sin \phi \cos \phi}{F}\right)$$

$$- R_z \left(\sum \frac{x^2 \sin^2 \phi}{F} + \sum \frac{y^2 \cos^2 \phi}{F} + 2 \sum \frac{x\, y \sin \phi \cos \phi}{F}\right)$$

$$+ M_x \left(\sum \frac{x \sin \phi \cos \phi}{F} + \sum \frac{y \cos^2 \phi}{F}\right) - M_y \left(\sum \frac{x \sin^2 \phi}{F}\right)$$

$$= - \epsilon \left(\sum \frac{M_o\, x \sin \phi \cos \phi}{F} + \sum \frac{M_o\, y \cos^2 \phi}{F}\right) \ldots\ldots\ldots (9b)$$

$$R_x\, \epsilon \sum \frac{x \sin \phi \cos \phi}{F} - R_z \left(\sum \frac{x \sin \phi \cos \phi}{F} + \sum \frac{y \cos^2 \phi}{F}\right)$$

$$+ M_x \sum \frac{\cos^2 \phi}{F} = - \epsilon \sum \frac{M_o \cos^2 \phi}{F} \ldots\ldots\ldots\ldots\ldots (9c)$$

and

$$R_x\, \epsilon \sum \frac{x \sin^2 \phi}{F} - R_z \sum \frac{x \sin^2 \phi}{F} - M_y \sum \frac{\sin^2 \phi}{F} = - \epsilon \sum \frac{M_o \sin \phi \cos \phi}{F} . (9d)$$

(The right-hand sides of Eqs. 9 for temperature are the same as in Eqs. 7. The summations $\sum \frac{\sin \phi \cos \phi}{F}$ and $\sum \frac{y \sin \phi \cos \phi}{F}$ are each equal to zero and have been omitted.)

Multiply each term of Eq. 9*c* by y, making the assumption that an average value of y can be introduced as a constant quantity under the summation signs, and subtract Eq. 9*c*, as thus modified, from Eq. 9*b*. Then multiply the resulting equation by $\epsilon\, K$ and subtract from Eq. 9*a*. The formulas for R_x are obtained directly, as follows: For applied loading—

$$R_x = \sum \frac{M_o\, y}{I} \div \sum \frac{y^2}{I} \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

and, for temperature—

$$R_x = (1 + \epsilon^2) \frac{E\, c\, t^\circ\, L}{I} \div \sum \frac{y^2}{I} \ldots\ldots\ldots\ldots\ldots (10b)$$

Therefore, $R_x = H$ and $(1 + \epsilon^2)\, H$, respectively, for the corresponding rectangular frame.

DISCUSSION

C. D. Geisler,[7] Esq.—The subject of this paper long has been more or less of a nightmare to structural engineers. When confronted with the task of designing a skewed frame or arch, the designer has been obliged to wrestle with intricate mathematical equations.

The writer favors Mr. Hodges' simple method of procedure, in which first the frame is analyzed as a rectangular structure in computing the horizontal reaction H, and then corrections are made for the varying effects of the skew at the different points along the arch axis. It is surprising, however, that the torsional moments, which determine the amount of transverse reinforcement, have no effect in concentrating the dead load reactions and the live load reactions at the outer edges of the frame, thus increasing the main stresses at the obtuse corner.

Mr. Hodges' analysis of the main stresses would appear to agree with the theory advanced by Rankine and other early authorities who assumed that the stresses in a skewed arch were similar to those in a symmetrical right arch of span equal to the span on the skew.

The presence of certain concentrated stresses at the corners of a skewed barrel structure long has appeared evident from various model tests, and also from the results of tests conducted from 1922 to 1925 by the Bureau of Public Roads[8] on model concrete arches, with skews from 15° to 60°. Investigations of actual failures of concrete arches likewise have led to this conclusion. However, many concrete rigid frames of considerable skew have been constructed during the fifteen years since about 1928, apparently without added allowance for concentration of stress at the obtuse corners, and no reports have come to the writer's attention of any indication of distress in the skewed barrel frames at or near these corners.

It is somewhat doubtful if extreme concentrations of reactions occur to the same degree in the rigid frame type of bridge as they apparently do in arches, since structures of the frame type generally are not founded on rock and any slight yielding of the soil under the loaded corners would cause the resultant of the reactions to shift toward the center of the abutment with a reduction rather than an increase in the critical stresses. On the other hand, a lateral movement of the abutments due to side thrust would tend again to increase the concentration of horizontal reactions at the obtuse corner.

Reports of failures in skewed barrel structures appear to be confined generally to true arches, which have a greater horizontal thrust, and depend for their stability on an unyielding foundation. A relatively thin section for the arch barrel above the springing lines will satisfy design requirements due to the absence of the high bending stresses which exist in a flat-top frame.

---

[7] Senior Highway Bridge Engr., Public Roads Administration, Federal Works Agency, Washington, D. C.

[8] "Progress Report on Skew Arch Tests," by G. W. Davis, *Public Roads*, Bureau of Public Roads, U.S.D.A., Vol. 6, No. 9, November, 1925, p. 185.

Since 1933 some twenty skewed concrete frames and arches (most of which have been constructed on National Park projects) have been designed under the writer's direction. These structures comprise a variety of barrel-type arches ranging from flat-top frames to circular and elliptical arches, with spans up to 114 ft and skews up to 45°. In these designs the final stresses were determined after calculating the eccentricity of reactions, based on the relative stiffness of elements radiating from the superimposed load to the abutments and involving the angle of skew, span length, and width. The frame or arch barrels were divided into longitudinal sections or strips usually from about 10 ft to 12 ft in skew width, and these extended without transverse construction joints from abutment to abutment. A 10-ft width was found to be about the minimum for practical construction operations. In the computations, the individual section was designed as an independent arch rib, and the stress analysis of the structure was based on the investigation of this rib section. The analysis involved the assumption that each section would deflect independently of the adjacent sections and that no load would be transferred across the longitudinal joints. Inasmuch as the concentration of reactions at the obtuse corners depends on the effective width of the structure, if the barrel is constructed in independent longitudinal sections, each section must transmit its own load to the abutments in much the same way as a timber roadway plank laid diagonally carries its load to the stringers. However, in order to provide lateral distribution for heavy concentrated live loads and to prevent deflections which might break up the roadway pavement in cases where a thin roadway surfacing lay directly on the frame, the sections were keyed together for a length of a few feet at the crown of the arch. Furthermore, to tie the structure transversely, particularly in structures retaining a spandrel fill, transverse reinforcement extended over the full width of a structure, crossing the joints between successive sections. To what extent the keys at the crown and the transverse reinforcement might act to upset the assumption of independent deflection of each strip is questionable, of course, but it is doubtful that the small keying action so furnished would be sufficient to cause deflections in the successive concrete sections which could produce serious overstressing in the outer edges.

The usual computations for reactions in a statically indeterminate reinforced concrete structure are based on relative deflections in which the concrete is assumed to be a homogeneous material in both tension and compression. This would be more nearly correct in an arch than in a frame, since, in a true arch, the line of resultant thrust departs but little from the middle third when only small tension stresses occur. In the computations for deflection of a reinforced concrete barrel frame in a diagonal direction, or along an element not parallel with the reinforcement on which the tension steel can act only directly, considerable error may be involved. An element crossing the reinforcing steel diagonally would likely develop tension cracks under flexural stress, resulting in a reduction in its effective moment of inertia. Its capacity as a beam for transmitting any considerable amount of load to the obtuse corners then would be decreased.

It is quite conceivable that in many existing skewed barrel structures local concentration of forces, if this occurs, could produce overstresses of as much as 50% without giving any indication of distress, particularly in a structure that otherwise is designed conservatively for moments and side thrusts.

Tentative conclusions drawn in 1925 at the completion of the foregoing tests made on models of true arches by the Bureau of Public Roads were as follows:

1. The vertical reactions varied uniformly from a maximum at the obtuse angle of the arch to a minimum at the acute angle.
2. The increase of the vertical reaction at the obtuse angle of the arch varies directly as the angle of skew.
3. The resultant pressures of both the horizontal and vertical components move toward the obtuse angle of the arch with an increase in the skew.
4. The direction of the side thrust is toward the face of the arch at the acute angle and the intensity increases with the degree of skew.

The foregoing conclusions could be interpreted to indicate that the reactions from a superimposed load on a span have a tendency to follow the shortest distance to the abutments. In a beam and girder bridge, the reactions are determined readily by the relative stiffness of the supporting members. In a slab or barrel arch structure, however, a close determination of the reaction distribution is more involved. Fortunately, in the latter structures a concentration of load is seldom serious.

ARTHUR G. HAYDEN,[9] M. AM. SOC. C. E.—The development of analysis for skewed arches and rigid frame bridges has been a subject of intimate interest to the writer from the beginning. Therefore, he can appreciate the value of Mr. Hodges' contribution to the literature on the subject.

Historical Perspective.—In 1920 the writer was assigned the duty of revising plans for an arch bridge on a very heavy skew (more than 50°) and sensed that something was wrong with the design. Undoubtedly, if the bridge had been built according to the original plans, the ends of the arch ring proper would have slipped along the very smooth skewbacks of the abutments which had already been built, and a collapse would have followed. There was comparatively little steel reinforcement projecting from the abutments for anchoring the arch ring thereto. A thorough search was made in technical literature, and it was discovered that several attempts had been made by engineer-mathematicians to solve the problem of the skew arch; but all of the analyses discovered were fallacious. All that the writer could do was to saw-tooth the skewbacks of the abutments already built to prevent slippage of the arch ring when built thereon. An accurate analysis of the stresses throughout such a structure seemed impossible, and the design of several skewed bridges on the parkways of Westchester County was in prospect. Happily, a paper by Professor Rathbun appeared in the February, 1924, *Proceedings* of the Society, giving a solution of the fixed skew arch problem.[3] This paper has proved to be epoch-making in the field of structural analysis. Unfortunately, the validity of Professor Rathbun's theory was questioned at first, and a practical test of the theory was authorized by the Society and assigned to a Special Committee on Concrete and Reinforced Concrete Arches.[10] The late George E. Beggs, M. Am. Soc. C. E., of Princeton University in Princeton, N. J., was engaged to make a deformeter analysis on models of skewed arches and Professor Rathbun, then of the South Dakota School of Mines, in Rapid City, was engaged to make mathematical analyses for structures of the same proportions. Professors Beggs and Rathbun were not in communication with each other during the progress of the work, but, when both parties had completed their assignments, results were compared and were found to be in very close agreement. This was accepted as a verification of Professor Rathbun's theory.

---

[9] Cons. Civ. Engr., New York, N. Y.; formerly Designing Engr., Westchester County Park Comm.

In 1925 Professor Rathbun was engaged to extend his theory of the skewed fixed arch to skewed rigid frame bridges, because the Bronx Parkway Commission and its successor, the Westchester County Park Commission, was faced with the design of a number of bridges of this type for grade separations on Westchester County parkways. Professor Rathbun's work was accepted, but, in order to make it more useful in the designing room, an adaptation was made.[11] Apparently even this "simplification," involving as it does the derivation and solution of four simultaneous equations, was not suitable for use by the average designer, and, because of this, many bridges have been built on poor alinement, or unsatisfactory types of structure have been used in order to circumvent the analysis of a skewed arch or frame bridge.

Early in 1942 Mr. Hodges and the writer were engaged on the analysis of a heavily skewed rigid frame bridge for the highway network near the new War Department Building at Washington, D. C. The writer saw that Mr. Hodges was near the goal of a genuine simplification, and urged him to prepare the paper now under discussion. The value of the work can be appreciated when one considers that the solution of four simultaneous equations is now reduced to the solution of two. Moreover, the author makes the subject much more lucid.

Observations and Suggestions.—Mr. Hodges assumes, to begin with, a "residual" or "transformed" system in which all redundant reactions have been removed except the horizontal thrust as calculated perpendicular to the abutments. He then analyzes the structure on the square span and derives the remaining redundant reactions. It appears that he could as well have taken a residual system in which all redundant reactions have been removed except the horizontal thrust as calculated parallel with the skew. Stresses would be analyzed on the skew span and the remaining redundants derived. Subsequent transformation of stresses in the square direction to stresses in the skew direction would be avoided. Although the same fundamental principles would be involved, the redundants would be derived differently. The final total reactions, however, would be the same. It has been shown that, in general, any one of a number of "transformed" or "residual" systems may be selected as a basis for deriving redundant reactions, the number of possible residual systems depending upon the number of redundants to be calculated.[12] Some of these systems involve much less mathematical work in solution than others. So, in the case of the skewed frame bridge, one of the residual systems may involve less work in solving for the redundant reactions due to the effect of skew than the other system. Such a comparison might yield valuable results.

---

[10] "Concrete and Reinforced Concrete Arches," Final Rept. of the Special Committee, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 1551–1573.

[11] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed., Chapter X.

A common misconception in regard to skewed arches or frame bridges is that the effect of skew on the stresses can be reduced by the use of T-beam construction. Unless the floor slab is distinctly separated from the ribs so that the two elements do not act as T-beams, skew stresses will exist in the structure to full extent and should be accounted for in design. As a matter of fact, Professor Rathbun has warned[13] that the problem is made even more complicated by the use of T-beam construction. If all the ribs did act independently, the eccentric loading from earth pressure at the ends of the bridge would become troublesome.

BERNARD L. WEINER,[14] M. AM. SOC. C. E.—The design of any structure, including that of a skew arch, is normally divided into two parts: (1) The computation of the reactions, and (2) the design of the sections to resist the internal stresses. Although this is the usual procedure, the two steps are not as separate and distinct as they are generally assumed to be. The major part of the paper is devoted to the first step, but the second step is covered sufficiently to indicate the general method of attack. Each step will be discussed separately and in the order given.

Step I. Reactions.—The purpose of this part of the paper is to show that permissible approximations may be made in the frame constants with the result that the equations developed on the basis of the accepted theory (proved by tests) are simplified. In addition, the equations break up into groups, each of course containing fewer unknowns. That permissible approximations usually result in simplifications is, of course, obvious; nevertheless, the results are approximate and the paper has been misnamed. Neither proof nor data are given for the degree of approximation.

The author states that certain reactions are affected but slightly by the skew and can be computed from a right arch of the same square span and with the same arch constants. Once this assumption is made, the so-called "skew stresses or reactions" can be computed independently. The author states further that, for a flat-topped bridge, these properties can be proved mathematically. This has been known for some fifteen years. In fact, all that is necessary is that the members have straight center lines which are either vertical or horizontal; the individual members may have variable moments of inertia and the strict mathematical proof is still valid. The shape of such a structure is not a practical one, but, for the purpose of studying the permissible approximations, this fact is of no importance.

---

[12] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed., Chapter IX.

[13] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1471.

[14] Designer, Parsons, Brinckerhoff, Hogan & Macdonald, New York, N. Y.

Many years ago the writer found that for a symmetrical structure the approximations were permissible and result in the simplifications stated. In 1931 the writer gave a complete mathematical derivation of the simplified approximate equations applying to the symmetrical rigid frame with straight center lines and with variable moments of inertia.[6] For practical structures, the approximations were shown to be negligible; influence lines obtained by the use of the approximate equations were plotted on the same chart with the corresponding curves from the more accurate analysis.[15] Two sets of curves were given—one for a 32° skew and another for a 50° skew. The square spans were 56 ft and 41 ft, respectively.

The question still remained, at that time, whether the approximate analysis was valid for unsymmetrical rigid frames and for single-span structures with sharply curved soffits. There was also no information available whether the approximations were valid for multiple-span bridges. From his own experience in the design of numerous bridges of various shapes and skews, and of one, two, and three spans, the writer has found that good approximations may be made with great reduction in the amount of numerical work involved. An experienced designer will soon find that he can make his own approximations very easily for any specific case provided he has a good basis from which to begin.

A word of caution is necessary, however. Since bridge design should be regarded as a science, an experienced and conscientious structural engineer should get a thorough understanding of the action of skew structures by completing at least one accurate analysis. He will then get the "feel" of the values involved, and will be able to find permissible approximations of his own which he can use with confidence. The design of skew bridges is not as difficult as seems to be assumed—the real difficulty, if any, is the solid geometry involved rather than the structural theory.

There is still much useful work to be done on a real and complete simplification of the skew structure. It is the writer's opinion that the governing stresses—those that exist in modified form in a right arch—may be computed in all cases by simplified equations regardless of shape and of the number of spans. The skew stresses, however, may not only be computed from approximate equations, but, since they are of lesser importance, it is the writer's belief that curves can be developed from which they could be estimated with sufficient accuracy for design purposes. The reason why this possibility suggests itself will be discussed more fully under Step II, "Stress Distribution."

The complete simplification of the skew structure is an urgent necessity, for modern highway speeds make good alinements obligatory. However, there is no excuse for distorting the alinement of a highway in order to avoid a skew bridge because the cost of even the most tedious "exact" design is less than the cost in human lives lost on sharp highway curves. As a matter of fact, the difficulties which the author and others deplore are largely exaggerated. With an orderly arrangement of the equations and with a simple system of signs, the work involved is only a small part of the total man-hours necessary to prepare a complete set of plans. More time has been spent attempting to avoid work, very often, then the work itself would require. Since short cuts seem so desirable, however, they may be had practically for the asking.

---

[15] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1286.

Much confusion indeed has been caused by the constant search for new "methods." The writer takes issue with the author's contention that his "method" is more simple than other "methods," whether approximate or exact. All the different methods are really not different at all; it is a misuse of the English language to refer to "different methods." The differences consist of such matters as the choice of the origin of coordinates, the "basic structure" chosen for the analysis, and in some numerical short cuts which are usually present in all numerical work.

Obviously, one should choose an origin of coordinates and a basic structure which will reduce the resulting equations to the simplest form. The writer has given complete equations[6] for both the symmetrical and unsymmetrical skew arch (single span) with the origin and basic structure chosen with this idea in mind. There should be no more question about this than there is, normally, about writing $x^2 + y^2 = r^2$ rather than $(x + a)^2 + (y + b)^2 = r^2$ for the equation of the circle. Most of the variations in the different methods consist of differences as profound as that in the two equations of the circle. Furthermore, experienced designers will soon find that, when they are doing the numerical computations themselves, they can apply such arithmetic short cuts as elastic loads to the skew arch as well as to the right arch.

The writer has found from long experience in the drafting room that it is more efficient to use a formal set of equations so that the computers need do nothing more than substitute in formulas. Even the experienced designer will find that, unless a formal setup is used, he will constantly be "thinking it out," which is not only time consuming but is also a constant source of error. When deadlines have to be met and with only juniors available to do the computing, one experience will convince a supervising designer that the proper and formal setup will save him both time and grief.

Furthermore, having well setup equations, it becomes a simple matter to make the angle $\phi = 0°$ and $90°$ and thus derive approximations of his own without going through a complete derivation each time. At the same time he has the more accurate equations available for checking the results of his approximations. Although this is true (using a formal set of equations) for the single span, it is an absolute necessity for multiple spans.

The question of signs is important in all structural analysis, but it is doubly important in skew-arch work. The author does not give a formal statement as to the directions of the positive moments and thrusts. A "picture" is not sufficient, for again experience shows that there is too much room for misinterpretation. In right arches of single span, the moments and thrusts scarcely need a sign system, but even in this case there is confusion about the shears. In a multiple-span and multiple-story analysis, a sign system stated in words is mandatory if error and confusion are to be avoided. It is equally mandatory in skew-arch work.

The writer has found that a system of signs based on the structure rather than a "space system" is very much easier to remember and to apply. In fact, it was partly because the original skew-arch theory was written on the basis of a space system that it had to be revised for efficient use in the drafting room, and this revision constitutes Part I of the writer's paper[16]—and is so stated therein. This system of signs (based on the structure) has been used by the writer for years in his subsequent work and the test in the drafting room has given the verdict. For example, in a space system, if the transverse shear, T_z or R_z, is defined as positive on the left section going away from the observer, it will reverse itself and be of opposite sense on a right section, and reverse again for a right-hand skew if the original definition is for a left-hand skew. As it happens, this shear almost always points toward the obtuse angle and, if it is defined as positive in this direction, the sign sense will not reverse either with the change in section or with the skew for the change from right to left. Such a system based on the structure is as much simpler than a space system as the ordinary convention of positive and negative moment in a horizontal beam is more simple than the clockwise and counterclockwise convention. The signs of the other forces and of the moments also are simplified similarly if the signs are based on the structure. It is a detail, but one of great importance both as a timesaver and as a way of avoiding errors. Perhaps the lack of a simple and easily applied sign system accounts for some of the confusion and difficulty which the author mentions.

The question of the assumption as to the degree of fixity of the footings is not unique with the skew arch. Actually, the footings are assumed neither fixed nor hinged; they are really designed for both conditions—for complete fixity and for free rotation about the z-axis. The theory is that, if the design is adequate for the two limits, it will also be sufficient for the actual condition of partial fixity. It was found, however, that either condition gives practically the same results except at the bottom of the "leg" or post, and even here there is sufficient dowel steel available, as a matter of practical detailing, to resist the moment produced by even full fixity. Hence, since it is simpler to design for the hinged condition and it is also slightly on the safe side for the critical crown section, the analysis is generally made on the basis of this assumption. The assumption of a virtual hinge for rotation about the z-axis is not so inconsistent therefore with fixity about the x-axis as would appear; in any case, rotation about this axis also can be taken care of easily, as the author shows. Rotation about the y-axis is hardly likely as friction and the restraint furnished by the wing walls will prevent it.

In recent years, this question of footing condition has been taken into account directly, especially where the correction is not so simple and obvious as in the single-span frame. The factor "k" or soil modulus has been introduced and thus the condition of the footing is taken into account directly in the analysis.

The writer cannot agree that unbalanced earth pressure is rarely used in practical design. Many designers have come to agree with the late William

---

[16] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1214.

Cain, M. Am. Soc. C. E., that a structure should not be designed so as to depend for its safety on the presence of the active earth pressure. The procedure has therefore been adopted of using the balanced earth pressure only when it is of the same sign as the dead and live load moments and to omit it when of opposite sign. Unbalanced earth pressure is also used but, for this condition, the basic unit stresses are increased by 10% or 15%. This condition often governs the design of the reinforcement.

The author states that such tests as have been made were for the effect of live load in various positions and that, since the effects of these loads are small, the implication seems to be that they are of little value. The writer fails to see the point of this reasoning. The tests still prove that the computed reactions check closely with the measured reactions for loads—any loads, not necessarily live loads—applied in various ways. That more tests would be of value is no doubt true, but that still does not explain why those already made do not prove what they were intended to prove.

The writer also fails to understand the reasoning on the earth-pressure distribution. Once equivalent fluid pressure is assumed, it acts normal to the surface of the wall and that is all that there seems to be to it. It is true, of course, that the equivalent fluid pressure theory leaves much to be desired in the way of a rational theory, but this rather unfortunate situation is not unique with the skew arch; it is true of all structures subjected to earth pressure.

Step II. Stress Distribution.—Even though in practice Step I comes first, it is really, from the analytical point of view, "putting the cart before the horse." Loads acting on a structure are resisted by internal stresses and not by moments, thrusts, and shearing forces. The internal stresses are tensions (or compressions which are merely of opposite sign to tensions) and shears. The so-called bending moments, torsion moments, thrusts, etc., are merely names for the summations or resultants of the primary internal stresses. As a matter of fact, in the rigorous derivation of the beam theory by the mathematical theory of elasticity, the internal stresses are obtained first—and it is then shown that the resultants can be summarized as "bending moments," "thrusts," etc. Since, in the general case, there are stresses in all directions, the total or resultant stresses in any direction will depend on the plane of reference.

It is only in the simple beam and in certain cases of symmetry that the moments, etc., can be found directly from the loads and the geometry of the structure. Even then, the moments and forces found tell something only about the applied loads and nothing at all about the internal stresses except that they can be equated to the resultants of the unit stresses which must be found independently by more complex methods. Since the simple beam was the first and the most common structure analyzed, the fact that it is the internal stresses which are the primary factors has been overlooked, with the result that there has been much confusion of thought about the so-called stress distribution.

Since the moments and the total thrusts and shearing forces are the resultants of the internal stresses, the reference plane chosen determines the values to be obtained. Since by the ellipse of stress, tensions on one plane correspond to shears and smaller tensions on another plane, and vice versa, the same analysis will result in different combinations of "torsion moments" and "bending moments" depending on the plane of reference. The same thing is true of the total forces.

The late George E. Beggs, M. Am. Soc. C. E., was fully aware of the confusion of thought and told the writer that he was very much in favor of discarding the use of the terms, bending moment, etc., even for the simple beam, and to refer only to couples and forces. He was encouraging his students to think along these lines.

The author gives as his reason for not using the procedure for the design of the sections, developed in the writer's paper,[6] the claim that it is too complicated. Simplification with no loss other than mere academic accuracy is not only justified, of course, but is and should be the goal of good engineering practice. It is not a question, however, of mere accuracy. The reason the writer discarded the method of design used by the author, which Mr. Hodges credits to Professor Rathbun, is that it is not correct. Two other design procedures were proposed at that time and not only did all three give results that differed radically in their numerical values, but they even differed in sign. It was for this reason that the writer undertook the study to find the principles involved.

The writer believes that the procedure developed in his paper, with the proofs given and with the qualifying considerations clearly stated, constitutes a logical design. Hindsight being better than foresight, experience has shown that some of the stress combinations derived cannot occur in a practical structure, which simply means that a few of the tabulated procedures are not used. The combinations that do occur happen also to be the most simple to compute as a study of the numerical examples will show. The writer has used the procedure for many years in the drafting room and the men have had no difficulty in following the reasoning, once they made the little effort necessary to a clear understanding.

In Step II as in Step I, much has been written and done in an apparent effort to "simplify," and the result has been confusion. Considering the fact that the design takes about only 10% of the total time necessary for the production of a complete set of plans and specifications, the writer fails to see the necessity for this constant search. Naturally, the design of a skew arch requires a little more effort for a thorough understanding than does a simple beam but the same is true of all present-day design. However that may be, there need be no mystery about the skew structures, and the work involved is nothing to worry about.

As far as the writer knows, the only tests that have been made for the purpose of studying the internal stresses were those by E. F. Gifford and were of qualitative rather than of quantitative nature. With the permission of Mr. Gifford, the writer published photographs of the models tested to destruction in his discussion of a paper by Professor Rathbun.[17] In a general way, the cracks in the models seemed to confirm the writer's stress distribution. Until less crude and more complete tests are made, however, designers still will have to depend on logical reasoning for their designs.

---

[17] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 46, Figs. 16–25.

For the sake of convenience, the reference plane, on which the total moments and thrusts of a skew arch are computed, is a radial plane parallel to the abutments. The plane chosen for the design of the sections is not the one that is most "practical" but the one on which the particular moments and forces under consideration are maximum. In other words, such recognized criteria as are furnished by the ellipse of stress must still be satisfied—regardless of the type of structure analyzed. In addition, the laws of equilibrium require that the boundary conditions be satisfied without stress on external surfaces. Neither of these conditions, among others, is fulfilled by the procedure given by the author.

To find the maximum moments producing the maximum tensions (or shears, for that matter), transformations similar to those which lead to the ellipse of stress must be made. Before this can be done, however, the moments and thrusts must be reduced to stresses (total over the thickness) per unit width. Although the author states that the width, b, does not enter into the analysis, it is only partly and approximately true; the truth is that everything is divided by b—but the structure, as the author states, is considered as a whole. Thus m/b and t/b result from the analysis, but whether or not these are stresses per unit width depends on whether the distribution is uniform or not. After the total stresses are reduced to stresses per unit width, they must be resolved to find the maximum bending moments and shearing moments ("torsion")—a procedure exactly similar to that of finding principal planes. This the author has failed to do and, however "practical" the omission may be, it is incorrect.

The author quotes Professor Rathbun incorrectly when he states that the transverse shearing force is distributed parabolically—like the shear in a rectangular beam. Professor Rathbun states quite correctly that the portion of the shear equal to the thrust multiplied by the tangent of the modified skew angle is distributed uniformly and the remainder is distributed parabolically. This procedure results from the fact that the boundary conditions must be satisfied; there would otherwise be a shear on the external face of the bridge —an impossibility, of course.

The author makes a similar statement about the shear produced by the torsion moment. In this statement, he quotes Professor Rathbun correctly, and both are in error. The reason is the same; in order to satisfy the boundary conditions for the tensions (and compressions) produced by moment, there must be shears equal to the unit tensions multiplied by the tangent of the modified skew angle. Since the resultant of the tensions is a couple, the resultant of the shears which are proportional to them must also be a couple. Therefore the terms "bending moment" and "torsion moment" are not sufficient; that is, they do not tell the whole story as far as distribution is concerned. At the boundary, the tension must be parallel to the face and the shear can be zero only on a section perpendicular to the external surface. Incidentally, the fact that this must always be true will enable the designer to eliminate some computation as soon as he gets the "feel" of the values involved.

This fact may also prove to be a valuable starting point for a simplified procedure for the design of the sections. Since the design of the sections is the ultimate goal, a simplification of Step II should also simplify Step I.

St. Venant's theory of torsion and such tests that are available show that Merriman's formula is very much in error for long and narrow rectangles. Although the more exact theory of St. Venant is supposed to be quite complicated for rectangles that are nearly squares, the hyperbolic functions reach their limit very rapidly for ratios of sides greater than 3 : 1. In a bridge, the ratio is at least 10 : 1 and there is no arithmetic hardship in the use of the results derived from St. Venant's theory, as a glance at the tabulated formulas for the reduction of total stresses to stresses per unit width presented by the writer[18] will show. In any case, it is time engineers realized that their profession is a science and that, on occasion, science requires something a little more advanced that the addition of 2 + 2.

In general, whether the designer is dealing with skew arches or with any other structure, it is well for him to check the results of an analysis to see that the fundamental laws of equilibrium are satisfied; that the boundary conditions are logical; and that such accepted criteria as the ellipse of stress are also satisfied. Much grief may be avoided by this precaution.

General Remarks.—In spite of the misleading statements that have been made time and time again, the skew arch must be analyzed as a complete unit and the division by b is merely a minor arithmetic convenience which has no structural significance. The various suggestions for using longitudinal strips parallel to the face of bridge, etc., all can be shown to be in error—as often as not, on the side of danger. The "trick" of using strips and then keying and tying the laminations together results in a structure that is usually "solid" in spite of the tricks. There is no need for these subterfuges.

If a systematic set of equations based on a simple system of signs is used (with the origin of coordinates at the point of symmetry), even the "accurate" analysis for the reactions is not a hardship; or at least it should not be in these days when engineers are really beginning to do designing. Least difficult of all is the solution of the four simultaneous equations; for those who are worried, it can be stated that the solution can be done on one sheet of paper. However, simplifications are possible, as the writer showed many years ago, and this fact is now confirmed by the author.

As for the stress distribution, a logical design which satisfies the laws of equilibrium, and, in addition, is also consistent with such accepted criteria as the procedure by which the "ellipse of stress" leads to the conception of "principal planes" is just as simple as other "methods" and also more correct. In this phase of the problem, as in other phases, the work must be well organized —it may be said that organization is the key to true simplicity. The difficulties involved in the design of skew bridges have been largely exaggerated. Such at least has been the experience of the writer over a period of years on the design of numerous skew bridges of various shapes and of single and multiple spans.

---

[18] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1297.

Any designer who makes the moderate effort necessary to grasp the subject need have no difficulty.

ALFRED L. PARMÉ,[19] JUN. AM. SOC. C. E.—A problem that is of vital interest to the engineering profession is treated in this paper. Unfortunately, Mr. Hodges has accepted present-day theory as adequate and has based his simplification on the existing formulas. The validity of these basic equations has not yet been completely substantiated. The writer is aware of only two quantitative experiments that have been conducted on skewed arches—one by the Bureau of Public Roads[8] and the other sponsored by the Committee on Concrete and Reinforced Concrete Arches in 1935.[10] In the former, the results obtained from tests made on concrete arches differ considerably from the values obtained by a mathematical investigation based on the method of Prof. J. Charles Rathbun,[10] M. Am. Soc. C. E. The latter series of tests, directed by the late Prof. George E. Beggs, M. Am. Soc. C. E., involved the technique of reciprocal deflections on rubber models. An apparently favorable agreement between experimental and theoretical values was obtained. However, this satisfactory trend was confined to loads at some distance from the center line of the arches. Marked deviations were noted for loads on the center line (and hence for loads symmetrical about the center line). The inconsistencies revealed by these investigations led to the careful examination (presented in this discussion) of the basic elastic equations advanced by Professor Rathbun. It will be shown that the latter contain highly questionable assumptions which militate against their usefulness.

It is not the simplification introduced by Mr. Hodges that is being questioned but the basic equations contained in the "Appendix." One difficulty with that presentation is that the equations are couched in terms not easily manipulated and having remote referents. This difficulty may be overcome by viewing the equations as extensions of the formulas derived for the analysis of curved beams. Incidentally, an excellent and lucid treatment of curved beams was presented[20] originally by A. V. Pippard and F. L. Barrow in 1926. On the basis of this mode of approach, it naturally follows that the prime difference between a two-dimensional and a three-dimensional analysis lies in the fact that displacements due to torsion are included in the latter. Thus, for the ordinary analysis of a right rigid frame:

$$\Delta = \int \frac{M\ m\ dx}{E\ I} \qquad \text{(11a)}$$

in which: Δ = the deflection at any point of the frame due to the loading on the frame; M = the bending moment at any section due to the actual loads (and reactions) acting on the frame; and m = the bending moment at any section due to the virtual or dummy load of unity (1-lb load) acting at the point whose deflection is desired, and acting in the direction of the desired deflection.

---

[19] Bronxville, N. Y.

[20] "Stress Analysis of Bow Girders," by A. V. Pippard and F. L. Barrow, *Technical Paper No 1*, Building Research Board, H. M. Stationery Office, 1926.

On the other hand, for a skewed frame or curved beam:

$$\Delta = \int \frac{M\,m\,dx}{E\,I} + \int \frac{T\,t\,dx}{G\,F} \quad \ldots\ldots\ldots\ldots\ldots (11b)$$

in which: T = the torsional moment acting at any section due to the actual loads (and reactions) acting on the frame; t = the torsional moment at any section due to the virtual or dummy load of unity acting at the point whose deflection is to be found and in the direction of the desired deflection; G = modulus of elasticity for concrete, in shear; and F = factor of torsion (a quantity corresponding to the moment of inertia).

It will be observed that Eq. 11*b* is a condensed statement of the relationship involved in Eq. 7*a*. Both statements (Eqs. 11*b* and 7*a*) are general and apply to any arch irrespective of its shape. To test the validity of this equation, it is best to deal directly with the simplest form of arch—namely, a straight slab clamped at each end. This selection has been made partly for the purpose of clearer exposition. However, the mathematical analysis to follow can be applied equally well to more complicated forms. In the example chosen (Fig. 7), the customary procedure of replacing the right support by redundant reactions has been followed. This operation transforms the slab from an indeterminate form to a determinate one. The number of equations due to displacement will be equal to the number of redundant reactions. Only the vertical displacement will be treated herein. The general equation of

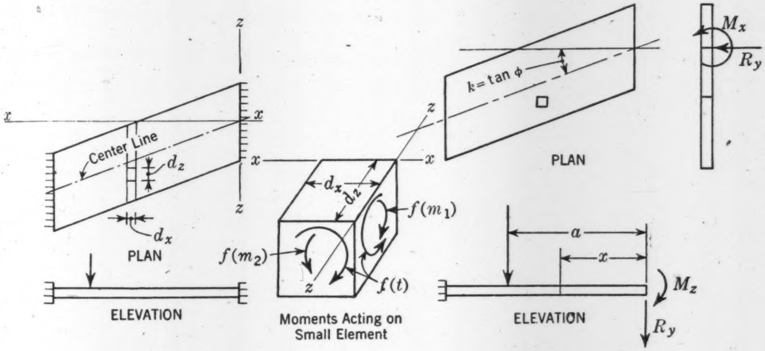


FIG. 7

$$\Delta = \int \frac{M\,m\,dx}{E\,I} + \int \frac{T\,t\,dx}{G\,F}$$

then becomes

$$\Delta = \int_a^l P\,(x - a)\,x\,\frac{dx}{E\,I} + \int_0^l [M_z\,x + R_y\,x^2]\,\frac{dx}{E\,I}$$

$$+ \int_0^l [R_y\,k^2\,x^2 + M_x\,k\,x]\,\frac{dx}{F\,G} + \int_a^l [P\,k^2\,(x - a)\,x]\,\frac{dx}{F\,G} \quad \ldots. (12)$$

In this case it must be borne in mind that the moments refer to the total moment acting on a cross section of the slab.

If the problem is attacked from the strain-energy conception utilized in the analysis of thin plates, the application of a vertical load Q at N will produce bending and torsional moments on a small element. (To avoid conflict with previous notation these moments will be labeled $f(m_1)$, $f(m_2)$, and $f(t)$.) Since the internal work and external work must be equal,

$$\frac{1}{2}\left[f(m_1)\frac{\partial^2\omega}{\partial x^2}+f(m_2)\frac{\partial^2\omega}{\partial z^2}+2\,f(t)\frac{\partial\omega}{\partial x\,\partial z}\right]dx\,dz=\tfrac{1}{2}\,(Q\,\Delta_1)\ldots\ldots(13)$$

in which Δ_1 denotes the deflection of N due to the deformation of the small element; and $\frac{\partial^2\omega}{\partial x^2}$ and $\frac{\partial^2\omega}{\partial z^2}$ = angular rotation of the small element due to the load Q. Let Q now be considered a unit load; then functions $f(m_1)$, $f(m_2)$, and $f(t)$ will be the moments created by a unit load at N. Hence,

$$\left[f(m_1)\frac{\partial^2\omega}{\partial x^2}+f(m_2)\frac{\partial^2\omega}{\partial z^2}+2\,f(t)\frac{\partial\omega}{\partial x\,\partial z}\right]dx\,dz=\Delta_1\ldots\ldots\ldots(14)$$

Eq. 14 expresses the relation between the deflection at N and the angular deformation at any point irrespective of the manner in which these angular deformations are produced. Now the angular deformations at any point are given by the expressions

$$\frac{\partial^2\omega}{\partial x^2}=\frac{F(M_1)+\mu\,F(M_2)}{(1-\mu^2)\,E\,I}\ldots\ldots\ldots\ldots\ldots\ldots(15a)$$

$$\frac{\partial^2\omega}{\partial z^2}=\frac{F(M_2)+\mu\,F(M_1)}{(1-\mu^2)\,E\,I}\ldots\ldots\ldots\ldots\ldots\ldots(15b)$$

and

$$\frac{\partial^2\omega}{\partial x\,\partial z}=\frac{F(T)}{(1+\mu)\,E\,I}\ldots\ldots\ldots\ldots\ldots\ldots(15c)$$

in which $F(M_1)$, $F(M_2)$, and $F(T)$ are the moments acting on the element due to actual loads (and reactions) acting on the slab. For simplicity, assume that μ, Poisson's ratio, is equal to zero; then

$$\frac{\partial^2\omega}{\partial x^2}=\frac{F(M_1)}{E\,I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16a)$$

$$\frac{\partial^2\omega}{\partial z^2}=\frac{F(M_2)}{E\,I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16b)$$

and

$$\frac{\partial^2\omega}{\partial x\,\partial z}=\frac{F(T)}{E\,I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16c)$$

Making the proper substitution,

$$[F(M_1)\,f(m_1)+F(M_2)\,f(m_2)+2\,F(T)\,f(t)]\,\frac{dx\,dz}{E\,I}=\Delta_1\ldots\ldots(17)$$

If Δ denotes the deflection of point N due to deformations in all the elements, then

$$\Delta = \int\int [f(m_1)\,F(M_1) + F(M_2)\,f(m_2) + 2\,f(t)\,F(T)]\,\frac{dx\,dz}{E\,I} \ldots (18a)$$

Since the designer is not interested in the effect of the transverse moment, the terms occurring because of this factor are omitted. The subscript used to differentiate the moment terms is also no longer needed. Eq. 18*a* then takes the final form

$$\Delta = \int\int [f(m)\,F(M) + 2\,f(t)\,F(T)]\,\frac{dx\,dz}{E\,I} \ldots\ldots\ldots\ldots (18b)$$

Eqs. 11*b* and 18*b* are two different expressions for the vertical deflection at N. The validity of the expression presented in the first part of the discussion rests on the hypothesis that a slab subject to torsion behaves like a beam. The latter has been established on a more fundamental basis. Both should yield the same result and therefore Eqs. 11*b* and 18*b* can be equated:

$$\int \frac{M\,m\,dx}{E\,I} + \int \frac{T\,t\,dx}{G\,F} = \int\int \frac{F(M)\,f(m) + 2\,F(T)\,f(t)}{E\,I}\,dz\,dx \ldots (19)$$

Recalling that M is used to designate the total moment acting on a section and that $F(M)$ expresses the moment at any point:

$$M = \int F(M)\,dz \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

and

$$m = \int f(m)\,dz \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

In a similar manner, it can be easily shown that

$$T = \int F(T)\,dz \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

and

$$t = \int f(t)\,dz \ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

Therefore,

$$\int\left[\int F(M)\,dz \int f(m)\,dz\right]\frac{dx}{E\,I} + \int\left[\int F(T)\,dz \int f(t)\,dz\right]\frac{dx}{G\,F}$$
$$= \int [F(M)\,f(m) + 2\,F(T)\,f(t)]\,\frac{dz\,dx}{E\,I} \ldots\ldots\ldots\ldots (22)$$

Since this is an identity, each term in the expressions must correspond. Therefore:

$$\int\left[\int F(M)\,dz \int f(m)\,dz\right]\frac{dx}{E\,I} = \int\int [F(M)\,f(m)\,dz]\,\frac{dx}{E\,I} \ldots (23a)$$

and similarly

$$\int\left[\int F(T)\,dz\int f(t)\,dz\right]\frac{dx}{F\,G} = \int\int\frac{2\,F(T)\,f(t)\,dz\,dx}{E\,I}\ldots\ldots(23b)$$

Concentrating on only one of the terms, the integration of expression on the right side of Eqs. 23 can be performed in any order. This means, therefore, that

$$\int F(M)\,dz\int f(m)\,dz = \int F(M)\,f(m)\,dz\ldots\ldots\ldots\ldots(24)$$

Eq. 24 can be satisfied only if the variation in moment across a section is constant; but this is in sharp contradiction to established fact. As a matter of fact, the primary purpose of an analysis of skew frame is to determine just what this variation is. Therefore, the basic equations used by Mr. Hodges are incorrect because they imply an assumption that is not consistent with the physical behavior of the skewed slab.

If the variation across a section is assumed to be triangular, the moment is a linear function of z. Then, if $F(M) = K\,z$ and $f(m) = k\,z$, Eqs. 23 become:

$$\frac{12}{E\,b\,t^3}\left[\int_0^b F(M)\,dz\int_0^b f(m)\,dz\right]$$

$$= \frac{12}{E\,b\,t^3}\left[\int_0^b K\,z\,dz\int_0^b k\,z\,dz\right] = K\,k\,\frac{12\,b^3}{4\,t^3\,E}\ldots\ldots\ldots\ldots(25a)$$

and

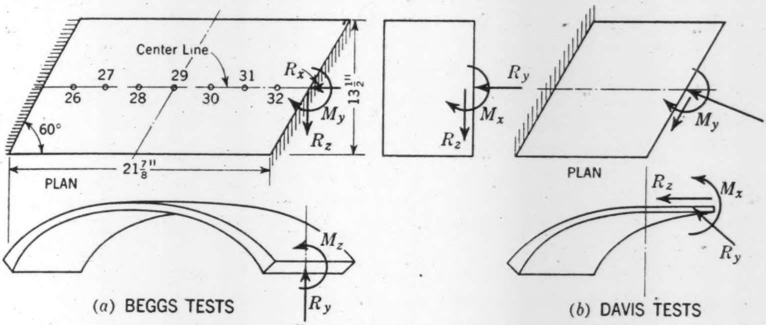
$$\frac{12}{E\,t^3}\int_0^b F(M)\,f(m)\,dz = \frac{12}{E\,t^3}\int_0^b K\,k\,z^2 = k\,K\,\frac{12\,b^3}{3\,t^3\,E}\ldots\ldots(25b)$$

A deviation of 25% is indicated in Eqs. 25. In general, $\int\frac{M\,m\,dx}{E\,I}$ is less than $\int\int F(M)\,f(m)\,\frac{dz\,dx}{E\,I}$.

It may be argued that the errors introduced apply to all terms and hence cancel. This would be true if the error in each term was a constant proportion. Such is not the case. The deviations will be more marked in terms involving the reactions than those due to loads. Therefore, the equations must lead to erroneous results. Because the final value of any reaction is the result of the difference of large numbers, it is difficult to predict in just what direction the error will lie. However, it appears that Professor Rathbun's method yields results smaller than those found experimentally.

How serious is this error? The method used to check the theory does not lend itself to an arithmetical solution. However, the results obtained experimentally and those obtained by Professor Rathbun's method can be compared, as shown in Table 12. Of all the differences to be noted between the experimental and theoretical values, the most significant is the complete lack of agreement in the values of the horizontal moment (M_y). The magnitude of the experimental values eliminates the possibility that the deviations are just due to chance and unavoidable inaccuracies in computing and testing. It would therefore seem that the theory does not represent the elastic behavior of the skewed arches.

TABLE 12.—Comparison of Reactions and Moments Determined Experimentally and Theoretically



(a) BEGGS TESTS (b) DAVIS TESTS

Reaction[a]	Type	(a) Beggs Tests, Arch "A"; Loads on Center Line							(b) Davis Tests, 60° Skew Arch	
		Distance s of Load from Reaction							Lateral restraint	Lateral displacement
		0.90 s	0.75 s	0.64 s	0.50 s	0.36 s	0.25 s	0.10 s		
R_x	Experimental	0.162	0.540	0.860	1.02	0.894	0.554	0.152	2,980	
	Theoretical	0.131	0.594	1.06	1.20	1.06	0.594	0.131	1,017	1,470
R_z	Experimental	0.002	0.046	0.192	0.27	0.188	0.036	0.026	....	....
	Theoretical	0.014	0.009	0.023	0.028	0.023	0.009	0.014	1,459	0
R_y	Experimental	0.020	0.106	0.258	0.492	0.704	0.890	0.972	2,100	
	Theoretical	0.018	0.109	0.281	0.499	0.719	0.891	0.983	2,100	2,100
M_x	Experimental	−0.010	0.32	1.14	1.56	1.14	....	0.18	83,200	
	Theoretical	0.008	0.054	0.133	0.76	0.133	0.053	0.008	9,540	53,400
M_z	Experimental	0.496	1.29	1.31	0.556	−0.575	−1.55	−1.49	....	....
	Theoretical	0.334	1.55	2.27	1.81	0.137	−1.46	−1.82	0	0
M_y	Experimental	0.088	0.096	2.44	3.02	1.80	0.46	0.050	143,100	
	Theoretical	0.016	0.110	0.244	0.39	0.73	0.40	0.31	9,432	37,080

[a] Reactions in pounds and moments in pound-inches.

In justice to the method described by Mr. Hodges it should be reemphasized that in the experiment conducted by the Committee on Concrete and Reinforced Concrete Arches much better agreement was obtained for loads near the free edges. The agreement obtained in this region does not constitute a sufficient proof of the reliability of the method however. For a theory to be tenable it must give reliable results at all points and above all for the conditions that govern the design; that is, for loads on the center line or loads symmetrical about the center line. As a matter of fact, loads symmetrical about the center line are handled as concentrated loads on the center line; that is, in the analysis identical results are obtained for these two types of loading. In the case of rigid frame bridges of average span, the weight of the structure forms a large part of the total weight. Because of this and moreover because wheel loads will usually be near the center line, the correctness of any analysis based on Professor Rathbun's equations or similar derivations will be measured by the reliability of the theory near the center line. Since appreciable differences exist for these loads, the results obtained are highly questionable.

It has been shown theoretically that the basic equations are not valid. Furthermore, experimental values have been compared with theoretical values. Both of these facts confirm the writer's contention that at present no simple correct solution exists. However, rather than close this discussion with a negative emphasis, the writer would like to make the suggestion that some means of correcting the equations by introducing factors to account for the variation of moment across a section be introduced. The revised method should then be checked by experiment or compared with results obtained by more precise methods. At present, this problem can be most readily handled by means of finite difference equations.

J. CHARLES RATHBUN,[21] M. AM. SOC. C. E.—There is a tendency among engineers to avoid reinforced concrete skew arches of the barrel type even when this type of structure would be more economical and would give a better alinement than the type adopted. Some designers have even used skew arches of rib construction with the floor slab cast integral with the rib. One such bridge was built with an added slab at the intrados. Of course those who try to dodge the problem not only fail in their purpose but introduce other problems, of a more difficult nature, in analysis.

Since 1924 a number of papers on the analysis of the skew arch have been published by the Society.[3, 6, 22, 23] These with their discussions give a complete view of the problem and its solution. Any engineer entrusted with a problem for which a skew arch would make a satisfactory solution owes it to his client to review this literature. A solution is offered for arches with fixed abutments.[3] The equations for hinged abutments were first developed by the writer in a report to the Westchester County Park Commissioners in 1925. Later they were published, along with the method of computing the steel reinforcement.[23]

In offering a practical application of that method, Mr. Hodges' paper is a distinct contribution to this series of papers. He has used it in the design of many skew arches for the past twenty years. It is hoped that this paper will impress the profession with the fact that an analysis of a skew arch can be made and that the stresses in the finished structure can be anticipated with design accuracy and at a reasonable cost.

The writer proved[24] that the "Rankine theory" (in which the force lines are parallel to the spandrel walls and stress distribution is uniform along the abutments) is very nearly correct for symmetrical arches loaded with symmetrically placed vertical loads. This is the condition for dead load and full uniform live load for symmetrical skew arches. The Rankine theory is not correct (often it is far from correct) for unsymmetrical arches, for unsymmetrical vertical loads, and for horizontal loads such as earth pressure and forces due to shrinkage and temperature.

---

[21] Prof., Civ. Eng., College of the City of New York, New York, N. Y.

[22] "Crown Stresses in a Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

[23] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *ibid.*, Vol. 98 (1933), p. 41.

[24] *Ibid.*, Vol. LXXXVII (1924), p. 620.

Soon after the publication of the writer's solution to the problem, A. G. Hayden, M. Am. Soc. C. E., suggested that the work of analyzing skew arches might be shortened by developing equations for the deviation of the stresses from those obtained by the Rankine theory. With these equations the preliminary analysis could be corrected. Mr. Hodges has used somewhat the same idea with the right arch as the basis of his work. The writer would have preferred to see Mr. Hayden's method used rather than Mr. Hodges' method because the magnitude of the corrections would have been less. Theoretically, the choice is of little importance, however, as both methods are correct. Perhaps Mr. Hayden's suggestion led to more involved equations and Mr. Hodges has found his solution more practicable.

The solution presented by the writer in 1924 has had considerable criticism because it does not conform to a preconceived idea that the "stresses take the shortest route to the abutments." The writer has yet to see a sound quantitative analysis of this tendency. Many tests have been made to show that the idea is incorrect. Three such tests have been made by the writer.[22, 25]

To test the correctness of the theory the Society had the late George E. Beggs, M. Am. Soc. C. E., conduct a series of tests on four arches. These arches all had fixed abutments and were tested for vertical loads. The reactions for these same loads were computed by the elastic theory under the writer's supervision. Contours of the influence surfaces of the components of the abutment reactions were drawn up for each arch and the two methods compared.[10] Models of these surfaces (see Fig. 8) were also made to assist in this comparison. The tests and the computations were in virtual agreement and the committee reported,[26]

> "For the practical design of skew arches, the limited number of tests which have been conducted indicate that Professor Rathbun's theory may safely be applied for the computation of the six components of the abutment reactions."

After these are computed the problem is a simple one in statically determinate stress analysis.

The models used by Professor Beggs were of rubber with the very high Poisson's ratio of 0.34, compared with about half this value for concrete. The committee decided that the discrepancy "would change the results of the calculations by only a small amount." [26] In fact the model surfaces showed that the change would occur in but few areas and for loads over most of the arch the effect would be very little. Designers of right arches, where the same effect occurs, have neglected to take Poisson's ratio into consideration, with no serious consequences.

---

[25] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 26 and 66.

[26] *Ibid.*, p. 1572.

The writer believes that the report of this committee has convinced the most skeptical engineers that the elastic theory, as developed, is sufficiently correct for design purposes.

One reason that the elastic theory, as developed for the skew arch, has not been popular is probably the writer's fault. In the analysis he used a geometrical approach instead of the method of virtual work; he includes horizontal and radial shear deformations as well as shortening due to thrust; he also developed the deformation due to the several components of the moments. Many of these terms have since proved to be negligible. He covered both the symmetrical and unsymmetrical skew arch for all types of loading. The result was that the paper became long, cumbersome, and laborious to follow. One critic stated that "No arch can follow that much mathematics," to which Professor Beggs replied, "Four Princeton arches did." Mr. Hodges seems to have overcome this objection.

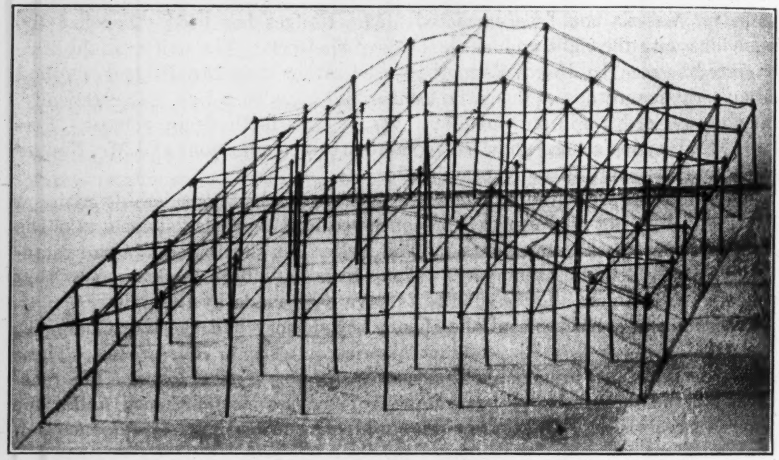


FIG. 8.—MODEL OF INFLUENCE SURFACE

With this paper of Mr. Hodges the question of the design of a reinforced concrete barrel skew arch should present little difficulty to those familiar with the analysis and design of the right arch of the same type. It is hoped that the paper will renew interest in the subject and encourage engineers who have arches to design to familiarize themselves with the basic theory and the literature on this subject.

MAURICE BARRON,[27] ESQ.—In 1932 the writer, under the author's supervision, made some of the computations for an investigation of the effect of variation in the skew angle on the reactions of skewed frames. The method then used was that first published in 1931 by Mr. Hayden[28] and the investigation was carried through the simultaneous solution of the elastic equations. In this investigation the writer was concerned with a structure of the standard concrete rigid frame type of bridge. Only the skew angle was permitted to vary. The conclusion was that, for all practical design purposes, and well within the limits of computational accuracy, the rectangular horizontal reaction is independent of the skew angle. This conclusion is the basis for the entire first part of the paper dealing with analysis.

---

[27] Designer, Office of Gilmore D. Clarke, New York, N. Y.

[28] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed.

The writer has investigated the validity of this conclusion when applied to two reinforced concrete elliptical skewed barrel arches (for convenience identified as bridges A and B). Bridge A, designed by the writer, is a 45° skewed arch with a square span of 56 ft. Bridge B has a 36.5° skew and a square span of 57 ft.

In designing bridge A many short cuts (not approximations) were used including the principle of the elastic centroid, ordinary and torsional elastic weights, the shear increment method of computing moments, and Maxwell's theorem of reciprocal deflections. The four elastic equations (see Appendix, Eqs. 7) were thus greatly simplified; one or more coefficients in each equation, a total of eight, were reduced to zero. The solution of the four equations was correspondingly simplified and resulted in the following equation for the rectangular horizontal reaction: $65.7574\, R_x = +\, \delta_x + 1.042448\, \delta_z - 3.197289\, \phi_y$; or,

$$R_x = 0.0152074\, \delta_x + 0.0158529\, \delta_z - 0.0486225\, \phi_y \quad \ldots\ldots\ldots (26)$$

in which R_x is the rectangular horizontal reaction; δ_x is the deflection (including torsional effects) of the base frame in the direction and at the point of application of R_x, due to the load under consideration; δ_z is the deflection of the base frame in the direction and at the point of application of R_z, due to the load under consideration; and ϕ_y is the angular rotation of the base frame in the direction of M_y, due to the load under consideration.

For the proposed simplified method:

$$65.37016\, H = \delta'_x \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which H is the rectangular horizontal reaction; and δ'_x is the deflection of the rectangular structure in the direction and at the point of application of H, due to the load under consideration. The coefficient of H is $\Sigma \frac{y^2}{I} \Delta s$ and does not contain torsional effects.

The values to be substituted in Eq. 26 for the unit vertical influence load are of interest because they show the contribution of each deflection to the total. Table 13 gives these values, as well as values for H, by the simplified method, for the unit vertical influence load, for earth pressure from the left, and for temperature change.

It is evident that, except for temperature change, δ_x is the predominant factor contributing to the reaction R_x. The other two values are not only of different order but are always of opposite sign. The close agreement in the data is added proof that the conclusion applies as well to elliptical skewed arches. The author has shown the proof that this conclusion is exact for flat-decked structures where the legs are vertical and the deck has a constant height.

The writer wishes to emphasize the manner in which Mr. Hodges has utilized this conclusion in his proposed simplified analysis. In step 1, the right arch or frame is analyzed for the redundant horizontal reaction, choosing as a base structure a statically determinate system, usually a simple span. This involves one unknown, H, and the solution of one elastic equation. In step 2, the final structure is analyzed using the foregoing system, with H and ϵH applied, as the base frame. Particular attention is called to the fact that the base frame in step 2 of the analysis is a statically indeterminate system; but it must be a system in equilibrium. Therefore, after H has been found and applied the second base system must be stabilized by the application of equilibrating reactions. This is possible only because the two elastic systems can be considered independently. In the analysis of bridge A the base structure chosen was a simple span with a rigid bracket from the hinge (on rollers) to the elastic centroid. For a symmetrical structure this centroid is on the center line. This procedure incorporates the desirable features of symmetry and agrees with the base structure ordinarily used in analyzing rectangular frames and arches—namely, a simple span. The writer believes this is another short cut that can be incorporated advantageously into the proposed simplified method.

TABLE 13.—SOLUTION OF EQ. 26

Equation	Points[a]								Earth[b] pressure from left	Temperature[b]
	3	4	5	6	7	8	9[c]	10		
	(a) ANALYSIS OF BRIDGE A									
$+0.0152074\ \delta_x$	0.0190	0.0692	0.1355	0.2092	0.2837	0.3501	0.3993	0.4249	132.92	21.21
$+0.0158529\ \delta_z$	−0.0000	−0.0002	−0.0013	−0.0033	−0.0060	−0.0092	−0.0120	−0.0136	−0.05	22.12
$-0.0486225\ \phi_y$	0.0025	0.0088	0.0171	0.0263	0.0353	0.0432	0.0489	0.0518	16.74	0.00
R_x	0.0215	0.0778	0.1513	0.2322	0.3130	0.3841	0.4362	0.4631	149.61	43.33
H	0.0216	0.0780	0.1516	0.2321	0.3118	0.3831	0.4317	0.4576	150.3	42.6
	(b) ANALYSIS OF BRIDGE B[d]									
R_x	0.0149	0.0587	0.1336	0.2215	0.3064	0.3759	0.4193	......	123.88	30.63
H	0.0149	0.0587	0.1336	0.2214	0.3059	0.3746	0.4176	......	124.0	30.4

[a] The points are equidistant and numbered from point 1 at the footing to point 10 at the crown. For points 1 and 2 the values are all zero since they are on the vertical part of the leg. [b] Values in kips. [c] Crown point for bridge B. [d] The details of the analysis for bridge B were the same as for bridge A and only the results are recorded.

The design of a skewed concrete rigid frame or arch bridge by the exact method is seldom attempted without the use of short cuts. Approximations should be accepted only where the limits of accuracy are understood and known. Any designer who has waded through this exacting work and the lengthy computations will appreciate any method that reduces the amount of computing or decreases the probability of error without sacrificing safety or economy. The proposed simplified method of analysis does this.

Comparisons, such as the foregoing, indicate that the approximation resulting from the neglect of the effect of the torsional elastic system on the rectangular elastic system is of the same order as the approximation usually made by neglecting shearing distortions when analyzing rectangular frames or arch bridges. The neglect of rib shortening due to axial thrust also falls in this category, except for flat arches.

Furthermore, in the solution of the simultaneous equations, analysis by the exact method encounters the troublesome problem of small differences. Only after considerable experience and numerous designs is it possible to know how many significant places are required in any particular case. Usually, therefore, the tendency is to carry the calculations to too many significant places. By using the proposed simplified method of analysis, this difficulty is overcome to a great extent. The problem is nonexistent in determining the horizontal rectangular reactions in step 1, since the solution of only one elastic equation is involved. In step 2, the number of simultaneous equations is reduced, and the occurrence of small differences is minimized.

Another interesting feature of this paper is its solution of the problem of foundation restraints. Rotation of the foundation because of compressibility of the supporting material is an unknown factor and therefore analysis for boundary conditions (full fixity and free rotation) is justified. Although the added refinement of including the soil modulus would show the effect of partial restraint, such an analysis would depend on the accurate determination of this modulus and its uniformity. For this reason the writer favors design for boundary conditions since the true condition must be somewhere between the limits. Rotation of the foundation about the vertical axis is usually impossible. Restraints due to friction and approach walls will prevent this rotation, and, even where the approach walls are separated by expansion joints, the foundations are usually poured monolithic.

Earth pressure, as a load, is much more important in a skewed structure than it is in a rectangular one. The analysis for this load is usually relegated to a minor rôle, but the author gives it some of the major consideration it deserves. In the proposed method step 1 is still the determination of the rectangular horizontal reaction, but step 2 is much more involved. The base frame for step 2 is again a statically indeterminate system with H (as found in step 1) applied at the hinges; and, exactly as before, this base frame must be stabilized by cross shear reactions before the computations for the torsional elastic effect of the structure are started. In previous methods this stabilizing cross shear reaction is not separated from the redundant, and the total is applied at the hinge. The author divides this stabilizing cross shear reaction into two parts, one applied at the hinge and the other applied along the back of the abutment. The first part is obviously the foundation restraint but the second part requires explanation. Since equivalent fluid pressure was assumed acting normal to the abutment, it seems logical that frictional resistance to distortion will develop along the back of the abutment, and that this part of the frictional equilibrating cross shear reaction cannot exceed the normal pressure times the coefficient of friction of earth on concrete. The author applies this part of the equilibrating cross shear reaction concentrated at the center of normal pressure. He then transforms it into a cross shear reaction applied at the hinge plus a moment about the x-axis. The net difference in results between the author's method and previous methods should be this additional moment of transformation. Since it is on the safe side to do so, this moment of transformation should be included in the analysis. There is also a transformation moment about the y-axis which has been neglected. The close agreement of R_x and H (see Table 13) for earth pressure from the left indicates that the basic conclusion stated in the beginning of this discussion is just as valid for horizontal loads as for vertical loads.

This simplified method of analysis may be applied to double-span skewed structures if the following logical deductions are accepted. The evidence shows that the two elastic systems of single-span skewed structures are practically independent for vertical and horizontal loads and equally independent of the shape of the structure. A double-span structure may be considered as a single span with an elastic bracket at the center, to the ends of which are applied forces equal to the redundant reactions. This logic may be extended to include three or more spans.

With all the available data, and this simplified method of analysis, there should be no necessity for avoiding the skewed barrel type concrete rigid frame or arch bridge.

PHILLIPS H. LOVERING,[29] ASSOC. M. AM. SOC. C. E.—The value to the profession of many papers in *Proceedings* is emphasized by predictions that the ideas and information advanced should prove important. The value of this paper does not rest upon prediction. Within nine months of its publication, the writer already had used this design method for four bridges and thereby saved at least seven days of design time on each bridge. The time required for checking the design is reduced about four days.

More important, for any method based on the trial of assumed sections, subsequent trial designs may sometimes have to be made; and, in the case of former procedure, the suitability of the sections tried could not be determined until the end of the design, whereas using Mr. Hodges' method the suitability of the sections is determined by the second day, or in about one tenth the time required by the former method. The value of dividing the design into two parts with the longer and subordinate part coming last is thus emphasized.

The writer has found that the difficulty of choosing correct sections increases with the skew, and is caused mostly by the high temperature stresses. As in other arches, the temperature stresses can be held to a minimum by choosing sections that have a moment of inertia just sufficient to satisfy the stress requirements throughout a large part of the arch. In applying this principle to the rigid frame bridge, it becomes evident that the $\frac{y^2}{I}$-values at the knee are so comparatively small that it is best to thicken the knee because, although this decreases these values still more, the reduction of moment obtained in the middle part of the arch allows a considerable reduction in I and a corresponding increase in the $\frac{y^2}{I}$-values. The difficulty of obtaining a smooth intrados curve under conditions of a thick knee and a thin section throughout the central part of the arch is partly overcome by using an ellipse instead of the arc of a circle for the intrados with possibly a short tangent at the crown. This expedient becomes valuable for skews of 45° and more. Lengthening the legs also somewhat lessens the temperature stresses.

---

[29] Structural Designer, W. Earl Andrews, New York, N. Y.

The high shears at the bottom of the leg are probably transferred across the construction joint into the footing mostly by friction. However, at least a large part of this uncertainty is removed by placing V-notches across the joint, and, if the longitudinal key occupies the middle third, the transverse V-notches should be set in the footing on either side of the key rather than across the bottom.

Jaroslav Polivka,[30] M. Am. Soc. C. E.—For some decades structural engineers and scientists[31,32,33,34,35,36] have grappled with the relatively simple problem of a rectangular slab or plate supported on two opposite edges and subjected to transverse loading—a problem that cannot be identified with the solution of a simple beam, due to the effect of Poisson's ratio. The problem of a similar plate, but one with skewed shape, has only recently been solved with any degree of exactitude.[37] These problems are much simpler than those involved in skewed frames and arches.

Structural engineers familiar with such analyses would be reluctant to apply any method which, in spite of the complex mathematics involved, yields only approximate results. Mr. Hodges is to be commended for simplifying the mathematical analysis and for bringing the subject for discussion. However, in the analysis of even less important structures, engineers are inclined to proceed with extreme caution, and in the matter of bridges they would seem certain to look with disfavor on a method of analysis which is only an approximation and which has not yet been corroborated satisfactorily by tests on models, and certainly not by tests on existing structures.

There are three possibilities for progress in the design of skewed frames and arches:

(1) To delve deeper into the study of the structural behavior and stress distribution in skewed structural members, developing an analysis on the basis of the exact theory of elasticity as it is used for plates and shells,[35] and checking such analysis with laboratory tests and field measurements;

(2) To use only the results of actual tests for the design of skewed frames and arches; and

(3) To discard the skewed slab in the design of frames and arches and to substitute another type of structure, more amenable to exact analysis, as, for

---

[30] Research Associate, Univ. of California; Cons. Engr., Kaiser Co., Berkeley, Calif.

[31] "Contribution à l'étude de l'équilibre élastique d'une plaque rectangulaire mince," by E. Estanave, thesis presented to the Faculty of Sciences, Univ. of Paris, Paris, 1900, p. 295.

[32] "Partial Differential Equation of the Fourth Degree," by M. Coialowitch, St. Petersburg, 1902 (in Russian).

[33] "Sur l'intégration de l'équation relative à l'équilibre des plaques élastiques encastrées," by G. Lauricella, *Acta mathematica*, Stockholm, Vol. 32, 1903, p. 189.

[34] "Sur l'intégration de l'équations biharmoniques," by S. Zaremba, *Bulletin de Cracovie*, Vol. 20, 1908.

[35] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1940.

[36] "Die Formaenderungen und Spannungen von rechteckigen elastichen Platten," by A. Nádai, *Forschungsarbeiten*, Berlin, 1915, Vol. 170, p. 171.

[37] "Ueber die elastiche Deformation parallelogrammfoermiger Platten," by A. Anzelius, *Der Bauingenieur*, Vol. 20, No. 35/36, September 8, 1939, p. 478.

example, frame girders combined with continuous slabs, or heavy wall girders combined with cylindrical or arched shells in reinforced concrete (which may even prove to be more economical).

A. A. EREMIN,[38] ASSOC. M. AM. SOC. C. E.—An interesting method of analyzing the stresses in skewed rigid frames has been demonstrated in this paper. The tabulation of the equations, as in Table 6, serves to clarify the procedure and results in a saving of time and labor. Before this method can be used with complete confidence, however, further experimental study is needed. The equation for the torsion factor, *F*, cited by the author in his definitions was originally developed by C. Bach for the purpose of computing stresses in reinforced concrete arch ribs. The ratio of breadth to depth in a concrete arch rib section is generally about 1 to 4, whereas in skewed rigid frames the ratio is often as great as 1 to 20.

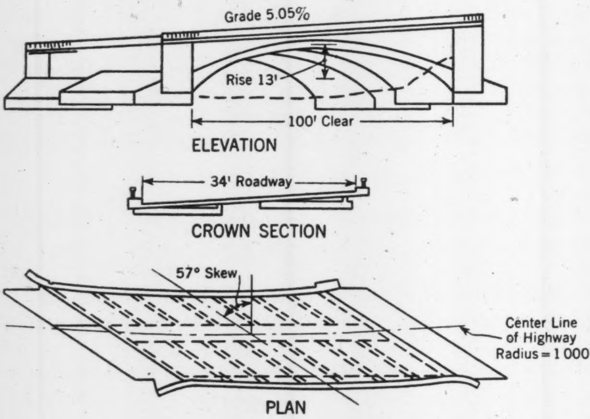


FIG. 9.—SKEWED ARCH BRIDGE

Loading on skewed rigid frames or arches often consists of concentrated forces eccentrically placed in relation to the longitudinal center line of structure. Such forces produce increased local stresses that are not considered in the equations of this paper.

In detailing skewed bridges the limitations of the torsion factor and local stresses may often be minimized conveniently. Consider, for example, the Gaviota Bridge in California, constructed in 1932, which carries a heavy traffic load on the main highway between San Francisco and Los Angeles.[39] This bridge, with an unusually large skew angle of 57°, was constructed with two separate skewed arch ribs as shown in Fig. 9. Therefore, the breadth-depth ratio was reduced considerably. The roadway slab is supported on the transverse rigid walls with resulting uniform distribution of the concentrated forces.

---

[38] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[39] *Engineering News-Record*, March 30, 1933, p. 399.

Mr. Hodges has stated that, in analysis of stresses in skewed arch ribs, the direct deformations may be neglected. In skewed rigid frames the direct deformations have negligible effect. However, in flat skewed arch bridges the direct deformations may often have a considerable effect and cannot be neglected safely.

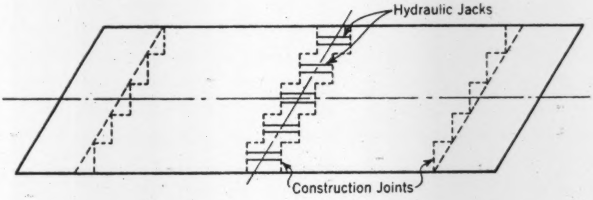


FIG. 10.—PLAN SHOWING HYDRAULIC JACKS METHOD OF POURING SKEWED ARCH

In designing skewed rigid frames and skewed arch bridges, attention should be given to the shrinkage stresses, which increase rapidly with corresponding increases in the skew angle—almost at the same rate as the temperature stresses shown by the author in his conclusion 3. A considerable part of the shrinkage stresses in skewed arch bridges may be reduced by a toothed construction joint, normal to the neutral axis, and in the arch and abutments,[40] as shown in Fig. 10.

RICHARD M. HODGES,[41] M. AM. SOC. C. E.—The writer is sincerely grateful to all who have gone to the trouble necessary to discuss this paper. In a few instances the comments seem to have particular reference to structures of such magnitude and cost that simplified methods would not be of much importance. It is to be hoped that all will lead, in one way or another, to a wider knowledge of what constitutes good practice in the analysis and design of concrete solid-barreled skewed frame and arch structures of moderate span.

The discussions plainly indicate the wide variation in the design methods now extant—from somewhat complicated attempts at theoretical precision at one extreme to rather rough approximations at the other. Perhaps it would be more correct to classify, at the latter extreme, the efforts being made to avoid the problem in various ways while still retaining the skew, such as the substitution of a ribbed structure for a solid-barreled structure, as mentioned in several of the discussions.

There is certainly something to be said in favor of attempting to make the design approach academic accuracy, in spite of resulting complications. (The writer is referring particularly to the effect on the design of the internal moments and stresses.) However, it is sometimes easy for those primarily interested in theory to overlook the practical elements of a problem. There is always a certain limit of justifiable precision in the practical design of a skewed frame or arch, or of any other structure. That limit is overreached when the degree of precision is such that (1) it cannot be warranted by the degree of accuracy attainable in the underlying data or in the basic assumptions; or (2) the cost of the design becomes unnecessarily disproportionate to the cost of the structure. There is even an element of danger in overprecision, in that it leads to a false sense of security. Another limiting factor of the utmost importance in connection with the design of moderate-span skewed frames and arches is established by the fact that, the more difficult or abstruse the design method, either to understand or to apply, the greater becomes the tendency of designers to go to the opposite extreme and to use inadequate approximations or rule-of-thumb methods, or even to shun this type of construction entirely. Of course any exact determination of any of the limiting conditions referred to is practically impossible—the least one should do about them, however, is to keep them in mind.

---

[40] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1329, Fig. 38.

[41] Chf. Structural Designer, Gilmore D. Clarke, Cons. Engr., New York, N. Y.

In several of the discussions, notably in those of Messrs. Parmé, Polivka, and Weiner, there has been a tendency to overlook the purpose and scope of this paper, which is merely an attempt, based on existing theory, to present a method of design, which, for the type of structure considered, will make it possible to meet the foregoing general requirements.

Mr. Geisler's discussion presents some interesting ideas. The tendency, to which he refers, of the forces to concentrate at the corners is shown graphically in Fig. 6. However, the effect on the longitudinal stresses is believed to be practically negligible, and, as stated by Mr. Geisler, there have been no indications of resulting distress in the numerous bridges designed and constructed without considering this effect.

It is common practice to construct the barrel with one or more construction joints parallel to the spandrel walls, as may be necessary, but the writer sees no advantage in dividing the barrel into comparatively narrow longitudinal strips, whether or not more or less effectively tied together. The only way to get rid of the increased corner reactions, as well as of other skew effects, would be to make the separations complete and effective all the way from the crown down to the bottoms of the vertical legs, and also to remove the earth pressure by some kind of open deck approach construction. The effect of Mr. Geisler's method seems to approximate the usual solid-barreled type of construction, with the transverse ties tending to perform the function of transverse reinforcement as designed by the method here adopted.

It is quite true that the corner reaction intensities increase with the width of the structure, but the effect is ordinarily not very serious. However, they should not be neglected entirely, as is too frequently the case. The increase in the vertical reactions at the obtuse corners is relieved by yielding, as Mr. Geisler states, when the structure is founded on compressible soil, and in such a case the bridge itself must resist the additional torsion. If the bridge is designed to withstand the extra torsion, as the writer has recommended, by assuming free rotation about the x-axes, there is no cause for concern. Incidentally, for severe skews extreme temperature rise leads to a reversal, and then the vertical reactions will increase toward the acute corners. A similar reversal also occurs in the normal direction of the lateral thrust along each abutment. These effects can become very important for a severely skewed frame founded on piles.

Mr. Parmé, as well as Mr. Geisler, mentions the tests made by the Bureau of Public Roads.[8] These tests were made on a model with one abutment hinged and the other fixed. The Society's Special Committee on Concrete and Reinforced Concrete[42] stated that "The results of these tests could not be used conveniently as a check of Professor Rathbun's theory." In fact, the tests were not of a skewed arch with either fixed or hinged abutments. This would seem to account for the discrepancies shown by Mr. Parmé. The tentative conclusions from these tests, as quoted by Mr. Geisler, may still be true (they agree in general with the writer's investigations), but no interpretation that could be derived from them would invalidate the elastic theory developed by Professor Rathbun.

In Mr. Parmé's discussion, and also in that of Mr. Polivka, the basic theory for finding the resultant reactions has been questioned in spite of the fact that its essential accuracy was confirmed, in 1926, by a comparison between theoretical analyses by Professor Rathbun and model analyses by the late Professor Beggs. Mr. Parmé's analysis appears interesting, but the preponderance of opinion seems to be rather solidly against his and Mr. Polivka's evaluation of previous investigations. Since Mr. Parmé attempts to show that Professor Rathbun's theory is incorrect, and, since Mr. Polivka seems to think that nothing that has been done to date is worth considering, the writer considered it proper that Professor Rathbun should be afforded an opportunity to present his views. Accordingly, in a letter addressed to the writer, he states:

> "*Transactions* [Vol. LXXXVII] 1924, p. 676 (3) covers the case analysed mathematically by Mr. Parmé and shows that his results are in 'sharp contradiction to established fact.' He will find, if he follows his analysis through all the terms, that he will arrive at the same results as I did, unless I misunderstand his rather short outline of his analysis."

Mr. Parmé cites but two experiments. One supports the elastic theory and the other does not. Four others that do support the theory are described elsewhere.[22, 43, 44, 45] Mr. Polivka cites no authority, but the foregoing also answers his statement that the existing method of analysis "* * * has not yet been corroborated satisfactorily by tests on models * * *."

The writer feels greatly complimented in having his paper approved, in general, by Mr. Hayden. An interesting feature of Mr. Hayden's discussion is his short history of the development of skewed arch and rigid frame design. The writer will take the liberty, when replying to Mr. Weiner, of filling in a few details which Mr. Hayden has omitted.

Mr. Hayden's idea (also mentioned by Professor Rathbun) of using the skew span, instead of the square span, as a basis of deriving the simplification, is well worth investigating. However, the writer thinks that it is more logical to develop a simplification along the lines of the universally applicable elastic theory than to attempt to use the Rankine approximation.

---

[42] "Concrete and Reinforced Concrete Arches," Final Rept. of the Special Committee, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1552.

[43] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *ibid.*, Vol. 98 (1933), p. 26.

[44] *Ibid.*, p. 50.

[45] *Ibid.*, p. 66.

Both Mr. Hayden and Professor Rathbun have called attention to the common fallacy that skew effects can be avoided by the use of ribbed construction. Many bridges have been designed and constructed in this happy belief, and no trouble has resulted, to the writer's knowledge, but nevertheless these bridges are incompletely designed, and headroom has been needlessly wasted in the process.

A more serious matter is the practice of using approximations without knowledge or discrimination. The writer has read several publications (although not very recently), which stated that the longitudinal reinforcement of skewed frames can be designed safely by multiplying the square span moments and thrusts by the square of the secant of the skew angle, without considering that temperature moments and thrusts (which are the controlling factors in proportioning a frame of moderately heavy skew) must be multiplied by the fourth power of the secant.

The responsibility for this type of advice can be placed squarely on those engineers who, knowing something about skewed arch and rigid frame design, not only have paid no attention to the necessity of making it simple enough for practical use in the design of small frame structures, but have seemed actually to resent any effort that may be made in that direction by any one else.

Mr. Weiner's discussion takes the form of a dissertation on the subject of skewed arches in general, in the course of which he often ignores the scope of this paper unnecessarily and takes frequent occasion to discuss his own paper[6] published in 1931. For instance, he states:

> "Much confusion indeed has been caused by the constant search for new 'methods.' * * * The writer has given complete equations[6] for both the symmetrical and unsymmetrical skew arch (single span) with the origin and basic structure chosen with this [simplification of equations] idea in mind."

and, subsequently:

> "* * * it was partly because the original skew-arch [Rathbun] theory was written on the basis of a space system that it had to be revised for efficient use in the drafting room, and this revision constitutes Part I of the writer's paper[16]—and is so stated therein."

It is significant that no mention of this revision by Mr. Weiner is made in the "Historical Perspective" part of Mr. Hayden's discussion. It was because Mr. Weiner's revision proved too laborious (except to Mr. Weiner and the men working in his squad) that it was considered necessary to make the subsequent adaptation of the original Rathbun method to which Mr. Hayden refers.

Mr. Weiner has brought up a number of points which are not directly related to the current paper but consist of criticisms of Professor Rathbun's previous contributions. In reply to an inquiry by the writer, Professor Rathbun states that his interest is purely in

> "* * * the development of the skew arch theory and its acceptance and use by the Profession. Mr. Weiner has written extensively on the skew

> arch. He stated that my 'theory [page 1330, 1932 *Transactions*] formed the basis of Part I of the writer's paper.' Mr. Weiner did revise the sign system and he changed the order of my equations. These were revisions which I consider unwise since they lead to confusion. He also introduced the idea of the ellipse of stress. As I have pointed out in previous discussions of papers, this idea is incorrect as here applied. To my knowledge Mr. Weiner has made no other contribution."

The writer is opposed to Mr. Weiner's ellipsoidal stress method for the additional reason that, even if it were theoretically correct in its application, it would still be unnecessarily complicated as a matter of practical design.

Opportunities to discuss the relative technical merits of Part II of Mr. Weiner's paper and of the design method originally proposed by Professor Rathbun were offered in their proper place and need not be extended in this closing discussion. The writer will not attempt to answer all of Mr. Weiner's numerous criticisms of this paper. He will answer the statements made in Mr. Weiner's opening paragraphs and will then attempt to use a sampling method on the others.

The first paragraph, under his side heading, "Step I. Reactions," is incorrect throughout. Mr. Weiner states,

> "The purpose of this part [computation of reactions] of the paper is to show that permissible approximations may be made in the frame constants with the result that the equations developed on the basis of the accepted theory (proved by tests) are simplified. In addition, the equations break up into groups, each of course containing fewer unknowns. That permissible approximations usually result in simplifications is, of course, obvious; nevertheless, the results are approximate and the paper has been misnamed. Neither proof nor data are given for the degree of approximation."

This paper is not concerned in the least with "permissible approximations" in "frame constants"; and it is misleading, at least, to state that the equations referred to are simplified. The "equations referred to" were not used at all. They were not simplified. New equations were derived on a simplified elastic basis.

Mr. Weiner has only described the process used by himself (and before him by Professor Rathbun) in arriving at approximate formulas for certain redundant reactions. These approximate formulas were derived simply by applying the original equations to a skewed frame with a perfectly flat top and vertical legs. When the writer first began to search for a reliable method of simplification, he tried this method and found it inadequate. Certain reaction components depending largely on deck curvature could not even be approximated roughly in this way. Others were close within certain limits of skew and curvature; but these limits were unknown. The only approximation that stood the test of comparative analyses was the well-known formula "$R_x = H$" (using the writer's notation), which was first published, not by Mr. Weiner, but by Professor Rathbun. In the "Appendix," the writer has given (as a matter of interest and not as an original contribution of theory) a derivation of this important relationship, as adapted to the system of coordinates used in this paper, showing the assumptions on which it is based. The method described by Mr. Weiner was found inadequate for obtaining the other redundant reactions. Instead of simplifying the existing equations as adapted from the original elastic theory, the writer derived new equations for Part 2, using only one instead of two elastic systems. There is no resemblance between the process used by the writer under the heading, "Derivation of Method for Determining Reactions," and the process attributed to him by Mr. Weiner.

The statements made in the last few lines of the paragraph previously quoted cannot be ignored. Before preparing this paper, the writer had obtained evidence of the closeness of the fundamental relationship that $R_x = H$ from comparative analyses of different frames of varying types. The writer stated this very clearly, and he had hoped that it would not be necessary to burden the reader with detailed descriptions and summaries of these comparisons, but Mr. Weiner has left no alternative. (Attention is called to Mr. Barron's discussion of this point, which represents a more constructive and useful form of skepticism than that of Mr. Weiner.)

The following summary is obtained from the results of comparative test analyses made in 1932 by the office force headed by Mr. Hayden, then Designing Engineer of the Westchester County Park Commission, under the direct supervision of the writer. Analyses were made of two bridges, representing different types of rigid frame construction. The Main Street Bridge, Central Westchester Parkway, White Plains, N. Y., is a rigid frame of average type (span 50 ft and rise 21 ft) with a comparatively small degree of deck curvature. The Loop Bridge, carrying the Bronx River Parkway Extension over the Taconic Parkway Connection, is an elliptical rigid frame, approaching a semicircular full-centered arch (clear span 42.5 ft and rise 23 ft). Both were constructed in 1931 and in both bridges the angle of skew was 38°.

Comparative analyses were made for various skew angles. For skews of more than zero, all reaction components were computed by Professor Rathbun's theory as adapted for the Park Commission. For skew angles of 0°, the reaction components R_x (or H) were computed by the ordinary elastic theory for two-hinged rectangular frames or arches. Calculations were made for a temperature rise of 50° F, for balanced earth pressure, and for unit vertical loads applied successively at three points which may be described approximately as near the haunch, quarter point, and crown. The results as given in Table 14 are for a unit width. The customary temperature corrections were applied in reverse, and the skew span reaction components were divided (instead of

TABLE 14.—SUMMARIES, VALUES OF R_x

Skew angle	Main Street Bridge					Loop Bridge (Elliptical)				
	Unit Load at			Earth pressure	Temperature	Unit Load at			Earth pressure	Temperature
	Haunch	¼ point	Crown			Haunch	¼ point	Crown		
0°	0.201	0.383	0.495	1.80	0.156	0.060	0.220	0.330	2.38	0.068
38°	0.202	0.384	0.499	1.75	0.158	0.059	0.221	0.332	2.33	0.072
45°	0.201	0.381	0.497	1.73	0.154	0.059	0.221	0.332	2.31	0.072
60°	0.202	0.376	0.510	1.76	0.154	0.058	0.221	0.337	2.24	0.072

multiplied) in this case by $\sec^2 \theta$ to make them comparable with rectangular frame reactions.

Near the beginning of his discussion, Mr. Weiner states:

> "Many years ago the writer found that for a symmetrical structure the approximations were permissible and result in the simplifications stated."

Presumably Mr. Weiner is referring to the simplifications stated in this paper. The only approximation used in the paper was found by Professor Rathbun. The others are not permissible for a symmetrical structure except under certain restrictions of small deck curvature and skew. The results of Mr. Weiner's analysis, as well as the methods used, are totally different from those advanced in this paper. According to Mr. Weiner (and Professor Rathbun before him), $R_z = \epsilon R_x$, using the writer's notation. This is a fairly close approximation for restricted deck curvature and skew. According to the method presented in this paper, $R_z = \epsilon R_x + R'_z$, in which ϵR_x is the static equilibrant of R_x, and R'_z is that part of R_z which is due to the torsional elasticity of the structure. Since M_x and M_y are both functions of R'_z, and since R'_z does not appear in Mr. Weiner's approximate formulas, Mr. Weiner's statement is incorrect.

Mr. Weiner favors the location of the origin of the coordinate system at the crown, rather at one of the hinges, which is permissible, but the writer disagrees emphatically with the contention that this is more logical for a hinged structure. For those who are accustomed to designing rectangular frames with the origin at a hinge, the hinge is the logical point of origin for the design of hinged skewed frames. One advantage of this arrangement is that the sign system is much simplified.

Some of Mr. Weiner's comments, regrettably, serve only to introduce complications and confusion. For example, the method under discussion needs no elaborate sign system. No one has ever considered the introduction of such a system in the design of skewed frames as finally adapted for the Westchester County Park Commission, on which the method under discussion is based. It would have been unnecessary, and that is one of the chief advantages of this method over Mr. Weiner's adaptation. The best sign system in any case is the one that requires the very least in the way of "formal" explanation. Mr. Weiner states that the writer "does not give a formal statement as to the directions of the positive moments and thrusts." The statements made in the paper (see "Derivation of Method for Determining Reactions: Details of Derivation," fifth paragraph) are perhaps not as formal as they might be, but, in conjunction with Fig. 1, to which they refer, they make the sign situation very clear. Mr. Weiner states that "a 'picture' is not sufficient," which is true, but a clear picture, together with a simple explanation, is certainly the best way to avoid misinterpretation. (There is an oriental proverb that states, in effect, that "a picture is worth ten thousand words.") This is just as correct for multiple spans and multiple stories (which were not open to discussion) as it is for the simple one span frame which is the subject of this paper. Definitions can mean various things to various people, but a picture always means the same to every one.

The system of setting up formulas for each step in a design process, and having "juniors" perform blind substitutions, may be temporarily efficient under certain conditions, although pretty hard on the "juniors." It is certainly a poor way to develop future designers. The writer is convinced that this procedure would be inadvisable as a general policy for any engineer or any organization.

Mr. Weiner's comment on one of the recommendations in the paper is that

> "The writer [Mr. Weiner] also fails to understand the reasoning on the earth-pressure distribution. Once equivalent fluid pressure is assumed, it acts normal to the surface of the wall and that is all that there seems to be to it. It is true, of course, that the equivalent fluid pressure theory leaves much to be desired in the way of a rational theory, but this rather unfortunate situation is not unique with the skew arch; it is true of all structures subjected to earth pressure."

This statement indicates a misinterpretation of the recommendation. There is no "unfortunate situation" involved, and the writer's recommendation certainly does not apply to anything but skewed bridges.

Unbalanced earth pressure is rarely used in the design of symmetrical frames under ordinary conditions. With proper specifications and proper inspection (which the designer has a right to expect), the chance of such a condition is too remote to justify the extra time and trouble. There are so many combinations of active, passive, and unbalanced earth pressure and restricted sidesway conditions to be considered, that there would be no end to the time that might be spent on this kind of investigation. Failure to take advantage of the neutralizing effect of balanced earth pressure is failure to benefit by one of the principal intrinsic economies of the rigid frame type of construction.

The writer does not agree that vertical concentrated load tests supply the information obtainable from earth-pressure tests and temperature tests. Vertical loads placed in various positions would be of little value in deciding, for instance, how much (if any) of the lateral earth-pressure reactions is actually carried along the abutments in friction, or whether it is actually true, as far as temperature is concerned, that the structure can be made stronger by using inferior concrete.

Mr. Weiner asserts that the writer "quotes Professor Rathbun incorrectly when he states that the transverse shearing force is distributed parabolically * * *." The writer did not quote Professor Rathbun, nor did he make any such statement. What the writer did state was that "the direct transverse shear is assumed to be distributed parabolically * * *," and the quotation is in fact from Mr. Hayden's book. Mr. Weiner's error is in overlooking the phrase "is assumed to be." This assumption (which was used for many years before this paper was written) is one of several in Part 3 of the design which have led to modifications in the original method. The purpose has been to simplify the design computations further, and also to increase the margin of safety in proportioning the transverse reinforcement, thus making allowance for the uncertainties (frequently referred to in the paper) affecting this part of the design.

In "General Remarks," Mr. Weiner states that designing by the 1-ft strip method "is merely a minor arithmetic convenience which has no structural significance." The same can be said of any short cut. Such a statement may even apply to a slide rule or a calculating machine, and certainly any convenience that saves time and eliminates possible errors is justifiable.

The writer hopes, with Professor Rathbun, that this paper will renew interest in the subject and perhaps be the means of inducing designers to familiarize themselves with the basic theory. It is only by such means that, among other advances, improved simplifications for practical design purposes can be evolved.

The writer is particularly gratified by the contribution of Mr. Barron, who immediately understood that the fundamental basis of the method under discussion is predicated on a complete separation of the two elastic systems which have always previously been considered together, and then proceeded to explore the subject on this basis. He not only found that the rectangular reaction components R_x are independent of the skew angles in the case of the two elliptical bridges which he analyzed, but also gave a general demonstration of why this condition holds true for every rigid frame investigated. The elastic-center, elastic-weight method employed by Mr. Barron reveals this fact much better than the method of analysis customarily used by the writer.

Mr. Barron has also made a very apt comparison between the method under discussion and the usual practice of neglecting shear deformations, and, in most cases, rib shortening, in the design of ordinary rectangular frames and arches. The writer is in agreement with this statement. He has not claimed that the basic relation is mathematically exact, although it proved so close in the case of some of his comparisons that he was almost tempted to believe so.

Mr. Barron speaks from experience when he mentions the difficulties encountered in considering the flexural and torsional elastic systems together in one group of equations. In such case, the same difficulties and small differences are always encountered, no matter what method is employed.

The writer agrees that rotation about the "y-axes" is highly improbable. However, there have been cases of separate footings for abutments and approach walls, and very little complication is added if such rotation is considered as a possibility. Incidentally, Mr. Barron is the only discusser who has paid much attention to the special problem of earth-pressure loading.

Mr. Lovering's discussion is short, but much to the point in discussing the question on which the practical value of the writer's method entirely depends,—namely, how does it work? The discussion emphasizes clearly the fact that skewed frame designs cannot wait for the solution of theoretical niceties and academic abstractions. They must be turned out quickly to meet present requirements, if they are to be used to any appreciable extent. Professor Rathbun showed that he realized this many years ago when he originated the very simple and straightforward method of practical design for these structures which has been used by the writer, with certain modifications for the sake of further simplicity. Mr. Lovering has stated the practical advantage of being able to establish the proportions of the structure once and for all before considering the somewhat more complicated torsional effects that apply only to the subordinate part of the analysis and design. He has emphasized the fact that temperature is the controlling factor in the design of all rigid frames of considerable skew, and has shown a very practical and economical way of minimizing the temperature effects in such cases, making it possible to extend the practical skew limits of any particular skewed frame or arch. He has also recommended an effective way of transferring the shears between the legs and the footings which should be standard practice. The writer had hoped that there would be more discussions of the type submitted by Mr. Lovering, dealing with the more practical and more neglected aspects of the problem.

Mr. Eremin's reference to the unreliability of the torsion factor calls attention again to the fact that there is no such thing, at present, as an exact method of design for the torsional effects on a skewed frame or arch structure. This fact has been previously referred to as a good reason for avoiding all misleading methods of unwarranted precision. It has also been mentioned that torsional effects are not neutralized by ribbed construction or by dividing the arch or frame barrel into separate strips, and that they can be taken care of safely and without undue loss of economy by proportioning the transverse reinforcement on the basis of extreme assumptions for this subordinate part of the design. As Mr. Lovering has stated, the controlling design, from which the structure itself and its longitudinal reinforcement are proportioned, can be made independently. It is not affected in any way by such assumptions.

The problem of local stresses caused by eccentric loading is an uncertainty which applies to rectangular as well as to skewed solid-barreled structures. The writer questions the necessity of any expedients either in the design or construction of solid-barreled skewed frames or arches. Mr. Eremin calls attention to the necessity of considering rib shortening stresses, which multiply themselves in skewed structures in the same way as temperature stresses, in the design of skewed flat arches. This comment should also apply to skewed frames of exceptionally long span.

Since the inception of this paper, it has become increasingly apparent that, with the modern trend toward divided traffic arteries, double span rigid frames may soon become the rule and single spans the exception. The rather common practice of designing such double span bridges by methods similar to Part 1 of this paper and then more or less guessing at the transverse reinforcement is probably a reasonably safe procedure in most cases; but it is evidently not a very reliable or satisfactory one, particularly in regard to the transverse reinforcement. Although the first part of the aforementioned procedure may be correct enough for practical purposes, an attempt should be made to establish conclusively by comparative analysis (as a further check on Mr. Barron's reasoning in the last paragraph of his discussion) the fact that it is proper to separate the two elastic systems in the same way as for a single span structure. If this is true, as expected, it will be possible to derive comparatively simple equations for the torsional and shear redundants, especially for two equal spans, and to arrive at a reliable method of completing the design with much less work than is now required.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2225

SLUDGE-DRYING DEVELOPMENTS AT CHICAGO, ILL.

By Lloyd M. Johnson,[1] Esq.

With Discussion by Messrs. C. W. Gordon, Paul Hansen,
Langdon Pearse, and Lloyd M. Johnson

Synopsis

Since the inception of the sewage treatment program, The Sanitary District of Chicago, Ill., has conducted many investigations on the dewatering and drying of sewage sludge. This paper is concerned with these investigations, particularly the results and conclusions as applied to the Southwest works.

The Sanitary District is served by four major sewage treatment works, as indicated in Table 1. The Southwest works and the West Side works are located on a common site although the treatment plants are about one mile apart. At the Southwest plant, complete treatment is provided for all the sewage from the Southwest area, and final treatment is to be provided for the partly treated effluent from the West Side works. This double function of the Southwest plant has caused it to be frequently called the West-Southwest sewage treatment plant. For the purposes of this paper it is to be identified as the "Southwest works."

TABLE 1.—Data on The Sanitary District of Chicago, Ill.

Plant	Design flow (mgd)	Method of sludge disposal	Date placed in operation
Calumet[a]	136	Incineration	December 3, 1935
North Side[a]	250	Pumped to Southwest works	October 28, 1928
Southwest[a]	400	Incineration	May 23, 1939
West Side[b]	472	Sand drying beds	June 2, 1930

[a] Activated sludge. [b] Imhoff tanks.

Note.—Published in September, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Commissioner of Streets and Electricity, City of Chicago, Chicago, Ill.; formerly, Engr. of Maintenance and Operation, The Sanitary Dist. of Chicago.

The West Side works serves an area of 150 sq miles, and the population equivalent reaching the works in 1941 was 2,244,000, based on the 5-day B.O.D. analysis.

The treatment plant consists of a pumping station, grit chamber and screen house, skimming tanks, and 108 Imhoff tanks with a sludge volume of 10,532,000 cu ft. Two of the batteries were designed for 134 mgd each and a settling period of 2 hr. The third battery was designed for a flow of 204 mgd and a period of 1 hr.

Digested sludge is dried on open-air beds 80 ft wide with lengths varying from 800 to 1,398 ft and having a total area of 27.3 acres. The beds are in pairs with a standard-gage railway track between each pair. Dried sludge is removed from the beds by cleaning machines that span the 80-ft width, discharging the air-dried material to 30 cu yd side dump cars. The loaded cars are hauled to dumping trestles on Sanitary District property about 3 miles west of the plant. Three heavy switching locomotives are available for hauling sludge and switching service.

The Southwest works serves an area of 82 sq miles. In 1941 the load reaching the works was equivalent to the sewage from 2,770,000 persons. The area served includes the stockyards and packing-house district.

The plant equipment includes a pump and blower house, preliminary settling tanks, aeration and final settling tanks, sludge concentration tanks, and a combined sludge-handling and steam generation plant.

The main prime movers, all driven by steam turbines, consist of four sewage pumps, three blowers, and two electric generators. Plant effluent is used as condensing water in the surface condensers of the turbines.

The sewage pumps discharge to aerated channels leading to the preliminary settling tanks. There are twelve of these tanks, each 101 ft by 103.5 ft inside, and with the depth varying from 10.66 ft to 11.66 ft. Sludge and scum removal equipment is provided. The detention period is 34 min at 400 mgd.

The aeration and final settling tanks are divided into two batteries, each having eight aeration tanks and sixteen settling tanks. The aeration units are the spiral flow type, each 434 ft long inside, and 135 ft wide, center to center of pressure walls. A depth of 15 ft over the diffuser plates is provided. Each tank is divided into four passes by three nonpressure bearing walls. The aeration period is 5 hr at 400 mgd and 20% return.

The settling tanks are the center feed, radial flow type, each 126 ft inside diameter and 11 ft deep at the side-wall. A settling rate of 1,200 gal per sq ft per 24 hr is obtained with 400 mgd and 20% return.

The plant was designed for an average flow of 400 mgd, although at present the activated sludge treatment capacity is limited by blower and sludge-drying capacity. With the present blowers a flow of 300 mgd could be treated. However, the sludge-handling capacity of about 200 tons per day, about half of which is from the North Side works, further limits the present useful capacity to about 225 mgd.

Plant extensions call for an additional battery of aeration and final settling tanks, eight additional final settling tanks for each existing battery, three additional blowers, and six more sludge-drying units. These extensions will provide activated sludge treatment for the West Side Imhoff effluent and sufficient sludge-drying capacity for the additional solids load.

The magnitude of the sludge-handling problem of the District may be visualized from the fact that at present the four major sewage treatment works have a combined average capacity of 1,258 mgd. For 1941 the actual combined flow averaged 1,064 mgd, carrying a daily load of suspended solids averaging 729 tons, dry weight. Of this about 95% reached the Southwest works, since the North Side sludge is pumped to the Southwest works for disposal. The completion of the extensions of the Southwest works will increase the solids loads for disposal at the combined plants to more than 600 tons daily.

As early as 1914 thought was given to the dewatering and incineration of sludge by the Engineering Department of the Sanitary District. Between 1922 and 1930, full-scale tests were made at the Calumet and the Des Plaines River sewage treatment works on dewatering, drying, digestion, and other methods of disposal of activated sludge. These experiments were confined principally to various types of filter presses, dryers, centrifuges, etc. The digestion of mixed activated and fresh sludge was tried first in the operation of the West Side works. Relatively large volumes of digestion capacity and large areas of outdoor drying beds were found necessary. Difficulties in securing adequate areas, and opposition from adjoining property owners, were encountered. Consequently, a procedure was sought which could be used the year round, in which the sludge would be conditioned, dewatered to a cake, and heat dried for incineration, or placed on sale as a fertilizer. At the time, the idea of lagooning was dismissed as a temporary measure that might produce odors.

In 1932 the first tests were conducted at the West Side works on dewatering, drying, and incineration of sludge in a pilot plant handling about 20 tons of dry solids per day. Various mixtures of North Side activated sludge and West Side fresh sludge were used. In all tests the wet sludge was dewatered to approximately 80% moisture by means of a rotary vacuum filter having a net filtering area of 570 sq ft.

In the first phase of the test a rotary drier 70 ft long and 8 ft in diameter was used. Heat was supplied by an incineration furnace equipped with a stoker having a grate area of 80 sq ft.

Dried sludge from the drier was burned in the furnace along with coal, as required, to provide the necessary heat.

Vapors from the drier and the products of combustion were passed through a vapor condenser and scrubber.

The principal disadvantages of this system were:

(1) The difficulty in securing quick and responsive control; and
(2) Ineffective and inefficient deodorization of gases and vapors.

In February, 1934, the first test plant using the flash-drying system was placed in operation. Three impact mills were used for sludge, and one was provided for firing coal when supplementary fuel was required. Gases from the drying circuit were passed through the combustion zone of the furnace, and heat was recovered from furnace gases by means of a vapor heater.

The impact mills operated satisfactorily but it was found that the high degree of comminution afforded was not necessary. The mills were replaced by a cage mill with an evaporating capacity of 5,000 lb per hr of water. A substantial saving in power was realized by this change. Changes were also made to permit use of a vapor cycle instead of an air cycle in the drying circuit.

The tests in this plant were completed in August, 1935. The Calumet Sewage Treatment Works, having a capacity of 40 tons of dry solids per day, was the first plant built after these tests. Experience gained from the operation of this plant aided in the design of the Southwest works. However, the Southwest sludge-handling plant is more complicated than the Calumet plant, as the function of steam generation is combined with that of sludge drying.

In planning the sludge-handling system at the Southwest works, provision was made for handling the North Side sludge, in amount about 100 tons of dry solids per day. The North Side sludge and the Southwest activated sludge are mixed, screened through mechanically cleaned bar screens with ½-in. clear openings, and passed to six concentration tanks, each 70 ft by 46 ft 9 in. inside measurements. A centrifugal pump is provided at each tank to pump the concentrated sludge to the sludge-handling building. The supernatant liquor is discharged to the incoming sewer by gravity.

The concentrated sludge is pumped into the conditioning apparatus on the top floor of the sludge disposal building. The sludge is coagulated by adding a concentrated solution of ferric chloride in a dosing flume. The flow of ferric chloride is proportioned to the flow of sludge by the operator as required by changing sludge conditions. From 6% to 9% of ferric chloride is used, based on the dry weight of the solids. The best guide is the pH-value of the conditioned sludge, which is recorded in a continuous graph. From the mixing flume the sludge is discharged directly to the vacuum filters.

There are thirty-two vacuum filters, each with a drum 11.5 ft in diameter and 16 ft long, with drain valves at each end (see Fig. 1). The filters are arranged in groups of four, each group discharging cake to a common conveyer belt leading to one drying unit. A constant effort is being made to reduce the costs of dewatering. Various substitutes are under investigation to replace the monel metal mesh for supporting the filter cloth on the outside of the drum. The use of plastics for interior piping in place of copper pipe and bronze fittings is being tested. Stainless steel has been found best adapted for the scrapers, although plastics are to be tried as a substitute. The most satisfactory filtering medium is a long-nap, virgin-wool cloth, weighing 13 oz per sq yd. The substitution of a coated iron wire for bronze or copper as winding wire over the cloth is under trial. The cake is peeled off the filters at 80% to 85% moisture, depending on the type of sludge. The sludge will filter more readily to a lower moisture as its volatile content decreases.

As the cake moves along on the belt conveyer, it is weighed automatically. It is then mixed with previously dried sludge to produce a mixture containing between 40% and 50% moisture. The mixing is done in U-shaped troughs lined with stainless steel in which paddles mounted on horizontal shafts mix the wet and dry sludge and move it along the trough. In these units severe abrasion occurs, particularly at the tips of the paddles. The use of a chromiumtungsten-cobalt steel alloy reduces the wear. The mixing has been improved by reversing some of the paddles on each shaft. The mixed sludge is passed to the drying units.

Each of the eight sludge-drying units has a normal capacity to evaporate 20,000 lb per hr of water, based on 1,150° F vapor inlet temperature and 300° F outlet temperature. The drying units operate at a pressure slightly below atmospheric to prevent escape of odors from the process. As a further assurance that no odors will be released to the atmosphere, all waste vapor is passed through the combustion zone of the furnace. The mixed sludge from each group of four filters is divided between the two halves of the drying tower for each drying unit, the division being controlled by the vapor temperature in the cyclone separators. Each tower is about 12 sq ft in total cross-sectional area. A 3-in. thickness of insulating block is used, lined with stainless steel containing 18% chrome and 8% nickel. It is here that the sludge is mixed with the superheated vapor from the vapor heater.

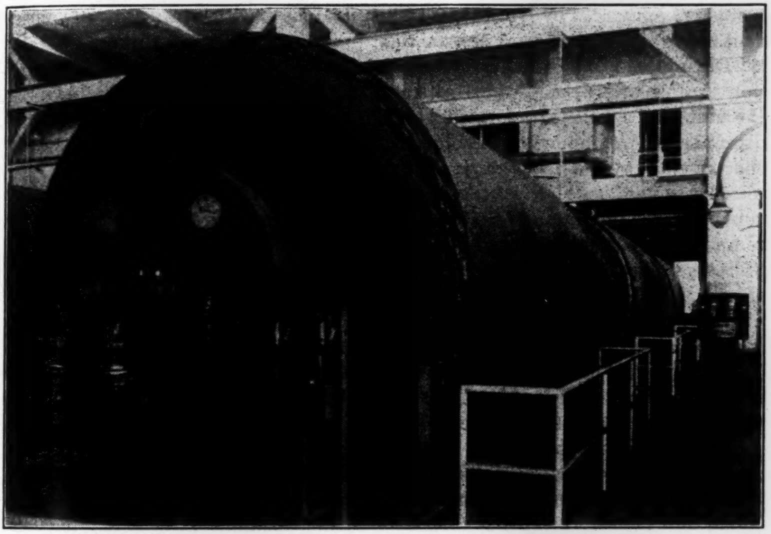


FIG. 1

At the bottom of the towers, the sludge and vapor mixture enters a squirrel-cage mill, which forms the flash drier. Each drier unit consists of two squirrel-cage mills mounted on a common shaft. The mill breaks up the sludge and brings the particles into intimate contact with the superheated vapor, resulting in very rapid drying. By the use of flash drying, cake at 80% moisture can be dried to 5% moisture in 5 min, as compared with 90 min in a modern rotary drier. The driers are subject to abrasion, but as yet this has not been serious. The wear is confined mainly to the liner plates and the cage bars. The renewal of cage bars has been made less frequent by the use of removable sleeves of alloy steel.

Each half of the sludge drier discharges to a cyclone separator from which the dried sludge is removed through rotary air locks and passed to a surge bin. The vapor passes from the separator to the circulating fan and thence to the vapor heater and through the circuit again.

The abrasion on the liner plates of the separators is serious. The original plates were made of ¼-in. steel and were very short lived. The separators were then relined by the gunite method, using lumnite cement and crushed trap rock (ganister) reinforced by No. 10 galvanized steel mesh with 3-in. square openings. This lining has proved very satisfactory.

During the first few months of operation the circulating fans required attention because of a deposit of grease and solids on the blades, which unbalanced the units. Deposits also formed in the ducts. This difficulty was reduced by diverting all preliminary tank solids to the West Side works, and handling only the activated solids in the drying plant.

The vapor heaters are subjected to high temperatures and require close supervision in operation. Room air is used as a tempering medium to hold the temperature of the inlet gas between 1,475° and 1,500° F. Particles of finely divided sludge may deposit on the plates during passage through the vapor side of the heater and may ignite if temperatures in the gas side become too high. The units are protected by thermostatically controlled steam valves which open into the hot gas duct if the temperature becomes excessive.

The surge bins receive the dried sludge from the cyclone separators and serve as distributors for supplying dried sludge to the sludge mixers and to the sludge burners or to the fertilizer conveyer. These units are insulated to prevent condensation on the walls.

Water-cooled butterfly dampers have been installed in the hot gas ducts between the furnaces and vapor heaters. The condensate from the sewage pump and blower turbines is used as a cooling medium, after which it is discharged to the de-aerating heater.

The heat required by the drying system is drawn from the four furnaces, each serving two drying units. They are of the water wall type, fired vertically through four burners for dried sludge and six burners for pulverized coal. They were designed for supplying heat under normal conditions to evaporate 40,000 lb per hr of water from sludge and also to generate 70,000 lb per hr of steam at 425 lb per sq in. gage and 750° F, total temperature, with feed water at 275° F. The boilers are the bent tube type with two drums. A superheater is provided.

The functions of steam generation and sludge drying were combined, not primarily with the recovery of heat from burning sludge as the end in view, but because most of the operating personnel and some of the physical structures of a steam generating plant are already provided in the sludge-drying plant. The only additional cost items for steam generation are fuel and fixed charges on steam generating equipment. Clarified plant effluent is used for condensing purposes. The ample supply has sufficient head to provide gravity flow and to eliminate the need for circulating pumps.

The waste furnace gases pass through an air heater, wherein the combustion air is preheated. After passing through this unit the gas mixes with the furnace gas withdrawn by the drying system. It passes through electrical dust precipitators and then to the stack. There are two dust precipitators, each rated at 180,000 cu ft per min of gas.

With fuel requirements dependent on two variables—steam load and drying load—the control of the furnaces is a delicate problem. Automatic control equipment is provided which ties in the control of the furnaces and drying systems.

In the tuning-up period, better control of steam temperature was secured by moving the superheater by-pass damper from near the top of the front baffle to the bottom of the by-pass passage. Furthermore, to secure higher superheated steam temperatures during high drying and low boiler loads, the side-wall tubes were covered with a plastic refractory, which permits higher furnace temperatures while reducing the heat absorbed by the water walls, thus increasing the heat passing through the main body of boiler tubes and the superheater.

Operating experience indicated the desirability of changing the control for the flow of combustion air. Originally the control acted through totalizing impulses from the flow of steam and waste vapor, which changed the position of the damper of the forced draft fan. This was altered to control the position of the damper by the rate of coal feed, balanced by the differential pressure across an orifice in the forced draft fan discharge duct. The new arrangement insures a definite quantity of air regardless of changes in the position of the secondary and tertiary air dampers.

A considerable amount of experimenting was done on the arrangements of secondary air ports. It was found necessary to reduce the number of ports and the total area of the openings to produce sufficient turbulence to insure complete combustion of the fuel.

Coal is delivered to the plant by rail to a track house south of the sludge disposal building. At this point it may be elevated to a conveyer leading to the bunkers or placed in storage in an adjacent area. A coal crusher is provided which may be used before the coal is elevated, if necessary. A belt conveyer with a traveling tripper distributes coal to the bunkers. This equipment has a capacity of 100 tons per hr. Each furnace is provided with a bunker holding 300 tons.

Coal for each furnace is drawn from the bunkers through two lines, each being equipped with a scale and feeder. Each furnace is served by two pulverizers of the bowl mill type with direct-connected exhausters.

When the firing equipment was designed and purchased the immediate production of fertilizer was not intended. Hence, with the entire sludge output being sold as fertilizer the pulverizing capacity provided is inadequate, since the heat value of each ton of fertilizer produced is equivalent to about 0.57 ton of coal. Also, the elimination of the preliminary solids presents a higher filter-cake moisture than would be the case with mixed solids, thus requiring more heat for drying.

Bottom ash collects in hoppers in the furnace bottom, below the superheater pass and below the boiler back pass, and also in the bases of the stacks. It is removed by hydraulic ash sluicing equipment and sluiced to a common sump, from which it is pumped to a dewatering hopper in the track house or to an ash lagoon. Ash pumped to the track house is loaded into railway cars and hauled to the sludge dumping grounds.

Fly ash is collected in hoppers below the air preheaters and dust precipitators. It is removed by a vacuum system discharging to storage hoppers in the track house. The material is moistened sufficiently to prevent blowing as it is drawn from the hoppers and then loaded into railway cars for hauling away from the plant.

CONCLUSION

Sludge disposal methods in use at the various works of The Sanitary District of Chicago were developed over a period of many years of operating experience and experimental investigation. Through the development of the flash-drying process, as installed at the Calumet and Southwest works, a method of handling sewage solids without the production of offensive wastes and odors was obtained. Although it is considered by those charged with the responsibility of design and operation that these installations have functioned, and will continue to function, in a very satisfactory manner, further improvements will no doubt be evolved that will decrease the cost of sludge disposal. There have been numerous problems attendant upon the operation of this new process, but in spite of these problems the method has proved its value as evidenced by the Sanitary District in enlarging the Southwest works along similar lines.

TABLE 2.—OPERATIONS RESULTS, 1941, MAJOR SEWAGE TREATMENT WORKS

Treatment works	Flow (mgd)	RAW SEWAGE (PPM)		EFFLUENT (PPM)		Solids removed; dry basis (tons)
		5-day B.O.D.	Suspended solids	5-day B.O.D.	Suspended solids	
Calumet	66.4	95	130	9	12	11,836
North Side	206.8	116	146	5.9	9	40,529
Southwest[a]	338	164	205	16[b]	17[b]	67,405
West Side	453.0	131[c]	198[c]	73	90	38,249

[a] Preliminary treatment only, 184 mgd. Activated sludge treatment, 154 mgd. [b] Activated sludge effluent. Weighted average of activated sludge effluent and preliminary tank effluent by-passed 72 ppm, both B.O.D. and solids. [c] Analysis includes preliminary tank sludge from the Southwest works.

TABLE 3.—SLUDGE-HANDLING PLANTS, CALUMET AND SOUTHWEST SEWAGE TREATMENT WORKS, 1941

Treatment works	PERCENTAGE OF DRY SOLIDS			Fuel value of dried sludge (Btu per lb)	Water evaporated (lb)
	Filter cake	Mixed sludge to driers	Dried sludge		
Calumet	17.3	73.4	93.9	5,920	95,380,000
Southwest	16.6	56.2	95.0	6,888	581,474,000

Although this paper deals chiefly with the Southwest sludge-handling plant, the magnitude of the sludge problems at the West Side works must not be overlooked. In 1941 the Imhoff tanks removed a total of 76,897 tons of dry solids, an average of 211 tons per day. The volume of wet digested sludge removed in 1941 was 644,055 cu yd. Of this amount 78,099 cu yd were discharged to temporary lagoons on the plant grounds, the remainder going to the drying beds.

An additional load was thrown on the plant in 1941 by the diversion of preliminary sludge from the Southwest works. This amounted to 132 tons of dry solids per day.

The capacity of the sludge-drying beds is inadequate even though worked to the limit. At times the air-dried sludge removed is rather high in moisture. The average for 1941 was 65.1%, compared to the previous maximum annual average of 54.0%. However, the sludge strippers handle this wet material easily.

Operating results for the year 1941 at the four major works of the Sanitary District are presented in Table 2. In Table 3 data on operation of the sludge-handling plants are presented for the same year.

DISCUSSION

C. W. **Gordon**,[2] **Esq.**—In presenting this paper Mr. Johnson has made available a wealth of design data, and he has discussed operating experiences frankly. This procedure gives the complete picture, and as such will be a great benefit to design and operating engineers. The Chicago Sanitary District was the pioneer in the field of sludge drying and incineration, and its method of sludge disposal is recognized as a success by sanitary engineers everywhere.

During 1940, multiple-hearth plants dried and incinerated 215,912 tons of filter cake. This is equivalent to 70,861 tons of dry solids. Since multiple-hearth plants are not generally designed for fertilizer production, only a small amount was produced on a casual basis.

A total of 651,869 tons of filter cake equivalent to 155,155 tons of dry solids was disposed of by drying and incineration in the United States during 1940. These data do not include the fertilizer production from rotary drier plants. Operations at Milwaukee, Wis., will make an impressive addition to this total. The production at Pasadena, Calif., Toledo, Ohio, Dayton, Ohio, and Grand Rapids, Mich., to mention only a few of the others, will undoubtedly swell the total dry-solids production to more than 200,000 tons.

At first glance one may well ask the question, "Is not this system unnecessarily complicated?" Taking into consideration the necessary functions will show that this is not the case.

First of all, "What is the problem?" The problem is to start with dewatered sludge ranging from 60% to 90% initial moisture, evaporate this moisture, burn the dry solids remaining, remove the odors and fly ash from the gases discharged to atmosphere, and pump the total ash to fill. The final products are then odor and dust-free gases and a sterile ash.

When only fertilizer is being produced, the problem is somewhat simplified. The problem then is to begin with dewatered sludge, evaporate the moisture, remove the odors from the gases discharged to the atmosphere, and transport the fertilizer to storage. The final products in this case are odorless gases and a valuable fertilizer. Thus in producing fertilizer the problem of ash disposal is eliminated. For large operations, such as at the West-Southwest works in Chicago, this is an important factor, since dry sludge normally contains at least 50% ash. Thus, when 300 tons of dry solids are incinerated per day, 150 tons of ash remain for disposal. This ash is quite abrasive, and the maintenance of a large ash handling system is no small item.

Each installation requires individual study, and economic considerations should govern the decision. When there is a question regarding fertilizer production, the better solution is to select a system which can produce fertilizer or incinerate with equal efficiency and without reduction in capacity.

---

[2] Chf. Engr., Drying and Incineration Dept., Raymond Pulverizer Div., Combustion Eng. Co., Chicago, Ill.

The value of the finished fertilizer will depend upon the origin and treatment of the sludge. Activated sludge, as produced at Chicago and Milwaukee, contains 6% nitrogen, whereas digested sludge contains about 2.5% nitrogen. This is not the entire story, since dry sludge fertilizer is a complicated compound; aside from being an excellent soil conditioner, it has numerous positive fertility values. The Research Department of the Chicago Sanitary District, through an extensive test program, has proved that its flash-drying system completely destroys all pathogenic organisms in the sludge. Thus it is a safe fertilizer.

To combine the functions of steam power generation with sludge drying and incineration is fully justified for large operations such as at the Southwest works. For smaller operations, the combination becomes less attractive, particularly when power is available at reasonable rates. The operation of the equipment is essentially automatic and does not require constant attention. For example, in one plant serving an equivalent population of 100,000, one operator supervises both the vacuum filters and the incinerator with entire satisfaction. This is possible in a properly designed plant when the filters and incinerator have a common operating floor.

This paper mentions abrasion at several points in the flash-drying system. Abrasive action at the points indicated is unavoidable. Experience at the Southwest works and all other plants indicates that maintenance costs from this source are very nominal, especially when related to the dry-solids production of the plant. For example, a set of mixer paddles will last from one year to three years, a steel cyclone lining from one year to five years, and a set of cage bars in the flash drier from one year to three years. These wide variations in life show the effect of silica content of the sludge. Obviously, it is desirable to remove all the silica in grit chambers.

Mr. Johnson has stressed the problems of the operating staff. This is perfectly natural for a front-line officer. To obtain the true perspective, it is necessary to balance results against costs and operating difficulties. The results speak for themselves at the West-Southwest works. Taking into account the fact that this is the largest sewage treatment and disposal plant in the world, together with the fact that this is the first time steam power generation has been combined with sludge drying and incineration, the accomplishments reported in this paper are amazingly good.

PAUL HANSEN,[3] M. AM. SOC. C. E.—To the extent that it details the problems resulting from sludge drying and incineration, this paper is of exceptional value. Such papers as this lead to a better understanding of sludge disposal by burning, and aid greatly in advancing current practice.

An installation whereby sludge, whether raw or digested, from the activated sludge process or from sedimentation only can be handled for the production of salable fertilizer, for steam production, and for the destruction of the sludge at high temperatures represents an almost ideal solution of the sludge problem, provided it can be done reliably and economically.

---

[3] (Greeley & Hansen), Chicago, Ill. (Mr. Hansen died February 6, 1944.)

The Chicago Sanitary District has been a pioneer in this field and has developed the process to a point of practicability and reasonable economy. On the other hand, many minor difficulties are still unsolved as suggested by the author. This is but natural, when the complex character of sewage sludge, and especially raw sewage sludge, is considered. Moreover, sewage sludges vary in different installations. In some cases, as at Chicago and a number of other cities in the Middle West, sewage contains large quantities of packing-house wastes introducing the grease problem. In other places, paper-mill wastes influence the character of the sludge. However, paper-mill wastes retard digestion because of their carbonaceous character; but they facilitate heat disposal. Some plants providing partial treatment by sedimentation and partial digestion, as at Buffalo, N. Y., produce a sludge low in fertilizer and heat value. Thus, the sludge varies greatly in its adaptability to incineration and in its value as a fuel and as a fertilizer.

There is also an interrelation, both economic and functional, between heat destruction of sludge, on the one hand, and prior treatment by storage and digestion, on the other. This relationship has not been exploited completely. At Buffalo, a cost analysis of three projects, comprising (1) incineration of raw sludge, (2) incineration of stored sludge (partly digested), and (3) incineration of fully digested sludge, indicated an economic preference for No. (2).

The experiences at Buffalo parallel somewhat the experiences at Chicago. At Buffalo, the cages in the flash driers wore considerably in the beginning. It is clear that the replacement of the bars in the cages must be considered as a regular maintenance problem. The labor and cost involved, however, have been substantially reduced by the utilization of sleeves of especially hard metal that would slip over the bars.

Another difficulty encountered was the wearing of bends in the piping of the ash handling and dry sludge feeding systems. Here again is another element that requires regular maintenance. Difficulties have been overcome by the use of a steam ejector, instead of a vacuum pump, with longer bends and couplings that can be taken apart readily, and with concrete jackets at the bends. Wear in the cyclones was encountered, as at Chicago, and this has been remedied by the use of renewable cement linings.

In a broad sense, the process of sludge handling by incineration, with provision for diverting the material of fertilizing value and with provision for utilizing heat, is in about the same status as the automobile was in the early years of its general use. It works, but it requires considerable tinkering with every hundred miles or so. Little by little the defects and limitations have been revealed and overcome. Today the automobile is reliable for thousands of miles and requires no more attention than systematic oiling and greasing. In the field of sludge incineration, it may now confidently be expected that the minor problems will be overcome and that longer life and more reliable service will result.

LANGDON PEARSE,[4] M. AM. SOC. C. E.—Sludge-drying development at Chicago has been presented briefly by Mr. Johnson. The West Side (Imhoff tank) plant provides a sludge digestion volume of 10,532,000 cu ft. It was expected to accommodate both the sedimentation sludge from the West Side and the excess activated sludge from the North Side. This proved inadequate, because the excess activated sludge would not concentrate within reasonable limits.

The sludge-drying beds were provided for the West Side only, with the expectation of immediate enlargement. The sludge stripping machines are unique.[5] The Sanitary District owns and operates four large and two small locomotives and forty-one side-opening dump cars for moving the air-dried sludge to a dump some 3 miles away. By moving the sludge in a wetter condition than heretofore, the output of the drying beds has been increased materially.

The Southwest works was left unfinished in 1939 because of lack of money. Two years later, when bonding power became available, construction was stopped because of the war.

The most recent data on the plant loading and over-all performance of each plant are given in Tables 4(*a*) and 4(*b*). The suspended solids record for the four plants was as shown in Table 4(*c*). At the West Side works, the record of dried sludge and scum removed from the drying beds was as given in Table 5(*a*).

In the operation of the Southwest works, certain changes have occurred. In the vacuum filters, plastic pipe is replacing the metal pipe in the pipe drainage system. The wire mesh underdrainage supporting the filter cloth is now replaced by a wooden drainage grid. The winding wire is being discarded for a system of crimping the cloth in grooves with locking splines.

Lately, the filter cake has been ranging from 83% to 86% moisture. This is fairly representative for past seasons at Milwaukee, Wis. Fly ash is disposed of in lagoons near the plant. It lacks commercial value.

Recently the grease collected from the preliminary tanks at the Southwest works was sold for 0.55¢ per lb, f.o.b. trucks at the works. The output is estimated from 5,000 to 20,000 lb of scum per day, with an average of 14,000 lb. This contains approximately 50% grease, of which 15% is unsaponifiable. The heat-dried activated sludge has been sold in bulk under a five-year contract to an exclusive purchaser, on an analysis basis. The average results are given in Table 5(*b*).

When sludge is sold as fertilizer, there is an additional expense to the Sanitary District for fuel and incidental charges, which reduces the gross receipts. At the Calumet works in 1941 this cost slightly exceeded the receipts. In 1943 the receipts will be somewhat greater than in 1942.

To take care of the large sludge load at the West-Southwest works, the Sanitary District has completed a 16-in. sludge pipe line, 4⅔ miles long, to discharge excess activated sludge for the duration of the war into 78 acres of lagoons, 16 ft deep, divided into 16 units. This will provide about 45,700,000 cu ft of storage. Sludge will be seeded with digested sludge from the West Side Imhoff tanks. The heat-drying plant will be operated to capacity. Any excess sludge will go to the lagoons. With the new blower just completed and with rehabilitated air-diffuser plates, the treatment of 300 mgd of sewage by activated sludge should be reached next spring at the Southwest works.

---

[4] San. Engr., The San. Dist. of Chicago, Chicago, Ill.

[5] *Engineering News-Record*, January 21, 1932, pp. 82–85.

Among the problems of the sewage works designer and operator is the tuning up of the works. The varied works of the Sanitary District have shown a marked difference. The West Side works went into service with but little difficulty. The North Side works also began operating smoothly, and continued with the least operating problems of any of the activated sludge works of the Sanitary District. This is due in part to the transfer of the sludge disposal to the Southwest-West Side works and in part to the character of the sewage.

TABLE 4.—SLUDGE

Line No.	Sewage works	(a) PLANT LOADING				
		Million Gallons Daily		Population		
		1941	1942	1940 census	1941[a]	1942[a]
1	Calumet[c].......	66.4	82.4	327,092	313,800	377,500
2	North Side[c]....	206.8	195.8	1,019,967	1,184,000	1,122,000
3	Southwest[c].....	338.0	331.0	1,127,682	2,770,000	3,014,000
4	West Side[d].....	453.0	423.0	1,487,773	2,244,000	2,180,000
5	Total.......	1,064.2[e]	1,032.2[e]	3,962,514[f]	6,512,300[g]	6,780,800[g]
6	Over-all[e]....	....	....	....	....	....

[a] Total equivalent population. [b] Including grit. [c] Activated sludge. [d] Imhoff tanks. [e] For four major plants.

TABLE 5.—ANALYSIS OF DRIED SLUDGE

Year	(a) WEST SIDE WORKS					(b) SOUTHWEST WORKS			
	Cubic Yards		Average Analysis (%)			Tons delivered	Receipts, in dollars	Average Analysis	
	Scum	Sludge	Moisture	Volatile	Nitrogen			Ammonia	P_2O_5
1936	577	36,158	42.0	41.8	1.96	....	....	....	....
1937	....	72,379	52.9	42.1	1.79	....	....	....	....
1938	....	83,156	54.0	43.8	1.82	....	....	....	....
1939	2,064	61,253	49.5	42.5	1.91	11,303	71,102	6.00	....
1940	....	129,064	65.7	44.0	2.12	58,464	346,307	6.40	2.25
1941	9,134	149,867	65.1	47.1	2.03	56,597	282,428	6.24	2.27
1942	8,666	164,973	70.3	44.3	2.22	51,729	391,386	5.97	2.45

The Calumet works was used as a pilot plant for the Southwest works in so far as dewatering, heat drying, and incineration were concerned. Various changes were made over a period of some three years before a complete working plant resulted. Industrial wastes presented other difficulties. The removal of waste-iron pickling liquor from the system greatly reduced the clogging of air diffusers.

The Southwest works began operation in May, 1939, as a partly completed plant. It remains such today. It is unable to give activated sludge treatment to the entire flow because of lack of air compressors. Further, it lacks adequate sludge-handling equipment. The diffusers have now become clogged to a degree requiring replacement in 1944. The air-cleaning devices need rehabilitation. Suitable grit chambers are required, as none were originally provided. The designers were handicapped by lack of funds. When funds became available, war conditions stopped construction. The operating force has done its best to cope with the difficulties of operating under war conditions.

DRYING AT CHICAGO, ILL.

(b) PERCENTAGE REMOVAL				(c) TONS OF SOLIDS:				Line No.
Suspended Solids		Five-Day B.O.D.		Arriving		Removed[b]		
1941	1942	1941	1942	1941	1942	1941	1942	
90.8	90.7	90.5	90.2	13,051	16,075	11,836	14,541	1
93.8	92.8	94.9	94.5	45,782	41,279	40,529	38,156	2
64.9	62.4	56.1	56.5	105,038	103,066	67,405	48,204	3
54.5	51.2	44.3	44.7	102,182	107,014	38,249	44,485	4
....	....	....	....	266,055	267,434	158,019	145,388	5
67.5	65.7	60.6	61.5	....	....	....	....	6

[f] Entire Sanitary District. [g] Including miscellaneous plants and corn products waste.

Those interested can follow the various problems in the Annual Budget Report of the Chief Engineer of The Sanitary District of Chicago for 1940–1943, inclusive. The period of tuning up and rehabilitation of the Southwest works apparently will continue until several years after the end of the war. Meanwhile, the plans for completion of the West-Southwest works are in process.

This discussion is intended to add certain phases of the sludge-handling developments, which Mr. Johnson has omitted, to complete the picture of the sewage treatment problem at Chicago and the relation of the sludge handling to it. The recent developments are noted in the sale of scum, in construction of lagoons and pipe line, and in disposal of fly ash, as well as in the difficulties in tuning up and operating an incomplete Southwest works under war conditions.

LLOYD M. JOHNSON,[6] ESQ.—Mr. Gordon has properly indicated that the problem of ash handling and disposal is minimized when fertilizer is produced. There is a further benefit at the Chicago Southwest plant in that the slag formation on the wall and roof tubes in the furnaces is held at a minimum. In burning a material like sludge with a high percentage of ash (compared with coal), the amount of slag is an important operating factor.

Although, essentially, operation appears automatic in a plant combining steam generation and sludge disposal, the need for properly trained and competent operators cannot be overstressed. The balancing of the various elements introduced in this combination results in a more delicate plant to operate than a straight steam generation plant.

---

[6] Commissioner of Streets and Electricity, City of Chicago, Chicago, Ill.

Mr. Hansen notes the economy of handling partly digested sludge, where the treatment is sedimentation alone. Sludge from such a plant has a limited market as fertilizer, except in wartime, whereas with activated sludge treatment the economics may be different. The possibility of recovery of sludge gas for development of power is important.

Mr. Pearse has added data on procedures and operating results. In addition, the recent sale of grease-containing scum from a pumping station tributary to the Southwest works proves worth while at least during these times. Under a 2-yr contract, the Sanitary District receives 1.10 ¢ per lb for the scum at point of skimming. The contractor furnishes all labor for skimming and loading.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2226

DEVELOPMENT OF THE CHICAGO TYPE BASCULE BRIDGE

By Donald N. Becker,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Alonzo J. Hammond, Clinton D. Hanover, Jr., Armour T. Granger, C. B. McCullough, and Donald N. Becker.

Synopsis

On the background of a description of the waterway system that serves the navigation needs of Chicago, Ill., the author describes the development of a bridge type that serves the peculiar local requirements. The evolution of the movable type of bridge is traced through several phases, including the early "jackknife," the vertical lift, the rolling lift, and, finally, the various developments of the bascule type.

Introduction

The Chicago River (see Fig. 1) lies along the north side of the central business district, branching just west of it, with the North Branch traversing the north side of the city in a more or less northwesterly direction and the south branch traversing the south side in a southwesterly direction. In the extreme south end of the city is the Calumet River system. In Fig. 1, the various bridges mentioned herein are given numbers that are identified as follows: 1. West Cortland Street (Clybourn Place); 2. West Division Street (River); 3. West North Avenue; 4. North Halsted Street (Canal); 5. West Kinzie Street; 6. West Washington Street; 7. West Grand Avenue (Indiana Street); 8. East 92d Street; 9. West 35th Street; 10. West Chicago Avenue; 11. West Webster Avenue; 12. West Belmont Avenue; 13. West Monroe Street; 14. North Franklin-Orleans Street; 15. West Adams Street; 16. East 106th Street; 17. West Roosevelt Road (Twelfth Street); 18. North Damen Avenue (North Robey Street); 19. South Damen Avenue (South Robey Street); 20. North Wabash Avenue; 21. West Ogden Avenue (Canal); 22. West Ogden Avenue (River); 23. Outer Drive (River); 24. Outer Drive (Canal); 25. South Ashland Avenue; 26. North State Street; 27. Weed Street Crossing of Chicago River; 28. Canal Street Crossing of Chicago River; 29. South Halsted Crossing of

---

Note.—Published in February, 1943, *Proceedings*. Positions and titles are those in effect when the paper or discussion was received for publication.

[1] Engr. of Bridge Design, Dept. of Public Works, City of Chicago, Chicago, Ill.

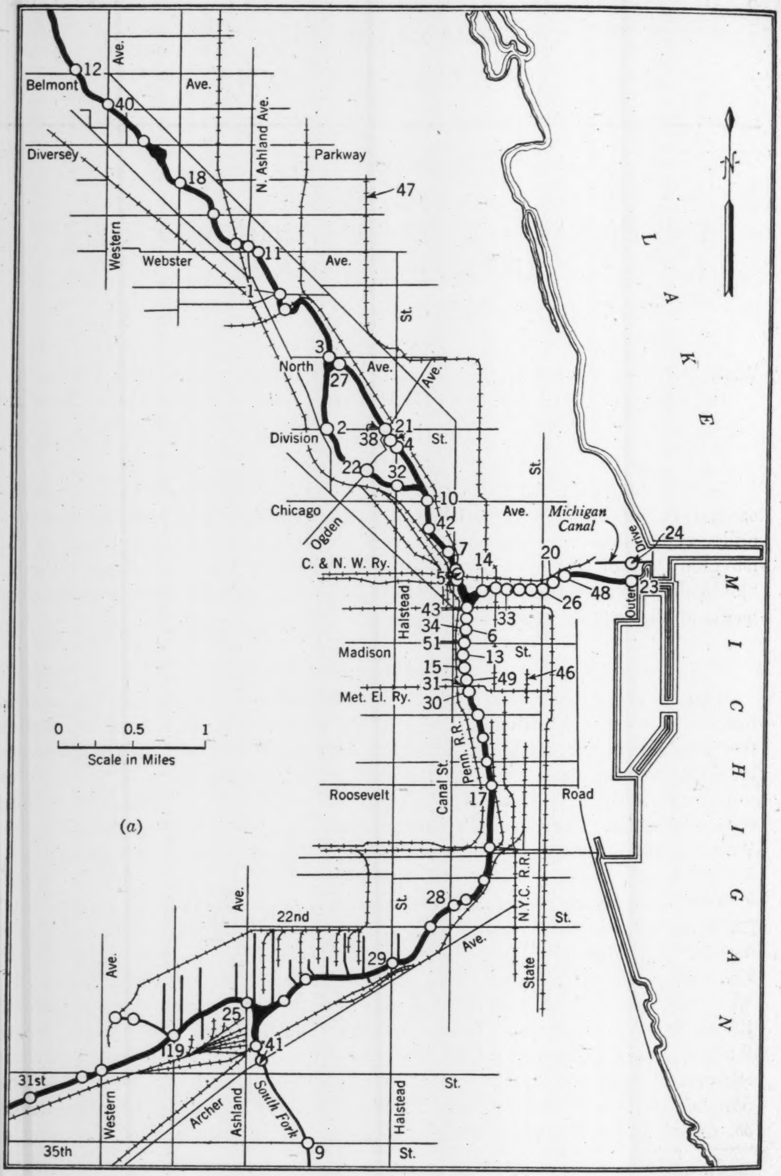
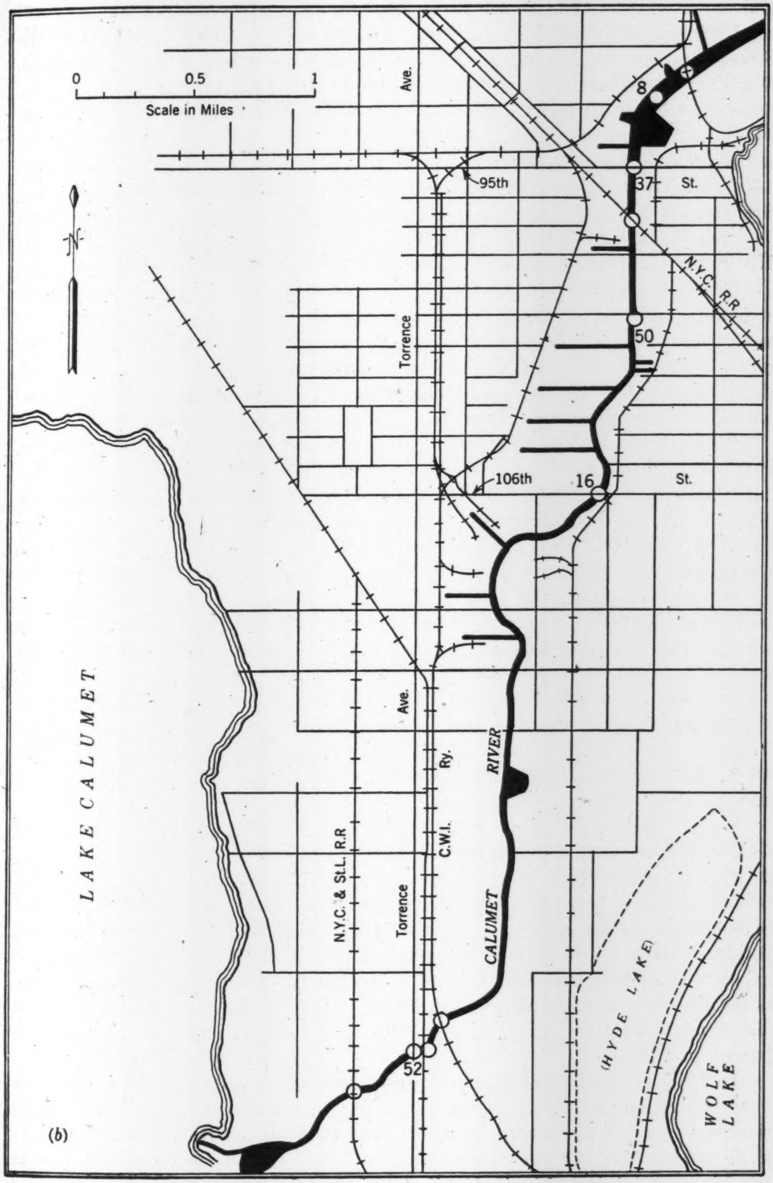

Belmont
Ave.
Diversey
Parkway
N. Ashland Ave.
Western
Webster
Ave.
St.
North
Ave.
Ave.
Division
St.
Chicago
Ave.
Ogden
C. & N. W. Ry.
Halstead
Madison
St.
Met. El. Ry.
Canal St.
Penn. R.R.
Roosevelt
Road
N.Y.C. R.R.
State
St.
22nd
Ave.
St.
Ave.
Ave.
31st
Western
Archer
Ashland
South Fork
Halstead
St.
35th
St.
Michigan Canal
Outer Drive
LAKE
MICHIGAN
0
0.5
1
Scale in Miles
(a)
12
40
18
47
11
1
3
27
2
21
38
4
22
32
10
42
7
14
20
24
5
43
34
51
15
31
30
33
6
13
49
46
26
48
23
17
28
29
25
19
41
9

FIG. 1.—DRAINAGE


0
0.5
1
Scale in Miles
N
Ave.
95th
St.
8
37
N.Y.C. R.R
50
Torrence
106th
16
St.
LAKE CALUMET
Ave.
Ry.
RIVER
C.W.I.
N.Y.C. & St.L. R.R
Torrence
CALUMET
52
(HYDE LAKE)
WOLF LAKE
(b)

System, Chicago, Ill.

Chicago River; 30. Van Buren Street Bridge; 31. Metropolitan Elevated Railroad Bridge; 32. North Halsted Street crossing of North Branch; 33. La Salle Street Tunnel; 34. Washington Street Tunnel; 35. Clark Street Bridge; 36. Wells Street Bridge; 37. East 95th Street and Calumet River; 38. West Division Street and North Branch Canal; 40. North Western Avenue and North Branch; 41. West Archer Avenue and South Fork of South Branch; 42. West Erie Street and North Branch; 43. West Lake Street; 44. Lake Street Elevated Railway; 45. West Lake Street and Elevated Railway; 46. Dearborn Street Subway System; 47. Northwestern Elevated Railway; 48. Michigan Avenue; 49. West Jackson Boulevard and South Branch; 50. East 100th Street and Calumet River; 51. Madison Street; and 52. South Torrence Avenue and Calumet River.

Before civilization reached Chicago these rivers were relatively shallow streams, navigable only in the sense that canoes and shallow draft vessels could enter them from Lake Michigan. With the development of the city, both of these rivers were deepened and widened so that they are navigable for considerable distances for the largest lake-going vessels. They are generally equipped with dockwalls providing channels varying in width from 100 ft to 250 ft.

When the early settlers traveling overland first reached the Chicago River, they established ferries, but these soon proved inadequate for the traffic, and as early as 1833 a crude float bridge, manipulated by cables, was installed. In 1834, a wooden drawbridge similar to a double moat-bridge was built across the river, only to be torn down five years later because it was an obstruction to navigation.

By 1840, however, the land traffic was so heavy that another bridge was built over the river. This time it was a pontoon type that could be pulled open when needed for river traffic. Several of this type were built in the next few years. In 1854, the first pivot swing bridge with wooden trusses was built across the river; and in 1856 the first all-iron bridge was built. By the time of the great Chicago fire (1871), there were twenty-seven movable bridges costing well over a half million dollars. The great fire destroyed eight of these. By 1890, the number of bridges had increased to forty eight, some through the annexation of adjacent towns to the city.

During this period many disadvantages of the swing-type bridge presented themselves, so that thought was directed to the new types that were being developed in the minds of embryo inventors. In 1890 a so-called "jackknife" bridge, built under patents issued to Capt. W. Harmon, was installed at Weed Street, the action of each half of the bridge resembling that of a loosely-hinged jackknife as it lowered from the vertical position. It was expected that, if experience showed that this type of bridge could be operated safely and economically, it would be preferable in most cases to the swing bridge. It was also thought that the raised floor, in the open position, would solve the street barrier problem.

In 1892 so much objection was raised to the swing bridge, built only two years previously at Canal Street at a bend in the river, that it was ordered removed and was replaced at a cost of $46,844.90 with a Harmon-type of jackknife bridge (Fig. 2) providing an 80-ft draw. From the beginning it was troublesome due to light construction and poor execution.

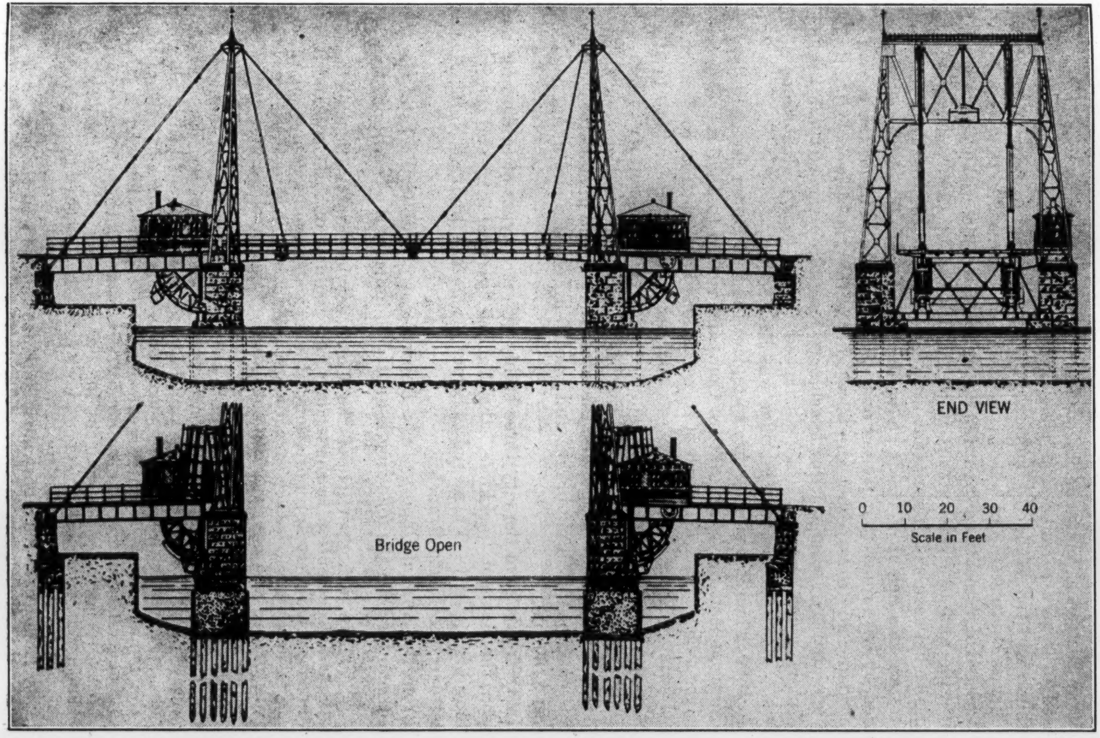

END VIEW
0 10 20 30 40
Scale in Feet
Bridge Open

FIG. 2.—SOUTH CANAL STREET FOLDING BRIDGE OVER THE SOUTH BRANCH OF THE CHICAGO RIVER, 1892

In 1892 when it became necessary to replace the old swing bridge at South Halsted Street, the navigation interests and the Drainage Board (later The Sanitary District of Chicago) objected so effectively that the War Department refused to issue a permit for a swing bridge. After considering the Harmon jackknife and the Waddell lift bridge, the city decided in favor of the latter, which was completed and opened to traffic on March 22, 1894 (see Fig. 3). It consisted of a steel truss span, 130 ft long by 58 ft wide, with a single roadway 40 ft wide and two sidewalks 8 ft wide. Its weight (280 tons) was lifted vertically between two towers that were braced by a truss between the top of the towers at a height of 155 ft above the river level. The lift span was suspended by wire ropes that passed over sheaves at the tops of the towers and were connected to cast-iron counterweights within the towers. The bridge was operated by steam hoisting machinery located on top of the lift span at its center, which operated cables attached to the tops and bottoms of the towers. The total cost of the bridge was $242,880.

In 1894, when it was necessary to replace the Van Buren Street bridge, and the Metropolitan Elevated Railroad Company wished to build a bridge across the river a short distance north of it, conditions being such as to prohibit the use of swing bridges, a new type was adopted (see Fig. 4) for both locations. This bridge was hailed as the embodiment of an eleventh century idea brought up to date, as the combination of the bascule (which was the most common type of medieval moat bridge) and the latest electric and pneumatic appliances. It was equipped with air brakes and two 50-hp electric motors on each leaf. It cost $169,700.

In 1895 this gave the city three highly different types of bridges for comparison, the Harmon "jackknife," the Waddell "vertical lift," and the Scherzer "rolling lift." In 1896 a second Scherzer bridge was built at North Halsted Street over the North Branch.

From the Annual Report of the Department of Public Works of the City of Chicago for 1898 the following is quoted to indicate the conditions then:

> "There is no portion of the City's service that is in such need of immediate attention as the bridges and viaducts of the City, and it is to be regretted that appropriations of sufficient amount cannot be secured to make proper and necessary repairs. Many of the structures are old and dilapidated and have reached that stage of decay where wise expenditure of money cannot be made to repair them, and they should be replaced by new structures. All the bridges to be constructed hereafter should be constructed on such a plan that center piers will not be necessary. The practice of placing obstructions of any character in the Chicago River should be discontinued, with a view to accommodate navigation and to conform to the policy of the statutes governing the construction of the Sanitary Drainage Canal. While the importance of the Chicago River appears to be appreciated by the citizens in the discussion of the development of Chicago, no practical steps have been taken by the City to preserve it as a harbor for modern vessels. A reference to the tables submitted by the City Engineer in his report must convince anyone that the greater facilities offered in the Calumet River are diverting a large portion of the business from the Chicago River, and while the Calumet interest should be con-

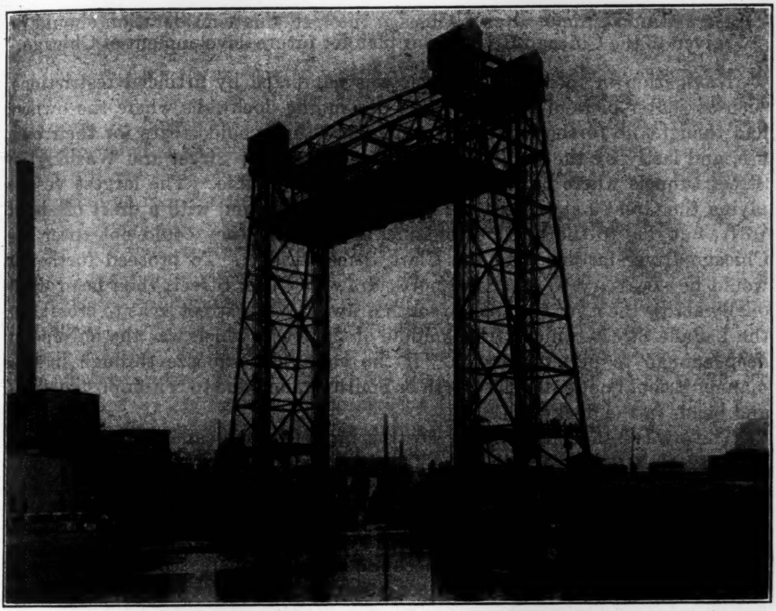


FIG. 3.—SOUTH HALSTED STREET BRIDGE, FACING NORTHEAST ACROSS THE CHICAGO RIVER, 1894

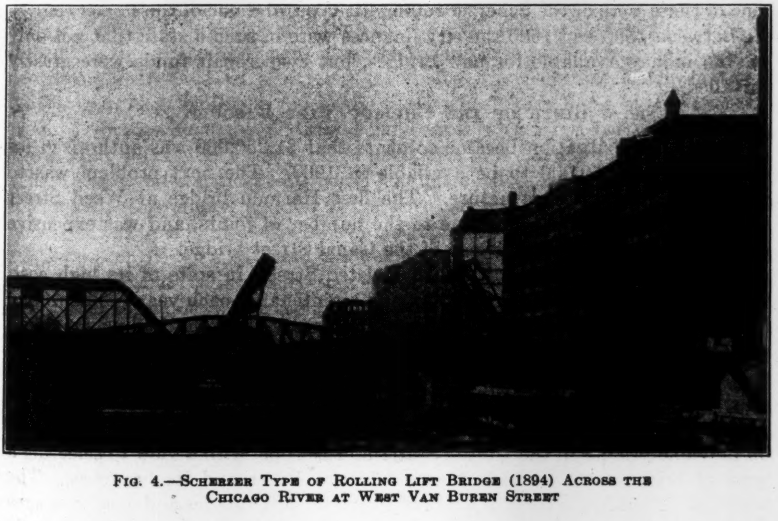


FIG. 4.—SCHERZER TYPE OF ROLLING LIFT BRIDGE (1894) ACROSS THE CHICAGO RIVER AT WEST VAN BUREN STREET

served and promoted as a Chicago interest, due consideration should be given to the Chicago River in our plan for future development of Chicago."

Navigation in the Chicago River was hampered by artificial restrictions, such as center piers of swing bridges, protruding dockwalls where the owners had thought no further than how much land they could create for their own use, and lastly by the shallow clearance over La Salle Street and Washington Street tunnels where only 17 ft of water was available. The largest vessels plying the Great Lakes were 432 ft long by 48 ft beam, with a draft of about 16 ft; but due to the tortuous channel in the river they could not enter the Chicago River farther than the Clark Street bridge. To proceed further it would be necessary to enter the north draw at Clark Street, then head diagonally across the river toward the south draw at Wells Street so as to cross over the LaSalle Street tunnel at the middle of the river, which was the only point deep enough to permit passage. If the vessel did squeeze through in this manner it could not enter the North or South Branch due to center obstructions and bends in the dockwalls.

The law creating the Sanitary District provided for a canal with a discharge capacity of 10,000 cu ft per sec. The District embarked on a policy of constructing a 200-ft channel in the South Branch with a center depth of 26 ft and a cross section of 4,800 sq ft so as to limit the current to about 1¼ miles per hr. Property was condemned and the docks were moved back. As the city was in no position to replace bridges to accommodate this increased flow, especially where bridges were in fair to good condition, an agreement was made whereby the District would rebuild certain structures and turn them over to the city. Under this arrangement ten bridges were completed between 1902 and 1907—nine of these were of the Scherzer rolling lift type, and one of the Page type.

Between 1896 and 1900 the city finances were in such a state that not only was no money available for new bridges, but even repair funds were greatly curtailed.

Birth of the Chicago Type Bascule

In 1900 the situation became so acute that $1,250,000 was authorized for eight bridges, $850,000 to be available in 1900. The next problem was to determine the type of structure. The first Harmon bridge at Weed Street was continually out of order, due to the number of joints, and was expensive to maintain. This was true also of the Canal Street bridge.

The vertical lift bridge at South Halsted Street, in spite of its high cost, was proving quite costly in operation and maintenance each year. The rolling lift bridge, although admittedly more satisfactory than the folding jackknife and the vertical lift bridge, had the very important disadvantage that the center of gravity traveled back and forth as the bridge opened and closed, thus requiring a solid foundation.

In 1899 the city's Bridge Division made a critical analysis of the literature on movable bridges in the United States and Europe, with a view to selecting a type of bridge most suitable to the Chicago River and its branches. The results were put in the form of a report giving the advantages and disadvantages of the various movable types in use. The type known as the trunnion bascule was considered to be the one that could meet Chicago requirements most fully and satisfactorily.

Three designs were prepared by the city, differing in appearance, method of mounting, etc., but all involving the principle of revolving on a fixed trunnion. The three designs were then submitted to a Board of Consulting Engineers, consisting of E. L. Cooley, the late Ralph Modjeski, M. Am. Soc. C. E., and the late Byron B. Carter. The Board selected and recommended the design designated as No. 3 with some modifications.

The design was described in the 1900 report of the Department of Public Works as a fixed-center, double-leaf, counterbalanced bascule bridge. Each leaf had three through trusses operated by racks on their curved tail ends, the tail ends descending into a pocket or tail pit in the abutments when the bridge was open. The total width of the structure was 60 ft, the trusses being 21 ft, center to center, and the sidewalks being carried by 9-ft cantilever brackets. The pivot bearings were 4 ft back from the center lines of the river piers and were carried by box girders spanning the tail pits in the abutments. The approaches were carried by I-beams, supported by cross girders resting on the box girders and extending from the river piers to concrete abutments independent of the abutments of the moving span. The bridge was designed to turn through a maximum angle of 76° 58′, which required a travel of 35 ft on the pitch line of the rack. The break in the roadway was on the river side of the pivot.

The machinery for operating each leaf was under the approach roadway, each leaf being operated by means of a pinion gearing with a rack on the curved tail end of each truss. The teeth on this rack had openings between them, preventing the clogging of the rack by dirt or ice. The teeth of the operating pinion dislodged any accumulation that would then fall through the rack. Along the top of the abutment a shaft carried three pinions and had two sets of driving gear, each set of driving gear being driven by an electric motor of 38 hp. On each motor-driven shaft was a wheel for a band brake, operated from the bridge house by a vertical hand lever with a latch handle. Besides these hand brakes there was a worm-resistance brake at the middle of the rack pinion shaft and a hand-brake wheel was fastened to the shafts of the worm. To prevent resistance or friction in the worm brake when not needed, the worm was revolved by a light continuous hemp rope that passed over grooved pulleys on the motor-driven shafts and the worm shaft. The band brake was applied to the worm shaft to introduce the resistance, thus checking the motion of the worm and consequently of the gearing by which it was connected to the main shaft.

The machinery was designed so that the opening of the bridge, from the moment it was closed to traffic to the moment when the leaves reached their highest position, would not require more than 1 min in calm weather or 2½ min with a 70-mile wind blowing in a direction unfavorable to the operation of the bridge. Pneumatic buffers arrested the motion of the leaf at each end of its travel.

When the bridge was closed, the center of gravity of each leaf was on the river side of the pivot, and as the movable part of the roadway did not extend back of the pivot it was only necessary to lock the leaf against further downward movement. This was done in connection with the bridge-closing buffers, by allowing the leaf to pull the lower buffer pistons against the cylinder ends, the cylinders being anchored to the masonry.

When the bridge was closed, the action of the buffer was as follows: As the leaf approached the end of its closing movement, a hooked lug at the lower end of the arched back of each truss picked up a 6½-in. pin, connected by eyebars to a combined cross head and piston. This pin forced the piston upward in an inverted cylinder, meeting the increasing resistance of the compressing air, which absorbed the energy of the closing leaf. When the pressure reached 350 lb per sq in., the air escaped through a relief valve and allowed the piston to rest against the cylinder head above it.

As the leaf began to open, a second lug near the aforementioned hooked lug returned the buffer and connections, by means of a hooked lever, to position in readiness for closing.

The action of the upper buffer in arresting the leaf at the end of the opening movement was similar to that of the lower buffer, the difference being that the upper buffer was operated through an intermediate lever pivoted to special pairs of stringers of the approach roadway, which stringers also supported the upper buffer cylinders. The rear end of the lever acted on the piston through a short link. The front end of the lever was operated in essentially the same manner as the pin of the lower buffer, by lugs in the upper end of the arched back of each truss.

Quoting from the Report of the Board of Consultants:

> "The principle of the trunnion bearing meets with our approval as a very simple solution of the problem, the chief advantages being the constant point and direction of application of the load on the foundation, whether the bridge is in motion or stationary, and the reduction of the number of moving parts to the minimum. The first advantage is of no great consequence if the piers are placed on an unyielding foundation, but with such foundations as can be obtained in Chicago at the majority of bridge sites, it is of great importance. The trunnion type of bridge used in these designs is an old and tried device, and is not covered by patents."

The consulting engineers called attention to the fact that the bascule design permits the center of gravity of the moving bridge to be placed in or near the trunnion axis. Placing the center of gravity a short distance from the axis of the trunnion toward the draw opening and the arrangement by which the fixed floor relieves the tail end from any possible live load, had the advantage of tending to hold the bascule firmly in position when closed, and so avoided the absolute necessity for heel locks. There was no tilting effect when a live load came on to the bridge.

The Board recommended a three-truss design, as had been prepared by the city, in preference to a two-truss design for the same width of floor, that the curved bottom lateral struts be omitted and a stiff lateral system used. Also, it was recommended that, when the bridge was raised, the floor beams be supported by angles running from the middle of each floor beam to the intersection of the lateral angles, and that the heel ends of the trusses be connected by a transverse brace, to add lateral stiffness to the racks and the counterweights.

The Consulting Board further recommended that adjustable resting blocks be placed in front of, and near, the trunnions so that when the draw was being closed, the load could be transferred from the trunnions to the resting blocks. This facilitated the lubrication and practically eliminated the starting friction of the trunnions. To meet all possible conditions of wind and incidental changes in the position of the center of gravity, pawls or heel locks were used with the resting blocks. They were capable of lifting the tail ends and holding them firmly against the anchorage. This arrangement was also claimed to facilitate the overhauling and adjustment of the trunnion bearings.

Air buffers were considered practical, provided they could be designed so that the pistons would not withdraw from the cylinders, and provided they were packed with cupped leather or other material, to prevent entrance of foreign matter or grit into the cylinders. Air was controlled in the cylinders by free entrance through check valves, and by allowing exit through adjustable openings, for buffing. Springs insured proper action of the pistons. Exceedingly strong racks on the heel of the draw, with through openings between the rack cogs, met with approval as they will not accumulate ice in cold weather. On those rare occasions when sleet may coat the racks with ice, it was considered that the shaft and pinions were strong enough to force the ice through the cog openings.

There was no practical difficulty in making the driving machinery strong enough to resist all probable strains, and, therefore, the engineers recommended the following general arrangement: The three pinions are mounted on one line of shafting, with specially strong bearings, being driven by two gears; these two gears are connected by a jackshaft and suitable pinions, with two gears mounted, and clutch-driven, on this jackshaft. The jackshaft gears are driven by two independent gearing trains to two motors. All these gears, shafts, and motors are arranged in a vertical or inclined train, so proportioned that all gears are between their respective shaft bearings, which were to have ample length and all were to be mounted, as far as possible, on one structure. The gearing could easily be made stronger and safer and more accessible for overhauling and repairs than that of any bridge then built in the city.

The worm-gear brake for emergency use had novel merits compared with the usual band brake on the motor shaft. If designed with large pitch and large angle of worm, running in an oil bath, the friction of the gears running light may not be large. By the ingenious application of a light rope belt (to revolve the worm), the friction due to the thrust of the worm was practically eliminated until the brake was applied. In the detail design, the desirability of friction brakes applied directly to the curved segments of the trusses can be developed. Finally, quoting the Report:

> "It is fair to state that the assumed wind loads against opening or closing the bridge may never prevail in the practical handling of the bridge, and may be regarded as so much additional security. The fact is that ships do not navigate when such winds prevail, and there would be

> no occasion to open the draw at such times. In the case of the Tower Bridge (London), the reserve power put in on account of extra wind loads was reported as 'never used' at the end of two years.
>
> "In general, our board is of the opinion that the flow of the river should be provided in the opening of the draw wherever practicable."

In the light of forty years of experience, the recommendations of this Board were almost prophetic, as so many of their recommendations have been adopted with the passage of years.

Early Examples of the Chicago Type

Meanwhile, the 95th Street swing bridge, over the Calumet River, had collapsed on August 17, 1899. This location was used as the site for a new bridge, the call for competitive bids to be opened on May 15, 1900. A similar call for bids was advertised to be opened on June 1, 1900, for a bridge at Division Street over the North Branch Canal.

The specifications were drawn very broad for the purpose of giving competitors as wide a scope as possible in their designs and selection of a new type of bridge. The only restrictions in regard to type were in both cases:

> "The bridge shall be a movable structure without center pier and shall leave a channel with a clear opening of 120 ft. at 95th Street and 80 ft. at Division Street. The bridge shall have two roadways 18 ft. in the clear, or one roadway 38 ft. in the clear between the wheel guards and two sidewalks 8 ft. in the clear, and shall have an overhead clearance of 16 ft. There must be an underbridge clearance of 16½ ft. above Chicago Datum for a space of at least 40 ft. at right angles to the center of the channel. The general appearance of the bridge shall be as graceful and attractive as the nature of the structure will permit."

Five designs were submitted by outside bidders, some of these bidders having two or more alternate designs differing in matters of detail. With one exception, the designs were for bridges revolving on fixed bearings.[2] The contract was awarded to a bidder following one of the city's designs.

Contracts for both 95th Street and Division Street bridges were awarded so that work was started in July, 1900. Meanwhile the city's design No. 3 was modified and arranged for a bridge at Clybourn Place (later Cortland Street) over the North Branch and contracts were awarded in the late fall. These plans embodied many of the alterations suggested by the Board of Consulting Engineers, especially, the use of a full tail pit instead of separate wells for each truss. Due to construction difficulties at both 95th Street and Division Street, the Clybourn Place bridge (Fig. 5) was completed first and opened to traffic on May 24, 1902; the 95th Street bridge was not opened to traffic until April 1, 1903; and the Division Street bridge on February 1, 1903. In the construction of the 95th Street and Division Street bridges, the bottom of the abutment was lowered to the elevation of the river piers but the separate pockets for the counterweight ends of the trusses were retained. These were made of sheet steel covered on the outside with concrete and have been a frequent cause of repairs. The machinery was set on structural supports, and it has been difficult

---

[2] *Engineering News*, January 10, 1901, p. 18.

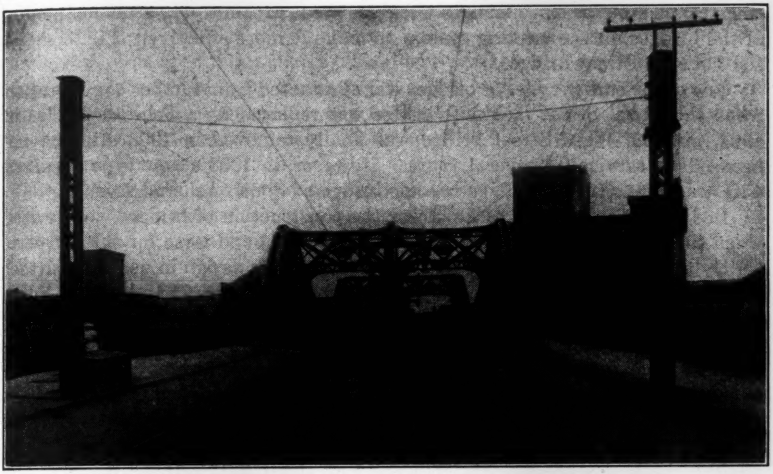
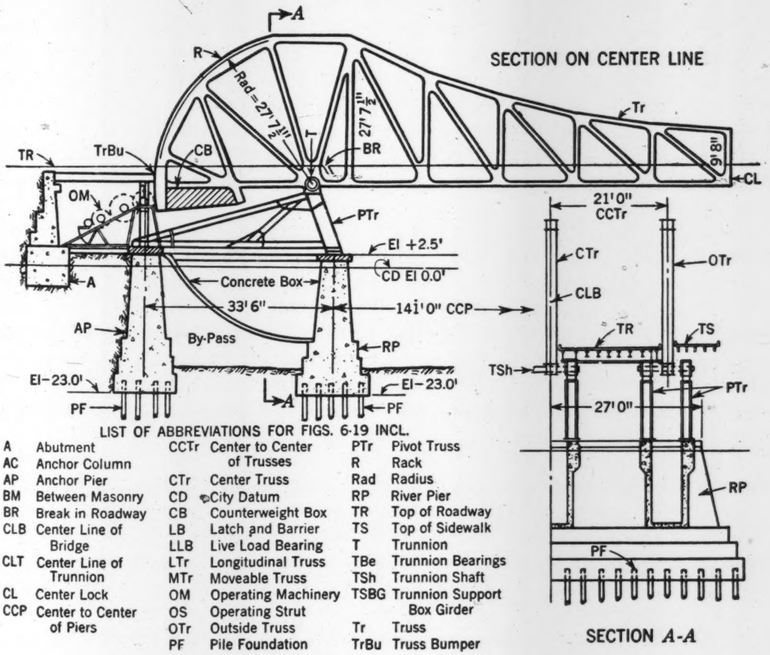


FIG. 5.—WEST CLYBOURN PLACE BRIDGE (1902)



A
SECTION ON CENTER LINE
R
Rad = 27' 7½"
27' 7½"
T
Tr
TR
TrBu
CB
BR
9' 8"
CL
OM
PTr
El +2.5'
CD El 0.0'
A
Concrete Box
33' 6"
141' 0" CCP
By-Pass
AP
RP
El−23.0'
El−23.0'
PF
A
PF
21' 0"
CCTr
CTr
OTr
CLB
TR
TS
TSh
PTr
27' 0"
RP
PF
SECTION A-A

LIST OF ABBREVIATIONS FOR FIGS. 6-19 INCL.

A	Abutment
AC	Anchor Column
AP	Anchor Pier
BM	Between Masonry
BR	Break in Roadway
CLB	Center Line of Bridge
CLT	Center Line of Trunnion
CL	Center Lock
CCP	Center to Center of Piers
CCTr	Center to Center of Trusses
CTr	Center Truss
CD	City Datum
CB	Counterweight Box
LB	Latch and Barrier
LLB	Live Load Bearing
LTr	Longitudinal Truss
MTr	Moveable Truss
OM	Operating Machinery
OS	Operating Strut
OTr	Outside Truss
PF	Pile Foundation
PTr	Pivot Truss
R	Rack
Rad	Radius
RP	River Pier
TR	Top of Roadway
TS	Top of Sidewalk
T	Trunnion
TBe	Trunnion Bearings
TSh	Trunnion Shaft
TSBG	Trunnion Support Box Girder
Tr	Truss
TrBu	Truss Bumper

FIG. 6.—WEST DIVISION STREET CANAL CROSSING (1903)

to maintain alinement. Changes were also made within a year after completion of the bridges, in the braking system to make it more effective. Fig. 6 shows the Division Street bridge.

One of the earlier types of bridges was eliminated from further consideration when the Canal Street jackknife bridge was replaced by a Scherzer bridge in 1903, and the Weed Street bridge was finally removed in 1905 after having been out of service for several years. However, in 1903 a new type appeared with the completion of a Page bascule bridge at South Ashland Avenue.

In 1904 a new method of financing the construction of bridges was devised when the State Supreme Court ruled in favor of a bond issue for this purpose.

By the end of 1904 the city had five bridges of its design in use with further improvements in the last two, one at Division Street (Fig. 7) and one at North Western Avenue, both over the North Branch. In both of the two later bridges, instead of using shaped iron castings bolted to the trusses for counterweight as was done in the first three bridges, a structural box was framed between the tail ends of the trusses. These tail ends were sloped downward back of the trunnions, whose supports were changed to tapering box girders sloping upward at the back so as to pass over the counterweight box. The box was filled with the counterweights consisting of about 22% shaped castings and the remainder a mixture of pig iron, punchings, and cement grout to produce a material weighing about 375 lb per cu ft. The bottom of this box was floored with a grillage of old rails to carry the weight of the extremely heavy counterweight material to the main girders. There were many advantages in this type of construction. First, it allowed the center of gravity of the counterweight to be placed farther back from the trunnion, thus saving material. Second, it eliminated the necessity for the specially shaped castings with their greater unit cost. Third, the counterweight material did not have to be bolted piece by piece to the trusses with the danger, later, of dropping if bolts rusted away.

Four of these bridges were on pile foundations, but, at North Western Avenue bridge, piles could not be driven below −22 ft Chicago Datum because of a very hard indurated clay, which could safely sustain a load of 4,000 lb per sq ft. Accordingly, piles were omitted and the masonry rested directly on the soil. This bridge also had the tail pits especially waterproofed with a 4-in. layer of Portland cement mortar on both the inside and outside walls below water level. A distinct improvement in the machinery consisted of taking the motors from the two gear trains and combining them in a unit in back of the center trusses. Two sets of hand brakes were operated by hand levers in the operator's room. Also the pneumatic buffers at each end of the travel were eliminated and wooden bumper blocks were placed on the under side of the trunnion girder with which the top of the counterweight box came into contact in the closed position to limit the final position of the leaf. The decelerating speed of the trusses was entirely controlled by the machinery brakes. It had been found that the pneumatic buffers were too slow in operating and the pressure relief valves were too temperamental.

In 1903 plans were prepared for a double-leaf trunnion bascule bridge at Archer Avenue, over the South Fork of the South Branch, but later the federal government issued a revised permit with the clear channel reduced from 136 ft 8 in. to 107 ft. Investigation showed that a one-arm structure would be feasible, especially if the type of machinery worked out for North Western Avenue were adopted. By increasing the radius of the pin rack from 30 ft, used in the North Western Avenue bridge, to 36 ft, it was found that the machinery would have to be only a little heavier. The leaf was placed on the west side of the river with the idea that the prevailing west or southwest winds of the navigating season would assist rather than hinder the closing of the leaf. The trunnion supports were again changed, this time to an inverted triangular truss with the apex under the trunnion. This was necessary as the load was rather large for the tapered box girder type used at North Western Avenue. The bumper for the bridge, when down, was attached to the under side of this truss and, in addition, a bumper for the bridge in the open position consisted of a rubber block 7 in. by 8 in. in section attached to the top of a bracket on the fixed part of the steel, placed so that a bumper bracket on the top chord of the trusses came into contact with it when the leaf reached the fully open position. This type of bridge proved more economical than a double leaf. The only disadvantage was the open street on one end. At this end a counterweighted barrier was installed that raised above the roadway level when the bridge leaf was raised.

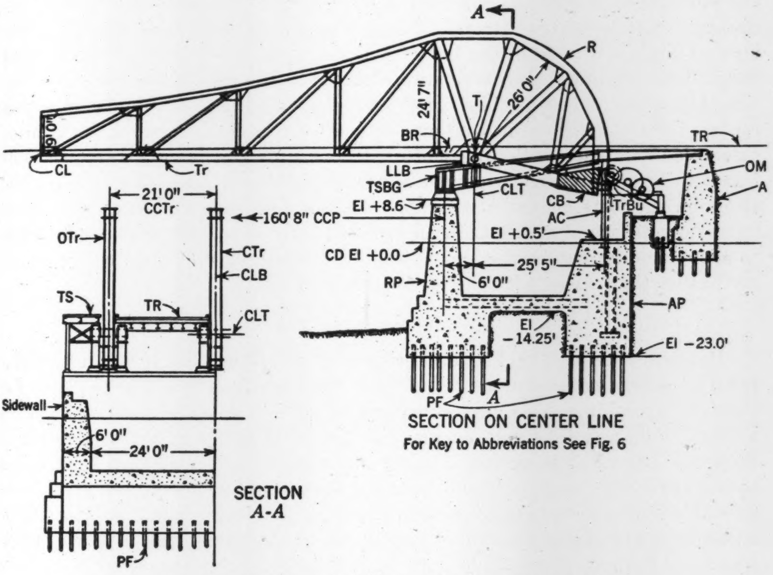

A
R
24' 7"
T
26' 0"
9' 0"
BR
TR
CL
Tr
LLB
TSBG
CLT
OM
A
CB
TrBu
AC
21' 0"
CCTr
160' 8" CCP
El +8.6
El +0.5'
OTr
CTr
CD El +0.0
CLB
25' 5"
RP
6' 0"
AP
TS
TR
CLT
El
-14.25'
El -23.0'
PF
A
Sidewall
6' 0"
24' 0"
PF
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
SECTION
A-A

FIG. 7.—WEST DIVISION STREET RIVER CROSSING (1904)

In 1909 a single-leaf bridge was built at West Kinzie Street (Fig. 8) over the North Branch that was almost identical with the Archer Avenue bridge, previously described, with some improvements in the barrier. Two more double-leaf bridges were built, one at North Avenue over the North Branch and the other at North Halsted Street over the North Branch Canal. These showed only minor variations from previous practice.

In 1910 a double-leaf bridge was built at West Erie Street (Fig. 9) over the North Branch that was radically different from previous practice. The grade of the roadway is about 6 ft higher than on the previous bridges; hence it was feasible to approximate the outline of an arch. Two trusses were used instead of three, as previously, without overhead bracing. They extend only about 10 ft above the roadway at the rear ends. Due to reduction in depth of the tail end, the rear external pin rack was no longer usable, so the bridge was operated with operating struts. These struts were connected to pins cantilevered out sidewise from the bottom chords just ahead of the counterweight box. The machinery gear trains were placed in the rear of the bridge, as before, but at the sides just back of the trusses with the main pinions engaging racks on the struts held in contact therewith by guide rollers. Each train is assembled complete, including motors, in one unit and mounted upon a unit frame to keep parts in permanent alinement. Similar to earlier bridges, the trunnion supports consist of trusses, with the difference that the truss shape now is quadrilateral rather than an inverted triangle. This change was made to accommodate the heavier loads, this bridge having only four supports per leaf rather than six as with the three truss bridges.

THE IMPROVEMENT PERIOD

Between 1908 and 1911 several bridges were designed but it was not until a bond issue was passed in the fall of 1911 that funds were made available for their construction. This gave considerable time to develop new ideas and as a result there was a distinct change with the introduction of this new group of bridges. During this period the city became increasingly conscious of the appearance of its structures and, as a result, the bridges have been given considerable study to enhance their appearance. In 1909 plans were drawn for a new bridge at West Lake Street to provide for the Lake Street Elevated Railway in addition to street traffic. A vertical lift bridge was designed but public opinion was so against its appearance that the plan was abandoned. Later in 1913 a double-deck bascule bridge was developed which will be discussed subsequently.

With this group the method of operating the bridge was changed. In 1908 Alexander von Babo, then Engineer of Bridge Design, invented the internal rack for which he filed an application for a patent on June 26, 1908, and which was issued on August 29, 1911. This patent provided for a rack in the plane of the trusses on the trunnion side of the counterweight box with the truss members arranged so that the bridge could rotate the approximately 80° required without any of the truss members fouling the pinion shaft of the gear train which drives the bridge. The radius of this rack is necessarily considerably less than that of the external pin rack used in the older bridges; but the science of machinery design and manufacture had advanced sufficiently to take care of the larger rack stresses involved.

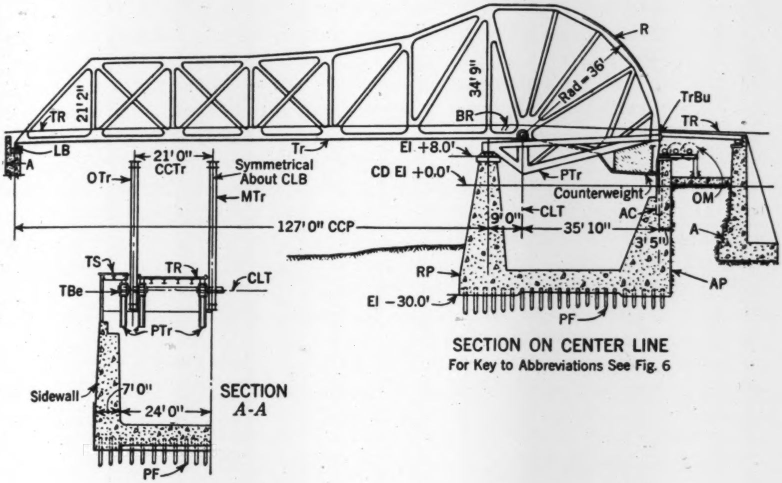

R
Rad = 36'
34' 9"
21' 2"
TR
TrBu
TR
BR
LB
A
Tr
El +8.0'
21' 0"
CCTr
Symmetrical
About CLB
OTr
MTr
CD El +0.0'
PTr
Counterweight
OM
CLT
AC
127' 0" CCP
9' 0"
35' 10"
3' 5"
A
TS
TR
CLT
TBe
RP
AP
El −30.0'
PF
PTr
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
Sidewall
7' 0"
24' 0"
SECTION
A-A
PF

FIG. 8.—WEST KINZIE STREET (1909)

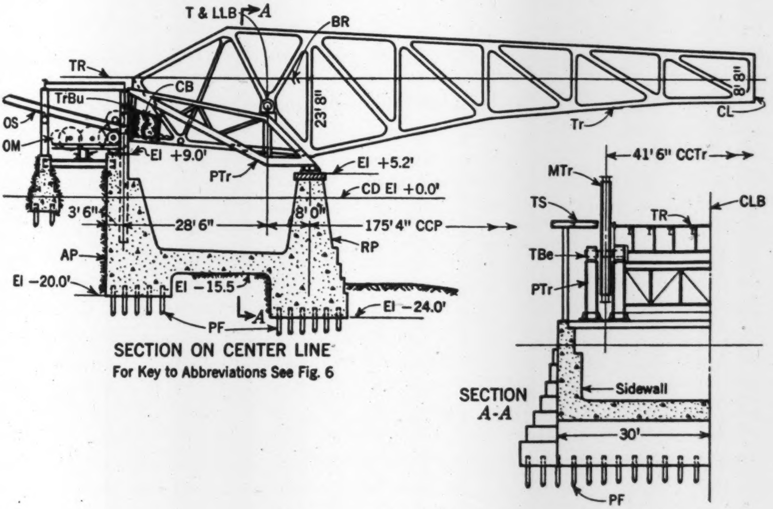

T & LLB
A
BR
TR
TrBu
CB
OS
OM
8' 8"
CL
23' 8"
Tr
El +9.0'
PTr
El +5.2'
41' 6" CCTr
MTr
CD El +0.0'
TS
CLB
3' 6"
28' 6"
8' 0"
175' 4" CCP
TR
TBe
AP
RP
El −20.0'
El −15.5
PTr
El −24.0'
A
PF
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
SECTION
A-A
Sidewall
30'
PF

FIG. 9.—WEST ERIE STREET (1910)

In this same patent another feature was incorporated, called the cross girder, which consisted of a member being inserted through the truss to carry the inside trunnion journals. This girder was supported in turn upon girders parallel to the trusses, spanning from the front to back piers. In later years, this feature led to a suit for infringement by a competitor who claimed this same feature was incorporated in a patent granted to him earlier than Mr. von Babo's patent. This claim of both patentees was doubtless void as a feature similar to it was incorporated in a bascule bridge over the Tiber River built near Rome, Italy.[3] This cross-girder feature is only practical with two-truss bridges, as it would be impracticable to carry the heavy reactions of a truss on a girder except near the ends. The fixed part-floor system consists of stringers, supported upon this girder, passing over the counterweight box to supports over the anchor pier, instead of being framed between floor beams carried by the longitudinal supports for the inner trunnions formerly used.

Another change was made in the counterweight by forming the box large enough so that the necessary weight could be obtained with stone concrete, or, with only slight amounts of heavier aggregates to produce the weight required to adjust the center of gravity to its proper location to balance the remainder of the bridge.

The truss outlines were made more graceful by the use of trusses of the pony type. They were only deep enough at the rear ends to provide an economical cross section for the truss members, and then they dipped gracefully toward the center of the channel. The bottom chords were also given as much arching as the required underclearances prescribed by the U. S. War Department would permit. At this time the government specified a 16.5-ft clearance above Chicago Datum (normal water level) for 85% of the clear channel from which point the lower chords could be sloped downward.

A machinery girder was placed outside of, and parallel to, the longitudinal girder supporting the cross girder, at a distance of 6.5 ft, which girder, together with the longitudinal girder, supported the machinery gear train. The gear train, in turn, was mounted on a cast-steel pedestal base. These girders also transmit the couple of the wind and operating moments to the masonry at the front and rear piers. Fig. 10 shows the East 92d Street bridge.

With this group of bridges another new feature appeared in the substructure. At West Washington Street, completed in 1913, it had not been feasible to use piles to support the tail-pit boxes, due to the presence of the street railway tunnel under the river. Rectangular piers were sunk from the bottom of the piers to a point well below the tunnel where they rested on a hard clay stratum at El. −57.0 (Chicago Datum), capable of carrying a greater unit load than at higher levels. At the West Chicago Avenue bridge (completed in 1914, see Fig. 11), cylindrical piers were sunk to El. −82.0, where they were founded on rock. The spacing was sufficient to permit a future double-track subway tube to pass between them. Reinforcing trusses then had to be placed in the front and rear walls of the tail pit to carry the weight.

Following the bond issue of 1911, six bridges were placed under contract in rapid succession—five of the new double-leaf bascule type and one, a single-

---

[3] *Industries*, London, England, Vol. 14, 1893, p. 316.

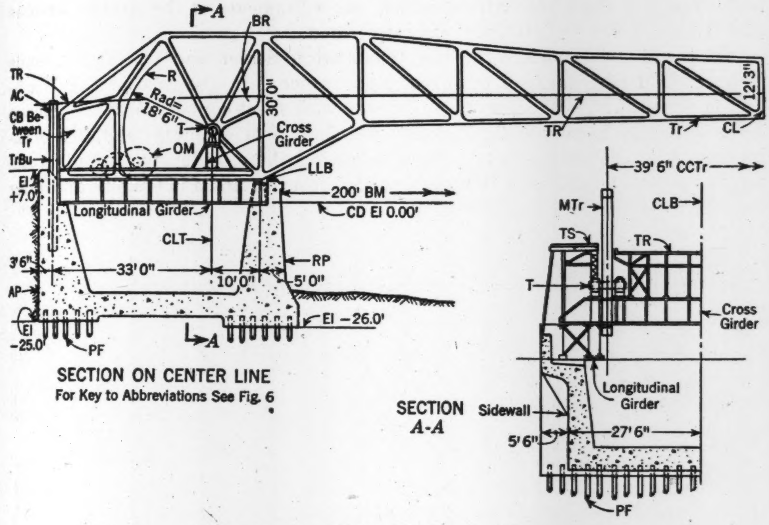

A
BR
TR
AC
CB Between Tr
TrBu
R
Rad= 18' 6"
T
OM
30' 0"
Cross Girder
12' 3"
TR
Tr
CL
LLB
El +7.0'
200' BM
Longitudinal Girder
CD El 0.00'
CLT
3' 6"
33' 0"
10' 0"
5' 0"
RP
AP
El -25.0'
El -26.0'
PF
A
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
39' 6" CCTr
MTr
CLB
TS
TR
T
Cross Girder
Longitudinal Girder
SECTION A-A
Sidewall
5' 6"
27' 6"
PF

Fig. 10.—East 92d Street Bridge (1913)

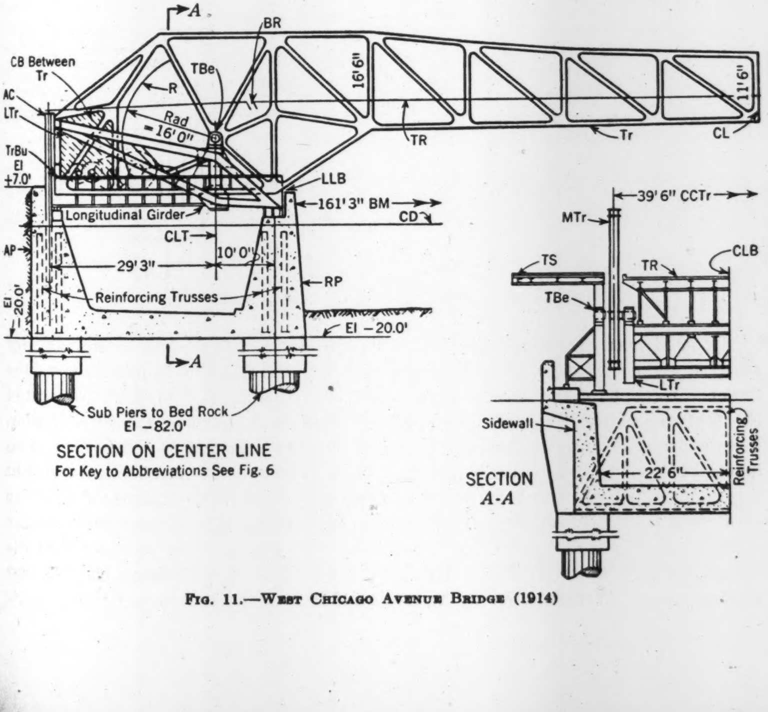

A
BR
CB Between Tr
AC
LTr
TrBu
El +7.0'
R
TBe
16' 6"
Rad =16' 0"
11' 6"
TR
Tr
CL
LLB
161' 3" BM
Longitudinal Girder
CD
CLT
AP
10' 0"
29' 3"
RP
Reinforcing Trusses
El -20.0'
El -20.0'
A
Sub Piers to Bed Rock El -82.0'
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
39' 6" CCTr
MTr
TS
TR
CLB
TBe
LTr
Sidewall
Reinforcing Trusses
22' 6"
SECTION A-A

Fig. 11.—West Chicago Avenue Bridge (1914)

leaf bascule at West 35th Street, which was a duplicate of the Archer Avenue and Kinzie Street bridges previously described.

Of the five bascules, two—the Washington Street and Chicago Avenue bridges—had enclosures of improved appearance. In the case of the Washington Street bridge, the house was covered with sheet copper, and the Chicago Avenue bridge houses and enclosures were made of concrete, suitably ornamented. In the case of the bridges following these, extensive architectural treatment was applied to the houses and enclosures, first in concrete and later in stone or other durable materials.

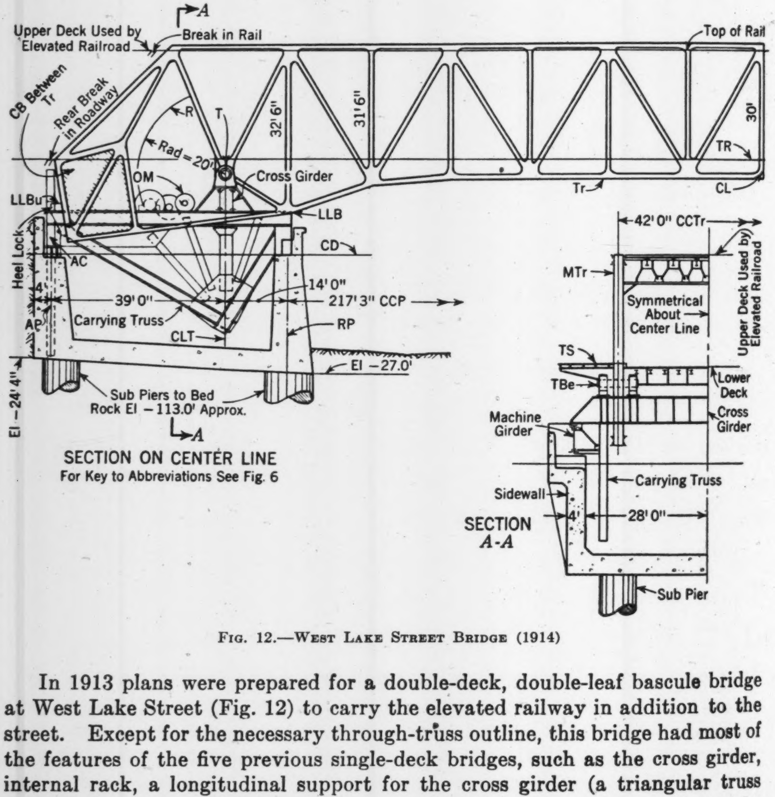

A
Upper Deck Used by Elevated Railroad
Break in Rail
Top of Rail
CB Between Tr
Rear Break in Roadway
R
T
Rad = 20'
32' 6"
31' 6"
30'
TR
OM
Cross Girder
Tr
CL
LLBu
LLB
Heel Lock
AC
CD
14' 0"
4'
39' 0"
217' 3" CCP
AP
Carrying Truss
CLT
RP
El −27.0'
El −24' 4"
Sub Piers to Bed Rock El −113.0' Approx.
A
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
42' 0" CCTr
MTr
Symmetrical About Center Line
Upper Deck Used by Elevated Railroad
TS
TBe
Lower Deck
Machine Girder
Cross Girder
Carrying Truss
Sidewall
4'
28' 0"
SECTION A-A
Sub Pier

FIG. 12.—WEST LAKE STREET BRIDGE (1914)

In 1913 plans were prepared for a double-deck, double-leaf bascule bridge at West Lake Street (Fig. 12) to carry the elevated railway in addition to the street. Except for the necessary through-truss outline, this bridge had most of the features of the five previous single-deck bridges, such as the cross girder, internal rack, a longitudinal support for the cross girder (a triangular truss instead of a girder), machinery girder, unit machinery frame, and counterweight box with relatively light materials. Lead slag was used instead of stone to produce a concrete weighing 165 lb per cu ft. About the only unusual feature in this case was that everything was heavier, as witnessed by the fact that the trunnion load was about double that at the East 92d Street bridge, the heaviest to that time. With these very heavy loads it was decided to rest the tail pits on cylinder piers carried to solid rock at El. −113.0 and with a clearance of 36 ft to provide for a double-track subway at some future date. Chicago's subway has since utilized this space for a single bore of the Dearborn Street subway system.

With single-deck bridges it had always been practical to have the break in floor of the deck on the river side of the trunnion so that the raised leaf provided a barrier in the street. With a double-deck bridge, this was not possible so that a rear break in the floor was necessary, which does not provide a natural barrier in the street. Accordingly, it was necessary to provide some kind of a barrier at each end of the bridge. These were not put in at once but were installed at a later date. First, there was a semi-rigid type consisting of a timber pole that could be lowered into place and which would slide up an inclined grade if struck. Later a yielding barrier of the net type was developed, which plays out against friction drums when struck.

In many of the earlier bridges street traffic was accommodated during construction with a temporary bridge, usually of the so-called pontoon type. A truss span was supported at each end upon pile supports, and a scow placed beneath and near one end. By pumping water out of the hull, the truss at this end could be raised and the span could be rotated about a pin at the other end to clear the channel for navigation. At Lake Street, however, the presence of the elevated railway made this impracticable. After attempting to detour via other streets it was decided to maintain traffic by leaving the old swing bridge in place, and to support the approaches upon shoring. The length of the movable span of the new bridge was designed so that it could be erected in the vertical position clear of the ends of the swing bridge, and, as the bridge was erected, sufficient floor system was omitted to allow the elevated trains to pass through. When the leaf was ready to lower, traffic on the elevated railway was interrupted, the old swing bridge was turned on to the center pier, and the center section was cut out. The new bridge leaves were lowered after the omitted section of floor had been placed, the decking for the elevated railway was placed, and traffic was then allowed to pass over the new bridge with only a few days of interruption. No attempt was made to maintain traffic on the lower or street level.

Due to the fact that the bridge was balanced about the trunnion for dead load and had a rear break in the roadway, it was possible for live load to pass on to the rear ends only, which might place live load on the machinery brakes. Accordingly, heel locks were installed to take whatever live-load reaction might come on to them. These consist of toggle levers between the rear pier of the tail pit and a shoe on the rear of the counterweight box that were forced in place by machinery.

In 1922 a similar bridge was completed at North Wells Street to accommodate the North-Western Elevated Railway. In this bridge the cross girder was supported directly upon a column embedded in the side-wall of the tail pit, with a sub-pier under the tail pit carrying the load down to rock at El. −117.0. This obviated the necessity of providing a heavy structural truss to distribute the heavy load to front and rear walls, the loading being 25% greater than at the Lake Street bridge.

Short girders framed into these trunnion columns spanning to the rear wall to carry the machinery gear train. These were not so heavy as formerly due to the omission of the bridge weight and the shortened span. With this bridge the machinery stresses induced by a pressure of 20 lb per sq ft on the upraised leaf were considered rather beyond practical bounds, so it was decided to scale it down to 16 lb per sq ft for the gear train, but to attach a brake directly to the tail end of the bridge trusses. This brake was to be of the pneumatic type and to consist of shoes working on a curved rail set in the masonry. After further consideration it was thought not to be very practicable, because of variable operating characteristics from moisture conditions; hence the regular machinery was left to handle extreme conditions. There has been some distress in the gear train, which has been corrected by replacing parts with higher strength steels than those originally used. Traffic maintenance during the erection of the trusses was similar to that at Lake Street, but with the added feature that the lower-deck traffic was maintained as well as the elevated traffic. The change over from the old swing bridge to the new bridge was made in 72 hr for elevated traffic, but the lower-level traffic was interrupted for about a month.

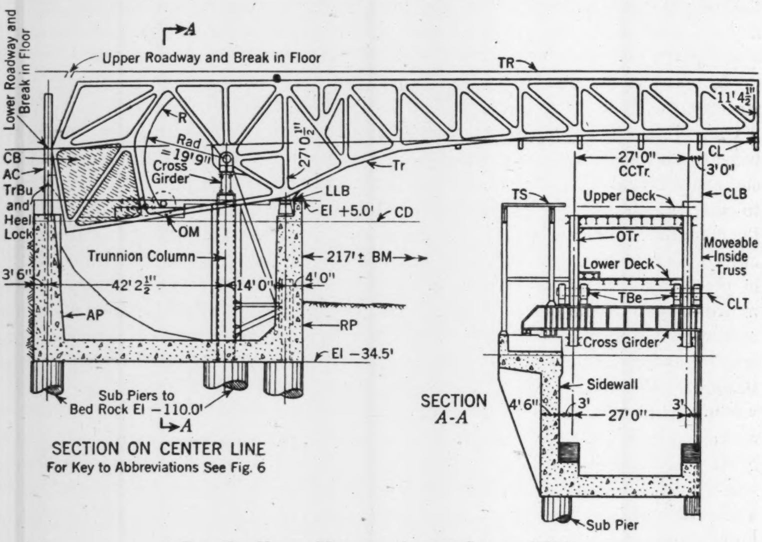
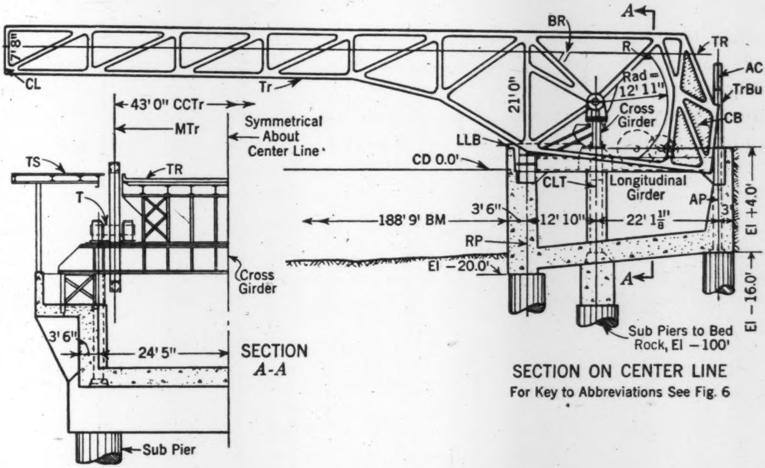

Lower Roadway and Break in Floor
A
Upper Roadway and Break in Floor
TR
11' 4½"
R
Rad = 19' 9"
27' 0½"
CB
AC
TrBu and Heel Lock
Cross Girder
Tr
LLB
El +5.0'
CD
OM
Trunnion Column
217' ± BM
3' 6"
42' 2½"
14' 0"
4' 0"
AP
RP
El −34.5'
Sub Piers to Bed Rock El −110.0'
A
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
27' 0" CCTr.
CL
3' 0"
TS
Upper Deck
CLB
OTr
Moveable Inside Truss
Lower Deck
TBe
CLT
Cross Girder
Sidewall
SECTION A-A
4' 6"
3'
27' 0"
3'
Sub Pier

FIG. 13.—NORTH MICHIGAN AVENUE BRIDGE (1920)

In 1920 the Michigan Avenue bridge was completed (see Fig. 13). This was the most unusual bridge built up to that time. As Michigan Avenue bridge was to be the link between the boulevard system of the north and south sides of the city, it was considered that a four-lane structure would not be sufficient. Furthermore, to separate commercial traffic from boulevard traffic, it was necessary to have two levels—the upper for boulevard traffic and the lower for commercial traffic. Accordingly, it was deemed necessary to have an upper level with six lanes and a lower level with four lanes. It was decided to use four trusses spaced 27 ft, 6 ft, and 27 ft, with separated roadways on both levels for each direction of traffic. In effect, there were two bridges, side by side, connected with diaphragms, that could be cut out to permit one half to be operated while the other was put up in the air for repairs. (This contingency has not been used in 20 years, mainly because necessary repairs have been on the deck which had to be left down to work upon.) The two bridges were supported upon separate cross girders supported near the center line of the bridge on columns to the floor of the tail pits and on trunnion columns in the side-walls. Yielding barriers were provided to protect traffic at both ends on both levels.


BR
A
TR
R
7'8"
CL
Tr
Rad = 12' 11"
AC
TrBu
21' 0"
Cross Girder
CB
43' 0" CCTr
MTr
Symmetrical About Center Line
LLB
CD 0.0'
TS
TR
CLT
Longitudinal Girder
T
AP
3' 6"
188' 9' BM
12' 10"
22' $1\frac{1}{8}$"
3'
El +4.0'
RP
El −20.0'
Cross Girder
A
El −16.0'
3' 6"
24' 5"
Sub Piers to Bed Rock, El −100'
Sub Pier

SECTION *A-A*

SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6

FIG. 14.—WEST MADISON STREET BRIDGE (1922)

Machinery for this bridge consisted of two gear trains per leaf, one at each outside truss. Due to the importance of this bridge it was considered desirable to have an extra pair of motors for stand-by use if the two regular motors should fail; all four motors were grouped in a unit at the rear of the bridge with a clutch to select the pair to be used at any time.

In 1922 another novel type of bridge was completed at West Madison Street (see Fig. 14). At this location there was a clear span of 188.75 ft and, although the street grade on the west side of the river was reasonably high, it was not high enough to permit a deck type of bridge and yet leave a sufficient depth for the trusses. Accordingly, a new idea was developed, called the "railing-height truss," which means that the top of the top chord is at about the same elevation as the top of the sidewalk handrail, about 4 ft above the walk. This gives the appearance of a deck structure from adjacent streets, as the top chord cannot be seen and, by the use of wide sidewalks, the trusses do not obstruct the walkways. Incidental benefits are that vehicles out of control on the roadway cannot get on to the sidewalk to crash into the river or injure pedestrians, and pedestrians are effectively barred from "jaywalking." By the use of nickel steel for main truss members, the sections were kept within reasonable bounds. Supports for the bridge were furnished by cross girders and trunnion columns as previously described for Wells Street.

Between 1911 and the end of 1923, thirteen city-type bascule bridges were built, three double-deck double-leaf bridges, nine single-deck double-leaf bridges, and one single-deck single-leaf bridge. The Sanitary District also built one single-deck double-leaf bridge on the South Branch at West Jackson Street. The total, to this time, was twenty-four city-type bridges.

THE REFINEMENT PERIOD 1923–1941

In 1920 funds were becoming scarce, with the result that in 1923, 1924, and 1925 no more bridges were added to the list. However, between 1924 and 1930 several more bond issues were approved by the electorate and in 1926 bridge development was resumed. This period, however, did not produce the radical improvements that marked the years 1911 to 1922. In 1924, when the 100th Street bridge over the Calumet River (see Fig. 15) was proposed, it was decided to avoid all patents, even the internal rack patent held by Mr. von Babo. Mr. von Babo's patent had been used by the city without compensation because he developed it while he was a city employee, although the patent had never been assigned to the city. To overcome this complication, a rack was attached to the side of the truss instead of in the plane of the truss. Instead of the cross

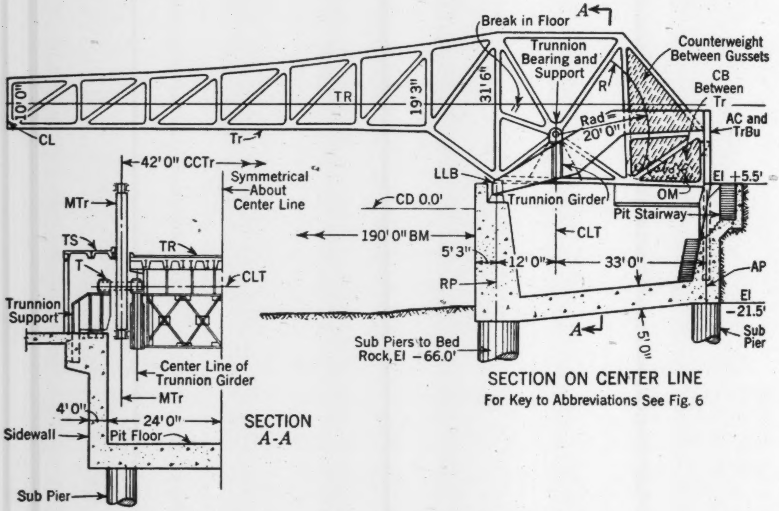
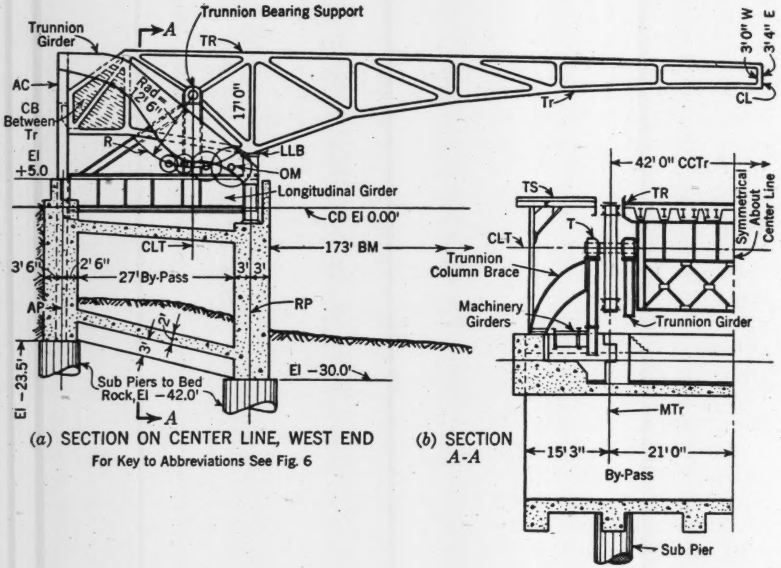

Break in Floor
A
Trunnion Bearing and Support
Counterweight Between Gussets
CB Between Tr
AC and TrBu
10'0"
TR
19'3"
31'6"
R
Rad= 20'0"
CL
Tr
42'0" CCTr
Symmetrical About Center Line
LLB
El +5.5'
OM
MTr
CD 0.0'
Trunnion Girder
Pit Stairway
TS
TR
190'0" BM
CLT
T
5'3"
12'0"
33'0"
AP
CLT
RP
El -21.5'
Trunnion Support
A
5'0"
Sub Pier
Sub Piers to Bed Rock, El -66.0'
Center Line of Trunnion Girder
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
MTr
4'0"
24'0"
Sidewall
Pit Floor
SECTION A-A
Sub Pier

FIG. 15.—EAST 100TH STREET BRIDGE (1924)

girder the inner trunnions were supported on a longitudinal **S**-shaped box girder in which the rear end of the girder passed over the counterweight box, but yet under the roadway to a support over the anchor pier. In principle this was similar to the idea used as far back as the North Western Avenue bridge completed in 1904, where a tapered box girder was used. In this new development, by using curves it was possible to deepen the section for moment under the trunnion. Near the ends where the depth is less than under the load and where only shear governed, the web plates were reinforced with additional side plates. This support was only used for the inside journals, the outer ones being supported upon short stubby cantilever girders anchored in the side-walls and set transversely across the bridge, thus simulating a cross girder with the center section passing through the trusses cut away. A change was also made in the machinery supports, the unit bed frame being now set directly upon the masonry of the side-walls, which are suitably reinforced with metal grillages embedded in the masonry and with holding-down bolts effectively anchoring the machinery into the masonry. Here another of the recommendations in the 1900 Report of the Board of Consulting Engineers was finally, and most effectively, adopted.

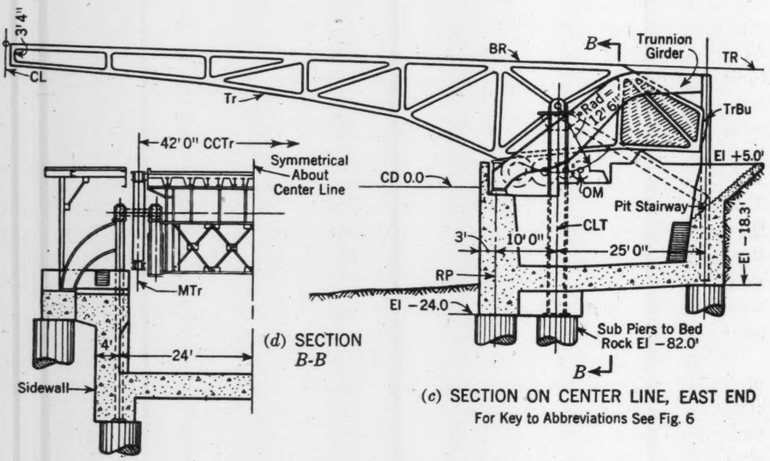
One of the bridges of this period, that at West Adams Street, is interesting in that local conditions on each side of the river forced a different tail-end design on the two sides of the river (see Fig. 16). This bridge is set at such a high grade that a deck bridge was feasible; but, due to the shallowness of the rear arms, an internal or side rack was not feasible, so the rack was placed outside and below the bottom chord in the plane of the member The machinery on the east side rested directly on the masonry as described in the previous paragraph, but on the west side it was impossible to use the usual pit as a by-pass through the masonry because added flow area in the river was required. Accordingly, the outer trunnion supports and the machinery rest on longitudinal girders spanning from front to rear piers, whereas the inner trunnions rest on **S**-girders. On the east side the outer trunnions rest on trunnion columns and the inner ones on **S**-girders. The counterweight arm on the west side was so short that only a shallow pit was required, because (1) the by-pass, and (2) a railroad track on the west dock did not permit extension that way. This made it necessary to reduce the volume of the counterweight box below the most desirable dimensions. To overcome this condition and to provide the weight required, concrete made with steel punchings to produce a unit weight of 275 lb per cu ft, combined with shaped iron castings, and a small amount of lead pigs were used, thus adjusting the center of gravity to produce an over-all unit weight of about 330 lb per cu ft.

When the South Halsted Street bridge (see Fig. 17) was being designed, a problem was presented because the U. S. War Department urged higher river clearances. Combined with the fact that the street passes through a subway under a railroad just south of the bridge, this requirement would have made almost prohibitive grades on the bridge approach unless the bridge grade were kept as low as possible. The grade was reduced by using three trusses and a very shallow floor system, the floor beams consisting of 30-in. Carnegie beam sections with the total depth of the deck only 3.5 ft from the roadway grade to


Trunnion Bearing Support
Trunnion Girder
A
TR
3'0" W
3'4" E
AC
CB Between Tr
Rad = 12'6"
17'0"
Tr
CL
R
LLB
El +5.0
OM
Longitudinal Girder
CD El 0.00'
CLT
173' BM
3'6"
2'6"
27' By-Pass
3' 3'
AP
RP
2'
3'
El -23.5'
Sub Piers to Bed Rock, El -42.0'
El -30.0'
A
42' 0" CCTr
TS
TR
Symmetrical About Center Line
CLT
T
Trunnion Column Brace
Machinery Girders
Trunnion Girder
MTr
15' 3"
21' 0"
By-Pass
Sub Pier

(a) SECTION ON CENTER LINE, WEST END
For Key to Abbreviations See Fig. 6

(b) SECTION A-A


3'4"
CL
Tr
BR
B
Trunnion Girder
TR
Rad = 12'6"
TrBu
42' 0" CCTr
Symmetrical About Center Line
CD 0.0
OM
El +5.0'
Pit Stairway
CLT
3'
10' 0"
25' 0"
El -18.3'
MTr
RP
El -24.0
Sub Piers to Bed Rock El -82.0'
4'
24'
Sidewall
B

(d) SECTION B-B

(c) SECTION ON CENTER LINE, EAST END
For Key to Abbreviations See Fig. 6

FIG 16.—WEST ADAMS STREET BRIDGE (1926)

the underclearance. As this bridge has two three-lane roadways, it does not give the same constricted appearance as the older three-truss bridges, with only 18-ft roadways.

These same plans were used at South Ashland Avenue bridge, a mile west of the South Halsted Street bridge (1938), not on account of local conditions but in order to use plans that would fit the location and provide a job under PWA without the delay of preparation of new plans. These plans, with the shallow floor, however, had the advantage of keeping the hump in the street profile at a minimum.

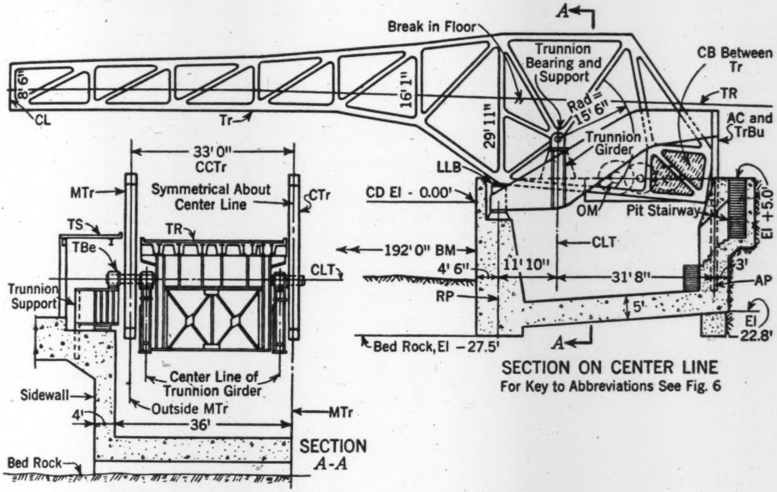

A
Break in Floor
Trunnion Bearing and Support
CB Between Tr
TR
8'6"
CL
Tr
16'1"
29'11"
Rad=15'6"
Trunnion Girder
AC and TrBu
33'0" CCTr
MTr
Symmetrical About Center Line
CTr
TS
TR
TBe
Trunnion Support
CLT
LLB
CD El - 0.00'
OM
Pit Stairway
El +5.0'
192'0" BM
CLT
4'6"
11'10"
31'8"
3'
AP
RP
5'
El -22.8'
Bed Rock, El -27.5'
A
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
Center Line of Trunnion Girder
Sidewall
Outside MTr
MTr
4'
36'
SECTION A-A
Bed Rock

FIG. 17.—SOUTH HALSTED STREET BRIDGE (1933)

The railing-height truss idea, first used for the Madison Street bridge (1922), was used again at the North Wabash Avenue bridge (1930) and the North State Street bridge (1942). However, silicon steel was used instead of nickel steel. The Wabash Avenue bridge (Fig. 18) was awarded the first prize by the American Institute of Steel Construction as the best looking bridge completed in 1930 in the class costing more than $1,000,000. This bridge has a six-lane divided roadway. The center island was used as a protection for a row of T-shaped trolley supports, thus avoiding the necessity for arches over the roadway. These T-shaped trolley arches were preferred to rather flat arches otherwise required.

The North State Street bridge (Fig. 19) is a three-truss bridge because it provides for eight lanes of traffic, and it is not feasible to design floor beams to span the width required for two additional lanes, without a further rise in grade. With such wide roadways the old danger that existed with four-lane roadways of vehicles running head on into the center trusses is not so great; at least, it is not as bad as crashing head on with traffic traveling in the opposite direction at speeds varying from 30 to 40 miles per hr. This bridge has the further innovation of roller bearings instead of phosphor-bronze bushed bearings. These bearings also have spherical ball seats so that they adjust themselves for correct alinement in case of unequal deflection of the supporting structures. This feature reduces bearing friction to a minimum.[4]

Some other new ideas are incorporated in the design of the South Canal Street bridge. The design calls for a double-leaf bridge with a four-lane roadway of the open grating type. There is a great economy in this type of floor for bascule bridges, because the saving in weight of the floor not only reduces stresses in the truss members, but effects a reduction in weight of the counterweight of two to three pounds for each pound saved in the floor weight. Frequently, due to limited space for the tail end, this slight increase in weight can only be attained by the use of heavy unit-weight material which is usually more expensive than plain concrete that could be used otherwise; such a case was described previously in relation to the Adams Street bridge. Another innovation in this bridge is the use of hydraulic motors, instead of the usual electric motors, for driving the gear train. The prime power will be electricity as in the past but the electric motors will operate at constant speed driving an oil pump. By varying the displacement of oil by this pump (which displaced oil is used to drive oil motors at the gear trains), the speed of the bridge can be controlled more effectively than with electric motors which require considerable control equipment to handle the large currents involved. In this case, a simple push-button control is all that is needed to start and stop the constant-speed electric motors.

A total of 39 city-type bridges have been constructed in Chicago. Some of these were built by governmental bodies other than the city. The two on the Outer Drive at the mouth of the Chicago River were built in 1937 by the Chicago Park District and were designed by a local firm of consulting engineers, but following, carefully, the city's practice on fundamental details. These bridges provide two four-lane roadways on the upper level, and the lower level provides for railroad tracks if future developments in this part of the city require them.[5]

As a side issue, not exactly within the province of this paper, is a note concerning the construction of the only movable bridge, not of the bascule type, constructed in Chicago in the past 25 years, at South Torrence Avenue over the Calumet River. A vertical lift type was used, at a location where the center line of the river channel forms an angle of 50° with the center line of the street. Here the U. S. War Department required a clear channel of 200 ft with a 21-ft headroom for the full channel. Hence a bascule bridge would have had to have a clear distance between masonry of about 320 ft. As this bridge is in a commercial or manufacturing district, a skewed vertical lift bridge was not considered out of place and comparative estimates indicated the latter could be built for less than half the cost of a bascule.[6]

---

[4] *Civil Engineering*, March, 1942, p. 140.

[5] *Engineering News-Record*, April 15, 1937, p. 546; also April 22, 1937, p. 583.

[6] *Ibid.*, November 17, 1938, p. 618.

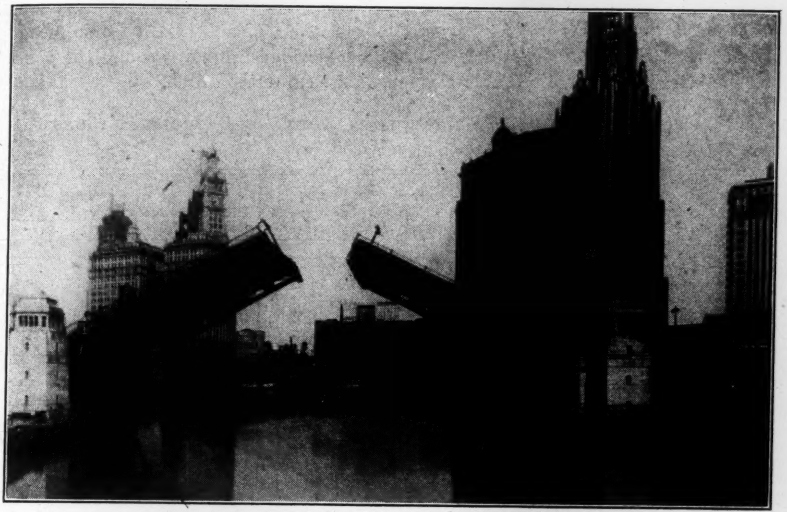


Fig. 18.—North Wabash Avenue Bridge (1930)

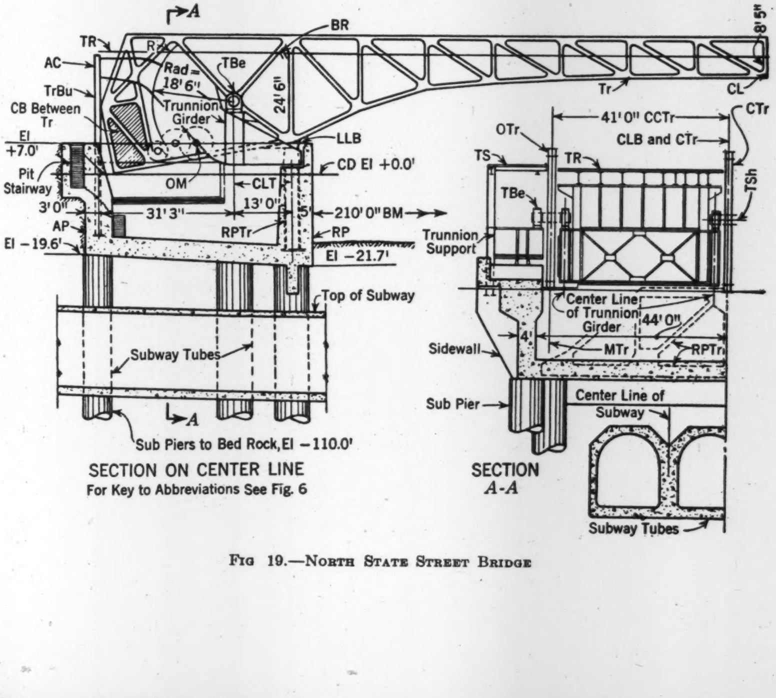

A
BR
8' 5"
TR
AC
R
Rad = 18' 6"
TBe
TrBu
Trunnion Girder
24' 6"
CB Between Tr
Tr
CL
El +7.0'
LLB
Pit Stairway
OM
CLT
CD El +0.0'
3' 0"
31' 3"
13' 0"
5'
210' 0" BM
AP
RPTr
RP
El −19.6'
El −21.7'
Top of Subway
Subway Tubes
A
Sub Piers to Bed Rock, El −110.0'
SECTION ON CENTER LINE
For Key to Abbreviations See Fig. 6
41' 0" CCTr
CTr
OTr
CLB and CTr
TS
TR
TSh
TBe
Trunnion Support
Center Line of Trunnion Girder
44' 0"
4'
Sidewall
MTr
RPTr
Sub Pier
Center Line of Subway
SECTION A-A
Subway Tubes

Fig 19.—North State Street Bridge

CONCLUSION

Figs. 5 to 19 indicate the development of the Chicago type bascule bridge from the Clybourn Place and Division Street bridges, down through the years to the latest structures. Table 1 gives statistical data, which, by comparison, show the development of this type of bridge better than meager words. This type of bridge has proved highly satisfactory and has solved most of the problems existing previous to its development, as discussed in the historical part of this paper previous to 1900.

TABLE 1.—DATA AND CHRONOLOGY, CHICAGO BASCULE BRIDGES
(Dimensions Are in Feet)

Year	Location (see Fig. 1)	Fig. No.	Bridge type[a]	Clear span	Between trunnions[b]	River channel	Bridge width	Trusses	
								No.	Type
1902	W. Cortland St. (Clybourn Place)	5	SD/DL	115.0	128.0	92.0	60.0	3	Through
1903	E. 95th Street	..	SD/DL	140.0	154.0	120.0	60.0	3	Through
1903	W. Division St. (Canal)	6	SD/DL	135.0	149.0	90.0	60.0	3	Through
1904	W. Division St. (River)	7	SD/DL	155.67	172.67	100.0	60.0	3	Through
1904	N. Western Ave.	..	SD/DL	184.58	205.58	121.0	60.0	3	Through
1906	W. Archer Ave.	..	SD/SL	121.0	136.0	100.0	60.0	3	Through
1907	W. North Ave.	..	SD/DL	153.67	172.67	128.0	60.0	3	Through
1908	N. Halsted St. (Canal)	..	SD/DL	184.58	205.58	90.0	60.0	3	Through
1909	W. Kinzie St.	8	SD/SL	121.0	136.0	100.0	60.0	3	Through
1910	W. Erie St.	9	SD/DL	170.33	191.33	140.0	60.0	2	Pony
1913	W. Washington St.	..	SD/DL	170.58	197.33	150.0	57.25	2	Pony
1913	W. Grand Ave. (Indiana St.)	..	SD/DL	162.75	188.75	140.0	60.0	2	Pony
1914	E. 92d St.	10	SD/DL	200.42	228.0	190.0	60.0	2	Pony
1914	W. 35th St.	..	SD/SL	120.0	136.0	120.0	60.0	3	Through
1914	W. Chicago Ave.	11	SD/DL	161.25	188.75	150.0	60.0	2	Pony
1916	W. Lake St.	12	DD/DL	209.25	245.25	190.0	70.0	2	Double deck
1916	W. Webster Ave.	..	SD/DL	161.25	188.75	120.0	60.0	2	Pony
1916	W. Jackson Blvd.	..	SD/DL	173.5	202.27	147.0	64.0	2	Deck
1917	W. Belmont Ave.	..	SD/DL	161.25	188.75	75.0	60.0	2	Pony
1919	W. Monroe St.	..	SD/DL	165.5	192.79	164.0	60.0	2	Pony
1920	N. Franklin-Orleans St.	..	SD/DL	220.0	251.83	178.0	62.0	2	Pony
1920	N. Michigan Ave.	13	DD/DL	220.0	256.0	200.0	91.79	4	Double deck
1922	N. Wells St.	..	DD/DL	231.0	268.0	220.0	72.0	2	Double deck
1922	W. Madison St.	14	SD/DL	188.75	221.23	168.0	72.0	2	Rail height
1926	E. 100th St.	15	SD/DL	200.0	233.5	190.0	62.0	2	Pony
1926	W. Adams St.	16	SD/DL	173.0	199.0	151.0	64.0	2	Deck
1928	E. 106th St.	..	SD/DL	220.0	250.6	192.5	62.0	2	Pony
1928	N. LaSalle St.	..	SD/DL	210.0	242.0	200.0	86.0	2	Pony
1929	N. Clark St.	..	SD/DL	215.0	245.5	200.0	72.0	2	Pony
1929	W. Roosevelt Road (Twelfth St.)	..	SD/DL	170.0	204.0	170.0	90.0	2	Deck
1929	N. Damen Ave. (N. Robey St.)	..	SD/SL	120.0	162.23	120.0	62.0	2	Deck
1929	S. Damen Ave. (S. Robey St.)	..	SD/DL	212.0	243.83	140.0	66.0	2	Pony
1930	N. Wabash Ave.	18	SD/DL	232.0	269.0	193.0	90.0	2	Rail height
1932	W. Ogden Ave. (Canal)	..	SD/DL	158.0	191.0	132.0	90.0	2	Pony
1932	W. Ogden Ave. (River)	..	SD/DL	150.0	182.0	130.0	90.0	2	Pony
1933	S. Halsted St.	17	SD/DL	192.0	224.67	165.0	90.0	3	Pony
1937	Outer Drive (River)	..	DD/DL	220.0	264.0	220.0	108.0	4	Double deck
1937	Outer Drive (Canal)	..	DD/SL	70.0	96.5	70.0	108.0	4	Double deck
1938	S. Ashland Ave.	..	SD/DL	192.0	224.67	189.83	90.0	3	Pony
1942[c]	N. State St.	19	SD/DL	210.0	245.0	200.0	108.0	3	Rail height
1943[c]	S. Canal St.	..	SD/DL	224.0	254.5	171.0	68.0	2	Pony

[a] Key: SD = single deck; DD = double deck; DL = double leaf; and SL = single leaf. [b] Distance, center to center of trunnions. [c] Completion deferred until after the War.

The navigation requirements can be met ideally by the proper selection of size and location of the channel, almost without limit. The requirements of street traffic are satisfied by the provision of a structure that can be operated reliably in a minimum of time and which provides a measure of protection to the traffic when open, not provided by a swing or lift bridge. Maintenance is kept at a low figure by avoiding pin-connected linkages except for the trunnions upon which the movable part rotates.

The extra cost (if any) of this type of bridge over swing or lift types has been justified by its far better appearance especially when placed in a congested section of the city. However, the development of this type of bridge has not yet reached a point of stagnation as demonstrated by the various bridges built during the so-called Refinement Period, beginning in 1923, and many more refinements can be expected in the years to come.

In the development of this bridge, exclusive credit cannot and should not be given to any one engineer as it represents a development over a period of more than 40 years during which time many engineers have had an opportunity to add their bit to what has gone before. The complexity of such a structure, involving complicated foundation design conditions, unusual structural conditions, special mechanical and electrical features and, finally, architectural details in operators' houses and enclosures, is beyond the capabilities of almost any one man. Accordingly, credit is given to each and every one ever connected with its development in proportion to his contribution.

However, most of the early development was due to the efforts of the late John Ericson, city engineer, Thomas G. Pihlfeldt, engineer of bridges for more than 40 years, until the time of his death in 1941, and Alexander von Babo, engineer of bridge design for more than 20 years, until his death in 1920.

DISCUSSION

Alonzo J. Hammond,[7] Past-President and Hon. M. Am. Soc. C. E.—A valuable contribution to the literature on movable bridges is found in this paper. It is especially fitting and appropriate as coming from a bridge engineer who for some thirty years has participated in the design of such bridges in the City of Chicago which has, no doubt, a greater number and a greater variety of these bridges than any other city in the world, due to the required navigability of its two rivers, the Chicago and Calumet.

The writer's introduction to the bridge situation in Chicago was in the spring of 1912 when he was appointed engineer of bridges and harbors, at a time when a large bond issue was available for new bridges, and it was necessary to enlarge the bridge department materially to cope with the demand for new designs and construction. It may be of interest to note that the first problem "dumped into his lap" was a determination of the best type of movable bridge to replace the old swing bridge at Lake Street over the Chicago River. Submitted were two types of vertical lifts, and the Rall, the Scherzer, the Strauss, and what was then known as the City type. On esthetic grounds the vertical lift was eliminated and, on the basis of cost and also esthetic grounds, the City type was adopted.

Previous to 1912, little attention had been given to the artistic appearance of the bridges; as a matter of fact the bridge engineer had to make his dollar give the maximum of strength and stability; but the Chicago Plan Commission came into existence in 1912, so that the opportunity came to the new engineer of bridges to inaugurate the beautification of the new bridges immediately. During 1912 and 1913, the lines of the bridge trusses were simplified and where possible given an arch effect; the masonry abutment, bridge houses, and approaches were built of more permanent material of greater artistic design, due to the close cooperation of the engineer of bridges (who during 1912 and 1913 was given a comparatively free hand) with the Chicago Plan Commission and the Illinois Society of Architects. This period may be termed the renaissance in the designing of Chicago's bridges. The immediate results were seen in the Michigan Avenue plans, Lake Street, Chicago Avenue, and Jackson Street bridges. The latter was not designed by the City Bridge Department but the general plans were developed under the writer's direction and approved by him.

Mr. Becker properly has given credit to the long-time employees of the City who were participants in the earlier developments of the movable bridges and who continued over a long period of years to meet the requirements of the traffic successfully. Alexander von Babo was a very able designer, modest and retiring, who gave freely to the City his patented ideas on which he reaped little or no financial reward.

A movable bridge is a large machine which requires constant attention for successful and satisfactory operation. For one thing, power is important, so there were installed in 1912–1913 duplicate sources of supply of electric power.

---

[7] Cons. Engr., Chicago, Ill.

One of the early difficulties in the Scherzer type of bridges was the size and shape of the teeth on the horizontal rack, and a more effective design was developed to assure a more safe meeting of the demand. As a matter of fact the Canal Street bridge almost "walked into the river" one night; this possible situation is now avoided by the better designing.

Under the heading, "The Improvement Period," Mr. Becker refers to the Chicago Avenue cylindrical piers as founded on rock at El. – 82.0. Bids were received on piers stopping in the hard clay with cone shaped bottoms, flaring out to give a sufficient spread to take the loads, and also on cylindrical piers extending to rock.

It so happened that the less excavation and less concrete made the cylindrical piers cheaper, but it gave a star reporter on the leading morning Chicago paper a chance to write an article about political favoritism to the successful bidder, the article appearing on the front page in large type. Later the said star reporter denied the truth of the article, but this was buried on the inside pages of the paper.

One day the Mayor, Carter H. Harrison, Jr., came into the office and complained about the shock he got going over the center break in the floor of the State Street bridge. This was a very light structure, so the complaint led to the designing of the continuous-rail joint at both center and heel breaks, which were adopted against the objections of the supervising engineers and have proved to be a great asset.

As a matter of interest to those bridge engineers who may be designing double-leaf bascule bridges, the following incident happened in the case of the West Lake Street bridge shown in Fig. 12: When the writer took charge of the bridge department he found borings had been made on both sides of the river at Lake Street, showing rock at El. – 75.0. Suspicious that the elevations should be the same, where there was a possible dip, new borings were ordered, and at El. – 75.0, sure enough, rock was found, but only 6 in. in thickness—so the borings were continued down until bedrock was found at about El. – 113.0, as shown in Fig. 12.

Mr. Becker refers to a new idea as developed in the Madison Street bridge, in 1922, called the "railing height truss." As a matter of fact two designs were prepared in 1912 for the Jackson Street bridge, one a railing height truss and the other a deck span, and the writer approved the latter. At about the same time, preliminary measurements and studies were made under his direction for the Madison Street bridge, and these preliminary sketches were for a railing height truss. It will be of interest also to note that a temporary bridge was designed, at this time, of the retractile type to carry traffic across the river during the construction of the proposed new bridge. This was a very simply designed structure, as railroad tracks were laid at right angles to the dock on vacant ground at dock height. The design included car trucks, on top of which were trusses extending far enough to the rear to carry a counterweight and far enough in front to provide a span of about 60 ft—a balanced affair planned to run out and back. It was not used, however.

The Michigan Avenue double-deck bridge was designed under the writer's direction in 1912 and 1913 and had a novel history—the construction was paid for by property assessments, and the drawings had to be reduced. The latter, with the specifications, were made a part of the council proceedings and were published in detail. A close examination of Fig. 18 will show the Michigan Avenue bridge in the background—a bridge that carried an enormous automobile and bus traffic until relieved by the outer drive bridge.

Since 1912 river traffic has dwindled tremendously and the need of movable bridges has seemed to be approaching an end; but with the present war some bridges that were not in operation have been provided with operating machinery by the federal government; so it appears that the movable bridge is in for a long lease of life on the Chicago River.

The designing engineer of Chicago's bridges had to meet two rigorous conditions which taxed his designing ability to keep within limits. The grades on the approaches were kept to a maximum of 4% and the under clearance above river level was required to be not less than 16.5 ft for a certain width of space. The type of bridge to meet these conditions was a two-leaf bascule, and that which is known as the City type has proved a very satisfactory design.

CLINTON D. HANOVER, JR.,[8] ASSOC. M. AM. SOC. C. E.—As stated in the "Synopsis," this paper describes the development of a bridge type serving peculiar local requirements. Mr. Becker's treatment of his subject, in chronological form, is interesting as a record of the evolution of this one type of bridge and as a record of the experiences of one agency in the design and operation of movable bridges. Unquestionably, other agencies—such as states, other cities, and railroads, who design and operate movable bridges—have similar and parallel records of experience and development on various types of movable bridges.

The original sources of some developments in the art of movable bridge design might be hard to trace, since not only engineers employed by agencies owning the bridges, but also engineers employed by contractors, manufacturers, and consulting engineers, have from time to time added their share to the general development. Many of these improvements in design, especially improvements in the design of details, are reflected in the latest specifications prepared under the auspices of various engineering societies. These specifications are based on the experiences of many practicing engineers and apply to all types of movable bridges.

However, the choice of the general type of movable bridge is still a question for determination on the basis of conditions at the site and on the basis of the requirements of navigation and highway or railway traffic. For this reason, a full discussion of the advantages of any one type of movable bridge is of interest to engineers responsible for the selection of types for use at various locations.

Mr. Becker states that, with the development of the City of Chicago, the rivers were deepened and widened, providing channels varying in width from

---

[8] Chf., Bureau of Bridge Design, Dept. of Public Works, New York, N. Y.

100 ft to 250 ft. Table 1 gives among other things, the clear span and the width of the river channel for each of the Chicago-type bascule bridges on these rivers. It is interesting to note that, for many of the bridges, the clear span exceeds the width of river channel considerably. As an example, a few of these are as follows:

Location	Width of river channel, in feet	Clear span, in feet	Ratio, clear span to channel
West Division Street (Canal)	90	135	1.50
West Division Street (River)	100	155.67	1.56
North Western Avenue	121	184.58	1.52
North Halsted Street (Canal)	90	184.58	2.05
West Webster Avenue	120	161.25	1.34
West Belmont Avenue	75	161.25	2.15

It is the writer's understanding that in each of these cases the clear span exceeds the river channel because the bridge does not cross the channel at right angles but crosses at a considerable skew, and that in each case a normal bascule, or bascule without skew, was used.

As the leaves of a normal bascule bridge rotate on an axis which is normal to the center line of bridge and center line of roadway, it has been necessary at skew crossings to provide spans with a clear distance between piers considerably in excess of the clear width of channel. Also this condition has required that the large rectangular bascule piers be placed at an angle to the direction of the channel flow, thus disturbing the flow of the current as well as blocking off a greater width of the waterway to the flow of current. At such locations, in order to protect river traffic, it also has been necessary at times to provide independent fender systems of considerable cost extending along the lines of the real channel.

In 1940 the Department of Public Works of New York, N. Y., began studies for a new bascule bridge at Hamilton Avenue over the Gowanus Canal in Brooklyn. The intersection of Hamilton Avenue and the canal makes an angle of 56°. The old bridge at this location was a skew bascule bridge of the rolling lift or Scherzer type. The main girders of this bridge were framed normal to the channel lines, the axis of rotation being parallel to the channel. This resulted in the main girders being placed rather close together, thus limiting the available space for counterweights and machinery, and in large triangular parts of the roadway framing being cantilevered outside of the main girders. Although the clear channel width was only 46.5 ft, in order to obtain any semblance of substantial framing, a double-leaf bascule bridge was used. The bridge was built in 1905 and had proved to be structurally somewhat flexible, resulting in high maintenance costs.

In making studies for the new bridge several schemes were tried, among them a skew bridge, similar in arrangement to the original one, and a double-leaf bridge without skew. The skew design was not favored because of the experiences with the former bridge. A double-leaf bascule without skew would require a clear span parallel to the bridge approximately twice as great as the width of the skew channel. The estimate of cost of the bridge without skew was so great that it, too, was not considered with favor.

On further study, the writer developed a new type of skew bascule bridge that has the advantages of a bascule without skew, without the disadvantages attended by the extra length of span. The forward leaf of the bridge is made to fit the skewed channel crossing precisely, and the rear leaf is built normal to the channel lines, thus making the forward leaf as short as possible and allowing the center line of trunnions to be parallel to and as near to the channel line as possible. This gives the shortest span from center line of trunnions to toe end of span that can be arranged. The main girders are spaced as far apart as may be desired, thus permitting complete and substantial bracing systems to be used, allowing ordinary concrete in the counterweights for counterbalancing, and providing maximum room for locating the operating machinery between the main girders. Since the piers are built in line with the channel, the channel flow is obstructed the minimum amount.

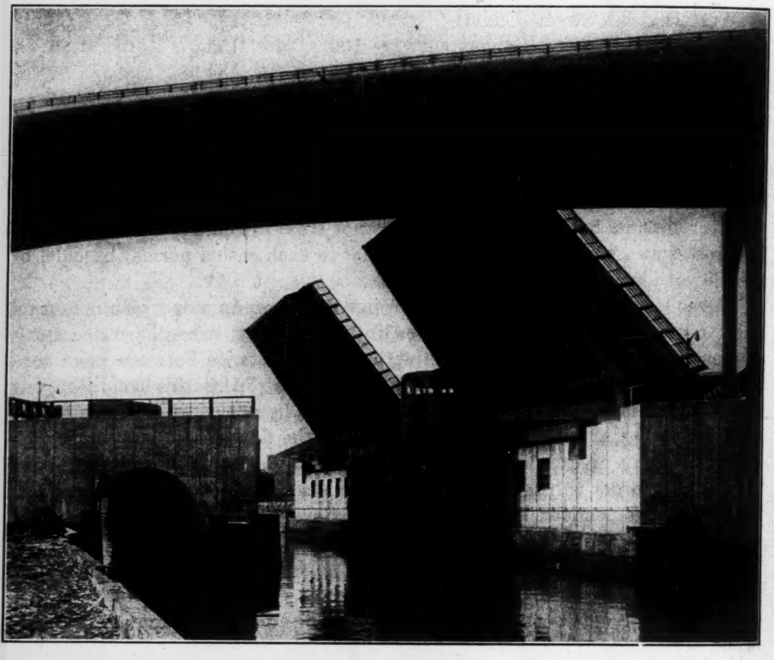


FIG. 20.—VIEW OF THE UNDER SIDE OF THE BASCULE LEAF IN OPEN POSITION

Although the center of gravity of the forward leaf is not midway between the trunnions or supports, the center of gravity of the counterweight can be midway between the trunnions, thus providing a counterweight of uniform section throughout its length. Fig. 20 is a view of the under side of the bascule leaf in the open position, showing the framing of the main girders as well as that of the other structural members of the forward leaf. The adoption of this type of bridge allowed the use of a single-leaf bridge and resulted in a great saving in cost. The same scheme can be used in a double-leaf layout for wider skewed channels.

This experience in the City of New York may be of interest in other cases where skew crossings are involved. It is the writer's belief that natural conditions of the relative alinement of roadway and channel result in considerably more skew crossings than square ones, especially since the tendency is to maintain a straight alinement on highways to accommodate modern high-speed traffic.

Mr. Becker states that in the design of the South Canal Street bridge some new ideas are incorporated, among them the use of open grating for the roadway pavement. This type of grating early was recognized as economical if used as a floor for movable bridges. The City of New York has nine bascule bridges with this type of decking installed, some of them older bridges on which the floor has been reconstructed. The Hamilton Avenue bridge shown in Fig. 20 is one of the new bridges on which this type of floor was installed. The writer agrees that its use saves weight not only in the floor but in the supporting members and in the counterweight. In addition, it reduces the operating forces required, as both friction and wind load are lessened, thus reducing the power and the weight of machinery required.

It is interesting to note that, on the South Canal Street bridge design, the use of an hydraulic drive is contemplated. In 1937 the writer made a study of this type of installation as it was recognized that speed control with the hydraulic drive could be very much more sensitive than with an alternating current electrical drive. However, at that time, the idea was rejected because the manufacturers of the various hydraulic drives available were not in a position to assist in the development of the automatic control necessary to duplicate the protection provided by limit switches in the standard electrical setup. Duplication of this protection required the redesign of the hydraulic units so that these automatic controls could be attached to them. Another factor against adoption of the hydraulic drive was that some systems worked on a high-pressure basis and others on a low-pressure basis, and, in case of emergency replacements being required, there necessarily would have to be full dependence upon only one or two manufacturers.

Armour T. Granger,[9] M. Am. Soc. C. E.—All engineers engaged in the design of movable bridges should be interested in this timely and valuable paper. The comprehensive account of the development of the Chicago bascules from the crude structures of the past century to the excellent and artistic spans of the present reveals strikingly the tremendous advances which have been made in the design of bascule bridges during this period.

Pound for pound and dollar for dollar, the bascule probably presents more intricate problems than any other type of bridge structure. The successful solutions of many such problems are presented in the paper. Certainly the engineers who have participated in the design and development of the Chicago bascules have contributed greatly to the science of designing movable spans.

---

[9] Prof. and Head, Dept. of Civ. Eng., Univ. of Tennessee, Knoxville, Tenn.

The writer is strongly of the opinion that the simple trunnion bascule, with the counterweight beneath the floor, as exemplified by the Chicago bridges, is in general the most desirable of all bascule types, including those types of double-leaf bascules which require the span to function as a simple span or as anything other than a pair of shear-locked cantilevers when the leaves are closed.

Among the many interesting and difficult problems requiring solution in the design of a bascule are the following: Method of supporting the inside trunnion bearings, details of floor breaks, rail joints, provisions for trolley wires during movement of span, span locks, arrangement of counterweights, arrangement of operating machinery (frequently in limited space), adequate provision for proper machinery alinement, prevention of moving interferences, and many others. Some of these have been discussed in the paper. Limitations of space doubtless have precluded the consideration of others of interest and importance. It is hoped that in his closure the author may comment on more of these details.

It would be of interest, for example, to have some information relative to the development of the center locks for double-leaf spans. Various types of locks have been devised and used and some of them have not proved satisfactory. (This statement is general and is not made in reference to the Chicago bascules.) A thoroughly satisfactory lock should grip both leaves firmly, without play or clearance, should be arranged to engage and disengage readily, and should be so designed that no part of the lock can foul either leaf. The lock mechanism, of course, should be so interlocked with the operating machinery that power to operate the span cannot be available until the locks are opened fully. Any lock which involves vertical play or clearances with the lock in closed position is likely to cause objectionable bumping or impact when loads pass over the floor break.

The details of the center floor break are also important. Such breaks should be designed to insure smooth riding over the joint under all conditions. The overlapping tooth joint, with teeth tapered in plan and rounded at the ends on a vertical radius, can be arranged so as to be entirely satisfactory for the center roadway break of a double-leaf bascule.

Some discussion of the choice of the type of trussing used would be of interest. Warren trusses were employed for the West Lake Street bridge (Fig. 12), but apparently Pratt trusses were used for all the other bridges. It has been the opinion of the writer for many years that, except for shallow spans with deep members where secondary stresses may be of consequence, Warren trusses are preferable to Pratt trusses for practically all riveted bridge spans and the great popularity of the Warren truss in recent years evidences the fact that many engineers are of this opinion. A statement of the reasons why Pratt trusses were selected for the Chicago bascules should therefore be of value to the profession.

Another detail of much interest is the use of the live-load bearing or forward support in the front wall of the bascule pier. It would appear that the desirability of using such supports is somewhat open to question. In designing double-leaf, deck-girder, bascule spans the writer has investigated their use on more than one occasion, but has omitted them. These supports will afford an appreciable saving in the weight of the forearm girders or trusses, but they have attendant disadvantages which tend to offset this. These disadvantages may be stated as follows:

(1) With the forward supports omitted, the double-leaf, trunnion bascule is statically indeterminate for live load to the first degree only, and the actual stresses can be determined quite closely. The introduction of a forward support in each leaf renders the span threefold indeterminate, and the computed stresses therefore are likely to be considerably more in error than if the forward supports are omitted. The fact that the inside trunnion support usually has a certain amount of flexibility contributes to the uncertainty as to exact stress distribution. Furthermore, since the relative simplicity of the trunnion bascule (as compared to other types) is one of its principal advantages, it seems rather a step in the wrong direction to detract from this simplicity.

(2) It is seldom, if ever, possible to make the forward support carry live-load reaction only. With full live-load covering both forearms between the forward supports, a part of the dead load on the trunnions will be transferred to the forward supports, and it is possible in some cases, where the live load is heavy, for the "live-load support" actually to be subjected to the full dead load on one trunnion in addition to the full live-load reaction.

The truth of this statement should be apparent from the following considerations:

With no live load on the span, the live-load bearing (forward support) and live-load anchor at the rear are adjusted presumably so that the parts of these details on the fixed and moving portions are just in contact. Now assume full symmetrical live load on both forearms—which condition will give zero shear at the center locks and negative moment for the full length of both leaves. As each leaf is held vertically at the forward bearing and rear anchor, that part of the span between these two points then will deflect upward. This upward deflection will reduce the dead load on the trunnion bearings; and, unless the trunnion supports are quite flexible, enough dead load will be thrown off the bearings to the leaf to balance the moment of the live load on the leaf about the forward support, with a consequent increase in the reaction on the forward support. As the distance from the trunnion to the forward support is small compared to the length of the forearm, this increase in the reaction is large, and, with some span dimensions and live loads, it is possible to take the dead load completely off the trunnion bearings and transfer it to the forward support.

Such behavior of the span calls for heavy forward supports. Although the total resultant load on the foundation, of course, is in the same location with or without the forward support, the manner in which it reaches the foundation is quite different in the two cases. It is sometimes difficult to care locally for the heavy concentrations resulting from the forward support and to insure that they are distributed properly over the foundation.

Transference of at least part of the dead load back and forth from the trunnion to the forward support seems rather objectionable and appears to detract from the simplicity inherent in the trunnion bascule.

The writer should state again that his investigations of this matter have been made only on girder spans, but a similar line of reasoning should apply to truss spans.

Probably more ingenuity has been exercised in devising methods of supporting the inside trunnion bearing than in connection with any other bascule detail. Some very ingenious ways of accomplishing this on the Chicago bascules are described by the author. Another method, similar to the longitudinal, S-shaped girders used for many of the later bridges, consists of supporting the inside trunnion bearing on a very short girder hung from a horizontal longitudinal girder overhead which is supported by the front and rear walls of the pier. This arrangement can be used where the distance from crown of roadway to center line of trunnion permits.

Another good arrangement (patented) consists of supporting the inside bearing on a bracket cantilevered longitudinally from an inside trunnion column which rests on the floor of the pier. The column is so shaped in elevation as to fit into a notch in the counterweight to clear the span in the fully raised position.

It is desirable, but sometimes rather difficult, to provide approximately equal rigidity in the supports for inner and outer trunnion bearings. If there is much difference in the dead-load deflection of these supports, trouble may be encountered during and after erection.

The writer has had occasion to investigate the matter of transverse centering devices for the leaves of girder bascules and always reached the following conclusions:

(*a*) Transverse centering was not needed; and

(*b*) Any device that would deflect the leaf laterally by any appreciable amount would need to be so large and powerful as to be impracticable.

There are a number of spans with wide roadways, having interlocking tooth center breaks and no transverse centering devices, and so far as the writer is aware there has never been any transverse fouling on these spans.

The author has made a few comments relative to the comparative appearance of bascule and vertical lift spans, and a word on this subject may be in order. Having been connected with the design of many spans of both types, the writer is decidedly of the opinion that the bascule span is not necessarily superior in appearance to the vertical-lift span. Certainly many deck or railing-height bascules are more attractive than many vertical lifts, but in the eyes of many engineers nothing is more hideous than some types of through bascules with overhead counterweights. With thought and care the lift span can be given a very fine appearance. In the opinion of the writer, the Harlem River (Triborough) lift span in New York, N. Y., is not excelled in appearance by any movable bridge in the world. Certainly, however, the required horizontal and vertical clearances at Chicago, in general, are not particularly favorable for lift spans.

In conclusion, the writer would like to state clearly that no statement made in this discussion is intended to imply unfavorable criticism of any point in the design of the Chicago bridges. They are certainly a worthy monument to the city and the engineering profession, and the paper is a valuable addition to the literature of movable bridges.

C. B. McCullough,[10] M. Am. Soc. C. E.—The development of the "Chicago type" bascule structure is an interesting and clear-cut illustration of the adaptation of design types to meet special or particular local conditions. Numerous advantages inhere in this type, principal among which may be mentioned:

(1) Exterior blocking of the channel is avoided during operation. This advantage over the swing bridge assumes considerable importance in localities where it is essential to maintain maximum docking facilities along the river close to the structure, especially in those relatively narrow waterways such as the Chicago River, where the movable span constitutes the greater part of the stream crossing.

(2) It is possible for river traffic to approach within a comparatively short distance of a bascule structure, whereas for swing spans, craft must stand off much farther on account of the difficulty in negotiating a split channel and swinging around the draw rest.

(3) A bascule may be raised slightly to permit the passage of small boats which fail to clear by only a small margin, whereas, in general, a swing span must be opened a full 90° for each vessel regardless of its vertical clearance. Because of this fact, bascule spans may be operated with much less interference to vehicular traffic, particularly in those instances wherein a great portion of the river traffic is relatively low-masted.

(4) For the bascule it is not necessary, as in the case of the swing span, to construct a large pivot pier in the center of the channel. In certain locations, such a pier structure operates to deflect shoreward currents, with consequent destructive erosion along adjacent banks.

(5) The bascule span, unlike the swing span, does not require the construction of a draw rest in the center of the channel, which construction always operates as a serious obstruction to the channel and a menace to water-borne traffic.

(6) It is possible to construct wide roadway structures (such as are necessary for traffic conditions in cities the size of Chicago) without the necessity of blocking the channel. This condition does not always prevail in the case of swing spans, for the reason that there is a definite relationship between the width of the roadway and the minimum dimensions that may be adopted safely for the pivot pier. If the roadway width is increased beyond a certain limit, the pivot pier becomes of such dimension as to obstruct the channel seriously.

(7) The bascule type lends itself to widening with a minimum of inconvenience to traffic because of the possibility of building additional roadway lanes immediately adjacent and alongside the existing structure, whereas such procedure is impossible in the case of a swing span because of the necessity for the maintenance of swing clearance during construction.

(8) In the bascule structure (in certain types at least), the raised leaves may be made to constitute an effective traffic barrier. This is a tremendous advantage as compared to the swing span, particularly in the case of heavy,

---

[10] Asst. Chf. Engr., State Highway Dept., Salem, Ore.

high-speed traffic arterials, whereon accident elimination and control pose serious and challenging problems.

(9) As compared with the swing span, in general, the bascule structure is safer from the standpoint of water-borne traffic in that it eliminates the possibility of collision with the open end of the span when it is swung.

Most of the foregoing advantages derive from a comparison of the bascule bridge with the ordinary swing span. However, some of them are also applicable when such type is compared with the vertical lift. In addition, it appears possible in the case of the bascule, particularly the double-leaf design, to develop a greater degree of esthetic or architectural excellence than in the case of either the swing span or the vertical lift.

The foregoing items constitute the principal advantages inherent in the bascule type, some of which were mentioned by Mr. Becker.

The paper covers the origin and development of one specialized form of bascule, which has justly been denominated the "Chicago type." The historical development which he traces is most interesting. Of special interest to engineers is his mention of the internal rack (heading, "The Improvement Period") developed by Mr. von Babo; the utilization of transverse trunnion girders; the gradual development of concrete as a counterweight material; the utilization in one instance of an operating rack attached to the side, instead of in the plane, of the truss; the utilization of roller bearings in lieu of phosphor-bronze bushings; and the use, in one instance, of hydraulic motors.

The Chicago type represents a particular adaptation to meet the demands imposed by local conditions. The entire movable ensemble rotates about one single trunnion. In comparison with the multiple-trunnion types, this development presents the advantage of simplicity in construction and at the same time operates to reduce to a minimum the number of parts needing lubrication. As compared to the rolling lift type, the Chicago type concentrates the center of gravity at one fixed point during all phases of the operation. This type also discloses certain esthetic advantages as compared to most of the other types in common use. As mentioned by Mr. Becker, it is possible through the utilization of an arched lower chord, a full deck structure, and an appropriate architectural treatment of the operators' houses to produce an ensemble much more pleasing than is generally possible in other types. Fig. 21 is a view of a structure in this category which spans the Siuslaw River on the Oregon Coast Highway. It is illustrative of the possibilities in double-leaf simple trunnion bascule design in combination with reinforced concrete approach construction. The flanking approach spans of this bridge are 154-ft reinforced concrete bow string arches.

Another advantage inherent in the Chicago type is the facility with which the raised leaf, in single-deck structures, can be converted into a traffic barrier. Fig. 22 is a line drawing of the shoreward end of a Chicago type span showing the location of the trunnion in reference to the break in the roadway floor. With a fixed total angle of opening, ϕ, the clearance C between the fully raised leaves and the fixed roadway deck may be varied simply by moving the roadway break fore or aft along the longitudinal axis of the deck; in other words, simply by varying the angle θ as indicated. In view of the difficulty in safeguarding the operation of modern high-speed highway traffic, a positive barrier of this kind presents an advantage that cannot be overestimated.

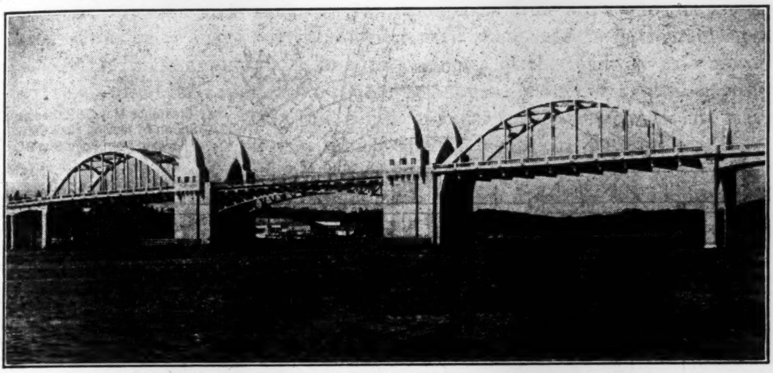


Fig. 21.—150-Ft Electrically Operated Double-Leaf Bascule Span Across Siuslaw River on Oregon Coast Highway

A most important feature in the design of bascule bridges is that of controlling the sequence of the various events that compose an operating cycle. In general, such a cycle comprises the following incidents, which should occur in the order given:

Opening Cycle

(1) Operation of warning lights, signs, or signals;
(2) Closing of remote roadway gates, if any;
(3) Closing of near roadway gates, traffic barriers, and the opening of derails, if any;
(4) Unlocking the leaves;
(5) Opening the span.

Closing Cycle

(1) Closing the span;
(2) Locking the leaves;
(3) Raising the traffic barriers, near gates, and closing the derails if any;
(4) Raising the remote gates.

The warning signs or signals may consist of bells, sirens, or whistles for the purpose of an audible warning, and the use of lights and semaphores for visual warning. The bells or sirens may be operated simultaneously with the closing of the gates or they may cease before the gates are lowered. In general, a prolonged siren is objectionable in that it may drown out signals from river

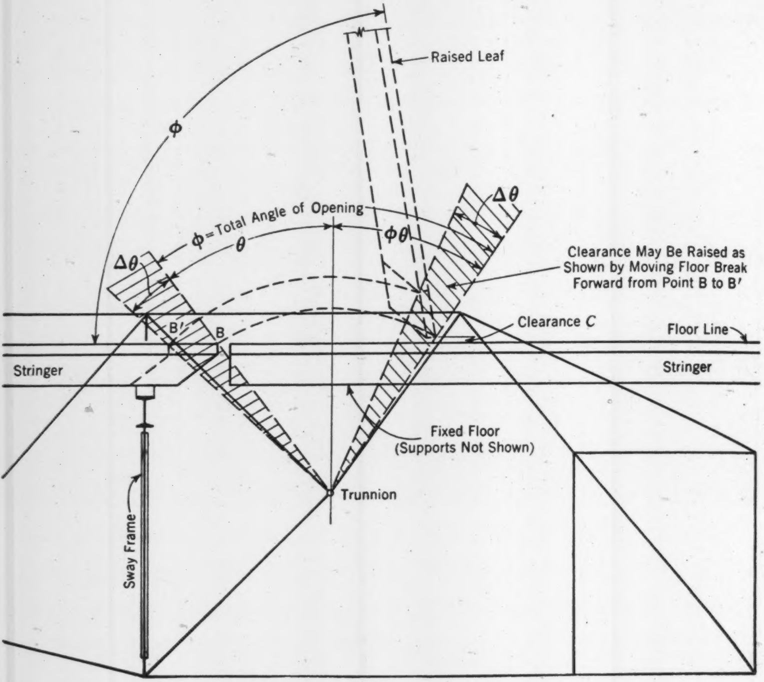

Raised Leaf
φ
Δθ
φ = Total Angle of Opening
θ
φθ
Δθ
Clearance May Be Raised as Shown by Moving Floor Break Forward from Point B to B′
B′
B
Clearance C
Floor Line
Stringer
Stringer
Fixed Floor (Supports Not Shown)
Trunnion
Sway Frame

FIG. 22.—EFFECT OF VARYING OPENING IN DECK

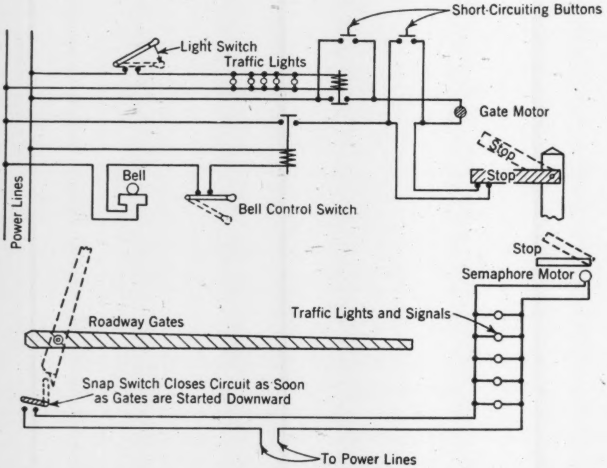

Short-Circuiting Buttons
Light Switch
Traffic Lights
Gate Motor
Stop
Stop
Bell
Bell Control Switch
Power Lines
Stop
Semaphore Motor
Traffic Lights and Signals
Roadway Gates
Snap Switch Closes Circuit as Soon as Gates are Started Downward
To Power Lines

FIG. 23.—METHOD OF INTERLOCKING CONTROL OF GATES AND WARNING DEVICES

craft, for which reason it is generally the part of wisdom to terminate the siren signals before the roadway gates are lowered. However, this objection does not hold in the case of a traffic bell or gong, which may be kept ringing during the entire operation of the roadway gates. Lights or semaphore arms may be interlocked electrically with the roadway gates in such manner that the gates cannot be operated until such lights or semaphore arms are set to "danger." For example, an ordinary door-type switch may be cut into the gate circuit and installed on the semaphore arm in such manner that the roadway gate motor will remain dead or inoperative until the semaphore is set in its warning position, as indicated in Fig. 23. In a similar manner the traffic-light circuit may be interlocked to operate a relay which closes the gate circuit when, and only when, the light circuit is energized. It is also possible to arrange the circuits operating semaphore arms and traffic lights in such a way as to energize such circuits the instant the gate moves from its fully open position. Both of these arrangements and also a system of short-circuiting switches are indicated schematically in Fig. 23. In any interlocked operation of this kind, a short-circuiting arrangement is necessary; otherwise it is possible for electrical interlocking to interpose dangers as great as those which the ensemble is designed to avoid. The foregoing constitutes the essence of the first step in the electrical interlocking of operations.

If, in addition to the roadway gates indicated in Fig. 23, the installation comprehends a second system of gates (near gates), traffic barriers or derails, the interlocking obviously should be extended so that it is impossible to close these devices unless and until the remote gates are closed.

It is obviously necessary to lock the leaves of a double-leaf bascule bridge together in order that the structure may transfer live-load shear. This locking is usually accomplished by a center-locking arrangement, and, in addition, tail locks are sometimes used, as described by the author. Such devices are, in general, operated by means of a master switch or controller, and the wiring should be so arranged as to interlock with previous warning signals, gates, and barriers so that such locking devices cannot be released until all the signals, gates, barriers, etc., have been properly operated and set to warning. A schematic arrangement of this kind is indicated in Fig. 24.

In addition to the motors controlling the locking devices it is necessary, of course, to install motors for the purpose of operating the leaves. Such motors generally are controlled through a master switch. This switch should be wired through a contact on the center lock which is closed only when the center-lock pin is fully released, thus avoiding the possibility of damage by attempting, negligently, to raise the leaves before they are unlocked. A schematic arrangement of this kind is indicated in Fig. 25.

In closing, the leaves of a bascule structure are generally lowered to a seat on an outboard or live-load shoe at the stream edge of the abutment and anchored at a rear bearing, as indicated in Fig. 24, to preclude the possibility of live load on the trunnion. At either of these points it is possible to install a device to complete an otherwise open circuit for the center-lock or tail-lock motors in the "drive" direction, so that these devices will remain inoperative in that direction until the bridge leaves are down, fully seated and correctly alined. If this interlock is not provided, the center or tail locks can be driven while the bridge is in the open position. Although this in itself is not particularly objectionable (since the main motor circuit is wired so as to be inoperative except for these devices fully open or drawn) there is some danger of jamming the center or tail-locking devices by attempting to drive them when the leaves are not fully seated and exactly alined, except in those cases where the locks are designed so as to draw the leaves together during the last part of their movement. When the wiring is such as to permit the operation of the center-locking pins with the leaves partly open, it appears that provision should be made for making the roadway gates, traffic barriers, etc., inoperative when the leaves are raised, since otherwise such gates or barriers could be raised with the leaves partly or fully open. On the other hand, if the gate and barrier circuits are led through a pair of contacts, one on the center pin and the other on its mating seat, then it is obvious that the leaves must be seated and the locking pins fully driven before the structure can be cleared for roadway traffic.

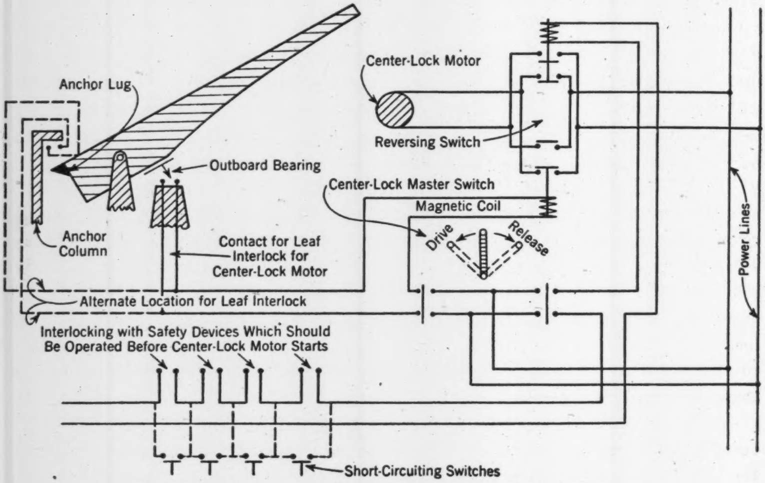

Anchor Lug
Center-Lock Motor
Reversing Switch
Outboard Bearing
Center-Lock Master Switch
Magnetic Coil
Drive
Release
Anchor Column
Contact for Leaf Interlock for Center-Lock Motor
Power Lines
Alternate Location for Leaf Interlock
Interlocking with Safety Devices Which Should Be Operated Before Center-Lock Motor Starts
Short-Circuiting Switches

FIG. 24.—WIRING OF CONTROLS ON THE CENTER-LOCK MOTOR

Fig. 26 is a schematic diagram showing interlocking contacts at the center lock for the barrier and gate motor circuits. If the two contacts are at A and B, it is apparent that the gate or barrier circuit can be energized whenever the pin is driven forward, regardless of the position of the leaves, in which case additional interlocking should probably be provided on either the live-load shoes or the anchor columns. On the other hand, if the contacts are located at A and C, no additional interlock appears necessary.

If the gate or barrier circuits are interlocked with the leaves but not with the tail or center lock, it is possible negligently to permit traffic over an unlocked span, thus inducing live-load stresses not contemplated in the original design.

In general, the sequence of all of the remaining operations should be controlled by arranging each circuit so that it remains open until all of those operations, which normally should precede it, have been completed.

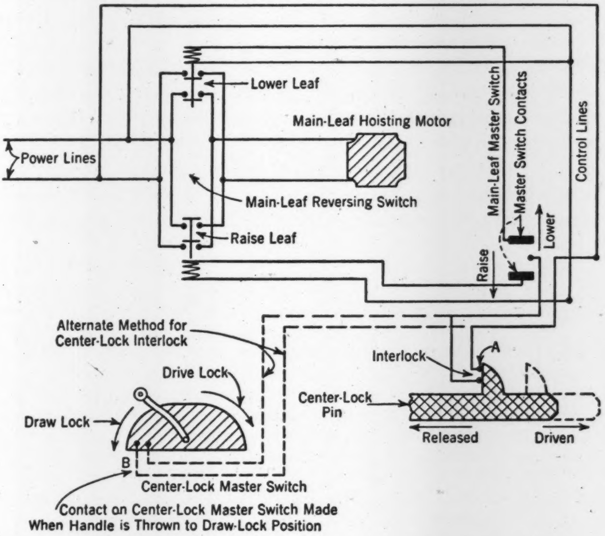

Lower Leaf
Main-Leaf Hoisting Motor
Power Lines
Main-Leaf Reversing Switch
Raise Leaf
Main-Leaf Master Switch
Master Switch Contacts
Control Lines
Lower
Raise
Alternate Method for Center-Lock Interlock
Interlock
A
Drive Lock
Center-Lock Pin
Draw Lock
Released
Driven
B
Center-Lock Master Switch
Contact on Center-Lock Master Switch Made When Handle is Thrown to Draw-Lock Position

FIG. 25.—INTERLOCKING OF MAIN MOTOR AND CENTER-LOCK PIN

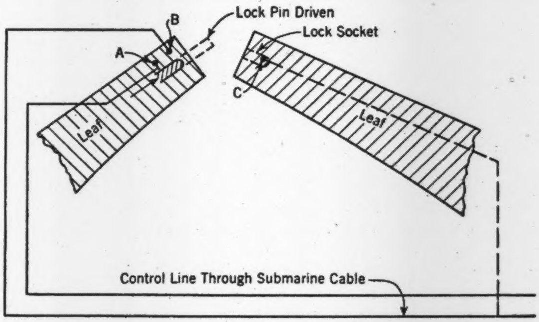

Lock Pin Driven
B
Lock Socket
A
C
Leaf
Leaf
Control Line Through Submarine Cable

FIG. 26.—METHODS OF INTERLOCKING CIRCUIT TO BARRIER OR GATE MOTORS

For low-power circuits, such as those for operating gates, signals, etc., the control switches may be cut directly into the line. For the main (high-power) operating circuits, however, magnetic contactors should be employed.

As an example of the dangers inherent in incorrect interlocking arrangements, assume that contact A in Fig. 25 were placed on the center-lock master switch instead of on the pin itself. In such a case, if the roadway gates or barriers were open, the circuit to the center-lock motor would be open and the handle on the center-lock master switch could be thrown to the draw-lock position with the pin still closed. Such an arrangement removes the safeguard of interlocking on the main hoisting motor circuit, thus making it possible for negligent operation to cause serious damage.

The foregoing are a few typical instances of electrical interlocking on bascule bridges. With the tremendous development in traffic densities and traffic speeds, the provision of adequate safeguards in this category becomes increasingly important. Each installation obviously presents its own interlocking problems. Sight distances, traffic speeds, traffic densities, etc., all operate to modify the arrangement of details. In a report covering this phase of bascule bridge design made by the writer in 1931,[11] the following general features of electrical interlocking were listed as necessary:

> "(1) Each control circuit should be so wired that the prior operation must be actually and completely performed before it becomes operative.
>
> "(2) Interlocking contacts must be positive in action and so located as to minimize the danger of short-circuiting or deterioration from moisture, ice, or mechanical injury.
>
> "(3) Contacts should preferably be made by a spring or snap device rather than by a sliding motion in order to avoid sparking and injury to contact points or tips, although a certain amount of wiping motion may be advantageous in tending to keep the contacts clean and bright.
>
> "(4) Make-and-break contacts for interlocking should not be placed directly in a heavily loaded power line but rather on an auxiliary or control circuit wired to operate a magnetic contactor cut into the main power line.
>
> "(5) Cross interlocking of operations should be provided wherever there is a danger due to the possibility of a failure of one phase of operation.
>
> "(6) Short-circuiting buttons or switches should be provided to permit operation in the event of a failure of any portion of the interlocking mechanism.
>
> "(7) In general, the entire interlocking arrangement should be as simple as possible, compact, sturdy, protected to the maximum possible extent, and so designed as to eliminate as far as possible every conceivable traffic hazard and every contingency or event that might expose the structure to undue stress."

In addition to the necessity for interlocking as a method of securing the correct sequence of operations, it is obviously necessary to provide devices in the nature of limit switches for controlling or limiting the movement of the leaves. Thus the wiring diagram for an electrically operated bascule structure becomes quite complicated.

It was with considerable interest that the writer noted the mention of Mr. Pihlfeldt (see "Conclusion"), who for many years was engineer of bridges for the City of Chicago. It was Thomas Pihlfeldt who took the writer on his first inspection trip over the bascule bridges of Chicago nearly a quarter of a century ago and who first impressed upon him the necessity for the bridge engineer becoming fully conversant with the principles underlying electrical operation and control.

---

[11] "Electrical Equipment on Movable Bridges," by C. B. McCullough, A. L. Gemeny, and W. R. Wickerham, *Technical Bulletin No. 265*, U.S.D.A., Washington, 1931, p. 38.

In closing, the writer wishes again to emphasize the importance of Mr. Becker's paper and its value to engineers engaged in this line of specialized endeavor. The writer has taken the liberty of adding these few pages in regard to electrical control and interlocking for the reason that this phase was not covered by Mr. Becker and for the further reason that credit for many of these features belongs to the Chicago group, to Mr. Pihlfeldt, to Hugh E. Young, M. Am. Soc. C. E., engineer of bridge design, and to others in the city bridge engineering force. Never has a group of engineers more successfully pioneered the development of a bridge type especially adapted to local needs. In the perspective of more than forty years, the wisdom of the early type selections stands out unchallenged.

DONALD N. BECKER,[12] M. AM. SOC. C. E.—Foundation conditions, especially the unreliability of some borings, are discussed by Mr. Hammond. For that reason borings made in later years have been continued 10 ft into rock to avoid the possibility that the first report of "rock" was merely a boulder. The accumulated mass of data that has been gathered through the years also tends to verify the results. Rock along the river in the downtown district is usually encountered at elevations slightly more than 100 ft below Chicago Datum (normal lake level) from Lake Michigan to the Lake Street bridge. The rock elevation then rises gradually southward with rock at about El. − 60 at the Roosevelt Road bridge and at about El. − 25 near 14th Street. Rock occurs at lower levels again south of 14th Street but not as deep as at the Lake Street bridge.

The rock is generally overlain with a fine water-bearing sand of varying thickness, covered by a hard clay layer of considerable thickness—say, as thick as 20 ft or 25 ft—which gradually softens as the elevation rises so that soft clay is reached usually above El. − 40.

The presence (in some cases, of as much as 15 ft or 20 ft) of water-bearing sand just above rock has made penetration to rock difficult and expensive. In some such cases sub piers have been belled out on hardpan after a test boring has been made to see that this layer is at least 15 ft thick. The belling out has usually extended until the unit pressure is reduced to 8 tons per sq ft. This practice has been found desirable in only a few cases in each of which rock was at, or more than, 100 ft below datum. No difficulties from settlement have been encountered.

Mr. Hanover presents a novel solution of the skew crossing problem, but the writer wishes to call attention to the relatively narrow channel in this case. Even if a similar leaf were placed on the other side of the channel to provide a wider opening, there would have been a channel of less than 100 ft. The bridges that Mr. Hanover refers to in Table 1 are rather old and in remote or narrow parts of the river system. In general, no channels of less than 140 ft are now permitted, and preferably channels should have from 170 ft to 200 ft clear span. With such great widths the Hamilton Avenue type bridge would not be practicable, especially where semithrough trusses are indicated instead of the deck girders at Hamilton Avenue.

---

[12] Engr. of Bridge Design, Dept. of Public Works, City of Chicago, Chicago, Ill.

In March, 1940, a bridge design was being developed for the proposed Congress Street superhighway where the skew is 62° 29′ and a 170-ft clear channel was required; the grade was high enough to permit deck trusses. As the proposed bridge was quite wide, 109 ft over-all, to provide two 44-ft roadways separated by a 3-ft center dividing fin and two 9-ft sidewalks, a square double-leaf bridge would have required from about 245 ft to 250 ft between masonry. A layout was finally developed in which two bridges will be placed side by side with a longitudinal joint in the center fin. In this layout one bridge will be set 31.61 ft ahead of the other. Thus, the clear span between masonry was reduced to 214.70 ft. These two bridges will be operated simultaneously by one operator on each side of the river as in the present Chicago double-leaf bridges. Arrangements will be made so that either bridge may be operated independently, which will permit one bridge to be placed up in the air for repairs and yet allow traffic on the superhighway to continue by using the other bridge for traffic in both directions on its 44-ft roadway until the repairs are completed.

Before this arrangement was developed, a study was made in which trusses were placed at right angles to the channel, with part of the outer truss in the acute angle of each leaf entirely outside of the bridge sidewalk. Since this plan produced a layout of questionable appearance, another layout was prepared in which the trusses were bent in a vertical plane with the trusses in front of the masonry parallel to the street but with the rear ends of the trusses normal to the channel. This layout was not considered desirable because of the difficulty of providing for the large kick reaction at the bend in the truss and the unequal power requirements on operating racks.

Mr. Hanover's remarks on the use of a hydraulic drive such as that adopted for the Canal Street bridge are not without merit. However, this installation is an attempt to develop a drive that will eliminate the complicated and expensive electrical control system of the present electric drives. It is being tried out as a transition from the usual electric drive to full hydraulic drive, and will have an auxiliary electric motor for emergency use. However, this electric motor drive will not have the complicated and expensive magnetic control equipment of the usual electric motor drives ordinarily specified in Chicago but will be controlled from a simple drum-type controller. If, after a few years of trial, the hydraulic drive proves reliable, future bridges may be equipped with it alone.

Chicago engineers also encountered Mr. Hanover's difficulties with control devices for the hydraulic drive and had to originate their own. A controller has been developed which operates the same as does the electric controller with which all the operators are familiar, in which the handle moves in a vertical plane—a pull on the handle toward the operator from a vertical position causes the leaf to rise and a push forward from the vertical causes the leaf to lower. The handle on the hydraulic drive is similarly arranged, and the greater the deflection of the handle is from the vertical the greater the speed will be. One great advantage of the hydraulic drive over the electric drive is that a return of the handle to the vertical automatically stops the leaf in its position without the application of the brake. A brake is provided, however, for emergency holding and for use with the auxiliary electric drive.

Professor Granger comments on the desirability of the simple trunnion bascule of the Chicago type. Its main advantages are: Freedom from linkages with pins that wear, and which may bind, in case of failure of lubrication; freedom from the hinged traps sometimes required with the rolling lift type; and other similar difficulties. Professor Granger is correct in his assumption that lack of space limited the discussion of details. However, these are so often affected by the requirements of each site that they can seldom be applied at another location and any experienced bridge engineer can easily develop details to suit his location. The main purpose of the writer was to chronicle the history and development of the Chicago type—the most significant phase of which was probably in support of trunnion bearings.

Professor Granger states that "A thoroughly satisfactory lock should grip both leaves firmly, without play or clearance, * * *." As against this need one should not lose sight of the necessity that the horizontal movement should allow for the temperature expansion of each leaf. It is not desirable to place one leaf on rollers so it can move in case of temperature expansion. (This was done in the case of the Soo Canal bridge where a full moment lock was provided to make the entire bridge function as a simple span under live loading. A failure of this lock for some undetermined reason was the cause of dropping one leaf of this bridge into the Canal on October 7, 1941.)

Locks on the Chicago bridges now consist of I-shaped bolts operated by electric-driven gear trains that force the bolts into the sockets on the other leaf. The sockets are provided with suitable adjustment features to allow for wear. The bolts are so designed that in case of necessity the leaves may be operated and the bolts can be withdrawn without damage to either bolts or sockets. Latest designs provide for the operation of these bolts by means of hydraulic pistons instead of motor-driven gear trains. Chicago's bridges have had their full share of difficulties with center locks.

Since before 1924, new bridges and some of the older ones have been equipped with intermeshing finger castings at the center roadway break. These do not foul each other when they mesh because the clearance tolerance of the teeth is greater than the clearance in the side cheeks of the center-lock castings which aline the bridge leaves before the teeth come opposite each other.

The writer admits that the general use of the Pratt type of trussing has never been given much consideration beyond the thought that with a curved bottom or top chord the difference in slope of the diagonals on each side of a vertical does not look well for Warren type trussing. A better appearance is obtained with the Warren type if parallel chords with attendant equal slopes are used. Also, the angle between the diagonals that were reversed and the sloping bottom chord would be so small that connection details would be difficult.

In regard to the use of a live-load bearing forward of the trunnion, several considerations have led Chicago engineers to this practice. First, the necessary supporting girders, whether of the **S**-type or cross-girder type, would otherwise have to carry the live load on the bridge and these girders are already of very heavy design. Second, in case of maintenance work the presence of the live-load support has provided a ready means of relieving the dead load from the trunnions by simply jacking up on the rear end. One such example was described by S. J. Michuda in 1942.[13] In this case because of a bump the entire east leaf of the East 106th Street bridge had shifted eastward and, by methods described in that article, the bridge was brought back to proper position and reanchored.

Unequal deflection between the supports for the inner and outer trunnion bearings is compensated by calculating the anticipated deflection of the **S**-girder and by setting the inner trunnion bearing higher. No ill effects have been experienced. Professor Granger comments on transverse centering and indicates that he has found no need for such devices. In Chicago bridges where the span is long enough to require trusses, designers usually provide cheeks on the center lock which guide the bridge into correct alinement within the allowed tolerances. Of course, where the end of the leaf is away from the trunnion 100 ft to 125 ft, only a relatively small force is necessary to deflect the leaves.

The writer agrees with Professor Granger on the merits of vertical lift versus bascule bridges. When a lift bridge can be placed out in a clear open location as was the Triborough Bridge in New York, it is definitely desirable. In the past too little attention has been given to the appearance of a structure and too much stress to the lowest possible cost, with the result that many monstrosities have been foisted upon the long suffering public. The writer believes that day is gone.

Mr. McCullough's comments are welcomed as helping to round out the entire topic by summarizing the numerous advantages of the bascule type bridge over other types and by outlining the necessity for, and the manner of, providing interlocking for the various operations.

Chicago's new bridges are thoroughly interlocked but by-passes must be provided so that a failure at any point will not stop the entire functioning. These by-passes are sometimes the cause of trouble if used inadvertently or otherwise when not actually needed, thus defeating the very purpose of interlocking. In this way one small human failure may defeat the best efforts of science.

---

[13] "Dislocated Bascule Rocked Into Position," by S. J. Michuda, *Engineering News-Record*, April 9, 1942, p. 85.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2227

SEDIMENTATION IN RESERVOIRS

By Berard J. Witzig,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. Joe W. Johnson, John W. Stanley, Stafford C. Happ, Thomas H. Means, Carl B. Brown, C. S. Jarvis, Hugh Stevens Bell, Harry F. Blaney, A. L. Sonderegger, L. Standish Hall, and Berard J. Witzig.

Synopsis

The principal phases of the problem of reservoir sedimentation are reviewed in this paper, including (*a*) the origin and nature of sediment, (*b*) its transportation to and through, or deposition in, the reservoir, and (*c*) remedies for reservoir silting. Quantitative study of sedimentation rates may be made from several approaches, none of which are sufficiently developed to permit more than approximate estimates of probable sedimentation rates. Such estimates may be of use to design engineers, who must allow for inevitable depletion of storage, but the real solution to the problem, both from the technical and social viewpoints, seems to lie in the prevention of sedimentation, and conservation of the available storage to the greatest extent possible.

Introduction

Studies by the Soil Conservation Service (SCS) of the U. S. Department of Agriculture (1)[2] show that, in the silt-carrying streams of the United States, the probable life expectancy of existing reservoirs is dangerously short. Detailed surveys of sediment volumes have been made in about 1% of all reservoirs in the United States. Assuming that these constitute a representative sample, that the long-term silting rate remains uniform, and that a reduction of 80% in the capacity of a reservoir terminates its useful life, these studies indicate that about 64% of all reservoirs have useful lives of less than 100 years, and only about 15% serve for more than 200 years.

Although it is a primary problem in badly eroding areas, silting of reservoirs is not the only manifestation of the silt problem. Among other aspects are sedimentation of navigable channels and harbors; aggradation of flood channels, causing increased frequency of flooding; silting of arable lands and destruction

---

Note.—Published in June, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Engr. (Civ.), U. S. Engr. Office, Buffalo, N. Y.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix II).

of fertility; increased cost of operating irrigation systems, due to the necessity of frequent dredging or provision of desilting works; increased cost of water supply treatment; damage to mechanical equipment by wear of gates, valves, turbines, etc. Some beneficial effects of sedimentation are decreased seepage losses from reservoirs and canals; decreased vegetation and algae growth due to absorption of sunlight; and fertilization of lands overflowed, when the silt has been produced by sheet erosion rather than by bank or gully erosion.

The factors involved in the production, transportation, and deposition of sediment are many and devious. Approach to the problems has been both empirical and rational: The former has generally seemed productive of the earliest applicable results, but often lacks a sound theoretical basis; and the latter seems more promising of establishing basic principles, but only a beginning has been made. Remedies for reservoir silting are costly and have been neglected, because of additional storage that ordinarily could be developed more cheaply, or because it was impracticable to evaluate the forces involved.

ORIGIN AND NATURE OF SILT

The problem of reservoir sedimentation and its solution are inextricably united with that of erosion. In regions where erosion is negligible, silting of reservoirs is generally not serious; and, where erosion is normally excessive, or has been accelerated by man's activities, reservoir silting is usually rapid.

The factors influencing the erosion cycle may be placed in two broad classes. "Eroding and transporting forces" include rate of runoff, turbulence of flow, fluid shear at boundaries and in eddies, fluid and particle impacts, surface and channel slopes, roughness, and sediment concentration. "Resisting and depositing forces" include gravity, adhesion, protective cover, impermeability, decrease in turbulence, local stilling, penetrable surface cover, and flocculation. Theoretical attempts to evaluate or relate more than one or two of these factors at a time end in practical confusion. The quantitative, empirical approach by way of field survey or reconnaissance gives probably the best data on erosion for engineering usage.

The major factors affecting erosion by water on land surfaces are precipitation and temperature. The impact of large raindrops is believed a significant fraction of the forces causing sheet erosion; the resulting overland flow is a vehicle for the loosened particles of soil. Changes of the overland sheet flow from laminar (streamline) conditions to turbulent flow result in additional erosion. The dynamic action of water in channels adds to the eroded burden. A drop in temperature to or below the freezing point may halt erosion temporarily, since frozen ground is more resistant than unfrozen, but the freezing and thawing cycle itself is a major force in comminuting large rocks and soil particles.

Origin of Sediment.—All sediment in streams originates in sheet erosion or in the various types of bank erosion. Robert Horton, M. Am. Soc. C. E., states that, of the total erosion in the evolution of a humid drainage basin, probably 1% was initially bank erosion, and 99%, sheet erosion (2). Studies of the SCS in the Piedmont (3) suggest that existing deposits within the flood plain of a stream are now often more important as immediate sources of sediment than concurrent sheet erosion, and it is evident in some cases elsewhere that bank erosion predominates over sheet erosion. Principles developed by the SCS indicate that gullying is more important than sheet erosion as a cause of harmful stream and valley sedimentation; that stream-bank erosion is an important factor in sediment production; and that valley trenching produces large amounts of fine sediment and facilitates downstream transportation.

The watershed characteristics most patently affecting sediment production, and hence the rate of sediment delivery to a reservoir are (4): Degree of erosion; relative extent and distribution of kinds of erosion (sheet, bank, gully, highway, etc.); soil type (size of particles); amount of runoff and character of discharge; and topographic form (colluvial and alluvial deposits).

Definition of Silt.—The erosion process ultimately reduces rock to the finest possible physical particles. The material that fills streams, valleys, and reservoirs actually varies from large boulders to grains just larger than colloidal size. In its widest application, the word "silt" applies to this entire range of sizes; in its more limited meaning, "silt" is a range of particle sizes between that of clay and that of fine sand. Only arbitrary definitions of size can be made, as shown (5) in Table 1.

TABLE 1.—SOIL SIZE CLASSIFICATION[a] (DIAMETERS IN MILLIMETERS)

Class	A.S.T.M. (1)	U.S.D.A. (2)	M.I.T. (3)	Park Service (4)
Sand:				
Coarse	2.0–0.25	1.0–0.50	2.0–0.60	6.0–2.0
Medium	....	0.5–0.25	0.60–0.20	2.0–0.6
Fine	0.25–0.05	0.25–0.10	0.20–0.06	0.6–0.2
Very fine	....	0.10–0.05	....	....
Silt:			....	
Coarse	0.05–0.005	0.05–0.005	0.06–0.02	0.2–0.006
Medium			0.02–0.006	
Fine			0.006–0.002	
Clay	<0.005	<0.005	<0.002	....
Dust	....	....	....	0.006–0.0002
Colloids	....	....	<0.0002	<0.0002

[a] In Col. 1 "A.S.T.M." refers to classification D422–35T of the American Society for Testing Materials; in Col. 2 "U.S.D.A." refers to the Bureau of Soils, U. S. Department of Agriculture; in Col. 3 "M.I.T." refers to the Massachusetts Institute of Technology; and the last column contains the modified classification of the National Park Service, U. S. Department of the Interior.

The size of the silt particle affects the degree of concentration of a suspension for given conditions of quiescence or turbulence. When the vertical components of turbulence are sufficiently strong, no settling takes place. Research (6) indicates that for each reach of each stream, a "dividing grain size" exists between material which moves as bed load and material which moves in near-permanent suspension. Table 2 indicates the rate of settlement (7) of a particle in still water at 50° F.

Colloidal particles, which include the "fine clay" of Table 2, will not settle out in any practical period of time, due to the effect of like-electronic charges on the particles. Under certain conditions of salt content and temperature, fine silts tend to flocculate and have higher rates of settlement than are shown in Table 2. The existence of graded suspensions results also in greater rates of deposition of the finer materials than when the latter are isolated, as the coarser particles appear to drag down the finer. The presence of organic matter slows up the deposition process, because its weight is more nearly that of water. Coagulation, or the production of flocculation by chemical means, as in water supply reservoirs and treatment tanks, hastens settling. Flocculation is aided by calcium and magnesium salts, and counteracted or prevented by sodium or potassium salts; obviously, the geologic composition of a watershed may favor a flocculated silt in some instances.

TABLE 2.—Rate of Sedimentation[a] in Still Water at 50° F

Line	Coarse Sand		Fine Sand				Silt	Coarse Clay	Fine Clay
1	1	0.20	0.10	0.06	0.04	0.02	0.01	0.001	0.0001
2	100	21	8	3.8	2.1	0.6	0.15	0.0015	0.000015

[a] Line 1 contains the diameters of particles, in millimeters; and, Line 2 contains the rate of settlement, in millimeters per second.

Silt deposited in a reservoir varies greatly in weight and volume, depending on its source, the depth of deposition, and the degree of submersion or exposure. The percentage of voids in a deposited silt sample appears to be the most important criterion of silt weight and volume. Assuming an average specific gravity of 2.6, which varies little in any case, J. C. Stevens, M. Am. Soc. C. E. (8), presents the values for silt deposits shown in Table 3.

TABLE 3.—Weight of Silt, in Pounds Per Cubic Foot

Weight	Percentage of Voids:									
	0	20	30	40	47.8	50	60	67.5	70	80
Dry...............	163	131	114	98	85	81	65	53	49	33
Saturated..........	163	144	133	123	115	113	102	95	93	83

Silt from exposed beds in Elephant Butte Reservoir (Rio Grande) has been found to weigh from 97 to 124 lb per cu ft, and the dry weight of samples from various points along the Colorado River has varied from 37 to 102 lb per cu ft. Samuel Fortier and Harry F. Blaney, M. Am. Soc. C. E. (9), recommend for the Lower Colorado a specific weight of 62.5 lb per cu ft for suspended silt, and 85 lb per cu ft for deposits in reservoirs.

The space occupied by silt after deposition in a reservoir changes with time and with the method of operation of the reservoir. Superimposed sediment compacts the lower deposits, as also do exposure and drying. The degree of shrinkage upon drying is dependent upon the relative sand and clay contents of the deposit. Sandy deposits shrink relatively little, but fine, uniform clay deposits are subject to considerable shrinkage. Deltaic deposits, worked over by wave action, may be well graded and quite dense.

Borings in exposed silt deposits in storage basins indicate that a definite water table exists commensurate with the water level of the reservoir; deposits below the water table are flocculent; and deposits do not shut off spring-water inflow on the beds of reservoirs.

NOTATION

The letter symbols used in this paper are defined when they first appear and are assembled for convenience of reference in Appendix I.

TRANSPORTATION OF SEDIMENT TO THE RESERVOIR

The sediment produced by erosion finds its way to the reservoir by movement in suspension, and movement as bed load. The former accounts for the transport of most of the finer sediment, whereas the coarser sands and gravels, and boulders, are moved by rolling along the bed of the stream. A third, or intermediate, stage of transportation exists—that of "saltation"—whereby movement becomes a series of steps or jumps; but this process is more predominant in littoral transportation. Hunter Rouse, M. Am. Soc. C. E., pertinently states that theoretically there is no fixed dividing line between the two primary modes of movement, but that limited knowledge of the correlation between the two precludes any but an independent treatment for the present (10). As previously stated, however, there now appears to be at least an empirical and unique division between grain sizes carried by each mode of transportation in each stream reach.

One of the earliest and best known attempts to evaluate the effect of flowing water on sediment was the formula developed by R. G. Kennedy for the Bari Doab Canal in the Punjab, India, about 1900. This formula is of the general form:

$$V_c = C_k \, (y_d)^x \qquad (1)$$

in which V_c = a "critical" velocity that will neither scour nor deposit material; C_k = an empirical coefficient, depending on the kind of silt; y_d = depth of water in channel; and x = an exponent, obtained from plotting observed data on logarithmic paper.

Eq. 1 has been used widely for canal design, with more or less successful results, but the wide range of values for C_k and x indicates that almost every application is unique. The coefficient, C_k, has been found to vary (11) from 0.38 to 1.83, and the exponent, x, from 0.44 to 0.73. There appears to be no relation, inverse or direct, between C_k and x. It may be concluded, therefore, that the range of velocities in the non-eroding, non-silting state is either little dependent on the depth, or highly dependent on the nature of the silt. Other formulas have since been proposed, but these also require the evaluation of empirical constants. At best, the available formulas are but guides to judgment. No general or certain analytical method is available for the determination of the "critical velocity," as defined by Mr. Kennedy.

Suspended Load.—After material has been taken into suspension, turbulence theory offers an explanation of subsequent sediment transportation. The theory of fluid turbulence also suggests the principle of vertical distribution of sediment in a stream, and therefore permits more accurate estimates of silt load from sampling observations.

Turbulence in a fluid represents the transfer of energy or momentum in random manner from one particle of the fluid to another. If such a fluid contains silt, some of the energy is transferred from fluid to silt, and from silt particle to silt particle. In maintaining suspension, the net transfer of energy is the resultant of all upward forces against the forces of gravity and other deposition factors. G. I. Taylor defines turbulence as "* * * an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces, or even when neighboring streams of the same fluid flow past or over one another" (12). Theodor von Kármán, M. Am. Soc. C. E. (12*a*), in substance defines the scientific term turbulence as implying irregular fluctuations, governed by laws of some statistical equilibrium between the random forces of turbulence and the force of gravity.

Various research workers have developed formulas to express the degree of turbulence, the turbulent energy, and its mode of transfer to other fluid particles and to the suspension of sediment. The formulas require a number of empirical constants, but the science rests on sound physical and rational foundations, a source of criticism against the purely empirical attack.

The present theory of fluid mechanics assumes a completely homogeneous and fluid substance. As soon as sediment is introduced into a stream, conditions depart from the ideal state, since both sediment and fluid are now moving, not only with respect to the boundaries of the fluid, but with respect to each other. Professor Rouse (10) as well as Morrough P. O'Brien, M. Am. Soc. C. E. (13), have reviewed the relation of turbulent flow to sediment transportation in some detail, and reference to their work is suggested.

Research has established fairly well that the concentration is dependent on sediment grain size, water discharge, and the degree of turbulence, at a particular instant. The total quantity of sediment is not necessarily related directly to discharge at all times, because of seasonal variations in the supply and source of sediment, and distribution of rainfall and runoff from the watershed, so that a measurement of the sediment load for a given discharge does not indicate the amount which may be carried by an equal discharge at another time. E. W. Lane, M. Am. Soc. C. E., and A. A. Kalinske, Assoc. M. Am. Soc. C. E. (14), have developed analytical expressions, based on the theory of the mechanics of fluid turbulence, for the distribution of sediment concentration in a vertical, and for the total suspended load in a unit width of a "wide" stream. Simplifying assumptions are that: The stream is "wide"; the kinematic viscosity is constant throughout the depth; the turbulence is due to an unconsolidated stream bed; and the sediment composition by sizes is known. The formulas permit estimating sediment distribution and load in a unit section from a single silt and velocity measurement in the section. The formula for distribution of sediment concentration in a vertical is:

$$N = N_a \, e^{-15t(z-y_a)} \ldots\ldots\ldots\ldots (2a)$$

and that for total sediment load in the unit section is:

$$W = V_m \, y_d \, P \, N_a \, e^{15 t y_a} \ldots\ldots\ldots\ldots (2b)$$

In Eqs. 2, N = sediment concentration at any point in the vertical, at a distance, y, above the bottom; N_a = measured sediment concentration in the vertical, at a distance, y_a, above the bottom; e = base of Napierian logarithms (2.718); t = a dimensionless parameter $= \frac{c}{\sqrt{g\, y_d\, S}}$; c = mean velocity of fall of sediment within a selected size range; g = acceleration of gravity; S = slope of energy gradient of the stream at the time of measurement; z = relative depth of point, above the bottom $= \frac{y}{y_d}$; y_a = distance of sampling point above bottom; y = distance above bottom of the point for which the concentration is desired; V_m = mean velocity in the vertical (or in the unit section); P = ratio of mean concentration in the vertical to the concentration at the bottom; n = Manning's roughness coefficient; and W = total weight of sediment within a given size range in a unit width of stream.

Eq. 2*a*, with z plotted arithmetically against N on semilogarithmic paper, results in a straight-line curve having a slope of $-15\,t$. In 1941 a report of the U. S. Engineer Office (15) showed the slope as $-16\,t$, and presented a plot of P versus t for three values, 0.01, 0.02, and 0.03, of the relative roughness. Eq. 2*a* may therefore be solved graphically if N_a is known. Eq. 2*b* represents the integration of Eq. 2*a*, after inserting a term for velocity. The term, P, is approximated by the following expression (15), and must be evaluated for the proper conditions:

$$P = \left\{ \left[1 + 9.50 \frac{n}{(y_d)^{1/6}} \right] \int_0^1 e^{-15tz}\, dz \right\} + \left\{ 9.50 \frac{n}{(y_d)^{1/6}} \int_0^1 e^{-15tz} \log_e z\, dz \right\} \qquad (3)$$

Eqs. 2 and 3 permit more accurate estimates of suspended sediment than by averaging a few measurements at arbitrary sampling points. However, very fine suspended sediment is so evenly distributed through the depth that a single measurement in a vertical or unit section suffices for silts and finer sediments as defined in Tables 1 and 2. It is dependent on the individual investigator whether the increased accuracy warrants the considerable increased time and expense involved in making the necessary measurements and analyses.

If a reasonably long record of suspended sediment measurements is available, a correlation between suspended sediment and discharge may be observed by plotting the data on logarithmic paper. A straight line usually fits the data as well as any other curve, and Frank B. Campbell, M. Am. Soc. C. E., and H. A. Bauder (16) call the relation a "silt-rating curve." However, the scatter of points is deceptive on logarithmic paper, and, for reasons previously stated, an estimate therefrom of silt load in an unsampled flow may be considerably in error. Nevertheless, the curve may be used as an indication of the probable average relation and, hence, it is sufficiently accurate for use in estimating the average annual silt quantities delivered to a reservoir. Messrs. Campbell and Bauder made such use of data for estimating the probable rates of silting for the reservoir above Denison Dam on the Red River, Texas. The equation of the curve is of the general form:

$$G_s = C_s\, Q^x \qquad (4)$$

in which G_s = suspended sediment, in tons per second; C_s = an empirical coefficient in suspended sediment rating curve; Q = water discharge, in cu ft per sec; and x = an exponent (the slope of the logarithmic plat).

Fig. 1 shows such "silt-rating curves" for the Black, Cuyahoga, and Grand rivers, in Ohio, based on several years of daily measurements taken in 1902–1904 (17). The data are admittedly of limited accuracy. The samples were obtained near the mouths of the streams by submerging rapidly a wide-mouthed 8-oz bottle at midstream to about 10 ft; and the discharges, based on a metered rating curve, are the averages of twice-daily readings. The silt measurements for the Denison River, on the other hand, appear to be of a higher order of accuracy, in that they represent averages of samples taken at 0.6 depth at 1/6, 3/6, and 5/6 of the river width, with the center sample given a weight of 2. The scatter of points above and below a certain range suggests the existence of different correlations. The lower values are of relatively little importance compared to the total annual silt load; the higher values accompany only extreme floods of comparatively rare frequencies and, although of weighty magnitude, also represent but a small part of the average annual load. The equations of the three curves of Fig. 1 are compared in Table 4 with that derived by Messrs. Campbell and Bauder, for the Red River (16).

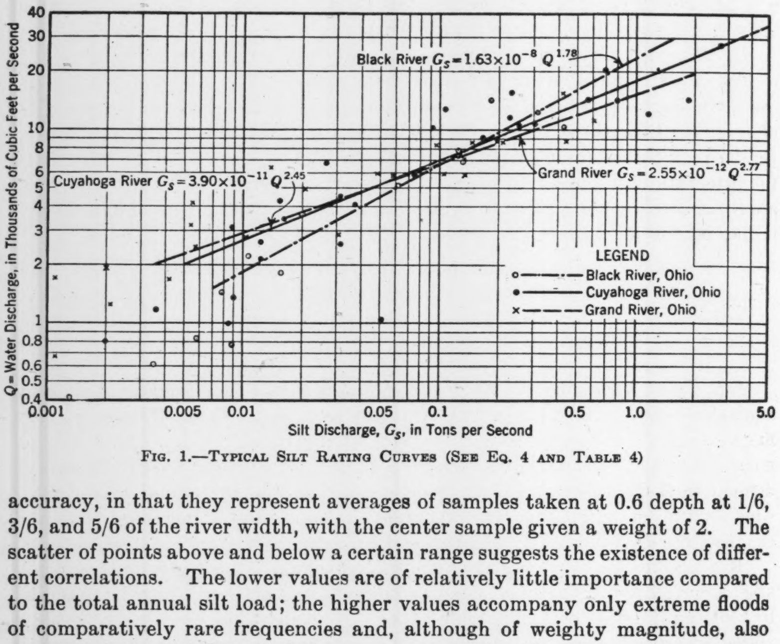


FIG. 1.—TYPICAL SILT RATING CURVES (SEE EQ. 4 AND TABLE 4)

The average annual suspended silt load can be computed by a combination of the "silt rating curve" and the discharge-frequency curve. It is first necessary to compute the ratio of silt load to corresponding discharge for a sufficient range of points, and then to plot the ratio $\frac{G_s}{Q}$ against the percentage chance of occurrence for the proper discharge, on arithmetic paper. The area under the curve represents the average annual suspended silt load per cubic foot per second of water discharge and is reduced to total average annual load by multiplying by the total average annual volume of discharge. Based on the equation in Item 3, Table 4, and on discharge-frequency relations adopted for an unpublished Engineer Department report, the area under the curve in Fig. 2 indicates that the probable average annual silt load of the Cuyahoga River at the mouth is about 0.0425 lb per cu ft of water, or, for a mean annual discharge of 794 cu ft per sec, about 532,000 tons per year. U. S. Engineer Department records show that about 650,000 cu yd of silt (or about 526,000 tons, assuming a concentration of 60 lb of dry sediment per cubic foot of deposit in place) are removed annually in maintenance dredging of the navigable areas of Cleveland Harbor, most of which comes from the Cuyahoga watershed. No reservoirs of adequate size exist on the Black, Cuyahoga, or Grand watersheds, to permit a comparison with actual rates of reservoir silting, and similar data for other areas are not available. Rates of silting in harbors are not directly comparable with those in reservoirs, and it is unlikely that the total load of the Cuyahoga is deposited in Cleveland Harbor. Some undetermined amount reaches the open lake. Also, part of the deposition in Cleveland Harbor is due to bed load

TABLE 4.—TYPICAL "SILT RATING CURVES"

No.	Geographical region	River	Place	Curve	LIMITS OF DISCHARGE (CU FT PER SEC)	
					Lower	Upper
1	Southwest	Red[a]	Denison, Tex.	$G_s = 9.5 \times 10^{-9} Q^{2.036}$	1,000	20,000
2	Great Lakes	Black	Lorain, Ohio[b]	$G_s = 1.63 \times 10^{-8} Q^{1.78}$	2,000	30,000
3	Great Lakes	Cuyahoga	Cleveland, Ohio[b]	$G_s = 3.90 \times 10^{-11} Q^{2.45}$	2,000	30,000
4	Great Lakes	Grand	Fairport, Ohio[b]	$G_s = 2.55 \times 10^{-12} Q^{2.77}$	1,500	25,000

[a] Reported by F. B. Campbell and H. A. Bauder (16). [b] At the mouth of the river.

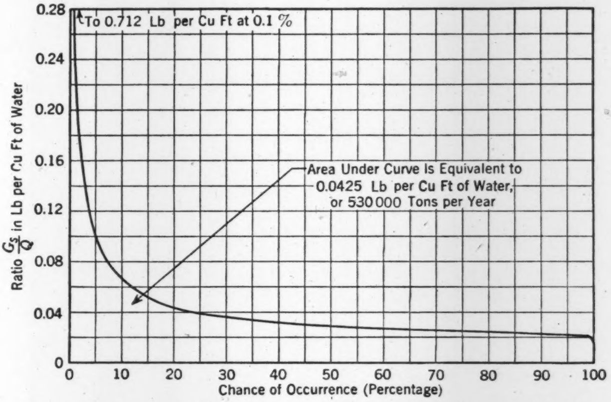


FIG. 2.—GRAPHICAL DETERMINATION OF PROBABLE ANNUAL SUSPENDED SILT LOAD OF CUYAHOGA RIVER, OHIO, AT THE MOUTH

from the river, subaqueous erosion from undredged areas, and possibly a small volume carried into the harbor by littoral forces. Considering the accuracy of the data in this example, the method seems reasonably applicable in determining the average annual suspended silt load of a stream.

Bed Load.—The problem of evaluating the bed load of the stream remains. Bed load appears to move first in a uniformly distributed layer, which begins when the tractive force reaches a certain value, and then in riffles which travel downstream, beginning at some larger critical tractive force (18). Bed-load sediment consists of those particle sizes which are too large for movement in suspension by the available suspending forces. The line of demarcation, as noted under the heading "Origin and Nature of Silt," is flexible, and, to the writer, it appears probable that this "limiting size" might have to be evaluated for various ranges of discharge, as well as for individual reaches in a stream. Investigation of the phenomenon indicates that bed load, unlike suspended load, is directly related to the discharge at all times, because the sediment supply for the bed-load forces is nearly always available, except on debris-clear rock-lined channels.

A greater amount of work appears to have been done on developing formulas for the amount of bed load than for suspended load. Experiments on flume traction by G. K. Gilbert (19) pointed the way for other workers. Two typical formulas are the Schoklitsch and the M.I.T. formulas. Investigations for the design of the All-American Canal (20) indicated that "the Schoklitsch bed-load formula is still a reliable instrument for the hydraulic engineer * * *." This formula, expressed in English units, is:

$$G_b = \frac{86.7}{(D_g)^{1/2}} S^{3/2} (Q_i - b\, q_o) \qquad \text{(5a)}$$

and the M.I.T. formula (18) may be expressed as follows:

$$G_b = C_b\, S^{x+1} (Q_i - b\, q_o) \qquad \text{(5b)}$$

In Eqs. 5, G_b = total bed load, in pounds per second; D_g = effective grain diameter, in inches; S = slope of the energy gradient; Q_i = total instantaneous discharge, in cubic feet per second; b = width of river, in feet; q_o = critical discharge, in cubic feet per second per foot width at which movement begins $\left(= \frac{0.00532\, D_g}{S^{4/3}}, \text{ in Eq. } 5a\right)$; and C_b and x = constants in Eq. 5*b*, depending on the specific gravity and mechanical composition of the bed material. The application of Eq. 5*b* is obscure since C_b and x "are yet to be related to sand mixtures so as to be determined directly from mechanical analysis" (18).

An approximate solution for bed load by the Schoklitsch formula can be made by determining or assuming a single representative grain size of the bed-load sediment, and mean values of the slope and discharge. However, it is obvious that the "critical" grain size varies within a given reach of a stream with different flows, so that only very limited accuracy can be expected. An estimate of the finest bed-load grain size for particular discharge and slope conditions may be made by analyzing the bed-surface material immediately after a flood. Assuming a mean grain size of 0.04 in. (about 1 mm) for the Cuyahoga River bed sediment near the mouth, an average slope of 1.0 ft per mile, and a mean annual discharge of 794 cu ft per sec, the total annual bed-load movement would be about 9,000 tons per year. Accurate computations of bed load are not possible because of variations of bed material from time to time, shifting of the effective width of bed that is actually scouring, and the necessary assumptions that must be made.

The practical and successful application of available formulas depends on expressing the nature of a mixture of grain sizes in some equivalent mean diameter, and evaluating the initial transporting force. No simple mathematical means of doing so are available. The theoretical functions presented by H. A. Einstein, Assoc. M. Am. Soc. C. E. (21), eliminate the idea of a "critical" force or state, and include the characteristics of the particles, but, although these functions describe experimental data, they are quite complex, and do not readily permit estimates of bed load for the varied conditions met in the field.

THE ACTION OF SEDIMENT IN RESERVOIRS

Some streams carry appreciable quantities of sediment only during floods, whereas others are burdened constantly with large silt loads. Whether the major part of silt damage is done intermittently or continually, the process of deposition is essentially the same. Some phenomena, however, are evident in reservoirs that are not so obvious in natural lakes or in the sea. Deposition of sediment results from the removal or dissipation of the transporting forces. The heavier silt sizes are naturally the first to be deposited, with the smaller sizes being carried farther out into the reservoir or not being deposited therein at all. The late O. A. Faris, M. Am. Soc. C. E. (22), partly describes the process thus:

> "Suspended silt settles to the reservoir bottom soon after entering the slack water and, having a greater specific gravity than water, flows, in the form of liquid mud, down the slopes into depressions and along the main channel until blocked by the dam. Owing to its greater density, silt-charged water entering a reservoir partly filled with clear water does not mingle with the clear, but forces it downstream toward the dam. No suspended silt is carried through the reservoir and over the spillway until all of the clear water has been discharged."

Deltas.—However, the sedimentary load of a stream is actually reduced considerably by valley aggradation above the reservoir. The remaining coarse-grained sediments, whether carried in suspension or as bottom load, tend to form deltas at the points of inflow. These deposits may be greatly altered from the ideal delta form by variations in reservoir level. Where the reservoir level is fairly constant, the characteristic bottom-set, fore-set, and top-set beds of a conventional delta are present, and the delta is subject to subsequent entrenchment by the stream with redeposition of the material farther out in the reservoir. If coarse sediments are lacking, no delta may be formed. The presence of deltaic deposits in a reservoir too frequently distracts attention from the unseen bottom deposits.

Bottom Deposits.—The fine-grained sediments carried by a stream into a reservoir spread over the bottom, and tend to concentrate in the deeper areas, especially immediately above the dam. This action is aided by the flow of the heavier turbid water along the thalweg beneath the lighter clearer waters of the lake. When the stilling effect of a large storage basin is available to an incoming sediment-laden stream, practically all of the sediment may be deposited in the reservoir cavity, and the outflow may be almost crystal clear. In such cases, an estimate of the probable average silt load of the tributary streams will indicate the rate of silting of the reservoir. Carl B. Brown of the SCS states that most of the reservoirs surveyed thus far trap between 70% and almost 100% of incoming sediment (23).

Effect of Reservoir Shape.—The distribution of sediment in a reservoir depends on the shape of the basin. If the reservoir is regular in shape, deposits from suspension will be distributed quite uniformly along its axis, decreasing in depth with distance above the dam. If the reservoir is irregular, there may be marked irregularity in the depths of the bottom-set beds. This is illustrated in the case of Elephant Butte Reservoir, where a section, known as "The Narrows," exists from about mile 15 to mile 19 above the dam. Profiles of silt deposits (1*a*) show that the silt surface here is almost horizontal, instead of following the grade of the original valley floor. However, the upper end of Elephant Butte is partly affected by seasonal drawdown.

Density Flows.—Considerable sediment is distributed to the deepest parts of a reservoir by the denser silt-laden water flowing beneath the lighter desilted water. Such flows sometimes dispense their load by diffusion and mixing or by settlement, thus losing their identity. However, it has been observed frequently in Lake Mead (above Boulder Dam), at Elephant Butte Reservoir, and in various reservoirs in Texas and in the Piedmont, that muddy underflows often extend to the outlet through a superficially clear lake. Ultimate understanding of the principles of density flows may lead to some control of reservoir silting. If the flow is allowed to stagnate, the silt will be deposited in the reservoir; if such flows could be induced and controlled, much of the sediment entering a reservoir could be voided before deposition. However, density flows are sometimes of quite low silt content, with the greater part of the difference in density from the reservoir water due to salt content or temperature.

Study of density flows in the past was generally neglected, because such flows are mostly invisible or hidden in the depths of the reservoir. However, two visual characteristics of underflow in reservoirs are the sharp line of demarcation near the head of the reservoir where the muddy water plunges below the clear water, and the localized zone of floating debris created at the upper end of the reservoir by a return current induced at the surface.

Density Flows in Lake Mead.—With reference to three observed occurrences of silty discharges at Lake Mead during 1935, Nathan C. Grover, M. Am. Soc. C. E., and Charles S. Howard (24) state that the flows apparently passed entirely through the reservoir, "essentially unmixed," and ascribe the phenomenon to the greater specific gravity of the incoming silt-laden water with respect to the clearer water at the surface of the lake. They estimate that these three silt flows carried some 6,000,000 tons of sediment, or about 2.5% of the average annual silt load brought to Lake Mead. Although measurements were made at Willow Beach about 10 miles below the dam, these quantities apparently do not include any materials scoured from the river channel below the dam. Mr. Faris (24*a*) disagrees with this theory and suggests that such discharge is the result "* * * of disturbance of silt which was virtually in place; that is, was no longer in suspension * * *." He implies that operation of the gates disturbed the equilibrium of this almost-liquid deposit and caused it to flow out of the reservoir. Ivan E. Houk, M. Am. Soc. C. E. (24*b*), suggests that lake temperature distribution is an important factor in density flows. In the case of a deep lake, movement may occur at some intermediate level rather than along the bottom of the lake, depending on the density of the silt flow. In order that silt may appear at the outlet of a lake, Mr. Houk believes that a relatively large inflow must continue until the silty flow has had time to reach the gates. On the other hand, Paul A. Jones, M. Am. Soc. C. E. (24*c*), suggests that the density flows of 1935 were due to the initial submergence and resulting erosion of unstable existing deposits as the reservoir filled. Hugh Stevens Bell (25) evaluates knowledge gained at Boulder Dam since 1935, and estimates that silt-laden density flows in Lake Mead carry about 24% of all sediment deposited in the lake.

Formation and Dispersion.—Experiments conducted at the California Institute of Technology, in Pasadena (26), show that "* * * density currents are not delicate or easily destroyed, but are difficult to prevent and more difficult to destroy." The loss of energy as the stream enters the reservoir is not generally sufficient to prevent the formation of a density current. Once the current has formed, only mixing across the "interface" can disperse it. The particles in suspension in the flow are believed to furnish most of the "driving force" down the slope of the reservoir bed, by virtue of their relatively greater specific gravity over that of water, so that the silt conveys the water rather than the water carrying the silt. Observation verifies the hardiness of density flows in the laboratory and in natural lakes and reservoirs. Density currents are known to have flowed for 100 miles through Lake Mead, and for 35 miles through Elephant Butte Reservoir. After storms on Lake Erie, the Niagara River carries considerable sediment into Lake Ontario. At the mouth, part of the river current flows out into the lake and part turns almost 90° to the east, flowing as a distinct stream at various distances offshore, depending on wind and wave conditions.

Mr. Einstein (27) infers that density flows may be considered laminar, and that dispersion, or mixing across the interface, will occur only when the flow is "unstable" and turbulence is caused. The turbulence must not be so great as to overcome the depth of the density current and break through the interface. He states that "* * * if the depth and concentration of the suspension are sufficiently high, and the sediment is flocculated, even violent turbulence will not reach the interface from the bed." Existing knowledge does not indicate the quantitative influence of dissolved matter, such as salts, and of temperature. No methods are known for quantitative analysis of density flows. Mr. Einstein states that "* * * not enough is known of turbulence structure to permit even estimates * * *." Additional field data are especially needed to substantiate the results of laboratory study. A fuller understanding of the phenomena involved may well lead to a utilization of density flows for controlling the rate of reservoir sedimentation to some extent.

Silting of Flood Control Reservoirs.—Few observations of silting have been made in flood control reservoirs. In general, however, silting therein appears to be a less serious problem than in storage basins. Due to drying and shrinking of silt beds in flood control reservoirs which are emptied after each flood, the silt consolidates and does not regain its original volume when submerged. Studies made on the Germantown and Englewood detention basins of the Miami Conservancy District by E. W. Lane and J. C. Kennedy (28) suggest that retarding basins do not silt up quickly, since a large part of silty and clayey sediment passes through without being deposited; that the proportion of clay to sand deposited increases with the time of detention; that recreational or other permanent storage pools within a retarding basin may be seriously silted within a short time; and that, unless the inflow contains high sediment concentrations, or the detention period is unusually long, sedimentary deposits are unlikely to become serious for many years. No evidence is available, but it seems improbable that density flows as previously described could exist in purely detention-type reservoirs, because of the more or less thorough mixing of the inflow.

ESTIMATING THE RATE OF RESERVOIR SEDIMENTATION

Methods of estimating the rate of sedimentation in a proposed reservoir are desirable, technically and economically, and may be approached at present from several angles. Comparison may be made of sedimentation rates in other reservoirs exposed to similar conditions; or an estimate may be made of the probable quantities of silt that will be delivered to and trapped by the reservoir. Unless specific remedies are to be undertaken to prevent sediment production and delivery, it is necessary to make proper allowance for silting, but even the fullest possible measure of control of sediment production will not eliminate partial depletion of storage.

Effectiveness of Reservoirs as Sediment Traps.—The quantity of sediment deposited in a reservoir is dependent on the extent to which deposition can take place. For complete deposition of the silt load, the capacity of the reservoir would have to exceed the volume of the incoming flood, and would have to retain the flood sufficiently long to permit all of the sediment to settle out. A criterion of reservoir effectiveness as a trap is the ratio of reservoir capacity to the drainage area above the dam. It appears that reservoirs having more than 15 acre-ft of capacity per square mile of drainage area (about 0.023-ft depth on the watershed) are comparable for rates of silting, whereas those of smaller capacity are not (29). The latter reservoirs either are filled by bed load, with little retention of flood waters, or a large part of the deposits are vented by operating the gates at the dam.

"Regional Indexes."—The SCS has given considerable study to sediment surveys in existing reservoirs, and the data thus obtained are presented in their official reports (1). Fig. 3 presents graphically the average sedimentation rates for various reservoirs reported by the Service (1*b*). The writer has drawn

envelope curves for storage reservoirs in the four geographic regions noted in Fig. 3 and in Table 5. The envelope curves have been drawn in "by eye"; the two points above curve I were neglected because of the small storage involved or the short time elapsing between surveys. The equation is of the general form:

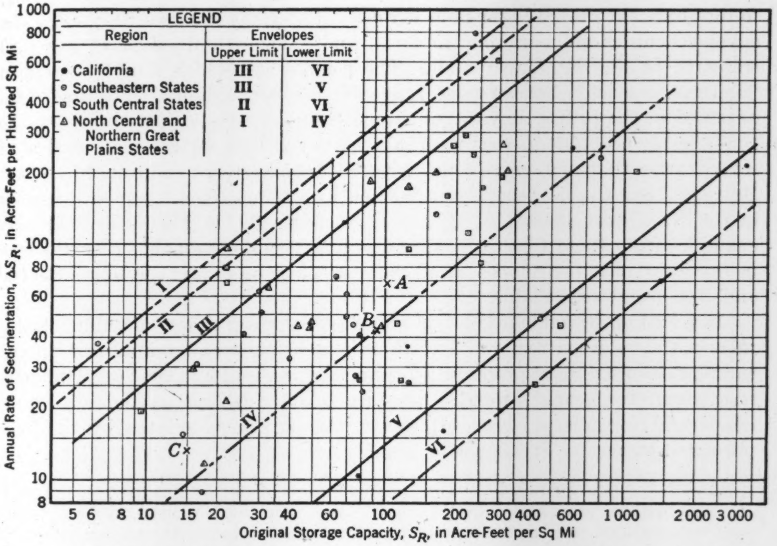


FIG. 3.—SEDIMENTATION RATES IN EXISTING STORAGE RESERVOIRS

$$\Delta S_R = I\,(S_R)^x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which ΔS_R = annual silting rate or depletion of storage, in acre-feet per 100 sq miles of drainage area; I = coefficient, herein termed the "regional index"; S_R = original storage, in acre-feet per square mile of drainage area; $x = 0.83$ = the graphic slope of the plat on logarithmic paper.

TABLE 5.—"REGIONAL INDEXES" (EQ. 6) OF STORAGE RESERVOIR SEDIMENTATION

Region (data are from reservoirs in the states indicated)	UPPER LIMIT		LOWER LIMIT	
	Envelope	I	Envelope	I
Southern Pacific Coast				
California	III	3.75	VI	0.167
Southeast				
Alabama, Georgia, Maryland, North and South Carolina, and Virginia	III	3.75	V	0.307
South Central				
Arkansas, Missouri, Oklahoma, and Texas	II	6.24	VI	0.167
North Central (Including the Northern Great Plains)				
Illinois, Kansas, Montana, Nebraska, and South Dakota	I	7.56	IV	1.01

The data of Fig. 3 are from large geographic regions, differing considerably in soil, erosion, meteorologic, and hydrologic conditions. It should be expected, therefore, that distinct indexes could be determined for each region; but, as the envelope curves of Fig. 3 indicate, there is considerable overlapping, so that only the probable range of sedimentation rates can be estimated. The envelope curves are parabolic in form, and a logarithmic slope of $x = 0.83$ seems to fit the data fairly well. The coefficients, which might be termed "regional indexes," have a wide range for a given region. Of course, data obtained in the future may either spread the envelopes, or narrow them; the latter seems probable in that reservoirs surveyed thus far are more typical of severe rather than of average conditions. It is of interest to note that Elephant Butte and San Carlos reservoirs in the Southwest, and Black Canyon Reservoir in the Northwest (points A, B, and C, respectively, in Fig. 3) are the only examples from these two regions; and yet they plot reasonably near the geometric mean of the envelopes for all the stated regions except the North Central and Northern Great Plains.

It would be desirable if the data indicated narrower ranges of sedimentation rates within a given region. Isolation of such factors as annual rainfall and runoff, rate of sediment production (erosion), variations in operation of different reservoirs, etc., would probably produce this result, but available data are insufficient for this purpose. However, the data accumulated by the SCS, presented in part in Fig. 3, are invaluable in that

> "* * * the principle of the limited range of debris production makes it possible henceforth to take this factor into account in design, and to predict from regional indices of sediment production developed through widespread reservoir surveys and sediment load measurements, the order of magnitude of the silting rate, and the relative need for sediment control measures within the limits of design to which a specific reservoir must be limited" (29).

Sediment Trap Efficiency.—In an attempt to narrow the range of sedimentation rates indicated by considering only the rate of silting and volume of reservoirs, as in Fig. 3, G. M. Brune and R. E. Allen, Jun. Am. Soc. C. E., of the SCS studied the effectiveness of a single type of storage pool in trapping sediment (4). The rate of erosion was estimated from erosion surveys or reconnaissances; the rate of silting, from reservoir studies; and the percentage of eroded soil trapped was computed. Data for twenty-five reservoirs in the Ohio Valley region (Indiana, Kentucky, Michigan, Ohio, and Tennessee), plotted logarithmically against storage volume in acre-feet per square mile, produced a "band" of values indicative of the "trap effectiveness" of storage reservoirs. The envelopes and median curve may be closely defined by:

$$100\left(\frac{\Delta S_R}{E}\right) = C_T\,(S_R)^x \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which: E = sediment produced annually on watershed (annual rate of erosion); and C_T = "trap efficiency" coefficient for the reservoir. For the Ohio Valley region, $x = 0.5$, which may also hold for other regions. The coefficient C_T has a range from 0.91 to 4.34, with a graphic mean of 2.00. In applying to tentative designs, it would be necessary to select a value of C_T that would properly allow for the contemplated operating conditions, and any abnormal soil, erosion, or reservoir conditions. If the reservoir were to be used primarily for storage, selection of a high value of C_T would be conservative; if an estimate of the desilting efficiency were desired, assumption of a low value of C_T would be on the safe side.

Eq. 7 would give only the probable percentage of eroded sediment that would be caught by a reservoir. The annual rate of depletion of storage may be computed by estimating the rate of erosion on the watershed, for those cases where survey data are available. An assumption must be made of the probable density of silt deposits under proposed conditions. As stated, the SCS has found that storage reservoirs trap "between 70 and almost 100 percent" of the sediment delivered to them (23).

Estimates of sediment delivery to a reservoir may be made also on the basis of suspended silt and bed load carried in by tributary streams, in accordance with procedure and formulas given in a previous section of this paper.

In designing a reservoir, estimates should be made of probable silting rates by all feasible methods. The general lack of data and knowledge concerning the process does not permit very great accuracy, but judgment may reach reasonable conclusions by considering such "regional indexes" as are now available, the effectiveness of vari-sized reservoirs as sediment traps, the rate of erosion on the watershed, and the rate of sediment delivery by tributary streams. These factors all can be measured or estimated. Data already accumulated are of great value, and the continuation of such studies is highly desirable from the practical engineering viewpoint.

REMEDIES FOR RESERVOIR SILTING

Authorities generally agree that redemption of lost reservoir capacity is practically impossible, due to the nature of silt, its distribution in a large reservoir, and the usually large quantities present. The most practicable means of avoiding this loss are to prevent the formation of permanent deposits, and to control the rate of sediment production from eroding areas. When this cannot be done, sufficient storage space must be provided to compensate for depletion by silting during a reasonable economic lifetime.

Two general methods in use to remove silt from reservoirs after deposition are mechanical excavation, as practiced in the debris basins of California, and in some small municipal and industrial reservoirs; and removal of silt by sluicing above the dam.

Excavation.—Removal of silt by mechanical excavation involves high maintenance costs. It has been found feasible in California only because of the severe damages caused by debris flows from arid eroded watersheds. The basins are built especially to trap the debris, often comprising as much as 90% of the total flood volume. The sediment is removed from the basin by steam shovel, or other mechanical means, at a cost of about 30¢ per cubic yard. A related problem affecting costs is the disposal of the excavated material, which sometimes must be hauled several miles from the basin (30). The debris problem in California is unique, and the social and economic aspects differ considerably from the reservoir silting question in other regions. Mechanical removal of silt does not appear to have widespread practicability.

Sluicing.—Silt may be sluiced from a reservoir by the provision of small gates to drain the reservoir when seasonal requirements are low, thereby cutting away the deposited sediment above the dam; or by the provision of large flood gates which would pass the entire silt-laden flood flow, thereby preventing most of the deposition. The first procedure has not been effective generally, except in scouring out the deposits for short distances above the dam. Although used with "considerable success" on the Hambra and Hamiz reservoirs in Algeria, it has had little success with deposited silt in Bhatgurk Reservoir in India, and Zuni Reservoir in the United States (8). The second procedure eliminates the flood control value of the reservoir and merely passes the flood and silt problem to downstream areas. However, it has been found practicable in the irrigation reservoirs of Egypt, where the population of lower areas desires the fertilizing silt brought down from the Upper Nile River.

Use of density flows, previously described, may be considered a possible remedy analogous to sluicing, but the silt carried by such currents must be voided, before the density current comes to rest. Mr. Bell (25) suggests some interesting uses that might be made of density flows, which not only would result in voiding some sediment from the storage cavity, but would permit operating a large reservoir to provide silty water for the repair of leaky canals and the control of vegetation in channels. Knowledge of density flows in reservoirs is insufficient at present to indicate the extent to which they may be made to serve a useful purpose, but further study is warranted in determining remedies for reservoir silting.

Erosion Prevention.—Prevention of erosion, by checking sediment production at the source, appears to be the best remedy for minimizing the silting of a reservoir. It has been estimated that if all possible erosion-control measures were applied to the Colorado River watershed, a possible reduction of 15% to 20% could be made in the present silt load of the river, thereby adding 100 to 200 years to the life of the reservoir system (31).

Erosion-control measures are correctly termed "upstream engineering," as distinguished from "downstream engineering," which involves large dams and reservoirs and channel projects. "Upstream engineering" consists of reforestation, halting of gully growth by check dams and planting, contour plowing, regulation of crop and grazing practices, proper treatment of high embankments and cuts, and stabilization of stream banks.

One phase of reforestation that may also be applied near the reservoir is the planting of proper vegetation screens and the construction of earth barriers on the flats at the head of a reservoir and above the mouths of large tributaries. Such screens reduce the velocity of silt-laden inflows and cause valley deposition before the silt reaches the main cavity of the reservoir. The accidental introduction of tamarisk above Lake McMillan on the Pecos River in New Mexico in 1912 produced a dense screen over the entire upper end of the reservoir and for some 200 miles upstream The annual rate of sedimentation in the lake dropped from 13.30 acre-ft per 100 sq miles of drainage area in the period 1910–1915 to 0.98 acre-ft per 100 sq miles in 1925–1932 (1).

Desilting Works.—The desilting works of the All-American Canal at Imperial Dam were designed to desilt 15,000 cu ft per sec of water. The silted water is passed through circular stilling basins, with a maximum velocity of 0.24 ft per sec, and a detention period of 14 min. Only material larger than 300-mesh is removed. The deposited sludge has a water content of about 90%, and is removed at the center of the basins by means of circular scrapers and effluent conduits. Desilting removes an estimated five sixths of the silt load, and saves about $1,000,000 annually in canal maintenance costs (32). The use of desilting works of this type does not seem applicable to the prevention of reservoir silting itself, and appears practicable only when reservoir sedimentation is not sufficient for the purpose. It seems probable that "desilting" on a large scale may be secured more economically by vegetation screens previously mentioned.

Therefore it appears that the most certain means of maintaining reservoir capacity is to prevent sediment from forming, or, after formation, to prevent its deposition in the lake.

Henry M. Eakin and Carl B. Brown (1) state that:

> "Main reliance for material and permanent conservation of reservoir resources must rest upon control of sediment production at primary sources through more widespread and effective application of established methods of erosion control."

SUMMARY

To a great extent, evaluation of the sediment problem on a particular watershed remains a qualitative one, when the accuracy of available methods of analysis is compared with that of the more rigidly established branches of hydrology. The general aspects of the principal phases of reservoir sedimentation have been reviewed, perhaps too briefly, in an attempt to summarize the present status of knowledge and investigation. Recent developments attempt, with some success, to combine the empirical and rational procedures; their present complexity warrants retaining those empirical procedures which have been used successfully in the past.

Additional data should indicate the possibility of developing accurate "regional indexes" that may be applied to proposed reservoirs. Certainly, it would be desirable that a fund of data be accumulated in this field as in others involving natural phenomena. Although present methods of collecting data give a reasonable accuracy considering the highly variable factors involved, it is doubtful that engineers will be satisfied until the most practical and accurate methods of measurement and analysis are evolved. The required research and the collection of data are necessarily expensive and slow, but the return that may be reasonably expected should justify the expense. It is doubtful that total research costs to date could exceed a small percentage of the value of irreplaceable reservoir storage capacity already destroyed.

Knowledge of the physical quantities involved must lead to decisions concerning whether to provide additional storage space in a reservoir to insure its returning the investment costs and charges during its lifetime, or to undertake such other remedies as may be possible. The former course, if physically possible at a given site, is strictly an economic problem and usually can be evaluated easily. The latter course usually entails considerable additional expenditures, but it is also a sociological problem which can scarcely be neglected by an enlightened and progressive engineering profession.

The literature on the subject is extensive, and the Bibliography in Appendix II barely indicates the work accomplished or in progress. Yet unified data are meager, or often unavailable when the practicing engineer is suddenly faced with a problem on reservoir sedimentation. Often, then, he must make use of such approximate methods of analysis as readily present themselves, or, if he has sufficient time and funds in a unique case, make an independent investigation.

CONCLUSION

The present methods of studying sediment transportation and reservoir sedimentation are laying the foundations and accumulating the basic data required for a better understanding of the problem. Continued study should accelerate progress. Available data and methods, and known remedies, must suffice for immediate problems. It has been the writer's intention to review the problems involved and some of the solutions for them.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to all those listed in the Bibliography, and to those not listed, whose thoughts or ideas he may have assimilated and appropriated in a rather extensive reading of literature on the subject. He is further indebted to Lt.-Col. Harland C. Woods, M. Am. Soc. C. E., U. S. Engineer Office, Buffalo, N. Y., for his critical review of this paper, which was originally prepared early in 1942 for use within the U. S. Engineer Department. Revision for publication has involved practically complete rewriting, and some additional data and comments. The opinions expressed herein are the writer's and do not necessarily reflect the attitude of the U. S. Engineer Department.

APPENDIX I

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

---

[3] ASA—Z10.2-1942.

b = mean width of a stream; as a subscript it denotes "bed load";

C = general symbol for coefficients:

C_b = a constant in Eq. 5b, depending on the specific gravity and mechanical composition of the bed material;

C_k = an empirical coefficient in Eq. 1, depending on the kind of silt;

C_s = an empirical coefficient in a suspended sediment-rating curve;

C_T = "trap efficiency" coefficient for the reservoir;

c = velocity of a falling particle of sediment; as a subscript, c denotes "critical";

D = diameter: D_g = grain diameter;

d = a subscript denoting "total depth";

E = annual rate of erosion, or sediment production, on a watershed;

e = base of Napierian logarithms = 2.718;

G = rate of sediment movement, subscripts, b and s denoting "bed load" and "suspended load";

g = acceleration of gravity; as a subscript, g denotes sediment grain;

I = "regional index," an empirical coefficient in Eq. 6;

i = a subscript denoting "instantaneous";

N = sediment concentration, by weight, at any point in a vertical, at a distance y above the bottom: N_a = measured sediment concentration, by weight, at a sampling point in a vertical, distant y_a above the bottom;

n = Manning's roughness coefficient;

o = a subscript denoting the state at which bed-load movement begins;

P = ratio of mean sediment concentration in a vertical to sediment concentration at the bottom;

Q = total rate of water discharge: Q_i = total instantaneous discharge;

q = rate of water discharge per unit width of stream: q_o = value of q at which bed-load movement begins;

R = a subscript denoting "reservoir";

S = slope of the energy gradient of the stream at the time of measurement;

S_R = storage capacity of a reservoir: ΔS_R = annual rate of reservoir sedimentation;

s = a subscript denoting "suspended sediment";

T = a subscript indicative of the "sediment trap efficiency" of a reservoir;

t = dimensionless parameter in Eqs. 2 and 3;

V = velocity, subscripts m and c denoting "mean" and "critical," respectively;

W = total weight of sediment within a given size range in a unit width of stream;

x = a general symbol for exponent, defined in each case;

y = distance of any point in a vertical above the stream bed, definite points being further defined by subscripts;

z = relative depth of a point above the bottom = $\dfrac{y}{y_d}$;

Δ = annual rate; see ΔS_R.

APPENDIX II

BIBLIOGRAPHY

(1) **"Silting of Reservoirs,"** by Henry M. Eakin (revised by Carl B. Brown), *Technical Bulletin No. 524*, U.S.D.A., 1939. (*a*) Fig. 19, facing p. 98. (*b*) Table 13, pp. 120–123.

(2) Discussion by Robert Horton of **"Dynamics of Water Erosion on Land-Surfaces,"** by Leonard Schiff and Robert E. Yodey, *Transactions*, Am. Geophysical Union, 1941, pp. 287–302.

(3) **"Some Principles of Accelerated Stream and Valley Sedimentation,"** by Stafford C. Happ, Gordon Rittenhouse, and G. C. Dobson, *Technical Bulletin No. 695*, U.S.D.A., May, 1940. (*a*) p. 632.

(4) **"A Consideration of Factors Influencing Reservoir Sedimentation in the Ohio Valley Region,"** by G. M. Brune and R. E. Allen, *Transactions*, Am. Geophysical Union, 1941, pp. 649–655.

(5) Adapted from **"Low Dams,"** National Resources Committee, Washington, D. C., 1938, pp. 247–248.

(6) **"A Distinction between Bed-Load and Suspended Load in Natural Streams,"** by H. A. Einstein, Alvin G. Anderson, and Joe W. Johnson, *Transactions*, Am. Geophysical Union, 1940, pp. 628–633.

(7) **American Civil Engineers' Handbook,** by Mansfield Merriman, Section 14, by Allen Hazen, John Wiley & Sons, Inc., 5th Ed. 1930, p. 1474.

(8) **"The Silt Problem,"** by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 207–288.

(9) **"Silt in the Colorado River and Its Relation to Irrigation,"** by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., 1928.

(10) **"Fluid Mechanics for Hydraulic Engineers,"** by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 331.

(11) **"Stable Channels in Erodible Materials,"** by E. W. Lane, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 123–194.

(12) **"Recent Developments in the Dynamics of Wind Erosion,"** by Frank J. Malina, *Transactions*, Am. Geophysical Union, 1941, p. 264. (*a*) pp. 264 and 284.

(13) **"Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation,"** by Morrough P. O'Brien, *ibid.*, 1933, pp. 487–491.

(14) **"Engineering Calculations of Suspended Sediment,"** by E. W. Lane and A. A. Kalinske, *ibid.*, 1941, pp. 603–607.

(15) **"Analytical Study of Methods of Sampling Suspended Sediment,"** *Report No. 3* in **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** published by St. Paul U. S. Engr. Dist. Sub-office, Hydr. Laboratory, Univ. of Iowa, Iowa City, Iowa, November, 1941.

(16) **"A Rating Curve for Determining the Silt Discharge of Streams,"** by F. B. Campbell and H. A. Bauder, *Transactions*, Am. Geophysical Union, 1940, pp. 603–607.

(17) *Annual Report*, Chf. of Engrs., U. S. Army, Pt. 4, Vol. 8, 1904, pp. 3803–3818.

(18) **"Bed-Sediment Transportation in Open Channels,"** by C. H. MacDougall, *Transactions*, Am. Geophysical Union, 1933, pp. 491–495.

(19) **"The Transportation of Debris by Flowing Water,"** by G. K. Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, 1914.

(20) **"Bed-Load Transportation and the Stable Channel Problem,"** by Samuel Shulits and W. E. Corfitzen, *Transactions*, Am. Geophysical Union, 1937, p. 456.

(21) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–577.

(22) **"The Silt Load of Texas Streams,"** by Orville A. Faris, *Technical Bulletin No. 382*, U.S.D.A., 1933.

(23) **Report of Committee on Sedimentation, 1940–1941,** Exhibit D, National Research Council, Washington, D. C., March, 1942, p. 26.

(24) **"The Passage of Turbid Water Through Lake Mead,"** by Nathan C. Grover and Charles S. Howard, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 720. (*a*) p. 733. (*b*) p. 736. (*c*) p. 734.

(25) **"Stratified Flow in Reservoirs and Its Use in Prevention of Silting,"** by Hugh Stevens Bell, *Miscellaneous Publication No. 491*, U.S.D.A., September, 1942.

(26) **"Density-Currents,"** by Robert T. Knapp and Hugh Stevens Bell, *Transactions*, Am. Geophysical Union, 1941, p. 257.

(27) **"The Viscosity of Highly Concentrated Underflows and Its Influence on Mixing,"** by H. A. Einstein, *ibid.*, pp. 597–603.

(28) **"A Study of Sediment in Miami Conservancy District Reservoirs,"** by E. W. Lane and J. C. Kennedy, *ibid.*, 1940, pp. 607–612.

(29) **"Dynamics of Entrainment of Erosional Debris and Sediment,"** by Carl B. Brown, *ibid.*, 1941, pp. 305–310.

(30) **"The Functions of Debris Dams and the Loss of Reservoir Capacity through Silting,"** by E. F. Kenyon, Jr., *ibid.*, 1939, pp. 16–21.

(31) **"Silt Problems of the West,"** by J. C. Stevens, *ibid.*, 1937, pp. 26–32.

(32) **"Colorado River Desilting at Imperial Dam,"** *Engineering News-Record*, October 17, 1935, pp. 538–541.

(33) **"Distribution Graphs of Suspended-Matter Concentration,"** by Joe W. Johnson, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 941.

(34) **"Use of Turbidity Determinations in Estimating the Suspended Load of Natural Streams,"** by R. G. Grassy, *Journal*, Am. Water Works Assn., Vol. 35, No. 4, April, 1943, pp. 439–453.

(35) **"Why Desilting Works for the All-American Canal?"** by C. P. Vetter, *Engineering News-Record*, Vol. 118, No. 8, 1937, p. 321.

(36) **"Missouri River,"** *H. R. Doc. No. 238*, 73d Cong., 2d Session, 1935, p. 1056.

(37) **"Transportation of Sediment in Suspension,"** by L. G. Straub, *Civil Engineering*, May, 1936, p. 321.

(38) **"High Silt Content of Colorado River,"** *Engineering News-Record*, **Vol. 130, No. 14, April 8, 1943, pp. 502–503.**

(39) **"The Flow of Water on a Movable Bed,"** by H. A. Einstein, 2d Hydraulic Conference, Univ. of Iowa, Iowa City, 1942.

(40) **"Bed-Load Transportation in Mountain Creek,"** by H. A. Einstein, Office of Research, SCS, U.S.D.A., 1943.

(41) Discussion by Joe W. Johnson of **"Statistical Analysis of Hydrology,"** by L. R. Beard, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1130.

(42) **"Effect of Turbulence on Sedimentation,"** by William E. Dobbins, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 632. (*a*) Discussion by Thomas R. Camp, p. 660.

(43) **"Standard Symbols and Glossary of Terms Used in Hydraulics as Applied to Irrigation,"** a report of the Society's Special Committee on Irrigation Hydraulics, 1933.

(44) **"On Sedimentation,"** by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 63.

(45) **"Terminal Velocity of Fall—Quartz Spheres in Air and Water,"** a chart by U.S.D.A., Cooperative Laboratory, SCS, California Inst. of Tech., Pasadena.

(46) **"Petrographic Methods and Calculations,"** by Arthur Holmes, Thomas Murby & Co., London, England, May, 1921, p. 207.

(47) **"Die rationell Klassifikation der Sande und Kiese,"** by A. Atterberg, Chemiker Zeitung, Johrgang 29, 1905, p. 198 (translated by I. B. Hosig).

(48) **"Die Mechanische Bodenanalyse,"** by E. C. J. Mohr, Departement de l'agriculture aux Indes Neerlandaises, *Bulletin No. 41*, Buitenzorg, 1910, p. 16 (translated by I. B. Hosig).

(49) **"Methods of Mechanical Analysis of Sediments,"** by C. K. Wentworth, Univ. of Iowa (Iowa City) Studies in Natural History, Vol. 11, 1925–1927, p. 41.

(50) **"Manual of Sedimentary Petrography,"** by W. C. Krumbein and F. J. Pettijohn, D. Appleton-Century Co., Inc., 1938, pp. 104–107.

(51) **"Advance Report on the Sedimentation Survey of Greenbelt Lake, Greenbelt, Maryland,"** by F. F. Barnes and C. B. Brown, SCS-SS-33, April, 1939, 12 pp.

(52) **"The Control of Reservoir Silting,"** by Carl B. Brown, *Miscellaneous Publication No. 521*, U.S.D.A., 1943. (*a*) pp. 116–117.

(53) **"The Silt Problem of the Zuni Reservoir,"** by H. F. Robinson, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 868.

(54) **"Lake Mead Density Current Investigations 1937–1940,"** Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., October 1, 1941, Vol. I, pp. 135–282; also unpublished Bureau of Reclamation data on samples taken monthly from September, 1940, to September, 1943.

(55) *Transactions*, Am. Soc C. E., Vol. 94 (1930), p. 1139.

(56) **"The Silt Load of Texas Streams,"** by Dean W. Bloodgood, mimeographed progress rept., Texas Board of Water Engrs. and U.S.D.A., September, 1940.

(57) *Water Supply Paper No. 844*, U. S. Geological Survey, 1942.

(58) **"Modifying the Physiographical Balance by Conservation Measures,"** by A. L. Sonderegger, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 285.

(59) **"Landslides and Related Phenomena,"** by C. F. Stewart Sharpe, Columbia Univ. Press, New York, N. Y., 1938, Chapters 4 and 5.

(60) **"Le Rhone et les Rivières à Lit Affouillable,"** par M. P. du Boys, *Annales des Ponts et Chaussées*, Vol. XVIII, 1879, Pt. 2, p. 141.

(61) **"Field Practice and Equipment Used in Sampling Suspended Sediment,"** *Report No. 1* in **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** published by St. Paul U. S. Engr. Dist. Sub-office, Hydr. Laboratory, Univ. of Iowa, Iowa City, Iowa, August, 1940, pp. 85 and 90.

(62) **"Silting of Reservoirs,"** by L. Standish Hall, *Journal*, A.W.W.A., Vol. 32, No. 1, January, 1940, pp. 25–42. (*a*) pp. 39–42.

(63) **"Erosion Control for Reservoir Protection,"** by Harry E. Reddick, *Soil Conservation*, Vol. VII, No. 3, September, 1941, pp. 77–79.

(64) **"Drop Structures for Erosion Control,"** by L. Standish Hall, *Civil Engineering*, May, 1942, pp. 247–250.

(65) **"The Use of Mass-Curves in Evaluating Suspended Load Observations on Small Piedmont Streams,"** by J. W. Johnson, *Transactions*, Am. Geophysical Union, 1942, pp. 664–672.

(66) **"Density of Sediments Deposited in Reservoirs,"** by E. W. Lane and Victor A. Koelzer, *Report No. 9* in **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** St. Paul U. S. Engr. Dist. Sub-office, Hydr. Laboratory, Univ. of Iowa, Iowa City, November, 1943. (*a*) Fig. 2, p. 37.

DISCUSSION

Joe W. Johnson,[4] Assoc. M. Am. Soc. C. E.—An excellent discussion of the origin, transportation, and deposition of the sediments in reservoirs has been presented in this paper. Of interest to the writer is the section of the paper concerning the transportation of sediment to the reservoir. Some of the phases of this problem well may be extended in the light of recent investigations.

For practical considerations the total sediment load of a stream consists of three parts—the wash load (6), the coarse material in suspension, and the coarse material rolling or sliding along the bed. The equipment and method of calculation used in estimating these loads differ greatly. The wash load may be estimated very simply by continuous and frequent sampling during stream rises. In some instances, the use of distribution graphs of suspended-matter concentration (33) or turbidity data (34) may greatly simplify the labor of determining the load of wash material. The wash load is composed of relatively fine material and constitutes much the greater part of the material in a suspended-load sample. It may be regarded as sediment in relatively permanent suspension. The settling velocity of the wash material is very small, and, once the material is washed into a stream, it is transported completely through the stream system, by even the most feeble velocities, to the point of deposition. It is the wash material which probably constitutes the bulk of sediment that passes through reservoirs in muddy underflows. The quantity of wash material transported in a stream bears no relation to the discharge because the conditions of supply, which vary from year to year and season to season, are independent of flow conditions.

The author states (see heading, "Transportation of Sediment to the Reservoir: Bed Load") that "bed load, unlike suspended load, is directly related to the discharge at all times, because the sediment supply for the bed-load forces is nearly always available * * *." This statement which infers that no direct relationship exists between suspended load and discharge is true only when total suspended load is considered, but needs qualification when the loads of sediment of various size classifications are considered separately. The suspended load, including both fine and coarse material, is not directly related to the discharge because of the variability of the wash load. The coarse material in suspension, however, is related to the discharge because the supply of this material is always available, except, as stated by the author (see same heading), "on debris-clear rock-lined channels." An excellent illustration, showing how the coarser grades of sediment are related to discharge, is given by the extensive measurements of C. P. Vetter, M. Am. Soc. C. E. (35).

The author suggests the use of "silt-rating curves" in estimating the suspended load of streams. In such a practice, of course, it is assumed that the suspended load is directly related to discharge. The curves given by the author, along with the data of Messrs. Campbell and Bauder (16), indicate that an approximate relationship may be developed between instantaneous values of discharge and suspended load, presumably including both fine and coarse material. In these instances, the period of observation has extended over only two or three years when conditions of supply may have been approximately constant. In connection with the extensive sediment investigations on the Missouri River and its tributaries by the U. S. Engineer Office, silt rating curves were plotted for twenty-two stations (36). Except for the curve of the Kansas City station on the Missouri River, which has been published elsewhere (37), the curves for the various stations have not been published. These curves may be examined, however, in the office of the Chief of Engineers in Washington, D. C. Some of these curves show a usable relationship, as found for the Kansas City station, whereas other plots show such a large scattering of data that their use is of questionable value. As additional data to the information given in Table 4, the equation of the Kansas City curve reported by L. G. Straub, M. Am. Soc. C. E. (37), is

---

[4] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

$$G_s = 30.42 \times 10^{-11} Q^{2.16} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The minimum discharge was 20,000 cu ft per sec and the maximum discharge was approximately 187,000 cu ft per sec.

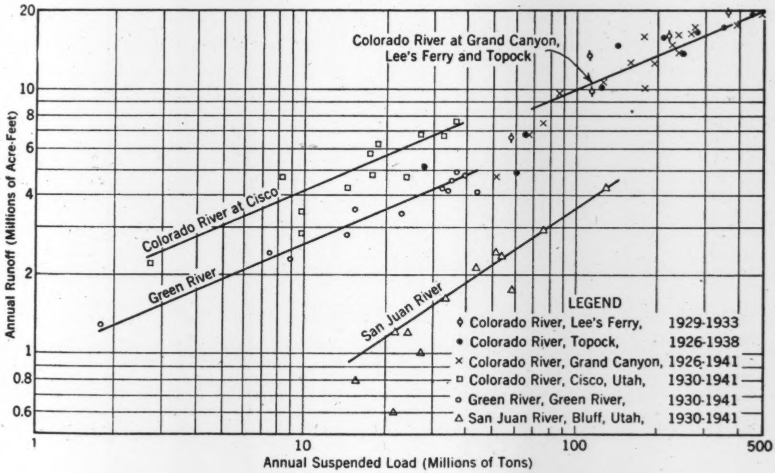


Fig. 4

For streams on which observations, extending over a period of several years, are available, a silt rating curve based on annual rates of runoff and sediment load shows some scattering of points but appears to give a more simple relationship for estimating the average annual suspended load. For example, Fig. 4 shows a plot of annual runoff and suspended load for the Colorado River and its tributaries as observed by the U. S. Geological Survey (38). These data were obtained for periods from twelve to sixteen years.

With the average annual runoff being estimated from long-term discharge records from the various streams, these curves can be entered directly to obtain the approximate average annual suspended load.

Although an estimate of the average annual contribution of suspended load of a stream requires considerable field and office work, an estimate of the average annual contribution of bed load can be made accurately and conveniently from the application of a suitable bed-load formula and a flow-duration curve. The author has suggested the Schoklitsch and the M.I.T. bed-load formulas, neither one of which has had the extent of verification in natural streams that the $(\phi-\psi)$-relationship (21) has had. The formulas expressed by Eqs. 5*a* and 5*b* include a "critical" discharge to describe the condition when movement of the bed material begins. "Unfortunately, this is a condition that does not exist in nature" (21*a*). In many instances, the magnitude of the critical discharge is negligible in comparison with the discharge under consideration and might just as well be discarded.

The author states (paragraph preceding heading, "The Action of Sediment in Reservoirs") that the Einstein functions "are quite complex, and do not readily permit estimates of bed load for the varied conditions met in the field." Admittedly, the functions are quite formidable in appearance on first examination. However, further consideration will show that many of the terms are constants, and calculations of the rate of bed-load transportation in a particular reach of a stream can be made quickly and conveniently from a few simply observed field data.

In the Einstein formula the intensity of transportation is given by the expression

$$\phi = \frac{q_b}{F(s_s - s_f)\,\gamma}\sqrt{\frac{s_f}{s_s - s_f}}\,\frac{1}{g^{0.5}\,D^{1.5}} \ldots\ldots\ldots\ldots\ldots (9a)$$

and is related to the hydraulic variable,

$$\psi = \frac{s_s - s_f}{s_f}\,\frac{D}{S_e\,R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

in which q_b is the rate of bed-load transportation in weight under water per unit width per second; F is a dimensionless factor in the Ruby equation for settling velocity and is a function of the representative grain diameter D; g is the acceleration of gravity; s_s and s_f are the specific gravity of the solid and fluid, respectively; γ is the unit weight of the fluid; S_e is the slope of the energy grade line; and R is the hydraulic radius of the bed.

For very wide channels—that is, where the energy dissipated along the banks is negligible compared to the energy dissipated along the bed—the ϕ-terms and ψ-terms can be used as follows to construct a set of curves or to prepare tables that show the rate of bed-load transportation per unit width as a function of either discharge or depth for various slopes and bed material. Thus, if $s_s = 2.65$, $s_f = 1$, and $\gamma = 62.3$ lb per cu ft, the ϕ-terms and ψ-terms may be written as follows:

$$\phi = \frac{q_b}{F\,D^{1.5}\,(1.65)^{1.5}\,(62.3)\,(32.2)^{0.5}} = \frac{q_b}{750\,F\,D^{1.5}} \ldots\ldots\ldots (10a)$$

and

$$\psi = \frac{1.65\, D}{S_e\, d} \qquad (10b)$$

in which d is the depth of flow. For a certain representative grain diameter (that is, the grain size at which 35% of the mixture is finer) and channel slope, values of ψ can be computed for various assumed depths of flow. From the Einstein curve showing the relation between ϕ and ψ, the value of ϕ corresponding to the computed value of ψ is obtained and the value of q_b is then computed. The total load, G_b, is $q_b\, B$ in which B is the channel width. A curve showing the relationship between G_b and depth of flow (or Q) may then be constructed. In the event that the friction along the side-walls cannot be neglected, the hydraulic radius of the bed should be used in place of the depth in the equation for ψ.

Field measurements by Mr. Einstein in 1941–1942 (39)(40) made on the rates of bed-load transportation in Mountain Creek near Greenville, S. C., and West Goose Creek near Oxford, Miss., using a portable measuring apparatus, have verified the $(\phi - \psi)$-relation previously developed from flume studies and have extended the range of application to $\phi = 10$. This range of application includes small streams and canals with fine sediment or large streams with coarse sediment.

A determination of the average annual volume of bed load transported by a stream may be obtained by using the relationship between discharge and rate of transportation in conjunction with the flow-duration curve of the stream (41). A table showing the frequency of occurrence of various discharges is prepared from the duration curve—that is, the total range in discharge is divided into a convenient number of parts and the percentage of time that the mean discharge in each part prevails is determined and tabulated. The number of seconds that each discharge prevails per year is computed. The amount of bed load that is transported per second for each discharge may be determined from the previously established relationship between discharge and rate of transportation. The rate of transportation for each discharge, multiplied by the total number of seconds that the particular discharge prevails per year, gives the annual amount of bed load for that discharge. The sum of the amounts of bed load for the various discharges gives the average annual weight of sediment transported as bed load. This annual load on a weight basis can be converted to volume by merely multiplying by the weight per cubic foot of the material.

On streams where discharge records are not available for the construction of a flow-duration curve, the duration curve for a neighboring stream (corrected for size of drainage basin) may be used because investigations have shown that the flow-duration curves are similar for various streams in a locality where topography, climate, and vegetation are similar.

JOHN W. STANLEY,[5] ESQ.—Studying the subject of reservoir silting and compiling data required much labor and time. Mr. Witzig is to be commended for his work in attempting to develop a method for predicting the sedimentation rate. It seems quite certain that any energy, time, and money spent in further study of the problem would be many times repaid by more efficient operation and longer life of reservoirs.

---

[5] Asst. Engr., Bureau of Reclamation, Denver, Colo.

Engineers and geologists have given much thought to the relationships among the many factors affecting the transportation and deposition of sediment, with the result that considerable progress has been made. Some facts have been established fairly well, and some promising theories have been advanced. In the process, however, a few conclusions, ideas, and suggestions have been submitted which later were found to be undesirable. Any investigator, in attempting to cover the subject by a perusal of pertinent literature, indeed would accomplish the impossible if he were able to weed out all the unacceptable statements. It is with this thought in mind, and with no intention of detracting from the value of Mr. Witzig's paper, that the writer wishes to call attention to the second paragraph under "Origin and Nature of Silt: Definition of Silt."

The statement in this paragraph that "When the vertical components of turbulence are sufficiently strong, no settling takes place" is somewhat misleading. The author undoubtedly intended the statement to mean that no sediment accumulates on the bottom. The present general belief, as pointed out by William E. Dobbins, Jun. Am. Soc. C. E. (42), and emphasized by Thomas R. Camp, M. Am. Soc. C. E. (42*a*), is that, except for possibly the very finest particles, there is a constant interchange between particles in suspension and those on the stream bed and that this will be true as long as there is any turbulence. It does not seem proper, therefore, to assume that no actual settlement occurs even when the vertical components of turbulence are very strong, although it is true that there might be no accumulation.

In the next sentence of the same paragraph Mr. Witzig refers to Mr. Einstein and to Alvin G. Anderson and Joe W. Johnson, Assoc. Members, Am. Soc. C. E., for this definition of bed load (6*a*): "* * * that part of the total sediment-load composed of all particles greater than a limiting size * * * whether moving on the bed or in suspension, and includes all bed-material in movement." Later in his paper, however, the author refers to bed load as the material moved by rolling along the bed, as distinguished from the material moving in suspension. The meaning actually used by the author is preferable. Bed load has been defined by a special committee of the Society (43) as "The silt, sand, gravel, or other detritus rolled along the bed of a stream," and long usage rather has fixed this meaning for the term. Although it is true that only material coarser than the "dividing grain size" is moved at a rate that is a function of the discharge, slope, etc., it is not necessarily feasible to ignore the fact that part of this material actually moves as suspended load and part as bed load. As someone has suggested, the natural term to use for material coarser than the "dividing grain size" would be bed-material load. By adopting this term and retaining the term bed load with its old meaning, much confusion may be avoided.

In the last sentence of the same paragraph Mr. Witzig refers to Table 2 for "the rate of settlement (7) of a particle in still water at 50° F." The rates in

Table 2 are questionable because those given for all particles except the 1-mm and 0.20-mm sizes are greater than the rates generally found for spheres with a specific gravity of 2.65. The rates are essentially the same as those given in a more extensive list by the late Allen Hazen, M. Am. Soc. C. E. (44). Mr. Hazen gave a table of "hydraulic values" for irregular grains (specific gravity approximately 2.65) of which "the diameters are taken as the diameters of spheres of equal volume." He determined these rates by actual experiment for particles from 1.0-mm to 0.10-mm diameter, by the Wiley formula from 0.02 mm to 0.0001 mm, and by interpolation from a connecting curve for particles between these two grades. His rates computed from the Wiley formula are approximately 200% of the rates given by the Stokes formula for spheres; those for particles 0.02 mm to 0.25 mm are greater, by varying amounts, than the rates for spheres (45); and those for 0.30 mm to 1.0 mm are lower than for spheres. Actually, any irregular particle should settle more slowly than a sphere of equal volume and specific gravity, because the irregular particle presents a greater surface area and hence has a greater resistance to motion than the sphere. In fact, if settling rates are to be used for denoting any apparent dimensions of irregular particles, extreme care must be exercised in selecting the rates for a particular type of material. A few workers besides Mr. Hazen have attempted to determine settling rates for irregular particles, and the results have varied widely.

Arthur Holmes (46) performed some experiments, in which he separated quartz grains by means of rising currents of water. The rates he determined are compared with the corresponding rates for quartz spheres in Table 6. A. Atterberg (47) determined rates for "fine sands" (Si O_2). He gave rates as the time required for particles to settle 10 cm in water and indicated that decantation or another elutriation process was used in making the determinations. He did not state the water temperature, but experiments on capillarity described in the same paper were performed at an approximate temperature of 17° C. The rates as indicated by his data, with corresponding rates for spheres, are shown in Table 6.

TABLE 6.—COMPARISON OF COMPUTED AND OBSERVED SETTLING RATES

(Units Are Millimeters per Second)

Diameter (mm)	Holmes[a]	Atterberg[a]	Quartz spheres
0.40	47	....	55[b]
0.30	32	....	38[b]
0.25	25	....	30[b]
0.20	20	....	22[b]
0.10	6.7	....	7.1[b]
0.06	....	1.8	2.8[b]
0.05	1.8	....	2.0[b]
0.02	....	0.22	0.32[b]
0.01	0.12	....	0.078[c]
0.006	....	0.028	0.03[c]
0.002	....	0.0035	0.0033[c]

[a] Holmes rates determined at 15° C and Atterberg rates at approximately 17° C. [b] See reference (45) in Bibliography. [c] Stokes' formula.

E. C. J. Mohr (48) gives results of his own determinations and those of four other investigators, including Mr. Atterberg, for particles 0.06-mm diameter and smaller. All values are widely different except those of Messrs. Mohr and Atterberg, whose rates for particles 0.02-mm diameter and smaller are very close. No temperatures are given, however, and the apparent agreement therefore might be better or worse if temperatures were known.

C. K. Wentworth (49) desired to extend his sieve analysis curves into the sedimentation range and for this purpose chose certain "hydraulic values." He commented that his scale "has no precise theoretical justification," but that it approximates "the mean of several published tables by hydraulic values." Since he did not indicate at what temperature the values were to apply, it is impossible to compare them with other data. W. C. Krumbein (50) gives a brief résumé of the work of H. Wadell on settling velocities of nonspherical particles.

The writer has done some investigating on the subject and has found values for particles from about 0.03-mm diameter to 0.12-mm diameter. His values are somewhat lower than those of Messrs. Holmes and Atterberg. In the process of investigating the subject, a method was found which apparently permits reliable determination of values for any given material.

The chart, equation, and table developed by Mr. Witzig for use in estimating the rate of reservoir sedimentation are interesting and should be useful, especially if further refinements can be made as more data become available. Perhaps it would be desirable to change the method of designating the "envelopes." For example, it seems probable that, by classifying drainage areas according to average annual rainfall and average susceptibility to erosion, the envelopes might be narrowed and the "regional indexes" possibly changed to indexes according to rainfall and erosion characteristics.

That phase of engineering termed "upstream engineering" by Mr. Witzig is one that has been neglected much too often in the past. The engineer should not be relieved of the guilt of failing to take steps, where possible, to prolong the life of a reservoir by means of erosion control, even though he may be able to prove that the cost of the reservoir will have been repaid by the time its useful life is ended by sediment accumulation. It is to be hoped that in the future the problem of erosion control, as related to reservoir life, will be given more nearly the amount of attention merited by its importance.

Stafford C. Happ,[6] Esq.—Sedimentation in reservoirs is a complex problem, and, in his attempt to cover the subject in a single brief paper, Mr. Witzig appears to have given some citations which require further clarification. One such reference concerns the findings of the Soil Conservation Service (SCS) reported (3) in May, 1940.

Mr. Witzig (see heading, "Origin and Nature of Silt: Origin of Sediment") appears to contrast the estimate by Mr. Horton that "of the total erosion in the evolution of a humid drainage basin, probably 1% was initially bank erosion, and 99%, sheet erosion (2)" with the SCS studies which Mr. Witzig states "suggest that existing deposits within the flood plain of a stream are now often more important as immediate sources of sediment than concurrent sheet erosion, * * *." These two estimates apply to different conditions, but the second citation appears to be based on a misunderstanding of the actual findings.

---

[6] U. S. Engr. Office, Ocala, Fla. (formerly Geologist, SCS, Washington, D. C.).

The findings of the SCS originally were based on studies in northern Mississippi, but they have since been tested and confirmed in many places from the southeastern Piedmont to the Great Plains, and in the Ohio and upper Mississippi basins. In all these areas, as a general rule, concurrent sheet erosion is the principal source of reservoir sediment, and stream-bank erosion is of minor importance. As an average for the Central States and Southern States, where present information is most complete, the studies of the SCS would not support an estimate of more than 10% for the proportion of reservoir sediment derived from stream-bank erosion.

The relative importance of bank erosion may be greater in forested or mountainous areas, which have not been studied extensively by the SCS, but only in the Southwestern States (where enlargement of arroyos or ephemeral channels is very active) has bank erosion been found to be a principal source of reservoir sedimentation. This latter region is in arid or semi-arid country and on that account cannot be compared directly with Mr. Horton's estimates.

The SCS research did indicate that stream-bank erosion and gullying were major sources of the relatively coarse alluvial deposits which often cause damage to farm lands on the flood plain, but these sands and gravels seldom form any large part of reservoir deposits in the humid region because there are much greater quantities of silt and clay contributed to the reservoirs from sheet erosion on agricultural lands.

THOMAS H. MEANS,[7] M. AM. SOC. C. E.—The results of much recent research on sedimentation are brought together in concise form by the author. All the references in Appendix II, except (7) (9) (17) and (19), were published in 1933 or thereafter—the period of greatest reservoir building in the United States. It is no wonder the two activities have coincided because many problems of sedimentation are created by reservoirs, and it will be many years before all of them are solved.

One lack in recent studies of sedimentation rates, as illustrated in Fig. 3 and Table 5, is the scarcity of information as to the quantity of sediment deposited above the reservoir level. Deltas of noticeable size have been built upstream from some reservoirs, and certainly the change in stream grade which is created by the reservoir will affect other streams. Bridges and towns and occupied valleys along channels upstream from reservoirs will be affected, and it is to be hoped records will be kept so that the extent of changes can be determined. The writer has examined a number of reservoirs constructed recently where bridges will be menaced in a few years if the to-be-expected sequence of events follows. Owners of these bridges may be able to establish the cause of the increased menace if records are collected.

There seems to be a growing tendency to summarize all investigations in one formula. This may be helpful in understanding general relations but too often, when such a formula is used, the underlying data are not considered.

---

[7] Cons. Engr., San Francisco, Calif.

In the present state of knowledge several of the formulas presented by Mr. Witzig should be used with extreme caution or not at all. For example, Eqs. 2 and 3 are suggested as a short cut to sampling to determine the volume of sediment carried by a stream. A few samples run through this mathematical "mill" might produce results but not such as to inspire much confidence in an engineer who has observed silt-bearing streams in action.

Another example is the "silt-rating curve," also designed as a short cut to long sampling periods. The original paper in which this method was described (17) gives the following values of G_s for two periods on the Red River near Gainesville and the Washita River near Durwood:

Year	Red River near Gainesville	Washita River near Durwood
1936–1937	$1.04 \times 10^{-8} Q^2$	$9.2 \times 10^{-8} Q^2$
1938–1939	$3.85 \times 10^{-8} Q^2$	$2.7 \times 10^{-7} Q^2$

The differences between these two years (about 1 to 3) are too large to give one much confidence in this method of determining the flow of silt. One of the first things learned by an engineer, studying silt, is that the silt percentage does not always change as the flow changes.

The distinction between suspended silt and bed load still seems to be a matter of controversy. No phase of sedimentation has received more study, and today no phase seems to be in a greater state of uncertainty. There are many bed-load formulas but none seems to be valuable for all conditions. The writer was present many times when G. K. Gilbert was conducting his classic experiments at the University of California at Berkeley, and ever since has lived in hopes that someone would produce a usable method of determining the movement of sediment along the stream or canal bed. So far no such method has come to his attention. Bed load becomes very important in any study of sedimentation in reservoirs. For example, the Arkansas River is a great carrier of sand, as the manager of any canal diverting from it knows. What will be the result of Caddoa Reservoir on the movement of this sand and how will the bridges across the stream and the town of Lamar, Colo., be affected? There is an opportunity to collect much usable data from these new reservoirs, and it is to be hoped the Army Engineers will collect information systematically on changes in the bed upstream from reservoirs and of the delta formations that will build up at the point where the flow is checked.

The stream downstream from reservoirs, cleared of sediment, also will create new conditions. The Bureau of Reclamation is making very careful surveys of the Colorado River below Boulder Dam, as revealed by its Eighth Annual Report.

Carl B. Brown,[8] Esq.—The control of reservoir silting will constitute one of our most important problems of conservation in the postwar period. Construction of impounding reservoirs during the decade 1933–1943 has been a major item of public works. Reservoirs under construction in the United

---

[8] Head of Sedimentation Section, SCS, U.S.D.A., Washington, D. C.

States in 1941 will have cost, when completed, more than $1,000,000,000, compared to a total investment in reservoir storage during the past hundred years of about $5,000,000,000, not including the dependent developments in filtration plants, power transmission lines, irrigation canals, etc. There is every reason to believe that reservoir construction will be accelerated beyond any previous levels as soon as the Nation can return to peaceful pursuits. For these reasons Mr. Witzig's paper is of timely interest.

The direct annual loss in investment from reservoir silting is not less than $10,000,000 and is more likely several times that amount. Therefore, a more widespread understanding of the silting problem is obviously needed. Control of silting begins when the dam design takes shape on the drawing board. Many reservoirs built in recent years have suffered rapid loss because their storage capacity, although adequate to meet water needs, was much too small to accommodate the sediment load of the inflowing stream—even until the project was amortized. Furthermore, the increasing knowledge of sediment-laden density currents and of other silting phenomena indicates that properly designed and operated outlets in many dams would reduce the rate of silting substantially. The design of future dams and reservoir projects should include a thorough study, so often lacking in the past, of the effects of silting. No less attention should be given to methods of conserving the reservoir storage already developed.

Of the topics which Mr. Witzig discusses, "Estimating the Rate of Reservoir Sedimentation" is, perhaps, of the most practical importance to the designing engineer. Unfortunately, Eq. 6 and the envelope curves shown in Fig. 3 are misleading. They imply that the annual rate of sedimentation, in acre-feet per 100 sq miles of drainage area, ΔS_R, is a function of the original storage capacity, in acre-feet per square mile, S_R, and increases with it. If this were so, the rate of sedimentation at a given site would increase indefinitely with the size of the reservoir. Obviously this is not possible since the upper limit of sedimentation depends on the amount of sediment brought in by the streams, which is independent of the size of the reservoir.

If a reservoir with $S_R = 100$ traps 90% of the incoming load and $\Delta S_R = 47$, as shown by median curve IV in Fig. 3, a reservoir at the same location with $S_R = 500$ could have a value of ΔS_R not greater than 52.2, which would represent 100% deposition of all the stream load. Curve IV would appear to indicate, however, that, where $S_R = 500$, $\Delta S_R = 175$. Actually, ΔS_R is a function of several independent or partly interdependent variables; for example:

$$\Delta S_R = f(E_n, S_R, S_h, S_c, Q_c, O_R, \cdots \text{etc.}) \qquad (11)$$

in which: ΔS_R = annual rate of sedimentation; E_n = annual net watershed erosion, or sediment inflow to reservoir; S_R = reservoir storage; S_h = shape of reservoir; S_c = sediment characteristics, including textural composition and behavior under influence of dissolved load, temperature, etc., in forming density currents; Q_c = discharge characteristics of inflowing stream; and O_R = manner of operation of the reservoir. The symbols ΔS_R, E_n, and S_R are expressed in the same volume units per unit of drainage area (such as acre-feet per square mile).

The trap efficiency coefficient of a reservoir is best defined simply as the percentage of incoming sediment trapped:

$$C_T = 100\left(\frac{\Delta S_R}{E_n}\right) \qquad (12)$$

in which case C_T cannot exceed 100. From this relationship it is apparent that:

$$C_T = f\left(\frac{S_R, S_h, S_c, Q_c, O_R, \cdots \text{etc.}}{E_n}\right) \qquad (13)$$

Of the factors in the numerator only S_R can be assigned a definite value from data generally available. Furthermore, few and none too reliable data have been collected on actual values of C_T as defined in Eq. 12. The data that are available to the writer have been plotted in Fig. 5.

It is apparent from general considerations that any curves defining trap efficiency must pass through the origin and approach $C_T = 100$ asymptotically. Envelope curves enclosing the data in Fig. 5 may be defined by an equation of the form:

$$C_T = 100\left(1 - \frac{1}{1 + dS_R}\right) \qquad (14)$$

The spread of the curves is due to the effect of the variables, other than S_R, on which C_T depends. Therefore, the sum of effects of these variables is expressed in the range in value of the coefficient d, from 0.046 for the lower curve to 1.00 for the upper curve. Existing information indicates that the middle curve should represent a satisfactory approximation of the average relationship of C_T and R_S for reservoirs other than those used solely for flood detention. This average, or design, curve has the equation:

$$C_T = 100\left(1 - \frac{1}{1 + 0.1\,S_R}\right) \qquad (15)$$

Values of d tend to lie near the upper curve for: (*a*) Reservoirs in regions of smaller and more variable runoff; (*b*) reservoirs whose length and shape tend to reduce the time of flow through them; (*c*) reservoirs on watersheds which produce mainly coarse or highly coagulated sediments; and (*d*) reservoirs which are so operated as to release little water from the bottom of the dam or to hold back and store flood flows. Where $S_R > 100$, such a large proportion of the sediment is trapped that this factor generally may be ignored.

Substituting the value of C_T from Eq. 12 in Eq. 15, and transposing, it is found that, for an average reservoir:

$$\Delta S_R = \left(1 - \frac{1}{1 + 0.1\,S_R}\right) E_n \qquad (16)$$

The difference between Eq. 16 and Eq. 7 is that the latter is based on data from a paper by Messrs. Brune and Allen (4). Their letter symbol E does not denote the sediment load entering the reservoir, but is an estimate of gross erosion based on conservation surveys made by the Soil Conservation Service (SCS). These surveys are not intended to measure net watershed erosion volumetrically, but rather to show, among other things, the degree of soil loss from the land surface as this affects land use for agriculture and other purposes. These surveys do not ordinarily show stream-bank erosion, road erosion, or fan, colluvial, and flood-plain deposition in a form by which the quantity so eroded or deposited can be calculated. However, a rough volumetric approximation of soil loss from the land surface can be made from these surveys, and this is a fair measure of gross erosion, E_g, in most areas. The relation of E_n to E_g, as estimated from conservation surveys, varies within wide limits. Strictly, E_g should be a measure of the volume of all soil moved from place within the watershed per unit area, per unit time, regardless of where the resulting sediment is deposited. In drainage areas where sheet and gully erosion are slight and most of the sediment comes from stream-channel erosion, E_n may equal or exceed $2\,E_g$. In most watersheds, however, E_n is much less than E_g. In some of the watersheds used by Messrs. Brune and Allen, $E_n < 0.2\,E_g$. In general terms:

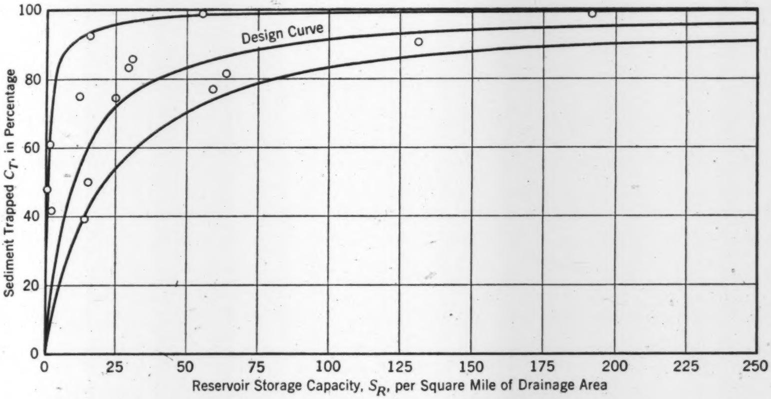


FIG. 5.—RELATION OF RESERVOIR TRAP EFFICIENCY, C_T, TO RESERVOIR STORAGE CAPACITY, S_R

$$E_n = C\,E_g \qquad (17)$$

In Eq. 7, therefore, C_T is not a true "trap efficiency" coefficient for the reservoir; it is a coefficient that depends on two independent variables—the true trap efficiency of the reservoir, defined by Eq. 14, and the coefficient C in Eq. 17. The relation between gross erosion on a watershed and rate of reservoir sedimentation may be defined by

$$\Delta S_R = \left(1 - \frac{1}{1 + dS_R}\right) C\,E_g \qquad (18)$$

but careful watershed surveys will be required to determine the value of the coefficient *C*.

It will be apparent from this discussion that there is no simple exponential relationship, as in Eq. 7, between ΔS_R and S_R. Regional envelope curves plotted on logarithmic paper would not be straight lines, as in Fig. 3, but curved lines approaching asymptotically some limiting value of ΔS_R with increase in S_R.

Someday, perhaps, data will be sufficient to develop an equation relating ΔS_R to all of the variables on which it depends. For the present, however, it seems desirable to plot ΔS_R against some one easily determinable variable, such as size of drainage area, A_D, for each geographic region of essentially similar soils, slopes, climate, and land use. The resulting spread of data represents the influence of other variables, but the spread is not too wide to preclude use for practical design purposes. In other words, there is a limit to the range of debris production per unit of drainage area.

In Fig. 6 is shown a logarithmic plotting of ΔS_R, in acre-feet per square mile per year, against A_D in square miles. Values for all the reliable data on reservoir sedimentation and suspended loads from the Southern Piedmont and immediately adjacent parts of bordering physiographic provinces have been plotted, except that reservoirs where $S_R < 15$ have been omitted to minimize the effect of C_T, for which no corrections have been made. Suspended-load data were converted to acre-feet by assuming a specific weight of 50 lb per cu ft. Envelope curves have been drawn to bound the data, and a median curve has been drawn "by eye." The single point outside the upper limiting curve represents Greenbelt Lake, Greenbelt, Md., where abnormal sediment production from an 0.83-sq-mile drainage area resulted from construction of a housing project (51). The envelope curves may be spread by future data, of course, but probably not much. The mean curve probably will not be changed appreciably by more

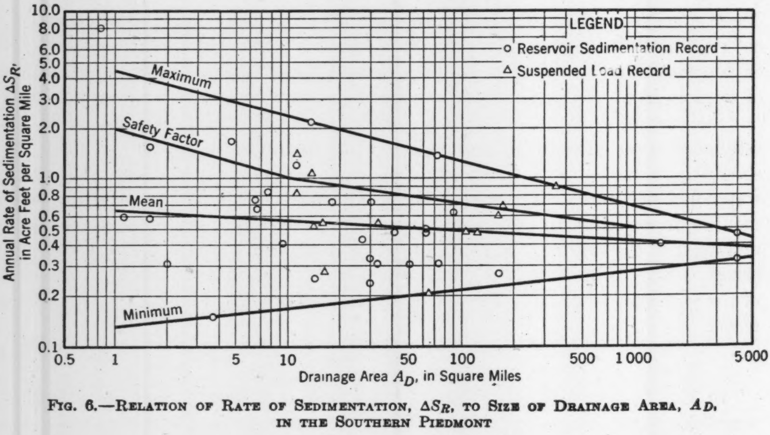


FIG. 6.—RELATION OF RATE OF SEDIMENTATION, ΔS_R, TO SIZE OF DRAINAGE AREA, A_D, IN THE SOUTHERN PIEDMONT

data. These data appear to show that, in the Piedmont, the larger the watershed, the more limited is the range of ΔS_R, and that the mean rate decreases somewhat with size of watershed.

It should be recognized that use of the mean curve in a design problem allows no "safety factor." An erosion and sedimentation reconnaissance of the particular watershed, comparison of observed watershed characteristics with those of watersheds from which sediment data are available, careful study of the methods of water release and waste, and other factors influencing trap efficiency of the proposed reservoir should indicate whether a value above or below the mean should be chosen. Lacking such a study, the writer would recommend use of the reasonable "safety factor" curve.

Once regional charts, such as Fig. 6, are compiled, the annual rate of storage loss can be estimated by substituting values for ΔS_R in the equation:

$$\Delta S_L = 100 \left[\frac{\Delta S_R \, (A_D)}{S} \right] \qquad (19)$$

in which: ΔS_L = annual percentage rate of storage loss; A_D = drainage area, in square miles; ΔS_R = annual rate of sedimentation, in acre-feet per square mile of drainage area; and S = storage capacity of reservoir, in acre-feet.

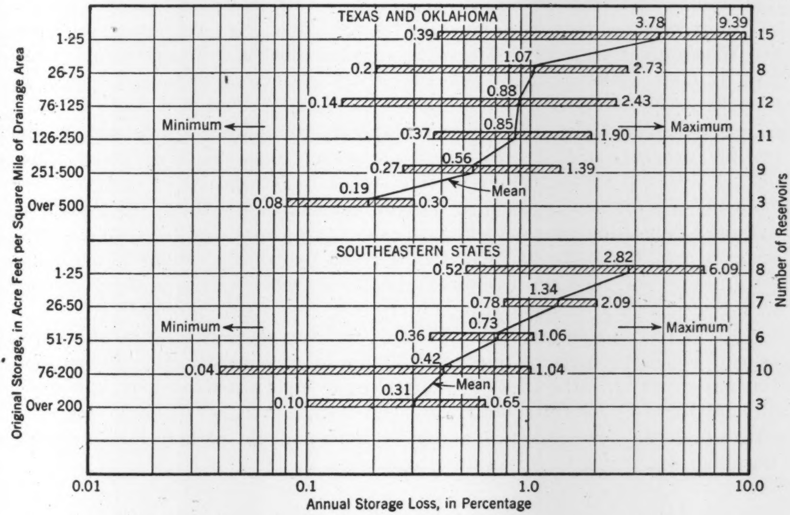


FIG. 7.—RELATION OF RESERVOIR STORAGE CAPACITY, S_R, TO RATE OF STORAGE LOSS, S_L

Except for flood detention basins and reservoirs where $S_R < 15$, there is generally no justification, in the light of present data, to consider trap efficiency separately. Its effect is well within the spread of the envelope curves, and it is taken into account in the "safety factor" curve.

In Fig. 7 is shown the relationship of the storage-watershed ratio, S_R, to the annual silting rate ΔS_L for two regions where most data are now available. This illustration reveals the range and mean values of annual storage depletion of representative reservoirs grouped according to values of S_R.

Mr. Witzig makes the statement (see heading, "Estimating the Rate of Reservoir Sedimentation: 'Regional Indexes'") "that reservoirs surveyed thus far are more typical of severe rather than of average conditions." Actually, the reservoirs that furnished the data used in Fig. 3 are a representative cross section of important reservoirs in the regions they represent. More than 150 detailed and 250 reconnaissance reservoir surveys have been made by the SCS since 1934 for the purpose of obtaining a representative sampling of silting conditions and developing the factual basis necessary for planning silting-control measures. The average regional conditions shown are conservative because the sampling did not give sufficient weight to the hundreds of smaller reservoirs with low values of S_R that are now more than 50% filled with sediment, where it is too late or uneconomic to undertake silting-control measures. However, only a few surveys, or none, have been made in New England, the Great Lakes region, the Northern Great Plains, and certain parts of the Rocky Mountains and Great Basin. With the exception of these areas, the existing reservoir data fairly represent silting conditions in the economically important impounding reservoirs of the United States.

Many methods have been tried for the control of reservoir silting, of which Mr. Witzig describes a few. The writer (52) has classified all methods of control into six major categories: (1) Selection of the reservoir site to avoid areas of excessive sediment production wherever possible; (2) design of the reservoir, which includes proper adjustment of capacity to watershed area and, where feasible, provision of outlets for sediment release; (3) control of sediment inflow by (*a*) settling basins, (*b*) vegetative screens, (*c*) location of reservoir off-channel, and (*d*) construction of by-pass canals and conduits; (4) control of sediment deposition by (*a*) venting density currents, and (*b*) control of waste water release; (5) removal of sediment deposits by (*a*) excavation, (*b*) dredging, (*c*) draining and flushing, (*d*) sluicing with controlled water, and (*e*) sluicing with hydraulic and mechanical agitation; and (6) watershed erosion control.

Mr. Witzig's paper is a valuable addition to the growing literature on various aspects of the problem of reservoir silting and its control. It is hoped that this paper will have the effect of stimulating interest among owners, operators, and designers of reservoirs, both in ascertaining the extent of silting in reservoirs with which they are concerned, and in studying possible methods for controlling the ill effects of this too often unrecognized menace.

C. S. JARVIS,[9] M. AM. SOC. C. E.—During more than two decades following the publication of the paper and discussions dealing with an irrigation reservoir that had lost 60.7% of its capacity in 12½ years (53), much progress has been made in the study of sedimentation phenomena and of the most promising means for their control or prevention in some degree. In spite of the notable progress accomplished by both federal and local agencies in the fields of soil conservation, channel stabilization, runoff regulation, and other activities designed to insure the utmost beneficial use of water resources along with adequate protection from the increasing flood menace, most of the required work yet lies ahead.

---

[9] Cons. Engr., Washington, D. C.

The paper under discussion affords a brief review of the situation and reflects diligent research and mature mental processes, along with an earnest effort to improve the outlook for the maintenance of reservoir capacities. Both Fig. 3 and Table 5 are forward steps in summarizing the findings of the Soil Conservation Service for various regions of the United States, indicating that the regional indexes are not so diverse as might have been expected in view of the wide range in climatic, rainfall, and runoff factors.

The alert irrigation farmer applying from 4 in. to 6 in. of turbid flood water on his field realizes that much fertility may be added thereby. If the thickness of the new sedimentary deposit happens to be ¼ in., he knows that from 4% to 6% by volume was carried by the irrigation lateral, in spite of all the deposits in the main canal system. Each farm is potentially a kind of hydraulic laboratory, if accurate and adequate records are kept.

Likewise with the stockwatering pond, especially if it is founded on gypsum-bearing or other highly permeable formation, the initial fillings may rapidly disappear through infiltration. The obvious corrective measure is to provide some kind of waterproof lining, such as clay or sand-clay mixture, preferably water-borne and deposited. Small reservoirs adequately lined in this manner have proved to be as retentive as if they had been waterproofed by much more expensive methods. Furthermore, the losses in the distribution canal system may be greatly reduced by the sedimentation processes. Where the need is great enough, it may be advisable to dump clay into a canal section for conveyance and puddling along the permeable reaches.

Wherever an irrigation supply has been deprived of its normal silt and sediment burden, either through intervening storage reservoirs or through the attainment of the ideal of effective erosion control or virtual prevention, the problem of restoring some of those elements for the aforementioned benefits is worthy of consideration. Some means of agitating the finest of the sediments within a storage reservoir, and delivering them to the irrigation canals serving lands that would be benefited by the addition of such sediment, might be devised with due regard to such regulations as the Biological Survey would necessarily impose.

In some locations, the gravel and sand deposits at the head of a reservoir might be used as a source of construction material; and the excavation resulting from such activity would serve as a trap for the next season's bed load, in the same manner as is intended for debris basins.

Now that fertile topsoil is becoming appreciated at somewhere near its true value, justifying its removal and storage in huge mounds, awaiting replacement as the final operation of some grading projects, it would appear that the finest sedimentary deposits in large storage reservoirs may have some commercial value within the range of cheap transportation, especially if waterways are available.

Hugh Stevens Bell,[10] Esq.—Under the heading, "The Action of Sediment in Reservoirs: Density Flows in Lake Mead," Mr. Witzig attributes to the writer (25) an estimate that "silt-laden density flows in Lake Mead carry about 24% of all sediment deposited in the lake." For the sake of accuracy it seems desirable to quote directly from the passage cited by Mr. Witzig (25), omitting any parts not pertinent to the present discussion:

> "Density currents * * * were transporting fine sediments into lower Lake Mead at the rate of 875,000 tons per week during the 61 months immediately following the closing of the (diversion) tunnel on May 1, 1936. * * * If it is assumed that these sediments will eventually compact to 60 pounds per cubic foot, the density currents are transporting material sufficient to occupy 1 percent of the original spillway-crest capacity each 8.2 years.* * *
>
> "To see what this might mean in terms of the total life of Lake Mead, assume that, under present conditions, the reservoir will become completely filled with sediments in 200 years. According to the estimates above, material transported by density currents will occupy approximately 24 percent of the original storage space at that time, provided that none is discharged, and the average concentration has become 60 pounds per cubic foot."

The 24% figure, then, is based upon two assumptions—(1) that density current deposits will contain 60 lb per cu ft of sediment and (2) that Lake Mead will be completely silted up in 200 years. These assumptions were not estimates based upon ample data. They were values chosen because they were convenient and conservative. Therefore, the 24% should not be used, as Mr. Witzig has used it, as a measure of the sediment carried by density currents in terms of the total sediment load brought to Lake Mead. The pertinent value is 875,000 tons per week, or, roughly, 45,000,000 tons per year. In 1928 Messrs. Fortier and Blaney estimated that a reservoir at Boulder Canyon would receive 253,628,000 tons of sediment per year (9). On this widely accepted estimate, the 45,000,000 tons transported annually by density currents would represent, roughly, 18% of the total.

Actually, there is no reason to expect that the fine sediment in lower Lake Mead will attain a mean specific weight that even approaches 60 lb per cu ft during the lifetime of the Boulder Dam project. The several hundred samples taken by the Bureau of Reclamation indicate that a value of 30 to 35 lb is a much more probable one (54).

Before the importance of density currents in the sedimentation of Lake Mead can be evaluated, a reasonable estimate must be had for the mean specific weight of all sediment deposited there. In 1930 Mr. Blaney stated that differences in opinion on this point had resulted in estimates ranging from 80,000 acre-ft to more than 250,000 acre-ft for the average annual volume of sediment that would be deposited. Although a value of 85 lb per cu ft had gained wide acceptance in engineering circles, Mr. Blaney expressed the opinion that 62.5 lb would prove to be approximately correct (55).

At 85 lb per cu ft the estimated annual sediment load of 253,628,000 tons would occupy 137,000 acre-ft. At 30 lb per cu ft, the 45,000,000 tons of this material that is carried into the lower part of the reservoir by density currents will occupy about 70,000 acre-ft, or more than half of the total. Therefore, the sediments not carried by density currents would occupy only 67,000 acre-ft, and would have a specific weight of 143 lb per cu ft. This is an absurd value.

---

[10] Associate Soil Conservationist, U. S. Dept. of Agriculture, SCS, California Inst. of Technology, Pasadena, Calif.

At 62.5 lb per cu ft the total sediment load would occupy about 186,000 acre-ft, of which the 70,000 acre-ft of density current deposits would constitute 37.6%. Sediments not carried by density currents would occupy the remaining 116,000 acre-ft, and would have a mean specific weight of slightly less than 83 lb per cu ft. Although the future may show that even this value is somewhat high, it seems to be a reasonable one.

It may be estimated with some confidence, then, that density currents are transporting not 24% of all sediment at Lake Mead, but from 35% to 40% of it in terms of the volume after deposition.

Not only do density currents transport those sediments that form deposits with specific weights that are very low, but also, by sorting and carrying away the fine particles that would fill a part of the interstices in the sediments left behind, they decrease the specific weight of even the coarser deposits. The economic importance of density currents, therefore, is tremendous. If it were possible to operate the reservoir so that 50% of the sediment carried by density currents could be vented, rather than stored, the life of the project would be increased by approximately 25%. The seemingly insurmountable obstacles that prevent this from being accomplished are human, not physical ones.

Mr. Witzig states (heading, "The Action of Sediment in Reservoirs: Formation and Dispersion"), "The loss of energy as the stream enters the reservoir is not generally sufficient to prevent the formation of a density current." This statement is correct, but may give a false idea as to the type of energy that produces density currents. It is kinetic energy that a river loses or releases upon entering a reservoir. Since this type of energy causes mixing between the river water and the reservoir water, the more of it there is available the less chance there is for a river to become a density current. If there is enough kinetic energy to make a homogeneous mixture of the river water and all the water stored in the reservoir, there can be no density current because there will not be two adjacent fluid masses of approximately equal density. On the other hand, if the kinetic energy is not sufficient to produce such a mixture, the river water may still possess great potential energy because, although it has been diluted by entrance mixing, it has a density that differs from that of the unmixed reservoir water. It is this potential energy, rather than any residual kinetic energy, that makes density currents possible.

HARRY F. BLANEY,[11] M. AM. SOC. C. E.—For many years engineers have recognized the need for more definite information concerning the silting of reservoirs under varying conditions. The author has raised this question again and his paper indicates the need for further research in the prevention of sedimentation in reservoirs. The writer believes, however, since determinations of silt content of river water usually are made on a weight basis, that the presentation of more data on the relation of volume and weight of silt would add to the value of the paper. Such information is needed to convert the results of silt samplings on a weight basis to volume, as the engineer is more interested in the volume of sediment than its weight.

---

[11] Senior Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.

Under "Origin and Nature of Silt: Definition of Silt," the author states that Mr. Fortier and the writer "recommend for the Lower Colorado a specific weight of 62.5 lb per cu ft for suspended silt, and 85 lb per cu ft for deposits in reservoirs." These estimates were made in 1926. Since then additional information has been collected by various agencies. In 1929 the writer called attention to the fact that differences in opinion as to the weight of 1 cu ft of silt have resulted in estimates of the average volume of suspended silt that would be deposited annually in reservoirs on the Colorado River ranging from 80,000 to more than 250,000 acre-ft per yr (55). At that time the writer was of the opinion that the specific weight of sediment deposited in the bottom of a reservoir created by a high dam at Boulder Canyon would be approximately 62.5 lb per cu ft, provided the deposits were not given an opportunity to dry out and thus destroy their colloidal condition.

The quantity of material deposited in a reservoir that will be affected by alternate drying and wetting is difficult to estimate. When the water level of a reservoir is lowered, material deposited in the upper part of the reservoir will be uncovered, and, upon drying, undoubtedly will shrink in volume. Mr. Faris reported (22) that, in Texas, material deposited under reservoir conditions, and remaining submerged, contained 30 lb or less of dry materials per cubic foot of deposit, whereas material similarly deposited and then subjected to shrinkage through exposure contained in excess of 80 lb of dry material per cubic foot of deposit. These silt load studies have been continued by Dean W. Bloodgood of the Division of Irrigation, SCS, in cooperation with the Texas Board of Water Engineers. In a progress report (56) Mr. Bloodgood states that

> "* * * the average weight of the dry material in silt deposits which are continuously submerged approaches 30 pounds per cubic foot. In those deposits which are occasionally exposed, the average dry weight approaches 70 pounds per cubic foot. In deposits where reservoirs are used exclusively for flood control, the average weight ultimately approaches 90 pounds per cubic foot. After a careful consideration of the volume-weight ratios of silt samples in different degrees of consolidation together with the fact that an indeterminable volume of vegetable matter in the form of logs and brush deposited in reservoirs becomes water-logged and lasts indefinitely, seventy (70) pounds was selected as the average ultimate weight of the dry material per cubic foot of deposit in reservoirs where the deposits are subjected to alternate wetting and drying."

The studies of stratified flow in reservoirs by Mr. Bell, mentioned by the author (25), illustrate the need for further research on the sedimentation in reservoirs. Investigations of density flows may provide the means of prolonging the life of large reservoirs.

Since the author's paper was published, the SCS has issued a new publication (52), in which all the known methods of silting control, as described in a rather voluminous but widely scattered literature, and as recorded in the sedimentation studies of the SCS, have been analyzed and classified. This bulletin includes references to some 138 reports. A review of this publication is recommended to those interested in information on methods of controlling or reducing excessive silting in impounding reservoirs.

A. L. Sonderegger,[12] M. Am. Soc. C. E.—The accelerated rate of reservoir construction in the United States since about 1910, together with the marked loss of storage space within a comparatively short period in many of these structures, has focused the attention of engineers on the problem of reservoir sedimentation. Two thirds of the thirty-two papers in the Bibliography of this paper were published since 1938. Kindred subjects were discussed at the July, 1943, meeting of the Society in Los Angeles, Calif., by the U. S. Geological Survey (57),[12b] and in recent special reports issued by the Soil Conservation Service.

The origin and nature of sediments have been described in this paper and in other papers, and records of the rate of debris deposition are available for a large number of representative watersheds. Hence, it is possible to forecast with some degree of probability the rate of sedimentation which may take place in structures, directly intercepting flood waters. However, a reconnaissance of physiographical and geological conditions and of the effect of the invasion of the watershed will be required.

Under the heading, "Remedies for Reservoir Silting," the author correctly states that authorities generally agree that redemption of lost reservoir capacity is practically impossible. In this connection, he mentions the removal of debris in Southern California by mechanical excavation, a process feasible with relatively shallow reservoirs in open terrain; the removal of silt deposits by sluicing which has been practiced with partial success; and last, but not least, the utilization of density flows. Since full redemption of lost capacity is not feasible, the question then arises as to what extent sedimentation may be prevented or controlled.

The basic causes of debris production are the disintegration and decomposition of rocks. These processes are as old as the hills, and nothing can prevent them; nor can the removal of debris by periodically occurring storms and concomitant runoff be stopped. Production of debris and erosion are at a minimum when a watershed is protected by its natural vegetal cover. Under such conditions a natural physiographic balance is maintained, and probably attempts by man to improve on these conditions would not be successful. Nature apparently is operating under the principle of self-conservation which automatically adjusts itself to varying conditions, climatic and physiographic (58).

Invasion of the watershed; effects of deforestation, overgrazing, cultivation, unstable slopes in road cuts; and other methods have generally tended to upset nature's balance and to increase sediment production and removal. The writer is impressed with the benefits that ultimately will result from the recent practice of erosion control initiated by the Soil Conservation Service to correct the destructive effects of land abuse. Many of the methods designed to restore the vegetal cover will become fully effective only after persistent efforts over periods of years. However, the long-range results should fully justify the expenditure.

---

[12] Cons. Engr., Los Angeles, Calif.

The author concludes (see "Remedies for Reservoir Silting") that "the most certain means of maintaining reservoir capacity is to prevent sediment from forming, or, after formation, to prevent its deposition in the lake." Although this theory is unquestionably correct, generally, it is not practicable. In this connection, the author has quoted Messrs. Eakin and Brown (1) as follows:

> "Main reliance for material and permanent conservation of reservoir resources must rest upon control of sediment production at primary sources through more widespread and effective application of established methods of erosion control."

This statement may apply to the disturbed watershed, but hardly to natural conditions. Since debris production cannot be reduced below its natural minimum, the preservation of water storage must rely mainly on the creation of separate space for sediment storage.

In many storage reservoirs, debris space has been created by raising the height of dams. On the other hand, storage and detention projects in large numbers have been constructed on silt-carrying streams without any provision whatever for preserving the integrity of the hydraulic function. The results have been little short of disastrous in many cases, not only because of the loss of the investment, but also because of the scarcity of reservoir sites.

In any storage project where sedimentation is a factor, provision for the long-time disposal of debris should be made part of the original project and should be included in estimates of costs and in budgets. The plans may contemplate additional height of dam, "upstream engineering" by the construction of a series of permanent debris barriers, provision for deposit grounds for sediments eventually to be removed, or other methods.

The author states (see heading, "Introduction") that "Studies by the Soil Conservation Service (SCS) of the U. S. Department of Agriculture (1) show that, in the silt-carrying streams of the United States, the probable life expectancy of existing reservoirs is dangerously short" and "* * * that about 64% of all reservoirs have useful lives of less than 100 years, * * *." Thus, a reduction of 80% in the capacity of a reservoir is assumed to terminate its useful life.

On the basis of these findings, the broad rule might be established that, in order to safeguard the storage supply of a community in the semiarid regions for 100 years or more, debris storage must be provided in volume equal to, or exceeding, the water storage. This is a "large order" and may be hard to fill; but it epitomizes the seriousness of the problem.

Although such provisions are not remedies, they tend to postpone the day of reckoning. Acting in accordance with the best available knowledge, engineers at least will escape the condemnation that was heaped on Louis XV when he shrugged his shoulder, saying "After us the deluge." It might be well to recall the fate that befell his grandson barely 35 years later during the French Revolution. The wise man has said that history repeats itself.

Coming generations may see more clearly and deeply into the true nature of things and find the solution that will save them from the fate suffered by some of the older civilizations in the semiarid regions.

The author is to be commended for the presentation of an excellent and condensed review of a complex problem.

L. STANDISH HALL,[13] M. AM. SOC. C. E.—By assembling the available data bearing on all phases of the subject of reservoir sedimentation, Mr. Witzig has rendered a valuable service to the profession.

Under the heading, "Origin and Nature of Silt," Mr. Witzig describes sheet and bank erosion as the principal sources of reservoir silting. The relative importance of these two factors on any watershed depends largely on the gradient of the main channel, although in some instances other effects may govern. In streams having a flat slope, the sediment carried by the tributaries will be dropped in the main channel resulting in aggradation. Then, reservoir sedimentation is caused by a reworking of these sediments in the stream channel. When the slope of the main channel is steep, the sediment fed into it by the tributaries will be carried along by the velocity of the current and the dynamic action of the water will cause excessive bank and bed erosion by abrasive action. In such cases, the erosion effect of the sediment is cumulative, increasing as the water proceeds downstream until a saturation of the sediment load is reached.

Other sources of erosion not mentioned by the author, but quite prominent in the Coast Range of California, are landslides and earth flow, particularly when these occur along stream channels. The toe of a slide which encroaches on the waterway is eroded by the flowing water causing a progressive movement of the slide into the channel. When landslides are not cut by water courses, the erosion of the slide may be negligible. In such cases, the landslip merely results in the mass movement of the soil from a higher to a lower level (59).[13b]

Erosion from highway construction is also an important factor. Much of the current highway practice of discharging water from small culverts with a free drop to the downstream end, or of failing to provide a basin at the outlet end of large culverts in which to dissipate the energy of the high velocity discharge through the culvert, has added many tons of sediment to stream channels.

Under the side heading, "Definition of Silt," the author cites a weight of 85 lb per cu ft for deposits in reservoirs. This value appears to be rather high unless the sediment in the reservoirs has been reworked and subjected to exposure and drying. Tests, by the writer, of samples of silty sand deposited by stream channels on their flood plains indicated a percentage of voids averaging 53% and a weight of the dry material of 77 lb per cu ft. For silty clay, the percentage of voids was 60% and the weight of the dry material 66 lb per cu ft. The weight of the dry material comprising the silt deposits in reservoirs, when not subjected to drying, is undoubtedly much less than that of material found in deposits on the overflow areas of stream channels. The void content of saturated sediments from reservoirs has been found by the writer to be as high as 80%, which corresponds to a dry weight of 33 lb per cu ft. The dry weight of deltaic deposits near the head of a reservoir, of course, would be greater and could equal the foregoing weight of the river bottom deposits, or in some instances it might be even greater.

---

[13] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

Under the heading, "Transportation of Sediment to the Reservoir," the difference between the method of transportation of suspended sediment and of bed load is indicated by a comparison of Eqs. 1 and 5*a*. In the Kennedy formula, V_c varies directly as a power of the depth, whereas, in the Schoklitsch formula, since $V_c = \frac{0.00532\, D_g}{y_d\, S^{4/3}}$, it varies inversely with the depth. Hence, it may be inferred that, to keep fine sediment in suspension, the velocity must increase with the depth of the water. On the other hand, the movement of bed load, by which erosion the stream bed continually deepens the channel and makes the bank slopes steeper, is the result of drag. This drag, in accordance with the du Boys theory, is proportional to the depth of water and to the slope or gradient of the channel (60). The movement of fine sediment depends on the supply available, which is variable, and this movement in a large measure is independent of the volume of flow. Approximate curves may be developed, but the actual proportion of sediment is normally greater on a rising than on a falling stage and, in addition, varies greatly between individual storms for the same drainage area (61). On the other hand, the movement of bed load is directly related to the volume of flow and for a given stream channel can be predicted with greater accuracy.

TABLE 7.—GENERAL CHARACTERISTICS, RESERVOIRS NEAR OAKLAND, CALIF.

Reservoir	Runoff, 1886–1887 to 1935–1936 (acre-ft per sq mile)	Drainage area (sq miles)	ORIGINAL STORAGE	
			Acre-ft	Acre-ft per sq mile, S_R
Chabot	409	42.05	15,860	377
Upper San Leandro	477	30.3	41,398	1,370
San Pablo	387	32.2	43,149	1,340
Temescal	409	2.75	656	238

Under the heading, "The Action of Sediment in Reservoirs," the author quotes Mr. Faris as follows: "* * * No suspended silt is carried through the reservoir and over the spillway until all of the clear water has been discharged." Observations of the writer indicate that this is not necessarily true in all instances. At the Pardee Dam on the Mokelumne River in California, the inflowing water is not of high turbidity at any time and during much of the year it is very clear. The maximum turbidity rarely exceeds 250 ppm. On one occasion in June, 1935, the sluice valves at the base of the dam had been opened and were discharging water of a turbidity of 20 to 30 ppm, while the river flow was between 3,000 and 5,000 cu ft per sec. These sluices were closed for a period of about a week. In a few days, turbid water was observed to rise through the reservoir near the spillway and to pass over the weir lip. By opening the sluice valves again to reduce the spillway discharge to a very small quantity, the release of the turbid water was transferred back to the sluice valves. The density currents in the reservoir at the time were such that the discharge of the turbid water could be transferred from the sluice valves at the base of the dam to the spillway, a vertical height of 327 ft, depending on the point at which the maximum volume of water was discharging.

TABLE 8.—SUMMARY OF SEDIMENTATION OF RESERVOIRS NEAR OAKLAND, CALIF.

Period	Interval, in years	Mean annual runoff (acre-ft per sq mile)	ANNUAL RATE OF SILTING			
			Mean annual (acre-ft)	Cu ft per acre	Total runoff (%)	Acre-ft per 100 sq miles
(*a*) CHABOT RESERVOIR						
1875–1900	25	447	60.4	98	0.32	144
1900–1910	10	526	131.0	212	0.59	312
1910–1923	13	303	37.5	61	0.23	89
1923–1942	19	232	17.3	28	0.18	41
1875–1942	67	387	54.2	88	0.33	129
(*b*) UPPER SAN LEANDRO RESERVOIR						
1924–1935	11	270	15.4	34	0.19	51
1935–1941	6	765	134.3	302	0.58	443
1924–1941	17	445	57.4	128	0.43	189
(*c*) SAN PABLO RESERVOIR						
1917–1936	19	226	12.4	26	0.27	38
1936–1938	2	732	378.5	820	1.60	1,210
1938–1943	5	616	111.0	234	0.56	345
1917–1943	26	340	64.8	137	0.59	202
(*d*) TEMESCAL RESERVOIR						
1869–1907	38	438	4.8	118	0.40	175

In connection with the section of the paper on "Estimating the Rate of Reservoir Sedimentation," the writer can add results on sedimentation obtained from surveys of storage reservoirs of the East Bay Municipal Utility District near Oakland, Calif. Surveys of sediment deposits in one of these reservoirs have continued over a period of 67 years (62). Additional surveys of the silt deposits have been made during the last few years and the results of these and previous surveys are summarized in Tables 7 and 8. The runoff per square mile from these drainage areas is relatively small and, on the other hand, the storage per square mile of drainage area is relatively large. Consequently, little runoff has been wasted over the spillways and the water that has been wasted was practically clear. Therefore, the deposits in the reservoirs represent all of the sediment produced on the watersheds during the periods recorded.

The relation between sediment deposited and the storage capacity per square mile of drainage area is plotted in Fig. 8 (in a manner similar to Fig. 3). The range in sediment deposits for the various periods between surveys is indicated. Based on the average deposit of sediment in the smaller reservoirs, Eq. 6 has been plotted using for *x* his value of 0.83. The application of this equation to the reservoirs listed in Tables 7 and 8 would require a much greater sediment deposit in the larger reservoirs than was actually recorded. The relation between the runoff by square miles of drainage area and the sediment deposits expressed as percentage of the volume of runoff has been plotted in Fig. 8(*b*). For these reservoirs, a straight-line relation appears to exist between these two factors. In the San Pablo Reservoir, the high deposits recorded between 1936 and 1938 were due to road construction and land subdivision above the reservoir during a period of heavy runoff. The subsequent survey in 1943 indicates that the rate of erosion from this watershed had returned to that indicated in Fig. 8(*b*).

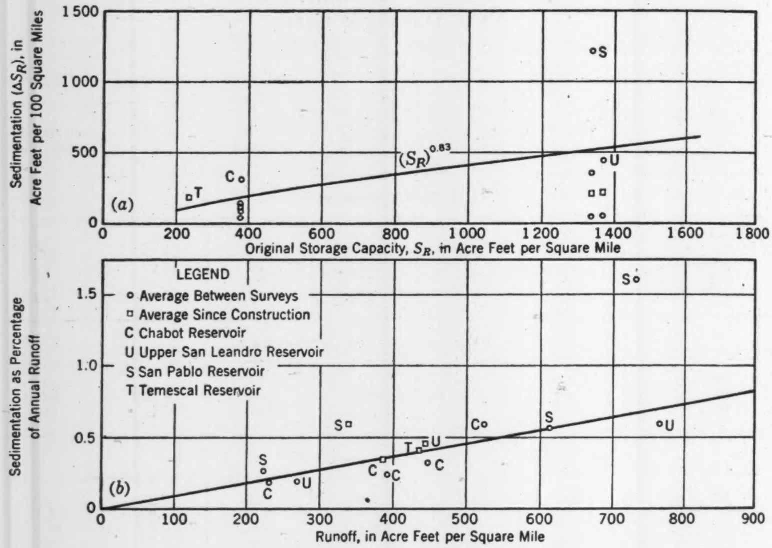


FIG. 8.—SEDIMENTATION OF RESERVOIRS NEAR OAKLAND

This relation between sediment deposits and volume of flow, if of general application, would be very useful in expanding short records of silting to indicate the true normal. Table 8 indicates that the rate of sediment deposits varies considerably during various runoff cycles. If the watershed erosion is proportional to the annual runoff, the mean rate of erosion could be determined from the ratio of the runoff during a period of sediment measurement to the long-time mean runoff from the watershed. Where appreciable quantities of sediment are discharged through sluiceways or over the spillways, this sediment should be combined with the quantity deposited in the reservoir to determine the entire watershed erosion.

Because of the relatively large size of the reservoirs of the East Bay Municipal Utility District near Oakland as compared with the discharge from the watershed, it has been estimated that their life will range from 500 to 800 years.

At Pardee Reservoir a record has been kept for several years of suspended sediment discharge through the sluiceways and turbines by taking samples of the river water immediately below the dam. The sediment discharge has been computed from these samples with a weight of 62.5 lb per cu ft for suspended material and a weight of 35 lb per cu ft for dry material if deposited in the reservoir. This light weight for deposited material has been used due to the fine colloidal nature of the sediment discharged through the sluiceways. The results of these measurements of the four years, 1940–1943, are as follows:

Year	Weight of discharge sediment, in tons	Equivalent volume, discharge in acre-feet
1940	4,806	6.3
1941	1,949	2.6
1942	6,951	9.2
1943	8,983	11.8

These quantities, added to the sediment deposits in the Pardee Reservoir, will give the total sediment load from the watershed. The drainage area above Pardee Reservoir is largely in granitic rock and the sediment load in streams is small. The mean annual runoff is 800,000 acre-ft from a drainage area of 575 sq miles, and the original storage capacity in the reservoir was 210,000 acre-ft. In the first ten years that the reservoir was in operation, the deposits were found to be approximately 1,000 acre-ft. Less than 10% of the total sediment was discharged through the sluice valves. The deposits consisted of about an equal proportion of delta deposits at the head reservoir, and of bottom-set beds of fine material in the main body of the reservoir. At this rate of sedimentation, the life of Pardee Reservoir will be longer than 2,000 years.

Eq. 7 expressing the relation between sediment deposits and the size of the reservoir appears to be defective in that it does not include the factor of runoff. A more suitable equation might be:

$$\frac{\Delta S_R}{E} = f\left(\frac{S_R}{Q_a}\right) \qquad (20)$$

in which, in addition to the notation of the paper, Q_a is the mean annual runoff. Other factors enter the problem which are not included in Eq. 20, but usually these are of secondary importance. These factors include: (1) The size gradation of the sediment; (2) the flow characteristics of the stream entering the reservoir; (3) the shape of the reservoir; and (4) the manner of operation of the reservoir, particularly the discharge of water through sluice gates or over the spillway.

Under "Remedies for Reservoir Silting," Mr. Witzig has presented four methods. Of these, sluicing of water through sluice gates and erosion control on the watershed above the reservoir appear to be the only practicable and economical means. The latter procedure offers the greatest possibilities as it controls the sediment at its source. The several types of "upstream engineering" listed by the author have been practiced by the East Bay Municipal Utility District on its local watersheds near Oakland. This work has been conducted since 1939 in cooperation with the U. S. Soil Conservation Service (62*a*)(63)(64).

Important causes of reservoir sedimentation on many watersheds, in addition to those mentioned by the author, are landslides or earth flow, particularly where the earth is carried into the stream channel. This condition is cured by draining the slide. However, in some instances where the toe of the slide has come to rest in a stream channel, the construction of a debris barrier below the slide will check the downstream movement of the sediment from the slide and result in its eventual stabilization.

To be more completely effective, debris barriers in stream channels must have a relatively narrow discharge weir with respect to the width of the channel at the dam. Otherwise, only the bed load will be trapped, since the velocity of the current will be only slightly reduced and the time of ponding will be so short that most of the suspended sediment will pass over the spillway. The discharge of a considerable portion of the suspended load is a fault naturally inherent in these small ponds. The growth of vegetation along the stream channel will increase the efficiency of the barrier by facilitating the deposit of fine material. If properly designed and maintained, a debris barrier will trap sediment greatly in excess of its normal water-holding capacity at the time of construction. This is made possible by the deposition of material well above the elevation of the spillway lip. An important function of debris barriers on main channels is the reduction of bank scouring by flattening the stream grade, and the reduction of erosion by localizing the drops at dams where the energy of the water can be dissipated in properly designed stilling pools.

An aid in the planning of the control of erosion on watershed lands is the taking of water samples from various tributaries entering the reservoir during storms. Channels carrying excessive sediment can thus be segregated and the source of erosion located later during more favorable weather. The erosion control work can then be planned to accomplish the maximum results with a minimum expenditure.

BERARD J. WITZIG,[14] ASSOC. M. AM. SOC. C. E.—The able engineers who discussed the paper have done much to clarify several aspects of the problem of sedimentation in reservoirs.

Messrs. Johnson and Stanley stress the confusion that exists in the lack of proper definitions and terms for material moving by the various modes of transportation. To eliminate this ambiguity in the paper, and to avoid confusion in the future, the writer suggests the following definitions:

---

[14] Engr. (Civ.), U. S. Engr. Office, Buffalo, N. Y.

(*a*) "Wash load" is the relatively fine material in near permanent suspension, which is transported entirely through a stream system without deposition.

(*b*) "Suspended load" is the coarser suspended material, derived from the stream bed, during changes in magnitude of the stream discharge.

(*c*) "Bed load" is the material derived from the stream bed, varying in size from silt to boulders, moved along the stream bed by rolling, sliding, or saltation.

(*d*) "Bed material" is the source of both the coarse suspended load and the bed load, and consists of the geologic formations and alluvial deposits through which the stream channel is cut.

The wash load, the greater part of which may be carried by the rising stages of the flood flow, bears no direct relation to discharge because of the seasonal variability of supply, affected by erosion, meteorologic, and hydrologic conditions on the watershed. However, because the coarse suspended load and the bed load are derived from an always available supply, relations between such loads and the water discharge exist, but may be different in different streams, or at different points on the same stream. Most total suspended-load measurements include the wash load. Hence, the lack of a relationship between the sediment loads so measured and the water discharge may mean only that the wash load constituted a preponderantly influential part of the total suspended load. Such measurements might be corrected, if the mechanical analyses of the sediment were known, and the boundary range of particle sizes were determined between the materials carried by the two modes of transportation.

No rigid natural distinction actually exists between the coarse material in suspension and the bed load. An arbitrary distinction is necessitated only by present limited knowledge of the phenomena of sediment transportation, and each condition must, therefore, be measured and evaluated by different methods. The writer believes that the so-called "dividing grain size" is only apparent, and may logically be expected to vary with change in stage or discharge at a given point on a stream, as well as with a change in location on the same stream or a shift to a different stream.

Mr. Stanley correctly interprets the writer's implication that no net settling occurs during sediment transportation in suspension. When the suspending forces are sufficiently strong, there is no accumulative deposition, but, because of the very nature of the modes of transportation, there must be an interchange of material between the load in suspension and that in movement or stationary on the bed.

Messrs. Johnson and Means add data on the so-called "silt rating curves." Their cautions in the use of such curves should be heeded, since these relations are not "rating curves" in the same sense as are stage-discharge relations above a real hydraulic control at a gaging station. They are inherently subject to the errors of including wash load, and to the approximations and errors unavoidable in measurement of suspended sediment. Moreover, they cannot include, intrinsically, any factor for bed load.

The writer believes that laboratory analysis of suspended-load samples could be refined to distinguish between wash load and suspended bed-material load, and, therefore, a relatively more accurate estimate could be made of the relation between the temporary suspended load and the water discharge. The breakdown might be made by comparison of suspended load samples with fresh bottom deposits sampled immediately after flood flows. However, there is a possibility that the wash load, varying independently of the discharge, may act either as a flux or, conversely, as a retardant, on the carrying capacity of a given discharge for the bed-material load. Only experimentation could settle this question.

With reference to the use of the silt rating curves, the writer reiterates his opinion that such relations are best interpreted not as actual physical relations but rather as statistical correlations, which may be validly used in estimating the probable average suspended loads carried by given water discharges. Perhaps doubt as to their validity for this purpose would be lessened if the envelopes enclosing the measurements were also drawn, so that it could be seen at a glance that the selected curve represents the probable average in the range of probable concentrations. Thus, the silt rating curve cannot be used to state the sediment concentration in a flood for which no sediment measurements were made, without probability of considerable error, any more than a flood-frequency curve can be used to state that a flood of a given magnitude will occur in a certain period. Both are statistical relations and should be used only as such. They can be employed to determine average annual occurrences. Whether the silt rating curve is used with a discharge-duration curve, as suggested by Professor Johnson, or with a flood-frequency curve, as in the writer's example, is immaterial, and would depend only on the extent of data available. Professor Johnson's method would require a longer period of silt sampling than the writer's; the data on the Cuyahoga River in Ohio cover too short a period to enable a comparison to be made.

Mr. Means properly warns against the indiscriminating use of "general relations" or formulas. The writer does not suggest that any of the formulas or procedures be so used. There is no adequate substitute for carefully observed data, but frequently, in their absence, or because time and expense prevent more thorough investigations, the short cuts provided by general formulas may give results of sufficient accuracy. Mr. Means questions Eqs. 2 and 3. Although these equations are intended to express what may prove to be a general law, reference to the source papers (14)(15)(21) shows that appreciable deviations exist. If this is admitted, proper use of these and similar equations can be made as engineering tools.

Professor Johnson clarifies the (ϕ-ψ)-relationship for bed load, developed by Mr. Einstein (21). The relationship seems based on and verified by a large mass of observed data. Equal substantiation of other methods of evaluating bed load is not known to the writer. However, the (ϕ-ψ)-relationship also requires evaluation of "a certain representative grain diameter." Because of this necessity, and the fact that many observations have varied considerably from Mr. Einstein's adopted curve, quite as much error may result in its use as in application of the simpler Schoklitsch bed-load formula.

Mr. Stanley carefully expounds his reasons for questioning the data on silt settling rates in Table 2, which was intended principally to illustrate indirectly the magnitude of forces required to keep sediment grains of different sizes in suspension. The additional data in Table 6 are of interest. These data illustrate the variation from theory of results obtained by different workers, which probably is due to laboratory procedure and control, material observed, and personal error in observation or interpretation, as well as some imperfection in the basic theory or formula. Because of the great variety in nature, such differences always might properly be expected.

Mr. Stanley suggests that the determination of reservoir silting "indexes" be further refined by consideration of the average annual rainfall and the average susceptibility to erosion. Evaluation of these factors would undoubtedly aid in presenting a more clear-cut delineation of the indexes, inasmuch as those derived in the paper are based on large geographical regions having considerable variation in rainfall and erosion characteristics. The writer has not had the opportunity to carry the study to this extent, however, because of lack of data.

A relation between rainfall and suspended load, on small watersheds, has been demonstrated by Professor Johnson (65). Mass curves of the deviation of suspended load from the mean closely parallel the mass curves of deviation of rainfall. The correlation obtainable therefrom may permit computation of suspended load from rainfall records for a period for which suspended load measurements are not available.

The writer does not know of any method of evaluating the average annual erosion from a watershed, except by surveys. As Mr. Brown states, the average annual erosion differs, depending on whether erosion is considered as the eroded material actually delivered to a reservoir or the material eroded from its source and deposited elsewhere on the watershed (colluvial and alluvial deposits). The former would be the factor to be evaluated in the present case. The magnitude of this factor can be stated in terms of annual accumulation of silt per square mile of drainage area, as shown by Mr. Brown in Fig. 6.

Mr. Happ clarifies the findings of the SCS relative to the influence of sheet and bank erosion. The writer was evidently in error in contrasting Mr. Horton's statement with the SCS conclusions. Identification of the principal source of erosion is of vital importance in attacking the problem. However, stream bank erosion is the principal culprit not only in the semiarid Southwest, but also in some humid basins in other parts of the United States. Thus, an SCS survey (52*a*) of the Buffalo Creek, New York, watershed, an area of 437 sq miles, indicates that stream bank erosion produces 80%, and land surface erosion, 20%, of the total sediment reaching lower Buffalo River. Buffalo Creek watershed can be called neither mountainous nor forested, although the headwaters are in the foothills of the Alleghenies. The annual rainfall on the watershed averages about 40 in., and is fairly well distributed throughout the year. A similar situation appears to hold for the Cuyahoga River. Although no erosion surveys of this watershed have been made, the writer's observations indicate that most of the eroded material carried to the lower Cuyahoga at Cleveland is produced from the stream banks and gullies in the lower half of the watershed. The river flats, the upland areas, and most of the headwaters area are of gentle slope, largely unwooded, and generally unaffected by sheet erosion.

Mr. Means pertinently draws attention to the important volume occupied by deposits above reservoir level. This was not emphasized sufficiently by the writer. Such deposits may be of serious consequence, if they should be eroded and again deposited in the reservoir cavity, or if the dam is later raised when they would detract from the added storage that might otherwise have been obtained. Use of vegetation screens, debris barriers, or desilting basins above the reservoir proper should therefore be planned with such future developments in mind. In this connection, Mr. Hall suggests several considerations for the design of debris barriers and vegetation screens. In addition, however, the designer should not disregard the possibility that resultant aggrading of channels and flattening of stream slopes upstream from such works may increase flood damages above the reservoir. If this damage cannot be prevented or avoided, then the damage, or its mitigation, should properly be included as a cost of the reservoir.

Mr. Brown epitomizes the problem when he states that "Control of silting begins when the dam design takes shape on the drawing board." Except for physical destruction of the dam itself, silting is the only force that can destroy the reservoir; the dam can be replaced, but loss of reservoir storage through silting has no completely efficient economical remedy. Mr. Brown properly questions the erroneous generalities that can be drawn from Fig. 3 and Eq. 6. Fig. 3, from which Eq. 6 was derived, represents an apparent correlation between rate of silting and reservoir capacity, and summarizes data that would otherwise be merely a mass of statistics. As evident from the plotted points, many of the smaller reservoirs have higher rates of sedimentation, when reduced to a common unit basis, than some of the larger. It was not intended to imply the existence of any natural law that the silting rate increases indefinitely with the size of the reservoir, but merely that, if a reservoir of a certain capacity per unit of drainage area were built, it might be expected to silt up at a rate similar to that of an existing reservoir in the same region. There must be a maximum limiting silting rate, regardless of the size of the reservoir, just as there is a limit to the rate of runoff from a watershed, regardless of the reservoir size that may be provided to store that runoff. If a reservoir of sufficient size to store the greater part of most floods from its drainage area silts at a certain rate, there probably would be but little increase in the rate of silting if greater storage had been provided at the same site.

Mr. Brown contributes a valuable discussion on sediment trap efficiencies. His distinction between net and gross erosion is important and should be kept in mind. Eq. 12 defines the trap efficiency coefficient in terms of silting and erosion rates only, which is simple and correct for a particular reservoir. Eq. 7, however, attempts to include the effect of reservoir size, so that known data on existing reservoirs may be applied to proposed reservoirs of different sizes. However, Eqs. 14 and 15, and Fig. 5, present the reservoir designer with a valuable tool for this purpose, and the writer especially appreciates Mr. Brown's contribution at this time.

Fig. 6 illustrates the relation between silting rates and drainage area. It seems reasonable to expect that, with an increase in drainage area, the unit silting rates on large drainage areas should decrease and become constant, since it would be unlikely that all parts of a large watershed would be eroding at the more intense rates found on certain of the small tributary areas. Use of Fig. 6, or of Fig. 3, should be based on thorough consideration of characteristics of and conditions on a basin. Neighboring watersheds often differ considerably; for example, Cayuga, Buffalo, and Cazenovia creeks in western New York, all join to flow into Buffalo Harbor. The three streams drain approximately equal areas, but Cayuga Creek with somewhat lower gradients contributes only about one tenth of the sediment deposited in lower Buffalo River, whereas the other two streams contribute the remainder.

In view of Mr. Brown's statement that SCS surveys have been made "for the purpose of obtaining a representative sampling of silting conditions * * *," the writer retracts his statement "that reservoirs surveyed thus far are more typical of severe rather than of average conditions." The latter statement was based on failure to find a specific utterance similar to Mr. Brown's in available reports of the SCS. Knowledge that SCS silting data constitute representative samples permits their study to proceed with some assurance that any results obtained are not influenced unduly by extreme conditions.

Mr. Brown has offered a valuable service to the profession in his paper (52) on the control of reservoir silting. It would be extraneous here to enlarge on control measures, and the writer seconds Mr. Blaney's suggestion that those interested review that publication.

Mr. Jarvis calls attention to some benefits resulting or obtainable from sedimentation in reservoirs and canals. However, some sediment deposits do not possess fertilizing value, especially those derived from bank erosion in areas of thin topsoil.

Mr. Bell objects to a misinterpretation of his paper (25) on density currents. Considering that both the original statement and the examples given by Mr. Bell in his discussion are only estimates based on assumed densities of silt in place, the magnitude of the quantities of material that apparently are carried through reservoirs by density currents is thereby illustrated. Whether density currents carry 24% or 35% to 40% of all sediment in Lake Mead, such currents evidently have considerable effect on the rate of sedimentation and may constitute an important factor in its control.

The writer does not agree entirely with Mr. Bell's statement that density currents decrease the specific density of the coarser deposits by removing the finer materials that would otherwise partly fill the interstices of sediments left behind. Density currents carry only the finer silts and clays. Curves given by E. W. Lane, M. Am. Soc. C. E., and Victor A. Koelzer, Jun. Am. Soc. C. E. (66*a*), indicate that the density of material deposited under water increases with an increase in the percentage of coarse material. Control of density flows for voiding silt from a reservoir before deposition might therefore result in final deposits of greater density and less volume. The deposits would also consolidate more rapidly because of the smaller silt or clay content.

Mr. Blaney emphasizes the importance of knowing the relation between silt volume and weight of deposited sediment. Messrs. Lane and Koelzer (66) have compiled all the available data on this subject, and have presented a procedure for estimating the density of deposits at any time after deposition, for various methods of reservoir operation.

Mr. Sonderegger distinguishes between the natural norm of erosion and that caused by man's activities, and states that erosion control can scarcely be expected to reduce sediment production to less than the undisturbed natural rate. The writer does not fully concur that nature cannot economically be improved on at times. When natural rates of erosion are due to the same causes as those accelerated by man, it does not seem an impossibility to modify them by proper remedial measures. Thus, burned-over areas can be reforested; eroding stream banks can be stabilized; and steep gully and stream slopes can be made more gentle by construction of dams and drop structures. The writer agrees, however, that sediment production, and transportation to reservoirs or navigable waters, cannot be prevented entirely at costs commensurate with the benefits.

Mr. Sonderegger properly emphasizes the necessity of providing adequate reservoir storage for silt loss to preserve the hydraulic functions of the reservoir. In many cases, it might be more economical to build a large dam immediately, rather than to raise the dam shortly after construction, but the decision to do so would necessarily rest on the engineering, financial, and natural conditions peculiar to a given project.

Mr. Hall discusses several special aspects of the erosion problem, and adds data on sedimentation rates in several reservoirs in California, as well as an example of the action of density currents in Pardee Reservoir. His comments on the effects of stream slopes seem to apply essentially to the reworking of flood plain and channel deposits. However, flat stream gradients do not necessarily imply that sheet erosion on the watershed outweighs bank erosion, for many streams of gentle slope flow through alluvial strata of low resistance to eroding forces, although the land surfaces themselves are amply protected by surface cover. A notable example is the Mississippi River below Cairo, Ill.

An interesting comparison is made between the Kennedy and the Schoklitsch formulas, with reference to the velocity and drag required, respectively, to maintain movement of sediment in suspension and along the bed. Although both these formulas are empirical in origin, the comparison would indicate that they are not unsound theoretically.

The extremely small concentration of sediment required to produce density flows is illustrated by Mr. Hall's description of the June, 1935, occurrence in Pardee Reservoir. This example also seems to demonstrate conclusively that density currents carrying silt can be controlled by proper manipulation of outlets. Neither laboratory research, nor large-scale experimentation with existing reservoirs, should be neglected until this presently speculative aspect of reservoir action is usefully applied.

With particular reference to Pardee Reservoir, the writer suggests that the current may have been of such density that it flowed along the top of a cold water layer of just slightly greater density. When the sluice-gates at the bottom of the dam were opened, the resulting hydraulic current drew the silty water to the outlet; when the gates were closed, the hydraulic current over the spillway drew the silty water to the top of the dam. Without further data, the writer would suggest that the concentration of silt in the spillway flow was somewhat less than that through the gates, inasmuch as the density current probably had to mix with a greater proportion of clear reservoir water in rising to the top of the dam, than in sinking to the bottom. Furthermore, the density flow must have been quite thick to have appeared so quickly over the spillway, or a considerable part of the stored water in the reservoir may have consisted of the silty suspension. The shape of the reservoir, and its relation to the location of the dam, as well as the shape of the back surface of the dam itself, may also have influenced the action of the current.

Mr. Hall suggests a method of correlating the rate of reservoir silting with the mean annual runoff from the tributary area. The linear relation illustrated in Fig. 8(*b*) between annual runoff volume per square mile and sedimentation as a percentage of annual runoff seems to fit the plotted data. The writer does not have the necessary information to test this correlation on the source data of Fig. 3. The method is worthy of further investigation, however, and may prove valuable in estimating sedimentation rates from runoff records. This tool may possibly be applicable to larger watersheds with more accuracy than is the correlation between mass deviation curves of rainfall and suspended load previously mentioned in this discussion. The latter method is unduly influenced by variable distribution of rainfall on large areas.

Eq. 20 is a particular statement of Eq. 11, but is more complete than Eq. 7. However, for practical use, it would require evaluation of a coefficient for the right-hand term. As Mr. Brown suggests in Eq. 12, however, the term E should be the net erosion, E_n, delivered to the reservoir. No practical method is apparent at this time which includes in a single quantitative equation the omitted factors mentioned by Mr. Hall. However, with reference to size gradation of sediment, few particles of the largest sizes are carried to the reservoir except in debris basins as in California, and most particles of the finest sizes are transported through the reservoir, except in the case of very large volumes which store the entire flood runoff for an appreciable period of time. The intermediate sizes, between "Cyclopean" sediment and colloidal silt, generally form the predominant components of reservoir deposits. The flow characteristics of the tributary stream are partly accounted for in the term, Q_a. Reservoir shape and manner of operation would seem to be qualitative factors.

By direct correspondence, C. S. Howard, Chemist of the U. S. Geological Survey, states that the greater part of the annual silt load in a stream may be carried in a relatively short time, with the stream flowing comparatively clear during the remainder of the year. This is illustrated in Fig. 1, where the quantity of 2.76 tons of silt per second was carried by a discharge of 27,800 cu ft per sec. A one-day flow of this magnitude would be equivalent to about 45% of the estimated average annual silt load of 532,000 tons. However, this concentration has about a 1.5% chance of occurrence, or may occur about once in 67 years. Thus, a flood of this magnitude and frequency would contribute over its period of recurrence only about 0.68% of the average annual silt load.

Mr. Howard also points out that the data in the basic paper on the efficiency of the desilting works for the All-American Canal were estimates before operation began. The writer has learned that up to August, 1943, the desilting works had not been put into operation, since it had been possible to obtain a comparatively silt-free diversion to the canal by reason of high reservoir levels and high sluiceway discharges.

The writer has found the discussions very instructive. Practically every topic in the basic paper has been discussed or commented upon, erroneous or doubtful points clarified, and interesting new data and suggestions added. Differences of opinion on theory and interpretation of natural phenomenon, of ourse, will continue. However, the travail common to the solution of all involved problems will undoubtedly produce valuable social and economic benefits, as the mechanism of sedimentation becomes better understood and practicable remedies are developed for reservoir silting.

In view of the historic fact that water resources are of utmost importance to a civilization and an economy, the present generation should exert every effort to use these resources wisely and to conserve them to the greatest degree for future generations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2228

THE HYDRAULIC JUMP IN SLOPING CHANNELS

By CARL E. KINDSVATER,[1] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOE W. JOHNSON, KARL R. KENNISON, J. C. STEVENS, C. J. POSEY, JEROME FEE, FRANK S. BAILEY, G. H. HICKOX, AND CARL E. KINDSVATER.

SYNOPSIS

Common forms of the hydraulic jump in sloping channels have been classified into three general cases, and an analysis is presented which leads to a practical method of computing the dimensions of the jump. Satisfactory agreement between analysis and experiment was obtained from laboratory tests on a channel with a 1 on 6 sloping floor. Conclusions drawn from this investigation indicate that experiments on other slopes might eventually yield a satisfactory treatment for hydraulic jumps on any slope within the practical range.

INTRODUCTION

Historical Résumé.—The inability to predict the occurrence or to compute the essential dimensions of the hydraulic jump in sloping channels has been a source of difficulty in hydraulic design.[2] Sufficient verification data now exist to substantiate the application of the momentum principle, derived from Newton's second Law of Motion, to the hydraulic jump in open or enclosed horizontal conduits of any practical cross-sectional shape. To apply the same principle to an analysis of the hydraulic jump in sloping channels, however, it is necessary to obtain floor pressures and certain elements of the jump profile from laboratory experiment.

In their early work[3] on the hydraulic jump, Bidone, Bazin, Bélanger, and Boussinesq were unable to verify their analyses largely because they neglected components of pressure on the sloping floor. Others, more recently, have made the same error. In 1934, the late D. L. Yarnell, M. Am. Soc. C. E., of the U. S. Department of Agriculture, Division of Drainage, attempted an analysis in which floor pressures under the jump were evaluated from an assumed straight-

---

NOTE.—Published in November, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Asst. Hydr. Engr., Flood Control Section, TVA, Knoxville, Tenn.

[2] "Hydraulic Tests on the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1086.

[3] "The Hydraulic Jump in Sloped Channels," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, A. S. M. E., Vol. 60, 1938, Paper HYD-60-1, pp. 111-118.

line profile from the beginning of the jump to the end of the roller. This approximation was not sufficiently accurate to give satisfactory results. In 1936, Mr. Yarnell resumed his investigation with a program of comprehensive laboratory tests, which were interrupted by his death on March 9, 1937.

In 1938, B. A. Bakhmeteff, M. Am. Soc. C. E., and A. E. Matzke, Assoc. M. Am. Soc. C. E., published[3] an analysis in which, by introducing a "form coefficient" based on profile measurements, they accounted for the effect of pressures on the sloping floor. Verification of their treatment was limited, however, to a single case of the jump in channels of very flat slope.

The present investigation, based on Mr. Yarnell's 1936 experiments, was undertaken by the Tennessee Valley Authority (TVA) in preliminary studies of spillway design for one of its dam projects.

Classification of the Hydraulic Jump.—The hydraulic jump occurs under many conditions in a great number of related forms. For facility of analysis it is desirable to classify into several general cases the most common forms of the jump in open rectangular conduits. Fig. 1 shows four basic cases selected for the present analysis, designated cases 1 to 4 in order of position relative to the sloping floor. Case 1, with the entire roller on the horizontal floor, is the hydraulic jump in horizontal channels. Cases 2, 3, and 4 are common forms of the jump in a sloping channel. Thus, in Case 2 the toe of the roller is on the slope and the end of the roller is on the horizontal floor; in Case 3 the toe of the roller is on the slope and the end is at the junction of the sloping and horizontal floors; in Case 4 the entire roller is on the slope. In each case, the water surface and channel bottom downstream from the jump, as well as the reference axis, are assumed to be horizontal. Messrs. Bakhmeteff and Matzke[3] investigated a fifth case, in which the water surface downstream from the jump was parallel to the slope. This case occurs in some natural channels and in long flumes of flat slope.

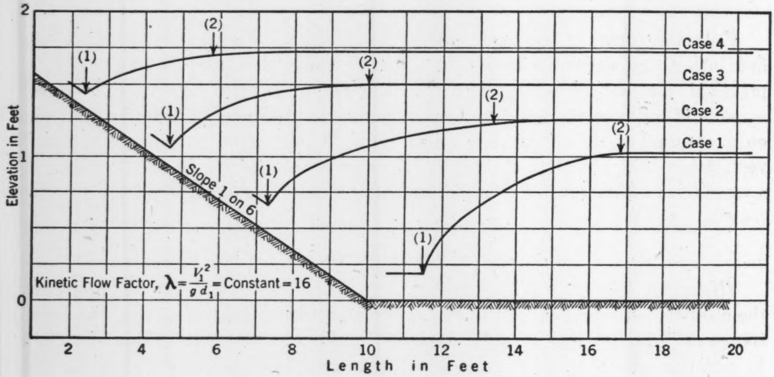


FIG. 1.—BASIC CASES OF THE HYDRAULIC JUMP IN SLOPING CHANNELS

NOTATION

The following letter symbols, selected for use in this paper, conform essentially to American Letter Symbols for Hydraulics, prepared by a Committee of American Standards Association, with Society representation, and approved by the Association in 1942:[4]

A = area of cross section, in square feet;
d = depth of flow, in feet, with appropriate subscripts to denote sections;
f = "function of";
g = acceleration due to gravity, in feet per second per second;
j = subscript denoting "hydraulic jump"; for example, "d_j = depth of hydraulic jump" and "L_j = length of hydraulic jump";
k = coefficient for Case 2 (dimensionless);
L = length along channel, in feet;
n = a subscript denoting "normal"; for example, p_n denotes "normal unit pressure";
P = total pressure, in pounds;
p = unit pressure, in pounds per square foot;
Q = total rate of discharge, in cubic feet per second;
r = radius; as a subscript, r denotes "roller"; for example, L_r = length of roller;
s = embankment or side slopes; as a subscript s denotes sloping floor; for example, L_s = length along a sloping channel floor;
V = average velocity of flow, in feet per second; $V = \frac{Q}{A}$;
y = vertical height of the pressure gradient above the floor, in feet; $\bar{y}$ = average height along a given length of channel;
α = slope angle ($\tan \alpha$ = slope of channel);
γ = unit weight of water, in pounds per cubic foot;
λ = kinetic flow factor, $\frac{(V_1)^2}{g\, d_1}$;
ϕ = pressure coefficient (dimensionless).

HYDRAULIC JUMP IN HORIZONTAL CHANNELS

The theory of the hydraulic jump in horizontal channels, Case 1, has been treated thoroughly by others,[5,6] and repetition is unnecessary except to review the important assumptions involved. Fig. 2 shows the hydraulic jump in a rectangular open channel with a horizontal floor. The body of the jump is delimited by section 1 at the toe of the jump and section 2 at the end of the jump. Since parallel lines of flow are assumed to exist at sections 1 and 2, pressure distribution in both sections is governed by hydrostatic principles. The vertical distances d_1 and d_2 are depths of flow at sections 1 and 2, respectively.

The momentum principle, from Newton's second Law of Motion, states that the time rate of change of momentum between any two sections in a steady flow is equal to the resultant of external forces applied on the mass of fluid between those two sections. Thus, taking the floor of the channel as reference axis, and neglecting the force of boundary friction, the momentum principle can be applied to Fig. 2:

---

[4] ASA—Z10.2—1942.

[5] "Theory of the Hydraulic Jump and Backwater Curves," by Sherman M. Woodward, *Technical Reports*, The Miami Conservancy Dist., Pt. III.

[6] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 630–680.

$$P_2 - P_1 = \frac{Q\,\gamma}{g}\,(V_1 - V_2) \quad \ldots\ldots(1)$$

in which P_1 and P_2 are the horizontal forces due to hydrostatic pressure, and V_1 and V_2 are uniform velocities across sections 1 and 2. The pressure-momentum equality in Eq. 1 is illustrated by the superimposed pressure diagrams in Fig. 2.

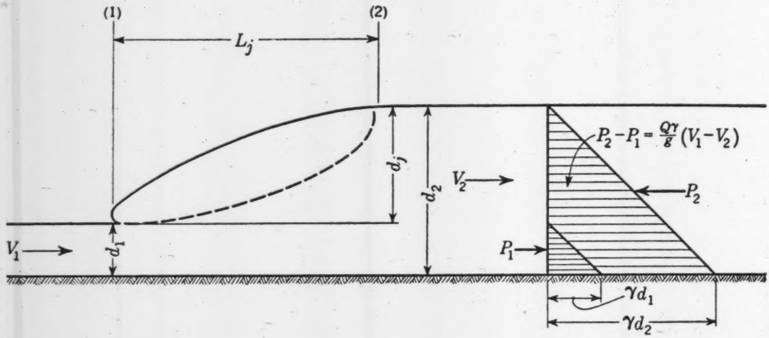


FIG. 2.—CASE 1; THE HYDRAULIC JUMP IN HORIZONTAL RECTANGULAR CHANNELS

Although Eq. 1 is an algebraic expression of the momentum principle, it is not in a form suitable for computation of the dimensions of the hydraulic jump. If the rate of change of momentum is evaluated on the assumption of uniform distribution of the average velocity, Eq. 1 can be written as an equation for d_2 in terms of V_1 and d_1:

$$d_2 = \frac{d_1}{2}\left[\sqrt{\frac{8\,(V_1)^2}{g\,d_1} + 1} - 1\right] \quad \ldots\ldots(2)$$

Messrs. Bakhmeteff and Matzke[6] have shown that the term $\frac{(V_1)^2}{g\,d_1}$, which they have called the kinetic flow factor, is the square of Froude's number, a criterion for dynamic similarity in fluid motion where forces of inertia and gravity predominate. Substituting λ for $\frac{(V_1)^2}{g\,d_1}$, Eq. 2 becomes

$$d_2 = \frac{d_1}{2}\left(\sqrt{8\,\lambda + 1} - 1\right) \quad \ldots\ldots(3)$$

THE HYDRAULIC JUMP IN SLOPING CHANNELS

Limitations to Analysis.—There are a number of practical limitations to the application of a generalized analysis of the hydraulic jump in sloping channels. The jump is commonly utilized as a means of dissipating energy below spillways and chutes, frequently as a supplement to impact devices in the stilling basin. Analysis of the latter case is not practical because of the difficulty in evaluating the impact forces on the auxiliary structures.

On very steep slopes on large structures, the problem is often complicated by the entrainment of air in the high-velocity stream before the jump or by the formation of traveling waves on the surface of the stream. In either case, it is practically impossible to compute accurately the momentum or the hydrostatic pressure in a cross section at the toe of the jump.

Hydraulic literature contains many references to differences between the level-floor jump and the jump in sloping channels, with the implication that similar analyses are not feasible for the two cases. In outward appearance, the hydraulic jump on flat slopes is quite similar to the jump in horizontal channels. On steeper slopes, however, the hydraulic jump has been dismissed as an imperfect form of the phenomenon, described as a "drowned nappe" and a "submerged jump." It is possible that these terms apply exclusively to jumps in the Case 2 classification.

Careful observations of several hundred jumps on slopes from 1 on 6 to 1 on 1 indicate that the sloping-floor jump is generally similar to the jump in horizontal channels. Fig. 3 is a mosaic of short-exposure photographs showing a Case 4 jump in a 1 on 6 sloping channel. Fig. 4 shows a Case 2 jump in a 1 on 3 channel. In both exhibits, when the high-velocity stream before the jump impinges on the tailwater, the impact produces a turbulent circulation of "white water" throughout the body of the jump. Immediately downstream from the toe of the jump, the high-velocity stream begins to expand, and the curve of expansion is apparently similar for all cases. However, as the slope of the channel increases, an increasing length of expansion is required before the curve of the upper boundary of the live stream changes from its initial negative slope to a positive slope, ascending to the tailwater surface. On the other hand, the difference in water-surface elevation between sections 1 and 2 decreases with increasing channel slopes.

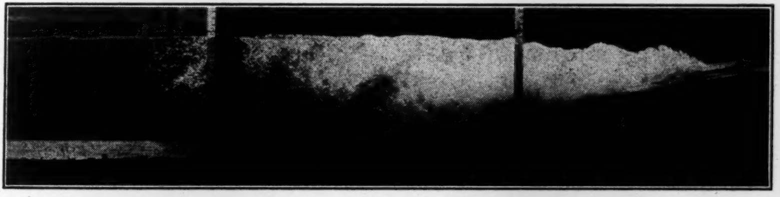


FIG. 3.—CASE 4; HYDRAULIC JUMP IN A 1 ON 6 SLOPING CHANNEL; KINETIC FLOW FACTOR $\lambda = 15.7$

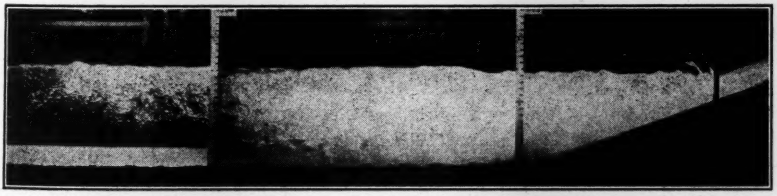


FIG. 4.—CASE 2; HYDRAULIC JUMP IN A 1 ON 3 SLOPING CHANNEL; KINETIC FLOW FACTOR $\lambda = 29.7$

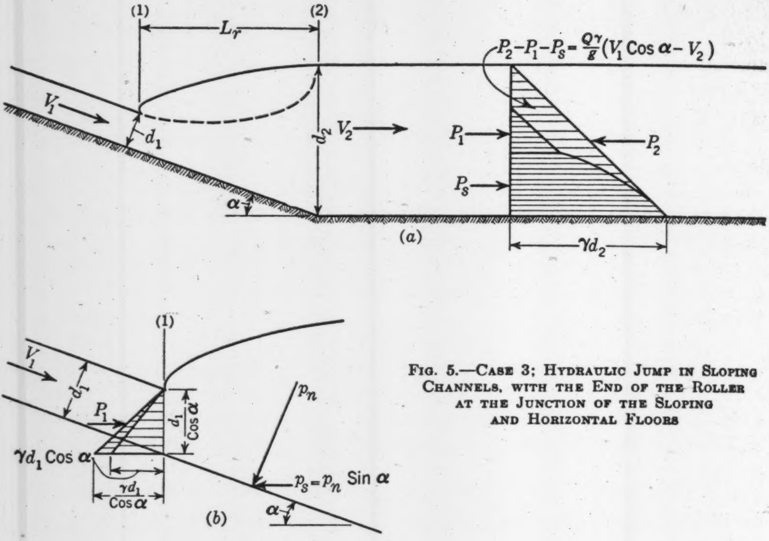

(1)
(2)
L_r
$P_2-P_1-P_s=\frac{Q\gamma}{g}(V_1 \text{ Cos } \alpha - V_2)$
V_1
d_1
d_2
V_2
P_1
P_2
P_s
α
(a)
γd_2
(1)
V_1
d_1
P_1
$\frac{d_1}{\text{Cos } \alpha}$
$\gamma d_1 \text{ Cos } \alpha$
$\frac{\gamma d_1}{\text{Cos } \alpha}$
p_n
$p_s=p_n \text{ Sin } \alpha$
α
(b)

Fig. 5.—Case 3; Hydraulic Jump in Sloping Channels, with the End of the Roller at the Junction of the Sloping and Horizontal Floors

Length of the Jump in Sloping Channels.—Although the length of the jump is not a factor in the analysis leading to Eq. 3 for the level-floor jump, it is a prerequisite to an analysis of the jump in sloping channels. If the channel slope is continuous, as in Case 4, some expansion may occur beyond the end of the jump. Thus, for analytical purposes, it is necessary to know the minimum length of expansion for which the depth of tailwater at section 2 is sufficient to insure the formation of the jump. It is possible to approximate the length of the jump with an assumed constant "safe length," but tailwater depths determined in this manner usually exceed the depth required to produce the jump.

In the present analysis, the length of the jump has been taken as the length of its surface roller. On the water surface, the end of the roller is marked as a narrow zone within which the downstream currents are divided from the upstream currents of the backroller. In the laboratory, the end of the roller was defined reasonably well as the point on the water surface from which confetti particles would float both upstream and downstream. Prolonged observations indicated that the end of the roller was the only point on the surface of the water near the end of the jump which could be determined conveniently and consistently by different observers. Experiments on a 1 on 6 slope showed that the depth of tailwater at the end of the roller was sufficient to produce a jump on the slope.

Since, as in Case 4 jumps, a slight expansion due to the recovery of velocity head may occur beyond the end of the roller, section 2 is not necessarily the point of maximum elevation on the water-surface profile. Nor is it necessarily true that the end of the roller marks a section of complete dissipation of destructive bottom velocities. However, experiments on a 1 on 6 slope showed a negligible error due to an assumption of uniform velocity distribution at section 2.

Generalized Analysis.—Case 3, with the end of the roller at the end of the slope, is obviously a primary form of the hydraulic jump in sloping channels, since the depth of flow at section 2 is definitely limited to the depth of tailwater. It is convenient for analysis to derive a general treatment from this case. Fig. 5 shows Case 3 in a channel with slope angle α, in which $V_1 \cos \alpha$ is the horizontal component of the average velocity in section 1 at the toe of the jump, and V_2 is the average velocity in section 2 at the end of the jump. The pressures P_1 and P_2 shown on the pressure diagram in Fig. 5(*a*) are the horizontal forces due to pressure on sections 1 and 2, respectively; and P_s is the horizontal component of the total pressure on the sloping floor between sections 1 and 2.

Neglecting boundary friction, the momentum principle applied to Fig. 5 gives

$$P_2 - P_1 - P_s = \frac{Q\,\gamma}{g}\,(V_1 \cos \alpha - V_2) \ldots\ldots\ldots\ldots\ldots (4)$$

The right-hand side of Eq. 4 is equivalent to the rate of change of the horizontal component of momentum between sections 1 and 2. If the velocity at section 2 is evaluated on an assumption of uniform velocity distribution, $V_2 = \frac{Q}{A_2}$, then for a channel of unit width,

$$V_2 = \frac{V_1\,d_1}{d_2} = \left[\frac{V_1 \cos \alpha \dfrac{d_1}{\cos \alpha}}{d_2}\right] \ldots\ldots\ldots\ldots\ldots (5)$$

Substituting this expression for V_2 in Eq. 4, the rate of change of momentum becomes

$$\frac{Q\,\gamma}{g}\,(V_1 \cos \alpha - V_2) = \frac{\gamma\,(V_1)^2\,d_1 \cos \alpha}{g\,d_2}\left(d_2 - \frac{d_1}{\cos \alpha}\right) \ldots\ldots (6)$$

If the pressure distribution in section 2 is assumed to follow hydrostatic principles, then P_2 in Eq. 4 becomes, for a channel of unit width,

$$P_2 = \frac{\gamma\,(d_2)^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Referring to Fig. 5(*b*), the equation of equilibrium[7] for section 1 gives

$$P_1 = \left(\frac{d_1}{\cos \alpha}\,\frac{\gamma\,d_1 \cos \alpha}{2}\right) = \frac{\gamma\,(d_1)^2}{2} \ldots\ldots\ldots\ldots\ldots (8)$$

but it is convenient for the present analysis to substitute for Eq. 8 an approximation,

$$P_1 = \frac{\gamma\,(d_1)^2}{2 \cos^2 \alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

---

[7] "Applied Fluid Mechanics," by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1937, p. 293.

As far as the final equation for the hydraulic jump is concerned, the difference between Eqs. 8 and 9 is negligible for slopes as steep as 1 on 2.

The term P_s in Eq. 4 must be evaluated from empirical relations established by laboratory measurements of floor pressures. Since experiments indicate that pressure on the sloping floor is proportional to the weight of the fluid body on the slope, P_s is a function of the dimensions and shape of the jump. In this analysis, it is convenient to express P_s in terms of a dimensionless parameter, ϕ, selected to facilitate algebraic operations:

$$P_s = \phi \left[\gamma\, (d_2)^2 - \frac{\gamma\, (d_1)^2}{\cos^2 \alpha} \right] \tan \alpha \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

As shown subsequently, ϕ is an experimental coefficient, distinct for each value of the kinetic flow factor and each channel slope.

Eq. 4 may now be evaluated in terms of Eqs. 6, 7, 9, and 10:

$$\frac{\gamma\, (d_2)^2}{2} - \frac{\gamma\, (d_1)^2}{2 \cos^2 \alpha} - \phi \left[\gamma\, (d_2)^2 - \frac{\gamma\, (d_1)^2}{\cos^2 \alpha} \right] \tan \alpha$$
$$= \frac{\gamma\, (V_1)^2\, d_1 \cos \alpha}{g\, d_2} \left[d_2 - \frac{d_1}{\cos \alpha} \right] \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Eq. 11 is a complete equation for the hydraulic jump in sloping rectangular channels.

For practical use it is desirable to have an equation for the depth of tailwater, d_2. Eq. 11 can be simplified by dividing both sides of the expression by $\frac{\gamma}{2} \left[d_2 - \frac{d_1}{\cos \alpha} \right]$, giving

$$\left(d_2 + \frac{d_1}{\cos \alpha} \right) - 2\, \phi \left(d_2 + \frac{d_1}{\cos \alpha} \right) \tan \alpha = \frac{2\, (V_1)^2\, d_1 \cos \alpha}{g\, d_2} \ldots (12)$$

and Eq. 12 can be solved for d_2:

$$d_2 = \frac{d_1}{2 \cos \alpha} \left[\sqrt{\frac{8\, (V_1)^2 \cos^3 \alpha}{g\, d_1\, (1 - 2\, \phi \tan \alpha)} + 1} - 1 \right] \ldots\ldots\ldots (13)$$

Letting the quantity $\frac{(V_1)^2}{g\, d_1} = \lambda =$ the kinetic flow factor, Eq. 13 can be written in another convenient form:

$$d_2 = \frac{d_1}{2 \cos \alpha} \left(\sqrt{\frac{8\, \lambda \cos^3 \alpha}{1 - 2\, \phi \tan \alpha} + 1} - 1 \right) \ldots\ldots\ldots\ldots (14)$$

Eq. 14 is a general equation for d_2, the depth of flow at section 2, for either Case 3 or Case 4 of the hydraulic jump in sloping channels. For Case 3, the depth at section 2 is equivalent to the tailwater depth.

Case 2, shown in Fig. 1, is a transition between cases 1 and 3, and neither Eq. 14 nor Eq. 3 for the level-floor jump can be applied directly to that case. As shown subsequently, however, experimental coefficients can be used to relate Eqs. 14 and 3 in a solution for Case 2.

LABORATORY INVESTIGATION OF HYDRAULIC JUMP IN SLOPING CHANNELS

Scope of Tests.—An intensive study of the hydraulic jump in sloping channels was begun in August, 1936, by Mr. Yarnell, in cooperation with the Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City, Iowa. Six hundred tests, on slopes of 1 on 6, 1 on 3, 1 on 2, and 1 on 1, were completed, and an analysis of the data had been started before Mr. Yarnell's death in March, 1937. The data from this investigation were loaned to the TVA in April, 1939.

Unfortunately, because of limitations in the apparatus, jumps tested on slopes greater than 1 on 6 were largely in the Case 2 classification shown in Fig. 1. Since it is basic to the present analysis to derive experimental coefficients from jumps in the Case 3 or Case 4 classification, data on the steeper slopes were insufficient to develop the necessary empirical relations.

Laboratory Setup.—The laboratory setup consisted of a glass-walled flume, 30 in. wide, 3 ft deep, and 30 ft long, with a large head-tank at the upper end. The sloping floor was built of wood, carefully treated and reinforced to produce a smooth, flat surface. The upstream end of the slope was fastened to a watertight bulkhead in the head-tank. An adjustable, sharp-edged, vertical slide-gate was used to regulate flow from the head-tank to the flume. Water depths in the flume were regulated at the lower end by a steel gate hinged at the bottom. Discharges were measured by means of a sharp-edged, suppressed, rectangular weir which emptied into the head-tank. Water-surface elevations in the flume were measured with a point gage on a sliding carriage. Point-gage readings were "zeroed" by readings on the floor after every observation. Three rows of piezometers were located on 6-in. centers over the entire length of the flume. Piezometers in the level steel floor consisted of $\frac{3}{8}$-in. pipe nipples, threaded into the floor and ground flush with the bottom of the flume. Piezometers in the wooden apron were made of $\frac{1}{4}$-in. brass tubing. Each piezometer was connected by means of rubber tubing to a manometer board.

VERIFICATION OF ANALYSIS FROM TESTS ON 1 ON 6 SLOPE

The Kinetic Flow Factor, λ.—In the foregoing analysis, the quantity $\frac{(V_1)^2}{g\,d_1} = \lambda$ has been referred to as the kinetic flow factor, a dimensionless criterion for dynamic similarity which is the square of Froude's number.[8] In problems of open-channel flow Froude's number is applicable when forces of inertia and gravity predominate. As applied to the hydraulic jump in channels of unit width, dynamic similarity implies geometric similarity of all common vertical and longitudinal dimensions. Since the kinetic flow factor is a dimensionless index of similarity, independent of the units of measurement or absolute size of the phenomena, it can be used to correlate experimental data obtained under many different test conditions.

The Pressure Coefficient, ϕ.—In the derivation of Eq. 11 the horizontal component of pressures on the sloping floor was expressed in terms of an experimental coefficient, ϕ (see Eq. 10). In this investigation, normal pressures on the sloping floor were measured by means of piezometers. An interesting fact observed by Messrs. Bakhmeteff and Matzke[3] and verified by the Yarnell tests was that the pressure profile of the hydraulic jump on a slope is very closely approximated by the point-gage profile. It would seem possible, therefore, to derive reasonably accurate pressure coefficients without the use of piezometric observations.

---

[8] "The Analytical Approach to Experimental Hydraulics," by Hunter Rouse, *Civil Engineering*, November, 1934, p. 563.

If p_n is the normal pressure at any point on the slope, then

$$p_n = \gamma\, y \qquad (15)$$

in which y is the vertical height of the pressure gradient above the floor. Thus, if $\bar{y}$ is the average height of the pressure gradient over a slope of length $\frac{L_r}{\cos\alpha}$, the total normal pressure on a unit width of channel is

$$P_n = \frac{\gamma\, \bar{y}\, L_r}{\cos\alpha} \qquad (16)$$

The quantity ($\bar{y}\, L_r$) in Eq. 16 is the area of the pressure diagram, shown by experiment to be proportional to the area of the jump body. The horizontal component of the total normal pressure is P_s, or

$$P_s = P_n \sin\alpha = \gamma\, \bar{y}\, L_r \tan\alpha \qquad (17)$$

Thus, from Eqs. 10 and 17,

$$\phi = \frac{\bar{y}\, L_r}{(d_2)^2 - \dfrac{(d_1)^2}{\cos^2\alpha}} \qquad (18)$$

Fig. 6(*a*) shows a plot of experimental data from tests on a 1 on 6 slope, indicating that ϕ is a function of the kinetic flow factor, or

$$\phi = f_1(\lambda) \qquad (19a)$$

An average curve is quite well defined for values of λ from 5 to 50. Future tests may extend the curve to higher values of the kinetic flow factor, unless a maximum of $\lambda = 50$ can be accepted as a practical limit. The curve in Fig. 6(*a*) has the straight-line equation,

$$\phi = 2.58 - 0.021\,\lambda \qquad (19b)$$

Length of the Roller, L_r.—It is convenient to express the length of the jump roller in terms of a dimensionless ratio. In treatments of the level-floor hydraulic jump, the length is commonly expressed in the ratio $\frac{L}{d_j}$, in which d_j is the difference in water-surface elevation at sections 1 and 2. For the case of the jump on a slope, however, d_j is not a suitable parameter since, as shown in Fig. 1, it varies with the jump's position on the slope. Experiments indicate that the length of the roller of the hydraulic jump in sloping channels is best defined by the ratio $\frac{L_r}{d_2}$ as a function of the kinetic flow factor, or

$$\frac{L_r}{d_2} = f_2(\lambda) \qquad (20)$$

Fig. 6(*b*) is a plot of Eq. 20 from experiments on the 1 on 6 sloping channel. An average curve is reasonably well defined.

Verification of Eq. 14, Cases 3 and 4.—To confirm the applicability of Eq. 14, all tests for which the nominal slope of the channel was 1 on 6 were classified as shown in Fig. 1. For each test classified in Case 3 or 4, the depth d_2 was computed from Eq. 14, using measured values of slope, discharge, and initial depth, with values of ϕ taken from Fig. 6(*a*). Actual slope of the channel varied from 0.170 to 0.176 for different installations. The depth at section 1, d_1, was taken as the average of a series of point-gage measurements made at the toe of the roller. Since the depth d_1 is not only difficult to measure accurately but is important to the evaluation of Eq. 14, it is frequently a source of inaccuracy in the computations. The depth at section 2, d_2, was taken for Case 3 jumps as the depth of tailwater and for Case 4 jumps as the depth at the end of the roller.

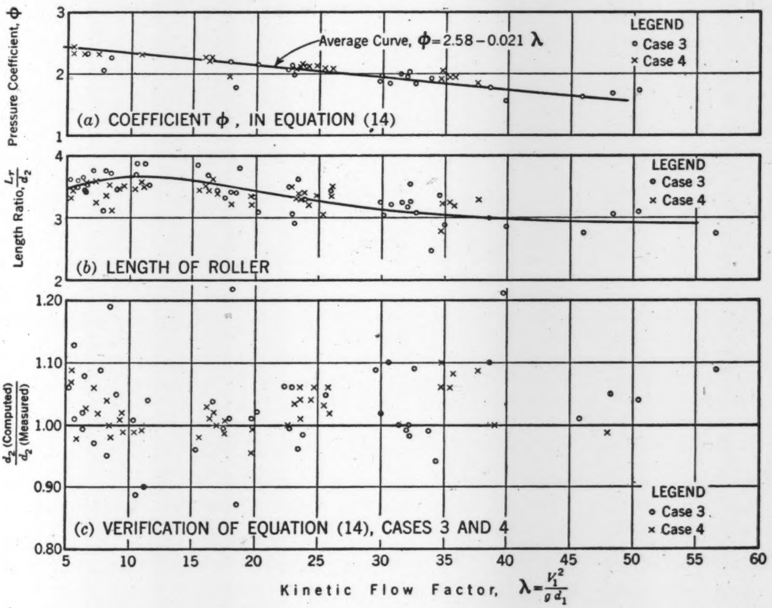


Fig. 6.—Dimensionless Relationships Derived from Experiment (Slope of Channel, 1 on 6) Eq. 14, Cases 3 and 4

A comparison for verification of Eq. 14 is shown in Fig. 6(*c*), in which the ratio of d_2-computed to d_2-measured is plotted against the kinetic flow factor. It is believed that deviations from an exact agreement between measured and computed values reflect to some extent the difficulties of observation.

Verification of Eq. 14, Case 2.—Fig. 7 shows Case 2 of the hydraulic jump which, being partly on the slope and partly on the level floor, is intermediate between cases 1 and 3. Obviously, if given values of slope, discharge, and initial depth for a Case 2 jump are evaluated in Eq. 14, the result, always exceeding the depth of tailwater, d_2, for the given case, is a depth $(d_2)'$ for an imaginary Case 3 jump. Similarly, if the same data are evaluated in Eq. 3, the result is a depth $(d_2)''$, less than d_2, for an imaginary Case 1 jump.

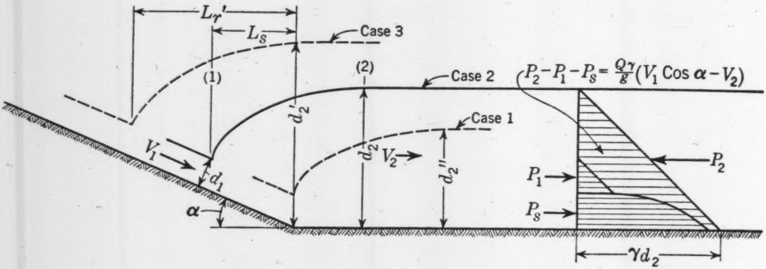


FIG. 7.—CASE 2; THE HYDRAULIC JUMP IN SLOPING CHANNELS, WITH THE TOE OF THE ROLLER ON THE SLOPE AND THE END OF THE ROLLER ON THE HORIZONTAL FLOOR

Experiments indicate that the depth of tailwater for any jump in the Case 2 classification can be expressed in an empirical relation,

$$d_2 = (d_2)' - k\left[(d_2)' - (d_2)''\right] \dots\dots\dots\dots\dots\dots (21)$$

in which k is an experimental coefficient, a function of λ and the ratio $\frac{L_s}{(L_r)'}$. The symbol L_s denotes the horizontal length from the toe of the jump to the lower end of the slope, and $(L_r)'$ is the length of the roller of the imaginary Case 3 jump from Eq. 20. Thus,

$$k = f_3\left[\frac{L_s}{(L_r)'}\right] \dots\dots\dots\dots\dots\dots\dots\dots (22)$$

Fig. 8 shows a plot of k against $\frac{L_s}{(L_r)'}$, from tests on the 1 on 6 slope. For all practical purposes, the average curve ($\lambda = 25$) is probably sufficient. Fig. 9 for Case 2 jumps in the 1 on 6 sloping channel shows a comparison of measured depths with depths computed from Eqs. 3, 14, and 21, using the average curve of Fig. 8.

Practical Application of the Analysis.—Frequently, when a large number of computations are required, solution for d_2 in Eq. 3, Case 1, can be simplified by writing the equation in dimensionless form,

$$\frac{d_2}{d_1} = \tfrac{1}{2}\left(\sqrt{8\,\lambda + 1} - 1\right) \dots\dots\dots\dots\dots\dots (23)$$

and the ratio $\frac{d_2}{d_1}$ can be plotted as a function of λ.

Similarly, for cases 3 and 4, when the slope of the channel is constant, Eq.

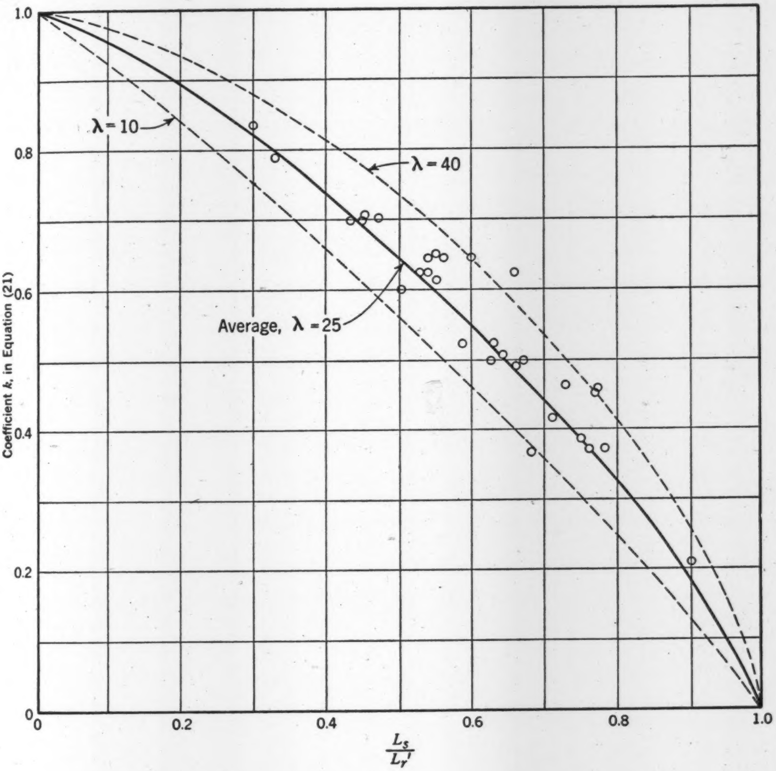


FIG. 8.—COEFFICIENT k IN EQ. 21, FOR A CASE 2 JUMP ON A 1 ON 6 SLOPE

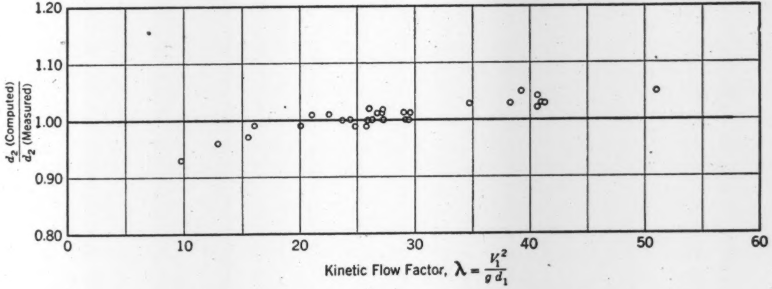


FIG. 9.—VERIFICATION OF EQ. 21, CASE 2, 1 ON 6 SLOPE

14 can be written,

$$\frac{d_2}{d_1} = \frac{1}{2 \cos \alpha} \left(\sqrt{\frac{8 \lambda \cos^3 \alpha}{1 - 2 \phi \tan \alpha} + 1} - 1 \right) \ldots\ldots\ldots\ldots (24)$$

and again, if the terms cos α and tan α are evaluated, the ratio $\frac{d_2}{d_1}$ can be plotted as a ratio of λ, using ϕ from Eq. 19*a*. Having curves of Eqs. 23 and 24 for cases 1 and 3, the solution of Eq. 21 for Case 2 is greatly facilitated.

In nearly every practical case, the location of section 1 at the toe of the jump must be assumed in first approximation for d_2, in order that d_1, λ, and ϕ can be evaluated from known conditions. If the jump falls in the Case 2 classification, the length L_s must also be assumed. Results obtained from the initial assumptions can readily be refined by successive approximations.

CONCLUSIONS

It is convenient to classify the most common forms of the hydraulic jump in open channels into several general cases based on the position of the jump in a sloping channel. Although the jump on a slope is similar in most respects to the level-floor jump, analysis of this case is dependent upon experimental coefficients to evaluate pressures on the sloping floor.

This paper presents a generalized analysis of the jump in sloping channels, leading to a practical method of computing the essential dimensions of the jump. The treatment was verified by application to experimental data on jumps in a 1 on 6 sloping channel. Dimensionless presentation of data on the 1 on 6 slope makes the results suitable for design use. It is hoped that further experimentation will make available similar data for use with steeper slopes.

ACKNOWLEDGMENTS

The analysis contained in this paper was made in the Hydraulic Laboratory Section of the Hydraulic Data Division, Tennessee Valley Authority. The work of the Hydraulic Laboratory is under the direction of A. S. Fry, M. Am. Soc. C. E., head hydraulic research engineer, and the direct supervision of G. H. Hickox, M. Am. Soc. C. E., senior hydraulic engineer.

As stated previously, the experimental data were obtained from the laboratory investigation made in 1936–1937 by Mr. Yarnell, whose notes and photographs were loaned to the TVA by the U. S. Department of Agriculture. The writer was employed as an assistant to Mr. Yarnell from August, 1936, until January, 1937.

Grateful acknowledgment is due Mr. Hickox and Sherman M. Woodward, Hon. M. Am. Soc. C. E., for consultation and advice. Many others have contributed with criticism and assistance in computations.

DISCUSSION

Joe W. Johnson,[9] Assoc. M. Am. Soc. C. E.—The problem of the hydraulic jump in a sloping channel is discussed in detail in this paper. The author has presented a formula which is applicable in most practical problems. The experimental coefficient in this formula has been determined over a fairly wide range of the kinetic flow factor.

From experiments in the hydraulic laboratory of the University of California, B. D. Rindlaub[10] developed a relationship for calculating the principal dimensions of a jump in sloping channels. The results of these experiments are of interest in connection with Mr. Kindsvater's paper.

The experiments were conducted in a glass-wall channel, 6 in. wide and 3 ft deep, in which a sloping bed could be set at various angles between 0° and 30°. The flume was equipped with an orifice for flow measurement, a forebay, a tail gate, and point gages for the determination of surface elevations. Most of the experiments were conducted with the bottom set at the angle $\alpha = 12°$; however, experiments also were made with slopes of 8°, 24°, and 30°.

In analyzing the experimental results, Mr. Rindlaub assumed the following equation to describe the jump in a sloping channel:

$$\frac{Q\,\gamma}{g}(V_1 \cos\alpha - V_2) = \frac{b\,\gamma}{2}(d^2{}_1 - d^2{}_2) + W \sin\alpha \cos\alpha \ldots\ldots\ldots (25)$$

in which W is the weight of the water in the jump. As in the case of the jump in a horizontal channel, the variables in this equation may be separated into dimensionless groups. If l represents the length along the floor from the toe of the jump to the point of change in grade $\left(l = \frac{L_s}{\cos\alpha} \text{ in Fig. 7 of the paper}\right)$, and d_1, d_2, α, and Q are as defined by the author, the following dimensionless groups may be written:

(a)..............α

(b)..............$\frac{d_2}{d_1}$

(c)..............$\lambda_1 = \frac{Q^2}{g\,d^3{}_1}$ and $\lambda_2 = \frac{Q^2}{g\,d^3{}_2}$

and

(d)..............$\frac{l}{d_1}$

The method of summarizing the experimental data by the use of these dimensionless groups is illustrated by Fig. 10. For the data taken with a bed slope of 12°, a plot of the ratio $\frac{d_2}{d_1}$ against the kinetic flow factor, λ_1, is shown for various values of $\frac{l}{d_1}$. The plotted points show a fairly consistent variation and, therefore, permit curves of constant $\frac{l}{d_1}$ to be constructed. For comparative purposes the relationship between $\frac{d_2}{d_1}$ and λ_1 for a jump in a horizontal channel also is shown in Fig. 10. Data taken on channels with slopes of 8°, 24°, and 30° gave curves similar to those in Fig. 10. In the experiments the value of l was varied between 0 and 4.11 ft, d_1 between 0.013 and 0.224 ft, d_2 between 0.25 and 1.18 ft, and Q between 0.056 and 0.670 cu ft per sec. All four of the author's classifications of the jump in sloping channels, consequently, were included in the experiments.

---

[9] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[10] "The Hydraulic Jump in Sloping Channels," by Bruce D. Rindlaub (unpublished), a thesis presented to the University of California, Berkeley, Calif., in 1935, in partial fulfilment of the requirements for the degree of Master of Science.

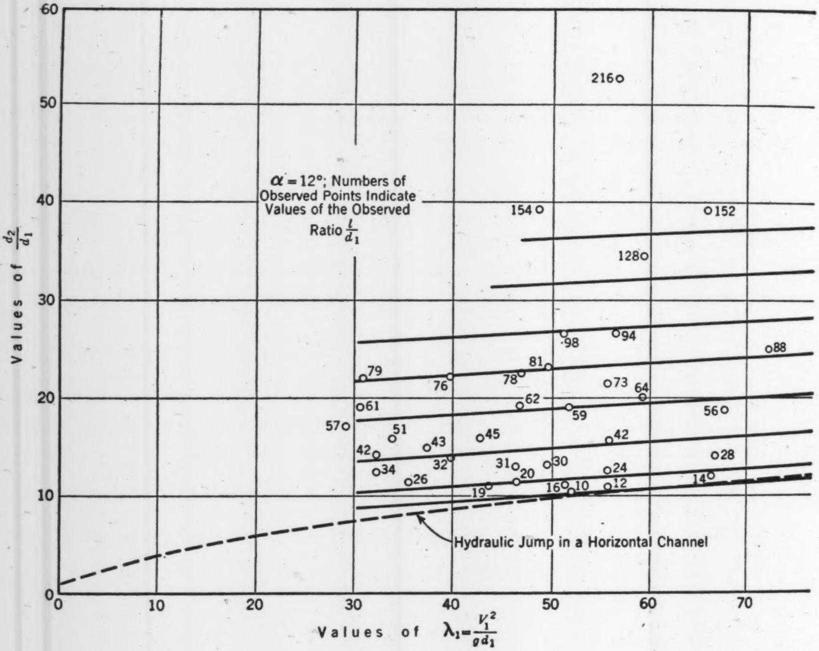
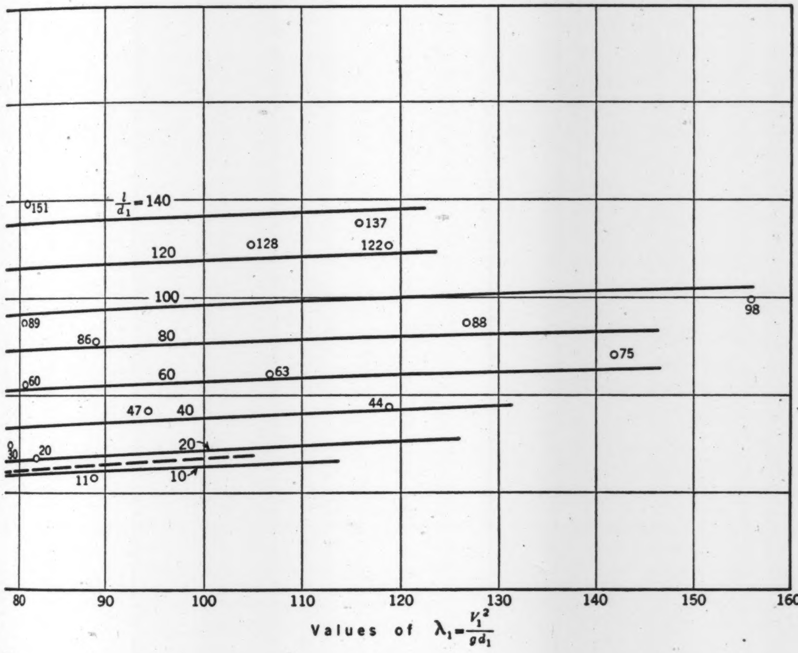


FIG.

The characteristics of a jump on a particular slope may be computed from plots similar to that illustrated in Fig. 10. Thus, by knowing the values of d_1 and Q, and by assuming a value of l, the values of $\frac{l}{d_1}$ and λ_1 can be computed. The plot of $\frac{d_2}{d_1}$ versus λ_1 for the particular slope can then be entered and the value of $\frac{d_2}{d_1}$ corresponding to the computed values of $\frac{l}{d_1}$ and λ_1 determined. Finally, the value of d_2 can be calculated from $\frac{d_2}{d_1}$.

This procedure is similar to the solution of Eq. 24 as described by the author. Mr. Kindsvater's generalized method of analysis, perhaps, is more practicable because curves of $\frac{d_2}{d_1}$ versus λ_1 (similar to Fig. 10) for the particular slope are not required. The author is to be complimented on his presentation of a simple and convenient method of computation that is suitable for most design problems.



10

Karl R. Kennison,[11] M. Am. Soc. C. E.—The author has done an excellent job in summarizing the available data in usable form. The subject of the paper seems to be somewhat of a misnomer, since it does not deal with continuous sloping channels, referred to as "a fifth case," but rather with the extent to which a sloping jet striking a horizontal apron can be backed up the slope. The results are very much worth while, particularly because they contain empirical factors resulting from important experimental work.

---

[11] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

The computations involved in the theoretical deductions of the formulas seem to be more complicated than warranted by the actual uncertainties in flows of this type. For example, these problems generally deal with a condition where the neglected friction loss is an important item. In the case of any formula for the jump in sloping channels, it should be borne in mind that this fact is certain to vitiate the results, particularly in the case of low Froude numbers.

The writer believes that the formula for the hydraulic jump in a sloping channel (made general to include the angle of slope, if any, of the channel bottom) can be simplified, both in the derivation and in the result, by equating the momenta before and after the jump, considering the forces parallel to the channel bottom, and by expressing the height of the jump as a function of its length. Referring to Fig. 11, the momentum equation is as follows, using the author's nomenclature:

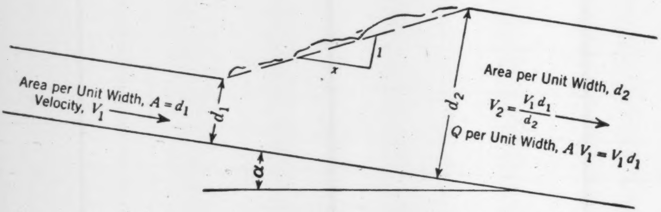

Area per Unit Width, $A = d_1$
Velocity, V_1
d_1
α
1
x
d_2
Area per Unit Width, d_2
$V_2 = \frac{V_1 d_1}{d_2}$
Q per Unit Width, $A V_1 = V_1 d_1$

FIG. 11

$$\frac{Q \gamma V_1}{g} - \frac{Q \gamma V_2}{g} = \frac{\gamma d^2{}_2 \cos \alpha}{2} - \frac{\gamma d^2{}_1 \cos \alpha}{2}$$

$$- x (d_2 - d_1) \left(\frac{d_2 + d_1}{2}\right) \gamma \sin \alpha \ldots\ldots\ldots\ldots\ldots (26)$$

This becomes

$$d_2 = \frac{d_1}{2} \left(\sqrt{\frac{8 \lambda}{\cos \alpha - x \sin \alpha} + 1} - 1 \right) \ldots\ldots\ldots\ldots (27a)$$

Instead of expressing the depth after the jump as the height normal to the bottom of the slope, consider the author's case of impinging on a horizontal apron and take d_2 vertical; then Eq. 26 becomes

$$d_2 = \frac{d_1}{2 \cos \alpha} \left(\sqrt{\frac{8 \lambda}{\cos \alpha - x \sin \alpha} + 1} - 1 \right) \ldots\ldots\ldots (27b)$$

which is to be compared with Eq. 14.

It is sufficiently accurate for practical purposes to assume that the length of the jump is three times its height, or that $x = 3$. Actually this simpler formula with $x = 3$, will give the same result as the author's, for a kinetic flow factor, λ, in the vicinity of 50. The choice of the value to assign to the multiplier, x, makes surprisingly little difference in the result. Using $x = 2$ gives a resulting value of d_2 which is 97% of that using $x = 3$, for a slope of 0.05; and 93%, for a slope of 0.10.

J. C. **Stevens**,[12] M. Am. Soc. C. E.—This paper touches a phase of the hydraulic jump about which comparatively little is known, and the author deserves the thanks of the profession for presenting an analysis of the data obtained by Mr. Yarnell. This analysis brings out a few anomalies for which there is no ready explanation. Perhaps it would be well to agree on a definition of the hydraulic jump before speculating too much about it.

The beginning of the jump is definite and well defined, but the end is open to conjecture. On a horizontal bed the end of the jump has been taken very properly as the section where the water surface reaches its maximum height[13] —in other words, where all the kinetic energy that is susceptible of conversion to potential energy has been so converted and before the surface profile is affected by the friction gradient in the channel below the jump. This end of the jump is not the end of the roller, as the author has considered it.

To define the length of the roller as the length of the jump, even on a sloping bed, slurs over the dominant characteristic of the jump, which is to convert kinetic to potential energy by highly concentrated turbulence.

The author's four cases are purely arbitrary, and scarcely can be considered as basic. With fluctuating tailwater or variable flow, the jump will oscillate through all four cases. The first three are applicable only where a slope merges into a horizontal bed.

Madden Dam was designed with an apron inclined downward into tailwater at a slope of 1 on 4.[14] The apparent theory was that, as the high-velocity water flows down the slope, it will encounter the proper tailwater level to produce the jump, and thus prevent undue scouring of the river bed. Actual experience has not substantiated that theory even though model tests seemed to be favorable to it. On Madden Dam, the upward sloping continuous baffle at the end of the apron has counteracted the effect of the 1 on 4 slope.

Certain it is that the model tests of a downward sloping apron for Bonneville Dam indicated that such an apron would be the worst that could be built as far as scour is concerned.[15]

General Formulas for the Hydraulic Jump in Rectangular Channels.—Some tentative formulas are presented herein for the solution of the hydraulic jump on level beds and on beds sloping both downward (negative) and upward (positive) in the direction of flow. Fig. 12 shows the elements of an hydraulic jump (a) for a negative slope and (b) for a positive slope. In each case the momentum relationships for a rectangular channel or for a unit width of a wide channel are: For a negative slope (downward in the direction of flow)—

---

[12] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[13] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 640.

[14] "Hydraulic Tests of the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *ibid.*, Vol. 103 (1938), p. 1080.

[15] *Ibid.*, p. 1117.

$$\frac{Q}{g} V_1 + \frac{d^2_1}{2} \cos \alpha + W \sin \alpha = \frac{Q}{g} V_2 + \frac{d^2_2}{2} \cos \alpha \ldots\ldots\ldots (28a)$$

and, for a positive slope—

$$\frac{Q}{g} V_1 + \frac{d^2_1}{2} \cos \alpha = \frac{Q}{g} V_2 + \frac{d^2_2}{2} \cos \alpha + W \sin \alpha \ldots\ldots\ldots (28b)$$

in which, in addition to the notation of the paper, W is the total weight of water in the jump prism between the two depths d_1 and d_2.

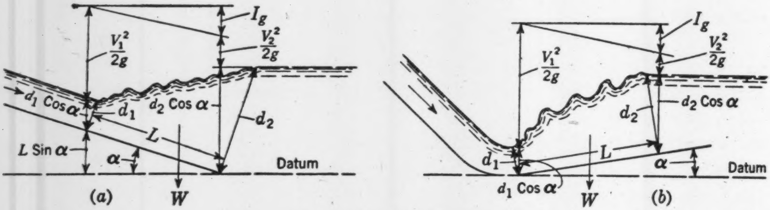


FIG. 12.—ELEMENTS OF AN HYDRAULIC JUMP

The length of the jump is quite arbitary except that it should not be assumed too short. The length should be chosen so that the energy conversion would be completed as indexed by the maximum height of the water surface. If a greater length is taken, the height of the jump is penalized by a slight drop due to friction. However, to include the force of friction for the relatively short length in which the jump develops is usually an unnecessary refinement.

The writer has adopted the length of jump given by Andréi Ivanchenko,[16] M. Am. Soc. C. E., in his discussion of the paper by Messrs. Bakhmeteff and Matzke. His formula, based on the lengths given in the paper for jumps on a level bed, is

$$L = 10.6\, \lambda^{-0.185} (d_2 - d_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

in which L = length of jump.

Weight of the Jump.—The weight of the jump prism has the force component $W \sin \alpha$ acting in the direction of flow for negative slopes and counter to the direction of flow for positive slopes. On a level bed this force disappears.

Any other than the simplest delineation of the jump boundaries seems an unwarranted refinement, therefore the body of the jump has been considered as a trapezoid bounded by the stream bed, a straight-line surface profile, and the two depths d_1 and d_2. Neglecting the curvature of the surface profile involves certain compensating consideration, such as

(1) The specific weight of the water is reduced by the entrained air;

(2) The water is being decelerated, which reduces the horizontal component of the weight on a negative slope and increases it on a positive slope;

(3) Deceleration of the vertical component tends to increase the weight on either slope; and

---

[16] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 668.

(4) In a strict sense, hydrostatic pressure within the jump does not exist although pressure measurements on the bed indicate that static pressures are closely approximated.

Considering all these factors, the effects of which are virtually unknown (or at best may be determined only by refined experimentation), to take into account the curvature of the surface profile seems hardly justified. However, if found to be desirable, the numerical factor in Eq. 29 may be increased to compensate completely for thus neglecting curvature.

The weight of the body of the jump for a unit width therefore becomes

$$W = 10.6\,\lambda^{-0.185}\,\tfrac{1}{2}\,(d^2_2 - d^2_1) \qquad (30)$$

Kineticity of the Jump.—There seems to be no particular virtue in tying the "kinetic flow factor" $\left(= \frac{V^2}{g\,d_1}\right)$ to the Froude number. The writer prefers to use half that factor, $k = \frac{V^2}{2\,g\,d_1}$, which has real physical meaning since it is the ratio of the kinetic to the potential energy at the beginning of the jump and therefore well may be called its "kineticity."

For any given bed slope, k is the only independent dimensionless parameter of the jump. It is measured easily and, once known, all other characteristics are derived readily from it. The jump characteristics on a level bed already have been outlined by the writer.[17] The kineticity k will be used in place of λ in what follows. Since its value is $\frac{\lambda}{2}$, Eq. 30 becomes

$$W = \frac{9.3}{k^{0.185}}\,\frac{d^2_2 - d^2_1}{2} \qquad (31)$$

Probably, the numerical factor in Eq. 31 should be increased for negative slopes and reduced for positive slopes. Only future experimentation can determine that. However, for this discussion, no change was made from that indicated for level beds.

General Formula.—Substituting Eq. 31 in Eqs. 28 and transposing,

$$\frac{Q}{g}\,(V_1 - V_2) = \tfrac{1}{2}\,(d^2_2 - d^2_1)\,(\cos\alpha \pm m \sin\alpha) \qquad (32)$$

in which for brevity m (a coefficient for the length of the jump) is written for $\frac{9.3}{k^{0.185}}$. In Eq. 32 the negative sign is for negative slopes (downward) and the positive sign for positive slopes (upward) in the direction of flow.

It will be simpler if each jump characteristic were to be put in dimensionless terms by expressing each as a ratio in which the denominator is the initial potential energy. Thus, let

$$k = \frac{V^2_1}{2\,g\,d_1 \cos\alpha} \qquad (33a)$$

and, the dimensionless height of the jump,

$$J = \frac{d_2}{d_1 \cos\alpha} \qquad (33b)$$

---

[17] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 660.

Substituting these in Eq. 32 and invoking the continuity relations $Q = V_1 d_1 = V_2 d_2$, Eq. 32 reduces to

$$J\,(J \cos \alpha + 1) = \frac{4\,k}{\cos \alpha \pm m \sin \alpha} \quad \text{.............. (34)}$$

For a level bed, Eq. 34 reduces to

$$J\,(J + 1) = 4\,k \quad \text{...................... (35a)}$$

and, from Eq. 34, the final depth is given by

$$J = \frac{1}{2 \cos \alpha}\left(-1 + \sqrt{\frac{16\,k}{1 \pm m \tan \alpha} + 1}\right) \quad \text{.......... (35b)}$$

which may be compared with Eq. 14.

Eq. 35*b* is a general formula for rectangular channels in which the negative sign is for negative slopes and the positive sign for positive slopes. When the slope is zero, it reduces to,

$$J = \tfrac{1}{2}\,(-1 + \sqrt{16\,k + 1}) \quad \text{.................. (35c)}$$

TABLE 1.—LIMITING VALUES FOR JUMPS ON SLOPING BEDS

No.	Description	TANGENT α					
		0.10	0.125	0.167	0.200	0.225	0.250
1	Final depth, J, is infinite when k =...........	0.85	2.25	10.96	28.5	54.2	98.9
2	Height of jump, J, is minimum when k =.....	1.70	5.59	26.7	70.6	134	219
3	Minimum height of jump, J =...............	6.14	11.6	26.1	43.3	60.5	80.5

Eq. 35*b* shows that the hydraulic jump on negative sloping beds has certain very definite limitations. The final depth becomes infinite for $m \tan \alpha = 1$. The values of k for which this occurs are found by substituting for m its value and solving for k (see Table 1, Item 1).

For larger values of k the depths diminish rapidly to some stable point. This point may be the minimum value of J, in either Eq. 34 or Eq. 35*b*, or an inflection point for a somewhat larger value of k. If either Eq. 34 or Eq. 35*b* is differentiated for negative slopes, the minimum values of J are found for

$$k = (11.0 \tan \alpha)^{5.4} \quad \text{...................... (36)}$$

and, for the foregoing negative slopes, the values given in Table 1, Items 2 and 3, apply.

The significance of Table 1 is that, for the slopes given, the jumps are wholly fictitious for values of k less than the limitations outlined therein. Fig. 13(*a*) shows graphs of Eq. 35*b* for both negative and positive slopes. Values of J were computed from assigned values of k. The minimum values of the curves for negative slopes are indicated by circles which have been connected by a curve.

It is likely that the zone of possible jumps on these curves is defined by the points of inflection rather than by the points of minimum values. A second line therefore has been drawn connecting the points of inflection (determined graphically). Between the two curves is a zone of doubt, but to the left of the minimum-values curve the jumps are fictitious.

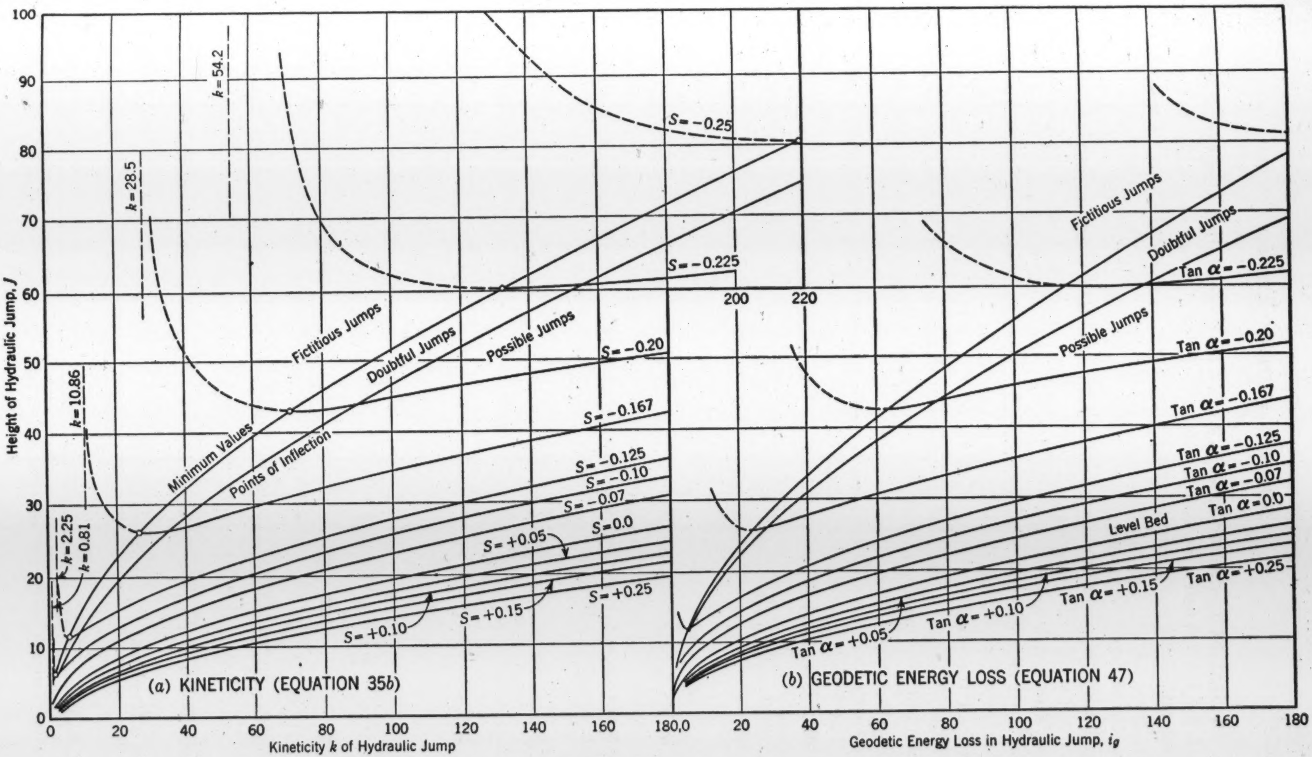


FIG. 13.—CURVES OF HYDRAULIC JUMP EQUATIONS

Analogous limitations exist in Eq. 14 from which it is seen that d_2 becomes infinite for $2\,\phi \tan \alpha = 1$ or for $(5.16 - 0.042\,\lambda) \tan \alpha = 1$ (Fig. 6(*a*)). This limitation will not apply to the 1 on 6 slope for which the author's formula is designed.

From the foregoing analysis it would appear that what have been called hydraulic jumps on inclined beds of 1 on 6 or steeper, projecting into tailwater, are not hydraulic jumps at all but a mere plunging of water into a pool. If a jet from a fire hose be directed into a pond, considerable turbulence results, but an hydraulic jump is not formed. This anomaly was noted by Messrs. Bakhmeteff and Matzke in their experiments.[3] The apparatus used by them limited the steepest slope to $S_0 = 0.07$, yet they state (speaking of steeper slopes), "The live jet 'plunges' into the tail water and within the steep section follows the slope downward with comparatively slow expansion and obviously relatively small losses."

Energy Losses.—There are two kinds of energy losses in hydraulic jumps on inclined beds: (1) Specific energy losses (that is, losses with reference to the stream bed); and (2) geodetic energy losses that are referenced to a horizontal datum.

The specific energy equation for either negative or positive slopes is

$$d_1 \cos \alpha + \frac{V^2_1}{2\,g} = d_2 \cos \alpha + \frac{V^2_2}{2\,g} + I_s \ldots\ldots\ldots\ldots (37)$$

in which I_s is the specific energy loss. (Coefficients,[18] strictly speaking, should be applied to the momentum flux and to the kinetic energy to obtain the mean momentum and energy of a unit weight of water. However, at this stage such refinements are hardly warranted.) Eq. 37 reduces to

$$\frac{V^2_1}{2\,g} = \frac{d^2_2}{d_2 + d_1} \cos \alpha + \frac{d^2_2}{d^2_2 - d^2_1} I_s \ldots\ldots\ldots\ldots (38)$$

The specific energy loss I_s is also to be put in terms of the initial potential energy—

$$i_s = \frac{I_s}{d_1 \cos \alpha} \ldots\ldots\ldots\ldots (39a)$$

Substituting Eqs. 33 and 39*a* in Eq. 38:

$$k = \frac{J^2 \cos^2 \alpha}{J \cos \alpha + 1} + i_s \frac{J^2 \cos^2 \alpha}{J^2 \cos^2 \alpha - 1} \ldots\ldots\ldots\ldots (39b)$$

The specific energy loss is then obtained by eliminating k between Eqs. 34 and 39*b* and solving for i_s:

$$i_s = \frac{(J \cos \alpha - 1)^3}{4\,J \cos \alpha} \pm \frac{(J^2 \cos^2 \alpha - 1)\,(J \cos \alpha + 1)}{4\,J \cos \alpha} \tan \alpha \ldots\ldots (40)$$

For a level bed this reduces to

$$i = \frac{(J - 1)^3}{4\,J} \ldots\ldots\ldots\ldots (41)$$

---

[18] "Applied Fluid Mechanics," by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1937.

—a formula developed by writer in 1925.[19,20] Eq. 40 gives the energy loss with reference to the stream bed. A more useful formula is one that gives the losses with reference to a horizontal datum plane which will be styled the "geodetic energy loss." Let the plane be one passing through the foot of d_2 for negative slopes and d_1 for positive slopes, as shown in Fig. 12. The geodetic energy equation then becomes, for negative slopes,

$$L \sin \alpha + d_1 \cos \alpha + \frac{V^2_1}{2\,g} = d_2 \cos \alpha + \frac{V^2_2}{2\,g} + I_g \tag{42a}$$

and, for positive slopes,

$$d_1 \cos \alpha + \frac{V^2_1}{2\,g} = d_2 \cos \alpha + \frac{V^2_2}{2\,g} + L \sin \alpha + I_g \tag{42b}$$

in which I_g = the geodetic energy loss. Eq. 29 in terms of k becomes

$$L = 9.3\,k^{-0.185}\,(d_2 - d_1) = m\,(d_2 - d_1) \tag{43}$$

and Eq. 39a for geodetic losses becomes

$$i_g = \frac{I_g}{d_1 \cos \alpha} \tag{44}$$

Substituting Eq. 43 in Eqs. 42:

$$\frac{1}{2\,g}\,(V^2_1 - V^2_2) = (d_2 - d_1) \cos \alpha \pm m\,(d_2 - d_1) \sin \alpha + I_g \tag{45a}$$

and

$$\frac{V^2_1}{2\,g} = \frac{d^2_2}{d_2 + d_1}\,(\cos \alpha \pm m \sin \alpha) + \frac{d^2_2}{d^2_2 - d^2_1}\,I_g \tag{45b}$$

Substitute Eqs. 33 and 44 in Eq. 45b and solve for k, thus:

$$k = \frac{J^2 \cos^2 \alpha}{J \cos \alpha + 1}\,(1 \pm m \tan \alpha) + i_g\,\frac{J^2 \cos^2 \alpha}{J^2 \cos^2 \alpha - 1} \tag{46}$$

The geodetic energy loss is then found by eliminating k between Eqs. 46 and 34 and solving for i_g, thus:

$$i_g = \frac{(J \cos \alpha - 1)^3}{4\,J \cos \alpha}\,(1 \pm m \tan \alpha) \tag{47}$$

For $\alpha = 0$, Eq. 47 reduces to Eq. 41.

Fig. 13(b) shows graphs of Eq. 47 in which the geodetic energy loss is plotted against values of J. The same zones of fictitious, doubtful, and possible jumps are evidenced in these graphs as in those of Fig. 13(a). For any given value of k, the height of jump J may be found from Fig. 13(a); then, entering Fig. 13(b) with this value of J, the geodetic energy loss as a multiple of the initial specific potential energy is readily found.

Perhaps a better picture of the energy losses in the hydraulic jump is obtained by expressing them as percentages of the initial energy. Only the percentage geodetic energy losses will be presented. From Fig. 12(a) for

---

[19] "Determining the Energy Lost in the Hydraulic Jump," by J. C. Stevens, *Engineering News-Record*, June 4, 1925, p. 929.

[20] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 661.

negative slopes,

$$\%\ \text{loss} = \frac{100\ I_g}{L \sin \alpha + d_1 \cos \alpha + \frac{V^2_1}{2\ g}} \quad \text{(48a)}$$

Substituting Eqs. 33*a* and 44,

$$\%\ \text{loss} = \frac{100\ i_g}{l \sin \alpha + k + 1} \quad \text{(48b)}$$

in which $l = \frac{L}{d_1 \cos \alpha}$, the length of the jump as a multiple of the initial specific potential energy.

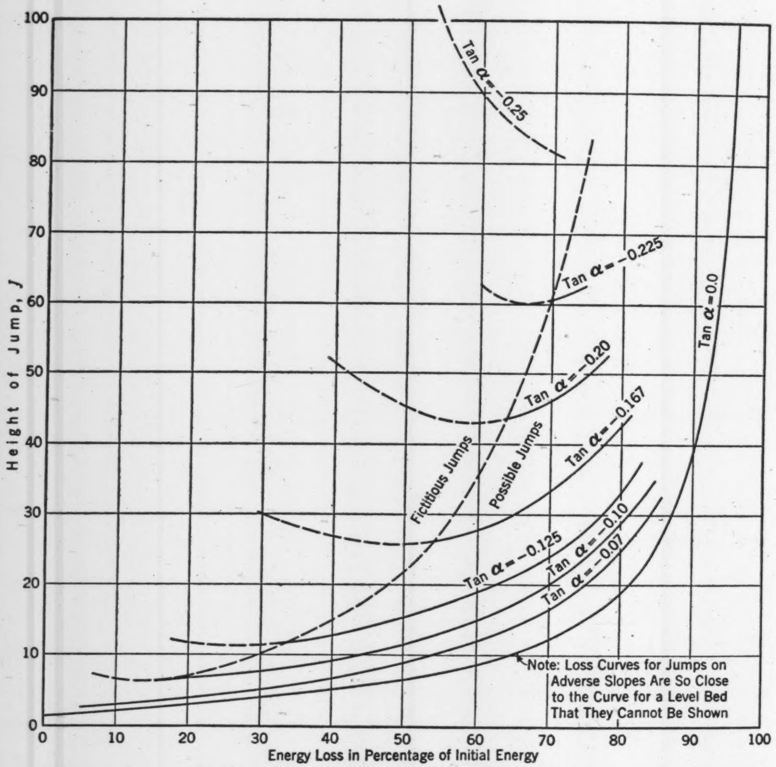


FIG. 14.—ENERGY LOSSES IN TERMS OF J (EQ. 48b)

Fig. 14 shows the losses computed from Eq. 48*b*, plotted to corresponding values of J. The loss curves for positive slopes are so near the curve for a level bed that they cannot be shown to the scale of this graph.

It is interesting to note that the percentage loss for any given height of jump increases as the negative slope diminishes; also that the zone of fictitious jumps is carried over from Fig. 13(*b*).

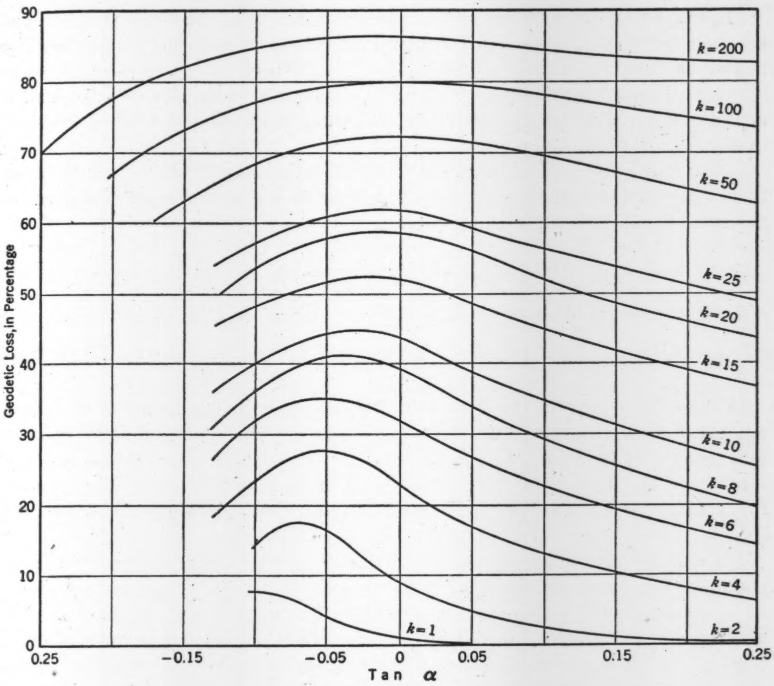


FIG. 15.—VARIATION OF LOSSES IN TERMS OF SLOPE AND KINETICITY

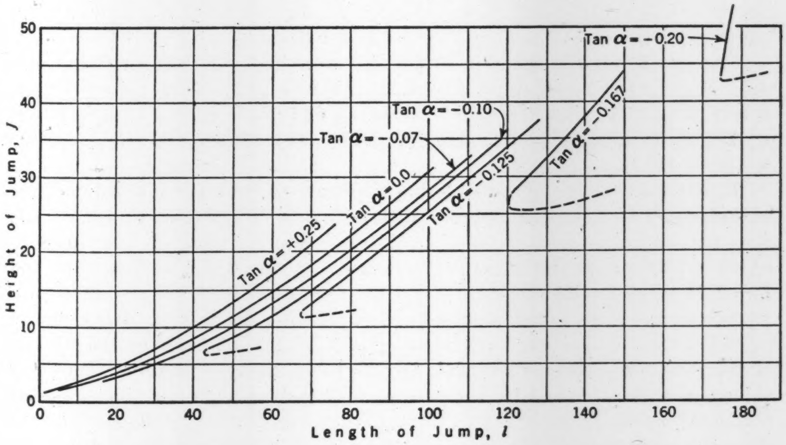


FIG. 16.—RELATION BETWEEN HEIGHT AND LENGTH OF JUMP

In order to show how the percentage loss varies with the slope and with the kineticity, Fig. 15 has been prepared. Loss (in percentages) is plotted against bed slope for given values of the kineticity k. Only values from the zone of "possible jumps" are plotted in Fig. 15. It appears from these curves that a negative slope of about 0.05 (1 on 20) is slightly more efficient than a level bed in dissipating the initial energy.

Length of Jump.—The length of the hydraulic jump for this discussion has been taken as indicated in Eq. 43. This may be put in dimensionless terms in the same manner as other jump characteristics. Thus, let

$$l = \frac{L}{d_1 \cos \alpha} \quad \text{(49a)}$$

Substituting Eqs. 33*b* and 49*a* in Eq. 43,

$$l = m \left(J - \frac{1}{\cos \alpha} \right) = 9.3\, k^{-0.185} \left(J - \frac{1}{\cos \alpha} \right) \quad \text{(49b)}$$

Curves for Eq. 49*b* are shown in Fig. 16 wherein the length l is plotted against J for various slopes. For negative slopes, the zone of fictitious jumps and lengths is evident. The lengths for positive slopes are so close to those for a level bed that only the curve for a 1 on 4 slope is shown. The lengths for a negative 1 on 4 slope could not be shown within the compass of the drawing.

Illustrative Examples.—In order to clarify the use of the curves presented, the following examples are offered:

Example 1.—On a level apron at the foot of a dam, the depth at the beginning of a jump is 1.25 ft and the velocity, 75 ft per sec. The velocity head is 87.0 ft; hence, from Eq. 33*a*, the kineticity $k = 69.5$. From Fig. 13(*a*), $J = 16.2$ and $d_2 = 16.2 \times 1.25 = 20.2$ ft. From Fig. 13(*b*), the energy loss $i_g = 54.5$; the head lost is $I_g = 1.25 \times 54.5 = 68.2$ ft; from Fig. 14, this loss is seen to be 77% of the initial geodetic energy. From Fig. 16, the dimensionless length of jump $l = 64.5$, whereas the actual length is $L = 1.25 \times 64.5 = 81$ ft.

Example 2.—If the apron in Example 1 were inclined downward into tailwater at a slope of 1 on 6, $d_1 \cos \alpha = 1.23$, and: $k = 70.5$; J (Fig. 13(*a*)) $= 31.1$; $d_2 = 1.23 \times 31.1 = 38.3$ ft; i_g (Fig. 13(*b*)) $= 62.0$; $I_g = 1.23 \times 62.0 = 76.4$ ft or 67% of the initial geodetic energy (Fig. 14); l (Fig. 16) $= 128$; and $L = 1.23 \times 128 = 157$ ft.

Example 3.—If the apron were inclined upward from the low point of the bucket on a 1 on 4 slope, $d_1 \cos \alpha = 1.21$; and: $k = 71.9$; J (Fig. 13(*a*)) $= 11.7$; $d_2 = 1.21 \times 11.7 = 14.3$ ft; i_g (Fig. 13(*b*)) $= 51$; $I_g = 1.21 \times 51 = 61.8$ ft, which is 69% of the initial geodetic energy (Fig. 14); l (Fig. 16) $= 45.5$; and $L = 1.21 \times 45.5 = 55$ ft.

Summary.—The writer would question (1) the conclusion of the author regarding the "convenience" afforded by his classification based on the position of the jump on a sloping channel, for the jump changes position with every change in kineticity; and (2) the statement that the jump on a slope is similar to the level-floor jump. The similarity only extends to mild slopes. As the slope steepens, the jump loses its identity entirely and becomes something else about which little is known. It is obvious from the author's four cases that he had in mind jumps at the toe of overflow dams, at which slopes obtain that are far steeper than any herein considered.

The writer's analysis is admittedly theoretical, but his formulas do have simplicity to recommend them. The existing experimental data apply to such mild slopes that the behavior of the jump on steep beds as outlined in this discussion is subject to experimental verification. As far as the writer knows, there are no experiments with jumps on adverse slopes.

The author's generalized formulas are based on experimental data obtained on only one slope. The data in the Yarnell experiments were not presented in usable form and the length of the jump used by the author, which is a vital factor on sloping beds, was greatly foreshortened. The soundness of the formulas may therefore be open to some question until verified by experiments on other and steeper slopes.

C. J. POSEY,[21] ASSOC. M. AM. SOC. C. E.—The mathematical analysis of the hydraulic jump on a sloping floor, presented in this paper, parallels logically the well-established momentum analysis of the jump on a horizontal floor. The final result is not obtained by purely analytical means, as is that for the jump on a horizontal floor, because of the effect of the unknown weight of the water in the jump itself, a major factor which cannot be eliminated from the momentum equation. The author's method of introducing this factor into the equation seems to be as rational as can be used and still permit a simple algebraic solution. Evaluation of the empirical function necessary is based on the large-scale experiments made by Mr. Yarnell.

The author's classification of the jump on a sloping floor into four cases is a convenience in studying the experimental data, but it should be noted that his subsequent analysis of the data shows that only three different classifications really may be necessary. For a floor with 1 on 6 slope, the data for Case 3 and Case 4 fit the same equation with no significant difference in average coefficients, as shown in Fig. 6. The points showing the maximum positive and negative deviations in Fig. 6(*c*) are all for Case 3, however, a fact which may indicate a slight difference between the two cases on the basis of variability. If further investigation shows no measurable differences, these two cases can be considered to be but one—a hydraulic jump on a sloping floor. Case 2 is a hydraulic jump forming across or "straddling" a change of grade from a steep slope to a horizontal or nearly horizontal slope. Case 1, as the author states, is the ordinary hydraulic jump on a level floor.

It seems likely that for the steeper slopes the phenomenon may change entirely, with the jet plunging under with little localized impact and continuing far downstream as a thread of high-velocity current. This happens to a jump below a sluice gate if the tailwater is raised so high that the jump is "drowned out." (Below a gate in a horizontal channel, an intermediate phenomenon known as the submerged jump can form; but it does not form over a very wide range of tailwater depths, and the sharply rising water surface over the submerged jump indicates considerable localized impact and energy loss.) A drowned-out jump cannot be classed as a true jump since it has lost its ability to lower velocities safely within a short distance along the stream. Therefore, it is important to know at what Froude number the jump on any given slope tends to plunge and become similar to a drowned-out jump. Both Froude number and slope evidently are involved because, if the slope is near zero, there seems to be no limit to the Froude number for which a true jump will form; whereas, if the slope is near infinity, the jet will plunge, no matter how low the Froude number. It may be that other factors also are involved, for air entrainment may play a vital part in the energy-destroying and velocity-evening capacity of the true jump. Certainly the change of grade which was a complicating feature of the Yarnell experiments would have an effect. Many investigators have observed that for particular setups there is an apron slope above which true jump action ceases. In the report on hydraulic tests referred to by the author, for example, occurs the statement, "The slope of 1 on 4 * * * is about the steepest that can be used and still obtain a satisfactory jump formation." [22]

---

[21] Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa, and Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

The Yarnell data include a large number of jumps straddling the change of grade. Although the author presents a method of computing the height of such jumps if the floor slope is 1 on 6, he gives no method for the steeper slopes because the data did not include enough jumps that were complete upstream from the change in grade for these steeper slopes. According to his method, coefficients for jumps complete on the slope must be determined before the heights of straddling jumps can be estimated. It may be possible to devise a more direct method of analysis that could be used to summarize Mr. Yarnell's data on straddling jumps. Such a method would have to make a clean break away from the type of analysis that has been successful with the jump on a level floor. For practical use in engineering design and construction, the final information desired is whether, in a given situation, the jump will form, and if it will form, where its location will be. For a nearly horizontal channel, this information can be obtained if, in addition to the upstream and downstream flow conditions, the height and length of the jump are known. For flat slopes, close to normal friction slopes, the height is the more important element, although J. W. Trahern, Assoc. M. Am. Soc. C. E., has shown that the length also must be considered in determining where the jump will form.[23] The importance of the height and the remarkably close agreement between theory and data, for jumps in channels that may be considered horizontal, naturally suggest that the height should be the first element for investigation in the case of the jump on a sloping floor. The steeper the slope, however, the less important is the height, for the more certain it is that the jump will form a short distance downstream from the intersection of the tailwater surface profile and the profile of the high-velocity flow on the slope. The information needed for design purposes is the location of the downstream end of the jump, where velocities have become sufficiently low to permit ending the apron. This point can be fixed by supplying its distance from the point of intersection of the upstream and downstream water-surface profiles, a distance that can be determined experimentally as a function of the Froude number and the profile of the channel bottom.

---

[22] "Hydraulic Tests on the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1090.

[23] "Location of the Hydraulic Jump," by J. W. Trahern, *Western Construction News and Highways Builder*, October 25, 1932, p. 608.

This method of attack would not be satisfactory for slopes so flat that the distance from the intersection of the headwater and tailwater profiles to the toe of the jump was large in comparison with the length of the jump but not flat enough to permit neglecting the horizontal component of the bottom pressure under the jump profile. The 1 on 6 slopes for which experimental coefficients are obtained by the author are apparently in this range. Even here, however, there may be an advantage in discarding the traditional "height-of-jump" approach. Using the author's coefficients for a 1 on 6 slope, Eq. 14 becomes

$$d_2 = \frac{d_1}{1.972}\left(\sqrt{\frac{7.67\,\lambda}{1 - \frac{\phi}{3}} + 1} - 1\right) \ldots\ldots\ldots\ldots\ldots (50)$$

The points determining the experimental coefficient ϕ, plotted in Fig. 6(*a*), seem to lie very close to the average curve. In the range $\lambda = 5$ to $\lambda = 10$, however, Fig. 6(*a*) shows a range of uncertainty of ϕ between 2.05 to 2.50. Substitution of the appropriate values in Eq. 50 shows that the corresponding uncertainty in the ratio $\frac{d_2}{d_1}$ is about 40%. Near $\lambda = 40$, ϕ may vary between 1.6 and 1.95, according to the data plotted in Fig. 6(*a*), giving a possible range of values for $\frac{d_2}{d_1}$ of from 12.5 to 14.5, a variation of about 16%. Moreover, the designer is not finished when he has found d_2 because d_2 is measured upward from a point on the steeply sloping bottom. The location of this point, at the downstream end of the jump, is also uncertain. According to the data in Fig. 6(*b*), $\frac{L_r}{d_2}$ may vary from 3.2 to 3.8 in the range $\lambda = 5$ to $\lambda = 10$, or from 2.4 to 3.4 near $\lambda = 40$. The corresponding uncertainty in water-surface elevation, from this source, is about 0.10 d_2 in the former case and about 0.16 d_2 in the latter. As far as can be ascertained from Figs. 6(*a*) and (*b*), these uncertainties are in addition to, and independent of, those due to possible variations in ϕ. In combination, they amount to a possible maximum of from 30% to 50% of d_2. Fig. 6(*c*) shows the actual total range for the data plotted to be about 36% of d_2. This uncertainty becomes especially significant when it is realized that, for a jump in a sloping flume, d_2 is the largest vertical dimension.

In view of the fact that the phenomenon has so much inherent variability, would it not be better to use a simpler method for representing the experimental results? The following relationships could be plotted so as to show the variation of the experimental values and give directly, without the necessity of substitution into intricate formulas, the information needed by the designer:

$$\frac{d_j}{d_1} = f_1\,(\lambda, \alpha) \ldots\ldots\ldots\ldots\ldots (51a)$$

and

$$\frac{L_r}{d_1} = f_2\,(\lambda, \alpha) \ldots\ldots\ldots\ldots\ldots (51b)$$

In these equations the variables are the same as those shown in Figs. 2 and 5.

It is suggested that the height and length of the jump be given in terms of d_1 rather than d_2 because d_2 is purely a dependent variable with little or no real utility in the case of the jump on a steeply sloping floor.

With regard to the author's use of λ, the "kinetic flow factor," it might be well to note that a factor of similar importance but of greater usefulness in practical hydraulic work is the ratio of the velocity head to the depth. It has much older standing in American literature than either the kinetic flow factor or the Froude number. Representing this ratio by ω, the equation of the hydraulic jump in a horizontal channel of rectangular cross section becomes

$$\frac{d_2}{d_1} = 2\sqrt{\omega + \frac{1}{16}} - \frac{1}{2} \qquad \text{(52)}$$

The factor ω could be used advantageously instead of λ in all of the equations, including Eqs. 51. Unfortunately, ω has no short convenient name. A state of confusion (apparent at least to the beginner) exists when hydraulic engineers refer to the Froude number and then plot charts in illustration showing values of λ, which is not numerically equal to the Froude number. Since ω is more convenient for practical work, why not use it, giving it some simple name such as "velocity-head ratio"? The writer knows of no other technical use of this phrase, and its meaning is at least as evident as is that of "kinetic flow factor" or "Froude number."

JEROME FEE,[24] ASSOC. M. AM. SOC. C. E.—Few investigations of any kind have been made of flow in open channels with steep slopes. Therefore, the author has performed a valuable service to the engineering profession by his study of the hydraulic jump in sloping channels. By further experiments the method easily can be extended to cover a wide range of conditions.

Fig. 6, showing the relation between the kinetic flow factor and several essential elements of the hydraulic jump, is particularly interesting. The pressure-coefficient curve is defined with surprising accuracy; and even the length of roller is more amenable to measurement than might be expected.

In view of the greater accuracy of the pressure-coefficient curve, it would appear desirable to obtain similar curves for other slopes by direct piezometric measurements, as was done in this case, rather than to derive values of the pressure coefficient from Eq. 18, which depends directly on the length of roller.

Although the momentum law is used instead of energy considerations to determine the dimensions of the hydraulic jump, the total loss of energy under various conditions is one of the most important features of this problem, particularly where the jump is used as a means of dissipating energy. The author's method can be applied directly to the determination of energy loss, and thus it will be possible to compare the efficiency, as energy dissipators, of the four classes of hydraulic jump described in this paper.

The writer has been interested particularly in the energy phase of flow in steeply sloping channels. The energy line in such cases is not the simple line with which the engineer is familiar under ordinary flat slope conditions. To borrow a term from modern physics, the energy line has a "fine structure" analogous to certain spectroscopic lines which were believed originally to be single lines and later each was found to consist of several lines, very close together, which could be separated under certain circumstances.

---

[24] Engr., Federal Power Commission, San Francisco, Calif.

This may be shown by the construction of Fig. 17. In Section A–A, normal to the bottom, particles indicated by the letters m, n, o, and p are all at the same energy level, Z–Z. For example, particle o has a pressure head equal to o–o′ and a velocity head equal to o′–o″, as determined by the velocity in the direction of flow, parallel to the slope. From an energy standpoint, particle o is in exactly the same condition as it would be at point o″. The same argument applies to Section B–B, also normal to the slope. Particle o at this section, of course, is not the same particle o as at Section A–A but a different particle at the same distance from the bottom. This particle is at energy level, X–X, being represented by point o″ on the horizontal line, X–X.

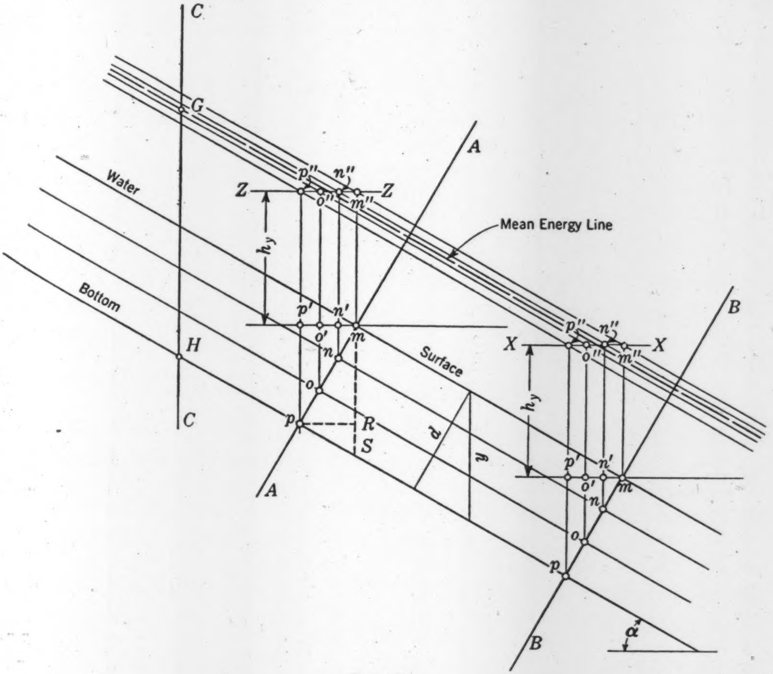


FIG. 17

The sloping line o″–o″, which for uniform flow is parallel to the water surface, is the energy line for all particles at the same distance above the bottom as particle o. Similarly, lines m″–m″, n″–n″, etc., are energy lines for particles at respective depths corresponding to points m, n, etc. Instead of a single energy line, there is a band of energy lines between m″–m″ and p″–p″.

The significance of these lines is apparent when one considers a vertical section, C–C. The point of intersection of line o″–o″ and Section C–C gives the energy level of the particle defined by the intersection of line o–o and C–C. Vertical sections, such as C–C, are the only practical means of defining the position of particles flowing in a conduit, and, to be of any value, energy lines must be correlated with vertical sections in the manner shown.

The average energy level of all particles in a vertical section is at the point of intersection of the "mean energy line" (shown in Fig. 17) with the section itself. The vertical distance from the bottom of the conduit to the mean energy line is readily determined as

$$\overline{HG} = y + \beta \frac{V^2}{2\,g} - \frac{d}{2} \sin \alpha \tan \alpha \qquad (53)$$

in which β is a constant greater than unity, introduced by inequalities in velocity across the section. The term $\frac{d}{2} \sin \alpha \tan \alpha$ is equal to one half of the distance RS, in Fig. 17.

In the foregoing analysis it is assumed that the reader is acquainted with the formula:

$$p = \gamma\, d \cos \alpha \qquad (54)$$

The author refers to the derivation of this equation[7] by Messrs. O'Brien and Hickox, who, in turn, ascribe the equation to Harald Lauffer. There is apparently some discrepancy in the text cited, since it contains the following erroneous equation:

$$(y - z) \cos \theta + z = y \cos \theta \qquad (55)$$

Eq. 55 leads to $z = z \cos \theta$, which is clearly incorrect.

The writer has not had an opportunity of learning the date and reference of Lauffer's work, which would doubtless be of interest to others. Possibly the author can supply this information.

The late John Hedberg,[25] Assoc. M. Am. Soc. C. E., presented an excellent proof of the equation, $p = \gamma\, d \cos \alpha$. Although he was evidently not the first, Mr. Hedberg gave a derivation that hardly could be improved upon—certainly not in point of brevity.

In developing Eq. 8 the author refers to "the equation of equilibrium for section 1" of Fig. 5(*b*). It would seem that Eq. 8 depends fundamentally on the equilibrium of the body of water between two sections; namely, section 1 and a section normal to the slope which intersects section 1 at the water surface. One section, alone, is not enough in this case to be the basis of an equilibrium equation.

It is a pleasure to find such a valuable contribution to the study of flow on steep slopes. The author is to be congratulated for successfully carrying out an investigation presenting unusual difficulties, both experimental and analytical.

---

[25] "Flow on Steep Slopes," by John Hedberg, *Civil Engineering*, September, 1937, p. 633.

Frank S. Bailey,[26] Assoc. M. Am. Soc. C. E.—Paraphrasing the Declaration of Independence, all men are endowed with certain rights, and among them are life, liberty, and the pursuit of knowledge.

The pursuit of knowledge often involves much labor and, when knowledge is attained by any one engaged in research and is presented for the consideration of other researchers, some advance in the understanding of the particular topic under consideration is almost certain to follow.

Many members of the engineering profession are engaged on work which must be pushed forward with as little delay as is consistent with reasonable accuracy. Thus, when a job is under way, they often must rely for basic information on the analyses and experimentation of others. The writer realizes the complexities and difficulties of the subject the author has undertaken to clarify and trusts that the paper will be of great value in its field.

G. H. Hickox,[27] M. Am. Soc. C. E.—The hydraulic jump as a means of dissipating energy has been known and studied for many years. The underlying theory is relatively simple, and has been stated clearly by Mr. Woodward.[5] A number of studies of its performance on a horizontal surface have been made, and the agreement of experiment with theory is gratifying, particularly with respect to the vertical dimensions. In many practical applications it is desirable, and sometimes necessary, to produce the jump on a sloping surface rather than on a horizontal one. The prediction of the final depth for this condition has not been entirely satisfactory heretofore. The phenomenon is governed by the same laws as when it occurs on a horizontal surface, but trouble has been experienced in applying them. The difficulty occurs in determining the horizontal component of pressure exerted on the mass of water within the jump proper by the sloping bottom.

Mr. Kindsvater presents an ingenious and rational method of accounting for the pressures on the sloping surface. The fact that his analysis includes an experimentally determined coefficient ϕ does not detract from its value. The paper was the outgrowth of studies, made at the Hydraulic Laboratory of the Tennessee Valley Authority under the direction of the writer, to establish the elements of the jump on sloping surfaces. The method of attack was suggested by an earlier paper by Messrs. Bakhmeteff and Matzke[28] in which it was shown that the horizontal and vertical elements of the jump could be expressed as functions of λ.

This discussion presents the results of later experiments made on a slope of 1 on 3 and shows that the author's analysis is applicable to any slope on which a satisfactory jump can be formed. The tests were made in a flume 3.5 ft wide with discharges varying from 0.760 to 3.966 cu ft per sec, initial velocities from 5.57 to 9.68 ft per sec, and initial depths from 0.028 to 0.179 ft. The values of λ ranged between 6.9 and 73.0. Fig. 18(*a*) shows values of ϕ as a function of λ for these tests.

---

[26] With Stone & Webster Eng. Corp., Boston, Mass.

[27] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[28] "The Hydraulic Jump in Terms of Dynamic Similarity," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 630–647.

Fig. 18(*b*) has been prepared to show the relationship between ϕ and λ for slopes between the horizontal and 1 on 3. Data for the horizontal slope have been taken from the paper by Messrs. Bakhmeteff and Matzke.[29] The curve for $S = 0.336$ is the same as that in Fig. 18(*a*). The other curves have been interpolated using the Kindsvater data as a guide. With the value of ϕ given by Fig. 18, it is possible to calculate d_2 for any slope by means of Eq. 14.

Calculation of Depth d_2.—In calculating the depth d_2 the writer prefers to use Eq. 24 which is dimensionless and therefore directly applicable to a large range of variables. When the relationship between ϕ and λ is known for a given slope, $\frac{d_2}{d_1}$ can be expressed as a function of λ for that slope. It has been calculated for slopes between 0 and 0.336 for values of λ between 5 and 60 using the curves of Fig. 18(*b*), and the results are shown in Fig. 19. The experimental values of $\frac{d_2}{d_1}$ found for the 0.17 slope by Mr. Kindsvater and for the 0.336 slope by the writer also are shown in Fig. 19 for comparison.

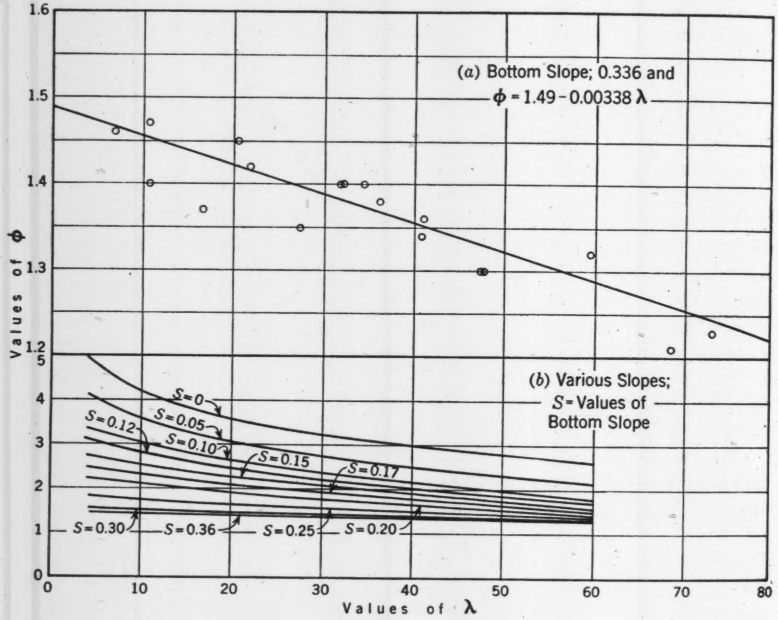


FIG. 18.—VALUES OF ϕ AS A FUNCTION OF λ

A paper by Messrs. Bakhmeteff and Matzke[3] includes data on four flat slopes, including the horizontal, from which the ratio $\frac{d_2}{d_1}$ can be calculated directly from experimental results. The points thus determined also are plotted in Fig. 19. In general, they agree reasonably well with the interpolated curves. It may be noted that the points for the horizontal surface fall slightly below the corresponding curve, indicating the possibility of experimental error. If the points for the other slopes are similarly in error, the agreement with the curves is improved accordingly. In general, the agreement between the calculated curves and the experimental results is good and it is believed that the curves are satisfactory for practical estimates of the jump height.

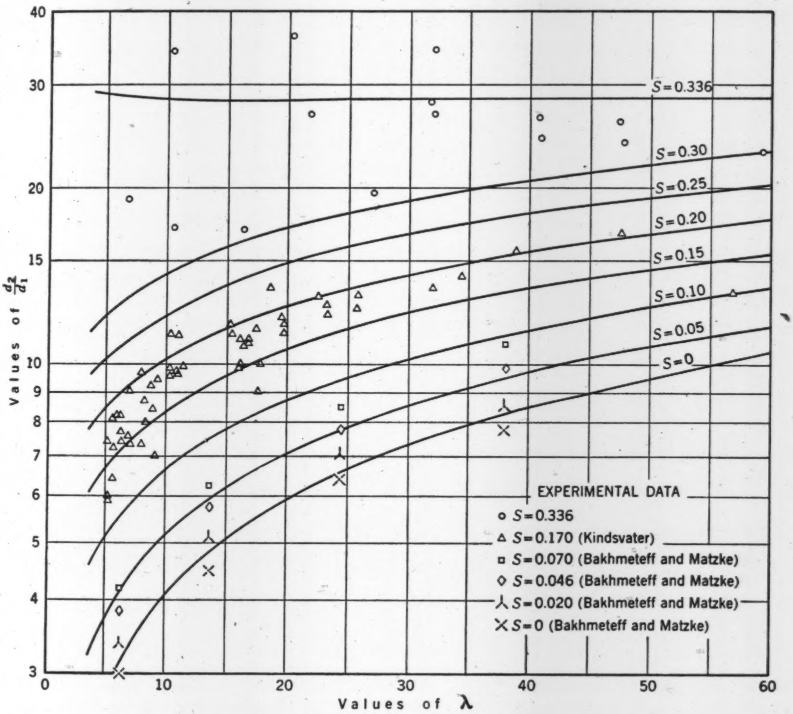


Fig. 19.—Relation Between λ and $\frac{d_2}{d_1}$

Location of End of Jump.—The location of the end of the jump is a matter on which there seems to be little agreement among investigators. The precise point to be chosen is open to question. Some authorities assert that the end of the jump is marked by the highest water-surface elevation. This is not true on a slope, however, since a part of the rise is not due to the jump itself but is caused by the conversion of kinetic to potential energy that occurs in any expanding channel. Hence, the highest elevation has no particular significance with respect to the jump. In addition, it is practically impossible to determine a corresponding point in observations of field installations because of the extreme surface turbulence and also because the recovery of velocity head varies considerably due to local variations in channel shape. Another criterion for determining the length of the jump is the distance to the point at which the bottom velocities are no longer dangerous to the bed. This distance is not a satisfactory measure as it depends on the resistance of the bed to erosion and has no absolute basis for determination. It also is extremely difficult to determine in a prototype structure. The location chosen by the author (the end of the roller) is one that can be determined with comparative ease both in the laboratory and in the field and is preferred by the writer for this reason. It is open to the objection that on steep slopes seriously high bottom velocities may exist beyond this point, and thus it does not necessarily indicate the length within which the high-velocity jet loses its destructive energy. On a horizontal surface the end of the roller is some distance from the end of the jump but on slopes as steep as 1 on 6 and steeper, the elevation at the end of the roller is practically equal to the highest water surface.

In practical applications of the hydraulic jump to the dissipation of energy, these differences in location of the end of the jump become unimportant. In most cases it is not economical to provide protective paving on the river bottom for the full length of the jump as defined by the highest water surface. It is frequently much cheaper and equally satisfactory to stop the apron at a point some distance upstream and to terminate it with a low sill having a sloping upstream face. Such a sill deflects upward from the river bed any high-velocity flows that still might exist. It also extends the range of depths over which a satisfactory jump will occur by maintaining a sufficient depth to produce the jump if for some reason the tailwater should fall below its expected elevation. An extended discussion as to exactly what constitutes the end of the jump is thus largely academic. The length of paving below a spillway is subject to economic considerations and seldom is determined finally by the theoretical length of the jump. Further discussion of this subject properly belongs to a study of stilling-basin design.

In the preparation of Figs. 18(*b*) and 19 two definitions of the end of the jump have been used. Values of ϕ for $S = 0$ in Fig. 18(*b*) are from data by Messrs. Bakhmeteff and Matzke[28] for which the end of the jump was taken at the highest tailwater elevation. For the Kindsvater data on the 0.17 slope and the writer's data on the 0.336 slope, the end of the jump was taken at the end of the roller. As stated, for slopes steeper than 1 on 6, the two elevations are practically identical. For the level floor, the highest elevation of the water surface is preferable since the elevation at the end of the roller is somewhat lower. It is believed that interpolation between the values used for the level floor and the 0.17 slope will give satisfactory results.

Calculation of Bottom Elevation.—The calculation of the depth d_2 at the end of the jump on a sloping surface has no meaning unless the depth can be measured at a known point along the slope. Messrs. Bakhmeteff and Matzke[28] have shown that the ratio $\frac{L}{d_2}$ is a function of λ for a jump on a horizontal surface.

The curve for $S = 0$ in Fig. 20 is taken from their data. The curve for $S = 0.17$ is reproduced from Fig. 6(*b*), and the curve for $S = 0.336$ is based on the experimental data on the 1 on 3 slope. Messrs. Bakhmeteff and Matzke considered the length L to be the distance to the highest water surface and the curve for $S = 0$ is constructed accordingly. Mr. Kindsvater, on the other hand, measured L to the end of the roller. The curves are thus inconsistent in this respect. However, for a 1 on 6 slope, the end of the roller and the highest water surface are practically at the same elevation. The broken lines of Fig. 20 represent an attempt to interpolate tentative curves that may be useful for design purposes. The plotted points represent the results of experiments by Messrs. Bakhmeteff and Matzke on relatively flat slopes. The agreement is not perfect but indicates that the curves are approximately correct. Fig. 20 is sufficiently accurate for practical design purposes.

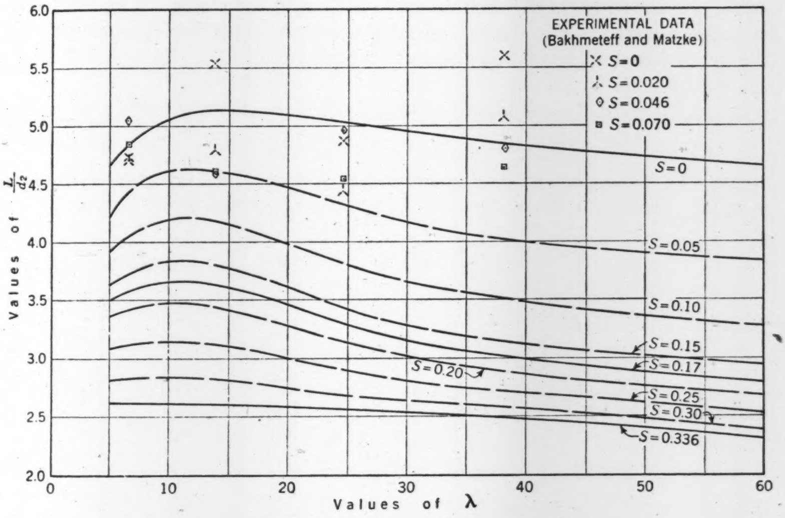


FIG. 20.—RELATION BETWEEN λ AND $\frac{L}{d_2}$

It is frequently necessary to determine the required bottom elevation to insure the formation of the jump with a given discharge and tailwater elevation. This can be done by assuming an elevation slightly below tailwater as the point of beginning of the jump. Having determined d_1 and λ for this point, the depth d_2 and the length L are determined in that order with the aid of Figs. 19 and 20, respectively. The elevation of the bottom at distance L from the beginning then is found by multiplying L by the slope (tan α) and subtracting the result from the bottom elevation at the assumed point of beginning. The difference between tailwater elevation and the bottom elevation thus calculated should be equal to d_2. If the difference is smaller than d_2, another trial must be made assuming a lower point of beginning. Only a very few trials are needed to determine the necessary dimensions with sufficient accuracy.

Case 2 Jump.—Data on the 0.336 slope for the Case 2 jump were not obtained. It was not possible, therefore, to supply a curve similar to Fig. 8 for other slopes.

Effect of Approximate Initial Pressure on Final Depth.—The effect of the approximation to the initial pressure (Eq. 9) on the final depth may be determined to a first approximation by assuming that the neglected difference in initial pressures affects the final depth so little that the change in final velocity may be neglected. The error in initial pressure, ΔP_1, is the difference between the approximate pressure and the correct pressure, or

$$\Delta P_1 = \frac{w\,(d_1)^2}{2\cos^2\alpha} - \frac{w\,(d_1)^2}{2} = \frac{w\,(d_1)^2}{2}\left(\frac{1}{\cos^2\alpha} - 1\right) = \frac{w\,(d_1)^2}{2}\tan^2\alpha \,.\,.\,(56)$$

Neglecting corrections to the momentum at section 2, the error in pressure P_2 is equal to the error in pressure P_1. The error at section 2 is

$$\Delta P_2 = \frac{w\,(d_2 + \Delta d_2)^2}{2} - \frac{w\,(d_2)^2}{2} = \frac{w}{2}\left[2\,d_2\,\Delta d_2 + \Delta(d_2)^2\right] \ldots\ldots (57)$$

Neglecting the square of Δd_2,

$$\Delta P_2 = w\,d_2\,\Delta d_2 \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Equating 56 and 58, $\frac{w\,(d_1)^2}{2}\tan^2\alpha = w\,d_2\,\Delta d_2$; from which

$$\Delta d_2 = \frac{(d_1)^2}{2\,d_2}\tan^2\alpha \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

It is more convenient to write the ratio of the error in final depth to the depth:

$$\frac{\Delta d_2}{d_2} = \left(\frac{d_1}{d_2}\right)^2 \frac{\tan^2\alpha}{2} \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

The error for a slope of 1 on 3 and a depth ratio of 0.4 is 0.9%. For smaller depth ratios and flatter slopes the error is less. For practical purposes it may be neglected.

The actual pressure at section 1 seems to be rather uncertain. The pressures on both sides of a vertical plane at that point should be equal. Approaching the section from upstream, the pressure on the section for uniform parallel flow is given by Eq. 8. On the downstream side of the section, experiment shows that the bottom pressure is $\frac{w\,d_1}{\cos\alpha}$. Since the vertical depth is $\frac{d_1}{\cos\alpha}$, the total pressure on the section is $\frac{w\,(d_1)^2}{2\cos^2\alpha}$ which is Eq. 9. This indicates a discontinuity of pressure at section 1 or, more probably, a rapid change in the type of flow. The use of Eq. 9 thus may be justified on other grounds than expediency.

Carl E. Kindsvater,[29] Jun. Am. Soc. C. E.—Observers apparently do not agree on a definition for the true hydraulic jump. Mr. Stevens discards the jump on a slope, because it is "a mere plunging of water into a pool," and because,

> "As the slope steepens, the jump loses its identity entirely and becomes something else about which little is known."

He quotes Messrs. Bakhmeteff and Matzke[3] to support his opinion:

> "The live jet 'plunges' into the tail water and within the steep section follows the slope downward with comparatively slow expansion and obviously relatively small losses."

A typical opinion, in which the jump below steep slopes is qualified as a "drowned-out" jump, was expressed by Professor Posey,

> "A drowned-out jump cannot be classed as a true jump since it has lost its ability to lower velocities safely within a short distance along the stream."

Thus, the criterion for a true jump would be its ability to destroy energy, and the reason for disqualifying the hydraulic jump in sloping channels is an opinion concerning its suitability as a stilling basin device. The disqualification is not based on proof that a similar manner of analysis will not apply to all jumps, free or "drowned out," in sloping or horizontal channels.

It is hazardous to attempt a general definition of a phenomenon having so many variations of form as the hydraulic jump in open channels of any slope. Fundamentally, the hydraulic jump is marked by an abrupt increase in the depth of flow. It is created when a stream of water flowing at less than critical depth is retarded by a stream of water flowing at a depth greater than critical. To form a stable jump in horizontal channels, the depth of flow at the downstream section must be exactly equal to the upper conjugate depth. When a jump is formed on a slope, the depth of water at the end of the slope may exceed the upper conjugate depth, but the stream will continue to expand beyond the end of the jump because of the increasing depth and the resulting transformation of energy. For any given conditions of initial depth and discharge, there is only one corresponding conjugate depth, and any change of slope beyond the point where this depth occurs will have negligible effect on the location of the beginning of the jump, its length, or its form. Thus, the selection of economic stilling basin dimensions might depend on the upper conjugate depth. This paper did not intend to propose the hydraulic jump in sloping channels as a stilling basin device. It has been used for that purpose, however, and apparently with some success. Therefore, regardless of a difference in opinion concerning the name of the phenomenon, computation of its essential dimensions is frequently desirable. The analysis proposed by the writer appears to serve this purpose. The length of the roller, as defined by the writer, is a fair approximation of the length from the toe of the jump to the section of upper conjugate depth. In addition, the agreement between measured and computed depths for jumps occurring on a 1 on 6 slope is adequate proof of the practical value of the analysis.

---

[29] Associate Hydr. Engr., Hydroelectric Section, U. S. Engr. Office, Little Rock, Ark.

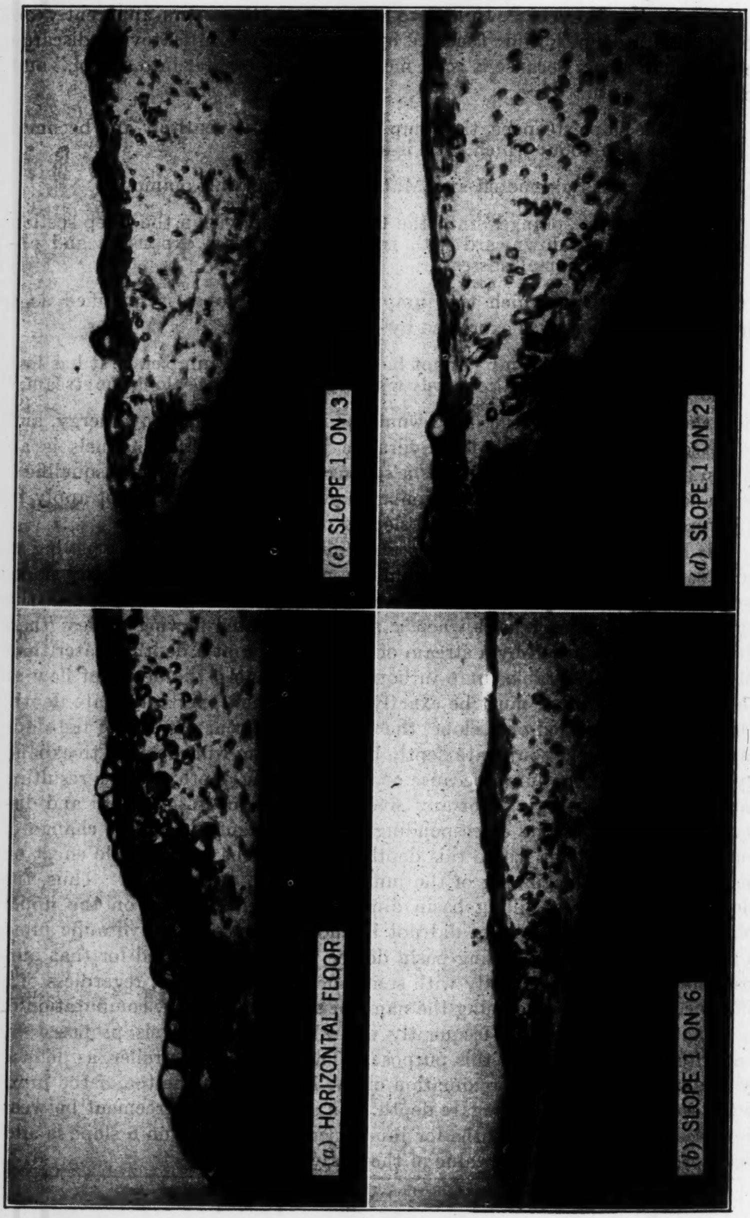

(a) HORIZONTAL FLOOR
(b) SLOPE 1 ON 6
(c) SLOPE 1 ON 3
(d) SLOPE 1 ON 2

FIG. 21.—VIEWS OF THE HYDRAULIC JUMP

The method described for determining the end of the roller might possibly yield unreliable results for very steep slopes. Mr. Hickox' discussion indicates, however, that this limit must be beyond a slope of 1 on 3. The writer's treatment of Case 2 jumps on 1 on 6 slopes may prove invalid for other slopes. Obviously, other limitations might be imposed by the form of the structure on which the jump must occur. For those cases where the path of the water under the jump is materially influenced by the shape of the structure, or where it is obstructed by impact devices (baffle piers, sills, etc.), the proposed analysis of the simple phenomena certainly is not a substitute for model studies.

In the foregoing quotations, the hydraulic jump which forms on slopes greater than about 1 on 10 is described as an occurrence in which the live jet plunges under the roller in a manner distinct from the behavior of the jump in horizontal channels. Under the heading, "The Hydraulic Jump in Sloping Channels: Limitations to Analysis," the writer expressed the opinion that the jump on a slope is generally similar to the jump in horizontal channels. In support of this opinion he submitted two large photographs which lost some of their effectiveness in reproduction (see Figs. 3 and 4). In another attempt for this closing discussion, he devised a miniature laboratory in his family bathroom and produced the four views in Fig. 21. The test flume was approximately 2 in. wide and 24 in. long. The photographs were taken with an exposure of 1/100 sec. The kinetic flow factor for all photographs was estimated to be approximately 35. In these short exposures, action is revealed which is not apparent to the naked eye. "White water" becomes a pattern of air bubbles and water. Slowly moving bubbles are caught as if motionless; rapidly moving bubbles make a visible streak, the length of which is a measure of local velocity components. Because the flume was so unusually narrow, the number of air bubbles involved in the photographs is comparatively small. The pictures illustrate instantaneous typical sections rather than average conditions. The four views in Fig. 21 show clearly a general confusion of path lines and a wide range of velocities throughout the jump. Fig. 21(*c*), for a 1 on 3 slope, shows a remarkable dispersion of high-velocity currents. Fig. 21(*a*), for the horizontal floor, is not a good photograph because it shows a temporary splash of bubbles near the toe. The distribution of velocities in Fig. 21(*a*) does not appear widely different from that for the other views. Any evidence of the distinctive plunging action described by some observers is difficult to find in these photographs.

Length of the Hydraulic Jump.—Argument concerning the definition of the downstream limit of the jump is made confusing by the tendency of some writers to think of the jump only in its relation to stilling basin design. As defined by the writer, the length of the jump is the horizontal length of the top roller. This definition was adopted after careful investigations showed that the end of the roller was the only section near the end of the rapid expansion which could be easily defined and readily observed, giving consistent results when recorded by different observers. For Case 3 jumps, the observed end of roller usually occurs near the point where the water surface first reaches its maximum height. Actually, this "point" is a fluctuating "zone," and argument concerning its exact definition is superfluous.

An important assumption in the writer's analysis, as well as in the classic analysis of Case 1, is that the velocity distribution at section 2 is reasonably uniform, in order that (*a*) the average velocity, $V_2 = Q/A_2$, will yield a satisfactory evaluation of the momentum in that section, and (*b*) the pressure distribution may be assumed to be hydrostatic. The agreement between computed and measured depths for both Case 3 and Case 4 is sufficient proof that the assumption is valid for practical use. As a matter of fact, velocity measurements at section 2 show that the distribution of velocities is not uniform at the end of the roller. Sporadic concentrations of higher-velocity currents prevail considerably beyond the end of the roller. Throughout the long series of tests described by the writer, repeated attempts were made to define a section where destructive bottom velocities ceased. The paths of entrained air bubbles were carefully observed to determine a section where the bottom velocities were no longer sufficient to carry the bubbles along the floor of the flume. Several hundred observations served only to prove that two observers could not agree on the location of that section, usually disagreeing by several feet. Attempts to analyze the observations were fruitless. The presence of local concentrations of erosive velocities beyond the end of the roller is a characteristic of the jump on a level floor as well as of the jump on a slope. Neither the end of the roller nor the highest point on the water-surface profile should be accepted as a zone marking the complete dissipation of erosive velocities.

There is no law of mechanics which requires that the end of the jump, the end of the roller, or any other section be taken as section 2 for the mathematical analysis of the hydraulic jump. The length of the jump is necessarily made as short as possible in order that the boundary losses may be neglected. This does not signify that the pressure-momentum equality could not be verified at any section between the beginning and end of the jump. Quite to the contrary, Kazimierz Woycicki,[30] working with the discharge of water below rectangular sluice gates, has shown that for both the free and the "covered" jump, the fundamental pressure-momentum equality can be verified for any intermediate section, as long as the direction and magnitude of the component velocities and pressures are properly considered. Any successful mathematical solution for the hydraulic jump in sloping channels hinges not only upon the analytical evaluation of forces and velocities at the end sections but also upon the experimental evaluation of forces on the sloping floor under the body of the jump.

Mr. Stevens' comments concerning the length of the jump appear to be somewhat contradictory. For example, he states that "The length of the jump is quite arbitrary except that it should not be assumed too short." Then, recognizing that the length varies with the kinetic flow factor, he adopts for his general analysis the involved formula proposed by Professor Ivanchenko for jumps on a level floor. If the quotation cited is correct, would it not be better to assume that the length ratio might be taken as a "safe" constant? Such was the assumption made by Messrs. Bakhmeteff and Matzke,[3] as well as by Mr. Kennison. However, as demonstrated by the experimental data—

---

[30] "The Hydraulic Jump and Its Top Roll and the Discharge of Sluice Gates," by Kazimierz Woycicki, translation by I. B. Hosig, *Technical Memorandum No. 435*, U. S. Bureau of Reclamation, Denver, Colo., Chapters III and IV.

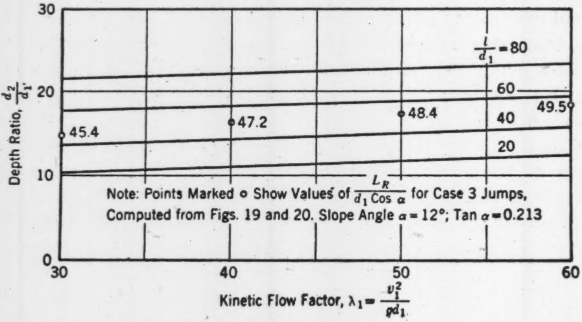
both Mr. Yarnell's and Mr. Hickox'—the length of the jump on a slope is dependent on the degree of slope as well as on the kinetic flow factor. Curves of L_R/d_2 derived from the experiments are quite well defined for a practical range of λ-values, making unnecessary the use of an arbitrary "safe" length, which to be safe must be excessive. Regardless of the definition for section 2, the slope of the apron is obviously an important factor governing the length of the principal expansion within the jump.

Fig. 16, based on an algebraic extension of Mr. Stevens' generalized analysis, does not show a true variation of length as a function of slope, since Eq. 43, taken from Eq. 29, assumes the length of the jump to be primarily a function of the kinetic flow factor.

Classifications of the Hydraulic Jump.—Mr. Kennison questions the title of the paper on the grounds that the "fifth case," in which the water surface downstream from the jump is parallel to the slope of the floor, has been omitted. Although the limits imposed on the length of descriptive titles often lead to ambiguous expressions, the writer believes that the scope of the paper is adequately described by the title. Cases 3 and 4 are concerned with jumps entirely on the sloping floor. Case 1 is necessarily included as a basic case. Case 2 is a practical intermediate classification. The fifth case, as defined by Mr. Kennison, occurs only infrequently and only on mild slopes, where d_2 is ordinarily the normal depth of flow. For practical purposes, any analysis which leads to a satisfactory solution of Case 4 will also yield a satisfactory solution for the fifth case. In the summation of pressure and momentum at section 2, the effect of substituting $V_2 \cos \alpha$ for V_2 would, because of the relatively small magnitude of $\frac{Q \gamma V_2}{g}$ compared to P_2, be negligible. Thus, the value of d_2 computed from Eq. 14 may be expected to agree satisfactorily with measured vertical depths at the end of a jump in a continuously sloping channel. All of the cases defined by the writer are basic to the extent that they include the four principal forms of the jump within a full range of tailwater levels. As stated by Professor Posey, and, as defined by the writer, Case 3 is really the primary case of the hydraulic jump in sloping channels. The agreement between experimental coefficients for Case 3 and Case 4 demonstrates that the distribution of velocities in section 2 can be neglected, and further obviates the fifth case suggested by Mr. Kennison.

Verification of the Analysis by Application to Other Slopes.—Mr. Johnson suggests that all of the writer's four basic cases are contained in Colonel Rindlaub's data in Fig. 10. Not only is this true, but Case 3, in which $l = \frac{L_R}{\cos \alpha}$, can be defined in Fig. 10 with remarkable success by applying Mr. Hickox' interpolated curves, Figs. 19 and 20. Fig. 22 shows a section of Fig. 10, comparing Colonel Rindlaub's data with computed points based on the Hickox curves. This verification of Mr. Hickox' interpolated curves is simultaneously a gratifying verification of the writer's analysis, since it indicates that the end of the roller is a satisfactory criterion for the location of the section of minimum depth for d_2.

Mr. Hickox' experiments on a steeper slope, and his attempt to develop a complete family of characteristic curves by interpolation of experimental results, are valuable contributions to the paper. The results of his computations are generally satisfactory when compared to the existing experimental data.



Note: Points Marked ○ Show Values of $\frac{L_R}{d_1 \cos \alpha}$ for Case 3 Jumps, Computed from Figs. 19 and 20. Slope Angle $\alpha = 12°$; Tan $\alpha = 0.213$

FIG. 22.—COMPARISON OF B. D. RINDLAUB'S DATA ON FIG. 10 WITH INTERPOLATED CURVES BY G. H. HICKOX

There is a danger in writing an equation for ϕ such as Eq. 19*b* or Mr. Hickox' equation in Fig. 18(*a*),

$$\phi = 1.49 - 0.00338\,\lambda \qquad (61)$$

There appears to be no reason why the relation $\phi = f_1(\lambda_1)$, Eq. 19*a*, should define a straight line. Although a straight-line equation does fit the data within the limits tested, the curves so defined should not be extended indefinitely.

Simplified Analyses.—Mr. Kennison and Mr. Stevens propose simplified general analyses of the jump on sloping floors. Both suggest the assumption of a straight-line profile between sections 1 and 2 to evaluate the volume of the jump body. This idea was explored by Mr. Yarnell in 1934 and by the writer in 1939. In both cases, the results of the simplifications were disappointing when computed data were compared with experimental measurements over a full range of λ-values. The explanation is probably suggested by drawing a straight line from the surface at section 1 to the surface at section 2 on the measured profiles shown in Fig. 1. The discrepancy thus illustrated is considerably greater than that shown by Mr. Kennison in Fig. 11.

Mr. Kennison objects to the various manipulations involved in the derivation of Eq. 14. The derivation is not particularly complicated; in fact, the equations were expressly set up in a manner to facilitate algebraic solution, and the result is an equation for d_2 quite similar in structure to the formula for the level-floor jump. For a full range of values of λ, the results from Eq. 14 are much more accurate than those to be expected from Eq. 27*b*. Considering the nature of the phenomenon and the difficulties of observation, the range of observational errors, as computed by Professor Posey, is not excessive. Average curves drawn from the experimental data are believed to be of sufficient accuracy for practical use.

Mr. Stevens' generalized analysis is based not only on the assumption of a straight-line profile of the jump, but also on the aforementioned erroneous assumption concerning the length of the jump. Using Professor Ivanchenko's formula for the length of a jump on a level floor, Mr. Stevens derives an equation for the jump on floors of any slope, in which the length factor becomes quite important. Thus, using $m = \frac{9.3}{k^{0.185}}$, he discovered a zone of "fictitious" jumps which might have been considerably different had he used another arbitrary assumption for the length term. Following this conclusion, Mr. Stevens constructed several diagrams showing zones of fictitious, doubtful, and possible jumps. From Fig. 13(*a*), it appears that all of the jumps taken from Mr. Yarnell's experiments on the 1 on 6 slope are fictitious, since values of *k* in every case are less than 30. When these data are plotted on a curve similar to Fig. 14, however, only about half of the observed jumps appear to be fictitious. This comparison is shown on Fig. 23. The manner in which a fictitious jump differs from a real jump is not clearly shown. Really fictitious is Mr. Stevens' case of a hydraulic jump on adverse slopes. A stable jump cannot exist on an adverse slope except, possibly, when the slope is relatively close to zero.[31]

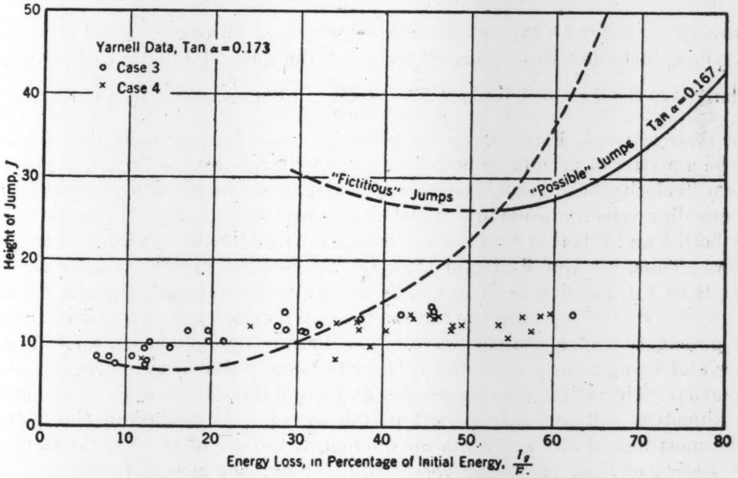


FIG. 23.—COMPARISON OF D. L. YARNELL'S DATA WITH J. C. STEVENS' FIG. 14

In the third paragraph following Eq. 36, Mr. Stevens states:

> "Analogous limitations exist in Eq. 14 from which it is seen that d_2 becomes infinite for $2\,\phi \tan \alpha = 1$ or for $(5.16 - 0.042\,\lambda) \tan \alpha = 1$ (Fig.

---

[31] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 301.

> 6(*a*)). This limitation will not apply to the 1 on 6 slope for which the author's formula is designed."

This statement is misleading, for, as previously noted, the algebraic expression for ϕ, Eq. 19*b*, is an equation of the curve in Fig. 6(*a*), derived from experiments in a channel with an average slope of 0.173. For this case, as observed by Mr. Stevens, a zone of fictitious jumps could not exist. The experimental data presented by Mr. Hickox subsequent to Mr. Stevens' discussion show that, similarly, for a slope of 0.336 a zone of fictitious jumps could not exist. It may be concluded, therefore, that, for every other slope, experiments will yield a separate curve of the ϕ-relation for "real" hydraulic jumps.

The Kinetic Flow Factor.—Both Mr. Stevens and Professor Posey prefer to substitute a ratio of the velocity head to the initial depth $\left(\frac{V^2_1}{2\,g\,d_1}\right)$ for the kinetic flow factor $\left(\lambda = \frac{V^2_1}{g\,d_1}\right)$ as defined by the writer. The proposed alternate, referred to by Mr. Stevens as the kineticity, k, and by Professor Posey as the velocity-head ratio, ω, is equal to $\frac{\lambda}{2}$. In recent years there has been a growing tendency to express the characteristics of all open-channel flow phenomena, including the jump, in terms of the general criterion of dynamic similarity, the Froude number, $\mathbf{F} = \frac{V_1}{\sqrt{g\,d_1}}$. For practical purposes, any of the three ratios, λ, $\mathbf{F}$, or k or ω, is applicable. The choice of definition appears to be a matter of personal preference. The writer objects to Professor Posey's term "velocity-head ratio," because it might be confused with a ratio of corresponding velocity heads in a model and its prototype. An advantage shared by both λ and $\mathbf{F}$ is that these ratios are equal to 1.0 for the condition of critical flow. Thus, for λ or $\mathbf{F}$ greater than 1.0, the flow is "rapid," and, for λ or $\mathbf{F}$ less than 1.0, the flow is "tranquil." A significant advantage of the Froude number is that it expresses the ratio of the initial velocity of flow to the celerity of propagation of a gravity wave. The Froude number has the additional merit of being widely accepted as a criterion for similarity of related phenomena. The writer, like so many who have followed the work of Professor Bakhmeteff, will probably persist in the use of λ as defined in this paper. The most logical alternative, in his opinion, is the use of the Froude number.

Energy of Flow on Steep Slopes.—In discussing the general problem of the energy of flow on steep slopes, Mr. Fee develops the concept of a "mean energy line," which is frequently misunderstood. Referring to Eq. 54, he notes a discrepancy in the cited derivation[7] and requests further reference to the work of Harald Lauffer. The discrepancy shown in Eq. 55, copied from the text cited, is distinctly a typographic error. A translation of Mr. Lauffer's work, on which the development was based, was published by the Society in 1937.[32]

Acknowledgments.—The writer is indebted to the discussers of this paper, especially to Mr. Hickox for his information on the 1 on 3 slope and his assistance in preparing Fig. 23.

---

[32] "Abridged Translations of Hydraulic Papers," *Proceedings*, Am. Soc. C. E., November, 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2229

RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING

By P. C. Rutledge,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Benjamin K. Hough, Jr., F. M. Van Auken, Jacob Feld, Raymond F. Dawson, Hamilton Gray, Karl Terzaghi, D. P. Krynine, D. M. Burmister, and P. C. Rutledge.

Synopsis

Techniques and equipment for undisturbed sampling of soil, laboratory testing of such samples, the mechanics of applying the results of laboratory tests to engineering soils problems, and field observations of the effectiveness of the solution methods through the behavior of full-scale structures have frequently been considered as separate entities. They should not be so considered because each is a series of links in a chain which must reach an anchor or end result to achieve its purpose. Undisturbed sampling and laboratory testing have as their prime objective the prediction of how natural soil deposits will behave under proposed loadings. The purpose of this paper is to explore one link in this section of the chain—namely the extent to which laboratory test results approximate the physical properties of soil in natural deposits.

Since successful undisturbed sampling operations have been largely confined to plastic cohesive soils, this paper is limited to analyses of the effects of sample disturbance on the results of tests on natural clays. The analyses are based on consolidation, direct shear, unconfined compression, and triaxial compression tests on specimens ranging in disturbance from the least disturbed samples obtained to samples that were completely remolded.

Sampling of Plastic Cohesive Soils

During the early years of soil mechanics all laboratory tests were performed on soil specimens remolded from small-diameter samples taken in borings. If larger samples were taken by hand in pits, they were also remolded before test to maintain a uniformity in laboratory technique. In 1932, Arthur Casagrande, Assoc. M. Am. Soc. C. E. (1),[2] discovered that natural clays have a complex

---

Note.—Published in November, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Soil Mechanics, School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

intergranular structure that is destroyed by remolding and cannot subsequently be duplicated in the laboratory. Professor Casagrande's tests showed that there is a marked difference in the physical properties of undisturbed clay as compared with the same material remolded. The importance of determining physical properties from tests on soil in its undisturbed condition was at once apparent and undisturbed sampling came into being.

Most of the identification and classification tests used today have survived from the earlier days of soil studies, and their performance requires preliminary remolding of the soil. Therefore, they need not be considered in relation to undisturbed sampling. Laboratory testing of undisturbed samples is confined almost exclusively to those tests which measure physical properties. The most important of these tests are the consolidation tests, which measure the volume change of soil under load, and the strength tests, which measure the change in shape of soil under load and its maximum resistance to compressive or shearing stresses. The results of consolidation tests are used to analyze and predict settlements of structures. The important strength tests applied to soils are the unconfined and the triaxial compression tests and the direct shear test. Their results are used in the analysis of all problems of stability of soil masses, pressures on walls and tunnels, and ultimate bearing capacities of foundations.

In the best undisturbed samples obtainable, which are large cubical chunks of soil carefully cut out by hand in excavations, some disturbance of natural conditions is inevitable, due only to the removal of pressures of overlying and surrounding soil and their replacement by internal capillary tension in the water in the soil voids. In boring samples the degree of additional disturbance varies widely with the sampling equipment and techniques and the type of soil being sampled. Consequently, one cannot assume that the structure and physical properties of the undisturbed samples tested in the laboratory are those of the soil in the ground.

What are the effects of slight sample disturbance on laboratory testing? If samples are taken with reasonable care, using the more modern types of sampling equipment, the disturbance in plastic soils is not visually apparent. It has little or no effect on the ease of preparation of specimens for laboratory tests or on the performance of the tests. The effects of slight sample disturbance become evident only in the analysis and use of test results.

The established fact that laboratory test results are affected in some degree by inevitable sample disturbance is a challenge to the validity of laboratory soil testing and the application of test results to the solution of foundation problems. Therefore, this paper has the following three purposes:

(1) To analyze the effects of sample disturbance on the significant results of laboratory tests;
(2) To determine to what extent laboratory test results can be used in the solution of foundation and earthwork problems; and
(3) To find means for extrapolation from laboratory test results to the corresponding behavior of soil in nature.

These purposes can be fulfilled only by determining, or approximating with reasonable accuracy, the true initial conditions of the soil in the ground and its subsequent changes in volume and shape under changes in stress. Only three methods are now available for investigating the relation between actual soil behavior and laboratory test results. These are: (*a*) Comparison of test results with field performance of full-scale structures, (*b*) direct reasoning based on the evidence in the test results, and (*c*) reasoning based on hypotheses for soil action which are supported by indirect field or test evidence.

Method (*a*) is greatly to be preferred, but only in a few cases are sufficient field observation data available for its use. As a single example, records of settlements due to compression of strata of soft clay underlying structures have been published and, in some cases, the settlements have been correlated with computations based on laboratory tests (2) (3) (4) (5) (6). The observations and correlations are sufficient to substantiate, in a general way, the methods of computing settlements taught by Karl Terzaghi, M. Am. Soc. C. E., for cases of heavy structures founded on or above reasonably uniform strata of very compressible clay or organic soils. The reported deviations of the admittedly approximate theory from observations are of the same order of magnitude as the deviations of stresses computed by conventional structural theory from published observed stresses in steel and concrete structures. The observational data are not sufficient to validate theoretical solutions for all cases of settlements or to confirm the inferences from laboratory tests in regard to the effects of sample disturbance on soils other than very soft clays.

Methods (*b*) and (*c*), which are not based on direct field observations, cannot establish the validity of applications of laboratory test results to field problems. Their function is to show whether the evidence now available supports or discredits the use of laboratory tests on undisturbed soil samples.

Sampling methods and techniques for laboratory testing of undisturbed samples have been applied successfully only to plastic cohesive soils. Therefore this paper will be limited to the principal tests for the physical properties of such soils; namely, the consolidation, direct shear, simple compression, and triaxial compression tests.

Sample Disturbance and Consolidation Test Results

The consolidation test is the laboratory means for measuring the relations between vertical stress, volume change, and rate of volume change of soil. The apparatus has been described in detail elsewhere (7). The test specimen is confined laterally by a rigid brass ring and is loaded vertically through porous stones. The load is usually applied in increments. For each load increment the sample changes volume slowly, and if this volume, or the void ratio of the sample, is plotted against the vertical stress after it has become substantially constant for each load increment, curves of the type shown in Fig. 1 are obtained. Fig. 1(*a*) is used in this paper because the semilogarithmic plot accentuates certain features of the test results. The void-ratio scale can be transformed linearly into a scale of volume or of volumetric strain if desired.

Fig. 2 shows an idealized test curve of void ratio plotted against vertical stress on a logarithmic scale. Over a substantial portion of the plot, the curve is a straight line for most natural clays with initial void ratios less than 1.5. The principal deviations from the straight line are found during the initial loading of the sample and at points where the load is reduced. As a preliminary hypothesis one may reason by extrapolation that, as the soil deposit was formed, it was gradually compressed by a growing weight of overburden and that the void-ratio, pressure relationship followed the same straight line prolonged, as shown in the dash-line portion of curve (2), Fig. 2. Hence the straight-line portion of the curve is called the virgin compression curve. The formula for this curve is

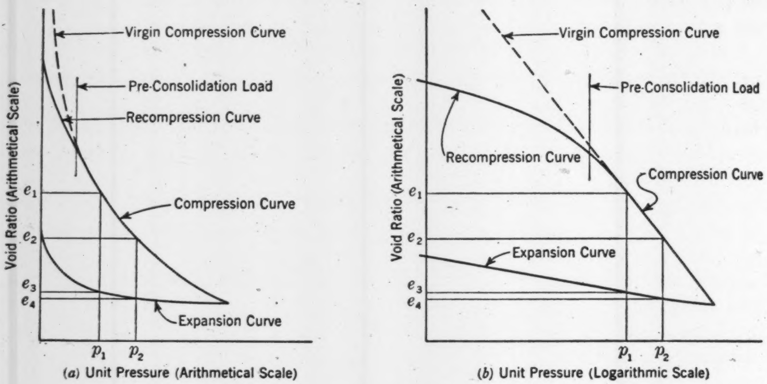


FIG. 1.—COMPRESSION DIAGRAMS

$$e_x = e_0 - C_c \log \frac{p_x}{p_0} \quad \text{.......................} (1)$$

in which C_c is a compression index and e_0 and p_0 are the void ratio and unit pressure, respectively, at an arbitrary point on the straight line, and e_x and p_x are the corresponding values at any other point in the curve. When the sample was removed from the ground it expanded with the release of the pressure of the surrounding and overlying soil (curve (3), Fig. 2) and was again compressed in the laboratory test (curve (1)). Following this reasoning, one can assume that the expansion during and after sampling, and the recompression during the initial part of the consolidation test, are identical to expansion and recompression which can be produced in the laboratory test, as shown in curves (5) and (4), Fig. 2. It is then reasonable to assume that the relation between the maximum vertical stress exerted on the soil in nature will be related to the initial laboratory compression curve (1) in the same way that some larger laboratory stress is related to its corresponding laboratory recompression curve (4). This maximum natural vertical stress is called the "preconsolidation load" (8) and can be determined approximately from the shape of the initial compression curve.

The effects of slight sample disturbance can be investigated by carrying the disturbance to an extreme—in other words, by remolding the test specimens. Figs. 3, 4, and 5 show the results of four sets of tests in each of which one test specimen, after being tested as undisturbed sample, was completely remolded with the addition of water to make its water content and void ratio as nearly equal as possible to the condition of the sample before the test. In this condition of being completely disturbed, or remolded, at the natural water content, these specimens were retested. The curves in Figs. 3 and 4 show the results for an organic clay and two glacial clays. The entire curves for the remolded

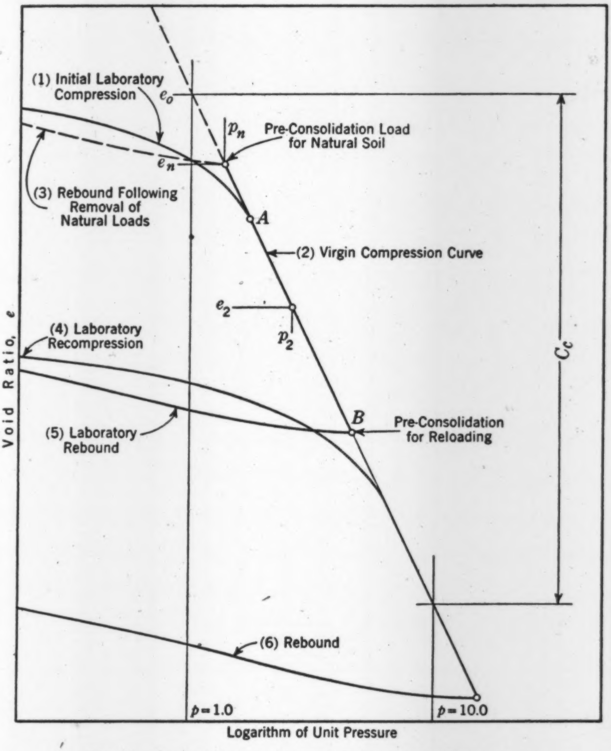

(1) Initial Laboratory Compression
e_o
p_n
Pre-Consolidation Load for Natural Soil
e_n
(3) Rebound Following Removal of Natural Loads
A
(2) Virgin Compression Curve
e_2
p_2
(4) Laboratory Recompression
Void Ratio, e
C_c
B
Pre-Consolidation for Reloading
(5) Laboratory Rebound
(6) Rebound
p=1.0
p=10.0
Logarithm of Unit Pressure

FIG. 2.—TERMINOLOGY FOR THE COMPRESSION DIAGRAM

tests are displaced downward on the scale of void ratio. Thus, the void ratios for any particular vertical stress are much smaller than the void ratios corresponding to the same stress on the undisturbed specimens. Straight-line compression curves are again obtained but, in each case, they have a flatter slope than the compression curves of natural soils obtained from the tests on the undisturbed specimens in the laboratory. Also the initial parts of the compression curves slope very gradually into the straight-line portions and no definite preconsolidation loads are indicated. Prolongations of the laboratory virgin compression curves for the undisturbed samples and the straight-line compression curves for the remolded tests tend to intersect. Thus, volume change under some load would tend to remold the soil completely.

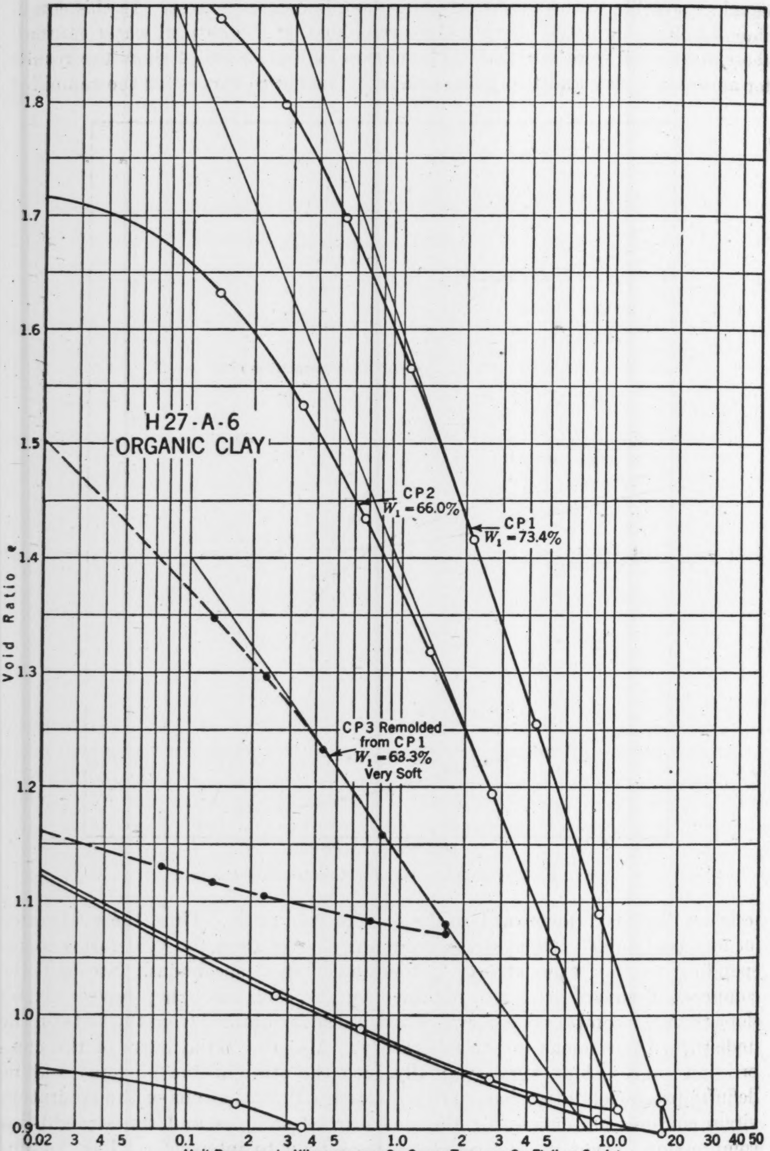


FIG. 3.—EFFECT OF REMOLDING ON THE COMPRESSION CHARACTERISTICS OF NATURAL ORGANIC CLAY (CLASS H27-A-6)

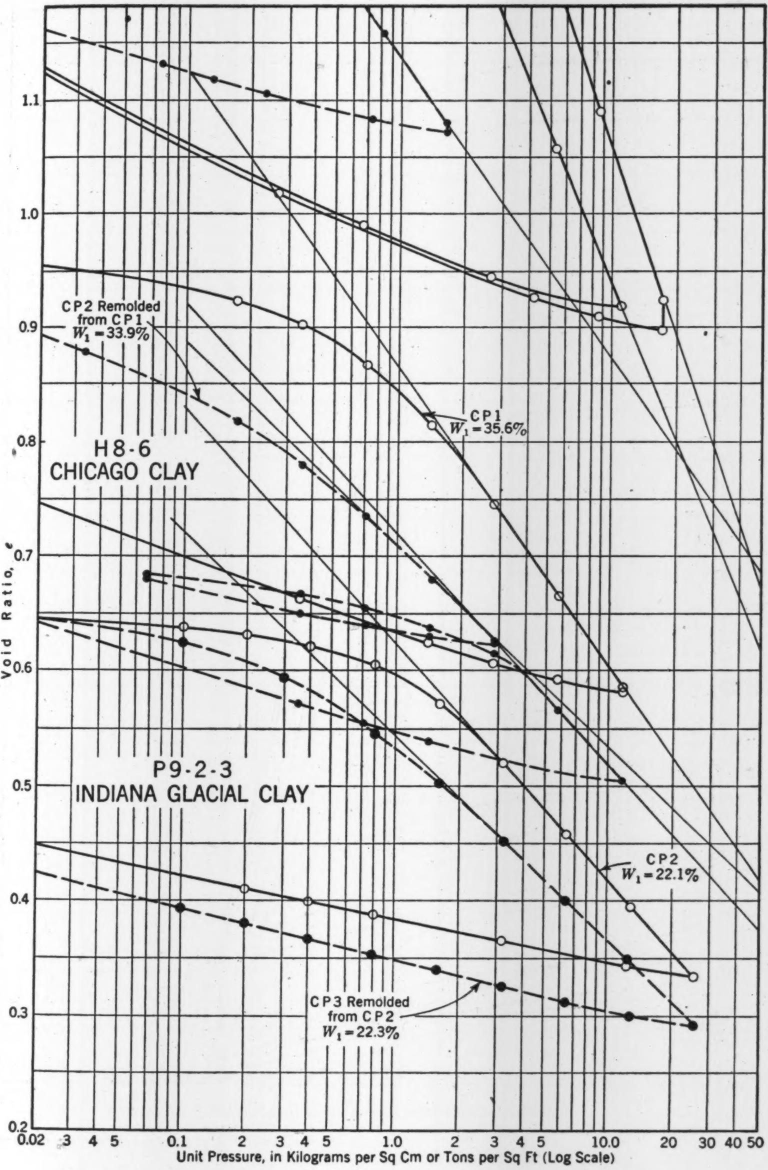


FIG. 4.—EFFECT OF REMOLDING IN THE COMPRESSION CHARACTERISTICS OF NATURAL CHICAGO CLAY (CLASS H8-6)

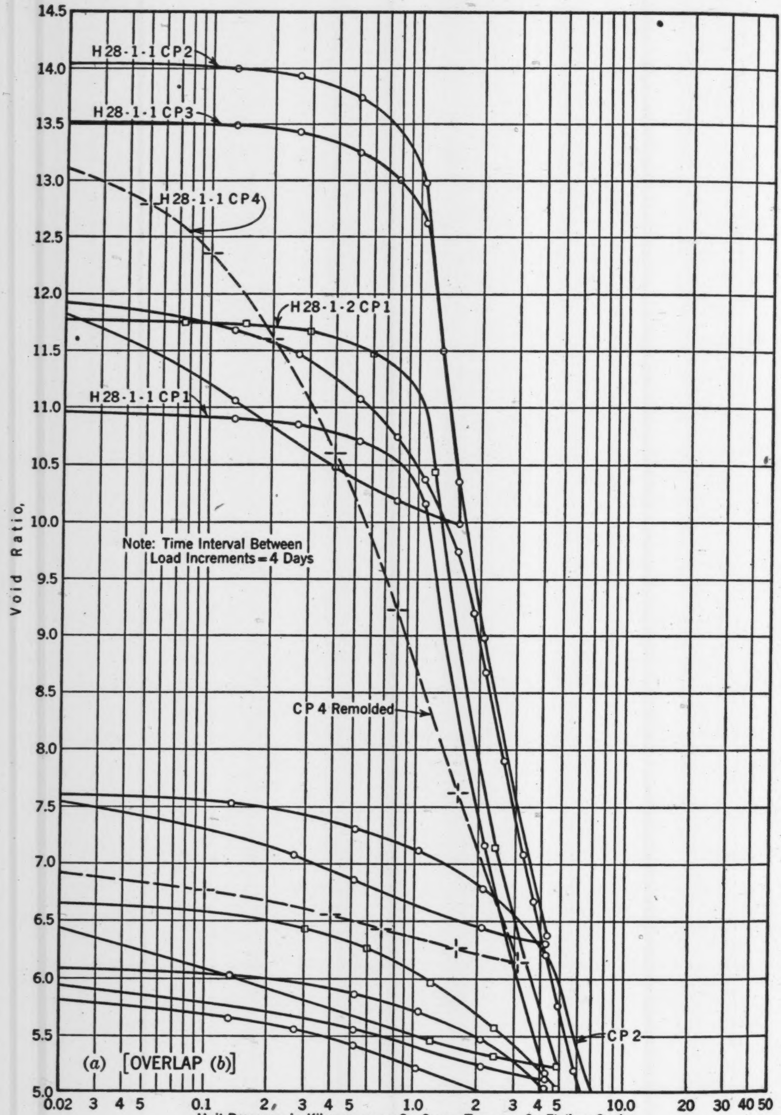
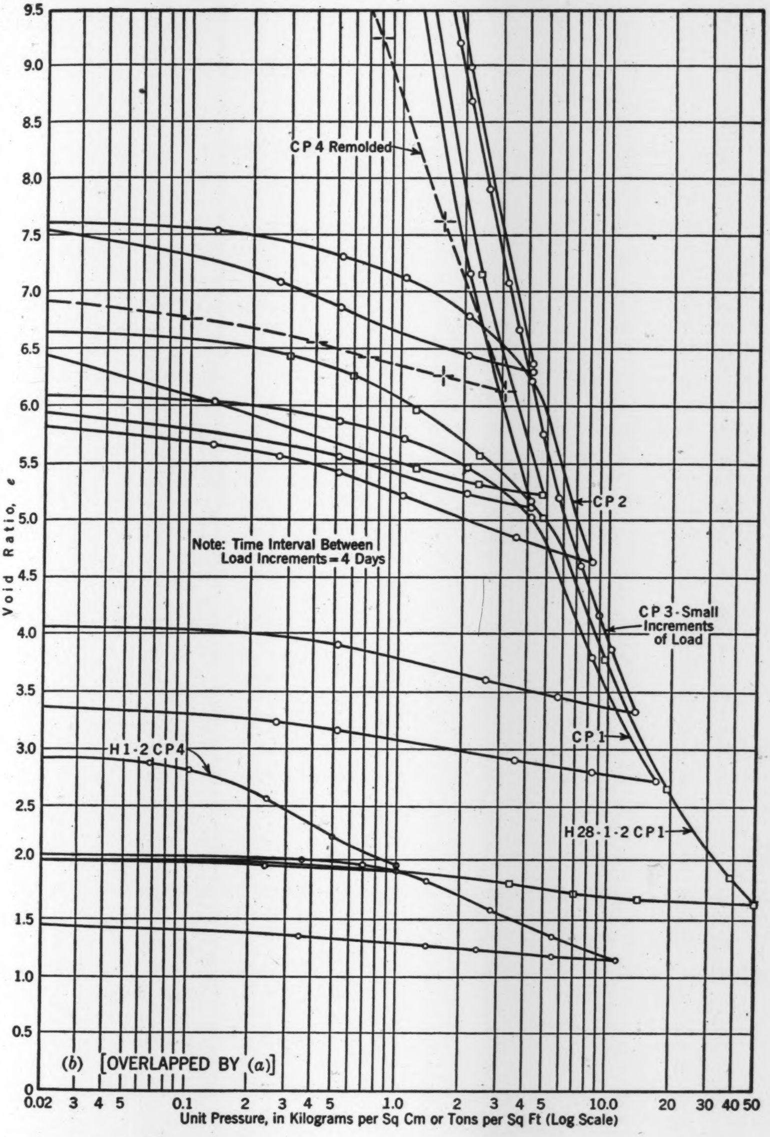


Fig. 5.—Effect of Remolding on the Compression



CHARACTERISTICS OF MEXICO CITY CLAYS

Fig. 5 shows the results of similar tests on a highly compressible volcanic clay from Mexico City, Mexico. The sample from which these test specimens were taken was a 12-in. cube carefully cut out by hand and shipped to the laboratory with great care. The soil was disturbed very little and, as a result, the initial compression curves for the tests on undisturbed samples break very sharply into the laboratory virgin compression curves. The tests show that the preconsolidation load should be almost exactly 1.0 ton per sq ft. The actual weight overlying the specimen was 0.97 ton per sq ft. The nature of this material is extreme in that, fully consolidated under 1.0 ton per sq ft, it had a void ratio of 14.0. In other words, only one part in fifteen of its volume was solid mineral matter. As the curves in Fig. 5 show, this material underwent tremendous volume changes under load and the resulting laboratory virgin compression curves are not straight lines. Compression curves of tests on natural soils which are slightly concave upward have also resulted from tests on other clays with initial void ratios greater than 1.5. Such curves probably are an indication of minimum sample disturbance. The compression curve for the test on a specimen remolded at its natural water content is a straight line as far as the test was carried. It falls below the curves for undisturbed tests and gives every indication of meeting tangentially and continuing with these curves.

The four sets of test results shown in Figs. 3, 4, and 5 are typical of many others and cover an extreme range in plastic soils. Remolding changes these compression curves for undisturbed samples in the following ways:

(1) It decreases the void ratio at which the soil will carry any given vertical stress;

(2) It obscures the previous stress history of the soil and its preconsolidation load; and

(3) The straight-line portion of the remolded compression curve is displaced downward from the laboratory virgin compression curve, and its slope, or the rate of decrease in void ratio with increasing stress, is less.

These results show definitely the effects of sample disturbance over the range from the best undisturbed samples which have been obtained for laboratory test to complete remolding. Any intermediate degree of disturbance must result in a compression curve which falls between the curves that would be obtained for the same soil in these two known limiting conditions. On this basis compression curves are plotted in Fig. 6 for (1) a good undisturbed specimen, (2) the same soil slightly disturbed, and (3) the same soil remolded. Curve (2), Fig. 6, is typical of actual test results on many specimens from samples taken in borings. In relation to curve (1) the effect of the previous stress history of the soil is less marked; but it is possible to estimate the preconsolidation load with decreased accuracy.

In Fig. 6, point B on curve (2) for a slightly disturbed specimen represents the preconsolidation load for the given soil. The corresponding void ratio is considerably less than the initial void ratio of the sample at point C. Without referring to curve (1), two possibilities are apparent: (*a*) The sample represented by curve (2) has expanded during and after sampling and been recompressed

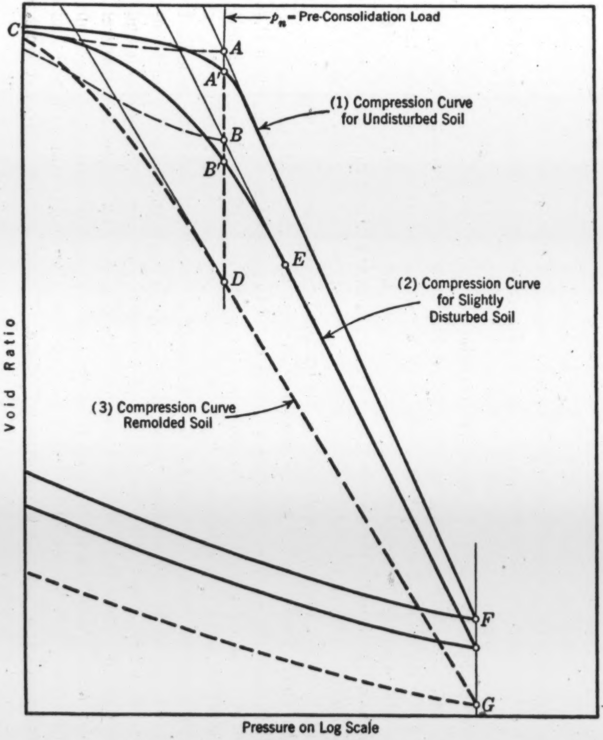


FIG. 6.—EFFECT OF REMOLDING AND SLIGHT DISTURBANCE ON THE SHAPE OF THE COMPRESSION CURVE

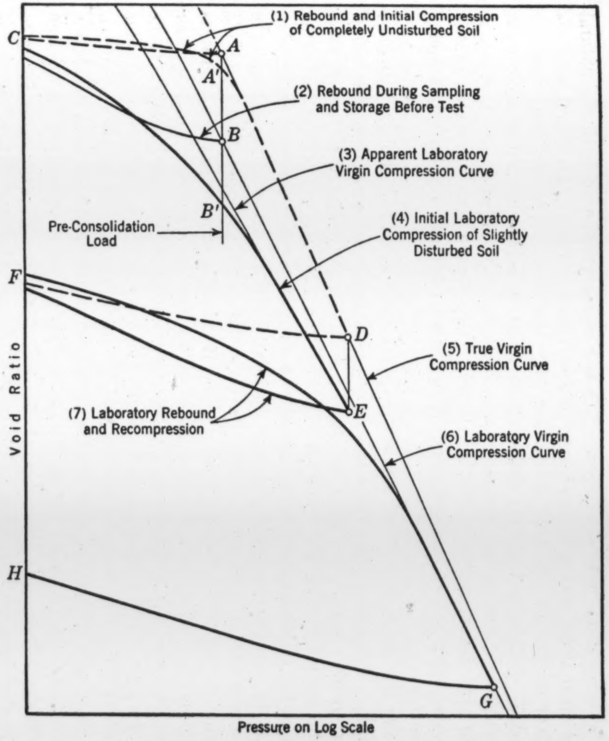


FIG. 7.—EFFECT OF DISTURBANCE DURING SAMPLING ON THE DETERMINATION OF THE PRECONSOLIDATION LOAD

in the laboratory test; or (*b*) point B is not representative of both the natural void ratio and preconsolidation load in the ground. Curve (1), Fig. 6, which is typical of test results that have been obtained for specimens from samples carefully taken by hand, indicates that point A represents more nearly the actual conditions of the soil in nature. Messrs. Casagrande (8) and M. J. Hvorslev, Assoc. M. Am. Soc. C. E. (9), have stated that saturated plastic cohesive soils probably swell very little during carefully conducted sampling operations.

Fig. 7 is a reconstruction of what may have happened to a slightly disturbed sample. Assume initially that the preconsolidation load is equal to the natural overburden pressure and that its relation with the natural void ratio of the soil in the ground is shown by point A. During the sampling operations the structure of the soil is weakened and it tends to compress under the preconsolidation stress to equilibrium at a new void ratio at point B. At the same time, the preconsolidation pressure is removed by the sampling operations and the sample is held at a constant volume by its small permeability which resists rapid outflow of water filling its voids and by the surface tension of the water at its free surfaces. The actual intensity of capillary pressure acting on the soil may be very small (it may be zero) because the reduction in strength due to disturbance removes the tendency of the mineral grain structure to expand. When a specimen from this sample is tested in the laboratory, it yields the heavy solid-line type of curve shown in Fig. 7. The preconsolidation load determined from this laboratory test falls on curve (6), Fig. 7, at point B. If point B can be determined with reasonable accuracy and if the initial void ratio of the test specimen (point C) is substantially that of the soil in nature, the intersection at point A of the preconsolidation load and the initial void ratio actually determines the condition of the soil in the ground.

Under these assumptions point A in Fig. 7 represents the natural conditions of soil with acceptable accuracy. The compression curve which will be followed actually by the undisturbed soil in the ground when it is loaded should then begin at point A. Beyond this point the investigator does not know its shape or its slope conclusively. He may assume that it is a straight line, parallel to the best laboratory virgin compression curve he can obtain. Laboratory investigations by the writer of many clays indicate that it may be a straight line slightly steeper than curve (6), Fig. 7. For practical purposes these two possibilities are identical and are shown by the broken line AD in Fig. 7.

In 1941, Professor Terzaghi presented a new hypothesis for the action of undisturbed clays which assumes a "rigid bond between the solid parts of the absorbed water films which surround the clay particles" (10). This hypothesis is based on, and explains some, field and test data not readily explained by existing conceptions. According to Professor Terzaghi, if the clay is loaded very slowly, as by the gradual deposition of overlying layers of soil over thousands of years, the "solid water" bond between particles is not broken and compression proceeds as shown by curve (2), Fig. 8. This very flat, natural compression curve has little or no relation to the results of laboratory tests on samples in which slight disturbance has broken the "solid water bond" and transformed the clay into a "lubricated" state. The possibility of its existence, however, does not affect the validity of the preceding analysis of the results below the point A in Figs. 6 and 7 of comparatively rapid laboratory tests. That analysis is based on concrete test evidence and is independent of the way the soil arrived at the condition indicated by point A. On the other hand, extrapolation of curve (5), Fig. 7, above point A may not correspond to natural phenomena. In a natural soil deposit loaded by a building, the clay may tend initially to follow some flat natural compression curve. Then, as the stress increase in the clay becomes larger, the bond might be broken and the clay might begin to compress as it does in the laboratory test. If this is the case, at stresses somewhat in excess of the preconsolidation load, the compression curve for the soil in the ground would break sharply downward toward the laboratory test curve. This is shown by the lower dotted curve (4) in Fig. 8. The tests on undisturbed specimens of Mexico City clay, shown in Fig. 5, re-

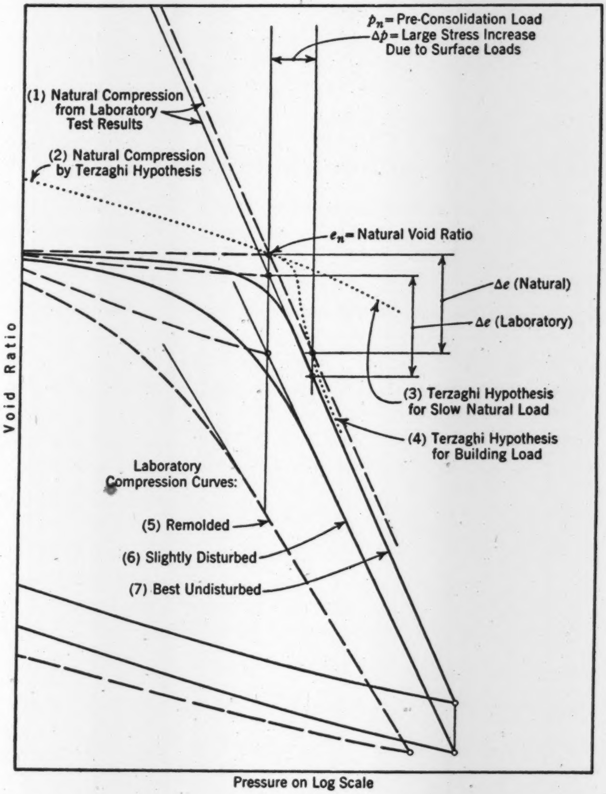

p_n = Pre-Consolidation Load
Δp = Large Stress Increase Due to Surface Loads
(1) Natural Compression from Laboratory Test Results
(2) Natural Compression by Terzaghi Hypothesis
e_n = Natural Void Ratio
Δe (Natural)
Δe (Laboratory)
(3) Terzaghi Hypothesis for Slow Natural Load
(4) Terzaghi Hypothesis for Building Load
Laboratory Compression Curves:
(5) Remolded
(6) Slightly Disturbed
(7) Best Undisturbed
Void Ratio
Pressure on Log Scale

FIG. 8.—LABORATORY CURVES VERSUS COMPRESSION CURVES

sulted in curves of this shape which break sharply and become almost vertical at a stress slightly in excess of the preconsolidation load. This is a phenomenon that has also been observed in a few tests on other clays with initial void ratios between 1.5 and 7.0.

The importance of these considerations is obvious. The engineer wishes to use the results of a consolidation test to predict the volume change in a natural undisturbed clay under an increase in stress caused by a building load, and consequently, the amount the building will settle. To do this, he must begin with the existing natural conditions and determine the actual relation between decrease in volume and increase in stress. The slight inevitable change in condition that occurs in the best undisturbed samples makes it inaccurate to apply the test results without modification. The following conclusions are a practical interpretation of the evidence now available:

(1) The preconsolidation load can be determined with reasonable accuracy from the results of consolidation tests on specimens from good undisturbed samples—for example, 8-in. to 12-in. cubical samples carefully cut out by hand in a test pit.

(2) Serious disturbance during sampling makes inaccurate, or prevents entirely, the determination of the preconsolidation load.

(3) If the Terzaghi hypothesis is accepted (and it seems to explain discrepancies between predicted and observed settlements under small stress increases as well as observed uniformities of water content over large depths of uniform clay strata) volume changes corresponding to changes in vertical stress in natural clay caused by foundation loads may be computed as follows:

(*a*) Volume changes caused by very small increases in stress will follow curve (3), Fig. 8, and may be neglected.

(*b*) Volume changes caused by larger increases in stress will break the rigid solidified water bond between clay particles. For all practical purposes the compression will then parallel the laboratory virgin compression curve (7) from the intersection with the preconsolidation load.

(*c*) The division point between these two possible actions has not been determined by field evidence. In present applications its selection is a matter of judgment, a division point at zero change in stress representing the present method of computation neglecting the Terzaghi hypothesis.

From these conclusions it is evident that a purely theoretical solution for settlement problems is not consistent with the character of the data derived from test results.

The effect of sample disturbance on test rates of volume change and prediction of rates of settlements of structures founded on clay soils has not been considered. There is little test evidence to show whether or not sample disturbance has any marked effect on rate of volume change except in so far as it affects the slopes of the compression curves. If the natural compression follows substantially the laboratory virgin compression curves, Professor Terzaghi's original theory of consolidation (11) (12) probably is adequate and well within the accuracy of field determination of drainage conditions. If the natural compression occurs without breaking the "rigid clay bonds," the rate will be entirely different from the rates obtained from laboratory tests, but the magnitudes of the settlements probably will be small and the rates unimportant.

STRENGTH TEST RESULTS

The strength tests are those tests which measure the resistance of soil specimens to change in shape and their ultimate strengths in shear, compression, or tension. The principal types of strength tests for soils are the direct shear, the simple or unconfined compression, and the triaxial compression tests. Within these three types many variations in test equipment and technique are found. They have been described in detail by many writers, including a symposium on shear testing published by the American Society for Testing Materials (13) and a recent comprehensive listing by B. K. Hough, Jr., Assoc. M. Am. Soc. C. E. (14).

The Direct Shear Test.—Direct shear testing of remolded clay has been studied exhaustively by Mr. Hvorslev (15); and L. Jurgenson (16) has studied direct shear tests on both undisturbed and remolded clays. These studies show that the shearing resistance of a clay depends on a number of factors, among which the previous stress history of the soil and its preconsolidation load are important. Consolidation test data have shown that sample disturbance modifies the effects of previous natural stresses in clays. Therefore, the shearing resistance must be affected in some degree. The direct shear-test data are inconclusive in regard to the magnitudes of effects of slight disturbance or remolding on the shearing strength of undisturbed clay. The only conclusion to be drawn from the existing data is that sample disturbance definitely affects the resistance of clay specimens to direct shear. The magnitude of the effect cannot be estimated with any assurance.

THE UNCONFINED COMPRESSION TEST

The unconfined compression test is easier to perform on undisturbed samples of plastic soil than the direct shear test and shows promise of becoming the more generally accepted of the two. It is performed on prismatic or cylindrical specimens in a manner almost identical to compression tests on concrete and other engineering materials. Precautions are taken to distribute the axial load uniformly to the ends of the specimen and the specimen is protected against evaporation. Longitudinal deformations are observed during the test. The results are plotted as longitudinal strain versus longitudinal stress acting on the average actual cross-sectional area for each load. The maximum longitudinal stress sustained by the sample in an unconfined compression test is called its "compressive strength." The slope of the initial portion of the stress-strain curve, which is frequently a straight line for the best undisturbed specimens, is called the "modulus of deformation" or the "modulus of elasticity" (17). Many natural clays are elastic in the sense that stress and strain are approximately proportional for continuously increasing compressive stress. They are not elastic in the sense that they will return to their original shape upon release

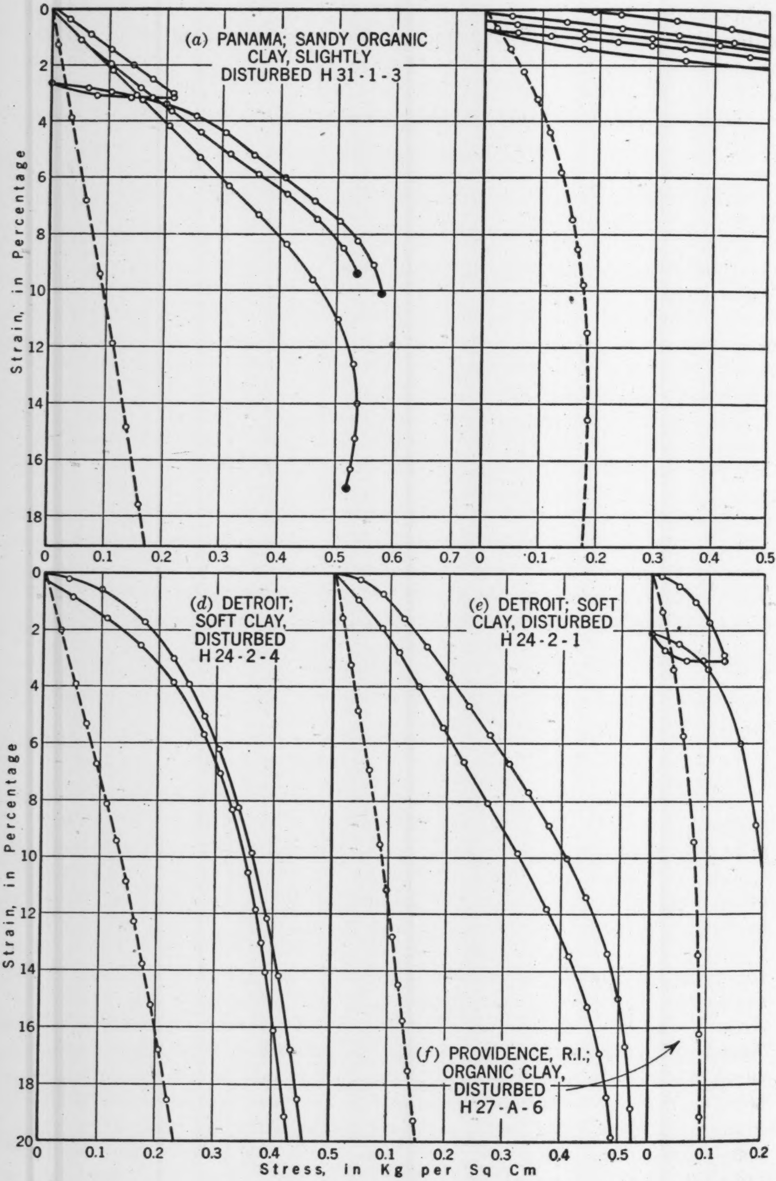
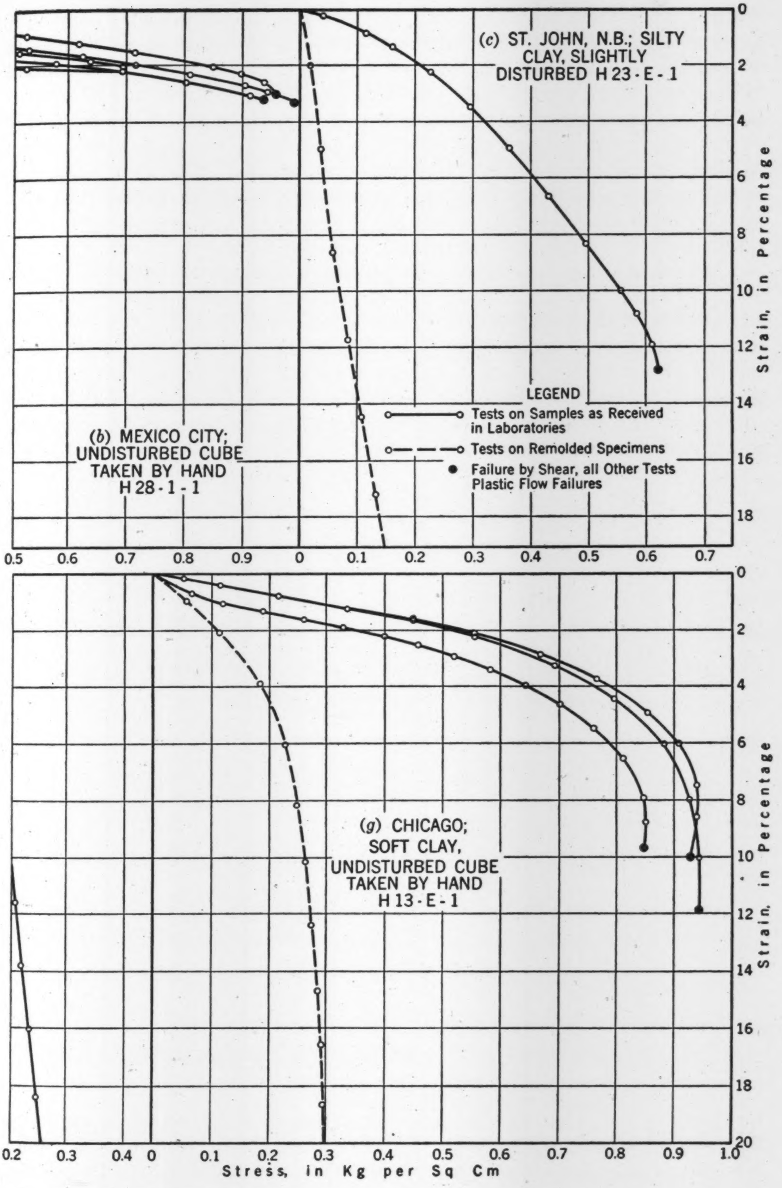


FIG. 9.—TYPICAL STRESS-STRAIN



Curves for Natural Soils

of stress. The term "modulus of elasticity" is used because it is the commonly accepted term for the coefficient of proportionality between stress and strain for specified conditions of loading, and because most field loadings of concern to engineers cause continuously increasing compressive stress in soil deposits (17). The use of this term should not imply that the modulus can be used in elastic theory without review of its limitations.

Fig. 9 shows a group of typical stress-strain curves for unconfined compression tests on natural clays. Specimens from each of these undisturbed samples were tested in the condition the samples were received. Then the undisturbed specimens were remolded to form one specimen for a test on the same soil in a completely disturbed condition. The difference between the results of tests on undisturbed and remolded specimens is striking. Specimens of Mexico City clay (Fig. 9(*b*)) were cut from the cubical sample described under consolidation tests and were little disturbed. The Chicago clay sample (Fig. 9(*g*)) was a similar cube cut out by hand in an excavation. The two sets of curves for these samples are representative of results of good tests on undisturbed samples. In the writer's experience tests on specimens from such samples invariably result in stress-strain curves which are straight lines to 30% or 40% of the maximum stress, whether the soil fails by rupture at 2% strain or by plastic flow at 20% strain. In contrast to these curves, the curves for the St. John, New Brunswick (Fig. 9(*c*)), Detroit, Mich. (Figs. 9(*d*) and 9(*e*)), and Providence, R. I. (Fig. 9(*f*)), clays are typical for tests on specimens disturbed during

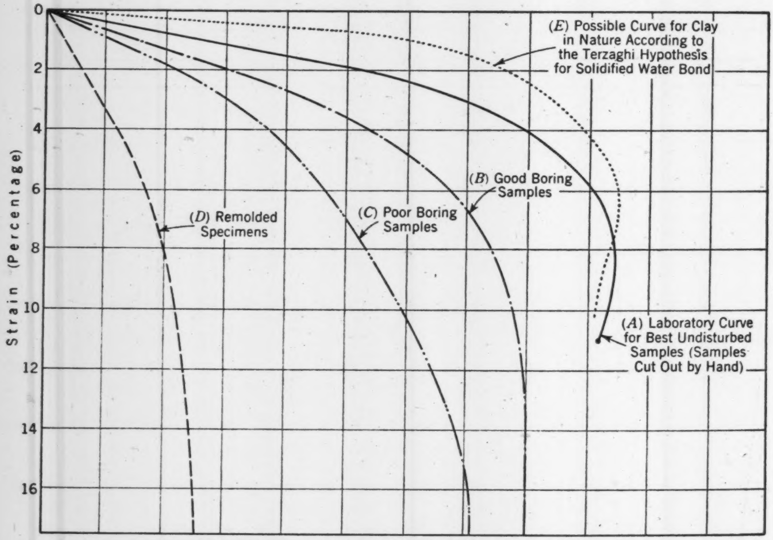


FIG. 10.—EFFECTS OF SAMPLE DISTURBANCE ON STRESS-STRAIN CURVES FROM UNCONFINED COMPRESSION TESTS

sampling. Curvature within the first 20% of the maximum load on a stress-strain curve is almost positive evidence of sample disturbance. Two series of tests by the writer on undisturbed specimens have shown that these results are not measurably affected by rates of application of strain varying between 1% per min and 1% per hr.

A specific study of the effects of disturbance during sampling on the results of unconfined compression tests was undertaken in connection with the soil investigations for the Chicago subway. The results were reported by R. B. Peck, Jun. Am. Soc. C. E. (18), in 1940. Mr. Peck has presented stress-strain curves which are remarkably similar to curves for approximately corresponding degrees of disturbance in Fig. 9. He plotted the test compressive strengths of remolded and of thin-walled seamless-steel tube samples against corresponding test compressive strengths for specimens from samples carefully cut out by hand, and found that compressive strengths varied between 65% and 75% of the "undisturbed" strength and the remolded strengths were 22% to 45% of the "undisturbed." The relative effects of sample disturbance on the initial slopes of the curves were much greater (18*a*).

As a result of the study of these test curves, and many others, with the same general characteristics, it is possible to formulate an hypothesis for the effect of sample disturbance on the results of unconfined compression tests. In Fig. 10, curve (A) represents typical results of laboratory tests on the best undisturbed specimens—for example, those cut from large cubical samples taken by hand in test pits or excavations; curve (B) represents typical results for tests on samples taken in bore holes by the most modern types of sampler; and curve (C), tests on disturbed samples taken by undisturbed sampling methods in bore holes. Tests on these same specimens, remolded at their natural water contents, would yield results shown approximately by curve (D), Fig. 10. Test measurements of the reductions in compressive strengths

TABLE 1.—EFFECT OF REMOLDING ON COMPRESSIVE STRENGTHS AND MODULI OF ELASTICITY OF CLAYS

(Kilograms per Square Centimeter)

Source	Material		Compressive Strengths			"Moduli of Elasticity"		
	Type of clay	Tested by[a]	Undisturbed	Remolded	Ratio, $\frac{\text{remolded}}{\text{undisturbed}}$	Undisturbed	Remolded	Ratio, $\frac{\text{remolded}}{\text{undisturbed}}$
Chicago	Glacial	Peck (18)	0.50	0.12	0.24	24	0.8	0.03
Chicago	Glacial	Rutledge[b]	0.94	0.30	0.32	28	5.7	0.20
Chicago	Glacial	Rutledge (19)	2.20	0.76	0.34	130	8.0	0.06
Chicago	Glacial	Rutledge (19)	4.1	2.1	0.51	295	44	0.15
Boston	Marine	Casagrande (1)	2.3	0.3	0.13	102	18	0.18
Laurentian	Marine	Casagrande (1)	6.1	0.4	0.07	740	28	0.04
Mexico City	Volcanic	Rutledge[b]	0.96	0.18	0.19	45	3.5	0.08

[a] Numerals in parentheses refer to the corresponding items in the Bibliography.
[b] Data taken from Fig. 9.

and in "moduli of elasticity" from curve (A) to curve (D) (Fig. 10) are shown in Table 1.

According to Professor Terzaghi's hypothesis (10) for the "solid" versus "lubricated" states of natural clays, the rigid cementing action of the solidified water bond between clay particles may cause the soil to offer much greater resistance to deformation in its natural condition in the ground than can be obtained in a laboratory test. Thus the true initial slope of the stress-strain curve may be much flatter and the "modulus of elasticity" much greater than the best test specimens indicate. Small deformations, however, will destroy this bond as effectively as small deformations during sampling and, beyond a strain of 1% or 2%, the true stress-strain curve for the soil should be essentially parallel to, or should approach, the curve obtained from a laboratory test on the least disturbed type of specimen. On the basis of this hypothesis the true stress-strain curve for natural clay may have a shape similar to curve (E), Fig. 10. The compressive strength will practically be the same as for curve (A) if the strain at the maximum strength is greater than 2%. This conclusion has been verified partly by correlation of unconfined compressive strengths of samples carefully cut out by hand with field observations of lateral pressures exerted on the sheeting and bracing of open-cut sections of the Chicago subway (20).

These observations can be summarized by the following conclusions in regard to the effect of sample disturbance on the results of unconfined compression tests:

(1) Serious sample disturbance is indicated if, (*a*) the stress-strain curve is not a straight line for the first 30% of the compressive strength, or (*b*) the stress-strain curve falls close to a stress-strain curve for the same specimen remolded.

(2) Compressive strengths determined from tests on specimens cut from large cubes taken by hand are probably equal, for practical purposes, to the compressive strengths of completely undisturbed soils.

(3) Test compressive strengths of samples which are slightly disturbed during sampling may be considerably less than the compressive strengths under conclusion (2).

(4) The "modulus of elasticity" of a clay in nature will not be less than that obtained from the tests described under conclusion (2), but it may be greater by an amount which cannot be predicted from the results of laboratory tests.

THE TRIAXIAL COMPRESSION TEST

In the triaxial compression test a cylindrical soil specimen, enclosed in a thin rubber membrane, is placed under a constant hydrostatic pressure and then tested in compression. A schematic diagram of typical equipment for this test is shown in Fig. 11. The additional compressive load in the direction of the axis of the specimen is applied by a piston which slides freely in an opening in the top of the pressure chamber. The axial compressive stress in excess of the constant hydrostatic pressure is called "deviator stress." The test results are plotted as a curve of axial strain versus deviator stress. Volumetric strains are measured by the quantity of water flowing into or out of saturated specimens and are also plotted against the deviator stress.

Triaxial compression tests on plastic, cohesive soils are divided into quick tests, consolidated quick tests, and consolidated slow tests. All have the advantage that the lateral forces of the hydrostatic pressure simulate the confining effect of surrounding soil in nature. Thus lateral expansion of the specimen with increasing vertical strain causes decreases in the radial and circumferential compressive stresses rather than tension as in the unconfined compression test. Observed increases in compressive strengths and in strains at failure of brittle clays in triaxial compression are due in part to the fact that lateral tensile stresses have been reduced or changed to compression.

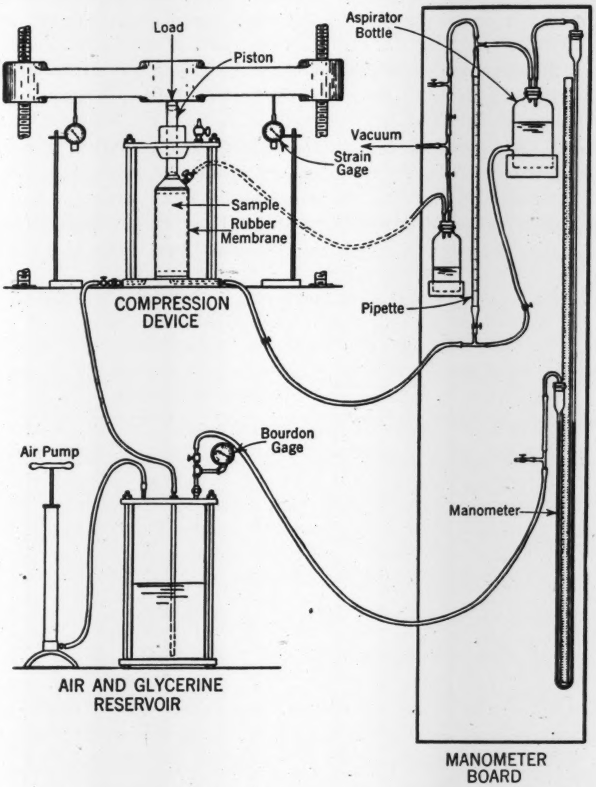

Load
Piston
Aspirator Bottle
Vacuum
Strain Gage
Sample
Rubber Membrane
COMPRESSION DEVICE
Pipette
Bourdon Gage
Air Pump
Manometer
AIR AND GLYCERINE RESERVOIR
MANOMETER BOARD

FIG. 11.—TYPICAL SETUP FOR TRIAXIAL COMPRESSION TEST

In the quick test hydrostatic pressure is applied to the specimen and it is tested immediately in compression, the entire process requiring less than thirty minutes. If the specimen tends to change volume during the test, and most undisturbed clays do, some of the hydrostatic and deviator stresses are carried by pressure in the water filling the clay voids which does not have time to flow out. Thus the actual stresses acting on the clay are difficult to evaluate. Sample disturbance increases the tendency of clay to consolidate and increases the error.

In both the consolidated quick and consolidated slow tests the specimens are placed under hydrostatic pressure and allowed to consolidate fully before being tested in compression. If the sample were perfect and undisturbed and the hydrostatic pressure were not greater than the preconsolidation load, little or no consolidation would take place and the test would begin with the soil in its natural condition. The results of consolidation tests show, however, that most undisturbed clay specimens in the laboratory consolidate under their preconsolidation loads. Therefore laboratory specimens under triaxial compression are not tested in their natural condition. As an example, the results of consolidated triaxial compression tests on undisturbed Chicago clay are shown in Table 2, each set of data being the average of two or three tests (21). All the specimens in these tests were prepared from one 12-in. cubical undisturbed sample, each specimen being consolidated fully before the test. The "modulus of elasticity," E, was computed from the formula

TABLE 2.—AVERAGE RESULTS OF TRIAXIAL COMPRESSION TESTS ON UNDISTURBED CHICAGO CLAY

(All Stresses Are in Kilograms Per Square Centimeter)

Description	HYDROSTATIC PRESSURES:									
	0		0.5		1.0		2.0		4.0	
Type of test	Quick	Slow	Quick	Slow	Quick	Slow	Quick	Slow	Quick	Slow
Natural void ratio	0.762	0.762	0.754	0.755	0.743	0.758	0.769	0.765	0.758	0.777
Deviator stress[a]	0.267	0.275	0.540	0.672	0.722	0.768	1.10	0.90	1.62	1.63
Modulus of deformation[b]	26.7	27.5	54.0	67.2	72.2	76.8	110.0	90.0	162.0	163.0
Void ratio[a]	0.762	0.762	0.708	0.685	0.687	0.690	0.635	0.645	0.570	0.575
Poisson's ratio,[a] μ	....	....	0.46	0.31	0.47	0.31	0.48	0.32	0.47	0.28
Stress in pore water[a]	....	....	0.52	0.50	0.69	0.64	1.09	0.78	1.74	1.29
Lateral soil stress,[a] σ_3	....	....	0.02	0.00	0.31	0.36	0.91	1.22	2.26	2.71
Vertical soil stress,[a] σ_1	....	....	0.52	0.67	1.03	1.13	2.01	2.12	3.88	4.34
"Modulus of elasticity"[c]	26.7	27.5	54	67	75	90	114	134	176	282
Maximum deviator stress	0.46	0.42	0.73	0.92	1.26	1.50	1.62	2.64	3.13	5.31
Void ratio at maximum stress	0.762	0.762	0.703	0.670	0.672	0.630	0.587	0.573	0.533	0.512

[a] These are values from tests at 1.0% strain. [b] Slope of the deviator-stress versus axial-strain curve. [c] Computed from Eq. 2.

$$E = \frac{1}{\epsilon} (\sigma_1 - 2 \mu \sigma_3) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which ϵ = unit strain; σ_1 = vertical soil stress; σ_3 = lateral soil stress; and μ = Poisson's ratio.

The preconsolidation load for this clay was approximately 1.3 kg per sq cm. The data in Table 2 show the decrease in void ratio of good undisturbed specimens under hydrostatic pressures less than the preconsolidation load and the difficulty of separating the effects of change in void ratio and of the applied hydrostatic pressures on the observed increases in strengths and "moduli of elasticity." The increases in test strengths with increasing lateral pressures, which have been attributed to an angle of internal friction in undisturbed clay, may, in fact, be due entirely to decreases in void ratio. Observations by Professor Terzaghi on the field behavior of undisturbed soft clays (22) (23) indicate a zero effective angle of internal friction in natural deposits of considerable depth.

The fact that most undisturbed clay specimens consolidate during triaxial compression tests apparently overbalances the advantage of the test as a method for measuring the strength and elastic properties of soil in its natural condition. It is possible that quick tests and tests under hydrostatic pressures less than the preconsolidation loads may be used successfully on good undisturbed samples to predict actual soil properties. With present methods the possible results do not represent an improvement over the results of unconfined compression tests.

Applications of Results of Laboratory Tests on Undisturbed Samples

The value of the preceding analyses of the effects of sample disturbance on laboratory test results, and indeed the value of laboratory testing of undisturbed samples, depend on the extent to which the results can be applied in practical foundation work. All existing methods for the solution of problems in earthwork and foundation engineering can be placed in one of three categories, each of which uses test results in a different way and to a different extent. In terms of their underlying philosophies these three arbitrary groups can be called empirical, theoretical, and semi-theoretical. If a solution method is based purely on mathematical analysis of an idealized material, it is theoretical; if it is based on theory modified by field observations, it is semi-theoretical; and if it has no theoretical basis or justification, it is empirical. This definition of "empirical" applied to foundation engineering is correct only in regard to the underlying conceptions of the solution methods, some of which (for example, the direct use of unit loads obtained from small-scale field loading tests in foundation design) do not have the support of direct observational evidence.

The true objective of the empirical methods is to establish statistical correlations between soil classes and the field behavior of structures. This is the approach adopted by the Society's Special Committee on the Bearing Value of Soils for Foundations in the early twenties, by highway engineers, and by agricultural soil scientists. Large volumes of data have been accumulated and synthesized into rules for the construction of highways, dams, and foundations and into building code tables of bearing values. The soil classes in these tables and construction rules are identified by verbal descriptions of their appearance, feel, and odor, and by numerical results of simple classification tests performed on remolded specimens. Therefore the application of empirical methods does not require undisturbed samples. The history of every science, and of foundation engineering, shows that progress solely on an empirical basis is painfully slow. F. M. Baron, Assoc. M. Am. Soc. C. E. (24), quoting long-forgotten engineering articles, demonstrates the fact that empirical foundation design has changed little in the past thirty years and, in fact, has changed only in details in two thousand years. Therefore, the fact that empirical methods do not require undisturbed samples cannot be considered a serious indictment of laboratory testing of such samples.

On the opposite extreme, purely theoretical methods proceed through mathematical analyses of forces, stresses, and strains to numerical solutions for deformations or for limiting conditions of stability. Problems in soils engineering, involving large, continuous, indeterminate masses, require the assumption of simplified physical properties for the soil for mathematical solution. The degree to which the assumed properties deviate from the real soil properties depends on the theory. The "classical" soil theorists, for example, Coulomb and Rankine, assumed that the limiting strengths of soils depended only on friction and cohesion without any reference to strains or deformations. Theoretical methods are increasingly popular today because they are the familiar approach of applied mechanics and because the powerful tools of theories of elasticity and plasticity, which are based on strains and deformations, have become widely known. A simple example is found in theory of elasticity solutions for the settlements δ_c at the centers of loaded areas carried by an infinite depth of homogeneous elastic material, all of which can be reduced to the form:

$$\delta_c = C \frac{P\,b}{A\,E} (1 - \mu) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which P is the total load on the area, A and b are the area and width of the loaded area, and C is a numerical constant which depends on the shape and rigidity of the loaded area. Both lateral squeezing and volume change of the soil contribute to settlements. As shown by Professor Casagrande (25), Poisson's ratio, μ, is equal to 0.5 when no volume change occurs and to zero when settlement is due exclusively to volume change. Thus, the equation can be written:

$$\delta_c = C \frac{P\,b}{A} \left(\frac{0.75}{E} + \frac{1.0}{K} \right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which K = the bulk modulus for the soil. In this form, E may be determined from unconfined or triaxial compression tests and K from consolidation tests, both on undisturbed samples. Laboratory tests on undisturbed samples have shown that the "modulus of elasticity" determined from either type of compression test may not be the effective modulus for the natural soil and furthermore that the modulus may increase with increasing depth and confinement of surrounding soil. The theory assumes that both E and K are constants throughout the soil mass. Derivation of an effective bulk modulus for the natural soil from consolidation test results poses a similar problem. Neither the theory nor its applied results are more accurate than are laboratory test results in relation to natural soil properties and both depend on the extent to which assumed physical properties approximate natural conditions. Rigorous theoretical solutions are deceptive because they have the superficial aspect of precision. Nevertheless, for progress and development in foundation engineering the theoretical method is preferable to an empirical approach which is unrelated to theory because, paradoxically, in a few years, theories have given foundation engineers a means of transmitting to others experience with the actual behavior of soil in construction operations which was lacking after hundreds of years of empiricism. The present situation in soils engineering is precisely parallel to that of structural engineering in the nineteenth century.

Cautious and intelligent use of theoretical conceptions and methods is the hope of the future in foundation engineering. The restrictive assumptions of rigorous theoretical methods mean that they can never be applied safely to soils without full-scale field observations to determine their limitations. This leads to a modified or semi-theoretical method in which rigorous theoretical solutions for idealized materials are used as a guide to the phenomena which may affect field performance and as a basis for field observations. Theoretical solutions are then modified and simplified into a workable form with a degree of accuracy compatible with the accuracy of determination of soil properties from tests. Tests on undisturbed samples are needed in this method both in conjunction with field observations and, later, in applications to design studies, because unpredictable, erratic effects of severe sample disturbance lead to unsafe inconsistencies in the correlation of tests with observations. The test results are corrected as well as possible for effects of minor sample disturbance and other sources of error and are used with the simplified theory. Field observation data sufficiently complete to be used in this way are rare and, in many problems, engineers must still struggle with empirical or theoretical methods supported by hypotheses based on indirect evidence. An outstanding example of the semi-theoretical approach is the Swedish circular-arc method for the analysis of stability of slopes. It is based on adequate field observations and sound mechanics and has been confirmed repeatedly at the expense of purely theoretical methods.

The problem of determining safe bearing values for footing design affords an excellent comparison of these three approaches to foundation analysis. Followers of the empirical or "practical" method consult tables of bearing value for the soil immediately below the footings or perform small-scale field loading tests on that soil and use the results directly in design. The theorists realize that stresses in the soil at considerable depths as well as immediately below the footings affect the performance of the structure. They solve for stresses and deformations by the theory of elasticity. From these solutions bearing values for design are selected which will cause the soil stresses and deformations to remain within acceptable limits, assuming that soil has all the properties of an ideal elastic solid. The semi-theoretical or realistic method accepts the conceptions brought out by the theoretical solution and retains its basic principles but, since soil is neither homogeneous nor elastic, it employs simple approximations to compute soil deformations and limiting footing loads, for example the Terzaghi, modified Prandtl, or Krey methods for approximation of ultimate bearing capacities. Undisturbed sampling and accurate laboratory testing of the samples are a vital part of the theoretical and semi-theoretical methods. The major difference between these two methods in regard to samples and laboratory testing appears in the way in which the test results are interpreted and used.

CONCLUSION

Thoughtful evaluation of possible methods for progress leads to the conclusion that laboratory testing of undisturbed samples will play an increasingly important rôle. As rational methods become more reliable the necessity becomes greater for evaluating the effects of sample disturbance on laboratory test results and for extrapolating from the test results to actual soil behavior. The effects of sample disturbance discussed in this paper show that laboratory testing becomes reliable only for samples which have undergone a minimum of disturbance. In the writer's experience only 8-in. to 12-in. cubical samples cut out by hand in test pits fall in this category. If sampling equipment and careful techniques can be evolved which produce samples which are comparably undisturbed, they are well worth additional effort and cost in sampling operations. The most costly sampling operations are those in which large boring samples are obtained in a partly disturbed condition because laboratory tests on such samples yield results that have little relation to reality and in which the effects of previous stress history and actual properties have been so distorted that a reconstruction of the probable true conditions is impossible.

An attempt has been made, in this paper, to extrapolate from observed data to the true natural properties of the soil, with some support from field observational evidence. This method leads to the following conclusions:

(1) Consolidation tests on good undisturbed clay specimens yield results that can be extrapolated to reasonably close approximations of the stress-volumetric properties of natural deposits.

(2) The effects of sample disturbance on direct shear tests have not been evaluated.

(3) Unconfined compression tests on specimens from undisturbed samples cut out by hand in pits or excavations yield compressive strengths that approximate natural strengths reasonably well, but may not produce stress-strain curves or elastic constants comparable with those of the natural soil.

(4) Triaxial compression tests on consolidated specimens of undisturbed clays do not yield properties comparable with those of natural deposits because even a minimum of sample disturbance results in consolidation under lateral pressures to test void ratios smaller than those of the natural deposit.

(5) Laboratory testing of undisturbed samples is valuable in the analysis of engineering soils problems only if its limitations are known and strictly observed.

In the writer's opinion, undisturbed sampling and laboratory testing of undisturbed samples are worthwhile provided:

(6) The economic importance of decisions based on the results of an analysis and the degree of reliability of the solution methods for the particular problem justify the cost of obtaining the best possible samples.

(7) Care is taken that samples undergo an absolute minimum of disturbance.

(8) The differences between test specimens and soil in nature are understood and can be evaluated.

(9) The test results are applied with an understanding of their limitations and the limitations of the solution methods.

APPENDIX

BIBLIOGRAPHY

(1) **"The Structure of Clay and Its Importance in Foundation Engineering,"** by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., Vol. 19, No. 4, April, 1932, pp. 168–209.

(2) **Progress Report of Special Committee on Earths and Foundations,** *Proceedings*, Am. Soc. C. E., May, 1933, pp. 795–814.

(3) **Settlement of Structures,** Section F, thirty papers and discussions in *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936.

(4) **"Settlement of Structures in Europe and Methods of Observations,"** by Karl Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 1432–1448.

(5) **"Settlement Studies of Structures in Egypt,"** by G. P. Tschebotareff, *ibid.*, Vol. 105 (1940), pp. 919–945.

(6) **"A Comparison Between the Results of Loading Tests, Settlement Analysis Based on Consolidation Tests, and Observed Settlements,"** by R. F. Dawson, *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, July, 1940, pp. 304–311.

(7) **"Recent Developments in Soil Testing Apparatus,"** by P. C. Rutledge, *Journal*, Boston Soc. of Civ. Engrs., Vol. 22, No. 4, 1935, pp. 226–231.

(8) **"The Determination of the Pre-Consolidation Load and Its Practical Significance,"** by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 3, 1936, pp. 60–64.

(9) **"The Present Status of the Art of Obtaining Undisturbed Samples of Soils,"** by M. J. Hvorslev, preliminary draft of report of Committee on Sampling and Testing, Am. Soc. C. E., March, 1940.

(10) **"Undisturbed Clay Samples and Undisturbed Clays,"** by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., Vol. 28, No. 3, July, 1941, pp. 211–231.

(11) **"Erdbaumechanik auf bodenphysikalischer grundlage,"** by Karl Terzaghi, Vienna, 1925.

(12) **"Theorie der Setzung von Tonschichten,"** by Karl Terzaghi and O. K. Fröhlich, Vienna, 1936.

(13) **"Symposium on Shear Testing of Soils,"** by M. J. Hvorslev, G. P. Tschebotareff, C. A. Hogentogler and E. C. Barber, J. D. Watson, D. W. Taylor, D. M. Burmister, W. S. Housel, and L. W. Hamilton, *Proceedings*, A.S.T.M., Vol. 39, 1939.

(14) **"Technique of Determining the Shearing Strength of Soils,"** by B. K. Hough, Progress Rept. of Special Committee of the Soil Mechanics and Foundations Div. on the Technique of Soil Tests, *Proceedings*, Am. Soc. C. E., February, 1942, pp. 255–263.

(15) **"Ueber die Festigkeitseigenschaften Gestoerter Bindigerboeden,"** by M. J. Hvorslev, *Ingeniorvidenskabelige Skrifter*, No. 45, Copenhagen, 1937.

(16) **"The Shearing Resistance of Soils,"** by L. Jurgenson, *Journal*, Boston Soc. of Civ. Engrs., Vol. 21, No. 3, July, 1934, pp. 242–275.

(17) **"Soil Mechanics Nomenclature,"** *Manual of Engineering Practice No. 22*, Am. Soc. C. E., April 20, 1941, p. 14.

(18) **"Sampling Methods and Laboratory Tests for Chicago Subway Soils,"** by R. B. Peck, *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, July, 1940, pp. 140–150. (*a*) p. 145.

(19) **"Regional Soil Study of Clays from Chicago, Illinois,"** by P. C. Rutledge, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 2, June, 1936, pp. 107–111.

(20) **"Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium":** "Earth Pressure Measurements in Open Cuts, Chicago (Ill.) Subway," by Ralph B. Peck, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), pp. 1008–1036.

(21) **"Shear Failure of Soils,"** by P. C. Rutledge, *Proceedings*, Fifth Texas Conference on Soil Mechanics and Foundation Eng., February, 1942, Pt. I, p. 18.

(22) **"Stability of Slopes of Natural Clay,"** by Karl Terzaghi, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Cambridge, Mass., Vol. I, June, 1936, pp. 162–163.

(23) **"Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium":** "Liner-Plate Tunnels on the Chicago (Ill.) Subway," by Karl Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 981.

(24) **"The Study of Earths—An American Tradition,"** by Francis M. Baron, *Civil Engineering*, August, 1941, pp. 473–476; and discussions by Karl Terzaghi and A. E. Cummings, October, 1941, p. 614.

(25) Discussion by A. Casagrande of **"Distribution of Stresses Under a Foundation,"** by A. E. Cummings, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1123–1126.

(26) **Slutbetankande 31**, Statens Järnvagärs Geotekniska Commission, 1914–1922, May, 1922.

(27) **"Kolvborn, ny borrtyp för upptagning av lerprov"** (The Piston Sampler, a New Sampler for Securing Samples of Clay), by J. Olsson, *Teknisk Tidskrift*, Vol. 55, 1925.

(28) **"Determination of the Consistency of Soils by Means of Penetration Tests,"** by K. Terzaghi, *Public Roads*, Vol. 7, 1927.

(29) **"Settlement Analysis—the Backbone of Foundation Research,"** by K. Terzaghi, *Paper No. 337*, World Eng. Cong., Tokyo, Japan, 1929.

(30) **"The Determination of the Pre-Consolidation Load and Its Practical Significance,"** by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics, Vol. III, 1936, pp. 60–64.

(31) *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 427. (*a*) p. 432.

(32) **"General Wedge Theory of Earth Pressure,"** by K. Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 68.

(33) *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1067.

(34) *Laboratory Manual of Soil Testing Procedures*, by W. S. Housel, Ann Arbor, Mich., 1939, p. 61.

(35) *Proceedings*, Am. Soc. C. E., September, 1942, p. 1184.

(36) **"A Study of Changes in Physical Properties of Putnam Soil Induced by Ionic Substitution,"** by H. F. Winterkorn, and R. B. Moorman, *Proceedings*, Highway Research Board, Vol. 21 (1941), pp. 415–434.

(37) **"The Energy Concept of Soil Moisture and Mechanics of Unsaturated Flow,"** by M. B. Russell and M. G. Spangler, *ibid.*, pp. 435–450.

(38) **"Effect of Time Rate of Loading on the Shearing Resistance of Natural Clay,"** by H. H. Ku, a thesis submitted to the Faculty of Purdue Univ., Lafayette, Ind., in June, 1941, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

DISCUSSION

Benjamin K. Hough, Jr.,[3] **Assoc. M. Am. Soc. C. E.**—Although the paper is entitled "Relation of Undisturbed Sampling to Laboratory Testing," the author has extended his remarks to include a discussion of the broader subject of the practical application of test results in soils and foundation engineering problems. This more comprehensive view of the subject is justifiable since soil sampling and testing are obviously only initial steps in solving practical soil problems and no matter how well they may be performed they cannot be considered an end in themselves. Furthermore, it can be maintained that the degree of precision which is necessary in sampling and testing depends on the nature of the analytical method in which the data are to be used. On this basis, it is possible that in some cases more time and money have been expended in obtaining undisturbed samples than can be justified in view of limitations in the method of applying the test data. For example, expense is often a major consideration in deciding whether to conduct exploration in test pits or by drilling. The writer does not entirely agree with the view that test-pit samples are superior to those from drill holes; but even if this were true, it would not follow that test pits are always preferable unless it is also true that they are, in general, more economical. It is the writer's experience that test pits more than about 5 ft deep, and particularly test pits that are extended below ground water, are considerably more expensive than even fairly elaborate drilling operations and, for this reason, test pits should not be used unless the results obtained clearly justify the additional expense.

In discussing ways of determining whether soil samples have been disturbed and, if so, to what extent, Professor Rutledge refers to data from consolidation and shearing tests on the best obtainable undisturbed samples and on the same material remolded. One of the materials referred to as being practically undisturbed is a volcanic clay from Mexico City which was obtained by manual sampling in an open pit. A characteristic of this clay is that it contains more than 90% voids. It is assumed by the writer that in addition to the care which was exercised in the sampling operation, some very special precautions were taken in packing and shipping this rather delicate material to the Harvard testing laboratory since considerable disturbance might well be caused by rough handling during transportation. In view of the real grounds for suspecting that some injury might have taken place during shipment, the writer feels that it might be advisable to omit this reference or to substitute data from tests made in the locality from which the material was obtained.

A further comment on the author's treatment of consolidation tests is that he has possibly created the impression that if a sample is practically undisturbed the compression-test diagram will have a certain more or less standard character such as that illustrated in Fig. 2. Two features of this idealized diagram, which are not always found in actual practice, are the straight-line "natural-soil" compression curve and the complete return to this curve of each cycle of reloading. That the "undisturbed-soil" curve is not always straight is evidenced by the author's tests on the Mexico City clay, and also by tests recently made by the writer on certain St. Lawrence River clays with an initial void ratio of about 1.6 instead of the unusual value of 14.0. Definite curvature in practically all the semi-log compression diagrams for these Laurentian clays was found to be characteristic. It has also been observed by the writer that, more often than not, release of load and subsequent recompression during a consolidation test has the effect of offsetting the compression diagram somewhat to the left so that the successive cycles fail to form an extension of the original or undisturbed-soil curve. It is not the writer's belief that these discrepancies between actual test curves and the idealized diagram in Fig. 2 are evidences of undue disturbance of the test specimens during sampling operations. The writer agrees, however, that sample disturbance does obscure the preconsolidation point and that great care in sampling must be exercised when a determination of this point is a matter of importance.

---

[3] Capt., Corps of Engrs., U. S. Army, U. S. Engr. Office, Boston, Mass.

In connection with the Terzaghi theory to which the author refers, that the amount of compression produced by application of a given load may be influenced by the rate at which the load is applied, the writer presents some data obtained during tests on the previously mentioned St. Lawrence River clays. At one point where subsurface investigations were made by the writer, the clay deposit is approximately 60 ft deep. The upper 10 ft or 15 ft of this deposit is altered somewhat by oxidation and has dried out to some extent. In the remaining depth, however, the material is remarkably uniform in appearance and even lacks the usual stratification and traces of bedding planes which mark many clay deposits. In this uniform section of the deposit, water-content determinations were made on samples taken at fairly frequent intervals, and it was found that the water content (and hence the void ratio) of the material remained substantially constant with depth. In fact, in one location there was a slight trend toward higher water contents near the bottom of the deposit than at the top. Water contents in all cases, incidentally, even at 60-ft depths, were found to be well above the liquid limit, a fact which is in contrast with the usual assumption that clays are laid down initially at the liquid limit. This clay is classified as a post glacial, marine deposit which has never been subjected to any loading other than its own weight and its age is sufficient to warrant the assumption that the clay is fully consolidated under its own weight. It would therefore appear reasonable to expect that samples taken from the bottom of the deposit would be in a more dense condition than those at the top instead of having practically uniform density as indicated by the data from water content determinations. This apparent discrepancy may be explained by the theory that rate of loading is a very important consideration and that the "sedimentation curve" may be nearly flat instead of an extension of the relatively steep compression curve obtained from laboratory tests. It also has bearing on certain aspects of the shearing strength of clays, a subject which the author discusses in connection with the effects of sample disturbance.

From the aforementioned evidence it appears to be a physical possibility for a clay deposit of considerable depth to exist without variation in void ratio from top to bottom. Despite this fact, it seems undeniable that intergranular stresses must increase steadily with depth in such a deposit, provided it is fully consolidated. Under these circumstances, it might well be expected that the shearing strength of the clay would gradually increase with depth in proportion to the increase of intergranular pressure; yet in the case of the St. Lawrence clay the evidence from drilling records, from visual inspection of the material in place in open test pits, and from an extensive series of direct, unconfined, and triaxial shear tests is that there is no appreciable change in shearing strength with depth. This finding, if valid, is a direct contradiction to the widely accepted Coulomb relation for the shearing strength of soils and establishes grounds for a major revision in many current soil theories. Space does not permit extensive treatment of this subject, but it is interesting to consider the possibility that at least in clay deposits of similar origin, shearing strength is to some extent a function of void ratio and hence water content, rather than being entirely due to intergranular pressure. Laboratory experimentation on this subject would be a difficult matter for, although nature can apparently produce deposits with uniform void ratio yet increasing intergranular pressure, any change of pressure in laboratory samples results in a change of void ratio. If shearing strength is suspected of varying with either void ratio or pressure, it is difficult to determine the basic relationship in a test in which both these factors vary.

To turn briefly to the author's suggested methods of applying test data, the writer agrees that purely theoretical methods are unsatisfactory and also that adherence to purely empirical methods is not always wise; but, of the two, the writer is inclined to favor the latter. The method designated by the author as semi-theoretical might perhaps be termed the rational method. Whatever name is used, however, for analytical methods which combine experience, judgment, and engineering ability, the writer feels that emphasis should be given to the need for careful substantiation of each new development before it is expounded. On the whole, the writer agrees that recent studies in soils engineering have given better insight into the behavior of soils than hundreds of years of empiricism, but it is also true that the path of progress has been considerably cluttered with theories which have been hastily proposed and, with equal or greater haste, discarded.

In closing, the writer suggests that the author's selection of the Terzaghi approximate relationships as an example of the preferred "semi-theoretical" method of applying test data is unfortunate. It is the writer's understanding that these approximations were developed for idealized soil types seldom if ever encountered in actual practice. Furthermore, the writer has found that the Terzaghi approximate relationships are not in agreement with the observed behavior of large-scale field tests, specifically tests to determine the effect of the size of loaded area on the bearing power of cohesionless soils.

F. M. VAN AUKEN,[4] ESQ.—An excellent treatment of the relation of undisturbed sampling to laboratory testing has been presented in this paper. The methods that can be used as "pilot" tests to determine whether or how much a sample is disturbed are particularly enlightening.

Engineers have long recognized that laboratory tests performed on disturbed and partly disturbed soils are not in themselves of great value. Fair interpretation of results is difficult; consequently, the data are usually relegated to the "files" where they well belong.

---

[4] Chf. of Soils Laboratory, U. S. Engr. Office, Denison, Tex.

It is fortunate for the soils engineer that only two major test divisions require the use of undisturbed soil specimens. Once the consolidation and strength characteristics of a soil are known completely, all other tests become mere pawns of the former. However, since the scope of the present paper is limited entirely to plastic soils—soils that under stress generally disclose great volume changes—its scope is somewhat narrow and cannot hope to cover, even by indication, the entire field of undisturbed sampling and testing. Unfortunately this is not the case. The choice of highly compressible clays as the subject for discussion, although they do furnish unusually fine examples for a study of this type, may indicate that it is relatively easy to determine the state of disturbance that invariably accompanies all attempts to sample soils.

Probably no one consideration is of more importance than a complete reconstruction of the geologic history of the soil before it is sampled and tested. In fact, a straightforward dependence on the so-called "preconsolidation" load, as determined by the consolidation test without a thorough knowledge of past geologic history, is probably as bad as not knowing whether a given material is disturbed.

Certain clays, such as the volcanic clays from the bed of Lake Texcoco, Mexico, D. F., for which the geologic history is well known and which are compacted to abnormally high void ratios, make perfect examples of the determination of the preconsolidated load because of the "classic, almost photographic enlargement," of the consolidation test features.

Other clays like the glacial clays from the Great Lakes region, in which the geologic history is more obscure, are difficult to rate with respect to determination of the preconsolidation load and condition of soil. Recently reworked alluvial, plastic clays also present problems identifying sample disturbance. Other soils that present even more difficulty in the determination of condition are marly and shaley clays. Although they are somewhat brittle in their initial state of consolidation, these materials become truly plastic when the structure is broken down and the material remolded. Soils of this type are usually laid down as sedimentary deposits and consolidation effected by the weight of material that the forces of nature have long since removed. It may be probable that periods of recompression have occurred, caused by deposition of additional material, with re-expansion upon its removal, so that the number of compression and expansion cycles to which the material has been subjected is problematic. One may never know whether a soil is now in a period of compression or expansion. Even with an accurate knowledge of that fact the determination of the preconsolidation load by means of the shape of the pressure, void-ratio curve is practically impossible. Additional complications arise when primary expansion occurs.

Although the effect of disturbance is pronounced on the determination of the consolidation characteristics of a soil, it is not a factor that will vitally affect a consolidation study which (because of the number of theoretical assumptions, drainage, and boundary conditions, etc.) is in itself only a fair estimate of expected settlement. On the other hand, the effect of disturbance on the strength of soils is profound. Soils under vertical stress undergo a gradual rearrangement of internal structure as the stress is slowly increased, thereby resulting in consolidation or reduction in volume. Assuming a somewhat flexible confining restraint such as would be afforded a loaded element in a soil mass, the investigator will note that, as the vertical stress is increased, although consolidation is occurring, a gradual approach to a state of shear is apparent. Hence, a semiconfined stressed soil is only in a transitory stage which, as pressure is increased, rapidly progresses into fully developed shear, and when no further strain can be taken either ruptures suddenly or undergoes metamorphosis and fails in plastic flow. Disturbance in sampling merely places a soil well along in that vague but nevertheless always present stage in which the application of even a minor stress could produce failure. Therefore, regardless of testing technique, tests that are used to measure the strength of soils should show a corresponding percentage decrease in strength if a material is disturbed during or after sampling.

The writer concurs with Professor Rutledge in passing over the direct shear test rapidly, not because the effect of disturbance cannot be ascertained, but because the test in itself furnishes data that are difficult to apply. Although the slope and shape of the stress-strain curve may furnish data from which possible disturbance of a soil specimen may be detected, it is quite probable that this is true only for sensibly plastic materials. Unfortunately the ordinary range of soils runs the gamut of plastic to brittle materials; hence, it may be that internal grain structure, pseudo structures set up by fusion or cyclic desiccation, relation of moisture content to capillary and chemical forces, have as much to do with the determination of the apparent shape of the stress-strain curve as possible disturbance. The simple compression test is probably a better test than the direct shear test, but since the application of a primary vertical stress sets up secondary stresses over which the investigator has no control it appears that the triaxial test would have more significance.

Professor Rutledge has condemned the triaxial test for the simple reason that some specimens tend to consolidate during the test, and since disturbance accentuates consolidation, as shown by consolidation test research, unmeasurable errors may be introduced. The writer takes exception to this theory on the basis that the approach to the shear problem absolutely necessitates volume changes. In a coplanar system, any loaded mass of soil will be subjected to major and minor stresses which, for given boundary conditions, have some definite ratio. To alleviate the effect of preconsolidating a test specimen prior to application of deviator stresses, there are a number of methods of doing the triaxial compression test. One such method is to set up a particular stress ratio $\frac{\sigma_1}{\sigma_3}$ and maintain that ratio until the volume of the soil is stabilized; then increase the major principal stress until failure occurs. In this manner the investigator has attempted to simulate what may happen in nature.

Professor Rutledge has shown a plot of stress-strain curves for the so-called quick and slow triaxial compression tests, but it is pointed out that no effect of disturbance can be ascertained from a singular comparison of these tests. In addition, the final strength obtained from the slow test is a function of time; that is, one can obtain increasing strength values depending on whether the strain is applied at the rate of 1% per minute, per hour, per day, or per month.

This fact has been substantiated by slow triaxial compression tests performed in the writer's laboratory in conjunction with the design of a dam where some tests were continued nearly 12 months before failures were obtained. Tests on remolded specimens, as in the case of the simple compression tests, disclosed far lower ultimate strengths and flatter stress-strain slopes.

If it is assumed that a perfectly undisturbed sample retains more elasticity than a disturbed specimen (that is, the structure set up by nature has not been sensibly altered), one can determine with a certain degree of success the effect of disturbance by cyclic loading, provided the strain has not exceeded 25% to 50% of the ultimate. Other triaxial compression-test series, performed in the writer's laboratory, have shown that the apparent preconsolidation load can be determined approximately by the proper combination of quick and slow tests, provided the samples were truly undisturbed.

A review of the various proposed methods of determining how much disturbance affects soil test results shows that all methods are still problematic in aspect. That disturbance does affect both consolidation and strength values is a fact that cannot be overlooked. However, during any complete soil investigation, the scope of the problem is often such that only pilot tests to determine how much disturbance is being caused by the adopted sampling methods can be performed. On the basis of those tests one must either discard or improve the sampling technique or recognize the limitations of the soil test data obtained. In years past, soils engineers, although probably recognizing the effect of sample disturbance on test data, were prone to pick the "yield point" from the stress-strain curve, resulting in the use of values far below the ultimate ones. However, during the past few years, ultimate values are more generally used, but the one redeeming feature that has covered many errors is the fact that sample disturbance generally lowers the obtained values.

Professor Rutledge's paper will have accomplished its prime purpose if only through stimulation of a timely subject. In closing, the writer wishes to emphasize two facts often overlooked and having a decided effect on the accuracy of soil-test data. Those facts are "too long" a time lapse between sampling and testing, and "too little" emphasis on the choice of sampling and testing technique.

JACOB FELD,[5] M. AM. SOC. C. E.—The careful analysis of the problem of undisturbed soil samples, in which the author lists proof of each point, is a contribution to the subject of soil mechanics. The identification of clays by the shape of the stress-strain curve may well be considered a means of classification sufficiently accurate to be used as a design tool. Many structures are planned to be located on clay soils, where the cost of expensive sampling and testing has not been warranted. If the engineer is certain that the samples and the tests will indicate correctly the future action of the structure, he might be justified in ordering the expenditure. The use of full-scale data from previous experience (and troubles) with foundations as a basis of design of other structures is not feasible without a guarantee that the soils encountered are similar. The author's method can be used to prove similarity, even without the exact sampling procedure.

---

[5] Cons. Engr., New York, N. Y.

The action of clay soils under loadings, in test samples and in nature, can be described by an analogy. Assume the existence of a large continuous bed of sponge living at the bottom of the sea. The density of the sponge is the result of the hydrostatic pressure above it, within it, and around it, together with its skeleton strength. Change in water depth will affect the density immediately, subject to rebound with time as the internal water pressure (from osmosis) changes to balance the new loading. A load imposed on the sponge bed will settle, immediately due to compression of the skeleton and of the water and additionally as the water under the load is pressed outward, to the extent that the adjacent sponge will absorb the surplus. The volume change in the sponge under the load will affect the adjacent materials.

Excavating a test pit for determining sponge conditions at any depth is likely to yield unreliable results, because of the expansion at the new surface as soon as the pit is dug. The job of removing an "undisturbed" sample for testing, without changing the physical characteristics, is almost hopeless. However, if a sample can be subjected to approximately the same exterior forces and moisture conditions in a laboratory test, its natural reactions under new loading can be determined. This paper indicates how one can be assured that the laboratory procedure reproduces original or natural conditions (natural void ratio and preconsolidation load). Such assurance will go far toward instilling confidence in the value of soil sampling and of soil testing.

RAYMOND F. DAWSON,[6] M. AM. SOC. C. E.—Because the expansive clays of the Southwest present unusual problems as foundation materials, a comparison of these soils with those included in Professor Rutledge's paper is of interest. The writer has never found an undisturbed sample with all the voids filled with water. The initial degree of saturation usually ranges between 94% and 99% and has been found as low as 89%. Since many of these samples were taken at a considerable depth below the surface, where ample water should have been available to fill the voids, this indicates a swelling of the specimen. Perhaps the initial degree of saturation is a measure of disturbance in these clays.

These samples continue to expand at a very slow rate after they have been removed from the ground. Specimens covered with paraffin and stored in the moist room will eventually expand enough to break the covering. With some samples this will occur in only a few weeks whereas others require many months to produce failure. This condition has caused local laboratories to apply two special procedures to the consolidation test: Samples are tested as quickly as possible after being removed from the ground; and the initial void ratio is held constant until sufficient load has been applied to offset the swelling pressure. The latter step is accomplished by applying small increments of load to the specimen after it has been placed in the consolidometer, the amount of load applied being sufficient to prevent movement of the dial gage. When no further load is required to keep the dial constant, the sample is loaded in the usual manner, starting with the next increment of load generally used in the consolidation test. On the pressure, void-ratio, curve this part will be a

---

[6] Associate Prof., Civ. Eng. Testing Engr., and Asst. Director, Bureau of Eng. Research, Univ. of Texas, Austin, Tex.

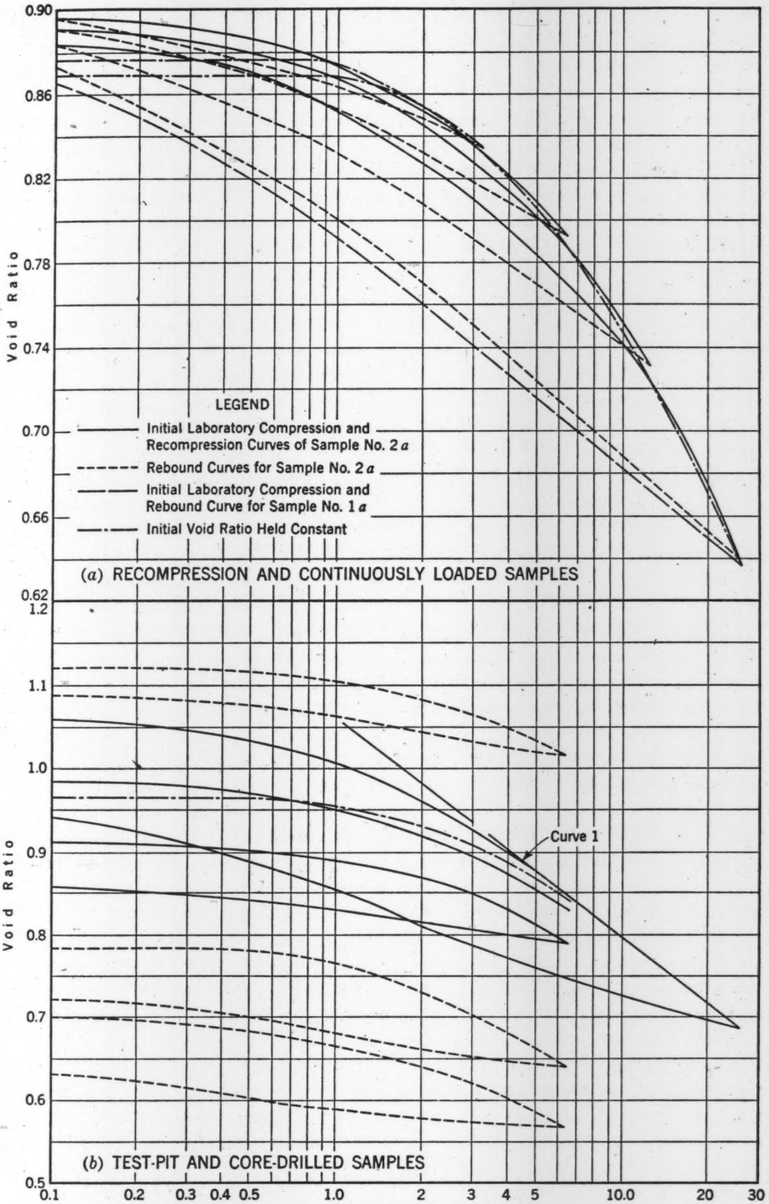


FIG. 12.—EFFECT OF SWELLING ON THE CONSOLIDATION CHARACTERISTICS OF EXPANSIVE CLAYS IN THE SOUTHWEST

straight line since the void ratio is constant and is usually indicated by a broken line.

In order to study the effect of swelling on the consolidation characteristics of these soils the test shown in Fig. 12(*a*) was made. A sample was prepared in the usual manner and the initial void ratio was held constant, as indicated by the dash-dot line. The soil was then loaded in increments to a total pressure of 3.2 kg per sq cm and the load removed to produce the rebound curve. With a load of 0.1 kg per sq cm (200 lb per sq ft) the specimen had expanded considerably above its original void ratio and the recompression curve started. In each cycle of recompression the maximum load was twice that used in the previous cycle. The recompression curves indicate that each cycle of loading changes the slope of the lower portion of the curve in a downward direction. The results of a consolidation test, made in the usual manner on a sample of similar soil, are shown in Fig. 12(*a*) by the dashed line. This indicates that the change in character of the curves due to recompression is not as important as the recompression curves alone would lead one to believe. The soil used in the recompression tests (sample 2*a*) had an initial void ratio of 0.869 and initial degree of saturation of 98.5%, whereas the continuously loaded specimen (sample 1*a*) had an initial void ratio of 0.877 and an initial degree of saturation of 97.5%. Continuous loading tests on other samples from the same site gave pressure, void-ratio, curves with shapes similar to sample 1*a* but with higher or lower initial void ratios.

Although it is agreed that cubical samples, taken carefully from a test pit, give more nearly undisturbed test specimens than those obtained by any other method, it may not be possible economically to use this procedure, especially in the case of a deep stratum. In 1942, the writer supervised the sampling of clay soils on two projects where it was impracticable to use test pits and where no special sampling spoon was available. On both jobs samples were taken by using what the drillers call a "poor-boy" sampler. This sampler consisted of a piece of seamless steel tubing, $\frac{1}{8}$ in. thick, with a 5-in. outside diameter. The lower edge was sharpened, but the 12-in. long tube had the same internal and external diameters for the entire length. Holes, 6 in. in diameter, were advanced by usual drilling methods until the desired depth was reached at which point the sampling tube was inserted for the drilling bit. The sampling tube was forced into the soil for a depth of 8 to 10 in. by means of a hydraulic jack and the bit rotated one turn to shear off the soil at the bottom of the bit. When it was necessary to advance the hole by wet drilling, the top 2 to 3 in. of the core was removed and discarded. As soon as the cores were removed from the tube, they were coated with several layers of paraffin and cheesecloth and tested immediately after being transported to the laboratory. These sampling tubes had an area ratio (9) of less than 10% as compared with about 40% for the usual sampling spoon. Therefore, they probably caused less disturbance than some of the more elaborate sampling devices. On one project a seamless steel tube only $\frac{1}{32}$ in. thick was used part of the time. This gave an area ratio of about 5% and proved satisfactory until a pebble which forced it out of shape was encountered. Visual examination showed very little disturbance of cores taken by this method.

On one project the investigators were able to compare the results obtained on cores taken as outlined, with the consolidation characteristics of the same soil as determined from cubical samples cut carefully from test pits; Fig. 12(*b*) gives this comparison. The solid-line curves were obtained for specimens taken from the test pits and the broken-line curves were for the core-drilled samples. There is little difference in the shape of the curves, indicating either (*a*) that there was practically no disturbance in taking the samples with the core drill; or (*b*) that the entire soil deposit is in a disturbed condition.

A further examination of all the curves shown in Fig. 12 might lead also to the conclusion that the soils were disturbed because they do not possess the general shape indicated by Professor Rutledge and most of them do not have a straight-line section. This is a result of the type of soil rather than disturbance at the time of sampling. When high consolidation loads are used on these soils, the lower portion does become a straight line as shown on curve (1), Fig. 12(*b*). Such a curve indicates a preconsolidation load of about 2 tons per sq ft, which is about twice the weight of the overburden; and, since the clays are of recent geological origin, one would not expect that any great amount of overburden has been removed. These clays have been badly fissured and break with slickensided surfaces, many of which contain thin layers of clay of a different color. It is assumed, therefore, that the clay dried and cracked after it was deposited and that the next flood filled the cracks with the lighter-colored clay; then, when the original clay expanded by taking up moisture, internal stresses developed because the cracks were filled, therefore causing the high preconsolidation load.

Engineers and investigators should underline Professor Rutledge's statement (paragraph following Eq. 4):

> "Cautious and intelligent use of theoretical conceptions and methods is the hope of the future in foundation engineering. The restrictive assumptions of rigorous theoretical methods mean that they can never be applied safely to soils without full-scale field observations to determine their limitations."

Certainly many more full-scale field observations accompanied by complete laboratory analyses must be conducted before final conclusions may be reached.

HAMILTON GRAY,[7] JUN. AM. SOC. C. E.—The principal steps involved in a foundation analysis have been aptly likened by the author to "links in a chain" of procedures which are intended to reach a definite point that may be considered the solution of the problem at hand. Conditions which produce inaccuracies in any link—figuratively changing the length of the link—tend to affect the length of the chain and hence the distance by which it fails to reach the desired terminal point. Such inaccuracies, of course, may be compensating, the effect of a "short" link being offset by that of a "long" one; but such an occurrence would be purely fortuitous. A close agreement between predicted and observed behavior associated with any physical phenomenon does not necessarily assure that each of the steps involved in making the prediction was flawless. Similar methods may not give as "satisfactory" results when applied another time. Hence, if the methods of soil mechanics are to achieve their full usefulness, the magnitude of the inaccuracies in the individual links must be known quantitatively so that the error in the length of the chain may be determined with reasonable certainty. Actually this chain consists of three major links, which, together with the chief source of error in each, may be listed as follows:

---

[7] Asst. Prof., Civ. Eng., New York Univ., New York, N. Y.

(1) Sampling (distortion of the soil and hence alteration of its physical properties during the sampling process);
(2) Testing (inappropriate testing methods for determining the pertinent physical properties); and
(3) Prediction (unsuitable analytic methods for predicting the behavior of the underground on the basis of values determined under Item 2).

If one were in a position to evaluate, confidently, the magnitudes of the errors inherent in each of these three steps, the science of foundations would be practically complete. Professor Rutledge has made an encouraging start toward determining the effect of imperfect sampling on certain of the physical characteristics which are determined by experimental means. It is clear, however, that the numerical and graphical comparisons are made between samples, and not between samples and the soil in place. Consequently, it remains difficult to know quantitatively how truly a sample represents its parent stratum.

Only by comparing properly conducted field observations with the solutions secured by means of steps 1, 2, and 3 in a large number of cases can one ever hope to determine the sources of discrepancy between predicted and actual observed behavior. In the present state of knowledge, an agreement between prediction and field behavior does not assure perfection of all links in the chain; nor may a discrepancy between prediction and behavior definitely be attributed to any one of them. One simply cannot state, in the light of present knowledge, what quantitative influence is exerted by inaccuracies in any or all of the three steps enumerated. Hence, only rough estimates can be made of foundation behavior.

The paper does suggest means of forming qualitative opinions as to the possible value of a sample in representing the stratum from which it was secured, and shows, further, that a given degree of soil disturbance influences certain physical properties more than others. This signifies, for example, that great caution should be employed in utilizing the modulus of elasticity as obtained from unconfined compression tests. On the other hand, the compressive strength secured in the same test is more nearly representative of the behavior in nature. Although it is easy to establish a numerical measure giving the degree of disturbance intermediate between complete remolding (100%) and the best available undisturbed sample (0%), comparison with the behavior of the parent stratum remains a matter of guess or judgment.

In connection with the consolidation test, the author leaves the impression that a pressure, void-ratio curve for a loose clay, which shows upward concavity (see Fig. 5) instead of being precisely linear (see Fig. 2), when plotted on semilogarithmic paper represents a minimum amount of disturbance. The writer has observed such upward concavity in loose clay samples of 2.5-in. diameter, which were taken in a sampling spoon (3-in. inside diameter) of a type no longer considered suitable for securing first-class undisturbed samples, and believes that there was no minimum of disturbance in these cases. Further, since a sample taken by spoon is generally subject to the greatest amount of distortion at its periphery, one is led to consider the extreme case of a "composite" sample consisting of a wholly undisturbed core of diameter, say 3 in., surrounded by a completely remolded soil annulus of the same material to give a total sample diameter of 4 in. When subjected to a consolidation test, the deformation of both core and soft annulus will be identical, but the annulus will be subject to a far lower stress because of its different pressure, void-ratio diagram (see Figs. 3, 4, and 6). An analysis of the loading and deformation of such a sample leads to the conclusion that the resultant pressure, void-ratio diagram would be slightly concave upward at the lower end just as are the curves in Fig. 5. Until further information is available, therefore, it would seem wise to accept, with definite reservation, the author's statement concerning the evidence of a minimum of disturbance. The foregoing inference should be easy to check experimentally for the behavior of composite samples of various clays.

In discussing the triaxial compression test the author touches upon the somewhat controversial subject of the proper type of test to be made in simulating certain field conditions. Similar differences of opinion exist in connection with quick and slow shear tests. The type of test to be used should be one which either simulates field behavior, or gives results that may be used somehow in analyzing conditions of soil in place. If two or more kinds of tests can furnish usable results in a given set of circumstances, that one which is performed more readily probably will be used the more widely. On the other hand, the method of testing always must be adapted to the type of analysis employed in predicting field behavior. For example, the author cites Eq. 4 for computing settlements. The use of this formula entails the previous determination of the soil properties, defined by K and E, which in turn require two types of tests. There are various other ways of estimating settlements, each with its proponents, and each necessitating knowledge of certain physical properties, determination of which may require more, or less, experimental work than do the constants of Eq. 4, depending on the details of the method. In all probability, the method that is best never can be decided by theory alone; but, in time, the compilation of observed behavior may give a clue as to the most reliable method.

A compilation of observational data of similar nature from several foundations which differ somewhat in such details as—(*a*) relative positions and thicknesses of soil strata; (*b*) size of loaded areas; (*c*) increase in net load; and (*d*) sampling technique—might permit a segregation of the inaccuracies inherent in the various steps involved in making a forecast of behavior. The data from a single foundation may or may not suffice to justify assigning the source of any discrepancy between prediction and observation to one of the steps employed in making the prediction. To cite a specific example, assume that a building imposing a uniformly distributed net increase in pressure of 1 ton per sq ft at the level of its foundation rests on a bed of dense sand. The sand extends to a depth of 15 ft below the foundation and is underlain by compressible clay overlying rock. If the building covers a very large area and the clay stratum is thin, any variation between observed and predicted subsidences may be attributed to disturbance during sampling or to improper testing technique with the former constituting the more probable source. On the other hand, if the area covered by the building is relatively small and the clay stratum thick, the variation of pressure increase with depth is introduced. This brings up the question of the applicability of the "Problem of Boussinesq," and, in addition, the more difficult matter of the effect of the sand layer in distributing loads to the more compressible clay. This latter problem in all probability cannot be treated satisfactorily by analysis alone, and hence it appears evident that a widespread collection of observational data, followed by an intensive study of its significance, would be required before an understanding of such conditions is reasonably complete. However, as suggested herein, a relatively small amount of data probably would suffice, under favorable conditions, to determine the influence of sample disturbance on the laboratory test results alone. That modification of the test results requisite to predict the actual behavior of the foundation then could be adopted readily. However, without the correlation between laboratory results and observed field behavior, the magnitude of the desired modification cannot be known with any certainty.

KARL TERZAGHI,[8] M. AM. SOC. C. E.—An up-to-date inventory of what is known or believed concerning the influence of the disturbance due to sampling operations on the physical properties of cohesive soils has been presented by Professor Rutledge. Since the growing realization of the importance of this influence constitutes a vital phase in the development of soil mechanics, some data concerning the historical background of this phase may be of interest.

The credit for the discovery of the mechanical effects of remolding belongs to the Swedish Geotechnical Commission. The final report of the commission was published in 1922 (26). The report contains many examples of the mechanical effects of remolding and the description of a method for expressing the sensitivity of the structure of the clay by numerical values. Opinion concerning the physical causes of the observed phenomena still is divided. In 1932, Professor Casagrande proposed a hypothesis according to which the sensitivity of the structure of the natural, undisturbed clays is due to a peculiar arrangement of the clay particles, which he called the clay structure. On the other hand, the writer believed—and still believes—that it is chiefly due to slow physico-chemical processes which take place in any clay. Fortunately the issue is only of theoretical interest.

The work of the Swedish Commission initiated the development of the technique for securing undisturbed samples. The first sampling tool of the modern type was proposed by John Olsson in 1925 (27). However, since undisturbed sample borings are much more expensive than the traditional wash or auger borings, it required many years for engineers and contractors to accept the new methods of sampling. In the meantime, investigators had to be content with whatever samples they could afford. Even the so-called drive samples were considered a luxury.

---

[8] Winchester, Mass.

Several years after the publication of the Swedish report, the writer investigated the influence of remolding on the modulus of elasticity of clays (28). In those days, consolidation tests still were made on completely remolded samples. In order to correct for the influence of remolding on the relation between pressure and void ratio, the graphical method illustrated in Fig. 13 was used (29). In Fig. 13 the symbol e_0 indicates the initial void ratio and p_0, the vertical unit pressure which acts on the clay in the ground at the depth where the sample was obtained. The curve C_1 represents the results of a consolidation test on a remolded sample. On this curve the pressure p_0 corresponds to a void ratio e_1 which is much smaller than the void ratio e_0. The difference between e_0 and e_1 represents the effect of remolding on the void ratio at a given unit pressure p_0. It seemed obvious that the clay in its natural state, with a void ratio e_0 at a pressure p_0, should be more compressible than the remolded clay, with a smaller void ratio e_1 at the same pressure p_0. Therefore, the writer assumed that the pressure, void-ratio curve for the clay in the ground can be determined approximately by multiplying the ordinates of the C_1 curves by $\frac{e_0}{e_1}$. Thus he obtained the curves C_2.

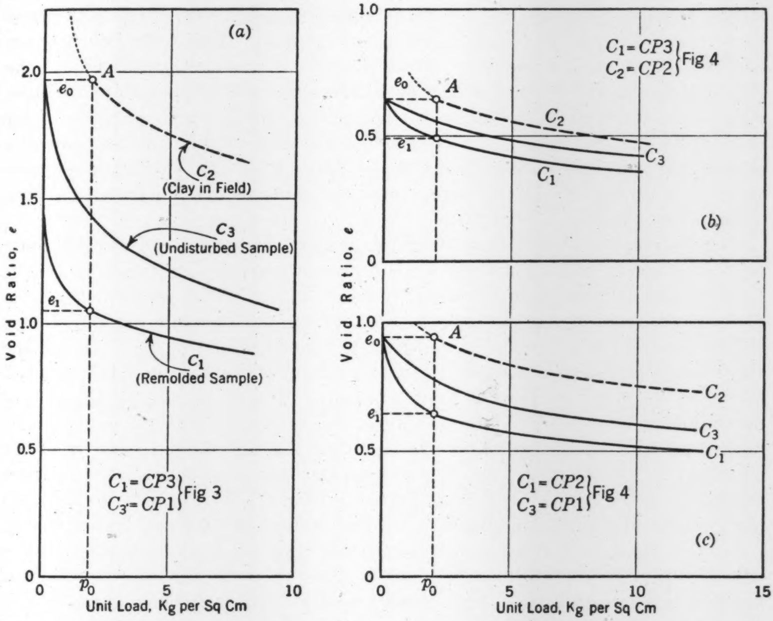


FIG. 13.—GRAPHICAL METHOD FOR ESTIMATING THE RELATION BETWEEN PRESSURE AND VOID RATIO FOR NORMALLY CONSOLIDATED CLAY IN THE FIELD (CURVES C_2) ON THE BASIS OF RESULTS OF CONSOLIDATION TESTS ON REMOLDED SAMPLES (CURVES C_1)

Curves C_1 in Fig. 13 were plotted by means of the data contained in Figs. 3 and 4. The corresponding pressure, void-ratio curves for undisturbed samples are represented by curves C_3.

In 1932 Professor Casagrande published, for the first time, the results of consolidation tests on undisturbed clay samples. Curves C_3, Fig. 13, are examples of the general character of the curves thus obtained. In the diagram they occupy a position intermediate between the pressure, void-ratio curves for the remolded samples and curves C_2 which represent very roughly the pressure, void-ratio curves for the clay in the ground. The difference between curves C_2 and C_3 is a measure of the increase of the compressibility of the clay due to the disturbance of the structure by sampling operations. Settlement computations require a knowledge of the relation between p and e for the clay in the ground. In order to determine this relation on the basis of a consolidation test on undisturbed samples (curves C_3, Fig. 1), certain assumptions must be made. In the paper these assumptions are illustrated by Figs. 6, 7, and 8.

To the general reader, these diagrams may be somewhat confusing because no distinction is made between what is known and what is merely believed. Each of the diagrams contains two rebound curves. The upper one, labeled (2) in Fig. 7, is assumed to represent the rebound during sampling and storage before test. It is generally agreed that the increase of the void ratio of the clay during sampling is insignificant and that, after the sample is taken out of the hole, its water content is kept constant. Therefore, the processes that occur during sampling and storage should appear in the diagrams as a straight, horizontal line whose ordinates are equal to the initial void ratio e_0 of the clay; yet in the diagrams the line which corresponds to these processes rises as steeply as the laboratory rebound curve (curve (7), Fig. 7), which represents the swelling of the clay in contact with free water after the removal of a load.

If the upper rebound curve is eliminated and replaced by a straight line with the ordinates e_0, all the other constructions shown in the diagrams become more or less meaningless. The writer proposes that they be replaced by the simple diagram Fig. 14, which leaves no doubt about what is known and what is believed. In Fig. 2(*a*) (which shows the relation between pressure and void ratio for different samples of a normally consolidated clay) the full-line curves represent empirical curves. The dashed and dotted curves are lines obtained by inference.

While the pressure on the clay increased in the field during the process of sedimentation from its initial value, zero, to the final value p_0, the void ratio of the clay decreased as indicated by the gently sloping curve ab. This curve is an empirical, and not a hypothetical, one, as suggested by the label in Fig. 8, because it can be constructed, without any assumptions, by combining two sets of empirical data. One of them is the water-content profile of the stratum, and the second is the profile that shows the relation between depth and the overburden pressure p_0. The process of sampling and the period of storing are represented by the horizontal line bc. Curve cd_1e_1 in Fig. 14 shows the results of a consolidation test on an undisturbed sample and curve cd_2e_2 those of a test on a remolded sample. In order to obtain the relation between pressure and void ratio for the clay in the ground, the straight line be is traced through point b, parallel to the straight part d_1e_1 of the pressure, void-ratio curve cd_1e_1 for the undisturbed sample. There is no theoretical justification for this method. Therefore, the procedure can be advocated only on the strength of empirical arguments. If the weight of a structure increases the load on the clay very considerably, the resulting disturbance of the structure of the clay is at least as important as the disturbance due to the sampling operations. Under this condition it appears reasonable to assume that the pressure, void-ratio curve for the clay in the ground should be roughly parallel to that for the undisturbed sample. By comparing the settlement of heavy structures computed on the basis of this assumption with the observed settlement, it was found that the agreement is usually satisfactory.

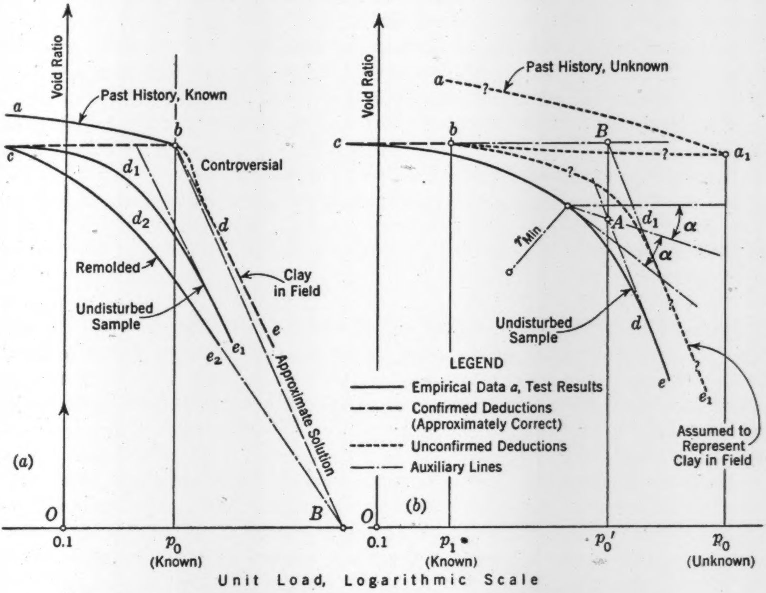


FIG. 14.—RELATION BETWEEN PRESSURE AND VOID RATIO FOR (a) NORMALLY CONSOLIDATED AND (b) OVERCONSOLIDATED CLAY

On the other hand, if the increase of the load on the clay due to the weight of a structure is small, the analogy between the clay in the ground and the undisturbed sample does not exist, and no reliable data concerning the settlement of light structures on clay foundations are available. Therefore the upper part of the line bd in Fig. 14(*a*) has been marked as controversial.

If a job involving a foundation above a stratum of normally consolidated clay is not important enough to justify the expense of securing undisturbed samples, the consolidation tests can be made on remolded samples. Each test yields a pressure, void-ratio curve similar to curve cd_2e_2 in Fig. 14(*a*). The continuation of the straight part d_2e_2 of this curve intersects the horizontal axis of the diagram ($e = 0$) at point B. In most instances, the slope of the line bB is somewhat greater than that of the line be, obtained from a consolidation test on an undisturbed sample; but the difference is not important enough to be of any practical consequence on a routine job. This approximate method of evaluating the relation between pressure and void ratio is identical with the method illustrated by Fig. 13. However, in Fig. 13 the pressures have been plotted on a natural, and in Fig. 14(*a*) on a logarithmic, scale.

Fig. 14(*b*) shows the relation between pressure and void ratio for an overconsolidated clay. The symbol p_0 denotes the highest pressure that acted on the clay during its geological history. It is greater than the present overburden pressure p_1. The changes of the void ratio which occurred during the gradual rise of the pressure from zero to p_0 and the subsequent decrease from p_0 to p_1 are not known. Therefore they are indicated by dotted lines, aa_1 and a_1b. The value of p_0 can only be guessed at on the basis of geological data. The changes that occurred during sampling and storing are shown by the broken horizontal line bc. The results of a consolidation test on an undisturbed sample are shown by curve cde.

By means of the graphical construction indicated in Fig. 14(*b*), a point A is obtained which is located on the straight-line continuation of curve de (30). The pressure $(p_0)'$ which corresponds to the abscissa of this point is assumed to be roughly equal to the preconsolidation pressure p_0. The controversial issues associated with this assumption have been discussed by the writer elsewhere (31). Since the author has made a considerable number of consolidation tests on shaft and tunnel samples from Chicago, Ill., and other localities, he could assist materially in clarifying these issues by presenting, in his closure, a list of the preconsolidation loads that were obtained for these samples, together with the data which determine the position of the samples with reference to the water table and to the ground surface. The importance of the difference between $(p_0)'$ and the real preconsolidation pressure p_0 cannot yet be estimated; but it is fairly certain that the ratio $\frac{(p_0)'}{p_0}$ depends to a large extent on the method of sampling. The greater the disturbance of the clay due to sampling, the smaller will be the ratio $\frac{(p_0)'}{p_0}$, all other factors being equal. The ratio $\frac{(p_0)'}{p_0}$ may also depend on the depth at which the samples were secured.

The next step consists in tracing a vertical line through point A. This line intersects the right-hand continuation of bc at point B. Finally Be_1 is traced parallel to de. The pressure, void-ratio curve for the clay in the ground is assumed to be tangent to Be_1, and the upper part bd_1 of this curve is assumed to have the same trend and curvature, roughly, as the upper part of the laboratory consolidation curve.

The most essential requirement for the application of the procedure illustrated by Fig. 14(*b*) is to secure first-class undisturbed samples, because a slight disturbance of the structure of the clay effaces almost completely the traces of the loading history of the clay. Even if the samples have been secured with utmost care in test shafts, however, the procedure still leaves ample margin for error. Therefore, it is not surprising that the agreement between the results of the settlement computations for structures on overconsolidated clays and the true settlements are rather unsatisfactory in most instances, whereas the settlement computations for heavy structures on normally consolidated clays consistently are found to be reliable.

The two diagrams in Fig. 14 are so simple that no special terminology is required; yet they show everything that is really essential.

Triaxial compression tests are an inexhaustible source of information concerning the behavior of soils subject to stress in space under laboratory conditions; but the practical benefits of these investigations cannot be reaped until the difference between field and laboratory behavior has been explored by means of field observations on full-sized structures. At present the evaluation of this difference still is based chiefly on opinion.

In the last part of his paper, the author mentions semi-empirical methods for dealing with foundation problems. The essence of these methods consists in adapting the fundamental assumptions of the theories to the results of observations in the laboratory and in the field. As experience increases, the methods are modified and generalized in accordance with findings. The Swedish circular-arc method is a good example. After the method was devised, it was found to be not applicable to the failure of fills above thin clay strata. Therefore it was necessary to supplement it by a method based on assuming the possibility of a failure along a composite surface of sliding. For similar reasons, Coulomb's theory of earth pressure had to be expanded into the general wedge theory (32). Because of its inherent adaptibility, a semi-empirical method does not deserve its name unless it agrees, at any time, with the experience which has been accumulated up to that time. The theories of bearing capacity which the author quotes as examples were published many years ago. Therefore they should be discounted until they are again brought up to date.

On account of the impressive diversity of its contents, the paper contains a vast amount of food for thought. Therefore, it will not fail to exert a stimulating influence on many readers.

D. P. KRYNINE,[9] M. AM. SOC. C. E.—The material testing laboratory is closer to the activities of an engineer when he deals with the foundations of a structure than when he works on the structure itself. This is because the medium surrounding the foundation, and acting together with it, cannot be purchased or ordered to satisfy certain specifications. Some evidence must be furnished by the soil laboratory in each particular case. The writer wishes to emphasize the word "some" in the preceding sentence. Every engineer understands very clearly that a soil laboratory does not test the earth material itself, but merely furnishes information on the properties of the samples tested, under the limitation, of course, that proper tests have been performed properly. In this connection an authoritative and frank explanation of the value of soil samples and of the tests themselves, as issued by Professor Rutledge, should be welcomed by the profession.

Consolidation Test.—To determine how much a deep clay layer will decrease in thickness (settle) due to the pressure from a proposed structure, a model of this layer (an "undisturbed" thin soil sample) is tested. It is subjected to the same conditions as the natural clay layer so far as the initial pressure p_1 (before construction) and the final pressure p_2 (after construction) are concerned. The main object of the test is to find the relationship between the "void ratio" of the clay (ratio of the volume of voids to the ratio of solids) and the applied unit pressure. On semilogarithmic paper, this relationship is approximately expressed by a straight line termed "virgin compression curve" (Fig. 1(*b*)). It is assumed that the change in void ratio, $e_1 - e_2$, due to an increment in pressure, $p_2 - p_1$, is the same for the model and the prototype. The final settlement (or the decrease in thickness) of the layer may be computed easily from the value $e_1 - e_2$; and the steeper the "virgin compression curve," the larger the expected settlement will be.

---

[9] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

The "virgin compression curve" is supposed to be tangent to the laboratory compression curve (Fig. 1(*b*)). The latter is obtained in the laboratory by applying rather large increments of load at a rather quick rate and the test cannot be done otherwise. According to Professor Terzaghi (10) in doing so, the laboratory worker breaks "rigid clay bonds" and transforms the clay of the sample into a soft ("lubricated") mass. In nature load increments are applied slowly—grain by grain, particle by particle. Hence the field "virgin compression curve," before construction, is flat (curve (2), Fig. 8) whereas the laboratory curve (curve (7), Fig. 8) is steep. What happens in the natural clay layer after the construction is not known; but it may be assumed that what may be termed the "true compression line" is somewhere between the continuation of curves (2) and (7) in Fig. 8. Professor Rutledge shows it under the form of curve (4), Fig. 8, close to the laboratory compression curve. The writer disagrees with this visualization for the simple reason that, although the load increments are equal in the model and in the prototype, the rate of application is about fifty times slower in the latter case than in the former. In fact, the same final unit load is reached perhaps in one week in the laboratory and roughly in one year on the construction job. Besides, no one can be sure whether the mechanics of this breaking of bonds, if any, is the same in a sample ½ in. or 1 in. thick and in a clay layer, say 40 ft thick. Hence the writer believes that curve (4), Fig. 8, should be flatter.

Thus the settlement analysis using laboratory data apparently furnishes exaggerated values of settlement, the order of magnitude of this exaggeration being uncertain. Of course, the settlements thus computed are always on the safe side; but this safety is of such a nature as to permit an experienced engineer to believe, and with some reason, that a simple guess of the value of the expected settlement may be not less correct than the result obtained in most elaborate tests.

However, the writer is hopeful about the future of the consolidation test. Professor Terzaghi is to be commended highly for publishing the theory cited (10). The theory may prove incorrect, in which case everything will remain as it was before the publication of Professor Terzaghi's paper. If the theory is correct (and probably it is correct), steps should be taken to correlate the laboratory tests with that theory, and this undoubtedly will be done.

Shear Tests.—Professor Rutledge discusses the direct shear test, the triaxial test, and the unconfined compression test, and apparently prefers the latter.

This is indeed a promising simple and nonexpensive test performed on prismatic or cylindrical specimens cut out from large cubes taken by hand or from undisturbed samples extracted from certain depths using samplers. Any disturbance of the sample furnishes results on the safe side; but evaporation (drying out of the specimen during the test) unduly increases the shearing strength of the sample and should be avoided.

During the Annual Meeting of the Society in January, 1943, M. Juul Hvorslev, Assoc. M. Am. Soc. C. E., demonstrated his apparatus for making the unconfined test on tiny prisms or cylinders cut from undisturbed samples. It would be interesting to know Professor Rutledge's opinion of this method.

A worth-while addition to the paper by Professor Rutledge is a discussion by A. E. Cummings, M. Am. Soc. C. E. (33), in which the comparison between the unconfined compression test and the ring test by W. S. Housel, M. Am. Soc. C. E., is thoroughly discussed. The ring method itself (34) consists in pulling out a slice from a cylindrical undisturbed sample. In other words, this is double shear as in the case of a rivet which works in that manner.

Coulomb Formula.—This is another important soil mechanics item which, together with the consolidation test, passes now (1943) through a crisis. It is rightly stated by Professor Rutledge (35) that: "The results of shear tests must eventually be reduced to the Coulomb equation or a simple modification of it." The Coulomb formula states that the shearing resistance (shearing strength) of a soil, s, is the sum of its unit cohesion, c, visualized for a given soil as a constant, and the product of the normal pressure, σ, acting at the given point, by a certain coefficient depending on the properties of the given soil. This coefficient is visualized as a tangent of an angle ($\tan \phi$), the angle ϕ being termed "angle of internal friction," "effective angle of internal friction," or "angle of shearing resistance." It follows from the Coulomb formula that the shearing resistance should increase with the depth of the deposit, since the normal pressure, σ, depends on the weight of the overburden; but this is not always the case. Some engineers believe that the angle of internal friction for plastic clays, ϕ, is equal to zero, which means that the shearing strength is constant throughout a mass of plastic clay. Some others, including Professor Rutledge (see paragraph following Eq. 2), believe that "the increases in test strengths * * * which have been attributed to an angle of internal friction in undisturbed clay, may, in fact, be due entirely to decreases in void ratio." In other words it is believed that the smaller the volume of the voids, the larger the shearing strength of the clay will be.

The writer believes that the shearing strength of a clay mass may be practically constant throughout, due to the peculiar structure of some clays which prevents consolidation. Another possibility is the reinforcement of a clay deposit by calcium carbonate (or other chemical) due to the action of fluctuating ground water as in some places in Arizona. Furthermore, the writer wishes to warn against attributing the increase in shearing strength during consolidation solely to the decrease of the volume of voids. Possibly, the increase in shear strength in such cases is due mostly to the interaction of mutually approaching surfaces and hence may be influenced by adding chemicals which change the surface conditions. In this connection acquaintance with the work by Prof. H. F. Winterkorn, Assoc. M. Am. Soc. C. E., may be helpful (36).

Runways and Highways.—The paper by Professor Rutledge considers primarily the tests used in the study of foundations of heavy structures such as bridges and tall buildings. The field of tests which are necessary for runways and highways is excluded from consideration. It is true that one of the most widely used tests in that field—the so-called "California bearing ratio test" does not require undisturbed samples. In this test moistened soil material is consolidated in a mold and tested for penetration; but capillary tests and tests of permeability may be made both on disturbed and undisturbed samples, and comparison of both may be of importance. In this connection some acquaintance with the theory of "capillary potential" used by soil scientists may be of interest (37).

Terminology.—In the opinion of the writer the term "idealized" is very correctly used in this paper. An "idealized" material or object is assumed to possess properties or qualities which in reality it possesses only in part or to some extent; or, possibly, it lacks entirely. The virgin compression curve in Fig. 2 may or may not be a straight line (as is the case of some clays); but in this paper it is "idealized" and visualized in the form of a straight line. Eq. 1 obviously refers to the "idealized" virgin compression line and not to the actual curve. If an able draftsman happens to trace a straight line, as in Fig. 2 or elsewhere, in an irreproachable manner, it will be an "ideal" straight line.

D. M. BURMISTER,[10] ASSOC. M. AM. SOC. C. E.—The extent to which the strength properties of clay may be affected by disturbance during sampling is treated in this worth-while contribution to soil mechanics. The author has suggested fairly reliable methods for determining whether a sample may be considered relatively undisturbed. There are some points, in connection with the curves and data presented, on which the writer wishes to comment.

The consolidation curve CP1, for example, in Fig. 3 represents a relatively undisturbed sample, which possesses the greater part of its natural structure at the preconsolidation pressure of approximately 0.5 kg per sq cm. This natural structure, which is the source of the strength of the clay, may be that visualized by Professor Casagrande or that due to physicochemical processes, as believed by Professor Terzaghi. The maximum effect of disturbance, resulting from a complete remolding of the material at approximately constant moisture content, is shown by the vertical decrease in void ratio under constant pressure (0.5 kg per sq cm) to curve CP3. The vertical void-ratio intercept represents the change in void ratio (and hence settlement) required before equilibrium can be achieved under the same pressure, as a result of complete disturbance. The remolded material now possesses an entirely different type of structure, which involves a complete breakdown in the natural bonds between grains. The structure is characterized by a homogeneous, dispersed state of the "remolded structure." The void ratio at any given pressure, which is consistent with this structure at the equilibrium condition, is much smaller than for the undisturbed material. Therefore, having reached equilibrium under this pressure at a smaller void ratio, the material will compress less in the next load increment than does the undisturbed material. This means that the consolidation curves for the undisturbed and remolded clays must always intersect at some pressure (40 to 50 kg per sq cm in this case), as shown in the upper part of Fig. 4, where the curves for this material are reproduced. The magnitude of this pressure depends primarily on the mineralogical character of the clay.

---

[10] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

The vertical intercept between consolidation curves for the undisturbed and remolded clay at the preconsolidation pressure is one important physical measure of the natural structure the clay possesses, and indicates the maximum effect of complete disturbance. It is to be noted that, at higher pressures, structure becomes less important and the effect of disturbance is less. This indicates that structure is more important for clays consolidated under relatively light loads, and that the effect of disturbance during sampling on the consolidation properties is likely to be greater than for clays consolidated under heavy loads.

It is believed that the consolidation curves intersect, not because the undisturbed clay becomes completely remolded by compression, as suggested by the author, but because a stage is reached at which a characteristic change in the compression phenomena occurs for the given clay (at least, laboratory consolidation phenomena). Curvature of the compression curve on a semilogarithmic plot, therefore, is not necessarily an indication of minimum disturbance of the clay. In the first stage of consolidation, the loose open structure and the chains of clay grains have been compressed in a vertical direction only, and possibly have been folded down upon each other, with some disturbance to the bonds, to form a structure under increasing pressure consistent with the void ratio, but still characteristic in form. No such complete remolding can possibly occur as a result of disturbance by compression which will cause the material to assume the dispersed remolded structure. During compression the remolded clay has also acquired structural bonds between grains (thixotropic effects). Beyond the intersection of the curves, the structure of the undisturbed and remolded clay has approximately identical effects upon compression, which takes place approximately along the flatter slope of the remolded consolidation curve, although the two structures are not identical in form.

This phenomenon is clearly evident in the case of the Mexico City clay in Figs. 5(*a*) and 5(*b*), where a break in the consolidation curve for the undisturbed clay occurs in the region of 5 kg per sq cm, and thereafter follows approximately along the curve of the remolded clay. The writer has observed such curvature in the consolidation curves of many different types of clays, the break occurring more frequently under rather low pressures for soft organic clays. This curvature or break in the slope of the compression curve of undisturbed clays is only one of several stages in the ultimate consolidation of soft clay deposits to shale. These stages are evident only on the semilogarithmic plot of void ratio against pressure.

Another point that should be mentioned is that disturbance to the natural structure tends to make the clay less permeable and hence also tends to make the coefficient of consolidation too small. The actual rate of settlement, therefore, will be more rapid than the estimates. The effect of disturbance cannot be detected in the form of the time-consolidation curves.

From the viewpoint of the application of the results of laboratory tests, the primary purpose of laboratory analyses is to furnish suitable representative averages for soil properties to be used in settlement and stability investigations. The writer has used a "Control Test Method" as an aid in selecting the least disturbed samples for test. Unconfined compression tests are made on all samples, as soon as received, to determine their quality, using prismatic specimens trimmed from the least disturbed central portion of the sample. It has been observed that the compressive strength of specimens, using specimens the full diameter of the sample (3-in. sampler or 2-in. seamless tubes), was 50% to 75% of the strength of small specimens obtained by trimming away the circumferential disturbed material. The ratio of the unconfined compressive strength, remolded to undisturbed, is used as a measure of the quality of the sample. This strength ratio should be less than some fraction, which is determined and revised as the testing progresses. For clays consolidated under light pressures, the ratio may be as low as $\frac{1}{5}$ to $\frac{1}{10}$, or less, depending on the character of the clay, but may be only $\frac{1}{2}$ to $\frac{1}{5}$ for clays consolidated under high pressures. However, this criterion alone is usually not sufficient, because its significance varies with the character of the clay. Frequently, the variations observed in the horizontal direction in the character of the clay of a given stratum are considerably less than those normally found in the vertical direction in this same stratum, due to changes in the geological and sedimentation conditions as the deposit was formed.

It has been observed frequently that the liquid limit, as a valuable measure of the character of the clay, increases consistently with depth, with a corresponding increase in the compression index in the consolidation test (a steeper compression curve denoting a clay that is more compressible) and with changes in the unconfined compressive strength and strength ratio, which may or may not be consistent. To provide a suitable and fairly reliable basis for judging the quality of the clays for the purpose of selecting test results and computing averages of properties from samples, which are judged to have suffered the least disturbance and to be most representative of subsurface conditions, the characteristics of the clay must be studied. Diagrams of soil properties are extremely useful for this purpose, and form a valuable part of any report of settlement or stability investigations. The diagrams should show the following:

(1) The variation with depth of the unconfined compressive strength, strength ratio (remolded to undisturbed), moisture content, liquid limit, and other test properties, if considered pertinent to the problem.

(2) The relationship between moisture content as a common argument and these same properties, in order to indicate the range of materials encountered and the characteristics of each type.

The writer believes that the triaxial compression test offers greater possibilities for the future than any other method of determining the shearing properties of soils. However, a direct application of test results cannot always be made, such as in the case of the unconfined compression test for determining the bearing capacity of clays, because of testing difficulties, but primarily because engineers do not know what the lateral pressures are in different types of deposits in nature and how they vary as the foundation loads are increased from zero to their ultimate value. In the unconfined compression test, unknown lateral restraints also exist, the magnitudes of which depend on the capillary pressure developed and the expansive properties of the clay at the preconsolidation pressure. These lateral forces probably are quite different from those existing in the deposit, being isotropic in character and also probably varying in magnitude as the vertical pressure is increased during the test. The issue has been merely side-stepped in this case. It is believed that curvature of the stress-strain curve in the first 20% of the unconfined compressive strength is not necessarily an indication of disturbance for clays consolidated under light loads and with compressive strength less than about 0.5 kg per sq cm, because of the variation in the magnitude of the lateral pressures that may be developed as loading progresses.

The fact that clays consolidate during triaxial compression tests should not be considered as a disadvantage of the test. This must be accepted as the normal phenomenon of clays, which are in a natural state of consolidation under their present overburden pressure. The primary problems are to reestablish the natural state of stress on the sample that existed in the deposit, and then to increase the stresses on the specimen in such a manner as to resemble those imposed on the actual material in place. Although the preconsolidation pressure determined by a consolidation test may fairly represent the natural vertical stress conditions, the lateral pressure may be of the order of 50% to 75% of this value, depending on the type of clay and the geological and load history of the deposit. It may be possible to obtain some approximate information on the natural pressure ratio—vertical to lateral—from careful measurements of the stresses set up in the consolidation ring after equilibrium has been reached under each pressure increment in the consolidation test.

The important question is: Are the final results the same and are they equally representative for the following test procedures?

(*a*) The clay specimen is preconsolidated by suitable increments under an all-around pressure, σ_3, equal to some fraction of the vertical preconsolidation pressure. The vertical stress is then increased until failure occurs.

(*b*) The clay specimen is preconsolidated by suitable increments, maintaining some constant stress ratio, σ_1/σ_3, until σ_1 becomes equal to the vertical preconsolidation pressure. Thereafter, the vertical stress is increased until failure occurs.

In either case, to simulate rapid failure conditions in a thick clay deposit, a slow triaxial compression test should be run as a closed system with measurement of pore water pressures as the vertical stress is increased to failure. If special care is taken in setting up the test and running it, and a balanced pressure system is used for measuring pore water pressures at the top and bottom of the specimen, fairly reliable information may be obtained on the effective stresses acting during the test and at failure. Tests run at pressures somewhat below and above the vertical preconsolidation pressure would provide a relation between the strength properties and void-ratio change to be used for estimating the natural strength properties. It is obvious that the triaxial compression test can never be reduced to the simple testing procedure and method of interpretation used in the unconfined compression test, and in this respect the unconfined compression test has a great advantage of simplicity. In the final analysis, it is the correlation with the actual behavior of full-scale structures that should determine the relative reliability of any test method.

Professor Rutledge has presented a very timely paper on this subject and has opened up a valuable field for further research.

P. C. Rutledge,[11] Assoc. M. Am. Soc. C. E.—The several discussers of this paper have contributed significantly to an understanding of the effects of sample disturbance on the results of soil tests. The discussions constitute ample evidence to support the underlying premise of the paper—namely, that the results of soil tests cannot be used directly to determine the behavior of soil in nature. In most applications, unmodified test results lead to solutions that have little meaning in terms of the true behavior of the soil and may result in unsafe conclusions. This is a significant factor in the application of soil mechanics to practical problems. It is an important reason why such applications cannot be reduced to simple terms and formulas, and is a serious obstacle to understanding by the average engineer working with structural materials. The errors introduced by differences in behavior between tests on structural materials and the finished product are usually small compared with the commonly used factors of safety; and, therefore, engineers have come to accept unquestioningly the direct use of the results of simple tests in machine and structural design. This fortunate situation does not exist for soils because neither the condition of the test specimen nor the stress conditions within the specimen are like those in nature.

Sample Disturbance and Consolidation Tests.—Three effects of sample disturbance on the results of consolidation tests have been emphasized by the discussers: (*a*) The significance of concave virgin compression curves; (*b*) the determination and significance of the preconsolidation load; and (*c*) the location of the most probable natural compression curve. These will be considered in order.

Concave Virgin Compression Curves.—Major Hough cites consolidation test results for St. Lawrence River clays with an initial void ratio of 1.6 in which the semilogarithmic virgin compression curves were concave upward. The writer has observed the same type of curves for many soils with initial void ratios larger than 1.5 but only when the test specimens had undergone a minimum of disturbance. For good samples with large initial void ratios, such curves are probably the rule rather than the exception. Major Hough also questions the undisturbed condition of the Mexico City clay which was used by the writer as an example. The samples were taken under the supervision of A. E. Cummings, M. Am. Soc. C. E., and J. A. Cuevas. Mr. Cummings arranged the transportation of the samples to the laboratory at Harvard University, Cambridge, Mass. Every precaution was observed to protect the samples and, as received in the laboratory, this clay had a rubbery consistency similar to artgum. It was not sensitive to shock or vibrations and was difficult to remold. The character of the clay, the fact that the laboratory preconsolidation load is within 3% of the existing pressure on the clay, and the sharp curvature of the compression diagram at the preconsolidation load—all indicate that this clay had suffered a minimum of disturbance, in spite of its extreme natural void ratio of 14.0.

---

[11] Prof., Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

Professor Gray has presented a challenging hypothesis for the compression of a "composite" specimen consisting of an undisturbed core surrounded by a disturbed or remolded annulus. The compression curve for such a specimen could well be slightly concave upward in the semilogarithmic diagram. The writer has never observed curves of this type for specimens known to be partly disturbed, possibly because the annulus of extreme sample disturbance was always trimmed from the test specimens. Professor Gray's example emphasizes: (1) The necessity for trimming off the outer annulus of bore-hole samples; and (2) the dangers of concluding that concave virgin compression curves represent minimum sample disturbance when there is a change in the techniques of sampling and sample preparation.

Professor Burmister's hypothesis for successive stages of compression of undisturbed clay and for the differences between the compression of undisturbed and remolded clays is based on a visualization of a chain-like structure of clay grains in undisturbed soil. He assumes that this structure is modified but not destroyed by compression whereas remolding causes a homogeneous dispersed state of the soil grains. These assumptions lead to an explanation for concave virgin compression curves and to the conception that the compression curve for remolded clay at large loads may intersect and lie above that for the same soil undisturbed. There is justification for the conception of successive stages of compression. At very high loads as the void ratio approaches zero, there must be a change in phenomena because compression can no longer be accomplished by reduction in void space. The slope of the curve should finally become the bulk modulus of the mineral making up the clay grains. What actually happens to compression curves at pressures greater than 25 kg per sq cm and to their relation to the curves for remolded specimens is speculative. The writer is not familiar with any tests that have resulted in either the joining or the intersection of the compression curves for undisturbed and remolded specimens.

The Preconsolidation Load.—Mr. Van Auken and Professor Dawson have shown the difficulties in determining the preconsolidation load from laboratory tests on clays that have been subjected to a complex geological history and on those that have been subjected to alternate shrinkage and swelling due to surface drying conditions. Mr. Van Auken recommends careful study of all geologic evidence. The value of study of all such information is unquestioned but Professor Terzaghi (31*a*) makes the restrained but pertinent statement: "* * * as a rule, geological evidence regarding the thickness of vanished strata leaves a wide margin for interpretation."

For "normally" consolidated clays, Professor Terzaghi recommends the use of the existing overburden pressure in place of the preconsolidation load determined from laboratory tests. It remains, of course, to determine whether or not any particular clay specimen is "normally" consolidated. Certainly expansive clays that have been subjected to forces of surface drying, for which Professor Dawson has drawn curves, are not normally consolidated even though they have never been subjected to an overburden greater than that existing at the time of sampling.

The writer is indebted to Professor Terzaghi for clarifying the historical background of knowledge of the effects of disturbance due to sampling on the test behavior of clays. Professor Terzaghi has also succinctly differentiated between those parts of the writer's compression diagrams that are based on experimental evidence and those parts that consist of extrapolations based on assumptions which have not been proved. The distinction should have been stated more clearly in the paper.

Professor Terzaghi states that the actual condition of most clays during sampling and preparation for testing is one of practically constant void ratio while the externally applied pressure is reduced to zero by taking the sample out of the ground. Hence, the horizontal line is derived from the natural void ratio at the preconsolidation load to zero pressure in Figs. 8 and 14. The writer cannot agree, however, that this simplified picture, which does represent what actually happens, renders meaningless points A and B in Figs. 6 and 7 or the constructions whereby they were obtained. Most consolidation tests result in curves of the type of the heavy solid line curves CB'E in Figs. 6 and 7. Point B is located by comparing the shape of the initial part of the test curve with laboratory recompression curves that follow rebounds from known laboratory preconsolidation loads. Hence, the significance of point B can be stated briefly as follows: Had the clay been preconsolidated under conditions identical to those of the laboratory test, point B represents approximately the pressure and void ratio that the clay would have attained during its preconsolidation. Therefore, the pressure at point B should be called the "laboratory preconsolidation load" because it is dependent on laboratory conditions of compression.

The meaning of the void ratio at point B is less directly apparent. Sampling operations weaken the soil structure simultaneously with the removal of natural external pressure. The void ratio at point B is the laboratory equilibrium point that would have been obtained had the soil structure been weakened without removal of the external pressure. As the writer stated in reference to Fig. 7, an imagined rebound from the fictitious void ratio at point B gives a direct measure of capillary pressure in soil test specimens. The hypothesis has been advanced that capillary pressure replaces in full the natural external pressure when a soil sample is removed from the ground. Analyses based on the void ratio at point B indicate strongly that any such capillary pressure is always less than the external pressure and is usually small if not zero.

The laboratory preconsolidation load is derived exclusively from similarity to laboratory precompression and recompression. Any relation between it and the maximum load to which a clay was subjected in nature requires the assumption that natural compression is similar in character or effect to laboratory compression. This assumption is not entirely without empirical justification. An example is the Mexico City clay for which the test results, shown in Fig. 5, yield a laboratory preconsolidation load almost exactly equal to the known pressure acting on the clay. A second example is a Chicago clay sample, for which consolidation and triaxial compression test results are shown subsequently in Fig. 15. Comparisons of approximate overburden pressures and laboratory preconsolidation loads for these soils and other soft saturated clays which contained no visible evidences of drying are shown in Table 3.

TABLE 3.—COMPARISON OF OVERBURDEN PRESSURES AND LABORATORY PRECONSOLIDATION LOADS FOR SOFT SATURATED CLAYS
(All Samples Cut by Hand)

Location	Sample No.	Initial water content[a] (%)	DEPTH, IN FEET		LOAD (TONS PER SQ FT)		
			From ground surface	From water table	Over-burden[b]	Laboratory Pre-consolidation	
						From	To
Mexico							
Plaza de la Reforma[c]	H28	460.0	24	14.5	0.97	0.95	1.05
Chicago, Ill.							
Peoria Street	H13E	45.5	22	12	0.95	0.8	1.0
South Wabash Street	H8B	35.6	35	21	1.30	0.8	1.2
South Canal Street	H13C	25.5	31	22	1.20	1.1	1.5
Elston Street[d]	H13A	25.9	24	18	1.25	1.4	1.6
North State Street	P9-3	28.0	34	22	1.30	1.2	1.4
South State Street	P9-5	31.9	25	13	1.10	0.9	1.1
Detroit, Mich.							
River Rouge[e]	P3-4	38.7 31.9	35	23	1.50	1.5	1.7

[a] Percentage of dry weight. [b] Approximate pressure. [c] Natural void ratio, 14.0. [d] One foot below stiff yellow clay. [e] Two consolidation tests.

Unfortunately the data referred to in the preceding paragraph are for samples at single depths in the upper parts of different deep beds of clay. The data from tests on bore-hole samples taken throughout the full depth of thick clay strata are not conclusive for the following reasons: (*a*) The soil pressure changes and mechanical sampling operations causing sample disturbance increase with the depth of sampling; and (*b*) increasing sample disturbance has a tendency to decrease the indicated laboratory preconsolidation load.

As a result of the several preceding considerations in regard to the laboratory preconsolidation load, this value, determined by extrapolation from the results of consolidation tests, should have definite physical meaning, and studies to ascertain correctly its relation to overburden pressure under varying conditions should be continued. For such research the best possible undisturbed samples are required.

Location of the Natural Compression Curve.—Much of the discussion regarding the most probable location of the natural compression curve for clays refers to the flat curve in Fig. 8 and also to curve ab in Fig. 14. This type of natural compression curve was proposed by Professor Terzaghi (10), who has presented strong arguments in its favor. The writer has not been entirely convinced, however, that water content and overburden pressure data are sufficient to establish the curve as empirical (as Professor Terzaghi states) rather than hypothetical. The effects of changes in either plasticity or natural water content or both on the laboratory compression characteristics of clays have not been sufficiently explored to constitute conclusive evidence in either direction.

Professor Krynine has argued in favor of the flat type of natural compression curve as opposed to curve (4) in Fig. 8. In the case of soft clay subjected to loads at least as great as those of a medium sized building, these arguments have been answered by Professor Terzaghi's discussion, in which observed agreements are cited between settlements and predictions based on curves similar to curve (4), Fig. 8.

An important utilization of simplified soil sampling and testing operations is the approximate method, described by Professor Terzaghi, for determining the natural compression of clay from the results of tests on remolded specimens. This method, published in 1929 (29), has not been widely known or used. It is an example of the "flashes of engineering intuition" to which Professor Terzaghi refers (31). The writer has applied the method to the results of tests on remolded specimens of soils for which test results for undisturbed specimens were also available. Except in the case of highly compressible volcanic and organic clays, the results were practically identical with the best estimates resulting from tests on undisturbed specimens. The writer agrees with Professor Terzaghi's evaluation of uses of this method. In particular, it is less expensive and no less reliable than results obtained from tests on inferior "undisturbed" samples. When the size of a job justifies the expense, the additional information obtained from the best possible undisturbed samples is a sound investment both for obtaining practical results and for increasing existing knowledge about the behavior of clays.

Sample Disturbance and Strength Tests.—Professor Burmister advocates triaxial compression tests in preference to unconfined tests on the basis of an unknown capillary pressure acting on the clay in the latter. The writer cannot agree with the arguments presented for the following reasons:

1. The application of lateral pressure through a rubber membrane does not eliminate or make known the stresses in the soil pore water.
2. Analyses of consolidation test results, previously described, indicate that initial capillary pressures in soil samples are small. Before testing they will be increased only by drying or by expansive tendencies caused by evolution of gas.
3. During the test, changes in capillary pore-water pressure can be caused only by volume changes in the specimen. No conclusive evidence is available, but existing data indicate very little tendency toward volume change of saturated clay tested in unconfined compression.
4. The unconfined compression test necessarily measures the applied vertical stress. This stress, when an unknown hydrostatic pore-water stress is considered, becomes the difference between major and minor principal stresses.
5. For a saturated plastic clay which does not change volume, the difference between principal stresses is substantially independent of small changes in the minor principal stress.

Mr. Van Auken and Professor Burmister both take exception to the writer's statements regarding the effects of volume change on triaxial compression tests of undisturbed natural clays. Both of them recommend preloading of triaxial compression test specimens by maintaining a constant ratio between major and minor principal stresses until the major principal stress reaches some desired overburden pressure. The writer does not question the fact that clays consolidate both during triaxial compression tests and in nature but wishes to emphasize that, because of sample disturbance, the consolidation, the void ratios at which the clay is tested, and the pore-water pressures are completely different from those which will result in nature. The effects of sample disturbance on consolidation occur whether or not the test pressure conditions are like those in nature.

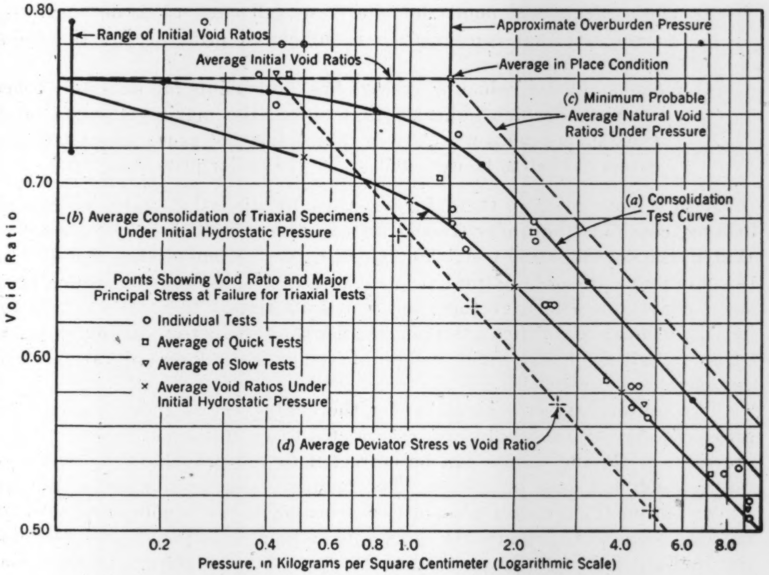


FIG. 15.—VOID RATIOS VERSUS MAJOR PRINCIPAL STRESSES

An example of the void-ratio changes during consolidated triaxial tests on soft clay is found in Table 2. The complete series of tests from which these data were abstracted was performed by H. H. Ku, Jun. Am. Soc. C. E. (38). In Fig. 15 the data are plotted in terms of void ratio. Curve (a) is the result of a consolidation test on one specimen from the cubic foot, hand-cut sample. The initial void ratio of the consolidation specimen is almost exactly the average of initial void ratios of all test specimens from this sample. Curve (b) is the average result of initial consolidation under hydrostatic pressure of all triaxial specimens. The maximum major principal stresses and the corresponding void ratios for individual tests and for averages of quick and slow tests are plotted as individual points. The consolidation time allowed in both the consolidation test and the preliminary consolidation of triaxial specimens was twenty-four hours, which was well beyond the theoretical 100% consolidation point for both. The maximum duration of the slow tests was also twenty-four hours.

Several facts should be noted in connection with Fig. 15:

(1) Curve (b) for consolidation under hydrostatic pressure falls below curve (a) for the consolidation test, indicating either that the preparation of triaxial specimens caused greater sample disturbance or that hydrostatic loading caused greater compression than consolidation test loading;
(2) The breaks in the compression curves (a) and (b) occur at almost the same stress and indicate the same laboratory preconsolidation load;
(3) The average final points for slow tests fall close to or on curve (b), signifying that consolidation is influenced primarily by the major principal stress; and
(4) None of the individual or average final test points fall definitely below curve (b), which indicates that shear (the maximum strain in all triaxial specimens was 20% or more) does not cause consolidation in excess of that to be expected under the major principal stress.

With the exception of the unknown cause for deviation under hydrostatic loading, consolidation under triaxial test conditions is found to be parallel to that in the consolidation test. Therefore, curve (c) was plotted representing the minimum probable natural void ratios that would occur in nature under stress conditions similar to those in the triaxial tests. In the tests, both the initial and final void ratios are much smaller than the corresponding values on curve (c) although the final values of the quick (20-min duration) tests approach curve (c).

Two options are now available for the evaluation of these triaxial compression tests: (*a*) The results can be corrected to a condition of zero pore-water pressure; or (*b*) the results can be corrected to the probable minimum void ratios which will exist in nature. The methods of correction are as follows: Under option (*a*) the maximum major principal stress is compared with the pressure required to produce the same void ratio in a comparable consolidation test and the difference is assumed to be pore-water pressure. Under option (*b*) the void ratio for any chosen minor principal stress is determined from curve (c), Fig. 15, and the difference between major and minor principal stresses corresponding to this void ratio is determined from the curve of deviator stress versus void ratio that is also plotted in Fig. 15. Both methods require radical assumptions. The first method assumes that consolidation under triaxial compression is exactly similar to that in consolidation tests. The data in support of this assumption have previously been presented. The second method assumes that all increase in strength is caused by decrease in void ratio.

The results of these two types of corrections are shown in Fig. 16. Corrections of the first type were made for each of the five series of individual tests. The total average correction line is also plotted for each of the five series. The close agreement supports this method for correcting for pore-water pressure. Corrections of the second type were made only for the average test results. There is a wide difference between the results of the two methods of correction. The evidence available (for example, the data cited by Major Hough) seems in favor of corrections by the second method although it is far from complete and

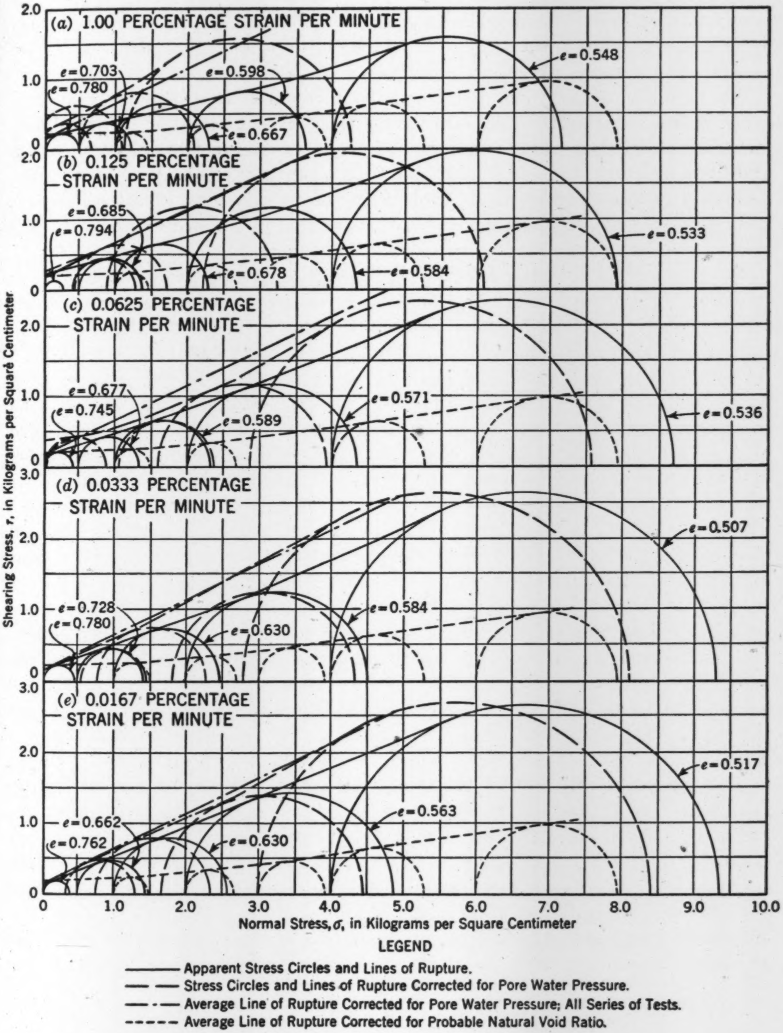


FIG. 16.—FAILURE DIAGRAMS—TESTS ON UNDISTURBED CHICAGO SOFT CLAY

(All Tests on Specimens Cut from 12-In. Cubical Sample with Average Initial Void Ratio = 0.759. Void Ratios Shown Are for Maximum Stress) correct. Therefore, the writer concludes that neither the strengths nor the pore-water pressures obtained from consolidated triaxial tests on undisturbed natural clays are representative of field conditions and that the results are unsafe when considering actual conditions.

It should not be concluded from these remarks that the triaxial compression test is without value. On the contrary, it is a tool of great usefulness. Even for plastic, saturated, undisturbed clays, testing in triaxial compression under small lateral pressures may be advisable for convenience in making the necessary test measurements. Another factor (which most of the discussers have agreed is important) that was not considered in the preceding analysis is the direct effect of sample disturbance on soil strength.

In connection with the effects of void-ratio changes, Professor Krynine's discussion of applications to the Coulomb formula conveys the impression that assumption (1) of a zero angle of internal friction for plastic clays and assumption (2) of void-ratio changes as a primary factor in observed strengths in triaxial tests are unrelated and have opposing results. This is not the case. Assumption (2) simply presents a means of explaining the results of triaxial compression tests in terms of assumption (1). Assumption (1) is based primarily on the results of field observations. As Major Hough has stated and the preceding analyses indicate, assumption (1) is difficult to substantiate by means of laboratory tests.

Summary.—Professor Gray's extension of the "chain" analogy to the solution of foundation problems appeals particularly to the writer. The clear statements of the limitations in present knowledge, contained in his first four paragraphs, cannot be improved. The writer wishes to re-emphasize the one statement that individual cases of numerical agreement between predictions and field observations do not constitute proof that the methods used are generally applicable. The validity of the several links of the chain (that is, sampling, testing, analysis of stresses, and prediction) can be established only through (*a*) the accumulation of large numbers of complete field and laboratory observations; and (*b*) analysis of all observations by coordinating agencies. Such a program will require the concerted effort of every one interested in foundation problems. In this connection Mr. Feld has suggested that the methods of extrapolation described by the writer may be used to establish similarity between soils of known and unknown behavior. The suggestion has merit but is perhaps too optimistic an adoption of the methods, considering their present limitations. The writer thanks the discussers of this paper for their contributions to the subject of the effects of sample disturbance and for the stimulus of their ideas.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2230

A METHOD OF COMPUTING URBAN RUNOFF

By W. I. Hicks,[1] Esq.

With Discussion by Messrs. C. S. Jarvis, S. W. Jens, W. W. Horner, and W. I. Hicks.

Synopsis

The purpose of this paper is to present the results of certain hydraulic investigations and rainfall-runoff gagings for improved urban drainage areas of different sizes and types of development, to analyze data from these results, to develop hydrographs of runoff for areas of various sizes, development, and times of concentration, to present methods of computing runoff for use in design offices, and to compare actual values of runoff with those computed by these methods. Much of the complete research, upon which this paper is based, has been placed on file for reference in Engineering Societies Library.

Basis of Runoff Computations

Rainfall-runoff relations in Los Angeles, Calif., are characterized by (*a*) a winter rainy season with consequent low evaporation and soil losses; and (*b*) wide variation in rainfall intensities on account of the mixed mountain, valley, and coastal plain terrain.

For urban areas, runoff is almost wholly that part of the rainfall which is not lost by infiltration into the soil or left in surface depressions and on vegetation surfaces to evaporate or sink into the ground.

The hydrograph of the net rainfall before it runs off the lot has the pattern of the rainfall intensities modified by the losses. Between this point and the point of concentration in the drain, the pattern is altered further by detention, which is the varying volume of water that must be built up to induce flow at the varying intensities of runoff. The general classification of detention is overland, gutter, and conduit.

This theory of runoff has been advanced by a number of engineers: The late C. E. Grunsky,[2] Past-President, Am. Soc. C. E., advocated the modifica-

---

Note.—Published in April, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Engr., Storm Drain Div., Bureau of Eng., Los Angeles, Calif.

[2] "The Sewer System of San Francisco, and a Solution of the Storm-Water Flow Problem," by C. E. Grunsky, *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 294.

tion of net rainfall by storage; Robert E. Horton,[3] M. Am. Soc. C. E., made a complete analysis of the theory; W. W. Horner, M. Am. Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E., applied the theory[4] to the runoff from a city residential block and suggested its projection into the drainage network.

I. Analysis of Rainfall Records

Intensity, Frequency, and Variation with Locality.—The Los Angeles County Flood Control District, H. E. Hedger, M. Am. Soc. C. E., chief engineer, has analyzed rainfall records operated by the District and other agencies to derive intensity-duration curves (Fig. 1) and an isohyetal map of the county showing the variation in the intensity of the 50-yr 1-hr duration over the mixed mountain and valley terrain. The map in Fig. 2 is based on studies of the Flood Control District, the U. S. District Engineer, and the writer (for the coastal region).

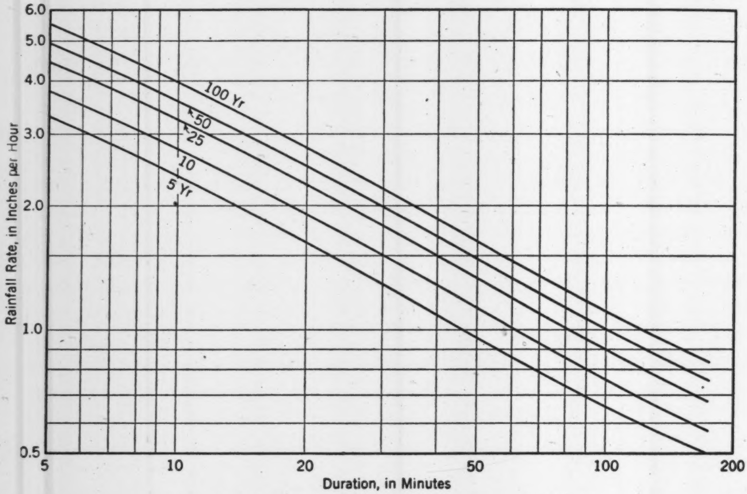


FIG. 1.—LOS ANGELES COUNTY FLOOD CONTROL RAINFALL

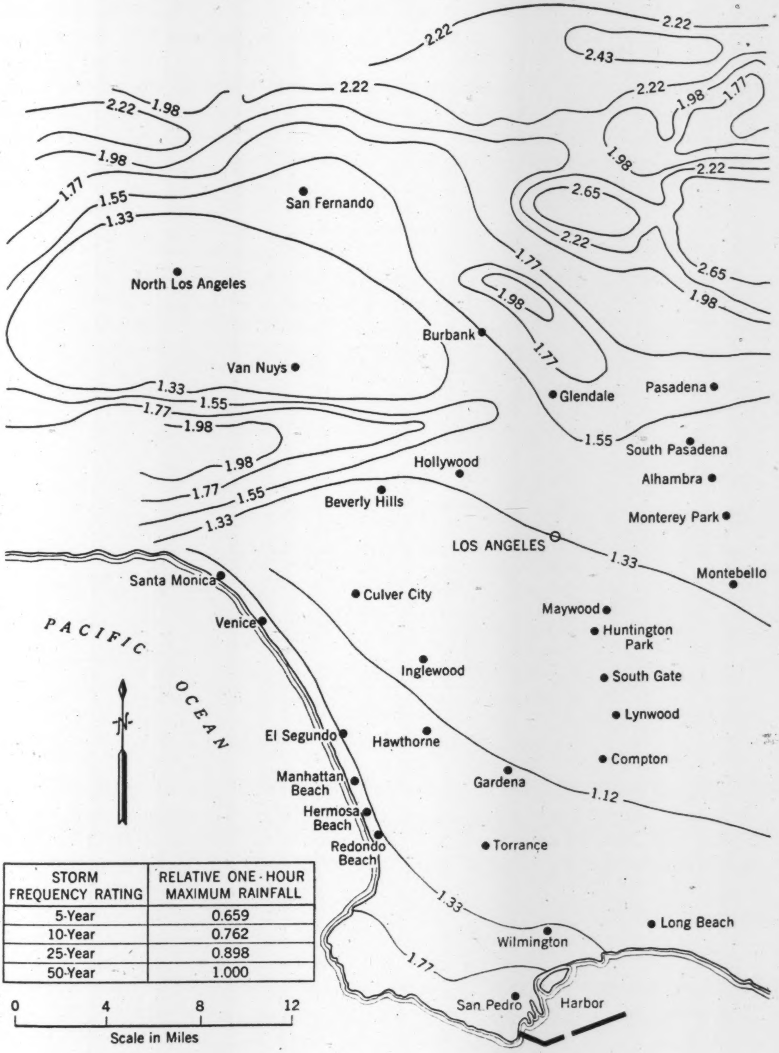
Storm Pattern.—The storm pattern was derived by plotting the major storms of the Los Angeles Station of the U. S. Weather Bureau as mass curves with the center of the most intense 5-min duration at a common point (90-min elapsed time for design purposes). The result was a pattern storm between "medium" and "delayed" and is illustrated in Fig. 3.

The rainfall intensity-duration curve (Fig. 1) is based on the analysis of a great number of individual storms; there is nothing in the construction of the curves which implies that a storm having a 10-yr frequency for any stated

---

[3] "Surface Runoff Phenomena," by Robert E. Horton, *Publication No. 101*, Horton Hydrological Laboratory, Voorheesville, N. Y., 1935.

[4] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.


2.22
2.43
2.22
2.22
1.98
1.77
2.22
1.98
1.98
1.98
2.22
1.77
1.55
2.65
1.33
San Fernando
2.22
1.77
2.65
North Los Angeles
1.98
1.98
Burbank
1.77
Van Nuys
1.33
Glendale
Pasadena
1.55
1.77
1.98
1.55
South Pasadena
1.98
Hollywood
Alhambra
1.77
1.55
Beverly Hills
Monterey Park
1.33
LOS ANGELES
1.33
Montebello
Santa Monica
Culver City
Maywood
Huntington Park
PACIFIC
Venice
Inglewood
South Gate
OCEAN
N
Lynwood
El Segundo
Hawthorne
Compton
Manhattan Beach
Gardena
1.12
Hermosa Beach
Redondo Beach
Torrance
1.33
Long Beach
Wilmington
1.77
San Pedro
Harbor
0
4
8
12
Scale in Miles

STORM FREQUENCY RATING	RELATIVE ONE-HOUR MAXIMUM RAINFALL
5-Year	0.659
10-Year	0.762
25-Year	0.898
50-Year	1.000

Fig. 2.—Isohyetal Map, Los Angeles Area

time duration will have the same frequency for any other time duration. A statistical study of the more intense of the storms shows that the intensities for the shorter durations are lower than would be indicated by the storm's frequency rating corresponding to the intensity for a longer duration; and that the greater the longer duration defining the frequency, the lower (relatively) will be the intensity corresponding to the stated shorter duration. This variation finds application in computing runoff hydrographs for various times of concentration.

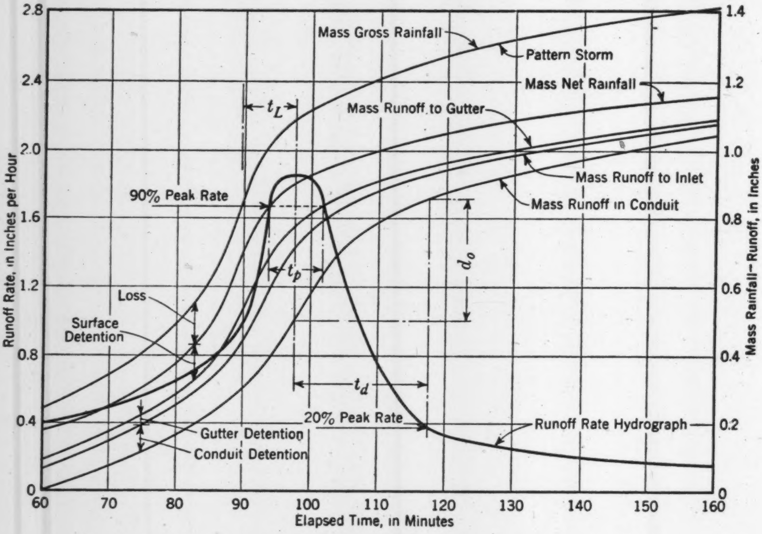


FIG. 3.—ELEMENTS OF THE HYDROGRAPH

Antecedent Precipitation.—For determining the values of antecedent precipitation, probability studies were made (using the method of H. Alden Foster,[5] M. Am. Soc. C. E.) of two composite factors of the rainfall records of the Weather Bureau, $I \times F$ and $I \times F \times M$. The composite factor M is defined by

$$M = \frac{E\,(d_{60} + 1)}{10} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and I = intensity of rainfall, in inches per hour (for a specific time duration). The rainstorm shape factor F is obtained by summing the values of the maximum 5, 10, 15, 20, 30, 45, and 60-min rainfall (d') in inches, in the order named and dividing by a corresponding summation of d''-values from the standard design rainfall curve (10-yr B curve, Fig. 1); or, $F_{15} = \frac{(d')_5 + (d')_{10} + (d')_{15}}{(d'')_5 + (d'')_{10} + (d'')_{15}}$

---

[5] "Theoretical Frequency-Curves and Their Application to Engineering Problems," by H. Alden Foster, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

for $t_c = 15$, etc. The evaporation factor E is an arbitrary value based on the average evaporation for the 60-day period preceding the storm, derived from seasonal variation of evaporation. Antecedent precipitation, in inches, is expressed as $d_{60} + 1$, in inches. This is equivalent to the summation of 100% of the rainfall for 0 to 4 days antecedent, 50% of that for 5 to 9 days, 25% of that for 10 to 14 days, 15% of that for 15 to 30 days, and 10% of that for 31 to 60 days, antecedent. The residual soil moisture is allowed for by "+ 1" in.

The application of the divisor 10 to the term, $E\ (d_{60} + 1)$ to derive M, a moisture factor, is made in order to correlate M to Fig. 4,[6] rainfall-rate loss-rate curves, in which the values of soil-loss rate were based on the local storm of March 2, 1938, when the value of $E\ (d_{60} + 1)$ was approximately 10 for the drainage areas subject to analysis.

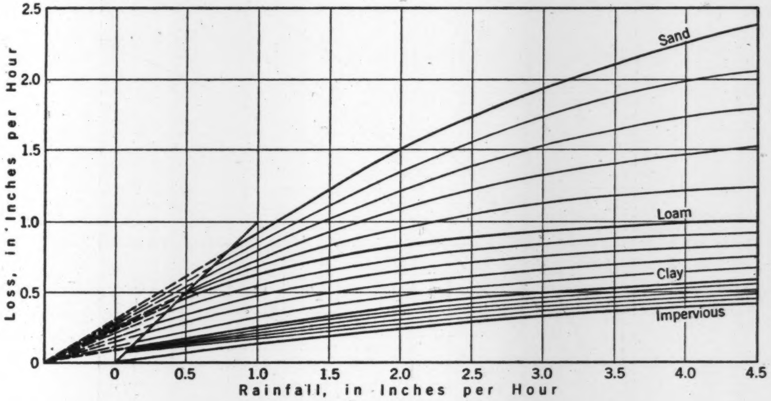


FIG. 4.—RAINFALL-RATE, LOSS-RATE CURVES

When $I\ F\ M$ for a given duration and frequency is divided by the corresponding product $I\ F$, the result is the value of the weighted moisture factor, M, equal to 0.39 on the isohyetal of 1.33 at the Weather Bureau station.

Because lawns and parks are irrigated between storms during the rainy season, an estimated 0.11 is added to the original value of M, making a total of 0.50 for isohyetal 1.33. The analyses show that this value applies to all frequencies of rainfall.

Rainfall Distribution.—The method of Frank A. Marston,[7] M. Am. Soc. C. E., for distributing rainfall intensities was applied to a large area centering in Pasadena, Calif., where records are available from a large number of automatic rain gages. These gages are located on isohyetals of varying intensity (see Fig. 2), and the records were corrected by the ratio of the value of the

---

[6] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), Fig. 16, p. 1098.

[7] "The Distribution of Intense Rainfall and Some Other Factors in the Design of Storm-Water Drains," by Frank A. Marston, *ibid.*, Vol. LXXXVII (1924), p. 535.

50-yr isohyetal on which the selected central gage was located to the value of the 50-yr isohyetal on which each individual gage was located. The results of the study are shown in Fig. 5.

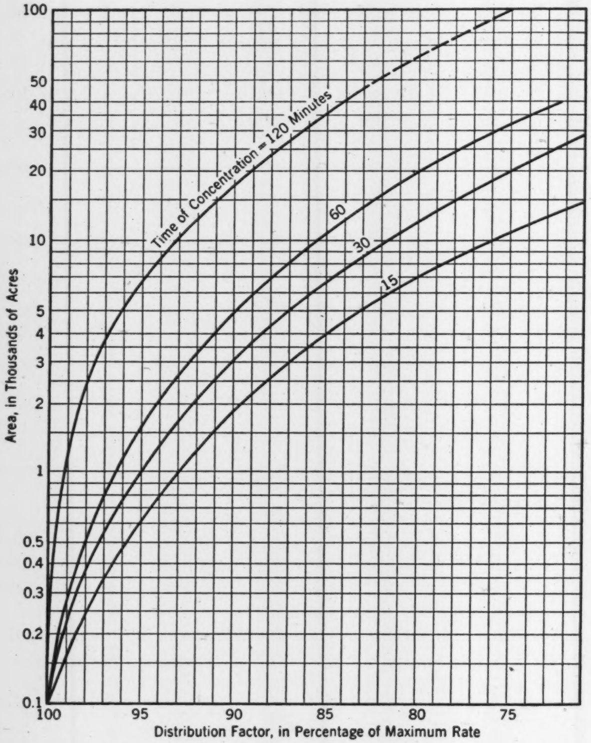


Fig. 5.—Rainfall Distribution Chart

II. Soil Infiltration Tests

A detailed report of soil infiltration tests on small soil plots with artificial precipitation was made in 1935 by L. W. Armstrong,[8] M. Am. Soc. C. E. Another method was the analysis of rainfall-runoff records for losses from large partly pervious drainage areas. From these sources, the rainfall-rate loss-rate curves for a moisture factor, *M*, of 1 were derived for use in preparation of design data (see Fig. 4).[6] A value of 1 for *M* denotes that the soil is well saturated and that the lower and more stable values of infiltration capacity prevail.

---

[8] Progress Report, Conference on Water Conservation, Los Angeles, California, by Committee on Conservation of Water, Irrig. Div., Am. Soc. C. E., December, 1935, p. 50.

In the paper by Messrs. Horner and Jens[4] and the discussion by the writer,[6] the question of variation of infiltration capacity with rainfall intensity was raised. Local experimentation shows predominant variation for bare soil as noted by the authors and a lesser amount of variation for sodded areas. The curves in Fig. 4,[6] used for design data, reflect the presence in urban areas of imperfectly graded lawns, bare soil, garden plots, and other departures from the plane surface on which any depth of detention spreads uniformly and utilizes the full infiltration capacity of the soil.

III. Experimental Analysis of Overland Flow and Gutter Flow

For a thorough treatment of this subject the reader is referred to the aforementioned paper by Messrs. Horner and Jens, on the development of the runoff hydrograph for a city block, and to the discussion by the writer introducing details of experimentation made in Los Angeles on overland flow on turf and various types of paving surface.

IV. Analysis of Rainfall-Runoff Records from Local Urban Areas

In attempting to solve analytically a problem with as many variables as runoff, the possibility of piling up accumulated errors through comparatively small errors in assigning values to these variables may be checked and minimized by collecting and analyzing rainfall-runoff records from drainage areas of various types and sizes. Further uses for rainfall-runoff records are the derivation of time and volume relations of the rainfall and the runoff, and the analyses of the shape and time elements of the runoff hydrograph.

Drainage Area Survey.—Drainage maps, improvement plans, and soil survey maps of the U. S. Department of Agriculture formed the basis of the survey. The drainage ridge was located by field surveys, and determinations were made of the percentages of pervious area A_p, impervious area A_i, and that portion of the impervious area directly tributary to the drain or gutter without flow over open soil A_d. Pertinent information is listed in Table 1. Runoff for Stations 10 and 11 was recorded with gutters calibrated from fire hydrant flow measured with a rated meter. All other runoff and rainfall were measured with automatic recorders using large scale charts.

TABLE 1.—Characteristics of Local Drainage Areas

Station	Location of runoff station[a]	Area (acres)	Soil type	Improvement (%)			Channel type
				A_i	A_d	A_p	
3	Eastern Ave. at Klamath St.	515.0	Clay and heavy loam	23.7	16.3	76.3	Gutter, pipe, reinforced concrete box
4	Broadway Channel[a]	1,549.0	Heavy loam	52.2	36.3[b]	47.8	Gutter, pipe, arch, lined channel
10	Spring St. at 5th St.	0.80	....	100	100	....	Gutter
11	5th St. at Spring St.	2.17	....	100	100	....	Gutter

[a] In Los Angeles, Calif., except Station 4 which is in Pasadena, Calif. [b] Estimated.

Time of Concentration.—This has been defined as the time for the storm water to flow from the ridge to the point of gaging, excluding finger-like pro-

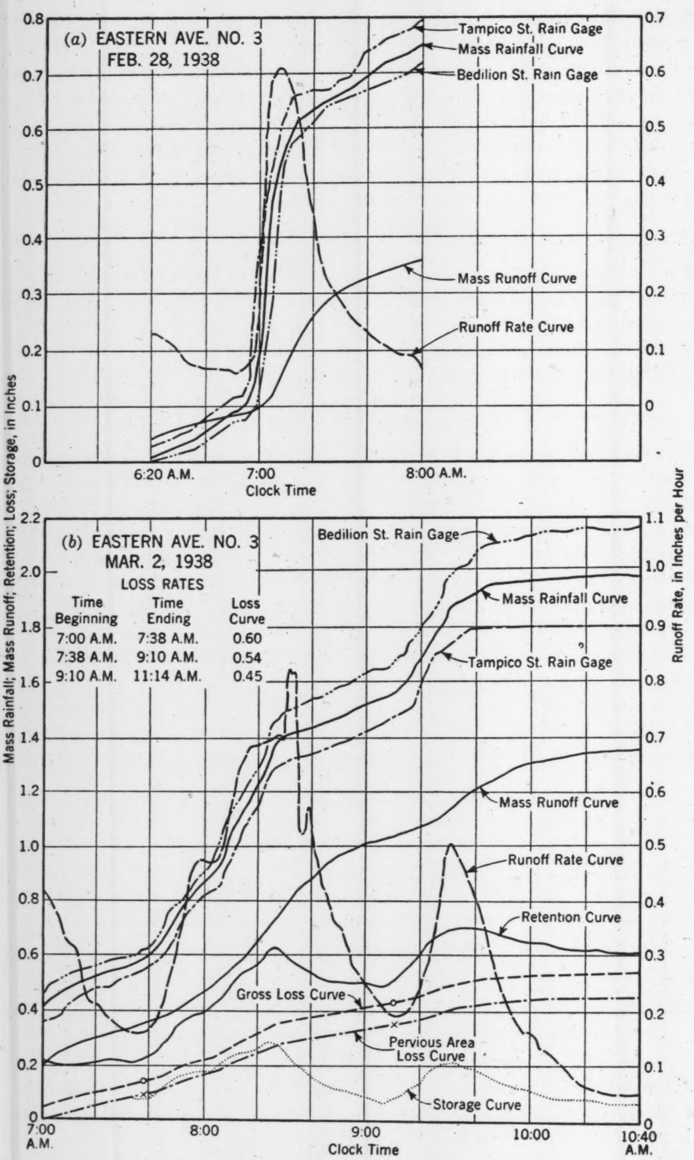


Fig. 6.—Analysis of Storms Recorded at Eastern Avenue, Station 3

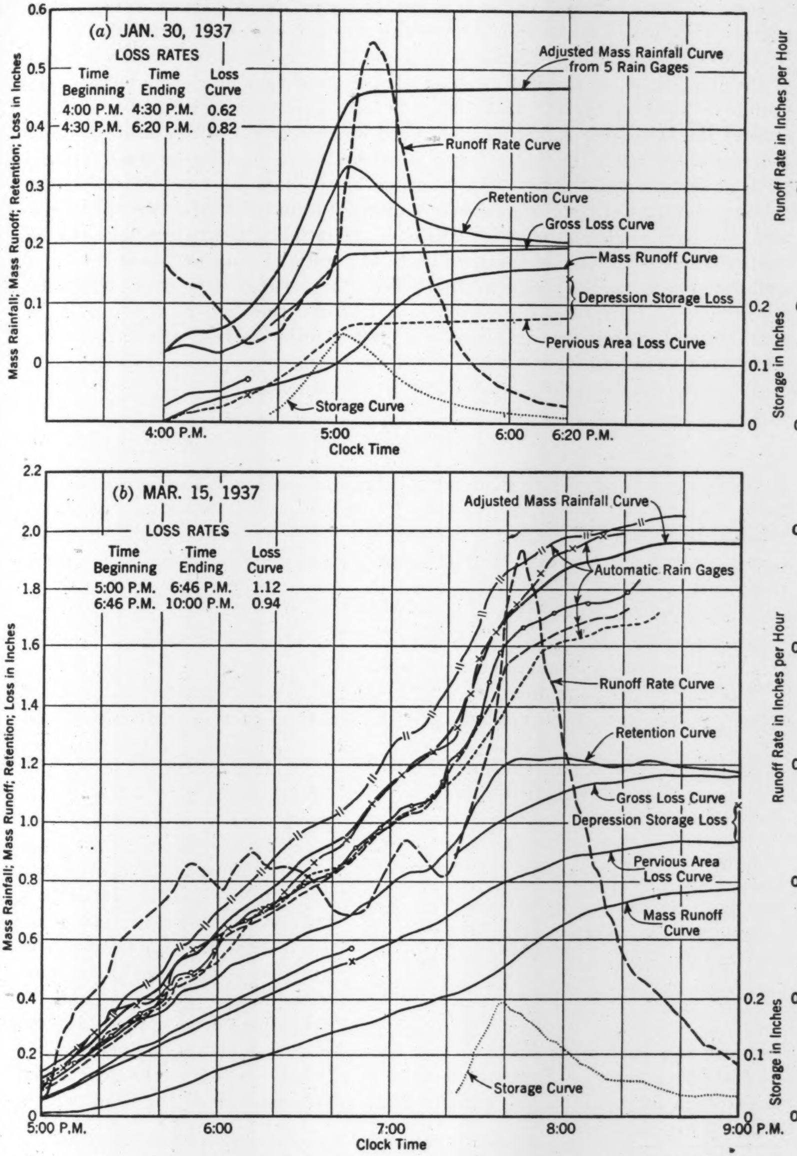


FIG. 7.—ANALYSIS OF STORMS RECORDED AT BROADWAY, STATION 4

jections of area. It is also the measure of an intercept on the mass rainfall curve which, with base flow, is effective in producing a peak of runoff.

In design work (for assumed conditions) the time of concentration consists of the inlet time for the initial block plus time of flow down the conduit (in minutes). For analysis of runoff records from partly pervious areas, the inlet time of the initial block is affected by the weights of the volumes of overland flow delivered from the lawns (slow flow) and from the impervious surfaces (fast flow). Account must be taken also of the fact that in normal design tabling, the time of flow in a closed conduit is computed for full-wetted perimeter (velocity = 86% of that for free flow); that the ultimate drainage system contemplates extension of the conduit to the initial block; and that flow in a well-designed drain is at efficient depth (free flow) except at points of hydraulic disturbance. Therefore, for analysis of records, the free flow velocities in conduits and gutters are multiplied by 0.86 in order to keep the data on the same basis as in design.

Time for overland flow is computed by the equation

$$t_c = \frac{60\ \delta_a}{\sigma} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which δ_a (average depth of flow) is computed for the various types of surfaces by formulas presented elsewhere;[9] and σ = rate of supply of excess rainfall, in inches per hour. Eq. 2 represents the horizontal (time) distance between the mass curves of rainfall supply and runoff. Gutter flow time is computed by[10]

$$t = \frac{0.0443\ l^{0.732}}{(S_o)^{0.39}\ \sigma^{0.252}} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which l = length of overland flow, in feet.

For partly pervious areas, such as Station 3 (Table 1), a series of time of concentration curves is needed, varying from a dry soil condition (about 18% runoff) to a saturated condition (about 76% runoff). Data for plotting these curves are presented in Table 2.

TABLE 2.—DATA FOR PLOTTING TIME-OF-CONCENTRATION CURVES

q_1	TIME, t_c, PERCENTAGE OF RUNOFF	
	76%	18%
0.15	59.7	32.3
0.20	54.8	30.9
0.40	43.2	27.0
0.60	36.5	24.3
0.80	32.3	22.7
1.00	28.7	21.7

Plotting.—Using a part of the storm of March 2, 1938, on gaging Station 3 (Fig. 6(*b*)) as an illustration, the mass rainfall curve (in inches), the runoff rate curve (in inches per hour), the mass runoff curve (in inches), and the retention curve or the difference between the mass rainfall curve and the mass runoff curve (in inches) need no further explanation for their method of plotting. The gross loss curve and the storage curve, both expressed in inches, are more involved and require more detailed explanation.

---

[9] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1100, Eqs. 17, 18, and 19.

[10] *Ibid.*, p. 1102, Eq. 20*b*.

First, select clock-time points on the record where the runoff rate is low and the rainfall rate has been low and fairly uniform for the preceding 20 min. With curves for the drainage area of depths of surface detention for low rates of rainfall and of depths of gutter and conduit storage for low rates of runoff, values of composite storage were plotted down from the retention curve; these points lie on the gross loss line and are control points.

Second, the value of impervious surface loss (experimentally determined as approximately 10% of the rain falling on the impervious surface) is computed for the reach between control points ($0.10\ A_i\ d_f = 0.0237\ d_f$ for Station 3, d_f being the volume of rainfall, in inches) and is plotted down from the control point at the right of the reach.

Third, the pervious surface loss is a function of the rain falling directly on the pervious surface plus the rain falling on the portion of the impervious surface discharging runoff on to the pervious surface; 10% of this latter rainfall is already lost on the impervious surface, leaving 90%, which will be subject to further loss on the pervious surface on to which it flows. The rainfall rate is adjusted therefore in the ratio: $\frac{A_p + 0.9\ (A_i - A_d)}{A_p}$, or 1.09, for Station 3.

Fourth, the reaches between control points are analyzed by the following check list:

(*a*) Time to time (for an intercept on the mass rainfall curve having an approximately uniform rate);
(*b*) Rainfall rate;
(*c*) Amended rainfall rate ($1.09\ I$);
(*d*) Infiltration rate (from Fig. 4[6]), for $1.09\ I$;
(*e*) Infiltration rate $\times\ A_p$ (76.3%);
(*f*) Impervious loss rate ($0.10\ A_i\ I$ or $0.0237\ I$); and
(*g*) Summation of items (*e*) and (*f*) (gross loss rates).

If the sum of the products of the rates in item (*g*) and the time intercepts in item (*a*) (in hours) does not equal the gross loss for the reach, recompute the tabulation with another infiltration curve from Fig. 4.[6] When a balance is secured, plot the pervious and gross loss curves. (The loss curves noted in Figs. 6 and 7 are identified by the loss intercept on a 3-in. rainfall rate in Fig. 4.[6])

Fifth, it is found frequently that, when a reach between control points contains high rates of rainfall and runoff, the infiltration curve required for balancing is greatly in excess of those used on both sides of the reach. Actually, the curve will lie between those for the reaches immediately before and after; the excess loss is depression storage. In this case, the tabulation is made for an intermediate infiltration curve; the products of items (*a*) and (*g*) are summed and deducted from the total divergence between control points; the result is a measure of depression storage loss. (For illustration, see Fig. 7 for Station 4.)

The storage curve, embracing surface, gutter, and conduit storage, is plotted as the vertical divergence between the retention curve and the gross loss curve. Analyses of other rainfall-runoff records are filed in the Engineering Societies Library.

Derivation of Hydraulic Phenomena.—A chart, showing a typical runoff hydrograph, its relation to the mass rainfall curve, and various factors to be analyzed in this paper, is delineated in Fig. 3. On the recession side of the hydrograph (see Figs. 6 and 7), when the depth of composite storage (D), in inches, is plotted against the coincident rate of runoff (q), in inches per hour, the resulting curve on rectangular coordinate paper is a straight line below 90% peak runoff rate. When the slope of this line $\left(\frac{\Delta D}{\Delta q}\right)$ is plotted against time of concentration, the result is also a straight line. Sample curves for Stations 3, 4, 10, and 11 are shown in Fig. 8. Fig. 9(a) shows the relation of the slopes

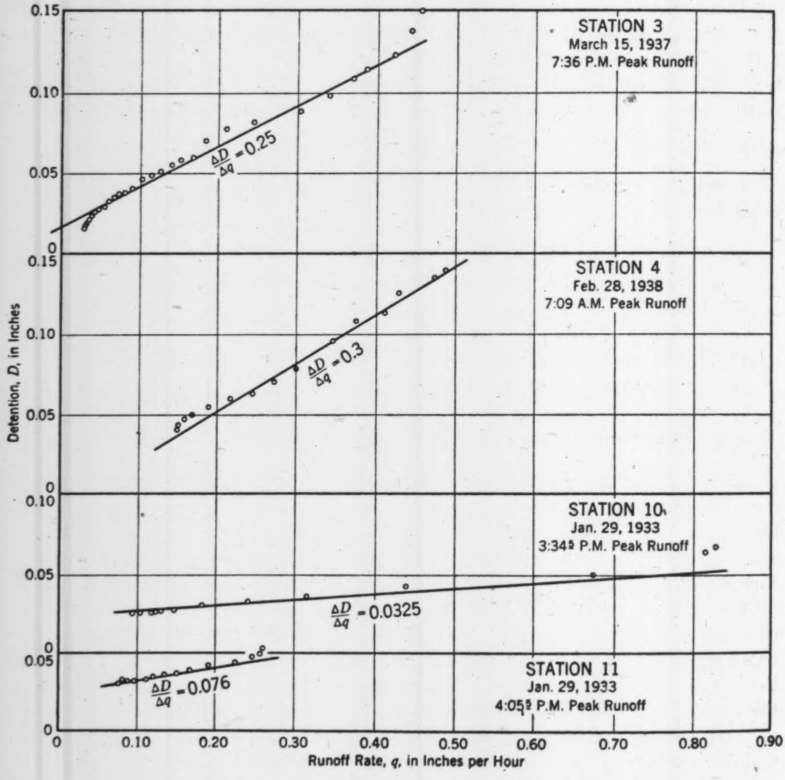


FIG. 8.—DEPTH OF STORAGE WITH COINCIDENT RUNOFF RATES, RECESSION SIDE OF HYDROGRAPH

of the storage-runoff rate curves and times of concentration as derived from analyses of records of Stations 3, 4, 10, and 11. Data are recorded in Table 3. In Col. 4, Table 3, percentage of runoff = ratio of the volume of runoff to volume of rainfall.

In the analysis of storm records, it is necessary to reproduce the recession curve by computation as it would occur if the rainfall following 90% peak of runoff were in the adopted standard pattern (see Fig. 10 for Station 4, February 28, 1938). The net rates of rainfall adjusted in ratio to the standard pattern rates are 0.110 and 0.036; the value of storage at 90% peak is 0.125 in.; the time of concentration is 44.3 min. Using a value of $\frac{\Delta D}{\Delta q}$ (Fig. 9(*a*)) of 0.311 for 44.3 min, the theoretical recession curve is plotted to a point where the runoff rate is less than 20% peak:

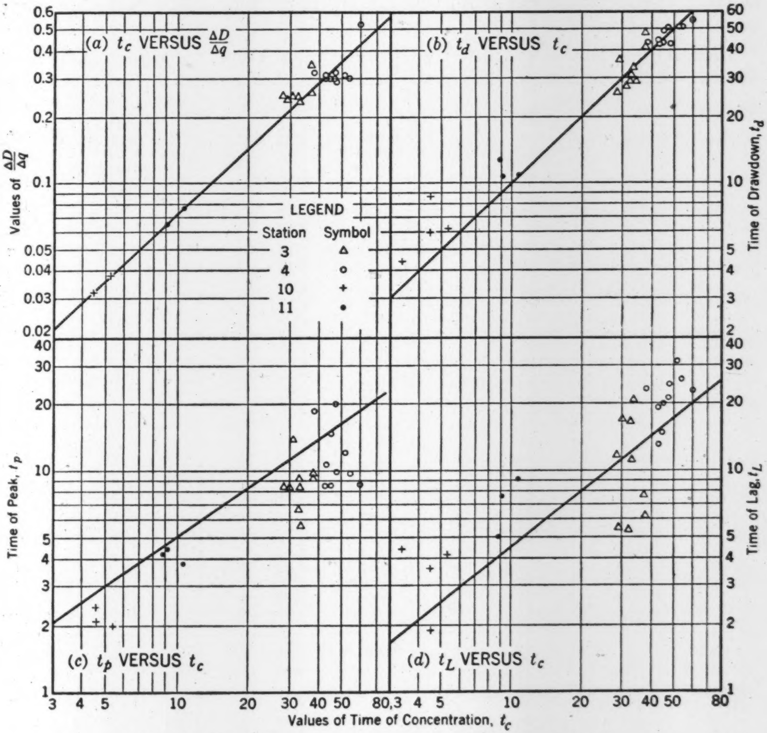


FIG. 9.—TIME ELEMENTS OF THE HYDROGRAPH FROM RECORDS

1. From 7:13 to 7:14 a.m., plot the 90% peak runoff rate as a mass runoff curve, distant below the mass curve of net rainfall 0.125 in. (the value of storage), measured at 7:13½.
2. From 7:14 to 7:16, prolong the mass runoff curve with a rate (0.4224 in. per hr) so selected that the value of storage measured at 7:15, the center of the reach, lies on the curve of $\frac{\Delta D}{\Delta q}$ (0.117 in.).
3. Continue with 2-min reaches.

From these curves, the time of drawdown (t_d), or the elapsed time (in minutes) between 100% and 20% peak runoff rate, is measured (see Table 3 and Fig. 9(*b*)).

Another derived phenomenon of the runoff hydrograph is disclosed by plotting the recession curve on semilogarithmic paper. The recession curve after 90% peak tends to be a straight line. In Fig. 11, recession curves have been plotted, both revised as indicated in the preceding paragraph and without revision. It will be noted that, for the records having small times of concentration, the curve tends to draw to the right, away from a straight line, at a point well above the 20% peak runoff rate; for the long times of concentration, the curve remains a straight line down to a low percentage of peak rate. An explanation is that, as the time of concentration increases, the greater depth

TABLE 3.—ELEMENTS OF THE HYDROGRAPH DERIVED FROM RECORDS

Date (1)	Hour (2)	Record peak runoff rate, in in. per hr (q_1) (3)	Percentage of runoff (4)	Time of concentration (min) (t_c) (5)	$\frac{\Delta D}{\Delta q}$ (6)	Time of drawdown (min) (t_d) (7)	Time of peak (corrected) (min) (8)	Time of lag (min) (t_L) (9)
(*a*) STATION 3								
1/ 5/35	3:44 a.m.	0.430	24.6	28.3	0.250	25.4	8.5	11.7
12/30/36	9:04 p.m.	0.320	34.2	33.4	0.240	29.2	5.6	20.6
3/15/37	7:36 p.m.	0.456	44.0	32.7	0.250	31.8	9.1	17.0
2/11/38	12:27 p.m.	0.315	30.4	32.2	....	29.3	6.7	5.4
2/28/38	7:08 a.m.	0.610	42.5	29.4	0.240	37.2	8.6	5.3
2/28/38	11:30 p.m.	0.560	56.0	33.3	0.240	33.4	8.4	11.4
3/ 2/38	8:32 a.m.	0.830	70.0	30.8	0.250	28.6	14.0	17.5
3/ 2/38	9:30 a.m.	0.505	69.0	37.5	0.260	49.2	9.8	6.2
3/ 2/38	3:54 p.m.	0.530	73.0	37.1	0.350	41.6	9.1	7.8
(*b*) STATION 4								
10/18/36	4:21 p.m.	0.407	31.8	43.2	0.300	44.6	8.5	13.0
12/28/36	12:29 a.m.	0.156	51.1	60.0	0.530	55.0	8.5	23.6
1/ 5/37	7:34 p.m.	0.224	46.7	54.2	0.300	50.4	9.6	26.4
1/30/37	5:12 p.m.	0.325	41.4	48.0	0.290	43.4	9.9	24.5
3/15/37	7:45 p.m.	0.480	40.0	43.0	0.310	43.4	10.6	19.2
2/28/38	7:09 a.m.	0.497	44.0	44.3	0.300	43.4	8.5	14.8
2/28/38	11:45 p.m.	0.376	44.3	47.6	0.320	50.8	20.0	21.0
3/ 2/38	8:38 a.m.	0.880	58.5	38.8	0.320	43.4	18.6	23.4
3/ 2/38	9:41 a.m.	0.690	70.0	45.3	0.300	49.8	14.7	20.1
3/ 2/38	4:00 p.m.	0.557	80.0	52.0	0.310	50.8	12.0	26.6
(*c*) STATION 10								
1/16/33	12:08 a.m.	1.570	90.0	3.4	....	4.4	1.3	4.4
1/16/33	3:36 p.m.	0.815	82.0	4.5	....	8.7	2.4	1.9
1/29/33	3:345 p.m.	0.828	78.5	4.5	0.0325	6.0	2.1	3.6
1/29/33	4:01 p.m.	0.560	69.0	5.3	0.0375	6.2	2.0	4.2
(*d*) STATION 11								
1/16/33	3:40 p.m.	0.472	69.0	8.9	....	12.9	4.2	5.0
1/29/33	3:38 p.m.	0.432	66.3	9.1	0.064	10.8	4.4	7.6
1/29/33	4:055 p.m.	0.260	56.0	10.7	0.076	10.9	3.8	9.1

of water stored in transit governs the pattern of runoff for a greater length of time.

An equation for peak runoff rate has been derived from a study of the elements of the hydrograph (see Fig. 3):

$$q_c = \frac{K\,d_o}{t_d} \qquad (4)$$

in which q_c = the computed peak runoff rate, in inches per hour, K varies from 96 for a 5-min time of concentration to 108 for 60 min, and d_o = volume of runoff on the recession side of the hydrograph between 100% and 20% peak runoff rate, in inches of depth on the drainage area.

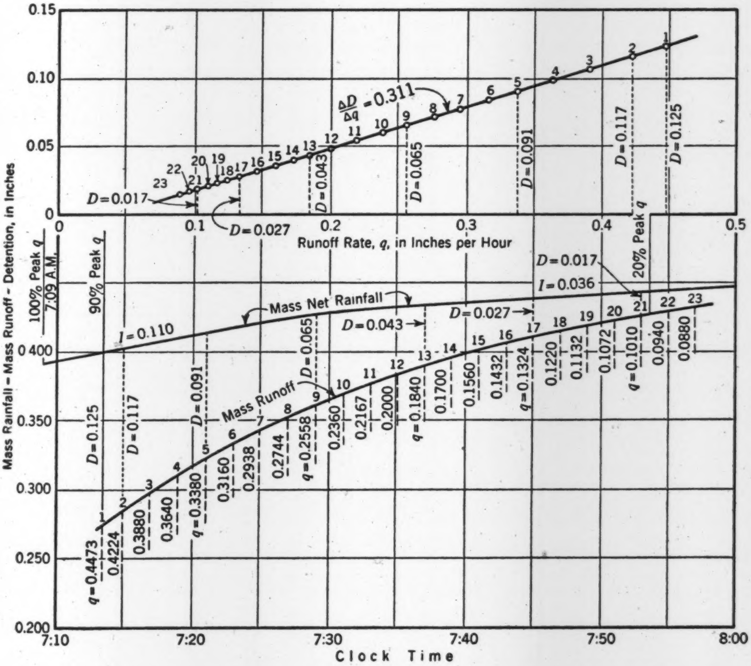


FIG. 10.—COMPUTATION OF THEORETICAL RECESSION CURVE (STATION 4, FEBRUARY 28, 1938)

The foregoing hydraulic phenomena are characteristic of urban runoff but are not found in certain types of mountain runoff.

Derivation of Time Factors.—The meaning of the time factors is shown graphically in Fig. 3. Values derived from records are listed in Table 3. Comparison of their values with corresponding times of concentration is shown graphically in Fig. 9. The curves used hereinafter in the synthesis of the standard hydrograph are delineated as solid lines.

The lag time (t_L) can be defined as the time distance (in minutes) between the centroid of the intensities of the effective rainfall curve, for the intercept thereon of the time of concentration, and the time of peak runoff rate. It is dependent on the shape and distribution of impervious development of the

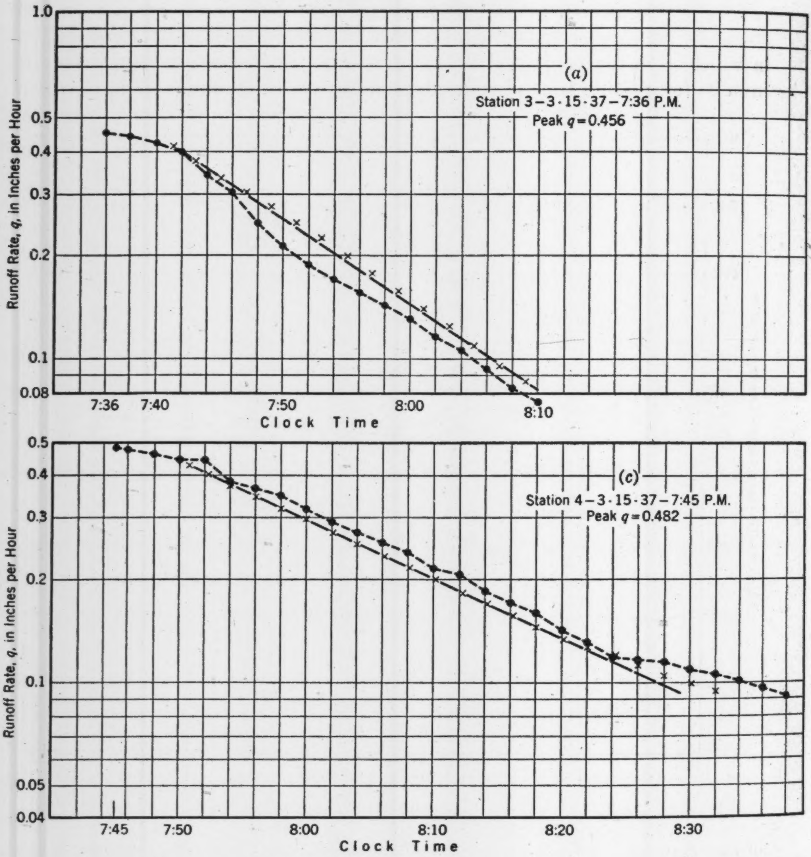
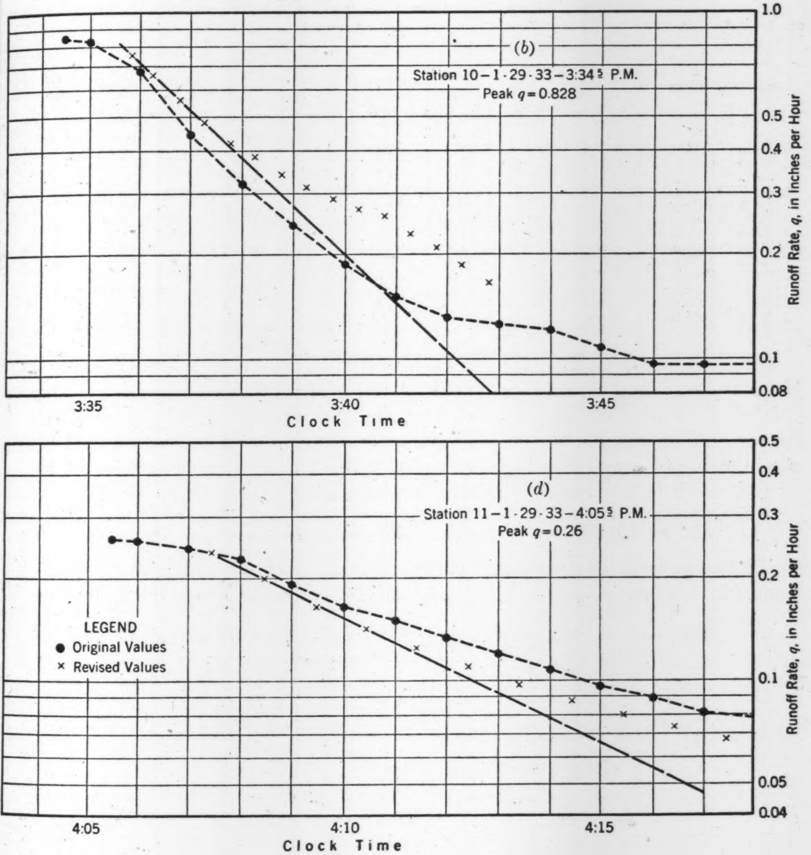


FIG. 11.—TYPICAL

drainage area and no curve of fixed values can be derived from the data (see Fig. 9(d)). The time of drawdown (t_d) is shown by Fig. 9(b) to be approximately equal to the time of concentration for the adopted rainstorm pattern. The time in minutes from 90% to 90% peak runoff rate (t_p) is weighted by comparison with the time from 80% to 80% peak (see Fig. 9(c)).

V. Runoff Frequency

The frequency of runoff is dependent in part on the amount of open soil in the drainage area and the wetness of that soil. This wetness is caused by precipitation antecedent to the high-intensity storm which produces the peak runoff. Infiltration tests and analyses of rainfall-runoff records show that runoff increases as the saturation of loams and clays progresses; this saturation is measured by the moisture factor M.



Recession Curves

Although sands, free of colloidal swelling, respond to an antecedent precipitation of much shorter duration, their infiltration capacity is so great that the use of the same moisture factor as for clays and loams causes no material error.

Analysis has been made of the factors in the equation

$$q_c = K_1\,(I\,F\,M^{A_p})^x \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

for the 515 acres of predominantly pervious area of Station 3 (see Table 4).

TABLE 4.—DATA FOR EQ. 5 AT STATION 3

Date	Hour	Record peak runoff rate, in in. per hr (q_1)	q_1 corrected for prior storm (q_2)	Percentage of runoff	Time of concentration, in min (t_c)	Rainfall rate, in in. per hr (I)	Rainstorm shape factor (F)	Moisture factor (M)	IFM^{A_p}	Item No.
1/ 5/35	3:44 a.m.	0.430	0.413	24.6	28.3	1.19	0.805	0.243	0.33	1
12/30/36	9:04 p.m.	0.320	0.300	34.2	33.4	0.68	0.425	0.441	0.16	2
3/15/37	7:36 p.m.	0.456	0.422	44.0	32.7	0.88	0.484	0.372	0.20	3
2/11/38	12:27 p.m.	0.315	0.275	30.4	32.2	0.59	0.451	0.393	0.13	4
2/28/38	7:08 a.m.	0.610	0.570	42.5	29.4	1.09	0.790	0.377	0.41	5
2/28/38	11:30 p.m.	0.560	0.510	56.0	33.3	0.77	0.497	0.575	0.25	6
3/ 2/38	8:32 a.m.	0.830	0.730	70.0	30.8	1.06	0.600	0.838	0.55	7
3/ 2/38	9:30 a.m.	0.505	0.455	69.0	37.5	0.64	0.416	0.902	0.25	8
3/ 2/38	3:54 p.m.	0.530	0.460	73.0	37.1	0.55	0.430	1.105	0.26	9
12/18/38	7:43 p.m.	0.192	0.182	....	32.4	0.61	0.400	0.442	0.13	10
1/ 5/39	12:12 p.m.	0.229	0.219	....	30.0	0.73	0.403	0.215	0.09	11
1/ 7/40	7:53 p.m.	0.227	0.201	....	30.2	0.63	0.440	0.231	0.09	12
2/29/40	3:15 a.m.	0.260	0.250	....	31.0	0.64	0.405	0.305	0.10	13
3/31/40	4:47 p.m.	0.147	0.147	....	33.7	0.50	0.353	0.228	0.06	14

If $I\,F$ is considered constant, q_c varies as M (or M^{A_p}) to the exponent x. For Station 3, $K_1 = 1.00$ and $x = 0.55$. Based on studies of this and similar curves and on results of the trial-and-error development of the method of computation of runoff (see Fig. 12), runoff from open soil has been assumed to vary as $M^{0.59}$ for the range of rainfall intensities used in urban design.

VI. SYNTHESIS OF THE HYDROGRAPH FROM DERIVED DATA

Preliminaries to computing runoff hydrographs for use in design are assumptions for rainfall intensities (Figs. 1 and 2), antecedent precipitation (see Section I and Table 5), and classification of drainage areas as to impervious development. This classification is given in Table 6, which is based on surveys

TABLE 5.—MEAN VALUES OF THE MOISTURE FACTOR M

Isohyetal	1.12	1.33	1.55	1.77	1.98	2.21	2.65
M	0.450	0.500	0.555	0.612	0.665	0.730	0.835
$M^{0.59}$	0.640	0.675	0.715	0.755	0.795	0.840	0.910

TABLE 6.—CLASSIFICATION OF URBAN DRAINAGE AREAS

Type of development	SURFACE CLASSIFICATION			Factor of equivalent pervious area rainfall $\frac{A_p + 0.9\,(A_i - A_d)}{A_p}$
	A_d	A_i	A_p	
Park	15	15	85	1.00
Level residence	40	60	40	1.45
Hillside residence	50	50	50	1.00
Bungalow court	60	80	20	1.90
Apartment house	70	85	15	1.90
Impervious; business and manufacturing	100	100	0	....

and zoning ordinances. For brevity, the percentage A_d and the soil type (1 for clay, 2 for loam, and 3 for sand) are used to designate an area (15 − 1 denotes "park, with clay soil").

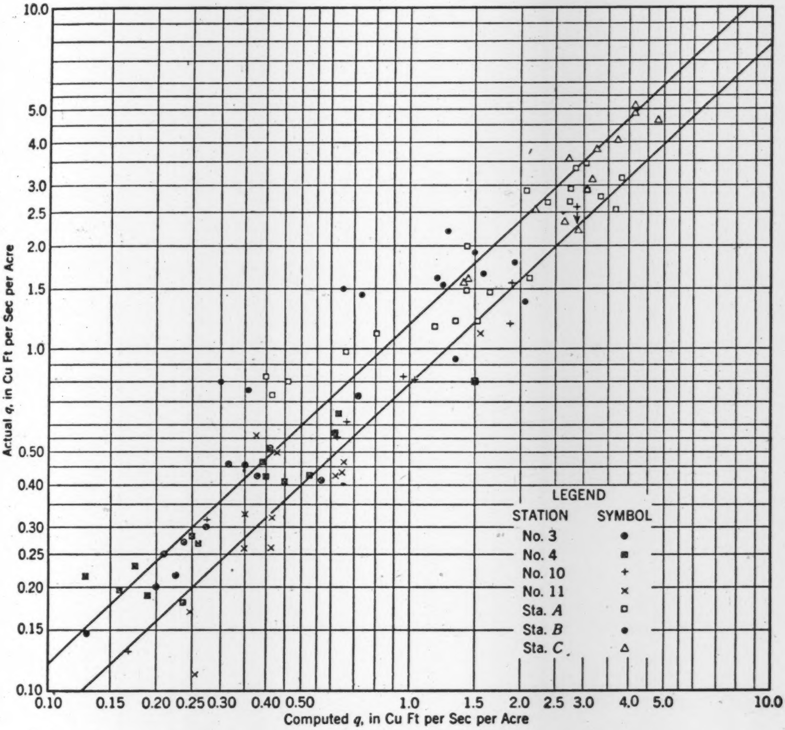


FIG. 12.—COMPUTED AND ACTUAL RUNOFF RATES—URBAN AREAS

The method of development of the runoff hydrograph follows (see Fig. 3):

1. Plot the gross mass rainfall curve (10-yr B curve, Fig. 1);
2. Deduct losses to form the net rainfall curve—
    (*a*) Impervious area losses;
    (*b*) Pervious area losses—depression storage and infiltration;
3. Deduct the varying depth of overland flow detention to form the mass runoff-to-gutter hydrograph;
4. Deduct the varying depth of gutter storage to form the mass runoff-to-inlet hydrograph; and
5. Deduct the varying depth of conduit storage to form the mass runoff-in-conduit hydrograph.

In the paper by Messrs. Horner and Jens,[4] the method has been developed down to, and including, the runoff-to-inlet hydrograph.

The writer has derived runoff-to-inlet hydrographs for the area classifications of Table 6 and for various types of soil. As the shapes of the resulting hydrographs were similar, they were plotted for the more intense portion of the storm on a common clock-time scale with percentages of accumulated depth of runoff as ordinates. The curves nested with slight variations; therefore the runoff-to-inlet hydrograph of the 10-yr rainfall curve on the 1.33 iso-

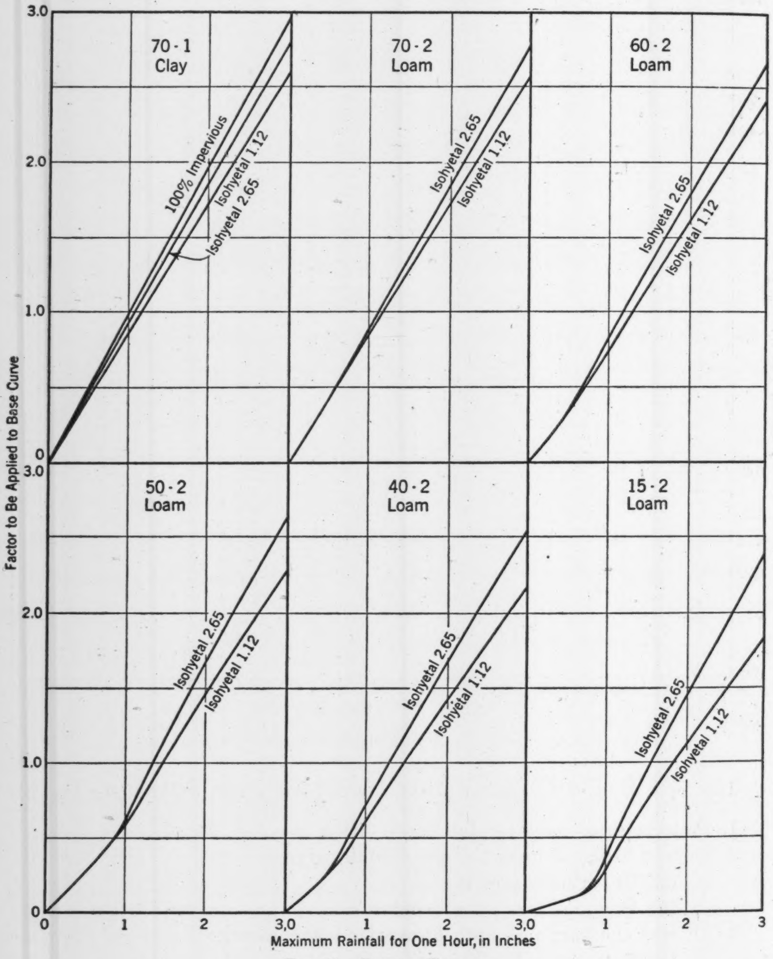


FIG. 13.—RUNOFF FACTORS

hyetal (Fig. 2) for 100% impervious development was selected as a base for the computation of hydrographs of runoff in the conduit, from which were derived the basic runoff rates for times of concentration from 5 min to 120 min (Table 7),

TABLE 7.—BASIC RUNOFF RATES AND REDUCTION FACTORS FOR CONDUIT DETENTION

(Col. A—Peak Runoff Rates Base Curve (Cubic Feet per Second per Acre); and Col. B—Peak Reduction Factor, Conduit Detention (Percentage per Minute of Flow))

t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)
5	3.40	12.00	14	2.17	4.10	26	1.65	2.04	42	1.34	1.18	58	1.17	0.80
6	3.13	10.00	15	2.10	3.80	28	1.60	1.88	44	1.31	1.12	60	1.15	0.77
7	2.93	8.60	16	2.05	3.50	30	1.55	1.75	46	1.29	1.05	70	1.07	0.64
8	2.76	7.50	17	1.99	3.30	32	1.51	1.62	48	1.27	1.00	80	1.01	0.55
9	2.63	6.60	18	1.94	3.10	34	1.47	1.51	50	1.24	0.96	90	0.96	0.47
10	2.50	5.90	19	1.90	2.90	36	1.43	1.42	52	1.22	0.92	100	0.92	0.42
11	2.41	5.40	20	1.85	2.75	38	1.40	1.32	54	1.20	0.88	110	0.88	0.37
12	2.32	4.85	22	1.78	2.45	40	1.37	1.24	56	1.18	0.84	120	0.85	0.34
13	2.24	4.45	24	1.71	2.25	....	....	....	....	....	....	....	....	....

the variation for other rainfall curves and states of areal development to be secured by applying to the basic values a runoff factor (Fig. 13) based on their volume of flow for the intercept of clock time containing the intense runoff to gutter.

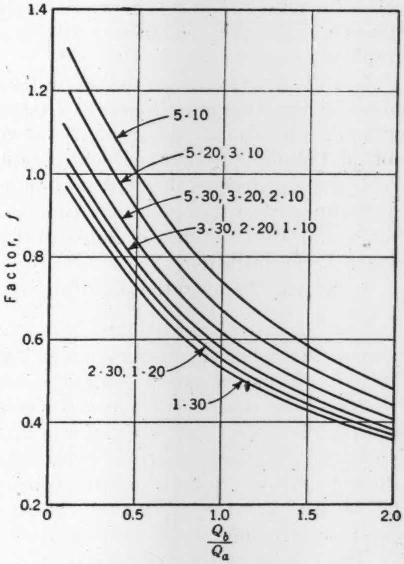


FIG. 14.—CHART FOR COMPUTATION OF POINTS OF CONFLUENCE

Construction of the Runoff-in-Conduit Hydrograph.—The drainage area for the computation is rectangular in shape with the unit areas consisting of five-acre blocks, 330 ft by 660 ft, with the long side north and south; drainage flow south and east. The trunk drain (13,860 ft long) flows south along the east side of the drainage area and intercepts a lateral (4,290 ft long) every 660 ft. Each lateral serves 14 blocks. The rectangular shape, although common in urban drainage, is a special condition of shape; a method of correction of time of concentration for variation of shape is developed subsequently herein (see Fig. 14).

The inlet time of the initial block is 5 min. The mean velocity of flow in the conduit (V), based on full wetted perimeter, is assumed as 5.5 ft per sec. A number of tabulations of conduit storage capacity using different velocities of flow showed the same depth of storage capacity, when spread over the drainage area, for any given time of concentration.

The factors assumed for computations are peak runoff rates (Table 7), lag time (Fig. 9(*d*)), time duration of 90% peak (Fig. 9(*c*)), and time of drawdown (Fig. 9(*b*)). The curves on these charts indicate the final values obtained from these studies, determined after numerous trial computations.

The steps of computation and construction are:

1. Plot the 5-min hydrograph from the initial block as a rate curve and as a mass runoff curve by altering the net rainfall curve for detention on the surface and in the gutters.
2. Plot a trial 10-min hydrograph as a rate curve.
3. At selected points on the clock-time scale, compute the cross-sectional area of conduit for the 5-min and 10-min hydrographs and calculate ratios thereof to full cross-sectional areas; the average of these ratios multiplied by the volume capacity of the conduits between 5-min and 10-min times of concentration equals the volume of water in the conduits at each selected clock time. These volumes (in cubic feet) divided by the square feet in the drainage area having the 10-min time of concentration yield depths of conduit storage in feet over the drainage area.
4. Plot the depth of conduit storage (in inches over the drainage area) below the mass runoff curve for the 5-min hydrograph for the clock time at the left side of the chart; beginning at this plotted point, plot the 10-min hydrograph as a mass curve.
5. Plot the remaining depths of conduit storage above the mass runoff curve for the 10-min hydrograph; if these points coincide with the mass runoff curve for the 5-min hydrograph, the assumed 10-min hydrograph is correct; if not, plot another 10-min trial hydrograph and repeat the operations.
6. Having derived the correct 10-min hydrograph, assume a 20-min hydrograph, and repeat the operations, computing volume of conduit storage between 5-min and 10-min times of concentration, and between 10-min and 20-min times of concentration.
7. Repeat the operation for 10-min increments in time of concentration.

For illustrating the method, the 20-min runoff-in-conduit hydrograph is constructed using computed data in Tables 8, 9, and 10, and the graphs in Fig. 15. Tables 8 and 9 contain computations for the storage capacity of conduits lying upstream from points having times of concentration between 5 and 60 min. Table 10 is a multiple table with a common horizontal scale of clock times of points selected for computing volumes of instantaneous conduit storage. These values of conduit storage depth are plotted in Fig. 15 above the 20-min mass runoff curve. These points have been balanced with the 5-min mass runoff curve to a reasonable but not mathematical accuracy.

The results of these analyses are summarized in the table of basic peak runoff rates (Table 7) and in a chart of the time elements of the basic hydrographs (Fig. 16). With these basic data and charts of runoff factors (Fig. 13) for various rainfall intensities and physical characteristics of the drainage area, peak runoff rates and hydrographs can be computed for use in design. In Fig. 16, the time of drawdown (100% to 20% of the peak runoff rate) is equal to the time of concentration, t_c, for any time exceeding 20 min. A table for plotting the basic standard hydrographs for times of concentration of from 5 to 60 min is filed in the Engineering Societies Library. Examples are shown in Figs. 3 and 15.

TABLE 8.—VOLUME INCREMENTS OF CONDUIT DETENTION CAPACITY

Time of concentration, in min (t_c)	Area acres (A)	Length, in ft (L)	Record peak runoff rate, in in. per hr (q_e)	Factor $\left(\frac{A\,L}{V}\right)$	Volume increment, in cu ft $\left(\frac{A\,L\,q_e}{V}\right)$	Cumulative totals, in cu ft
5	5	330	3.40	300	....	
6	10	330	3.13	600	1,020	
7	15	330	2.93	900	1,878	
8	20	330	2.76	1,200	2,637	
9	25	330	2.63	1,500	3,312	
10	30	330	2.50	1,800	3,945	12,792
11	35	330	2.41	2,100	4,500	
12	40	330	2.32	2,400	5,061	
13	45	330	2.24	2,700	5,568	
14	50	330	2.17	3,000	6,048	
15	55	330	2.10	3,300	6,510	
16	60	330	2.05	3,600	6,930	
17	65	330	1.99	3,900	7,380	
18	70	660	1.94	8,400	7,761	62,550
20	140	660	1.85	16,800	16,296	78,846

TABLE 9.—AVAILABLE CONDUIT DETENTION BETWEEN STATED TIMES OF CONCENTRATION

Time of concentration, in min (t_c)	Area acres (A)	Unit[a]	TIMES OF CONCENTRATION						Total detention, in in.
			5–10 min	10–20 min	20–30 min	30–40 min	40–50 min	50–60 min	
10	30	MCF in.	12.8 0.117	....	....	....	....	....	0.117
20	140	MCF in.	25.6 0.050	115.8 0.228	....	....	....	....	0.278
30	490	MCF in.	89.5 0.050	364.6 0.204	283.3 0.159	....	....	....	0.413
40	840	MCF in.	153.5 0.050	613.4 0.201	283.3 0.093	553.2 0.181	....	....	0.525
50	1,190	MCF in.	217.5 0.050	862.2 0.200	283.3 0.066	553.2 0.128	771.7 0.179	....	0.623
60	1,540	MCF in.	281.5 0.050	1,111.0 0.199	283.3 0.051	553.2 0.099	771.7 0.138	957.7 0.171	0.708

[a] MCF = 1,000 cu ft.

Runoff Factors.—The derivation of runoff factors (F_{ro}) is based on the volume of mass runoff-to-gutter curve between 80 and 120 min clock time shown as intercepts for various hourly rates of rainfall in Fig. 17, which is constructed as follows: Using the standard pattern of mass rainfall curve with hourly intensities varying from 0.7 to 6 in. per hr, net runoff-to-gutter hydrographs are computed for sand, loam, clay, and impervious surfaces with loss rates for $M = 1$ from Fig. 4,[6] and with depression storage losses of 0.20, 0.15, and 0.10 in. for sand, loam, and clay.

TABLE 10.—COMPUTATION OF RUNOFF HYDROGRAPH WITH CONDUIT DETENTION

t_c	q_c	CLOCK TIME, IN MINUTES														
		60	70	80	85	90	93	96	100	105	110	120	130	140	150	160
(a) RUNOFF RATE AT CLOCK TIME																
5	3.40	0.40	0.51	0.72	0.90	1.40	3.39	1.58	1.04	0.70	0.50	0.28	0.19	0.16	0.16	0.16
10	2.50	0.40	0.50	0.67	0.80	1.06	2.46	2.44	1.20	0.73	0.57	0.36	0.25	0.19	0.16	0.16
20	1.85	0.40	0.49	0.65	0.79	1.00	1.44	1.83	1.83	1.25	0.77	0.36	0.28	0.22	0.19	0.16
(b) PERCENTAGE OF PEAK RUNOFF RATE AT CLOCK TIME																
5	3.40	11.8	15.0	21.2	26.5	41.2	99.7	46.5	30.6	20.6	14.7	8.2	5.6	4.7	4.7	4.7
10	2.50	16.0	20.0	26.8	32.0	42.4	98.5	97.7	48.0	29.2	22.8	14.4	10.0	7.6	6.4	6.4
20	1.85	21.6	26.5	35.2	42.7	54.1	77.9	98.9	98.9	67.5	41.6	19.5	15.1	11.9	10.3	8.6
(c) PERCENTAGE OF AREA AT CLOCK TIME																
5	....	17.8	21.6	27.6	32.1	43.7	87.3	47.6	35.3	26.9	21.3	13.8	10.2	8.8	8.8	8.8
10	....	22.8	26.2	32.4	36.3	44.3	85.9	85.4	48.7	34.2	29.0	20.6	15.4	13.0	11.3	11.3
20	....	28.0	32.2	39.2	44.7	52.5	70.0	86.4	86.4	62.3	43.8	25.8	21.8	17.8	15.7	14.3
(d) AVERAGE PERCENTAGE OF AREA AT CLOCK TIME																
5-10	....	20.3	23.9	30.0	34.2	44.0	86.6	66.5	42.0	30.5	25.1	17.2	12.8	10.9	10.1	10.1
10-20	....	25.4	29.2	35.8	40.5	48.4	77.9	85.9	67.5	48.2	36.4	23.2	18.6	15.4	13.5	12.8
(e) CONDUIT DETENTION IN USE AT CLOCK TIME, IN 1,000 CU FT																
5-10	25,584[a]	5.2	6.1	7.7	8.7	11.3	22.2	17.0	10.7	7.8	6.4	4.4	3.3	2.8	2.6	2.6
10-20	115,812[a]	29.4	33.8	41.5	46.9	56.0	90.2	99.5	78.2	55.8	42.2	26.9	21.5	17.8	15.6	14.8
Summation		34.6	39.9	49.2	55.6	67.3	112.4	116.5	88.9	63.6	48.6	31.3	24.8	20.6	18.2	17.4
(f) CONDUIT DETENTION IN USE AT CLOCK TIME, IN INCHES																
20	....	0.068	0.079	0.097	0.109	0.132	0.222	0.229	0.175	0.125	0.096	0.062	0.049	0.041	0.036	0.034

[a] Total D_c available between times of concentration, in cubic feet.

Referring to Table 6, the rainfall on the pervious area is that falling directly upon it, plus 90% of that falling on impervious area and running on to pervious area. The runoff rate from pervious areas, which varies as $M^{0.59}$, is different for the different isohyetals (see Table 5 and Fig. 2). A sample computation of the runoff factor for an area in level residence classification, loam soil, on isohyetal 1.77, for an hourly rainfall rate of 1.77 × 10-yr factor 0.762 (Fig. 2) or 1.35 is as follows: Pervious area rainfall = 1.45 (Table 6) × 1.35 = 1.96, the converted hourly rate; intercept (Fig. 17) (hourly rate 1.35) for impervious area = 0.97; intercept (Fig. 17) (converted hourly rate 1.96) for pervious area = 1.02; and runoff factor

$$= \frac{(A_d \times \text{impervious intercept}) + (A_p \times M^{0.59} \times \text{pervious intercept})}{\text{Impervious intercept for basic hourly rate}}$$

$$= \frac{(0.40 \times 0.97) + (0.40 \times 0.755 \times 1.02)}{0.71} = 0.98 \text{ (see Fig. 13 for 40-2).}$$

The impervious intercept (0.71) for the basic hourly rainfall rate, 1.01 (10-yr B curve, Fig. 1) is used as a divisor in the foregoing equation for the runoff factor to make the results applicable to the basic runoff rates (Table 7). Runoff factors for various drainage area classifications are shown in Fig. 13.

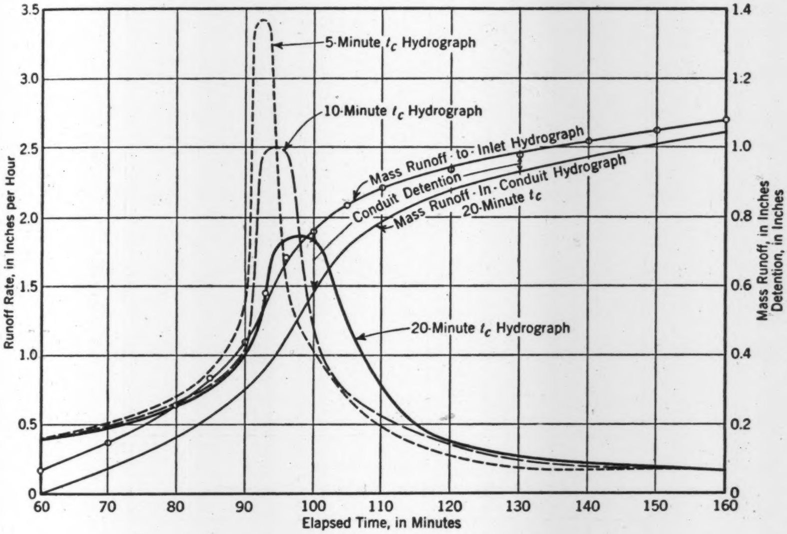


FIG. 15.—BASIC HYDROGRAPH FOR RUNOFF IN CONDUIT

A complete set of charts for the classifications listed in Table 6 has been filed with the Engineering Societies Library. The product of the runoff factor for a given area and the basic runoff rate for the time of concentration is the rate of runoff (q_c) in cubic feet per second per acre.

The values 1.12 and 2.65 in Fig. 13 indicate the 50-yr isohyetal on which the drainage area is located (see Fig. 2).

VII. HYDRAULICS OF THE PROCESS OF SUMMING HYDROGRAPHS

The process of summing hydrographs involves a hydrograph at point A on a drain being routed through a reach of conduit to point B where it is combined with a hydrograph from an incoming lateral and flows on to another junction point C, and so on.

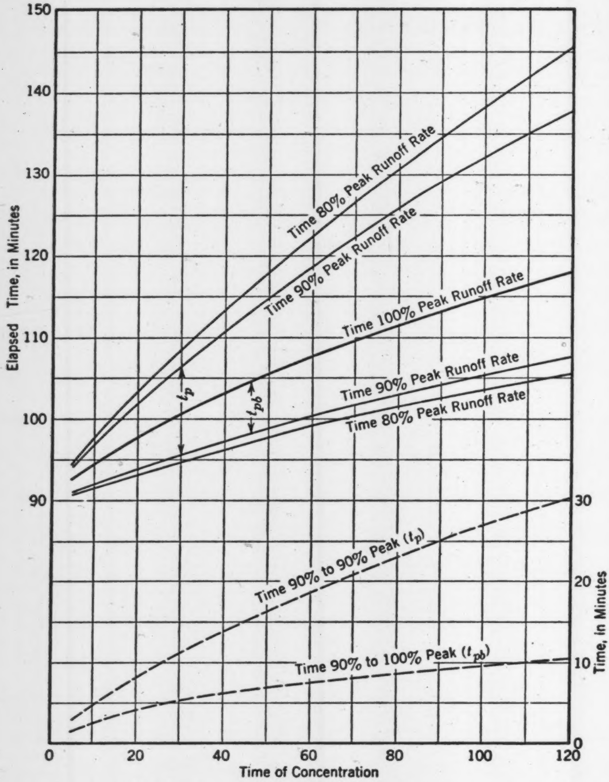


FIG. 16.—TIME ELEMENTS OF THE HYDROGRAPH

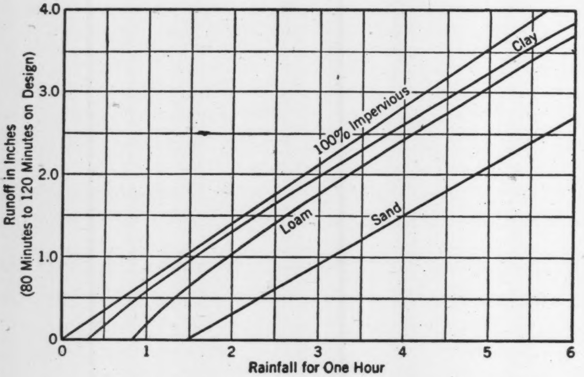


FIG. 17.—INTERCEPTS OF RUNOFF VOLUME, RUNOFF-TO-GUTTER CURVE

From studies of the basic hydrographs previously discussed, a method has been devised for transforming the rate hydrograph for the travel through a reach of conduit—first by a reduction of peak rate, and second by adjustment of the times of occurrence of various percentages of the new peak rate in the front and the back of the hydrograph.

Reduction of Peak Rate.—For this purpose, percentages of reduction of peak rate per minute of flow were computed for various times of concentration.

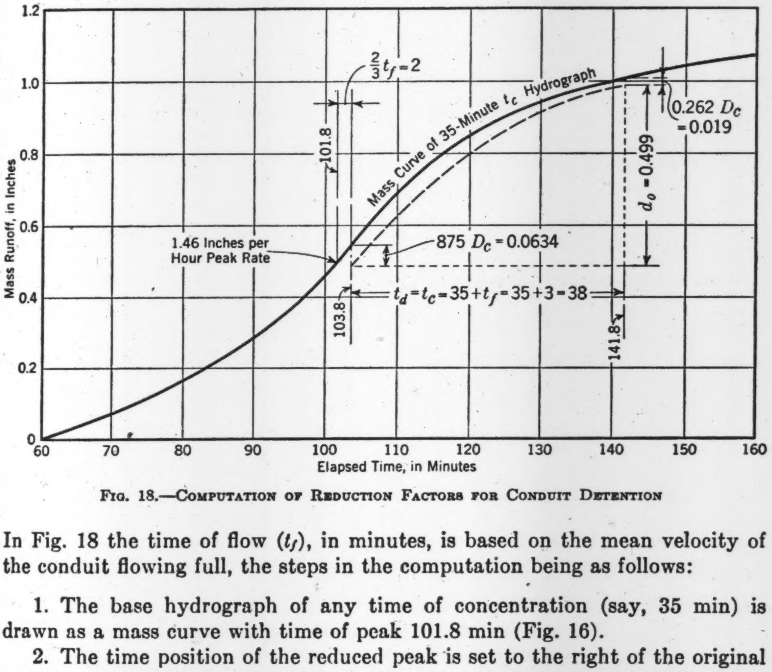


Fig. 18.—Computation of Reduction Factors for Conduit Detention

In Fig. 18 the time of flow (t_f), in minutes, is based on the mean velocity of the conduit flowing full, the steps in the computation being as follows:

1. The base hydrograph of any time of concentration (say, 35 min) is drawn as a mass curve with time of peak 101.8 min (Fig. 16).

2. The time position of the reduced peak is set to the right of the original peak by two thirds the time of flow (for a wave velocity 1.5 times the mean full-flow velocity) or at 103.8 min; the position of 20% peak is at 141.8 min or 103.8 min plus the time of drawdown (original t_c plus 3 min t_f).

3. The volume of available added conduit detention, in inches depth on the drainage area (D_c), is $(t_f \times 60 \times V)\left(q_c \frac{A}{V}\right)\left(\frac{12}{43{,}560\,A}\right) = 0.0165\,t_f\,q_c = 0.0724$ in. This volume, the product of the length of conduit and its cross-sectional area, is independent of both the velocity of flow and the drainage area.

4. As the flow in the drain is ordinarily free flow, the approximate added detention for peak flow is 87.5% of capacity or 0.0634 in., and for 20% peak, 26% or 0.0190 in.

5. Using $d_o = 0.499$, $t_d = 38$ min, and K for a t_c of 38 min = 106.9 in Eq. 4, a value for q_c of 1.40 is obtained.

6. The reduction for 3 min time of flow is 4.3% or 1.43% per min. Based on a series of similar computations, the values in Table 7, Col. B, were obtained.

Travel Velocity of the Flood Wave.—The velocity of travel of the front and peak of a flood wave is higher than the mean velocity computed for the peak depth while the velocity of the recession side recedes to less than the same mean velocity.

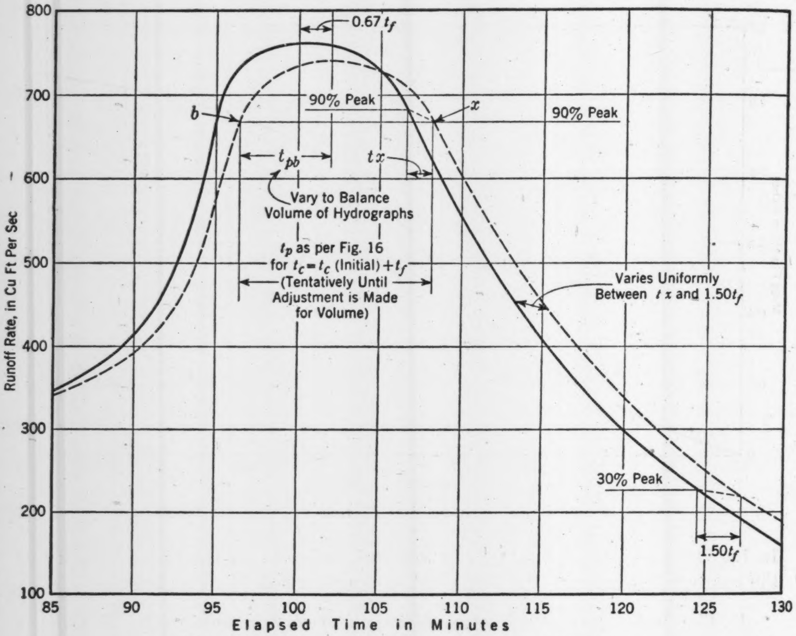


FIG. 19.—TRANSPOSITION OF A HYDROGRAPH FOR FLOW

An experiment along these lines by the Los Angeles County Flood Control District[11] was performed in an open rectangular channel 13,071 ft long between two reservoirs; the slope varied between 1.078% and 1.625%. Four releases of water were made, the last being in the form of a flood wave with a peak of 1,400 cu ft per sec. Gagings were made at five stations. In part the conclusions are:

> "7. For routing flood hydrographs down channels, a travel time based on a wave velocity affords a better approximation to true conditions than a travel time based on normal velocities. From the present test, the wave velocity ranges between 120% and 150% of the normal velocity for sudden changes of quantity."
>
> "11. The limitations of the data and conditions of flow must be kept in mind when applying these conclusions to other sets of conditions. * * *."

---

[11] "Field Test of Discharge from Puddingstone Dam and Flow in Puddingstone Diversion Channel," Los Angeles County Flood Control Dist. (unpublished).

In the experiment the normal velocity was based on open channel flow. If the percentages in conclusion 7 are expressed in terms of full-wetted perimeter, mean velocity, upon which time of flow is based in this paper, they become 140% to 175% (or 120% to 150% divided by 0.86).

From this experiment and a study of the time relations from rainfall-runoff records, the selection of 150% of mean full flow velocity of a closed conduit for the wave velocity of peak runoff rate at efficient free flow depth is reasonable and consistent with current engineering practice.

For the time movements of all phases of the hydrograph, the graph in Fig. 19 was developed from analyses of the time relations from rainfall-runoff records as recorded in Figs. 9 and 16.

Method of Summing Hydrographs.—This method is illustrated by an example. Two areas A and B have inlets 2 min time of flow apart. Area A has 700 acres of 50-2 development located on 50-yr isohyetal 2.22, $t_c = 40$ min; B has 300 acres of 40-2 on isohyetal 1.77, $t_c = 25$ min. A table filed in the Engineering Societies Library is used for plotting hydrographs.

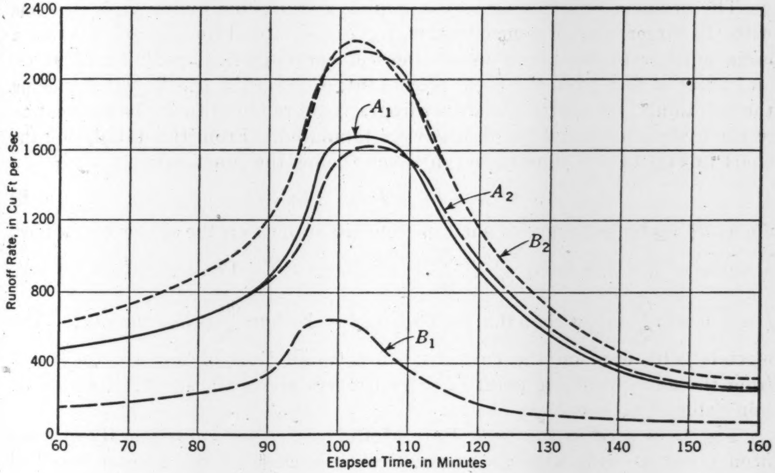


FIG. 20.—TRANSPOSITION AND SUMMATION OF HYDROGRAPH

For designing the conduit A to B, Q (peak runoff rate in cubic feet per second) = 700 acres × 1.80 (Fig. 13) × 1.37 (Table 7, Col. A) × 0.966 (Fig. 5) = 1,668 cu ft per sec. This quantity, applied to the 40-min basic hydrograph, yields hydrograph A_1 in Fig. 20.

For plotting hydrograph A_2 at the end of the reach A–B, the total reduction of peak for conduit detention is 2.45% (Table 7, Col. B), and the distribution factor becomes 0.954 because of the added 300 acres at point B; the peak rate for hydrograph $A_2 = 97.55\% \times \frac{0.954}{0.966} \times 1{,}668$ cu ft per sec = 1,607 cu ft per sec. Curve A_2 is plotted by the method of Fig. 19.

For plotting the hydrograph for area B, $Q = 300 \times 1.32$ (Fig. 13) $\times 1.68$ (Table 7) $\times 0.954$ (Fig. 5) $= 634$ cu ft per sec. The distribution factor is based on the combined areas A and B. Curve B_1 is the result.

Curves A_2 and B_1 are summed to form the tentative curve B_2. Because the mechanical process of summing does not evaluate the effects of hydraulic disturbances of the confluence, correction above 80% peak is made with the aid of Fig. 16:

1. By inspection of the uncorrected hydrograph at the points of 90% and 80% peak, select by trial a time position and value of 100% peak. Reconstruct the curve between points of 80% peak.

2. Check the revised curve against the original curve for agreement in volume between points of correction. The final value of Q for B_2 is 2,150 cu ft per sec.

Variation of Time of Concentration.—In summing hydrographs it is found frequently that the resulting hydrograph has time and peak characteristics materially different from those of either of the incoming hydrographs.

The problem was analyzed by summing a trunk-line hydrograph (the line with the larger time of concentration, t_{ca}) and a lateral hydrograph (t_{cb}) for a large number of combinations of times of concentration, peak runoff rates, and times of flow from the point of concentration on the trunk next preceding the junction. By trial, standard hydrographs were found to fit the summation of the hydrographs and the results were tabulated. From this tabulation the chart in Fig. 14 was constructed in which (below the junction):

$$t_{cx} = t_{cb} + f\,(t_{ca} - t_{cb}) \qquad (6)$$

To avoid undue influence of small increments of flow on the major trunk flow, a value of $f = 1$ is used when $\frac{Q_b}{Q_a}$ is less than 0.1. For $t_{ca} - t_{cb} = 40$ min, $f = 1$ when $\frac{Q_b}{Q_a}$ is less than 0.2; for 50 min, $f = 1$ when $\frac{Q_b}{Q_a}$ is less than 0.1. The legends 2-10, etc., on the curves mean a 2-min time of flow on the trunk from the next preceding point of concentration above the junction, and a 10-min value of $t_{ca} - t_{cb}$.

Fig. 14 is used in the Peak Rate Method described subsequently and has been given as wide a range as practicable because of the greater speed of operation of that method.

VIII. METHODS AND FORM OF RUNOFF COMPUTATION

Use of the Method of Summing Hydrographs described previously is confined to cases in which the Peak Rate Method is not applicable.

Peak Rate Method.—This method is subject to the following conditions:

1. When the hourly intensity of rainfall for the selected frequency is not constant over the entire drainage area (see Fig. 2), the hourly intensity for each point of concentration is computed by weighting the intensities within the drainage area by the effective areas to which they apply.

2. When a lateral and a trunk line combine with times of concentration and peak rates of runoff falling outside the limitations imposed by the notes

in the text, following Eq. 6, the Method of Summing Hydrographs is used for the continuation of the trunk line; the laterals to the continuation of the trunk are designed by the Peak Rate Method.

3. The outfall from a retarding reservoir is designed by the Method of Summing Hydrographs, after the inflow to the reservoir has been routed through by use of the reservoir formula (inflow = outflow plus storage).

4. Other limitations may be imposed by the judgment of the designer.

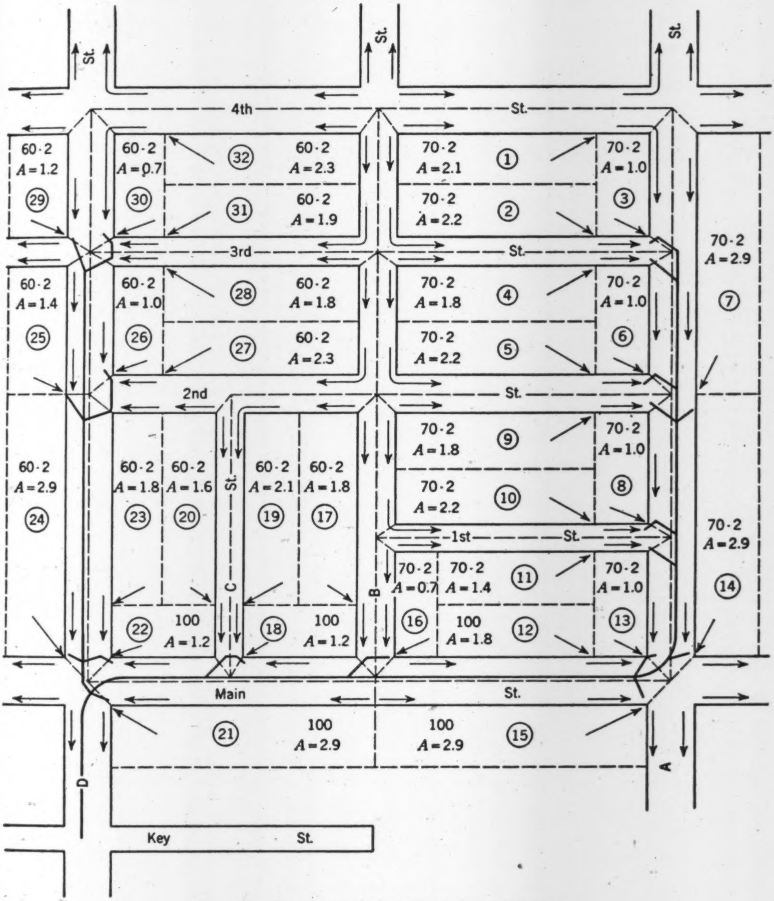

St.
St.
St.
4th St.
60·2 A=1.2 ㉙
60·2 A=0.7 ㉚
㉜ 60·2 A=2.3
70·2 A=2.1 ①
70·2 A=1.0 ③
㉛ 60·2 A=1.9
70·2 A=2.2 ②
3rd St.
70·2 A=2.9 ⑦
60·2 A=1.4 ㉕
60·2 A=1.0 ㉖
㉘ 60·2 A=1.8
70·2 A=1.8 ④
70·2 A=1.0 ⑥
㉗ 60·2 A=2.3
70·2 A=2.2 ⑤
2nd St.
70·2 A=1.8 ⑨
70·2 A=1.0 ⑧
60·2 A=2.9 ㉔
60·2 A=1.8 ㉓
60·2 A=1.6 ⑳
St.
60·2 A=2.1 ⑲
60·2 A=1.8 ⑰
70·2 A=2.2 ⑩
1st St.
70·2 A=2.9 ⑭
C
B
70·2 A=0.7 ⑯
70·2 A=1.4 ⑪
70·2 A=1.0 ⑬
㉒ 100 A=1.2
⑱ 100 A=1.2
100 A=1.8 ⑫
Main St.
㉑ 100 A=2.9
100 A=2.9 ⑮
A
D
Key St.

FIG. 21.—DRAINAGE AREA, PEAK RATE METHOD

Tables 6, 7, and a table of inlet times, and Figs. 2, 5, 13, and 14, are used in this method.

A sample computation for the drainage area in Fig. 21 is shown in Table 11, which is largely self-explanatory:

TABLE 11.—PEAK RATE METHOD APPLIED TO MAIN STREET

No.	Drainage Area			Classification, Areas and Runoff Quantities								1-hr rainfall rate	Distribution factor
	No.	Acres	Symbol	100	70-2	60-2							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
													Main
1	1-2-3-4	7.1	A		7.1								
			ΣA		7.1								
			q_e		2.75								
2			Q		19.5							1.01	1.00
3	5-6-7-9	7.9	A		7.9								
			ΣA		15.0								
			q_e		2.60								
4			Q		39.0							1.01	1.00
5	8-10-11	4.6	A		4.6								
			ΣA		19.6								
			q_e		2.49								
6			Q		48.8							1.01	1.00
7	12-13-14-15	8.6	A	4.7	3.9								
			ΣA	4.7	23.5								
			q_e	2.76	2.40								
8			Q	13.0	56.4							1.01	1.00
9	16-17	2.5	A		0.7	1.8							
			ΣA	4.7	24.2	1.8							
			q_e	2.62	2.28	2.10							
10			Q	12.3	55.2	3.8						1.01	1.00
11	18-19-20	4.9	A	1.2		3.7							
			ΣA	5.9	24.2	5.5							
			q_e	2.54	2.21	2.03							
12			Q	15.0	53.5	11.2						1.01	1.00
13	21 and D St. lateral	21.4	A	4.1		17.3							
			ΣA	10.0	24.2	22.8							
			q_e	2.49	2.17	2.00							
14			Q	24.9	52.5	45.6						1.01	1.00
15			A										
			ΣA										
			q_e										
16			Q										
													D St.
17	28-29-30-31-32	7.9	A			7.9							
			ΣA			7.9							
			q_e			2.37							
18			Q			18.7						1.01	1.00
19	25-26-27	4.7	A			4.7							
			ΣA			12.6							
			q_e			2.26							
20			Q			28.5						1.01	1.00
21	22-23-24	5.9	A	1.2		4.7							
			ΣA	1.2		17.3							
			q_e	2.66		2.13							
22			Q	3.2		36.8						1.01	1.00
23			A										
			ΣA										
			q_e										
24			Q										

AND D STREET STORM DRAIN (10-YR DESIGN CURVE)

Amended 1-hr rain-fall rate (14)	ΣQ (15)	Friction slope, S (16)	$A\ C\ \sqrt{r}$ (17)	Pipe size (in.) (18)	Pipe length (ft) (19)	V (20)	t_c (21)	Factor f (22)	Station (23)	Storm drain locations (24)	No.
LINE											
											1
1.01	19.5	0.01	195	24	375	7.1	6.0		36+15	A St. & 3d St.	2
							0.9				3
1.01	39.0	0.01	390	30	335	8.4	6.9		32+40	A St. & 2d St.	4
							0.7				5
1.01	48.8	0.01	488	33	325	9.0	7.6		29+05	A St. & 1st St.	6
							0.6				7
1.01	69.4	0.01	694	36	650	9.5	8.2		25+80	Main St. & A St.	8
							1.1				9
1.01	71.3	0.01	713	36	320	9.5	9.3		19+30	Main St. & B St.	10
							0.6				11
1.01	79.7	0.01	797	39	285	10.1	9.9		16+10	Main St. & C St.	12
							0.5				13
1.01	123.0	0.01	1,230	45	325	11.1	(10.4) 10.2	0.81	13+25	Main St. & D St.	14
							0.5				15
							10.7		10+00	D St. & Key St.	16
LATERAL											
											17
1.01	18.7	0.01	187	24	340	7.1	7.0		9+80	D St. & 3d St.	18
							0.8				19
1.01	28.5	0.01	285	27	570	7.8	7.8		6+40	D St. & 2d St.	20
							1.2				21
1.01	40.0	0.01	400	30	70	8.4	9.0		0+70	D St. & N/S Main St.	22
							0.1				23
	40.0	....	....	....	....	....	9.1		0+00 = 13+25 M.L.	....	24

1. List the areas at their respective points of concentration (Cols. 1, 2, and 24).
2. Subclassify the areas (Table 6 and Cols. 4 to 11, Table 11).
3. Select the 1-hr rainfall rate from Fig. 2, with respect to the isohyetal and frequency (isohyetal 1.33 × 10-yr frequency factor 0.762 = 1.01, Col. 12).
4. When the gross drainage area exceeds 100 acres, adjust the 1-hr rate by the distribution factor (Fig. 5 and Cols. 13 and 14).
5. Determine the runoff factors from Fig. 13 in relation to the adjusted 1-hr rainfall rate and apply them to the base runoff rate for the time of concentration in Table 7 to derive runoff rate per acre (q_c in Col. 3).
6. Follow through with the routine tabling computations.
7. When a lateral is added, the time of concentration is adjusted in accordance with Fig. 14 (Col. 22).

IX. Comparison of Computed Runoff with Record Runoff

The methods of computation outlined herein are based on rather complete analyses of gagings taken in and near Los Angeles. Other records have become available, but have not been analyzed. Instead, runoff factors (F_{ro}) have been determined for the rainfall data of both local and outside records, both of the intense storm producing the runoff and of the antecedent precipitation, and these runoff factors have been applied to the basic runoff rates of Table 7 to produce computed values of runoff to compare with the actual values.

The runoff factor is based on surveyed percentages of the specific drainage area, namely A_i, A_d, and A_p. A series of curves is necessary to reflect the varying percentage of pervious runoff, which varies as $M^{0.59}$. A set of curves of runoff factors for Eastern Avenue (Station 3) is shown in Fig. 22.

Table 12.—Comparison of Computed and Actual Values of

No.	Date	Hour	Percentage of runoff	Time of concentration, in min (t_c)	Rainfall rate, in in. per hr (I)	Rainstorm shape factor (F)	Standard rainstorm shape factor (F_s)
1	1/ 5/35	3:44 a.m.	24.6	28.3	1.19	0.805[a]	0.767[a]
2	12/30/36	9:04 p.m.	34.2	33.4	0.68	0.425	0.485
3	3/15/37	7:36 p.m.	44.0	32.7	0.88	0.484	0.618
4	2/11/38	12:27 p.m.	30.4	32.2	0.59	0.451	0.410
5	2/28/38	7:08 a.m.	42.5	29.4	1.09	0.790	0.716
6	2/28/38	11:30 p.m.	56.0	33.3	0.77	0.497	0.549
7	3/ 2/38	8:32 a.m.	70.0	30.8	1.06	0.600	0.720
8	3/ 2/38	9:30 a.m.	69.0	37.5	0.64	0.416	0.485
9	3/ 2/38	3:54 p.m.	73.0	37.1	0.55	0.430	0.415
10	12/18/38	7:43 p.m.	....	32.4	0.61	0.400	0.429
11	1/ 5/39	12:12 p.m.	....	30.0	0.73	0.403	0.486
12	1/ 7/40	7:53 p.m.	....	30.2	0.63	0.440	0.422
13	2/29/40	3:15 a.m.	....	31.0	0.64	0.405	0.435
14	3/31/40	4:47 p.m.	....	33.7	0.50	0.353	0.357

[a] Factors F and F_s are probably above average for area.

A set of computations for Station 3 is shown in Table 12, in which F_s = the standard rainstorm shape factor, which is the shape factor, F, for a standard shaped storm having a rainfall rate equal to that of the record storm for the specific time of concentration; F_s may be obtained by dividing the record intensity of rainfall (I) for this time of concentration by the rate for the same time duration on the standard 10-yr B curve (Fig. 1); I_H = the hourly rate of rainfall on the standard shaped rainfall curve corresponding to the rate for the specific time of concentration used, 1.01 $(F_s \times F_s)^{0.5}$, modified by the shape factor F, or 1.01 $(F \times F_s)^{0.5}$. The other headings have been defined or are self-explanatory.

The physical characteristics of local urban areas, Stations 3, 4, 10, and 11, are listed in Table 1; for Blocks A, B, and C, St. Louis, Mo., as reported[12] by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E., in Table 13. Record data, computations, and derived values of runoff, except for Station 3 (Table 12), are shown in tables filed in the Engineering Societies Library.

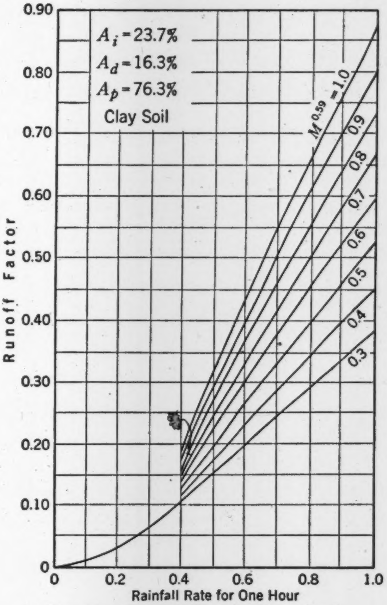


FIG. 22.—RUNOFF FACTORS, STATION 3

Record values of peak runoff rate for Stations 3 and 4 and for Blocks A and C have been corrected for overlap of preceding storms on the major storm (q_3). The hydrographs of the other areas were too irregular to justify this correction. In addition, the area for Station 4 has an inadequate lateral system

PEAK RUNOFF RATE; STATION 3, LOS ANGELES, CALIF.

Hourly rainfall rate I_H = 1.01 × $(F \times F_s)^{0.5}$	$M^{0.59}$ (M = moisture factor)	Runoff factor (F_{re})	Record peak runoff rate, in in. per hr (q_1)	q_1 corrected for prior storm (q_3)	Computed peak runoff rate, in in. per hr (q_e)	No.
0.793	0.44	0.36	0.430	0.413	0.57[b]	1
0.459	0.63	0.19	0.320	0.300	0.28	2
0.550	0.56	0.25	0.456	0.422	0.38	3
0.435	0.59	0.16	0.315	0.275	0.24	4
0.760	0.57	0.39	0.610	0.570	0.61	5
0.528	0.73	0.28	0.560	0.510	0.41	6
0.664	0.90	0.47	0.830	0.730	0.72	7
0.455	0.95	0.25	0.505	0.455	0.35	8
0.427	1.06	0.23	0.530	0.460	0.32	9
0.418	0.62	0.16	0.192	0.182	0.24	10
0.447	0.41	0.15	0.229	0.219	0.2[illegible]	11
0.436	0.43	0.13	0.227	0.201	0.20	12
0.424	0.50	0.14	0.260	0.250	0.21	13
0.359	0.42	0.09	0.147	0.147	0.13	14

[b] One automatic rain gage only.

---

[12] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

which causes overflow into street channels leading to the gaging system at about 0.45 in. per hr runoff rate, and causes overflow into another drainage area at about 0.60 in. per hr; this condition distorts computed corrections of the peak runoff rate.

TABLE 13.—Characteristics of Urban Drainage Areas, St. Louis, Mo.

Station	Location	Area (acres)	Soil type	Improvement (%)			Channel type
				A_i	A_d	A_p	
A	Belt and Ridge avenues	2.3 to 1923 2.2 after	Clay	42.0 49.0	34.0[a] 38.0[a]	58.0 51.0	Gutter
B	City block No. 4841	3.27	Clay and loam	30.0	21.8[a]	70.0	Gutter
C	Ewing and Washington avenues	4.34	Clay	72.0	60.0[a]	28.0	Gutter, pipe, egg-shaped sewer

[a] Estimated.

Even with these deficiencies in the records, the comparison of the corrected record values of peak runoff rate with computed values (Fig. 12) shows a majority of the points lying within the lines of 20% variation. The mean of the points indicates that the computed values are a little below actual, especially when the correction for the preceding storm is considered.

Conclusions

1. The peak runoff rate for a given storm pattern is proportional to the volume of runoff resulting from the intense portion of the storm. The volume of runoff may be predicted—

(*a*) For pervious areas, by infiltration tests on small soil plots and by analyses of rainfall-runoff records from partly or fully pervious drainage areas;

(*b*) For impervious areas, by similar tests on impervious areas.

2. Rainfall-runoff records in which the high intensities of rainfall occur at or near the end of the storm (delayed pattern) disclose hydraulic phenomena and time relations which, when secured from drainage areas of various sizes and physical characteristics, enable the investigator to predict the approximate intensity and pattern of the runoff hydrograph that will occur under selected conditions of drainage area and rainfall.

When the method of computation derived from the delayed pattern storms is applied to the peak runoff rates for advance pattern storms, the approximate agreement obtained indicates that the same phenomena are present but are obscured by the runoff resulting from rainfall occurring after the peak of runoff.

3. The runoff hydrograph is the result of the travel and confluence of hydrographs from small units of area. As long as the urban drainage area is homogeneous in development and conforms to the topography of the natural drainage valley or swale, the hydrograph has a characteristic pattern of

smoothly varying rates; when hydrographs from dissimilar areas combine at a point of confluence, the outflow hydrograph has an irregular shape and a peak intensity unpredictable by a simple computation involving the size and physical characteristics of the drainage area and the intensity of rainfall. In such cases, the outflow hydrograph may be constructed by summing the hydrographs from each of the dissimilar areas.

4. Further accumulation of data is needed, emphasizing runoff records defining all stages of flow, accurate time correlation between rainfall and runoff, and experimentation on the storm flood wave.

5. For urban areas, in both Los Angeles and St. Louis, a comparison of actual runoff rates with those computed by this method from rainfall records (Fig. 12) shows most of the points falling within a 20% tolerance; agreement of points within the design range (above 1 cu ft per sec per acre) is more nearly perfect.

ACKNOWLEDGMENT

This investigation was conducted by the Bureau of Engineering, Department of Public Works, Los Angeles, under the direction of L. W. Armstrong, M. Am. Soc. C. E., principal civil engineer. Lloyd Aldrich, M. Am. Soc. C. E., is city engineer, and Merrill Butler, M. Am. Soc. C. E., is assistant city engineer (design). Prior to 1932, the work was supervised by F. L. Boyle, assistant civil engineer, and G. S. Tapley, Assoc. M. Am. Soc. C. E., civil engineering associate. At that time, the writer undertook the work, with H. W. Fraim, civil engineer, who supervised field and laboratory activities. Analyses and computations were performed by H. B. Brown and H. P. Larsen, civil engineering associates. Operation of automatic recording instruments was by F. J. Doran and A. A. Weston, civil engineering associates.

The Los Angeles County Flood Control District made its voluminous records and analyses available. Maxwell F. Burke, Assoc. M. Am. Soc. C. E., of that organization was very helpful in discussing the many phases of the problem.

The Water Department and the Engineering Bureau of the City of Pasadena cooperated in gaging rainfall and runoff in that city and by making available other records collected by them independently.

DISCUSSION

C. S. JARVIS,[13] M. AM. SOC. C. E.—The extensive plottings, analyses, and calculations presented by the author should insure more than a casual consideration of his contribution. The orderly arrangement in logical sequence, step by step, facilitates the understanding of both the objective and the scientific approach. After a careful reading of the paper, the writer's interest centered on Fig. 12, showing the computed and actual runoff rates, the computed quantities being plotted against the observed.

It is certainly gratifying to find that most of the plotted points fall within a 20% tolerance zone. Without intentionally detracting from this accomplishment, it seems necessary to observe the extreme range of variation—for example, using the ordinate value of 1.5. The envelope of plotted values extends from abscissa value 0.65 to 2.15, showing an actual range from 0.43 to 1.43, representing 43% to 143% of the actual runoff. Similarly, for the ordinate 0.8, the range of computed values extends from 0.30 to 1.50, or from 37.5% to 87.5% of the actual value. Concentrating on the symbols representing data from Station 3, shown in Table 12, the range between envelope curves at the ordinate value of 0.45 extends from about 0.33 to 0.59, or from 73% to 131% of the observed quantity.

In view of these fairly wide dispersions, it seems appropriate to inquire as to the advisability of adopting an average factor, say 40% or 50% in Table 12, by means of which the computed peak runoff rates might be derived directly from the recorded rainfall rates, in the sixth column, under (*I*). Exercise of the same ingenuity and personal judgment required in assigning appropriate values for the shape and moisture factors in Table 12 should insure as close agreement between computed and actual peak runoff rates as was attained in either Fig. 12 or Table 12.

Fortunately, reports on research projects need not include the explorations that end in blind alleys, unless a brief account of such fruitless endeavor will serve as a warning to others. It appears that full recompense in results of practical value has been realized for every operation thus far reported by the author. After setting up such definite forms and procedures, it is a simple matter to introduce refinements and revisions promptly after acquiring additional authentic data. Whether all new data shall be subjected to exactly the same processing or whether some abbreviated reductions shall be devised is relatively unimportant at this stage of development. However, with such a background of accomplishment as belongs to both the City and County of Los Angeles in the analysis and solution of flood and runoff problems generally, the conduct of operations from now on is practically assured as an example worth following under conditions largely comparable.

One of the greatest services that could be rendered by those inclined and equipped to make such elaborate analyses would be to render the results in the simple forms involving the proportion of rainfall amount as well as rate appearing with the runoff phenomena. Admitting that there is some reason for departing from the use of runoff coefficients and for adopting a more rigorously scientific approach, it does not seem to be too great a concession to those who have become accustomed to the simpler methods to translate results into the terms and expressions most familiar to them. Likewise, the use of some of the suggested scales for indicating the relative yields per unit of area would facilitate comparisons. Perhaps the author has some preferred method of rating the unit yield relation to drainage area, which would facilitate application of his method to much greater areas.

---

[13] Cons. Engr., Washington, D. C.

S. W. Jens,[14] Assoc. M. Am. Soc. C. E.—A very important contribution to the literature covering the application of the newer hydrology to practical design is contained in this paper. It is of particular significance as an indication of what a progressive municipal engineering organization can accomplish in the conduct and application of research to its design problems. The first few paragraphs of the paper present a very clear and succinct statement of the fundamentals involved.

The author rightly emphasizes the statistical meaning of the rainfall intensity-duration curves. It is easy to slip into careless thinking about this, with consequent erroneous use of the data. It seems preferable not to use the term "10-yr frequency storm" but rather "10-yr frequency rainfall rate" because the intensity-duration data merely give an average rainfall rate over the time of duration and may or may not represent the entire actual storm. In other words, the 10-yr, 30-min rainfall rate may actually be part of, say, a 60-min storm; and the 60-min average rainfall rate for that storm may be of a different frequency.

The techniques indicated by Mr. Hicks for the determination of the storm pattern and as a measure of the influence of antecedent precipitation are very interesting. Mr. Hicks states that the value of M determined by his studies applies to all rainfall frequencies. Can it be inferred that this value of M also applies to all durations? For areas having rains distributed throughout the year, it is presumed that varying values of M would need to be developed to reflect properly seasonal differences in antecedent precipitation.

The considerable importance of the factor expressed by the percentage of impervious area directly tributary to the collecting system is given proper emphasis but, if possible for the particular areas in Table 1, it would be of value to have the average slopes indicated for overland flow, gutters, and drains. In discussing lag time the author (see Section IV, heading, "Derivation of Time Factors") states "It is dependent on the shape and distribution of impervious development of the drainage area * * *." Surface slopes seem to be fully as important a determinant.

Recent studies in this office of actual open channel recession curves show a straight-line relationship between discharge rate in cubic feet per second and volume of storage in cubic feet. In view of Mr. Hicks' findings that the same general relationship holds for the composite storage involved in urban drainage, it may well be that all ordinary recession curves can be expressed in some such form as:

---

[14] Partner, Horner & Shifrin, Cons. Engrs., St. Louis, Mo.

$$q = K\,V^{n}{}_{L} \qquad (7)$$

in which q = rate of discharge or runoff in cubic feet per second or in inches per hour; V_L = volume of storage in cubic feet or in average inches of depth over the tributary area; K = a constant reflecting the physical characteristics of the area and the collecting system as well as the particular units being employed; and n = an exponent (found by the writer to be approximately 1). Additional similar study of rainfall-runoff data from a great number of different areas should be made.

In describing his technique of analyses, the author states that impervious surface loss has been experimentally determined as approximately 10% of the rainfall. It is difficult to understand why this should be so because analytical reasoning points to a fixed value for such loss for any particular surface. The amount of rainfall required to fill the undrainable depressions plus the surface film remaining after rainfall ceases would seem to be all the losses that can occur on impervious surfaces. Furthermore, both of these losses seem to be fixed amounts for any specific area, and all rainfall in excess of their sum should appear as runoff. It would be of interest if the author presented the experimental data that support his use of a constant percentage loss, irrespective of the rainfall intensity.

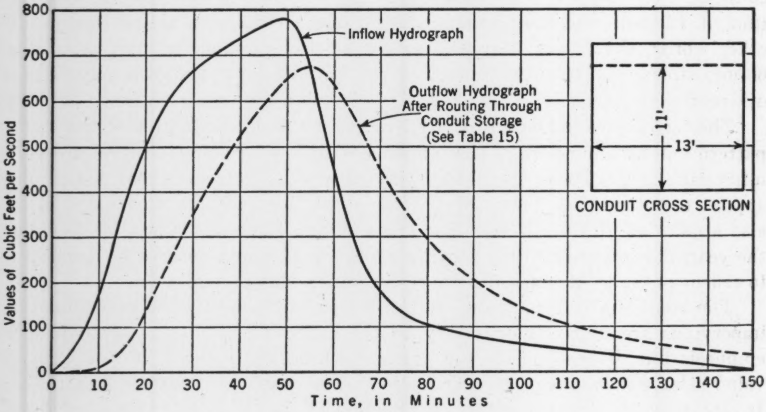


FIG. 23

For routing a known inflow through a conduit to determine the effect of conduit storage, the storage equation has been found applicable. The following example illustrates the technique: For a peak inflow rate of 780 cu ft per sec, a time of 49 min and a hydraulic slope of $S = 0.0003$, with $n = 0.012$, a box section 13 ft wide with free flow 11 ft deep is indicated. This section is shown in Fig. 23 and was chosen to keep the excavation as shallow as possible to minimize the extent of the trench in wet ground. The section prevails for $l = 5{,}585$ ft and serves as an outfall to a pump station.

Fig. 23 shows also the hydrograph of inflow at the upper end of the box. For a short finite increment of time, Δt, the equation can be written: Average inflow rate times Δt equals outflow rate times Δt plus or minus the change in storage in the time interval Δt. Let: I = average inflow rate in cubic feet per second for interval Δt; Q_1 = outflow rate at beginning of time Δt, in cubic feet per second; Q_2 = outflow rate at end of time Δt, in cubic feet per second; Δt = time interval, in minutes; V_{L1} = cubic feet of occupied volume in conduit at beginning of Δt; and V_{L2} = cubic feet of occupied volume in conduit at end of Δt. Then:

$$60\, I\, \Delta t = 60\, \frac{Q_1 + Q_2}{2}\, \Delta t + (V_{L2} - V_{L1}) \ldots\ldots\ldots\ldots (8)$$

Rearranging terms, Eq. 8 becomes

$$f Q_2 = I + f Q_1 \ldots\ldots\ldots\ldots (9a)$$

with

$$f Q_1 = \frac{V_{L1}}{60\, \Delta t} - \frac{Q_1}{2} \ldots\ldots\ldots\ldots (9b)$$

and

$$f Q_2 = \frac{V_{L2}}{60\, \Delta t} + \frac{Q_2}{2} \ldots\ldots\ldots\ldots (9c)$$

Table 14 gives the computations for the storage function curves shown in Fig. 24. Storage in the conduit has been assumed equal to the volumes under hydraulic gradients parallel to the flow line for varying depths of flow. This is not rigorously true, but, since change in storage is wanted, it is believed that the resulting curve represents the discharge-storage relationship with practical accuracy. The Manning formula has been used for discharge computations with n assumed as 0.012 for monolithic concrete. To catch the peak outflow more accurately, a curve for Δt equal to 1 min has been developed; for the remainder of the routing $\Delta t = 5$ min is used. As will be shown in describing the routing procedure, it is necessary to plot only the curve for fQ_2.

TABLE 14.—COMPUTATION OF STORAGE FUNCTIONS

Depth d (ft)	Area A (sq ft)	Wetted perimeter (ft)	Hydraulic Radius (Ft)		Flow (Cu Ft Per Sec)		$\Delta t = 5$		$\Delta t = 1$	
			R	$R^{2/3}$	Q	$\frac{Q}{2}$	$\frac{S}{300}$	fQ_2	$\frac{S}{60}$	fQ_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	26	17	1.53	1.33	74.1	37.0	484.3	521.3	2,420.2	2,457.2
4	52	21	2.48	1.83	208.5	104.3	968.6	1,072.9	4,840.5	4,944.8
6	78	25	3.12	2.14	357.4	178.7	1,452.9	1,631.6	7,260.8	7,439.5
8	104	29	3.59	2.35	523.0	261.5	1,937.2	2,198.7	9,681.0	9,942.5
10	130	33	3.94	2.50	695.6	347.8	2,421.5	2,769.3	12,101.0	12,449.1
11	143	35	4.09	2.56	785.0	392.5	2,663.6	3,056.1	13,311.4	13,703.9

Table 15 gives the routing computations. Cols. 1, 2, and 3 are taken from the inflow hydrograph. It should be noted that Col. 3 is the average inflow rate for the period and that Col. 4 is the outflow rate at the end of the time interval read from the proper storage curve. These outflows are obtained as follows:

(*a*) The first average inflow is laid off on the horizontal scale and the outflow rate at the end of the first 5-min period is read on the vertical scale (using, of course, the fQ_2 curve for $\Delta t = 5$); and

(*b*) The value shown in Col. 8 for the preceding line is placed in Col. 5; to this is added the inflow for the period under consideration (Col. 6); then the outflow arrived at in the previous interval is inserted in Col. 7 and this is subtracted, resulting in the value given in Col. 8. This latter figure is laid off on the horizontal scale and the outflow read on the vertical.

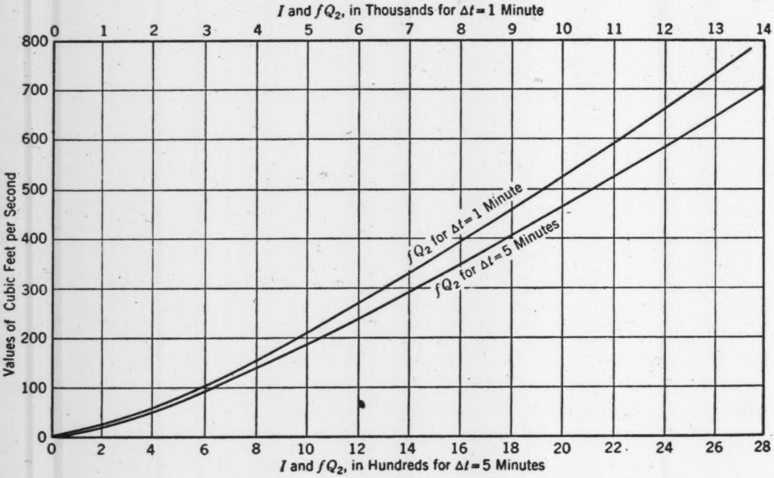


FIG. 24

The process is repeated throughout the hydrograph and is merely a combined graphic and arithmetic solution of the storage equation. Transition from a curve for one-time interval such as 5 min to that for another Δt is accomplished by selecting an abscissa value for the new Δt representing the same outflow value (Col. 4) as was obtained for the prior Δt; the procedure is then reversed upon changing back to the original Δt.

The foregoing computation suggests that, because of the peak reduction caused by the storage effect, the box section, 11 ft high by 13 ft wide, required for the inflow peak of 780 cu ft per sec could be replaced by an 11-ft by 12-ft box. A check routing through the smaller section is necessary to make sure that the lesser total storage accomplishes the requisite peak reduction.

Similar analyses were made of the storage effect for several other lengths and conduit sizes with various inflow peaks. These are shown in Table 16. These conduits all are in flat river bottoms and would be expected to exhibit more than ordinary storage effects. It seems significant, however, that only the extremely long, large box with a relatively short inflow hydrograph (49 min to peak inflow) gives an appreciable storage reduction. This is understandable from the viewpoint of the ratio of inflow volume to the available storage volume. The longer time hydrographs with greater volumes of inflow are more likely to fill the conduit before the occurrence of the peak.

TABLE 15.—Routing Computations (Units, Cubic Feet Per Second)

Time, in Minutes		Average inflow	Outflow at end of interval	Col. 8[a] +	Col. 3 −	Col. 4[a] =	fQ_2 (see Fig. 24)
From (1)	To (2)	(3)	(4)	(5)	(6)	(7)	(8)
0	5	17	2	0	17	0	17
5	10	100	8	17	100	2	115
10	15	260	45	115	260	8	367
15	20	430	128	367	430	45	752
20	25	557	237	752	557	128	1,181
25	30	642	345	1,181	642	237	1,586
30	35	687	444	1,586	687	345	1,928
35	40	722	525	1,928	722	444	2,206
40	45	750	594	2.206	750	525	2,431
45	46	765	607	11,030	765	594	11,201
46	47	769	618	11,201	769	607	11,363
47	48	776	628	11,363	776	618	11,521
48	49	777	637	11,521	777	628	11,670
49	50	777	647	11,670	777	637	11,810
50	51	774	656	11,810	774	647	11,937
51	52	760	665	11,937	760	656	12,041
52	53	741	667	12,041	741	665	12,117
53	54	715	673	12,117	715	667	12,165
54	55	678	673+	12,165	678	673	12,170
55	56	640	672	12,170	640	673	12,137
56	57	600	666	12,137	600	672	12.065
57	58	558	658	12,065	558	666	11,957
58	59	510	647	11,957	510	658	11,809
59	60	467	636	11,809	467	647	11,629
60	65	347	548	2,568	347	636	2,279
65	70	215	449	2,279	215	548	1,946
70	75	150	362	1,946	150	449	1,647
75	80	118	295	1,617	118	362	1,407
80	85	97	244	1,407	97	295	1,209
85	90	86	206	1,209	86	244	1,051
90	95	76	170	1,051	76	206	921
95	100	67	144	921	67	170	818
100	105	58	124	818	58	144	732
105	110	51	106	732	51	124	659
110	115	45	92	659	45	106	598
115	120	40	80	598	40	92	546
120	125	34	71	546	34	80	500
125	130	28	62	500	28	71	457
130	135	22	55	457	22	62	417
135	140	16	48	417	16	55	378
140	145	10	42	378	10	48	340
145	150	4	35	340	4	42	302
150	155	....	29	302	0	35	267
155	160	....	25	267	0	29	238
160	165	....	21	238	0	25	213
165	170	....	18	213	0	21	192
170	175	....	16	192	0	18	174
175	180	....	14	174	0	16	158
180	185	....	12	158	0	14	144
185	190	....	11	144	0	12	132

[a] From the line preceding a given time interval.

All the foregoing leads to the conclusion that the effect of conduit storage must be evaluated for each specific reach, and it is difficult to understand how reductions accomplished by any particular system can be applied as indicated by the author to any other system with different conduit sizes, lengths, and times.

The author has kept to the basic fundamentals throughout his development of practical tools for use in urban storm sewer design. His fine paper should encourage others to analyze rainfall-runoff records in the light of the hydrologic knowledge of the past few years.

TABLE 16.—RESULTS OF ROUTING THROUGH CONDUIT STORAGE

No.	Cross Section (Ft)		Hydraulic slope	Length (ft)	Peak Flow (Cu Ft Per Sec)		Time[a] (min)
	Height	Width			Inflow	Outflow	
1	6.0[b]		0.0009	850	127.3	127.3	30.6
2	7.5[b]		0.00056	840	181.0	181.0	33.7
3	8.0[b]		0.00055	1,040	215.4	213.0	37.1
4	9.0[b]		0.00055	900	297	295	41.1
5	10.0[b]		0.00047	1,020	361	358	44.3
6	9.0	8.0	0.00039	1,020	332	327	47.4
7	9.0	8.0	0.00048	1,250	369	361	86.3
8	9.0	8.5	0.0005	1,470	430	421	90.2
9[c]	11.0	13.0	0.0003	5,585	780	673	49.0
10[d]	10.0	12.0	0.00037	5,650	1,375	1,361	117.2

[a] Time to peak inflow. [b] Circular cross section. [c] See Table 15 and Fig. 23. [d] Two sections, each 10 ft high by 12 ft wide.

W. W. HORNER,[15] M. AM. SOC. C. E.—Within the limits of his allotted space, Mr. Hicks has attempted to present fundamental data that have been derived over a period of years; and he treats a wide variety of urban storm drainage design situations. As a result, the paper is too condensed to give the reader an adequate picture of the design methods developed by the Bureau of Engineering of the City of Los Angeles. It is extremely interesting to note that Mr. Hicks and his associates had anticipated the design procedure which the writer and Mr. Jens outlined in a previous paper.[4]

The writer has read Mr. Hicks' paper carefully several times and has discussed it, but he may not have fully appreciated all of the elements involved in the procedure. Accordingly, it would seem desirable for him to outline his understanding with the request that Mr. Hicks clarify certain of the matters that were presented much too briefly in the paper.

Mr. Hicks and his associates have apparently analyzed a great many actual runoff hydrographs in the Los Angeles storm drainage system, and have found that they could correlate certain of the parameters in a manner which would permit the preparation of a base hydrograph of correct form for any particular time of concentration.

Separately, extensive probability studies have been made resulting not only in the duration-intensity diagram of Fig. 1, but also in the determination of the most probable pattern of variation of rainfall intensity. The use of these studies permits the plotting of gross mass rainfall as indicated in Fig. 3.

The curve of mass rainfall is then modified through infiltration, surface detention, gutter storage, and finally conduit storage to a diagram of mass runoff in conduit, from which the runoff rate hydrograph can be derived. The writer understands that a large number of such hydrographs have been prepared and may be modified by the methods described to produce the actual hydrograph of runoff for any set of specific conditions.

---

[15] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

From the study of actual hydrographs and a correlation with the basic hydrographs, the author has been able to derive Eq. 5 and finds that this equation may be used satisfactorily as a basis for design in many instances. In actual practice it is understood that he uses either the summation of the derived hydrographs or Eq. 5, depending on certain limiting conditions.

From the foregoing, it would appear that, as the result of research work, the present actual design procedure may not be much more difficult or cumbersome than the sound application of the old rational method. If so, this answers one of the questions commonly raised; namely, can the engineer afford to take the time to evaluate infiltration, surface detention, gutter storage, etc., in the course of typical design practice. Apparently Mr. Hicks' research work makes it possible to evaluate all of these pertinent factors in an orderly manner and to reduce the procedure to a satisfactory working standard.

The writer would particularly like to have a more extended explanation of the moisture factor M. In part this factor as presented in Eq. 1 parallels somewhat the procedure, which the writer described,[16] in which he derived a moisture factor and gave it the same symbol M in order to relate infiltration capacity to antecedent precipitation. However, Mr. Hicks has introduced an evaporation factor which is fundamentally desirable. His value E apparently is not an evaporation rate, but some inverse function if the moisture factor is to increase with E. Further explanation of this factor would be desirable.

In recent design practice in the writer's office, particularly for the drainage of large airports, pattern storms or "design storms" have been derived very much in the manner Mr. Hicks describes. These are set up in hydrograph form and are routed successively from rainfall rate through surface detention to gutter inflow; then through gutter storage to inlet inflow; and finally through a process of routing through conduit storage and offsetting hydrographs to a determination of peak rates along the drainage lines. The writer has found, just as Mr. Hicks did, that, after a number of such designs had been prepared, correlations could be perfected which would give satisfactory, approximate values without going through the routing process for each particular subsection of the system.

Mr. Hicks is to be commended greatly for the fundamental work that he describes. This indicates that one city engineering department is now in a position to design storm drainage on the basis of a full evaluation of infiltration and detention of all classes.

W. I. HICKS,[17] ESQ.—The discussion of this paper includes commendation, requests for additional information and data, and constructive criticism. The writer is grateful for the commendation and will answer the queries and criticism to the best of his ability.

---

[16] "Rôle of the Land During Flood Periods," by W. W. Horner, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1269.

[17] Civ. Engr., Storm Drain Div., Bureau of Eng., Los Angeles, Calif.

Mr. Jarvis has noted certain divergences from agreement of the data plotted in Fig. 12. The writer selected data from seven drainage areas covering a wide range of urban development. As noted in the fifth paragraph of Part IX, there were irregularities in the records for Stations 4, 10, and 11, and Block B, which prevented full analysis of the peak runoff rates.

The record of Station 4, abscissa 1.50 and ordinate 0.80, is found on recheck of computations to be 1.33 and 0.80. As this drainage area is subject to heavy street flow at runoff rates greater than 0.45 in. per hr and to overflow from the drainage area at more than 0.60 in. per hr, agreement of record and computed flow would have been closer if the drainage system had been fully adequate to handle the storm.

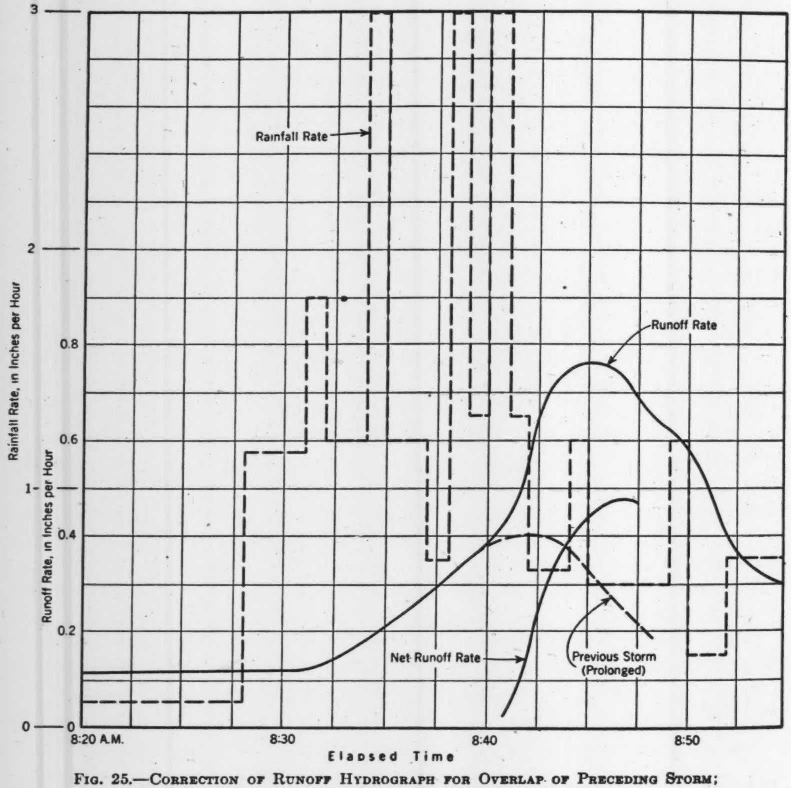


FIG. 25.—CORRECTION OF RUNOFF HYDROGRAPH FOR OVERLAP OF PRECEDING STORM; CITY BLOCK 4841, ST. LOUIS, MO., SEPTEMBER 28, 1919

Several of the records of Block B are subject to correction for overlap of preceding storms; one of these is analyzed in Fig. 25 for the point with an abscissa of 0.36 and an ordinate of 0.76. The peak at 8:45 a.m. is the result of rainfall from about 8:36 a.m. to 8:45 a.m. ($t_c = 9.2$); the rain prior to 8:36 a.m. would have caused a peak, if the rainfall had ceased at that time, with approximate shape and position as delineated ($t_c = 12 \pm$). Although no conclusive evaluation can be placed on its value nor on its recession curve, since no peak actually was formed in the record, it is apparent that the computed peak is more nearly in agreement than the plotted point indicates.

Station 11 is subject to heavier losses than would normally be expected on impervious areas, which accounts in part for the excess of computed peaks over actual peaks for that station. Although it may have been an error to include these divergent records in Fig. 12, they were included as being a part of the data; the writer considers that the computation method is substantiated by the predominance of points falling within the 20% tolerance zone and the possibility of explaining the divergence of some of the other points.

In compliance with Mr. Jarvis' suggestion of the possibility of computing a value of C from the data in Table 12 in the Rational Formula,

$$C = \frac{q_1}{I} \qquad (10)$$

Table 17 was prepared. It is largely self-explanatory: Cols. 1 to 5, inclusive, are prepared as Mr. Jarvis suggests, and the remaining columns are devoted to computation of a coefficient C_1 modified by the moisture factor M and the

TABLE 17.—INVESTIGATION OF COEFFICIENTS OF RUNOFF, STATION 3

No. (1)	I (2)	q_1 (3)	C (4)	$\frac{C}{C_{avg}}$ (5)	I_H (6)	t_c (7)	I_m (8)	q_3 (9)	$M^{A_{px}}$ (10)	C_1 (11)	$\frac{C_1}{C_{1\,avg}}$ (12)
1	1.19	0.430	0.362	0.684	0.793	28.3	1.230	0.413	0.550	0.610	0.87
2	0.68	0.320	0.470	0.888	0.459	33.4	0.640	0.300	0.708	0.660	0.94
3	0.88	0.456	0.520	0.982	0.550	32.7	0.780	0.422	0.660	0.817	1.17
4	0.59	0.315	0.534	1.008	0.435	32.2	0.621	0.275	0.672	0.659	0.94
5	1.09	0.610	0.560	1.04	0.760	29.4	1.048	0.570	0.670	0.812	1.16
6	0.77	0.560	0.727	1.37	0.528	33.3	0.739	0.510	0.790	0.875	1.25
7	1.06	0.830	0.783	1.48	0.664	30.8	0.981	0.730	0.880	0.843	1.20
8	0.64	0.505	0.790	1.50	0.455	37.5	0.596	0.455	0.915	0.834	1.19
9	0.55	0.530	0.961	1.82	0.427	37.1	0.563	0.460	1.035	0.790	1.13
10	0.61	0.192	0.315	0.595	0.418	32.4	0.593	0.182	0.710	0.432	0.62
11	0.73	0.229	0.314	0.593	0.447	30.0	0.665	0.219	0.524	0.630	0.90
12	0.63	0.227	0.360	0.680	0.436	30.2	0.645	0.201	0.542	0.596	0.85
13	0.64	0.260	0.416	0.786	0.424	31.0	0.620	0.250	0.604	0.666	0.95
14	0.50	0.147	0.294	0.555	0.359	33.7	0.501	0.147	0.538	0.545	0.78
Averages (numerical)			0.529	....	....	....	....	....	....	0.698	....
Averages (percentages):											
High values			137	....	....	....	....	....	....	118	....
Low values			72	....	....	....	....	....	....	85	....

shape factor F embodied in I_H (Col. 6, Table 17). The factor I_m (Col. 8) is the rainfall rate for the time of concentration (Col. 7) for the standard rainfall curve having an hourly intensity I_H. The modified C_1 is expressed in the equation,

$$C_I = \frac{q_3}{I_m\, M^{A_{px}}} \qquad (11)$$

The divergence between average high and average low ratios of q_c/q_1 in Table 12 is from about 118% to 87%; that for C is from 137% to 72%; and that for C_1, from 118% to 85%. Thus, for an area of stated physical and rainfall characteristics, Mr. Jarvis is correct.

However, to follow through with computation of runoff coefficients based on the ratio of observed runoff to observed rainfall without analysis and synthesis of the various factors affecting runoff, it will be necessary to gather and analyze data from a much greater number of drainage areas of different sizes, soils, and states of physical development. Although the possession of such data is greatly to be desired, it is doubtful whether personnel and money will be available to acquire it except through the passage of many years. In the meantime, the method of analysis and synthesis, refined and revised, as Mr. Jarvis suggests, by the accumulation of additional authentic data, holds forth the greater promise for current and future design. It is more flexible by reason of the use of the distribution factor (Fig. 5) and the factor for adjusting confluence of dissimilar areas (Fig. 14); when the designer has mastered its not too difficult method of operation, he will approach his problem with a clearer understanding of the fundamentals involved.

The area limitation in the paper has been set by the 120-min time of concentration set up in Figs. 5 and 16 and Table 7; larger areas can be handled by a reasonable extrapolation or by use of the Method of Summing Hydrographs.

In answer to Mr. Jens' questions on the factor M, the relations found by Eq. 5 led to the statistical analyses of rainfall data from the local Weather Bureau of the two composite factors, $I\ F$ and $I\ F\ M$. For loam or clay pervious areas, the maximum value of $I\ F\ M$ (not necessarily of $I\ F$) will cause the maximum runoff. Therefore, the seasonal variation of M is embodied in the data and need not be considered further in design. These analyses were made for several time durations and it was found that the resultant M was slightly greater for the short durations. On account of the small variation, it was ignored for the sake of simplicity in computations. However, in the analyses of individual rainfall-runoff records, M was computed from the data for each storm.

TABLE 18.—SLOPES FOR DRAINAGE AREAS IN FIG. 1 (PERCENTAGES)

Station No.	Overland			Gutter			Drain		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
3	0.6	50.0	15.0	0.50	22.0	4.25	0.75	8.51	1.036
4	0.6	10.0	2.6	0.25	7.50	1.61	0.33	2.67	0.946
10	1.75	6.00	2.42	1.13	1.13	1.13	....	....	....
11	1.27	6.30	2.70	0.20	2.00	0.92	....	....	....

The additional slope data requested by Mr. Jens for drainage areas in Table 1 are given in Table 18. It is true that lag time is dependent on these slopes as well as on the shape of the drainage area and the distribution of impervious development therein. In Fig. 9(d), lag time is shown as a function of the time of concentration which in turn is a function of the slopes of the surfaces and the channels of flow. The shape of the area and the distribution of impervious development determine the point of the maximum quick concentration of runoff; the elapsed time between this point and the gaging station is the lag time.

It is gratifying to learn that Mr. Jens finds the straight-line relationship between volume of storage and rate of discharge in his study of open-channel recession curves. In Eq. 7, K corresponds to the inverse of $\frac{\Delta D}{\Delta q}$ in Figs. 8 and 9(a).

From a theoretical standpoint, Mr. Jens is correct in his contention that the loss of rainfall on an impervious surface is a fixed amount. On broad drainage areas, however, absolute imperviousness is a goal reached only in varying degrees of perfection. In determining the amount of loss of rainfall on impervious areas, data were used from impervious Stations 10 and 11, and from experiments on overland flow on impervious surfaces. In the case of Stations 10 and 11, analyses were made as in Figs. 6 and 7, and the amount of rainfall and of loss between control points was determined. The average rate of loss is plotted as an ordinate against the corresponding average rate of rainfall in Fig. 26. The results indicate an increase of loss rate with an increase of rainfall rate. The higher loss for Station 11 offers some possible explanation: Paving grades are flatter with a consequent wider spread of flow; the pavement is not fully impervious; numerous utility and sanitary sewer manholes furnish small avenues of escape; and sidewalks around basement openings and area lights allow leakage into the basements. These deductions are substantiated by the facts that street railways use ballast subdrains, that public utilities pump out their conduit systems and the city outfall sewer runs under heavy pressure for an extended period after heavy rains, and that numerous buildings use basement pumps. The use of 10% instead of the higher percentages indicated in Fig. 26 was determined by the smaller percentage of loss in the

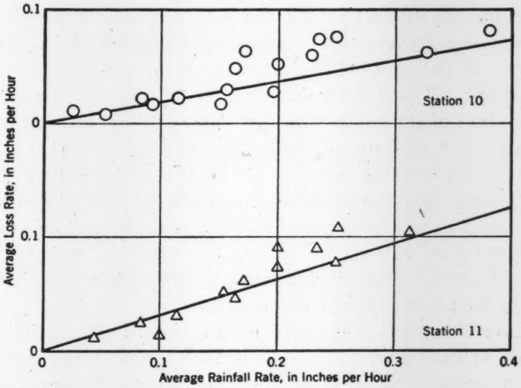


FIG. 26.—VARIATION OF LOSS RATES WITH RAINFALL RATES ON NOMINALLY IMPERVIOUS AREAS

overland flow experiments, the comparatively pervious soil under Stations 10 and 11, and the better surface drainage which an adequate storm drain system offers.

Mr. Jens' analysis of the storage equation is the most effective which the writer has encountered in that it avoids the tedious trial-and-error method so commonly used. Its use will furnish relief in reservoir problems where the inflow is translated to the gates and spillway of the dam by a high-velocity deep-water pressure wave. The complex pattern of velocities in the cross section of a conduit makes difficult the accurate determination of the changing shape of the hydrograph as it progresses downstream. In the studies leading to the method of peak-rate reduction used in this paper, investigations were made of a number of mathematical solutions of the storage equation; none seemed to fit the behavior of the hydrograph. The storage equation method, when applied to conduit flow, has the following inconsistencies: (*a*) It tends to shorten the near-peak-rate duration of flow instead of spreading the reduction in the vicinity of the peak; (*b*) computation for a 5-min reach of conduit shows much more than five times the reduction computed for a 1-min reach; and (*c*) the lag between the peaks of the inflow and the outflow hydrographs seems to bear no definite relationship to the time of flow in the reach of conduit selected. For example, in Fig. 23 the lag is about 5 min, whereas the time of flow is about 17 min in the 5,585-ft reach.

Hydrographs of storm drain flow, as gaged (Figs. 6 and 7) and as standardized for storm drain design (Fig. 15), are characterized by the sharp rate of rise on the accretion side of the curve and a comparatively short duration of near-peak rate, which duration increases with the time of concentration. These characteristics suggest a greater rate of peak-rate reduction for the slender hydrographs with small times of concentration than for the broader peaked, longer time hydrographs. The volume of available storage in a conduit t_f minutes long is the product of the length and the cross-sectional area, or $60\, t_f\, V \times Q/V = 60\, t_f\, Q$ cubic feet (see step 3 under the heading in Part VII, "Reduction of Peak Rate"). Since this volume of storage is independent of the slope or velocity of the conduit, the percentage of peak-rate reduction, computed by any method, remains constant regardless of the slope or slopes of the conduit. Based on these considerations and on the fact that Eq. 4 was derived from a study of the elements of actual hydrographs, the method of computing peak-rate reduction used in this paper was adopted.

The method of reduction is based on a standard shaped hydrograph. If, in the process of summing, the resultant hydrograph becomes so irregular as to not conform in shape to a standard hydrograph of any time of concentration, then Mr. Jens' objection to a standard method of reduction is correct. However, if the time of concentration is large, the percentage indicated in Table 7 is so small that no great error will be incurred by its use.

In discussing Eq. 5, Mr. Horner has assumed that either the summation of hydrographs or Eq. 5 is used in design practice. Under the heading, "VIII. Methods and Form of Runoff Computation," the Peak Rate Method is recommended for use except for stated conditions which require the Method of Summing Hydrographs. Eq. 5 has a limited special use on areas for which rainfall and runoff gagings are available.

Symbol M is an empirical factor derived from studies of rainfall-runoff data on pervious and partly pervious areas for the purpose of gaging the effect of antecedent precipitation, wind, temperature, etc., on the loss of storm precipitation into the soil. The writer also noticed its resemblance to the M derived by Mr. Horner.[16] Analyses of its effect on runoff were made with Eq. 5 for areas having different percentages of pervious area. It was used in the form M^{A_p} because, as A_p approaches 0 for impervious areas, M^{A_p} approaches unity.

The factor E is introduced to weight M for the seasonal variation in the amount of soil moisture loss. It was derived arbitrarily as follows: (*a*) Evaporation quantities for Fallon, Nev., California, Ohio, and Kingsbury, Calif.,[18] were averaged and summed for the calendar year; (*b*) the summation was converted into percentages of the total for the year; (*c*) for dates at regular intervals, the percentages covering 60 days antecedent were tabulated and plotted as ordinates on abscissas of the dates; and (*d*) these percentages, increased by a uniform 10%, were subtracted from 100% to form the E-curve.

In regard to the time required for design procedure by this method, the writer has encountered the same objections mentioned by Mr. Horner. In practice, the new method requires no more time than the Rational Method when applied to the routine peak-rate design which constitutes the bulk of municipal work, and it offers a comparatively simple method of solving special problems, such as retarding reservoirs, etc.

As the method of design in this paper was formulated for the solution of metropolitan Los Angeles drainage problems, informal discussion has raised the question of its applicability to other localities. The method is based on the following local fundamental data:

A. Rainfall
  1. Intensity-Duration Curves (Fig. 1)
  2. Variation of Rainfall Volume with Location (Fig. 2)
  3. Pattern of Major Storms (Fig. 3)
  4. Antecedent Precipitation (Moisture Factor M) (Table 5)
  5. Rainfall Distribution Factor (Fig. 5)

B. Soil Losses (Fig. 4)

C. Classification of Drainage Areas (Table 6)
  1. Percentages of A_d, A_i, A_p.
  2. Lengths and Slopes for Overland Flow
  3. Typical Block Dimensions

D. Climatic Conditions
  1. Temperature
  2. Humidity
  3. Frozen Ground—Snow
  4. Dry Weather Soil Cracking

Since these local data are used in constructing the table of basic peak runoff rates (Table 7), the charts of runoff factors (Fig. 13), and other tables and charts used in design, it is apparent that changes in the basic data would involve varying degrees of change in such tables and charts. The process of developing a runoff design method in another locality would be to analyze the data for that locality, derive the fundamental information indicated by the numbered figures and tables in the foregoing outline, and, by the methods of this paper and the previous paper by Messrs. Horner and Jens,[4] construct new tables and charts for design.

---

[18] "Handbook of Hydraulics for the Solution of Hydraulic Problems," by Horace Williams King, 3d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 444.

In other localities, use may be made of the charts and tables of this paper for peak runoff rates (not necessarily for shape of hydrograph, which is dependent on storm pattern) from areas having negligible or no percentages of pervious area (impervious business and manufacturing, apartment house, and bungalow court classifications, Table 6). As it is possible that intensity-duration curves for other localities will not have the same shape and pattern as those in Fig. 1, it will be necessary to use the following equation to secure the hourly rainfall rate for determining the runoff factor (Fig. 13):

$$\frac{I \text{ (for stated } t_c \text{ from their local rainfall curve)}}{I \text{ (for stated } t_c \text{ from 10-yr curve, Fig. 1)}} \times 1.01 = I_H \ldots\ldots (12)$$

The value of I_H will be amended by the distribution factor (Fig. 5), if the area exceeds 100 acres, before it is used to determine the runoff factor. The product of the runoff factor and the basic peak runoff rate (Cols. A, Table 7) is the peak rate of runoff, in cubic feet per second per acre. Because of possible variation of the shape of the intensity-duration curve and of the rainfall distribution factor, the solution will be subject to some error.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2231

RÔLE OF THE LAND DURING FLOOD PERIODS

By W. W. Horner,[1] M. Am. Soc. C. E.

With Discussion by Messrs. L. K. Sherman, Raphael G. Kazmann, W. G. Hoyt and W. B. Langbein, Charlie M. Moore, Waldo E. Smith, P. B. Rowe, C. S. Jarvis, and W. W. Horner.

Synopsis

The rôle of the land discussed in this paper is that which determines, from any specific precipitation period, the stream-channel inflow, which will result directly therefrom, and which will occur during or immediately following that period. Stream-channel inflow of normal ground water, from the water table, is not discussed in detail. This paper recognizes that this "rôle" is not a matter peculiarly related to "flood" periods, but that the land effect during flood periods is merely a special case of its effect during any period of precipitation.

The rôle of the land is discussed in its abstractive or retentive effect, determining the volume of related runoff, and also in its detentive effect as a partial determinant of the shape and timing of the hydrograph of channel inflow.

The principal controls of the land are given as surface retention, infiltration capacity, surface detention, subsurface detention, subsurface storage, and the facility for quick subsurface return outflow to the stream. It is stated that for the greater part of the land area, infiltration capacity is the most important determinant, but that for other parts, subsurface retention and detention capacity may be the limiting element. Quantitative values are discussed and an illustrative example is presented; some space is given to the type of basic data needed for a true evaluation of the controlling elements, and the extent to which the data are becoming available is the subject of comment.

PART I.—INTRODUCTION

The objective of this paper is to discuss the manner in which a rainfall pattern, during a generally continuous period of precipitation, is modified and transformed by the land into a pattern of occurrence of stream-channel inflow

---

Note.—Published in May, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

for a directly related rise of the hydrograph. For the sake of brevity, it is necessary to limit the scope of this paper to precipitation falling as rain, and to unfrozen ground.

The effects of the land with its vegetal cover fall into two general classes—(*a*) the retention effect, by which some or all of the rainfall is abstracted and stored and does not appear as stream-channel inflow or contribute to any directly related rise of the stream hydrograph, and (*b*) the detention effect through which the diagram of channel inflow is offset or delayed and modified with respect to the pattern of excess rainfall.

The paper traces the paths followed by water from its state as precipitation to its state as channel inflow, and the character of the controls that the land exerts on the water along these paths. It is recognized that the quantitative values of these controls, such as infiltration capacity, surface retention and detention, subsurface retention and detention, and subsurface return flow, are what they are, at the beginning of a storm period, because of the forces and effects that have been active during an antecedent period. During the antecedent period, the processes of evaporation, transpiration, and of percolation and subsurface discharge to the stream are the pumps which are evacuating ground storage space removing surface storage from the ground, and permitting, in varying amounts, changes in soil structure. These forces and effects are in a sense the "stagehands" which prepare the "setting," and what they accomplish prior to the rain has a very large relation to the type of performance which the principal "actor," precipitation, may produce with respect to storm runoff. Since this paper involves only the relationship between the precipitation and the channel inflow resulting from that precipitation, no consideration is given to channel inflow from the normal ground-water table, and the rise of the hydrograph is visualized as occurring from the excess channel inflow over and above normal ground-water inflow or base flow.

PRECIPITATION PATTERN

For the purpose of this presentation, it is assumed that precipitation occurrence may be treated in its pattern form. This pattern, depending upon the relative magnitude of the areas involved, preferably may be considered in intervals of 10 min, 1 hr, or even 3 hr. Mean values of precipitation for periods appreciably in excess of 1 hr are so different from the actual intensities that make them up, that their relation to controls, such as infiltration capacity rates, becomes obscured, and they may not be used except for generalization. This criterion becomes the more obvious when it is recognized that hydraulic procedures related to the flow in the streams must be based upon reasonably accurate patterns of inflow, and that only for the very large stream systems can mean inflow rates for periods greater than 6 hr be used to advantage in such procedures as flood routing, or in evaluating the effects of channel storage.

LAND CONTROLS

The effects of the land in modifying the amount and pattern of precipitation into an amount and pattern of channel inflow may be classified into five steps. There are two paths "a" and "b" which the precipitation at the soil surface may take in the direction of contributing to the stream-channel inflow, and along these paths the principal controls which the land exerts are:

I. Interception.—Storage, retention by and on vegetation, commonly referred to as interception. The precipitation not retained by vegetation will be referred to as rainfall at the soil surface.

II. Separation at the Soil Surface.—This is controlled by the relation between rainfall intensity at the soil surface and infiltration capacity. It divides the precipitation at the soil surface into (*a*) excess rainfall producing surface runoff and (*b*) accretion to soil moisture.

III. Storage.—

(*a*) The excess rainfall produced and remaining above the surface in Step II(*a*) may be retained in part in surface depressions, and is referred to as "depression storage." Excess rainfall minus depression storage becomes surface runoff as overland flow.

(*b*) That part of the rainfall at the ground surface which is infiltrated through the ground surface (Step II(*b*)) increases soil moisture initially up to the point of satisfying capillary deficiency.

IV. Detention.—

(*a*) Surface runoff in overland flow (Step II(*a*)) requires the detention of some part of the runoff in order to build up a film of water through which runoff can occur. Such surface detention does not of itself reduce the amount of channel inflow but modifies the time and rate of inflow occurrence. However, after the cessation of precipitation some water will infiltrate out of the surface detention film and surface runoff will be reduced further by the amount of such post precipitation infiltration.

(*b*) Accretions to soil moisture, in excess of capillary deficiency, represent free soil water and may be referred to as subsurface detention.

V. Runoff.—

(*a*) Surface runoff (Step II(*a*)) as reduced by infiltration out of surface detention (Step IV(*a*)) becomes channel inflow.

(*b*) Free soil water (Step IV(*b*)), when accumulated in sufficient amount and where the ground-water gradients are steep, may return to the surface, at stream margins or at the foot of steep slopes, in time to contribute to the rise of the hydrograph. For such conditions it may be referred to as quick subsurface return flow.

Values of these determinants are functions of soil surface, soil depth, soil composition and structure, and of soil slope; but they are also functions of the vegetal cover. Quantitatively they can be discussed only in terms of characteristic "complexes" or combinations of soil and cover.

SURFACE STORAGE OR RETENTION

The terms "surface storage" and "surface retention" in this paper include that part of the precipitation which does not appear either as surface runoff or as infiltration during the period of precipitation. It includes: (*a*) Interception by vegetal cover; (*b*) depression storage; (*c*) evaporation during storm period; and (*d*) surface retention.

(*a*) *Interception by Vegetal Cover.*—This item may be negligible with respect to major floods, but may affect the magnitude of minor floods appreciably.

Published values of interception generally are annual, seasonal, or monthly, but some information for precipitation periods is available. The areal holding capacity of the vegetation must vary with the density, percentage of canopy cover, character of leaf and stem surface, and similar factors. It is also subject to precipitation intensity and to wind effect. It will vary, therefore, with seasonal, crop, or vegetation conditions.

Interception for bare or fallow land, of course, is zero. Studies of interception by crop vegetation at Clarinda, Iowa, indicate that for corn, alfalfa, and clover, it may be as much as 0.5 in. for some types of storm and maximum vegetative development. It will generally range[2] from 0.05 in. to 0.20 in. when the precipitation in a storm period exceeds 0.5 in.

Studies at Bethany, Mo., indicate that for storms having a precipitation in excess of 0.5 in., and during a period of maximum vegetative development, alfalfa will intercept 35% of the precipitation, corn and soybeans from 13% to 17%, and oats 6%. Blue grass, exclusive of the holding power of the litter, seems to have a value between the latter two groups. Between the fallow and the maximum vegetative condition of the fields, the values probably would have a seasonal trend.[3]

TABLE 1.—PERCENTAGE OF RAINFALL INTERCEPTED BY TREES

Item	Variety	Precipitation, in Inches			
		0.50	1.00	2.00	4.00
1	Hardwoods[a]......	0.07	0.10	0.13	0.14
2	Pines[a]............	0.09	0.13	0.19	0.24
3	Various trees[b].....	0.14	0.25	0.48	...

[a] Unpublished manuscript, Appalachian Forest Experiment Station. [b] "Rainfall Interception," by Robert E. Horton, *Monthly Weather Review*, September, 1919.

Studies by the Southern Appalachian Forest Experiment Station, shown in Items 1 and 2, Table 1, indicate the range of canopy interception, in inches of depth, for storms throughout the year. The values in Item 3 were determined by Robert E. Horton, M. Am. Soc. C. E., for summer and fall conditions in New York State; whereas Items 1 and 2, Table 1, are for the forest as a whole, Item 3 is for the projected area under the tree crowns.

(*b*) *Depression Storage.*—This represents water trapped on the surface in natural depressions, or artificial depressions, such as furrows under ordinary land management. When, in addition, water is trapped or stored by structures provided for that purpose, such as closed terraces, pasture furrows, or ridges, it is generally referred to as "mechanical storage." Water retained as depression storage at the end of the precipitation period is disposed of by evaporation and infiltration, and does not contribute to channel inflow during the "flood" period.

(*c*) *Evaporation During Storm Period.*—The definition of surface storage in this section includes evaporation during the precipitation period. In general, this is materially less than 0.01 in. per hr, and is of little importance even with respect to minor floods. For long rains and for heavy vegetal cover, evaporation from canopy interception may cause an appreciable apparent increase in interception losses.

---

[2] *Miscellaneous Publication No. 397,* U.S.D.A., p. 19.

[3] "Interception of Rainfall by Vegetative Canopy," by J. L. Haynes, Annual Meeting of the Am. Soc. of Agronomy, 1937 (unpublished).

(*d*) *Surface Retention; General.*—In the course of deriving infiltration-capacity values for small gaged watersheds, rainfall and surface runoff are segregated for separable portions of the hydrograph, calling Δt the mean time of infiltration opportunity for such a portion. Then infiltration capacity is

$$f = \frac{P - Q - s}{\Delta t} \qquad (1)$$

in which precipitation P, surface runoff Q, and the surface storage or retention s, are values for that portion. The retention is the sum of interception and depression storage during that portion. The quantities P, Q, and Δt are known; by trial and elimination, values of s may be obtained which will give a rational set of progressively reducing values of f for successive portions of the hydrograph. This procedure is quite sensitive to the value of s, and determines it within rather narrow limits for any storm period producing surface runoff. For the watersheds at Edwardsville, Ill., and Garland and Tyler, Tex., the value of s for crop land and ordinary meadow was between 0.05 in. and 0.15 in. and up to 0.25 in. for Bermuda grass pasture. There were characteristic variations depending on the extent to which interception was filled out of prior rain.

Interception represents a definite abstraction from precipitation and a small but definite reduction in channel inflow for good vegetal cover as compared to bare or fallow land. It appears to make little difference whether it is filled out of light, immediately antecedent rain, or out of the main body of flood-producing precipitation. In the first case, it prevents increase of soil moisture to the extent of its value, with a concurrent prevention of a decrease in initial infiltration capacity. In the second case, it is equivalent to a decrease in precipitation intensity at the ground surface.

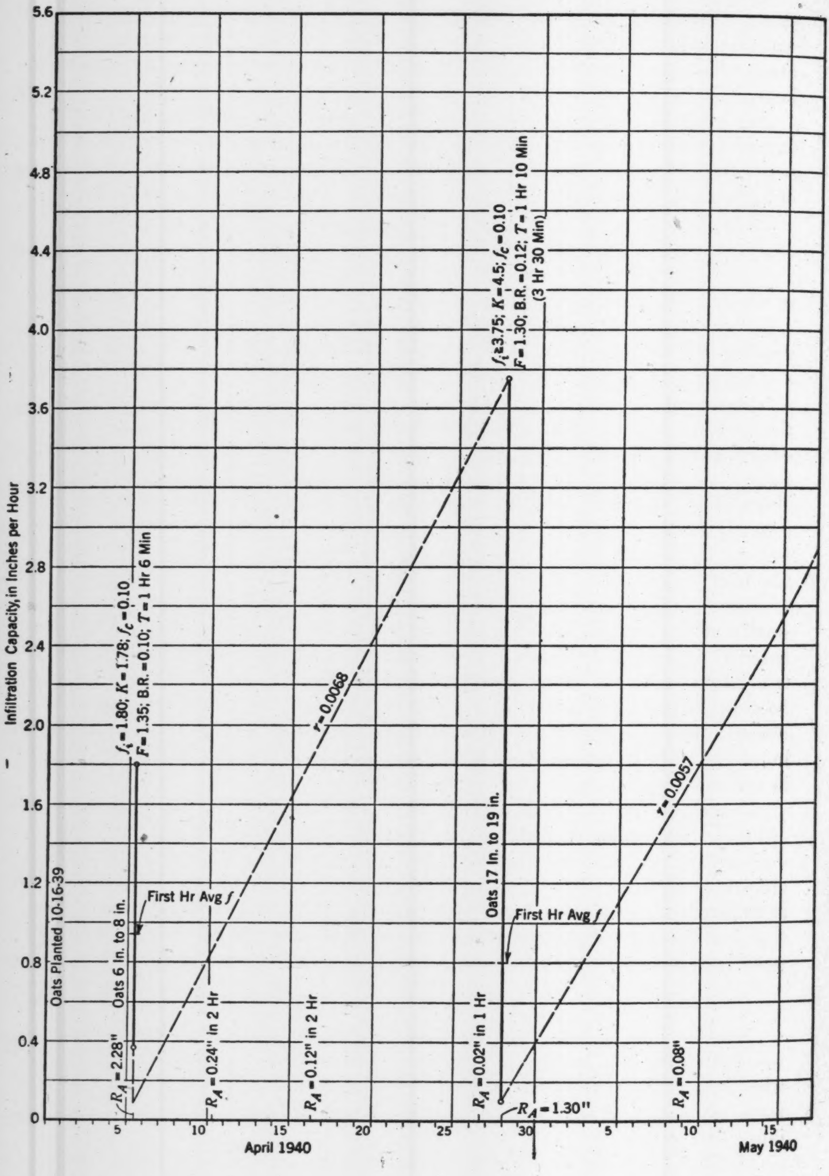
Depression storage can appear only after surface runoff begins. For ordinary cultivation or pasture, it seems rarely to exceed 0.10 in. This value does not include surface runoff into large sun checks or shrinkage cracks which appear in clay soils with low moisture content. Such water is treated herein as infiltration, even though not immediately disseminated into the soil mass, thus leaving depression storage as a value related to the soil in its swollen or uncracked state.

INFILTRATION CAPACITY

The conception of infiltration capacity is relatively new in hydrology. It was defined by Mr. Horton in 1935 as—"The maximum rate at which the soil, when in a given condition, can absorb falling rain."[4] Thus it is seen that the term "capacity" in this connection has no relation to volume, but is a limiting infiltration rate.

---

[4] "Surface Runoff Phenomena," by Robert E. Horton, *Paper No. 101*, Horton Hydrologic Laboratory, February 1, 1935.


Infiltration Capacity, in Inches per Hour
5.6
5.2
4.8
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
5
10
15
20
25
30
5
10
15
April 1940
May 1940
Oats Planted 10-16-39
Oats 6 In. to 8 in.
f_i = 1.80; K = 1.78; f_c = 0.10
F = 1.35; B.R. = 0.10; T = 1 Hr 6 Min
First Hr Avg f
r = 0.0068
R_A = 2.28"
R_A = 0.24" in 2 Hr
R_A = 0.12" in 2 Hr
f_i ≅ 3.75; K = 4.5; f_c = 0.10
F = 1.30; B.R. = 0.12; T = 1 Hr 10 Min
(3 Hr 30 Min)
Oats 17 In. to 19 in.
First Hr Avg f
R_A = 0.02" in 1 Hr
R_A = 1.30"
r = 0.0057
R_A = 0.08"

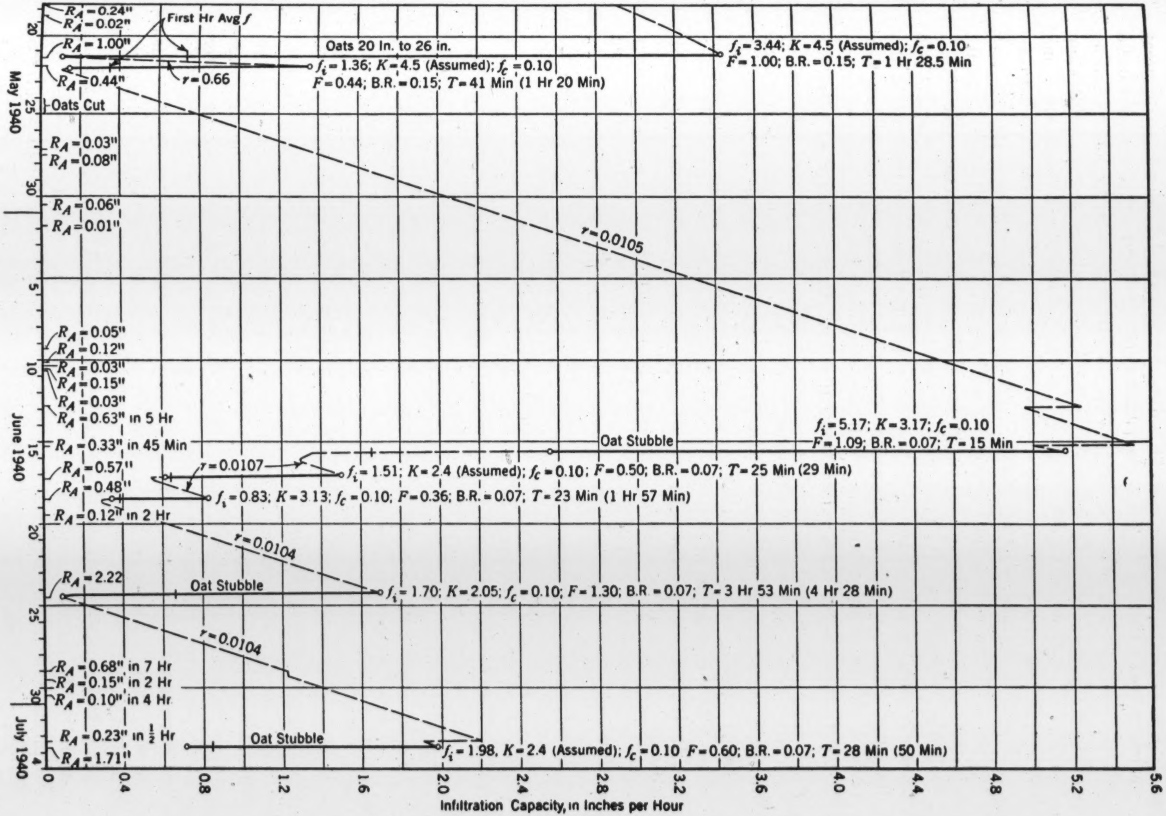


FIG. 1.—RECOVERY OF INFILTRATION CAPACITY BETWEEN RAINS, WACO, TEX. (EXPERIMENTAL WATERSHED SW 16), APRIL 6 TO JULY 3, 1940

Since 1935, infiltration-capacity values have been determined for a great variety of conditions. It seems clear that every soil and cover complex has a related characteristic curve of decreasing infiltration capacity values during a precipitation period. Mr. Horton also stated[4] that "as a rule, infiltration capacity of a given soil passes through a cycle from storm to storm," and illustrates the character of the recovery of infiltration capacity between precipitation periods; further "as the character of the soil and its moisture history are known for a time preceding a given rain, the infiltration capacity which it will have at the time of the rain can, in general, be closely predicted."

These conceptions have been verified by more recent investigations. The understanding of this writer with respect to the mechanics of infiltration and the elements which control infiltration capacity have been presented in another paper.[5] Much additional research will be needed for a complete understanding of the factors that control infiltration capacity. It is sufficient at this time to accept as a fact that each soil and cover complex has a related infiltration-capacity curve, that the values will follow a definite decreasing progression during a period of precipitation where intensities are in excess of infiltration-capacity values, and a somewhat modified curve during the period of precipitation where intensities are less than infiltration capacity.

Mr. Horton's statement with respect to the recovery of infiltration capacity between rains may be illustrated from Fig. 1, the results of a fragmentary study of one watershed at Waco, Tex. The symbols in this curve are defined as follows:

f_i = initial infiltration capacity, inches per hour;
f_c = minimum infiltration capacity, inches per hour;
F = mass infiltration in a storm, inches of depth;
T = compact time of storm (actual time shown in parentheses);
BR = basic retention, inches of depth;
r = rate of recovery, in inches per hour per hour;
R_A = rainfall, in inches of depth; and
K = constant in the equation.

$$f_1 - f_2 = K\,(\Delta F - f_c\,\Delta t) \qquad (2)$$

For lack of better knowledge, the rate of recovery r is shown as a uniform straight line between rains. Further studies will indicate the type of curve which this recovery really follows. It is interesting to note that these average rates of recovery are of the same general order as evaporation from water surface, and show the related seasonal change.

Characteristic curves of infiltration capacity which were derived in the course of investigations on the Trinity River basin in Texas (see Fig. 2) are shown in Fig. 3.

It will be noted that the original definition used the phrase "soil * * * can absorb." Mr. Horton undoubtedly referred to the actual absorption of precipitated water minutely into the soil mass. For certain soils which develop large cracks or sun checks when dry, it seems more convenient to treat infiltration as the disappearance of water below the general ground surface. This results in including, as part of the infiltration capacity, the rate at which surface water runs into open cracks. The effect of this inclusion produces relatively high infiltration-capacity rates at the beginning of the storm, decreasing sharply when the cracks are filled or become closed. However, this inclusion appears to be treated as part of depression storage, and this element would then be quite different in character for a particular ground surface in a normal uncracked condition, and for the same surface after large cracks have developed. The effect of this inclusion is particularly reflected in the high initial capacities and the rapid drop in capacities shown by the curves in Figs. 3(*a*) and 3(*b*).

---

[5] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.

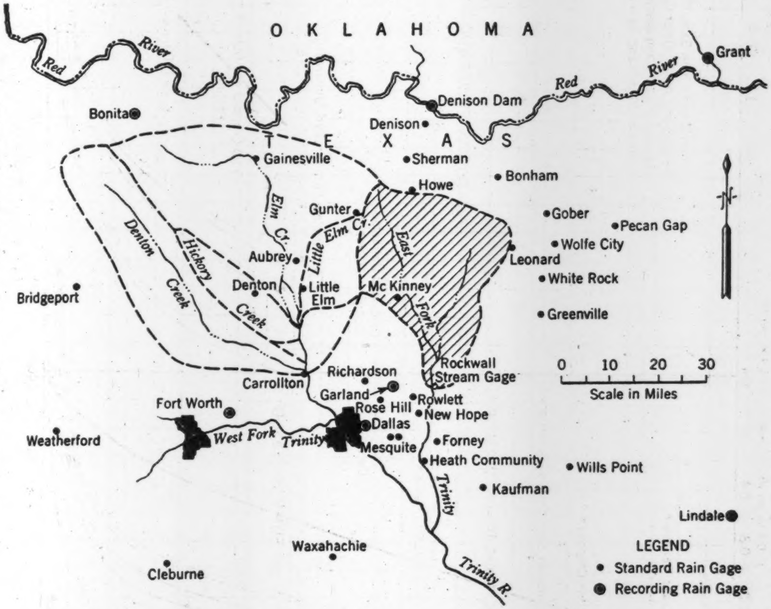

OKLAHOMA
Red River
Grant
Red River
Denison Dam
Bonita
Denison
TEXAS
Gainesville
Sherman
Howe
Bonham
Elm Cr.
Gunter
Little Elm Cr.
East Fork
Gober
Pecan Gap
Denton Creek
Hickory Creek
Aubrey
Wolfe City
Leonard
White Rock
Bridgeport
Denton
Little Elm
Mc Kinney
Greenville
Rockwall Stream Gage
0 10 20 30
Scale in Miles
N
Richardson
Carrollton
Garland
Rowlett
Rose Hill
New Hope
Fort Worth
Dallas
Weatherford
West Fork
Trinity
Mesquite
Forney
Heath Community
Wills Point
Trinity
Kaufman
Lindale
Waxahachie
LEGEND
● Standard Rain Gage
● Recording Rain Gage
Cleburne
Trinity R.

FIG. 2.—LOCATION OF RAIN GAGES IN THE VICINITY OF THE EAST FORK OF TRINITY RIVER, TEXAS

SUBSURFACE RETENTION AND DETENTION

It is obvious that that part of the soil moisture which satisfies capillary deficiency is definitely held in the soil and cannot become an element of stream-channel inflow. Accretions to soil moisture in excess of this amount represent free water, subject to the laws of ground-water movement. If the ground-water gradients are sufficiently steep and the permeability relatively high, such water may pass laterally to the stream margins or back to the surface at lower

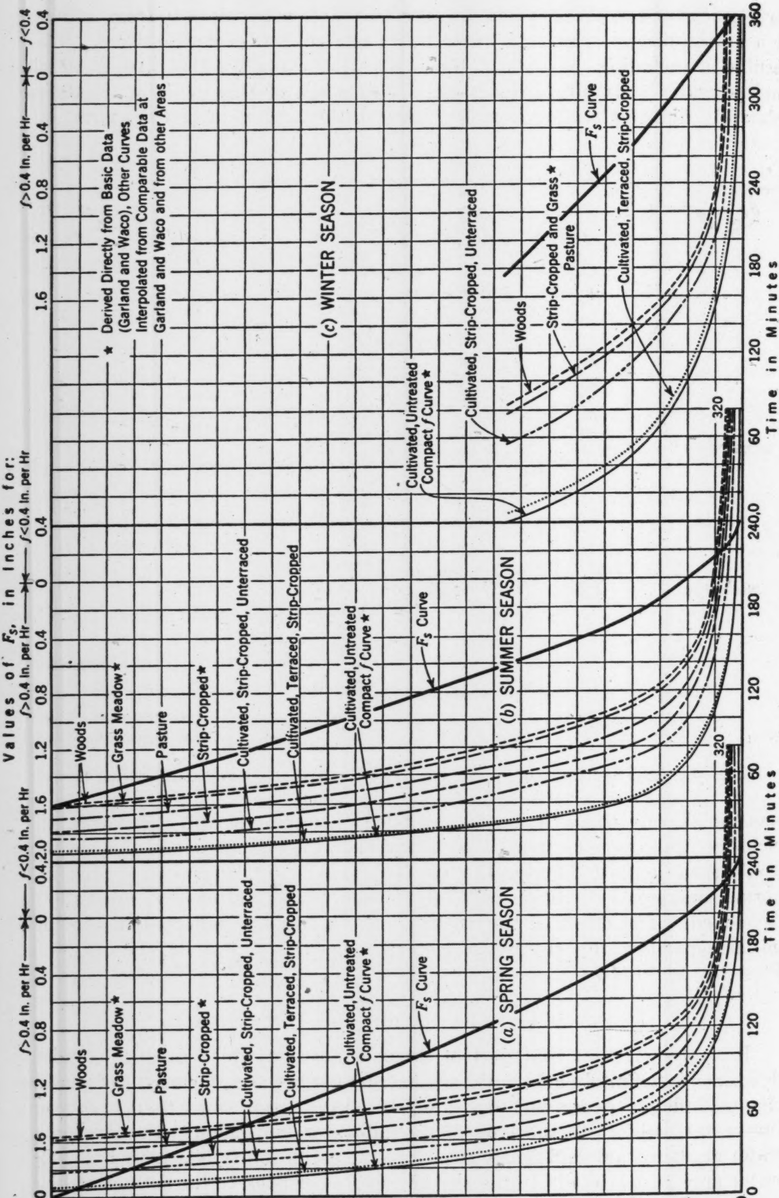


FIG. 3.—INFILTRATION CAPACITY AND CONTROL CURVES DERIVED DIRECTLY FROM BASIC DATA OBSERVED AT WACO AND GARLAND, TEX.; AND OTHER CURVES INTERPOLATED FROM COMPARABLE DATA

lying areas, and may become an important element of the stream channel inflow during the rise of the related hydrograph. On thin-soiled watersheds in mountainous regions a large part of the rise of the hydrograph may be produced by such "quick return subsurface flow." In some such areas, surface runoff may be extremely rare; in other cases where ground-water gradients are flatter, subsurface contributions to stream flow may come later than those from surface runoff resulting in an extended or possibly a double-peak hydrograph. Much information has become available with respect to characteristics of the hydrograph rise where inflow is largely "quick subsurface return water," but, as far as the writer is aware, no practical technique has been presented to the engineering profession that would permit quantitative evaluation of channel inflow from precipitation where subsurface inflow predominated, and for many drainage basins basic data are inadequate for this purpose. It is understood that basic data have been obtained and that such techniques have been developed by the flood control technicians of the U. S. Department of Agriculture. It is to be hoped some information of value in this field may become available in the course of the discussion of this paper.

ENGINEERING TECHNIQUE

As basic data have become available, engineering techniques have been developed for the evaluation of the flood flow which will result from specific precipitation occurrences; these utilize the quantitative values of the controls exercised by the land. In general, depending on the character the land controls, the techniques will fall into two classes.

Class 1.—Infiltration capacity is small to moderate in class 1, and stream-channel inflow occurs predominately as surface runoff. For this class a satisfactory technique is described in its application to the Trinity River basin in a succeeding section of this paper. This situation applies quite widely to cultivated and grass lands.

Class 2.—For class 2, infiltration capacity is generally large. Stream-channel inflow as surface runoff is a small part of the whole. Stream-channel inflow occurs predominately as "quick subsurface return flow." Infiltrated water is retained up to the amount of capillary moisture deficiency. This quantity has been referred to herein as "utilizable storage," but it may be retained also in part by percolation to deep water tables, and may arrive at the stream channel later as sustained base flow. Quantity and rate of stream-channel inflow also will be controlled by soil permeability and ground-water gradients. This situation is found principally in thin-soiled areas of steep topography. Many basic data are becoming available from which quantitative values of the land controls can be derived, but satisfactory techniques appear to be still in the process of development.

PART II.—THE RÔLE OF LAND IN DETERMINING STREAM-CHANNEL INFLOW WITHIN THE EAST FORK OF THE TRINITY RIVER DRAINAGE BASIN IN TEXAS

The physical conditions underlying the problem presented by the East Fork of the Trinity River drainage basin are typical of much of the land area which is susceptible to continuous agricultural use. Under these conditions, subsurface inflow to the stream channel is generally small during the rise of the hydrograph, and commonly constitutes less than 10% of the "flood" volume. Runoff is almost entirely surface runoff. The land abstracts a certain quantity of water above the ground surface—initially through interception and later through depression storage—and removes a considerable part of the precipitation into the soil through infiltration. For moderate storms producing small "floods," the effect of surface storage may be important. For great storms, the reduction effect of the land is determined almost entirely by the contemporary relative values of the ratio of precipitation "I" and infiltration capacity "f."

The drainage basin to which the application herein described was applied is outlined in Fig. 2, and has an area of 831 sq miles. The objective of the investigation was to determine the reduction in flood flow that would be expected to result in the application to this area of the Department of Agriculture program for change in land use and management. An investigation was conducted by the writer with the advice of the other members of the Engineering Consulting Board of the Department of Agriculture (R. E. Horton and L. K. Sherman, Members, Am. Soc. C. E.). The staff available for this work was furnished in considerable part by the flood control organization of that department under the supervision of Howard L. Cook, M. Am. Soc. C. E.

BASIC DATA

The basic data available consisted of (*a*) stream flow, (*b*) precipitation, (*c*) infiltration capacity and surface retention, and (*d*) soil and land use.

(*a*) *Stream Flow.*—The stream flow data consisted of a 16.5-yr record at Rockwall, Tex. The last 5.5 yr of this record was selected for the purpose of this study. During this period, 35 significant rises of the stream occurred; and these records then were found to represent, satisfactorily, the range of flood stages contained in the longer record.

(*b*) *Precipitation.*—Information concerning precipitation was available from the rain-gage stations indicated in Fig. 2. During the first part of the 5.5-yr period the only recording rain gages were those at Fort Worth, Tyler, Garland, and Dallas, Tex. During the latter part of the period, records were also available from the recording gages in Texas at Bonita, Denison Dam, Grant, Mount Pleasant, and Lindale. For many of the nonrecording gages the observers' records were adequate to determine the rainfall in periods of less than one day. On the whole, however, the precipitation information was much less adequate than was desirable, and this inadequacy accounts for some of the discrepancies in the results.

(*c*) *Infiltration Capacity and Surface Retention.*—Detailed rainfall and runoff records were available for several classes of land use at the experimental stations of the Department of Agriculture at Garland and Waco. From these could be derived the regression curves of infiltration capacity, relation of initial infiltration capacity to antecedent conditions and approximate values of "surface retention" taken to include interception and depression storage.

(*d*) *Soil and Land Use.*—Except for a negligible percentage of the drainage basin area the soil is typical of the Texas Blacklands. A detailed survey of the land use had been made and the Department of Agriculture had prepared each program for the revision of land use and management. The distribution of land use with respect to vegetal cover for "present" conditions and for "after" adoption of the Department's program is shown in Table 2. A study of the land-use survey and of the proposed program showed that the various classes of land use were reasonably well distributed throughout the basins and the percentages in Table 2 would apply equally well to each of the ten subdivisions.

TABLE 2.—DESCRIPTION OF LAND USE

Description	"BEFORE"		"AFTER"	
	Acres	%	Acres	%
Cultivated untreated and contour cultivation and idle	380,003	72	40,607	7
Terraced and strip cropped	0	0	182,330	34
Strip cropped with grass or small grain	0	0	78,551	15
Present pasture	82,387	15	......	..
Future pasture	......	..	167,624	32
Present woodland	37,540	7	......	..
Future woodland	......	..	30,818	6
Farmstead and non-farm	31,910	6	31,910	6
Total	531,840	100	531,840	100

METHODOLOGY

The methodology employed involved eleven steps, as follows:

1. Collection and correlation of all available precipitation data for each of the thirty-five storms;
2. Preparation of the most probable precipitation intensity patterns;
3. Derivation of infiltration capacity curves for the Blackland soil and various classes of vegetal cover;
4. Correlation of infiltration capacity with antecedent conditions;
5. Application of derived data to the evaluation of surface runoff;
6. Computation of the excess rainfall, hour by hour, throughout the storm;
7. Transformation of precipitation excess into mean values on each of ten sub-areas;
8. Comparison of mean computed surface runoff with actual surface runoff;
9. Determination of excess rainfall for "after" conditions;
10. Computation of flood-flow reduction resulting from the effective adoption of the program; and
11. Determination of flood-stage reduction, and reduction in area inundated.

Step 1.—The character of the available precipitation data has been described previously. The correlation from the information in the various standard gage stations with that at the available recording gage stations involved considerable tedious detail and some actual detective work in the interpretation of the cooperative observers' reports. Some of the observers made excellent records giving the time of the beginning and end of each such part of a storm period and the amount of precipitation in each part. Others gave only daily totals and the reports were often misdated.

Step 2.—The preparation of the most probable precipitation intensity patterns at each of the precipitation stations utilized for the specific storms (generally using five to eight stations) involved first, the plotting of the precipitation pattern by hourly intervals at the recording gage stations; and then the data given by the cooperative observers were placed in a similar pattern form. When the cooperative observers gave the time of the beginning and ending of each precipitation period, the shift in timing and in amount of each such period between the recording and cooperative station was evident. When this information was not available the timing was proportioned between recording gage stations or between stations where the timing was given on a basis of relative distance and position. For full-area storms it is believed that the resulting patterns are quite representative of the actual occurrence. For partial-area storms the problem was more difficult, particularly that of determining the total area covered by the storm. If two recording gage stations had actually been available within the drainage basin itself, the work required under Steps 1 and 2 would have been reduced by at least 75% and the significance of the resulting probable precipitation patterns would have been increased materially.

Step 3.—The problem of deriving infiltration-capacity curves for the Blackland soil and various classes of vegetal cover was simplified by the fact that, to all intents and purposes, only one soil type occurred in this basin, and that the variations in infiltration capacity were primarily those growing out of the difference in vegetal cover, and of antecedent conditions.

Infiltration-capacity curves were derived from the rainfall and runoff records available for a large number of small watersheds at Waco, and for several similar watersheds at Garland. The soils at both places were considered to be fundamentally the same. A review of the resulting infiltration-capacity curves, however, indicated a slight difference, between those from the two stations, which was adjusted in the final averaging.

The basic data were available for cultivated untreated land—that is, normal corn and cotton fields—for cultivation under the strip-cropping method and for grass meadow. Infiltration-capacity values were derived from all storms for a 2-yr period at Waco, and a 1.5-yr period at Garland. For some short storms, on some watersheds, only one point on an infiltration-capacity curve could be determined; but for many of the storms relatively complete curves could be derived. The derived curves were first condensed in time, to the form they would have if produced under continuous excess precipitation, on the hypothesis that

$$f_{t1} - f_{t2} = K\,(\Delta F - f_c\,\Delta t) \qquad (3)$$

in which f_t is infiltration capacity at time t_1, ΔF is mass infiltration in time $\Delta t = t_2 - t_1$, and f_c is the minimum final value of f. The condensed curves then were averaged by seasons for each station and then averaged for seasons for the entire data. Some information of value was also secured from infiltrometer runs.

The average curves were prepared directly from the basic data for the aforementioned three classes of cover. For those classes of cover which did not occur at the Waco or Garland stations, intermediate values were determined from the study of data on other experimental stations where records from such additional cover class were available and comparable. The curves for the remaining complexes were correlated in time with that for the "cultivated untreated" land by observing the time differences between occurrences of the same infiltration-capacity values under different vegetal covers, in each of the storms studied. The resulting curves for all classes of cover under consideration are shown in Fig. 3.

Step 4.—In addition to the derivation of the regression curve a correlation was developed between an index point ($f = 0.4$) on the master curve (that for "cultivated untreated" land) and the antecedent conditions which appeared to control the position of this curve at the beginning of the storm.

The correlation of antecedent conditions is an operation essential to the use of infiltration-capacity curves and is a matter which has been given too little attention.

For each of the storms studied the mass infiltration, occurring in the storm, prior to the time infiltration capacity reached a value of 0.4 in. per hr, was calculated. This is the value for F_s which for the average curves was later endorsed on Fig. 3. These values of F_s were plotted as ordinates on calendar paper for the full year, and a preliminary average curve was drawn through the resulting points.

The departure of the points from the average curve was then studied in terms of antecedent precipitation, and after a number of trials it was found that a new and much smoother curve would be produced if, to the values of F_s, a weighted value of precipitation was added during the antecedent thirty days. This weighted value took the final form of $M_a = \sum (F_a K_{t_d})$, in which F_a was the mass infiltration during a particular storm occurring t_d days prior to the storm under consideration; K varied from 100% for a time, t_d, of one day down to 10% for a period of thirty days.

This second correlation curve was quite significant. A study of the remaining departures from it indicated that these were primarily related to more general variations in soil moisture in different years, and could be reconciled by adding a final value, M_y, equal to 5% of the excess or deficiency of precipitation for ninety days preceding the aforementioned thirty days. The final M-curve in Fig. 4 has been referred to as an "effective moisture index." It is generally rational in form to the extent that it represents the antecedent accretions to soil moisture. It is theoretically defective in that it relates the loss of soil moisture only to elapsed time and does not reflect, directly, temperature or wind. However, these factors are involved in the seasonal position of the curve. Although the M-curve as a whole is entirely empirical in form, a test of its use indicated that it would place the infiltration-capacity curves quite accurately in the storms from which they were derived. It was used with confidence, therefore, in the application procedure described in the subsequent steps. Later studies have shown that, because of the small amount of data originally available for the midsummer and early fall periods, the position of curve should be revised for July, August, and September, and should be raised somewhat in these months. In particular, this reflects a sharp change in soil structure which occurs during these months and which produces excessive soil checking and aggregation.

Step 5.—For each of the thirty-five storms in the selected record for the Trinity basin, antecedent precipitation was reduced to antecedent infiltration and was weighted, as described in Step 4, to determine the value of M_a. The ninety-day value of M_y was taken also from the Weather Bureau records. Subtracting the sum of these two from the ordinate on the M-curve (Fig. 4) gave the value of F_s, or the mass infiltration which would have to occur during the storm period before the infiltration capacity reached the value of 0.4. With this information it was an easy matter to place the correct seasonal average infiltration-capacity curves in their proper position on the rainfall patterns. Highly variable or somewhat intermittent precipitation required the expansion of the infiltration-capacity curves, in time, reversing the procedure for condensing them, described in Step 3. Where there were gaps in the precipitation pattern of several hours or more, recovery of the infiltration capacity across the gaps had to be evaluated separately. An example of the resulting work diagram for the first 6 hr of a 7-hr storm is shown in Fig. 5, with computations for three of those hours in Table 3.

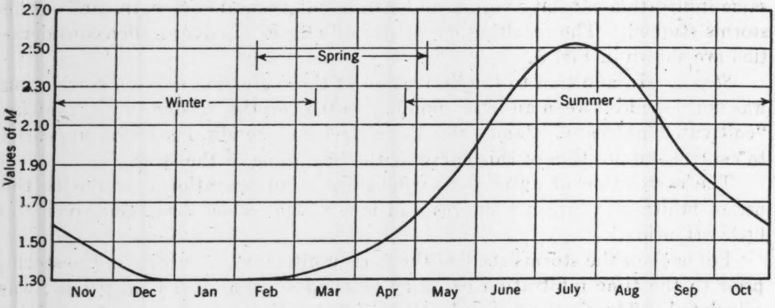


FIG. 4.—SEASONAL VALUES OF M, THE "EFFECTIVE MOISTURE" INDEX

Step 6.—From the related precipitation intensity rates and infiltration-capacity values, the excess rainfall was computed hour by hour throughout the storm. This excess rainfall for "present conditions" (conditions before adopting the land-use program) was modified by making deductions for interception and depression storage, the resulting net value being taken as equivalent to surface runoff.

Table 3 shows the values of surface storage in Col. 5, entitled "Retention," and the percentage of occurrence of various complexes in Col. 9 entitled "Weighted Values of Land Use." The values of surface runoff R_u by hours for each condition are shown in Col. 12, Table 3. This illustrative computation is for only one of the rainfall stations used in the basin. Similar computations were made for the remaining stations, and excess rainfall by hours, for the

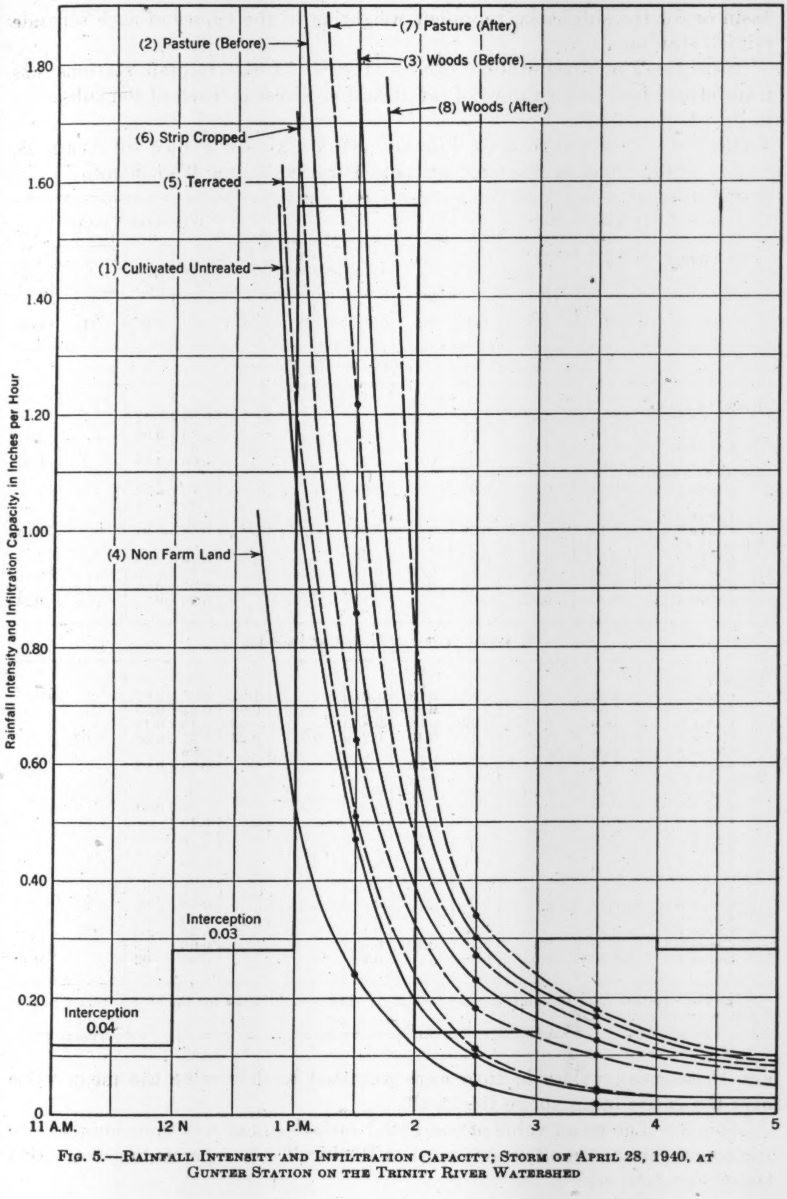


FIG. 5.—RAINFALL INTENSITY AND INFILTRATION CAPACITY; STORM OF APRIL 28, 1940, AT GUNTER STATION ON THE TRINITY RIVER WATERSHED

basin or for the sub-basin, represent weighting of the values of each separate rainfall station.

Step 7.—The precipitation excess at each of the rainfall stations was transformed into mean values of precipitation excess on each of ten sub-areas, and these mean values in turn were weighted to determine the mean value over the entire basin above Rockwall.

TABLE 3.—COMPUTATION OF PRECIPITATION EXCESS, STORM OF APRIL 28, 1940; GUNTER STATION ON THE TRINITY RIVER WATERSHED

Time range[a]	Precipitation I (in. per hr)	Infiltration capacity f (in. per hr)	$I-f$	Retention (in.)	Net P_e (in.)	Mechanical storage (in.)	R_u (in.)	WEIGHTED VALUES			Total net R_u (in.)
								Land use (%)	R_u (in.)	Mechanical storage (in.)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(a) BEFORE ADOPTING LAND-USE PROGRAM											
1 p.m. to 2 p.m.											
Curve 1	1.56	0.47	1.09	0.06	1.03	...	...	0.72	0.74	...	...
Curve 2	1.56	0.86	0.70	0.08	0.62	...	...	0.15	0.09	...	...
Curve 3	1 56	1.74	...	...	0	...	...	0.07	...	...	...
Curve 4	1.56	0.24	1.32	0.03	1.29	...	...	0.06	0.08	...	0.91
2 p.m. to 3 p.m.											
Curve 1	0.14	0.10	0.04	...	0.04	...	...	0.72	0.03	...	...
Curve 2	0.14	0.23	...	...	0	...	...	0.15	...	...	...
Curve 3	0.14	0.30	...	...	0	...	...	0.07	...	...	...
Curve 4	0.14	0.05	0.09	...	0.09	...	...	0.06	0.01	...	0.04
3 p.m. to 4 p.m.											
Curve 1	0.30	0.04	0.26	...	0.26	...	...	0.72	0.19	...	...
Curve 2	0.30	0.13	0.17	0.03	0.14	...	...	0.15	0.02	...	...
Curve 3	0.30	0.17	0.13	0.06	0.07	...	...	0.07	0.01	...	...
Curve 4	0.30	0.02	0.28	...	0.28	...	...	0.06	0.02	...	0.24
(b) AFTER ADOPTING LAND-USE PROGRAM											
1 p.m. to 2 p.m.											
Curve 1	1.56	0.47	1.09	0.08	1.01	...	1.01	0.07	0.07	...	...
Curve 5	1.56	0.51	1.05	0.09	0.96	...	0.96	0.34	0.33	...	...
Curve 6	1.56	0.64	0.92	0.09	0.83	...	0.83	0.15	0.12	...	...
Curve 7	1.56	1.22	0.34	0.10	0.24	0.02	0.22	0.32	0.08	0.02	...
Curve 8	1.56	2.34	...	...	...	...	...	0.06	...	...	...
Curve 4	1.56	0.24	1.32	0.03	1.29	...	...	0.06	0.08	...	0.66
2 p.m. to 3 p.m.											
Curve 1	0.14	0.10	0.04	...	0.04	...	...	0.07	...	...	...
Curve 5	0.14	0.11	0.03	...	0.03	...	...	0.34	0.01	...	...
Curve 6	0.14	0.18	...	...	...	...	...	0.15	...	...	...
Curve 7	0.14	0.27	...	...	...	...	...	0.32	...	...	...
Curve 8	0.14	0.34	...	...	...	...	...	0.06	...	...	...
Curve 4	0.14	0.07	0.07	...	0.07	...	...	0.06	...	...	0.01
3 p.m. to 4 p.m.											
Curve 1	0.30	0.04	0.26	...	0.26	...	...	0.07	0.02	...	...
Curve 5	0.30	0.04	0.26	...	0.26	...	...	0.34	0.09	...	...
Curve 6	0.30	0.10	0.20	...	0.20	...	...	0.15	0.03	...	...
Curve 7	0.30	0.15	0.15	...	0.15	...	...	0.32	0.05	0.01	...
Curve 8	0.30	0.18	0.12	0.10	0.02	...	...	0.06	...	...	...
Curve 4	0.30	0.02	0.28	...	0.28	...	...	0.06	0.02	...	0.20

[a] The curve numbers refer to points plotted in Fig. 5. The computations for the excess from 4 p.m. to 5 p.m. were not included in the table.

Step 8.—The mean value of computed surface runoff was then compared to the actual surface runoff as measured at Rockwall, and a percentage correction factor was determined.

Step 9.—The parallel procedure for "after" conditions (after adopting the land-use program) results in a determination of excess rainfall for "after" conditions. This computed excess rainfall is modified, not only by making deductions for interception and depression storage, as in Step 6, but must be modified further by making deductions for storage in mechanical structures on the ground. The resulting value of surface runoff when thereafter revised, through the application of the foregoing correction factor, becomes comparable with the actual measured surface runoff under "present conditions" (Table 3(*a*)).

Step 10.—The difference between these values represents the calculated flood-flow reduction in inches, resulting from the effective adoption of the program, and the percentage relationship between the two represents the percentage of flood reduction produced by the program.

Step. 11.—For all of the thirty-five flood rises the actual stage hydrographs at the Rockwall Station were available. Through the use of hydraulic engineering procedure, involving distribution graphs, comparable hydrographs at Rockwall were prepared as representative of the stages at Rockwall.

Using cross sections, the stream slopes between them, and hydraulic factors derived from the Rockwall Station, and other pertinent records, distribution graphs were prepared for some eleven stations upstream from Rockwall.

General.—In the course of the procedure described in Steps 5 to 7, inclusive, surface runoff was evaluated hour by hour throughout the storm at each of the rainfall stations utilized. The values of excess rainfall in this pattern form were then transferred to mean values of excess rainfall not only for the basin as a whole but also on each of ten sub-areas. From these mean values for the sub-areas, mean values were determined for each hour for the area as a whole above each of the cross-section points. These records become the "excess precipitation patterns" which were applied to the appropriate distribution graphs in order to produce the most probable hydrographs of flood flow at each particular cross-section point.

In the subsequent step these discharge hydrographs are transformed to stage hydrographs, representative of the stages for both "before" and "after" conditions. A comparison between them results in an evaluation of flood-stage reduction at each of these stations.

In a later stage of this project "before" and "after" flood stages were utilized as a basis for determining the reduction of acreage inundated for each reach of the river and these figures were used as a basis for the determination of the reduction in flood damage.

PRESENTATION OF RESULTS

The results of the study for the thirty-five storms are presented in Table 4, which extends the study through to the evaluation of the reduction in flood flow as a percentage of volume. In order to obtain a reasonably accurate representation of both rainfall depth and area covered for partial-area storms (Table 4, Nos. 2, 3, 6, 7, 8, 9, 15, 18, 19, 32, 33, 34, and 35) the preparation of isohyetal maps was necessary. For several of the storms the rainfall information was fairly satisfactory for this purpose and resulting maps prepared entirely from rainfall information were adequate for the purpose of the study.

For storms 6, 7, 13, 34, and 35, however, the rainfall information was not adequate and the measured runoff was used as a guide in preparing the maps. For all except these five storms excess rainfall was computed from precipitation and infiltration-capacity information, and the resulting values, reduced to mean inches of depth, are shown in Cols. 4 and 5 for comparison with the measured runoff at Rockwall shown in Col. 3. Col. 6 shows the correction factor by which the computed runoff would have to be multiplied to make it equal to the measured runoff. This factor is the test of the validity of the procedure and has been restudied for the twenty-three storms having a volume of runoff of more than 0.25 in. For the purpose of such analysis the factors have been grouped by seasons and by amounts of runoff, and results are shown in Table 5.

TABLE 4.—SUMMARY OF BASIC AND DERIVED DATA,

Storm No.	Date	Total surface runoff from gaging station (in.)	COMPUTED P_e FOR PRESENT CONDITIONS[a] (Inches)		Correction factor	COMPUTED P_e FOR CONDITIONS		
						Thiessen Method (Inches)		Propor Met
			Thiessen method	Proportional method		Total P_e	Storage deducted	Total P_e
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	May 3-5, 1935	2.28	2.74	2.63	0.87	2.42	2.35	2.32
2[d]	May 13-19, 1935	1.61	1.83		0.88	1.32	1.27	1.31
3[d]	May 28-30, 1935	0.58	0.58		1.00	...	0.40	
4	June 1-2, 1935	0.31	0.25	0.38	0.82	0.17	0.16	0.31
5	June 13-15, 1935	3.64	3.90	4.61	0.79	3.33	3.26	4.01
6[d]	Sept. 24-26, 1935	0.09	0.15		...	...	...	0.07
7[d]	Oct. 25-27, 1935	0.12	0.18		...	...	...	0.08
8[d]	Dec. 5-6, 1935	0.51	(0.95)		...	...	...	...
9[d]	Sept. 25-28, 1936	0.35	(0.35)		...	...	...	...
10	Oct. 22-25, 1936	0.47	0.92	0.83	0.57	0.28	0.26	0.61
11	Dec. 1-6, 1936	0.15	0.15	0.15	1.00	0.09	0.09	0.09
12	Jan. 6-9, 1937	0.14	0.18	0.18	0.78	0.10	0.10	0.09
13	Jan. 11-15, 1937	0.87	0.87		...	...	0.80	
14	Jan. 24, 1937	0.17	0.16	...	1.06	0.10	0.10	...
15[d]	Mar. 3-6, 1937	0.25	0.24	0.25	1.00	0.15	...	0.16
16	Mar. 23-26, 1937	0.17	0.07	0.10	1.70	0.04	0.04	0.06
17	Dec. 13-17, 1937	0.21	0.17	0.23	0.91	0.14	0.13	0.18
18[d]	Dec. 26-29, 1937	0.47	0.36	0.50	0.94	0.24	0.23	0.36
19[d]	Jan. 20-23, 1938	2.61	(2.25)		(1.16)	1.38	1.33	1.57
20	Feb. 16-17, 1938	4.02	3.91	3.83	1.05	3.38	3.31	3.29
21	Mar. 26-28, 1938	2.61	2.79	3.01	0.87	2.47	2.38	2.67
22	Apr. 6-7, 1938	0.51	0.53	0.67	0.79	0.38	0.37	0.53
23	Apr. 14-15, 1938	0.41	0.58	0.73	0.56	0.38	0.35	0.50
24	June 8, 1938	0.27	0.72	0.58	0.47	0.52	0.50	0.41
25	Mar. 25-29, 1939	0.30	0.47	0.38	0.79	0.34	0.32	0.27
26	Apr. 5, 1939	1.08	1.15	1.15	0.94	1.06	1.02	1.03
27	Apr. 15-18, 1939	1.62	2.19	2.19	0.74	2.02	1.95	1.98
28	Apr. 5-6, 1940	1.75	2.10	1.93	0.91	1.91	1.82	1.76
29	Apr. 28, 1940	0.78	0.93	0.96	0.81	0.70	0.67	0.74
30	May 17-18, 1940	0.26	0.20	0.29	0.90	0.12	0.12	0.21
31	May 22-23, 1940	0.35	0.61	0.64	0.55	0.49	0.48	0.55
32[d]	May 26-28, 1940	0.48	0.48	0.51	0.94	0.35	0.34	0.33
33[d]	June 8-12, 1940	0.19	0.28		0.68	...	0.11	
34[d]	June 13-18, 1940	0.70	0.70		...	...	0.48	
35[d]	June 28-July 3, 1940	0.87	0.87		...	...	0.57	

[a] Both Thiessen and proportional weightings shown if computed. [b] Surface runoff only. [c] Base flow included.

In general, it appears that the use of the derived infiltration-capacity and surface storage data, applied in this manner, will reproduce the measured flood flow to a reasonable degree and can be made equal to it through the use of a rather uniform correction factor. For this project the rainfall data were relatively unsatisfactory, particularly in the first part of the 5.5-yr period. No recording gage was within the basin, and considerable use had to be made of the timing shown on the observers' reports.

It is interesting that the computed values were less than the actual stream flow during the winter and spring months, and exceeded the actual stream flow during the summer and fall months. The most probable explanation is that the watersheds for which infiltration-capacity values were determined were largely on the uplands, and that during the winter period a part of the drainage area located on the lower slopes of the bottom lands had a higher average moisture content, a lower infiltration capacity and consequently a greater amount of excess rainfall than that reflected in the computations. The reverse difference for the summer and the fall storms probably reflects the generally

EAST FORK OF TRINITY RIVER ABOVE ROCKWALL, TEX.

"AFTER"[a] tional hod Storage deducted (10)	Col. 6 × Col. 9 (11)	"After" correct P_e storage deducted (12)	Calculated reduction in surface runoff (in.) (13)	Reduction in percentage by volume (14)	Calculated 9-hr precipitation excess (in.) (15)	Crest rate[b] "before" (1,000 cu ft per sec) (16)	Mean rate;[c] maximum computed daily (1,000 cu ft per sec) (17)	Storm No. (1)
2.25	2.02	1.95	0.33	14.5	1.26	28.85	22.3	1
1.27	1.14	1.10	0.51	31.7	0.68	11.20	9.3	2
...	...	0.40	0.18	31.0	0.43	7.76	5.2	3
0.30	0.25	0.24	0.07	22.6	0.31	4.18	2.6	4
3.94	3.16	3.09	0.55	15.1	2.85	69.60	52.0	5
...	...	0.07	0.02	22.2	0.05	0.78	0.74	6
...	...	0.08	0.04	33.3	0.07	0.93	0.91	7
...	...	0.36	0.15	29.4	0.50	6.00	5.5	8
...	...	0.28	0.07	20.0	0.35	4.63	3.1	9
0.59	0.35	0.33	0.14	29.8	0.36	5.00	3.6	10
0.09	0.09	0.09	0.06	40.0	0.09	1.10	1.2	11
0.09	0.07	0.07	0.07	50.0	0.08	1.02	1.2	12
...	...	0.80	0.07	8.0	0.40	7.90	7.3	13
...	0.11	0.11	0.06	35.3	0.17	1.20	1.1	14
...	...	0.16	0.09	36.0	0.15	1.20	1.2	15
0.06	0.10	0.10	0.07	41.2	0.16	1.95	1.7	16
0.17	0.16	0.15	0.06	28.6	0.21	2.00	1.6	17
0.36	0.34	0.34	0.13	27.7	0.27	5.20	3.7	18
1.52	2.14	2.09	0.52	19.9	1.28	18.50	14.9	19
3.22	3.46	3.39	0.63	15.7	1.61	57.33	47.8	20
2.57	2.32	2.22	0.39	14.9	1.02	32.40	25.3	21
0.51	0.42	0.40	0.11	21.6	0.35	3.92	3.7	22
0.48	0.28	0.26	0.15	36.6	0.28	3.20	2.9	23
0.40	0.19	0.18	0.09	33.3	0.27	3.10	2.8	24
0.26	0.21	0.20	0.10	33.3	0.10	1.40	1.3	25
1.00	0.97	0.94	0.14	13.0	1.08	15.64	10.9	26
1.92	1.46	1.40	0.22	13.6	0.88	26.00	15.8	27
1.70	1.60	1.54	0.21	12.0	1.24	32.60	18.6	28
0.71	0.60	0.57	0.21	26.9	0.78	9.70	8.0	29
0.20	0.19	0.18	0.08	30.8	0.26	3.60	3.3	30
0.51	0.30	0.26	0.09	25.7	0.32	3.30	3.3	31
0.32	0.31	0.30	0.18	37.5	0.25	5.30	4.1	32
...	...	0.11	0.08	42.1	0.16	2.30	1.9	33
...	...	0.48	0.22	31.2	0.38	6.58	5.6	34
...	...	0.57	0.30	35.0	0.78	8.89	8.6	35

[d] Storms analyzed by use of isohyetal maps; no correction factor used.

low moisture content and the fact that additional infiltration occurred out of overland flow or into the valley fills so that the excess computed rainfall did not all reach the gaging station during the main rise of the hydrograph.

TABLE 5.—CORRECTION FACTORS GROUPED BY SEASONS AND BY RUNOFF VOLUMES

RANGE OF FLOOD VOLUME (INCHES)		DECEMBER 1 TO MARCH 15		MARCH 15 TO DECEMBER 1	
From:	To:	No. of floods	Q/P_e	No. of floods	Q/P_e
2.0	4.0	2	1.10±0.05	3	0.84±0.04
1.0	2.0	None	......	4	0.87±0.06
0.5	1.0	1	1.00	3	0.87±0.09
0.5	4.0	3	1.07±0.06	10	0.86±0.06
0.25	0.5	2	0.94±0.02	8	0.70±0.16
0.25	4.0	5	1.03±0.06	18	0.79±0.12

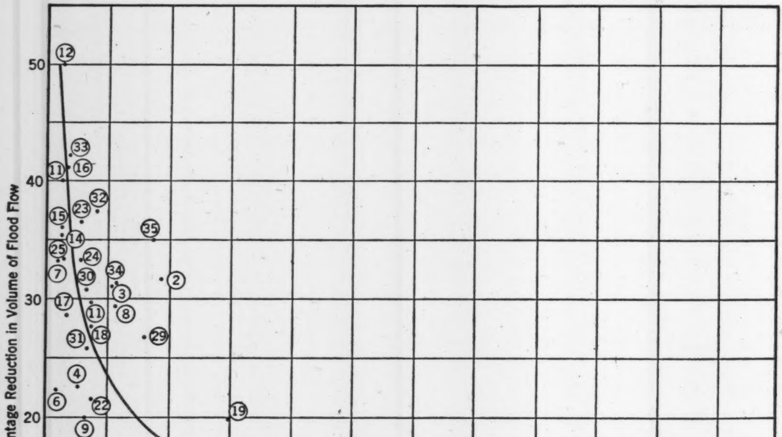


FIG. 6.—FLOOD REDUCTION CURVE, EAST FORK OF THE TRINITY RIVER NEAR ROCKWALL, TEX.

Referring to the infiltration-capacity curves in Fig. 3(b), it will be noted that they are similar in shape, that they are offset in time, and that they have a small but appreciable difference of infiltration-capacity values at the end of several hours. For the shorter storms, the difference in time position probably accounts for the major part of the difference in runoff—as, for example, between grass lands and "cultivated untreated" lands. For the long storms, the relatively small differences in the lower part of the curve account for material difference in total runoff.

The relation of flood reduction to the size of the storm is shown by Fig. 6. In this diagram no effort has been made to distinguish between storms of different seasons. Storm 13 was omitted because the precipitation was partly snow and sleet.

It is of speculative interest to determine from these data what the relative difference in runoff would be if the entire drainage basin had been uniformly in one type of vegetal cover. As an illustration, the values of excess rainfall or total runoff for one storm have been summarized separately for each class of cover. The results of this illustrative computation for the East Fork of Trinity River at Rockwall (storm of April 28, 1940) are as follows:

Discharge, in Cubic Feet per Second:	
Crest	9,850
Bank full	1,500
Rainfall-Runoff Depths, in Inches:	
Weighted rainfall	2.38
Measured runoff	0.78
Computed runoff, P_e	0.96
Correction factor to be applied to all computations (%)	81

The computed and corrected volumes of runoff are:

Present conditions	0.78
Department of Agriculture program	0.57
Entire Basin:	
Cultivated untreated	0.89
Present pasture	0.45
Managed pasture	0.16
Present type woods	0.12
Managed woods	0.08

Undoubtedly a similar computation for the larger floods would show somewhat less difference between runoff under the different types of cover. The foregoing is illustrative of the effect of cover on the retention of the water, for one single soil type.

TRANSLATION TO CREST RATE AND STAGE REDUCTION

The procedure described in Step 11 is outside of the scope of this paper since it is in the field of the channel phase of runoff and not of the land phase. It has been described merely to show the continuity of the technique and the relationship of the end-point results to the objective of the particular study. As a matter of interest the summary of the results of the applications under Step 11 are shown in Fig. 7 in terms of the reduction in crest-discharge rate at Rockwall for storms having the average recurrence intervals indicated as abscissas. Comparing Fig. 7 with Fig. 6 it will be noted that there are material differences in the position of the individual points, as would be expected from the differences in the shape of different hydrographs. The resulting mean curves, however, on the two figures are quite similar. For the larger storms the percentage reduction in crest rate is a little greater than the reduction in flood volume; whereas, for the smaller storms, the reverse is true.

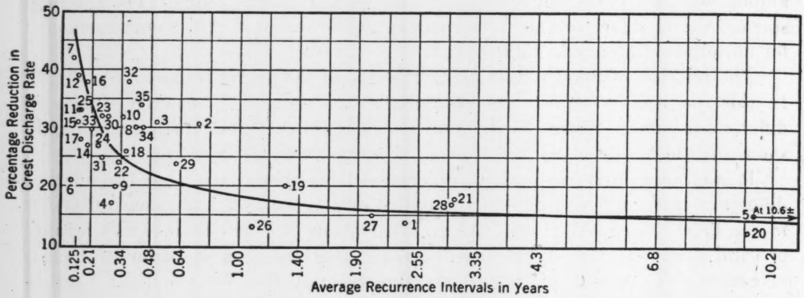


FIG. 7.—REDUCTION IN CREST-DISCHARGE RATE

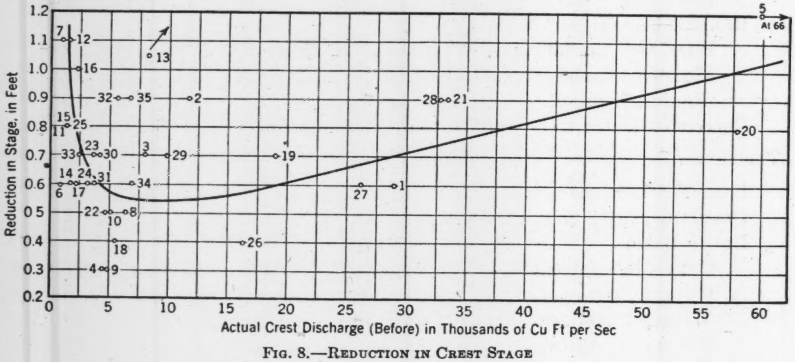


FIG. 8.—REDUCTION IN CREST STAGE

The application of the Rockwall rating curve to the "before" and "after" crest rates produces the values in crest stage reduction shown in Fig. 8. Similar determinations were made at other cross sections along the stream.

PART III.—RÔLE OF THE LAND DURING FLOOD PERIODS

GENERAL

All of the land controls have a common background and a characteristic related to the geology and the climate of the region, which has been well stated[6] by W. G. Hoyt, M. Am. Soc. C. E., and Walter Langbein, Assoc. M. Am. Soc. C. E.:

> "The ability of the soil and vegetation to serve as agencies for the disposal of potential flood-water is in a large sense a climatic factor. It depends

---

[6] "Some General Observations of Physiographic and Climatic Influences on Floods," by W. G. Hoyt and W. B. Langbein, *Transactions*, Am. Geophysical Union, Vol. 20 (1939), p. 166.

> not only upon the composition and gradation of the soil and the density and type of the vegetation, but also upon the degree to which their absorptive capacity has been utilized. All of these factors are largely reflections of their particular climatic environment experiencing seasonal changes in accordance with the established climatic regimen. Two principal factors that influence the soil and vegetal characteristics are those of precipitation and temperature. Thus it is observed that not only do precipitation and temperature affect the supply but also the ability of the soil to serve as a vehicle for its disposal."

It is necessary to recognize that "outstanding flood periods" are the result not only of great storms, but of the coincidence of great storms, and sometimes lesser storms, with the occurrence of either low infiltration capacity or small utilizable storage condition. In the Trinity basin of Texas (not in the semi-arid section) autumn storms of 7 in. often produce a fraction of an inch of runoff. In the southern Appalachian forest region it is reported that heavy storms, in July and August, of from 8 in. to 10 in. sometimes have not satisfied field moisture deficiency.

Studies restricted to "major flood periods" disregard the potential flood-producing storms that have not produced major floods. Only by studying the mechanics of storm runoff at all levels of flood magnitude separately by climatic provinces, and more definitely by soil and cover complexes, can the hydrologist begin to isolate the controlling factors and their quantitative ranges.

A system of bookkeeping is needed that will reveal the values of infiltration capacity and of utilizable soil storage capacity in their occurrence pattern over a considerable period of years. When these have been disclosed from studies of uniform areas, and sufficiently recorded by complexes, engineers can match them with storm precipitation of magnitude and pattern related to specific frequency, and thus use a sound hydrologic basis for the evaluation of storm runoff. In no other way can the effect of vegetal cover be recognized. The results will be helpful also in rationalizing statistical records of "flood" occurrence, many of which must reflect the nonuniform soil and cover conditions throughout their period.

Recent Investigations

During the ten years since 1933, basic precipitation and stream-flow data have been becoming available in a steadily increasing volume and in improving detail. This has inspired engineers and hydrologists to new efforts toward correlating cause and results, particularly in the flood phase of the hydrologic cycle. For example, the U. S. Geological Survey has presented an increasing number of Water Supply Papers of a descriptive and analytical character. Analyses are disclosing quantitative values of retention, in terms of subsurface storage capacity utilized, for particular basins of different general character. The quantitative values of storage capacity utilized, however, are mean values for complete drainage basins and rarely disclose the storage quantities that may be related to particular complexes. To permit the identification of the various controlling elements, particularly infiltration capacity and subsurface retention values with specific soil and cover complexes, they must be studied for areas where they are relatively uniform throughout. This may restrict the investigations to the smaller drainage basins, to the so-called small control watersheds and the small control plots, under either natural or artificial rainfall. For such areas, a wealth of fundamental data has been accumulated through the research work of the Soil Conservation Service and Forest Service of the U. S. Department of Agriculture, and some of the smaller scale investigations of the U. S. Geological Survey.

The new basic data which have been collected by the research agencies are becoming available rather slowly in compiled form. Some of them have been published. From them, essential secondary data reflecting the elements of control by the land are being derived, and satisfactory methodologies for this purpose have been described in departmental bulletins and in the *Proceedings* of the American Geophysical Union. A tremendous further program of analytical work will be needed before the data collected will all be in shape for widespread use in hydrologic engineering practice; and, although some of this work may be done under research programs in engineering schools, and some of it by engineering offices under pressure of specific need of information, the great bulk must be done by the federal bureaus—either those which have collected the basic data or those charged with programs to which the derived data are essential.

Although flood flow cannot be unscrambled from large basins, the engineer can reverse the process and use measured flood flow as a check on synthetic computations. When the values of infiltration capacity or of subsurface storage capacity are determined for the commoner soil and cover complexes, when these are applied to a series of actual precipitation patterns, and when the excess water is weighted in proportion to the distribution of the complexes in a basin, a synthetic value of retention by the land and vegetation, and of the complimentary runoff, can be obtained. If these computed values reproduce, significantly, measured flood flows over a sufficient range of storm and antecedent conditions, then both the unit values for the complexes and the propriety of the methodology used would appear to be sufficiently confirmed.

Thereafter, both the derived data as to controlling elements and the form of procedure may be re-used, for example, to evaluate:

(1) The changes in flood-flow characteristics which probably have occurred during the duration of a gaging station record on account of changes in land use and cover;

(2) The changes in flood flow that would occur under an expected future plan of land use and management; and

(3) The most probable flood-flow values for a basin for which no stream-flow records are available, either with regard to some specific storm or in the form of values for storms of various frequencies.

Lack of space precludes the use of illustrative material, other than the technique applied to the Trinity River; it may be hoped that other applications will become available in the course of the discussion. It is also to be hoped that at a later time the somewhat different effect of the land under snow and ice conditions will be presented in some detail.

ACKNOWLEDGMENT

The writer wishes to express his appreciation to the U. S. Department of Agriculture for permission to present certain heretofore unpublished material.

DISCUSSION

L. K. SHERMAN,[7] M. AM. SOC. C. E.—The subject of applied hydrology has been advanced appreciably by the publication of this paper. The writer has worked for several years along similar hydrologic lines relating to the measured effect of land, soil moisture, and vegetal cover on runoff and infiltration from natural rainfalls. His researches have been conducted independently of those made by Mr. Horner and on some basins differing materially in location and physical character from the basins described in the paper. The agreement between the two studies, as well as the lack of agreement, therefore, serves as a check or as a measure for the adoption of findings and procedures. The author's introduction gives a concise statement of the concepts and findings now generally approved in the science of hydrology.

Under "Infiltration Capacity" Mr. Horner defines the curve of infiltration capacity (sometimes called f-curve). This diagram is one of the most useful devices in the realm of hydrology. Rainfalls on the same basin or plot, occurring in the same month or season, will not give identical curves or parts of curves. However, these curves are generally in such agreement that they will furnish a composite f-curve which is sufficient for all practical purposes. A composite f-curve is more reliable than any curve from a single storm. The author assembles the curve at the point $f = 0.4$ to draw the average. The writer starts with a curve for a high value of f and successively moves the curve of next high f_o to the average point until the curve of lowest f_o is included. Tests made by the writer indicated that both methods furnish composites in close agreement. The important thing is to assemble all f-curves so as to maintain all of the highest f-values (f_o) at their original value on the f-scale. Vertical movement of position is not allowable. The writer finds that averaging f-curves by placing f_o at time zero, and then deriving average values at time intervals, is irrational and gives meaningless results. The same statement applies to the averaging of f_a-values.

Many infiltration-capacity curves have been derived from infiltrometers. The ultimate minimum values of f_c on "wet" runs agree well with f_c-values from natural rainfall. The agreement on the f-curves is not very good. In the final analysis, field comparisons must be based on natural rainfalls.

Time condensation is mentioned by Mr. Horner, but its importance in the derivation of infiltration-capacity curves from natural rainfalls deserves re-emphasis. By definition, during a short time interval, the infiltration-capacity curve gives the infiltration volume at the "capacity rate." Frequently, in nature, the rain intensity may fall below the capacity rate. The rain during a certain hour may be only 0.25 in. per hr, whereas the infiltration capacity f at that hour may be 0.75 in. per hr. To meet the f-curve requirement of capacity, this hour of 0.25-in. rain must be shown on the f-curve as 20 min. The latter is condensed time—symbol T.

---

[7] Cons. Engr., Chicago, Ill.

The procedures for deriving curves of infiltration capacity from natural rainfall include the following:

1. The Horner method, mentioned in the paper. Requires hydrographs of runoff closely synchronized with rain sequences or pattern. The method applies to plots and small basins.
2. The Sharp and Holtan method.[8] Requires hydrographs of runoff. Applies to small plots and large basins. First, the mass curve of infiltration is derived, and, from this, the f-curve is found.
3. The Sherman and Mayer method.[9] Requires the derivation of f_a (average infiltration capacity for the storm) and f_c. Requires only the total volume of surface runoff. The f-curve is found first, and the mass curve of infiltration is derived from it. Applies to small plots and large basins.

All methods require the sequence of rain intensities in time periods of one hour or less. All of the foregoing methods give f-curves that are in fair agreement.[10] (This applies to the Sharp and Holtan method after time condensation is applied. This was not done in their examples.[8])

The writer has compared Eq. 3 with the type of f-curves used in the Sherman and Mayer diagram. The term $\Delta F - f_c\,\Delta t$ represents the volume of infiltration under an f-curve and above the horizontal line of f_c. Eq. 3 therefore may be written:

$$\text{Volume above } f_c = V_f = \frac{f_{t1} - f_{t2}}{K} \dots\dots\dots\dots\dots (4a)$$

In the Sherman and Mayer type of f-curve the volume above f_c between f_1 and f_c is:

$$V_f = (f_1 - f_c - 0.01)C_b \dots\dots\dots\dots\dots\dots\dots (4b)$$

in which C_b is a constant. When Eq. 4b is applied to a segment of the f-curve between f_{t1} and f_{t2}, it becomes:

$$V_f = (f_{t1} - f_{t2})C_b \dots\dots\dots\dots\dots\dots\dots\dots (4c)$$

Equating Eqs. 4a and 4c, the author's ratio $\frac{1}{K}$ is equal to C_b introduced by Messrs. Sherman and Mayer.

The Sherman and Mayer type of f-curve originally was derived graphically to conform to the requirement that, at any point on an f-curve, the hypotenuse of a triangle with $f - f_c$ as altitude and C_b (constant base) as base on the f_c-line was tangent to the curve at that point. The type of f-curve to fit the given data of f_a and f_c is selected by the equation:

$$C_b = \frac{f_a - f_c}{f_a} \dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

---

[8] "The Analysis of Hydrographs of Control-Plots and Small Homogeneous Water-Sheds," by A. L. Sharp and H. N. Holtan, *Transactions*, Am. Geophysical Union, Pt. II, 1942, p. 578.

[9] "Application of the Infiltration Theory to Engineering Practice," by L. K. Sherman and L. C. Mayer, *ibid.*, Pt. III, 1941, p. 666.

[10] Paper by L. K. Sherman, presented at the meeting of the Section of Hydrology, Am. Geophysical Union, April, 1943.

There appears to be a definite relation between the f-curve arcs in the paper and those introduced by Messrs. Sherman and Mayer. The composite f-curve is generally used in practical applications.

The author's M-curve and his stress upon the importance of considering antecedent soil moisture are of prime importance. Runoff from rainfall is meaningless if this factor of initial soil moisture is not specified.

Attention is directed to Table 6, containing data on brown loam soil in the basin of the Little Tallahatchie River of northeastern Mississippi.[11]

TABLE 6.—RUNOFF, IN INCHES, AT INITIAL SOIL MOISTURE; GROWING SEASON IN THE BASIN OF THE LITTLE TALLAHATCHIE RIVER

Cover	Rain	Very dry	Medium	Very wet
Bare	2.5	0.5	1.2	2.0
	3.5	0.8	1.9	2.6
Cultivated corn cotton	2.5	0.2	1.1	1.8
	3.5	0.4	1.7	2.4
Native grass	2.5	0.0	0.4	1.0
	3.5	0.0	0.7	1.4

The M-curve, as shown by Mr. Horner, is probably the most accurately devised scale or index of initial soil moisture yet presented. It may be susceptible of simplification, especially for purposes of comparing runoffs from different cover complexes on the same basin. The writer has used the following procedure with satisfactory results: Derive a mass curve of infiltration from the composite f-curve of the basin for a particular cover, season, or month. The base or time scale is the same as the composite f-curve. The volume of infiltration scale, or ordinates, in inches, is at the left. Zero of this scale corresponds to the point of tangency of the f-curve at f_c. The scale shows the capacity for absorbing infiltration; hence it is expressed in terms of soil moisture deficiency. The deficiency is 0 at f_c when the soil is very wet. The deficiency is greatest at an f_o for which the soil is very dry. The range on the mass scale between very wet and very dry runs from about -2.0 in. for bare soil, 3.0 in. for grass, and -4.0 or -5.0 in. for old forest cover. Points above zero, on the f_c-line are marked $+1.0$, $+2.0$, etc.

"Medium" initial soil moisture is halfway between 0 or very wet and the maximum or very dry. "Wet" and "dry" are the quarter points. "Medium" is used generally for cover comparisons of runoff. It has a different initial soil moisture for different covers. This is as it should be. Different vegetal covers evaporate and transpire different amounts of soil moisture for identical antecedent meteorological conditions. This scale was used in the foregoing tabulations of initial soil moisture and runoff. When the observed runoff is known, the numerical figure for initial soil moisture can be determined. This, as the author states, is simply the reverse case of computing runoff. With either scale, correlation with antecedent meteorological conditions (used in forecasting runoff) has not yet been accomplished.

In line with the author's findings on the Trinity River basin of Texas, the writer presents the following for the Little Tallahatchie River basin of Mississippi:

---

[11] "Manual on Reduction of Runoff and Floods Through Agricultural Operations," by L. K. Sherman, 1942 (unpublished).

Description	Hudson Creek basin	Cane Creek basin
Drainage area, in miles	9.35	23.8
Soil	Brown loam	Pontotoc and Flatwoods
Initial soil moisture	Medium+14% toward dry	Medium
Date of observation	June 17, 1939	June 17, 1939
Basin f_c	0.17	0.11
Rainfall, in inches	1.34	3.11
Observed runoff, in inches	0.285	1.81
Computed runoff, in inches	0.26	1.86
Correction factor	1.09	0.97

A comparison of conditions, before and after certain assumed cover changes and gully controls, is as follows:

Description	Hudson Creek basin	Cane Creek basin
Medium Initial Soil Moisture Conditions:		
Rainfall, 2.70 In.—		
Runoff before assumed changes	1.09	1.46
Runoff after assumed changes	0.53	1.16
Rainfall, 4.35 In.—		
Runoff before assumed changes	2.40	2.52
Runoff after assumed changes	1.48	2.14

Some of the complexes used, like increased crop acreage, may increase the "after" runoff. The benefits under some plan designed by the U. S. Department of Agriculture may exceed this showing materially.

RAPHAEL G. KAZMANN,[12] JUN. AM. SOC. C. E.—The words "permeability" and "porosity" are sometimes used interchangeably by engineers. As this is incorrect, it may be of general interest to differentiate between these concepts and illustrate the difference by concrete examples. The porosity of a material is no indication of its ability to transmit water through its interstices. The ability of a material to function as a "conduit" for the transmission of water generally is known as the material's permeability.

As used by the Geological Survey, U. S. Department of the Interior, the coefficient of permeability of a water-bearing material is defined as the rate of flow of water, in gallons per day, through a cross-sectional area of 1 sq ft under a hydraulic gradient of 100% at a temperature of 60° F.

The porosities of water-bearing materials bear little relationship to their permeabilities. This is illustrated by Table 7, adapted from *Water-Supply Papers Nos. 596-F* and *887*[13,14] of the Geological Survey. Table 7 demonstrates that the water-transmitting properties of materials generally are not related directly to their porosities. The materials listed in Table 7 were obtained in various sections of the country and at various depths below the land surface. The coefficients of permeability were determined in the laboratory by passing water through samples of the material by means of techniques which are described in detail in *Water-Supply Paper No. 887*.[15]

---

[12] Asst. Hydr. Engr., Ground Water Div., U. S. Geological Survey, Washington, D. C.

[13] "Laboratory Tests on Physical Properties of Water-Bearing Materials," by N. D. Stearns, *Water-Supply Paper No. 596-F*, U. S. Geological Survey, 1928, pp. 164–169.

[14] "Methods for Determining Permeability of Water-Bearing Materials," by L. K. Wenzel, *Water-Supply Paper No. 887*, U. S. Geological Survey, 1942, pp. 13–14.

Mr. Horner's paper would have been strengthened by including a treatment of the movement of ground water. In dealing with the problem of subsurface retention and detention, there is no reference to the rise and fall of the water table during or following any of the storms; nor does any attempt appear to have been made to obtain data of this sort. Furthermore, a study of the trends and magnitudes of ground-water fluctuations would have corroborated or disproved a fundamental assumption upon which the paper is based—that the infiltration capacity of the Trinity River basin is small to moderate. Inasmuch as this is a qualitatively determinable factor, a generalization based on observations of water level should have been substituted for this assumption. In the absence of such observations it would appear impossible to evaluate the technique described in this paper.

TABLE 7.—POROSITIES AND PERMEABILITIES OF WATER-BEARING MATERIALS

Description of material (1)	Porosity (%) (2)	Coefficient of permeability (3)
Terrace gravel	23.6	1,230
Gravel mixed with some coarse sand	24.1	4,500
Gravel	25.1	20,000
Alluvium gravel (gravel and silt)	29.3	70
Sand and gravel	29.7	1,820
Sand	35.8	2,600
Fine gravel, coarse sand mixed with clay balls	37.0	30
Glacial gravel	38.0	90,000
Washed gravel	38.5	10,500
Red clay	42.4	20
Sand	44.4	1,100
Sand	44.8	560
Silty sand	45.1	6
Sand	45.8	410
Loess	53 2	7
Subsoil	54.2	20
Subsoil	55.5	0.2

W. G. HOYT,[16] M. AM. SOC. C. E., AND W. B. LANGBEIN,[17] ASSOC. M. AM. SOC. C. E.—With the passage of the Flood Control Act of June 22, 1936, an obligation was imposed on the engineering and associated professions to determine the magnitude of reduction in flood stages and discharges that could be effected through changes in land use. The author describes a technique for evaluating the rôle of land during flood periods with no snow or ice, and limited to areas where (see heading, "Part I.—Introduction: Engineering Technique") "Infiltration capacity is small to moderate * * *, and stream-channel inflow occurs predominantly as surface runoff," a situation which the author indicated "applied quite widely to cultivated and grass lands."

Through the use of infiltration-capacity curves for small experimental catchment areas, homogeneous in soil, cover, and cultural treatment, surface runoff volumes as the result of changes in land use are determined, and finally the runoff volumes are translated into river stages and discharges. To the extent that flood rises caused by rain are composed of surface runoff only and to the extent that "correct seasonal average infiltration curves" can be derived and placed "in their proper position on the rainfall patterns," the method enables one to evaluate a reduction in surface runoff through changes in land use. Such quantitative determinations are necessary if the feasibility of flood control through treatment of land is to be evaluated.

---

[15] "Methods for Determining Permeability of Water-Bearing Materials," by L. K. Wenzel, *Water-Supply Paper No. 887*, U. S. Geological Survey, 1942, pp. 50–71.

[16] Hydr. Engr. (Prin.), Water Resources Branch, U. S. Geological Survey, Washington, D. C.

[17] Associate Hydr. Engr., U. S. Geological Survey, Washington, D. C.

Flood control operations of any nature involve the expenditure of millions of dollars, and it is essential that the degree of control to be effected should be well demonstrated. The hydraulics of flood control through the use of engineering structures can be evaluated by verifiable and comparatively well-founded technical processes. The evaluation of the hydraulics of land use during flood periods is accomplished by techniques that are quite complex and speculative. This is true because of (1) a lack of basic knowledge concerning the land phase of runoff, (2) wide variations in infiltration capacity and other hydrologic conditions within a drainage basin, and (3) a lack of demonstrated beneficial change in flood-runoff régime of any large stream because of a change in land use.

The author attempts to supply the first deficiency and is to be congratulated for breaking down[18] the process by which rainfall is translated into runoff, for outlining the manner in which meager observational data may be used to give quantitative expression to the factors involved, for computing the runoff for a typical drainage basin in Texas in accordance with the concepts outlined, and for comparing it storm by storm with the measured volumes and rates. This comparison between computed and observed results may be necessary, but is it sufficient to prove the correctness of the postulates? The derived relations may furnish good empirical working tools, but the writers suggest that other postulates may evolve equally good checks.

The author presents curves of infiltration capacity for soils typical of the Texas Blacklands that show marked differences in position, and, therefore, for a given rainfall pattern and antecedent conditions, marked differences in surface runoff for different kinds of land use These curves are based on measurements of runoff on small homogeneous catchment areas at the Waco watershed and the Garland Runoff Studies Project. There is no reason to believe that these curves do not reflect the qualitative effects of cover on the surface runoff of the small homogeneous areas for which they were derived. There is nothing in the report, however, to demonstrate that a river such as the Trinity River would respond to changes in land use in the same way as the small homogeneous plots. Thus, although the author has devised a technique which will show quantitative difference in surface runoff (the immediate task before him), there is still lacking a process for evaluating the effect of land-use change on flood stages and flows which is not open to considerable doubt.

Some may feel, as do the writers, that the author has placed too much dependence on the concept of "infiltration capacity," arrived at, as it is, indirectly through the measurement of surface runoff on small experimental areas in a procedure designed to simulate flood rises from large natural drainage basins under assumed combinations of land use.

---

[18] "See also "Hydrology," Physics of the Earth—IX, edited by Oscar E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 507–513.

In so far as the engineering technique is concerned and as the author quite clearly implies, an expanded title for the paper might well read, "Rôle of the Land During Flood Periods in Those Areas Where Infiltration Capacity Is Low to Moderate, Where Infiltrated Water Does Not Appear in the Stream as 'Quick-Subsurface Return Flow,' and Only During Periods of No Snow, Ice, or Frost." These restrictions in the use of the technique impose quite serious limits on described procedures from the viewpoint of practical application to general flood problems. They do not necessarily detract from the author's conclusion with respect to the East Fork of the Trinity River provided that its basin characteristics can be definitely determined to be such that the foregoing limitations apply. Apparently, at one time, conditions in the Trinity River basin were such that the author's technique would not apply. It is stated in the final report on the Trinity River watershed:[19]

> "There is evidence that floods occurred on the Trinity before settlement but they were vastly different from the present floods. Water rose and receded so slowly that many streams which are now intermittent then flowed all year."

This description implies an area with large infiltration capacity, much groundwater storage, little surface runoff, and a slow release of infiltrated water to the stream as sustained flow—conditions that in the main are seemingly inherent to a basin and not modified so materially by human occupation.

The writers know of no simple method to determine, on an areal basis, whether subsurface inflow constitutes less than 10% of flood volume. Although the author may be correct in his assumption that 90% of the present flood volume in the East Fork of the Trinity River is surface runoff, no basic data have been submitted to prove it. The total runoff (surface) at the Rockwall gaging station for the thirty-five storms, May 3, 1935, to July 3, 1940, as reported in Table 4 is 31.29 in. The total flow past the Rockwall gaging station for the same period is about 40 in. In other words, 78% of the total runoff that appeared at Rockwall is assumed by the author to be surface runoff. The difference between these two amounts (8.7 in.), except for a few flood rises not included and which approximate 0.70 in., is presumed to represent all the infiltrated water that reached the stream channel at Rockwall.

Based on an extensive knowledge of hydrologic conditions at the North Appalachian Experimental Watershed near Coshocton, Ohio, a research project of the Hydrologic Division, Office of Research, Soil Conservation Service, U. S. Department of Agriculture, H. S. Riesbol, Assoc. M. Am. Soc. C. E., concluded[20] substantially as follows: Extensive study of ground-water levels, percolation rates, hydrograph recessions, and normal ground-water depletion curves should lead to dependable and practical knowledge concerning when and where infiltrated water joins channel flow as seepage and spring flow from perched water tables, temporary zones of saturation, or from deep seepage. No such basic information is submitted for the Trinity River basin.

---

[19] *H. R. Doc. No. 708*, 77th Cong., 2d Session (letter from the Secretary of Agriculture transmitting a report of a survey of the Trinity River watershed based on an investigation authorized by the Flood Control Act of June 22, 1936).

[20] "Some Aspects of Subsurface Water in Hydrologic Research Agricultural Watersheds," by H. S. Riesbol, *Proceedings* of the Hydrologic Conference, Pennsylvania State College, State College, June 30 to July 2, 1931, p. 107.

The writers do not question the accuracy of the various infiltration-capacity curves shown in Fig. 5 in which infiltration capacity is shown to increase progressively from untreated farm land to well-managed forests. Through the use of such a family of curves under any given rainfall pattern, computed reductions in surface runoff as a result of betterment in cover or land use are obtained. Whether or not these computed reductions in surface runoff can be assumed to be equivalent to reduction in the flood runoff of the stream draining the particular area, however, seems open to question. The writers believe the author fully understands this situation, but he fails to delineate or give an expression to the area where his technique would apply except the broad statement (see heading, "Part I.—Introduction: Engineering Technique") that the "situation applies quite widely to cultivated and grass lands." It would seem quite the contrary with respect to much of the cultivated land in the United States since the agriculturist would tend to select, among other factors, soils that combine high infiltration capacities with satisfactory moisture-retention properties. The real criterion underlying the author's technique is not so much whether the cover is cultivated or in grass but rather, the subsurface storage characteristics. Will the subsurface storage spaces retain the infiltrated water, or are they of such limited extent that infiltrated water passes into stream channels where it forms a part of the direct flood runoff of all stream rises?

The author suggests that the ratio between observed surface runoff and computed rainfall excess is a test of the validity of his procedure. It is stated that for five storms the rainfall data were inadequate and that measured runoff was used as a guide in the determination of rainfall excess. Examination of the author's unpublished reports on his study as a consultant to the Department of Agriculture indicates that the computation of rainfall excess in the other thirty storms was influenced to a considerable extent by knowledge of the actual volume of runoff as measured at the gaging station maintained by the U. S. Geological Survey at Rockwall. Some of these influences included reinterpretations of the rainfall patterns, adjustments in the position of infiltration-capacity curves, and deviation from the seasonal *M*-curve values. The correction factor (Col. 6, Table 4) therefore reflects the errors that remain after all readily explainable adjustments have been made. It is obvious that wide departures from observed runoff have been adjusted in order to arrive at a result more closely in conformance with observed runoff. The test is not the same as would be met by a forecaster who has no knowledge of the runoff to be produced by the storm other than that provided by his working formulas. If partial advantage is taken of the runoff as it was, why not take full advantage? The technique might not be so different. If the author had wished, he might have continued his adjustments until conformance of computed and observed runoff was obtained. Perhaps a method varying the *M*-curve or other factor to this end can be devised. Whether or not the author's procedure can be considered practicable for ordinary engineering practice will depend on the man-hours involved in its application. Some idea as to the number of man-hours required to simulate the thirty-five flood peaks would be enlightening.

An examination of Fig. 5 and a study of the method by which the family of infiltration-capacity curves was placed on the rainfall graphs suggests that the results are highly sensitive to rainfall intensity. Hourly rainfall amounts were estimated from data so scant as to be almost nonexistent; but, assuming that the hourly pattern shown in Fig. 5 is correct, minute-to-minute variations in intensity necessarily have been neglected. For example, consider the hour interval, 1 p.m. to 2 p.m., during which there was 1.56 in. of rainfall: Suppose 0.20 in. of this amount fell during the first 20 min, 0.52 in. during the middle 20 min, and 0.84 in. during the last 20 min; then the amount of mass infiltration that occurred prior to rainfall reaching the infiltration capacity would be increased by 0.20 in., and the initial infiltration capacity on curve (1), Fig. 5, would be 0.80 in. per hour instead of 1.09. The net result would be that the rainfall excess during this hour would be 1.03 in. for cultivated untreated land and 0.91 in. for strip-cropped, both important land classes. The author's values are 1.09 in. and 0.92 in., respectively. It is significant that, by assuming a different rainfall distribution, the computed reduction in runoff is less. The author's difference between the two values is 0.17 in.; that computed for a possible distribution of rainfall assumed by the writers is 0.12 in. Seemingly, the effect of any error that would advance the period of intense rainfall would increase the computed difference between "before" and "after"; an error that would retard the period of intense rainfall would diminish the effect.

The report explains the influence of antecedent conditions as they affect soil moisture on the infiltration capacity during a storm. Increasing the volume of infiltration prior to a storm decreases the capacity of the basin to retain water. Therefore, under the changed conditions, to what extent would the presumed increase in infiltration operate adversely to lower infiltration capacities during subsequent storms? Moreover, are there not economic implications of the reduction in flood runoff, particularly with respect to the rights of downstream riparian owners or appropriators? In this particular case, using the values given in Col. 13, Table 4, the 20.4% reduction in total surface runoff means a reduction of 16% in total stream flow.

The annual runoff in inches at the Rockwall gaging station is greater than that at other stations in the upper Trinity River basin.[21] It is much greater than at any of the basins at the Waco Experiment Station.[22] Does this difference in annual runoff indicate difference in land types or land use or does it conform to differences in rainfall?

Query is raised as to the use of the term "regression curves." Apparently this term as used refers to the die-away curves of infiltration capacity shown in Fig. 5. Lines or curves of "regression" are statistical terms devised by Sir Francis Galton[23] to describe lines or curves of best fit between a single independent variable and one or more dependent variables. In this sense the die-away curves of infiltration capacity may be regression curves; but, then, so are all the other empirical lines or curves shown. A more descriptive term should be devised.

---

[21] "Surface Water-Supply of the United States," *Annual Reports*, U. S. Geological Survey, 1924–1942.

[22] "Hydrologic Data, Blacklands Experimental Watershed, Waco, Tex.," *Hydrologic Bulletin No. 8*, U. S. D. A., SCS, 1942.

[23] "An Introduction to the Theory of Statistics," by G. Udny Yule, C. Griffin and Co., Ltd., London, 1924, p. 176.

The *M*-curve shows that the spring season overlaps the adjoining winter and summer seasons. This is understandable, but what procedures are used to determine which seasonal curve shall govern in a particular condition?

CHARLIE M. MOORE,[24] JUN. AM. SOC. C. E.—The actual application of a method for determining the measured effect of land, soil moisture, and vegetal cover on runoff and infiltration from natural rainfall is presented in this enlightening and interesting paper.

In discussing Step 1 (heading, "Methodology") the author has applied rainfall data collected by the writer at Lindale and Mount Pleasant, Tex., during the 5-yr period from 1935 to 1940. At each of these locations the federal government installed three Ferguson recording gages, and five standard gages within demonstration project areas of some 23,000 acres. The recording gages were supervised by junior engineers, whereas it was necessary to rely upon cooperating farmers to measure and record the precipitation collected by the standard gages. A diligent effort was made to keep the recording gages in good operating condition and to obtain records as accurate as possible for all gages. Cooperative observers were selected carefully in almost every instance before installing the gages. However, some of the original observers moved or became disinterested after a period of 2 yr or 3 yr. Where they were interested it was not difficult to obtain a satisfactory record that could be depended upon. In the monthly collection of data from cooperative observers, it was often necessary to redate the reports and to compare the recorded amounts of the rains with the nearest recording gages. Quite often several hours would elapse before the observer would remember to measure the rain. At times, after small rains, the precipitation would be evaporated before measurements were made. This fact, no doubt, made it difficult to analyze the data at a later date.

As illustrated by the author, the application of the method to drainage basins as large as 831 sq miles gives some valuable information to engineers interested in the determination of maximum runoff to be expected from watersheds, and the reduction that can be expected from the application of the Department of Agriculture program, as related to erosion control, land use, and management.

The engineers of the Soil Conservation Service, in Region Four, of necessity use the rational method in determining the runoff to be expected for a given frequency of rainfall. This method, which is based on the formula—

$$Q = C\,I\,A \qquad (6)$$

—is used for comparatively small agricultural areas and is limited to a maximum of approximately 1,000 acres. It is realized by engineers that some such method as the author has given is needed to evaluate properly the rôle of the land and the part that it plays in reducing the runoff where proper land use and management are employed. Also, a method similar to that of the paper, which can be applied to larger watersheds, is needed.

In the application of Eq. 6, a constant runoff factor is generally chosen and assigned to certain land-use groups within a soil group. However, it can be seen that such a procedure does not take into account the effect of previous rainfall, type of cover, condition of cover, degree of erosion, or variations in soils within a soil group. The method presented by the author will evaluate most of these factors and their effect upon the runoff.

---

[24] District Engr., SCS, Crockett, Tex.

It is noted in Step 3 that the problem of deriving infiltration-capacity curves for the Blackland soil and various classes of vegetal cover was simplified in that, to all intents and purposes, only one soil type occurred in the basin, and in that the variations in infiltration capacity were primarily those growing out of the difference in vegetal cover and of antecedent conditions. This problem would be very acute in the sandy lands of eastern Texas, and other areas, where some sixty to seventy soil types occur, ranging from deep sands to very heavy, almost impervious, clay soils. Many of these soils occur within very small drainage basins in the eastern Texas area.

The writer agrees with the statement that a system of bookkeeping is needed that will reveal the values of infiltration capacity and of utilizable soil-storage capacity in their occurrence pattern over a considerable period of years. This will help to form a sound hydrologic basis for the evaluation of storm runoff.

In considering the rôle of the land and its effect upon runoff, the writer has found that, by treating drainage basins properly with sound conservation practices, such as terracing, contour cultivation, contour ridges, retiring steep slopes to grass and woodland, controlling gullies with dams, and sloping and sodding gullies, considerable expense can be saved in the capacity of bridges, culverts, spillways, etc., in the lower reaches of the watersheds. These practices help to retain the water on the land for a period sufficient to cause most of it to be absorbed. By proper land use and management in a drainage basin, the reduction of runoff, as illustrated by the author, can well be expected. The mere changing of a system of cultivated rows by a farmer can cause almost total runoff as compared to a very small amount of runoff if the rows are laid off on the contour. The writer has observed that flood flows on drainage basins ranging in size from 100 to 1,000 acres or more—on which the Department of Agriculture program had been applied almost 100%—were reduced considerably. On one of these basins it had been impossible for the county government to construct semipermanent bridges across the main channel without having them overflowed or washed out. County officials have been troubled very little with this problem since the watershed has been treated.

The writer is anticipating the time when sufficient information is compiled and made available to engineers for use in the application of this method to various sizes of watersheds or drainage basins. The author has advanced this method appreciably by the publication of this paper.

WALDO E. SMITH,[25] M. AM. SOC. C. E.—Many questions that hitherto have been raised concerning the infiltration approach have been answered in this paper, the most notable of which is perhaps the placing of the infiltration curve in proper time position with reference to the rainfall. To the writer's knowledge, this is the most definite work of this sort yet published. The setting down of the step by step procedure clarifies still other questions. A related question that the author himself inspired in a previous paper[26] that yet remains unanswered is the validity of the use of average hourly intensities in the face of the great normal variations in intensity from the hourly average. About one half of the rain within the hour normally falls at two times or more the average hourly intensity, and about one fourth falls at three times or more the average hourly intensity. Thus, the actual pattern of the rainfall that is shown only in terms of hourly averages in Fig. 5 would normally result in decidedly different amounts of rainfall excess. The writer feels, however, that the development of the infiltration technique has been advanced considerably in this paper. An attempt should be made to test this technique as soon as possible, utilizing better basic data.

---

[25] Head, Section of Hydrology, Div. of Water Conservation & Disposal Practices, Soil Conservation Service, Washington, D. C.

It is unfortunate that the data for the testing of this method and for this extensive work using the infiltration approach were so inadequate. As the author states, only daily records of precipitation, some of doubtful validity, were available on the watershed for the storms studied. There was no recording rain gage on the area and, for that matter, none was very close to the area until the Garland gage was established in the latter part of the period. Streamflow records consisted generally of twice daily visual observations which, on the watershed of 831 sq.miles, do not define the hydrograph very well. The writer would like to see the author's technique tried with more adequate data, such

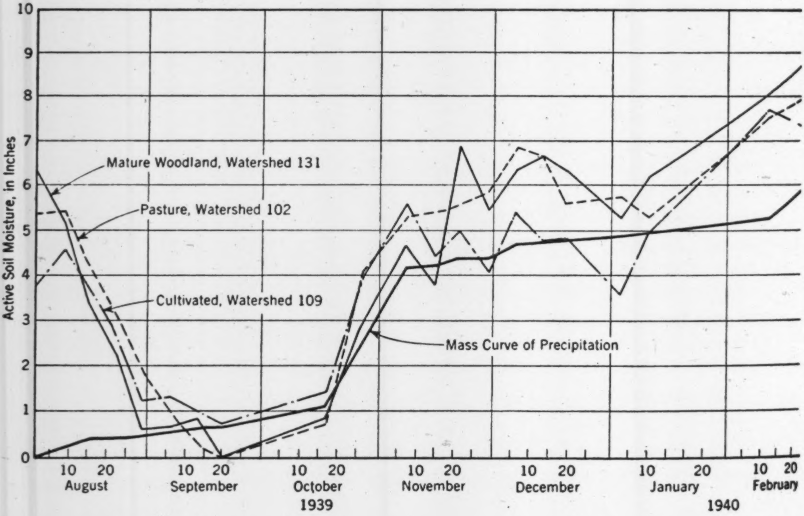
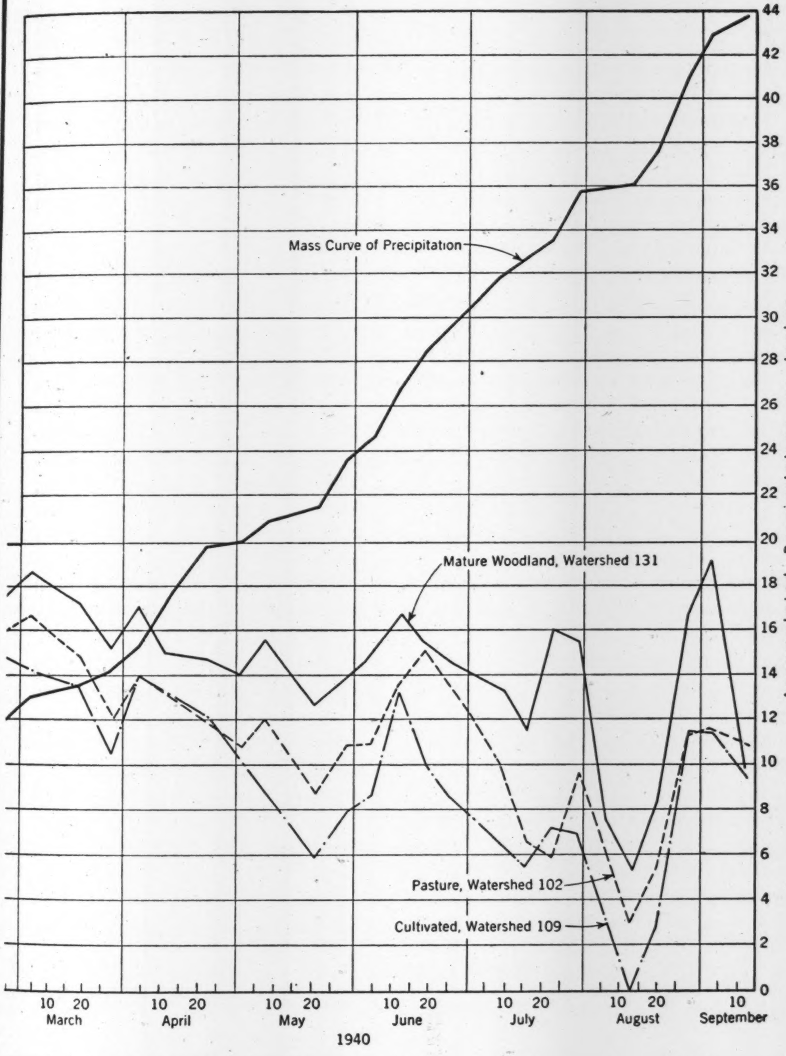


FIG. 9.—CHARACTERISTICS OF "ACTIVE SOIL

---

[26] "Surface Runoff Determinations from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1041.



Water" in the Upper 40 In. of Soil Mantle

as those that are available concerning the basin of the Licking River above Zanesville, Ohio. On this basin there are quite a number of recording rain and stream gages with records of 3 yr or 4 yr or longer. These data would afford a much more satisfactory test of the adequacy of this method. Attempting to establish refinements in records, such as hourly values from already uncertain daily amounts of rainfall and such as the sketching in of the gaps between the twice daily stream gaging observations, seems questionable indeed for the uses of the author's analysis.

Inasmuch as the derivation of infiltration curves is by no means standardized and, furthermore, is largely an unknown technique to many hydraulic engineers, a repetition of the citations on the procedure for their derivation presented in the earlier paper,[26] and the addition of more recent ones, is in order. A list, in so far as the writer has it at hand, is appended to this discussion.

The author may have a file of infiltration curves that he has derived for various soil-cover complexes for various seasons, but as yet such curves are not generally available. Derivation is laborious and within the writer's experience there is frequently little uniformity in product as derived by the different methods for the same single watershed complex and storm.

With regard to the author's discussion of outstanding flood periods in Part III of this paper and to his statement that floods are usually caused by the coincidence of storms and low infiltration rather than by either of these things alone, a paper by F. R. Dreibelbis and F. A. Post [27] should be called to the attention of hydraulic engineers. Messrs. Dreibelbis and Post showed that the "active soil water" in the upper 40 in. of the soil mantle that they studied varies as much as 9.39 in. in depth of water in a year. Three curves of the "active soil water" prepared from their data [28] are shown in Fig. 9. On this illustration also is given the mass curve of precipitation. Volume-weight soil samples were taken at intervals of a week to a month on the dates generally shown by the break points in the curves, and the actual quantity of water in each sample was determined. That water greater than the minimum for the year was defined as "active soil water." There was still considerable moisture in the soil at the time of the minimum value. which, in the case of watershed 109 (cultivated), was 6.28 in.[29] of depth. The "active soil water" in the graphs shown is probably considerably more than the subsurface retention (capillary and pellicular water) in the 40-in. layer, and includes subsurface detention (gravitational water). The soil probably approached saturation at the wettest time of the year. Infiltration values are low at such times in these soils and those of the Texas Blacklands. This method may possibly be used as a quantitative indicator of low infiltration rates and approach of the soil to saturation and thus of incipient flood conditions. More detailed information on the vertical distribution of the water in the 40-in. layer might prove helpful.

It is interesting to note that in general the mass curve shows light precipitation during periods of decreasing active soil moisture, and heavy precipitation during periods of increasing active soil moisture. This is a logical deduction supported by the data. Furthermore, due to high evapo-transpiration in the summer and due, perhaps in part, to the usual nature of many summer storms (high intensity and short duration), much more precipitation is necessary to support a constant value of active soil moisture in summer than in winter. Still a further fact is worthy of note—namely, that the greatest depth of active soil moisture occurred on woodland (9.39 in.), next on pasture (8.36 in.), and lowest on cultivated land (7.82 in.). (In the paper cited,[29] this value is given as 7.09 in., but the period used ended August 7, 1940. Extending the period to September 12 gave a new low value for the cultivated watershed, increasing the range between lowest and highest values of soil moisture to 7.82 in.) Data on retention and detention will probably prove to be very useful in analyses made using the infiltration approach.

---

[27] "An Inventory of Soil-Water Relationships on Woodland, Pasture, and Cultivated Soils," by F. R. Dreibelbis and F. A. Post, *Proceedings*, Soil Science Soc. of America, Vol. 6, 1941, p. 462.

[28] *Ibid.*, Table 1.

[29] For information concerning the watersheds involved, and for much of the data used by Messrs. Dreibelbis and Post, see "Hydrologic Data," *Hydrologic Bulletin No. 1*, North Appalachian Experimental Watershed, Coshocton, Ohio, 1939, and "Hydrologic Data," *Hydrologic Bulletin No. 4*, North Appalachian Experimental Watershed, Coshocton, Ohio, 1940.

It is further interesting to note that the active soil moisture fluctuations for the 13-month period conform to the trend of precipitation-evaporation ratios in so far as evaporation data are available for their determination. These ratios, the usefulness of which were established by C. W. Thornthwaite,[30] are simply the monthly precipitation divided by corresponding monthly evaporation. They may also be derived by empirical means from the data on temperature and precipitation. This was not done. The ratios for the available data might have been added to Fig. 9 but were omitted to avoid confusion. For the months for which they can be found they are given below:

Month	Precipitation-evaporation ratio
1939	
August	0.20
September	0.18
October	0.76
1940	
April	1.34
May	0.71
June	1.34
July	0.57
August	1.44
September	0.42

The ambiguity of the term "infiltration capacity" is indicated in the closing paragraph of the "Synopsis" of the paper in which "infiltration capacity," a rate, is compared with "detention capacity," a volume. As a substitue for the confusing term "infiltration capacity," the writer proposes the term "potential infiltration rate" as being more descriptive of this phenomenon. Furthermore, in the place of "infiltration regression curve," the simple term "infiltration curve" might well be used; or, if one must be more explanatory, the term "the curve of potential infiltration" could be used.

Bibliography of Procedure for Deriving Infiltration Curves.—

(1) **"Analysis of Runoff Plat Experiments with Varying Infiltration Capacity,"** by R. E. Horton, *Transactions*, Am. Geophysical Union, Pt. IV, 1939, p. 693.

---

[30] "The Climates of North America According to a New Classification," by C. W. Thornthwaite, *Geographical Review*, Vol. XXI, No. 4, October, 1931, p. 633.

(2) **"Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois,"** by W. W. Horner and C. L. Lloyd, *Transactions*, Am. Geophysical Union, Pt. II, 1940, p. 522.

(3) **"A Graphical Method of Analysis of Sprinkled Plot Hydrographs,"** by A. L. Sharp and H. N. Holtan, *ibid.*, p. 558.

(4) **"Sprinkled Plot-Infiltration and Runoff Experiments on Arizona Desert Soils,"** by E. L. Beutner, R. R. Gaebe, and R. E. Horton, *ibid.*, p. 550.

(5) **"Derivation of Infiltration Capacity from Average Loss Rates,"** by L. K. Sherman, *ibid.*, Pt. I, p. 541.

(6) **"Analysis of Hydrologic Data for Small Watersheds,"** by W. W. Horner, *TP–30*, SCS, Washington, D. C., February, 1940.

(7) **"Infiltration Capacities of Soils in the Macoupin Creek Basin in Illinois,"** by L. K. Sherman, release of the Illinois State Planning Commission, Chicago, Ill., 1940.

(8) **"Determination of Areal Average Infiltration Capacity from Rainfall and Runoff Data,"** by R. E. Horton and Richard Van Vliet, Office of Land Use Coordination, U.S.D.A., 1940.

(9) **"An Approach Toward a Physical Interpretation of Infiltration Capacity,"** by R. E. Horton, *Proceedings* of the Soil Science Soc. of America, Vol. 5, 1940, p. 399.

(10) **"Application of the Infiltration Theory to Engineering Practice,"** by L. K. Sherman and L. C. Mayer, *Transactions*, Am. Geophysical Union, Pt. III, 1941, p. 666.

(11) **"Derivation of Infiltration-Capacity Curve from Infiltrometer Experiments,"** by R. E. Horton, mimeograph release of the Office of Land Use Coordination, U.S.D.A., Washington, D. C., January, 1942, 18 pp.

(12) **"Extension of Graphic Methods of Analysis of Sprinkled Plot Hydrographs to the Analysis of Hydrographs of Control Plots and Small Homogeneous Watersheds,"** by A. L. Sharp and H. N. Holtan, *Transactions*, Am. Geophysical Union, Pt. II, 1942, p. 578.

(13) **"Simplified Method of Determining an Infiltration-Capacity Curve from an Infiltrometer Experiment,"** by R. E. Horton, *ibid.*, p. 570.

(14) **"A Simplified Method of Determining the Constants in the Infiltration-Capacity Equation,"** by R. E. Horton, *ibid.*, p. 575.

P. B. Rowe,[31] Esq.—The hydrologic literature concerned with evaluating land effects on stream flow is measurably enhanced by this valuable and stimulating contribution. An approach developed by the California Forest and Range Experiment Station,[32] similar to that presented by Mr. Horner, has been used in the hydrologic analyses of some of the U. S. Department of Agriculture Flood Control Surveys in California. Because of variations in procedures used and in the watershed conditions encountered, a comparison of the two methods and of the results is of interest.

---

[31] Silviculturist, Div. of Forest Influences, California Forest and Range Experiment Station, maintained by the U. S. Dept. of Agriculture, Forest Service, in cooperation with the Univ. of California, Berkeley.

[32] "A Method of Hydrologic Analysis in Watershed Management," by P. B. Rowe, Twenty-Fourth Annual Meeting, Am. Geophysical Union, April, 1943 (in process of publication).

In the Horner method an average (f_a) infiltration-capacity curve of each infiltration "complex" was derived from the rainfall and runoff records of small watersheds. In the California approach, infiltration-capacity curves of the complexes were obtained from small plot (infiltrometer) measurements. These permitted—

(*a*) Reliable determinations, not only of the relative differences of infiltration capacities of the different complexes but also of the range of infiltration capacities within complexes; and

(*b*) Computation of surface runoff from individual complexes by use of the individual f-curves or of a series of f_a-curves representing the full range of the infiltration capacities of the complex. The use of such infiltration-capacity curves has proved important in making possible the calculation of surface runoff (often of considerable volume in large complexes) from rainfall at rates below the average infiltration capacity of the complex. Before being used in computing surface runoff the infiltration-capacity curves were adjusted to measured rainfall and runoff of representative sub-drainages comprising single or similar complexes, and the adjusted curves, in turn, were checked against measured rainfall and runoff relations of the watershed as a whole.

In Mr. Horner's analysis, stream-channel inflow was visualized predominantly as surface runoff. After deductions were made for such factors as interception and depression storage, this assumption permitted the direct computation of surface runoff from the infiltration-capacity curves and storm precipitation patterns. In the watersheds studied in California more than 10% of the flows at flood crest during major storms, and much higher proportions during minor storms, were composed of "quick subsurface storm flow." Further, under watershed conditions with an assumed improved land-use program in operation, the proportion of channel flow at flood crest composed of quick subsurface storm flow was normally much greater than under present watershed conditions.

TABLE 8.—COMPUTED SURFACE RUNOFF AND FLOOD CREST DISCHARGES FOR PRESENT AND ASSUMED LAND-USE CONDITIONS, SANTA YNEZ RIVER DRAINAGE, CALIFORNIA

Watershed land-use conditions	3.50-IN. STORM		6.00-IN. STORM	
	Runoff[a]	Discharge[b]	Runoff[a]	Discharge[b]
Present observed	1.08	51,600	2.15	105,000
Assumed without program	1.17	55,700	2.27	110,200
Assumed with program	0.89	43,600	1.77	90,000

[a] Surface runoff, in inches of depth. [b] Crest discharge, in cubic feet per second.

In some watersheds considerable amounts of surface runoff were also yielded to channel inflow from complexes of shallow soils that, owing to impervious or partly impervious soil and rock formations or limited water-storage capacities of the substrata, became saturated during the storm. In applying the infiltration-capacity curves to the computation of surface runoff from these complexes, it was necessary to determine not only the rates of the quick subsurface storm flow throughout the course of the storm but also the depletion rates in storage capacities of the soil and substrata of the complexes reaching saturation. This was accomplished largely by analysis of rainfall and runoff records of subwatersheds composed of representative complexes and by establishing correlations between rates of quick subsurface flow and the total amounts of water retained within the soil and substrata.

In both the Horner and the California methods the infiltration-capacity curves were correlated with soil moisture and other antecedent watershed conditions to determine the correct position of the curves with respect to the storm rainfall patterns used in the surface runoff calculations. Mr. Horner wisely stresses the importance of this operation and, for analyses in which storm patterns are fitted to average watershed conditions, presents a very workable method for accomplishing it. Of equal importance in California is the placement and use of the correct infiltration-capacity curves in complexes comprising large areas of recently cultivated or freshly burned-over lands. On such lands the infiltration capacities may be decreased rapidly by changes in the physical properties of the soil through exposure to the beating effects of rain, surface runoff, and soil erosion.

Table 8 presents a comparison of the computed surface runoff and flood crest discharges during mid-winter storms in the Santa Ynez River drainage under present and assumed land-use conditions, with and without a remedial program. The Santa Ynez watershed comprises an area of 790 sq miles in the chaparral (brush) type of the Coast Range Mountains in Santa Barbara County, California. Owing to the high soil-moisture content and comparatively low rate of water use by vegetation during the winter flood periods, the computed reductions in surface runoff were accompanied by almost equivalent increases in subsurface and ground-water flows. The reduction in flood crest discharges expected from the proposed remedial land-use program results, therefore, not so much from a reduction in total water yield as from the more equable distribution of the yield.

C. S. JARVIS,[33] M. AM. SOC. C. E.—It is much more satisfactory to peruse the report of a consulting engineer on the results actually achieved and recorded than to scan the prospectus of intended goals in soil and water conservation. Many of the technical papers thus far dealing with the rôle of the land in regulating runoff in general and flood flow in particular have perforce depended on assumptions, estimates, and specifications which may or may not represent the changed conditions after the contracts are approaching completion. The author has carefully presented his official observations and interpretations in such a manner as to convince the most skeptical of the following facts:

1. The East Fork tributary of the Trinity River above Rockwall—not specially favored as to topography, soil, natural vegetative cover, rainfall distribution, and other significant elements of the geologic and climatic background, but somewhat deficient in some of the characteristics that would assist

---

[33] Cons. Engr., Washington, D. C.

in conserving moisture where it falls—is apparently amenable to the improved land use and management, with a corresponding reduction in peak discharge ranging usually from 10% to 40%.

2. With such beneficial disposal of considerable percentages of the most notable storms of record during the past few years, the conclusion is inescapable that greater percentages of retention and infiltration would occur with the more numerous storms of less severity.

3. Contrary to the conservative forecasts of prominent hydrologists (who claimed to be friendly toward conservation principles, yet disavowed any hope of influencing the catastrophic severity of the flood by the surface treatment and land-use changes included in the conservation program), it appears that measurable reduction can be assured in the event of long-period maxima, with correspondingly greater effect on flood happenings of greater frequency and less intensity.

4. Wherever the geologic and climatic background is more favorably inclined for response to conservation measures, correspondingly greater reduction of flood crests may result from comparable treatment, as estimated for the North Concho River, Texas,[34] according to preliminary studies and the actual behavior of several thousands of acres of newly treated valley lands. After additional months of study involving the use of infiltrometer equipment of various types on many sample areas throughout the valley, the conclusion was reached by the consultants in charge that the preliminary estimates were conservative as to assumed infiltration capacities and rates that would prevail. The later findings would have supported a more liberal estimate of flood-peak reduction, as well as the decrease of total volume, than that shown under the hydrographs presented by the writer on a previous occasion.[34]

W. W. HORNER,[35] M. AM. SOC. C. E.—The discussion of this paper is extremely interesting both with respect to the corroborative data presented and with respect to pertinent questions raised. Some of the discussers allowed their curiosity to carry them beyond the scope of the subject matter in the paper, and many of the questions relate to other phases of hydrology.

Mr. Kazmann suggests that the paper could be strengthened by including a treatment of the relation of infiltration to ground-water movement, particularly to changes in water table elevation. Such a study would undoubtedly have been extremely interesting. At one phase of the investigation the writer made a quick review of observation well data available at the Black Lands Experiment Station at Waco, and found that water table elevations could not be correlated satisfactorily with changes in infiltration capacity.

The writer's detailed presentation was restricted to surface runoff and related stream flow. The quantitative value of the infiltration capacity of the soil surface seems to be related to the rise and fall of the water table only when the water table is high enough to bring the surface soil within the capillary fringe. Undoubtedly, an investigation such as Mr. Kazmann suggests would be of real value to the general knowledge of hydrology, and possibly sufficient data are now available at Waco and Coshocton to justify such an undertaking.

---

[34] "Analysis of Run-Off Characteristics," by Otto H. Meyer, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 132, Fig. 13.

[35] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

Messrs. Hoyt and Langbein ask a number of questions, some of which are pertinent to the subject matter of the paper. These questions are discussed herein in the order of their appearance and with reference to the paragraphs.

In the seventh and eighth paragraphs Messrs. Hoyt and Langbein offer an expanded title for the paper, part of which is "Where Infiltrated Water Does Not Appear in the Stream as 'Quick-Subsurface Return Flow,'" and suggest that basic data might have been presented to prove that "subsurface inflow constitutes less than 10% of the flood volume." Both of the foregoing facts were stated by the writer in the course of a general description of the basin; in the methodology used and described in the paper, ground water was separated out from the hydrographs of flood rises. An inspection of these hydrographs affords an approximate verification of the statements quoted.

The writer has no responsibility for the statement quoted by Messrs. Hoyt and Langbein.[19] Their interpretation of that statement might apply to parts of the Trinity basin outside the Blacklands, but not the East Fork above Rockwall.

In the ninth paragraph Messrs. Hoyt and Langbein make their own computation that 78% of the total runoff at Rockwall was surface runoff. The writer made no assumptions or computations of the percentage of total surface runoff at Rockwall. Surface runoff was determined for each rise of the hydrograph for the thirty-five storms through the elimination of base flow by generally accepted methods.

In the tenth paragraph Messrs. Hoyt and Langbein seem to question the results of the writer's presentation because it did not include research studies similar to those carried out at Coshocton. Undoubtedly such a research study can be made from data available at Waco; but it is entirely outside the scope of the subject matter presented.

In the eleventh paragraph the first question raised by the discussers, if properly related to the Trinity basin and to the material presented in the paper, is whether reduction in surface runoff can be assumed to be the equivalent of the reduction in flood runoff of the stream when there is no considerable subsurface return flow. With regard to the use of cultivated lands, if agriculturalists restricted their operations to "soils that combine high infiltration capacities with satisfactory moisture-retention properties," the nation would probably be on the verge of quite a famine.

The questions raised in the twelfth paragraph and thereafter are directly pertinent to the paper, but they also reflect the natural difficulty of fully understanding new methodologies and their implications until after a personal effort has been made to apply them. Messrs. Hoyt and Langbein state that they reviewed the writer's report as consultant to the Department of Agriculture, but apparently they did not distinguish between the two principal parts of the methodology as related to the objectives of that investigation. For each storm the report contained discussions of the probable reasons for differences between computed surface runoff and the surface runoff in the actual hydrograph. To clarify the natural resulting confusion, the following more extended discussion of Steps 7 and 8 is presented:

The hourly pattern of excess rainfall was computed for each rainfall station in the manner indicated in Table 3. These patterns then had to be transferred to patterns for each of the ten sub-basins. These in turn were weighted to produce the average hourly excess for the basin as a whole. The values of excess rainfall for most of the storms varied materially between rainfall stations. Many of the storms were actually partial-area storms. Initially the pattern transfer was made on the basis of Thiessen area weighting. Later, it was made on the basis of inverse distance ratios which seemed more nearly to reflect the actual variation across the basin. The latter method was adopted for determination of the correction factors and gave reasonable results for all the more uniform storms.

For the more highly varying and for some of the partial-area storms, neither of these arithmetical procedures could give a satisfactory indication of the actual distribution of excess rainfall over the sub-areas, and for such conditions the drawing of isohyetal maps was essential. The use of such maps throughout the study would have appreciably increased the significance of the results.

In all cases, excess rainfall was computed initially and transferred to a mean value for the entire basin by the proportional method. It was then compared to the measured surface runoff as determined from the Rockwall gage. Although this gage was a chain gage read twice daily, and, although for one or two of the greater floods the record included estimated values of discharge, it was necessary to assume that the complete record was valid and it was so accepted as a basis for comparison.

If the computed excess rainfall was greatly different from the measured runoff, this fact was taken as a "red flag" and the possible reason for the difference was explored and discussed. The same procedure was followed if the later computed hydrograph was seriously out of phase with the recorded hydrograph. Restudies of this kind resulted in several revised computations for some of the storms—for example, such a comparison served to clarify ambiguities of dating the cooperative observers' rainfall reports. For some of the storms the discordance could be largely accounted for by the method of transfer of values to the sub-basins; in others, the drawing of isohyetal maps giving rational consideration to the distribution of rainfall between stations quite generally produced weighted values for the basin having closer accord with the measured values. For those storms in Table 4, which were selected for inclusion in Table 5, isohyetal maps were initially made up on the basis of rainfall information only. For use in the second steps, such maps were in some instances readjusted to produce excess rainfall exactly equal to the measured runoff. This procedure was in lieu of the use of a correction factor.

For example, for storm 2 under the initial procedure the weighted excess was 1.77 in., requiring a correction factor of 0.91, but the later computed hydrograph was somewhat out of phase. Thus, a revision of the interpretation of dates on certain ambiguous rainfall reports was indicated. On this basis the recomputed weighted excess was 1.83 in.; and the correction factor, 0.88. This factor is shown in Table 4 and is based on transference by isohyetal map. This second computation was not influenced by the measured amount of runoff. For the succeeding "before and after" study, the pattern was shifted to produce better conformance with the shape of the hydrograph.

Table 5 is the compilation for those storms where excess rainfall was computed on the basis of the *M*-curve and transferred to the sub-areas either by the proportional method or by isohyetal maps. The only influence of the known "measured" runoff on this procedure was the indication of the possibility of error in the proportional method of transference. If (as was later concluded) such maps could have been used to advantage throughout this study, no reference whatever to measured values would have been involved.

A recheck of the data used in Table 5 discloses that one additional flood (No. 9) was excluded from this table besides those stated. Also one flood indicated in the range from 0.5 to 1.0 in., in the December 1 to March 15 period, was incorrectly included in that range. This table therefore is based on twenty-two floods for which the computation of excess rainfall and the indicated correction factors were completed before any comparison was made with measured runoff.

With respect to the question about the *M*-curve, the initial *M*-curve was developed from a study of the small watershed data from which the infiltration-capacity curves were derived.

Subsequent to the analysis of the thirty-five storms above Rockwall, determinations were made of the *M*-values (assuming all other data valid), the use of which would have resulted in reproducing the value of surface runoff taken from the hydrograph at Rockwall. This series of *M*-values departed somewhat from the *M*-curve shown in Fig. 4, placing this curve somewhat lower during the winter season and somewhat higher in July and August. Some composite of these two *M*-curves undoubtedly can now be used in this basin for forecasting the flood flow from any specific storm pattern.

Table 5 is the test of the ability of the described method to produce values of flood flow in fair conformance to measured values. Messrs. Hoyt and Langbein confuse the character of the data in Table 5 with the procedure in the succeeding stage. They ask, "If partial advantage is taken of the runoff as it was, why not take full advantage." This is exactly what was done in the second stage of the study in which the original computed values are adjusted by use of the correction factor or by revised or original isohyetal maps to an exact equality with the actual surface runoff. The reweighting of excess rainfall in accordance with the proposed program of land-use revision then produces "after" values of flood runoff. Thus, the present or "before" value having been adjusted to the measured value, the "after" value resulting from a computation identical in every respect (except for the land program) makes possible the determination of the reduction in flood flow which that land program would cause.

The writer is in full agreement with the questions concerning the use of hourly rainfall values. The procedure is quite sensitive to rainfall intensity. Mean rates for, say, 30 min could have been used to advantage if they had been generally available. It is interesting, however, that, even with only hourly values of rainfall derived from poor precipitation data, the results are generally consistent. In this investigation the objective is the determination of the average flood reduction with relation to flood magnitude. Possibly the poor rainfall data explain some of the scatter of points in Fig. 6; but nevertheless a rather significant trend is indicated.

In the fourth paragraph from the end of their discussion Messrs. Hoyt and Langbein state:

> "The report explains the influence of antecedent conditions as they affect soil moisture on the infiltration capacity during a storm. Increasing the volume of infiltration prior to a storm decreases the capacity of the basin to retain water. Therefore, under the changed conditions, to what extent would the presumed increase in infiltration operate adversely to lower infiltration capacities during subsequent storms?"

This question appears to be answered by the actual data presented. Infiltration-capacity curves, for each separate complex, were derived from small watershed data taken under a natural sequence of precipitation periods. Therefore the curves for pasture, for example, indicate the value of infiltration capacity for pasture when the soil moisture is related to antecedent infiltration for pasture. Since the infiltration for pasture is higher than for cultivated lands subjected to the same rains, the soil surface under pasture grass, and among other things protected from the effect of rain impact, seems to retain a higher capacity even though the moisture content be greater than that for adjacent cultivated fields.

The economic implications again are beyond the scope of the paper. However, the paper did not cover the subject of flow reduction as a whole, but only the "reduction in flood flow." Any adverse effect would seem to apply only to consumptive use of impounded flood waters. This, eventually, conceivably might be offset in some part by improvement in base flow.

Mr. Sherman's discussion presents the results of similar investigations in which he has used a methodology paralleling in many respect that which the writer applied to the Trinity River study. Mr. Sherman appears to be in thorough agreement with the practice of time condensation by which infiltration-capacity curves derived from natural rainfall data may be reduced to the approximate equivalent of curves that would have resulted from rainfall-intensity rates which exceeded infiltration-capacity rates in every case. This method permits a correlation between the infiltration-capacity curves derived from the two types of data. If such a condensation of the original derived curves is satisfactory, the condensed curves could, with some assurance, be expanded in the same manner, in order to be matched appropriately with any particular rainfall-intensity pattern.

Mr. Sherman also independently relates infiltration capacity to soil moisture through a compilation of antecedent infiltration. In his practice he has developed a somewhat simplified procedure as compared to the writer's *M*-curve, by relating infiltration-capacity values to three conditions of soil moisture deficiency.

It is of interest that, since the presentation of his discussion in May, 1943, Mr. Sherman has made a much more detailed analysis of the mechanics of infiltration and has developed a relationship between infiltration-capacity values and soil moisture deficiency as separated into capillary and noncapillary deficiency.[36]

As stated in the paper and in connection with the discussion by Messrs. Hoyt and Langbein, a second series of M-values was determined for the whole of the East Fork of the Trinity basin above Rockwall. On the basis of these additional values, a modified M-curve was developed. All of the data and the original and modified M-curves are shown in Fig. 10. The placing of the original M-curve, in the period from July to November, was based on a relatively few infiltrometer runs. Using the basin-wide data, satisfactory results cannot be secured from an infiltrometer during the period when these black soils are extensively cracked. Therefore, probably much of the applied water leaves the area of the infiltrometer plot through wide cracks and supplies noncapillary deficiency over a large area around the plot.

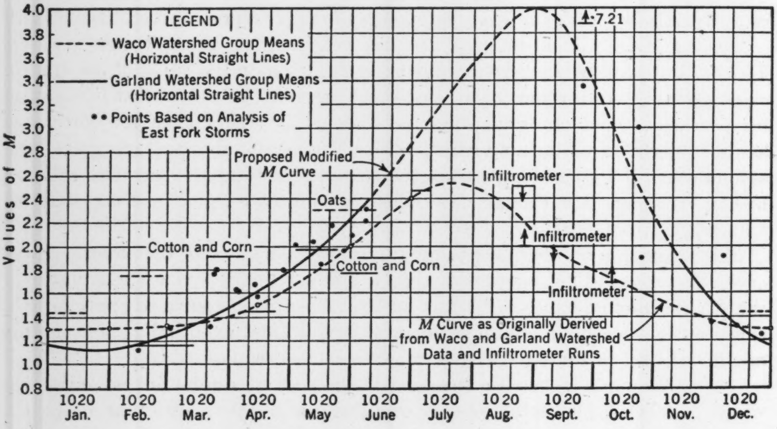


FIG. 10.—VARIATION, DURING ONE YEAR, OF THE VALUE OF M REQUIRED TO PRODUCE AN INFILTRATION CAPACITY OF 0.4 IN. PER HR

An inspection of the modified M-curve, even though it is sketched in, indicates that M-values of between 3 and 4 in. would control surface runoff for the months of July to October, inclusive, and further indicates that the probability of a major flood during those months is extremely remote. For example, for normal precipitation conditions in the three months prior to the last 30 days before a storm, infiltration capacity would not drop to 0.4 in. per hr until either—

(*a*) Infiltration in the storm had exceeded about 3.5 in.; or

(*b*) Some combination as, for example, 4 in. of infiltration from the preceding 30 days plus 1.5 in. of infiltration in the specific storm period had occurred.

---

[36] "Infiltration and the Physics of Soil Moisture," by L. K. Sherman, presented at the regional meeting of the Am. Geophysical Union in Berkeley, Calif., on February 15, 1944.

A study of the 5-yr record indicates a number of excessive precipitation periods that produced only insignificant rises of the hydrograph of stream flow. One such period involved about 7 in. of rainfall within 3 or 4 days.

Mr. Moore presents an interesting account of the difficulty in maintaining adequate supervision over a large group of rainfall gage observers. The writer made an analysis of some of the small watershed and runoff plot records at the Tyler Station, involving certain of the east Texas sandy soils. Infiltration capacity was determined for these soils for a number of complexes but the results were not used in the Trinity River investigation, since it was found that a negligible percentage of the soils in the basin above Rockwall was of that type. Out of these studies, however, some tentative conclusions were reached. One of these was that the infiltration capacity of terraced watersheds was not appreciably higher than that for watersheds having the same type of cover but without such structures. The greatly increased infiltration in the terraced channels probably resulted in a generally higher moisture content which offset the other advantages of the treatment. Another interesting result of these studies was finding that the terraced watershed in Bermuda grass and another watershed in native protected forest rarely produced any surface runoff.

Mr. Moore refers to the use of the rational formula by the Operation Division of the Soil Conservation Service with the choice of a constant value of *C* for each land use and soil group. The procedure which the writer has outlined would permit the choice of a value of *C* for each such group. This would properly reflect maximum soil moisture conditions and consequently would probably produce maximum values of runoff. Such an application may be justified where the ultimate purpose is to design structures of adequate capacity, as, for example, outlet channels for terraced areas.

Mr. Smith again raises the question of the use of average hourly rainfall intensities and comments further on the unsatisfactory character of the data. Apparently he would like to see an investigation of this type in which all the desirable data are of a highly accurate character. Such a study would then fall into the field of a high-grade research project. The investigation reported by the writer was one which needed to be made and had to be made on the basis of the data available. The results appeared to be highly significant, probably more so than any one would have anticipated before the effort was undertaken. It is to be hoped that at some future time, when technical personnel again becomes available for extensive hydrologic research, the methodology may be applied to some such basin as that of the Licking River in Ohio.

Mr. Smith's presentation of the data in Fig. 9 is extremely interesting. The use of information of this kind when properly correlated with corresponding infiltration-capacity curves would probably permit further material improvement in controlling soil moisture factors such as the *M*-curve which the writer has used. Mr. Smith has performed a real service in bringing up to date a brief bibliography for deriving infiltration-capacity curves.

The writer is highly gratified with Mr. Rowe's discussion. In a first draft of the paper the writer made an attempt to include a discussion of the analysis of drainage basins in which stream flow was materially affected by "quick subsurface return flow." He reviewed some of the excellent information which has become available from the Southern Appalachian Experiment Station of the Forest Service, U. S. Department of Agriculture, but was unable to cite good examples of the use of these data in the quantitative determination of the relation of stream flow to land factors. For this reason, the writer found it necessary to restrict his detailed presentation of methodology to the situation in which stream flow resulted primarily from surface runoff and in which infiltration capacity of the soil surface was the primary control. Under the heading, "Subsurface Retention and Detention," the writer expressed the hope that information might be presented with respect to a satisfactory methodology in which quick subsurface return flow represented a material part of stream flow.

Mr. Rowe has described briefly the approach in California to an analysis of this latter situation. He states that infiltration-capacity curves are derived from small plot infiltrometer measurements. Such derivations must be used with extreme caution. The writer has seen infiltrometer runs made on shallow surface soils and steep slopes where no surface runoff whatever was produced, and yet the streams in this area were subject to floods of considerable magnitude. In one particular instance, further investigation indicated that subsurface flow was preceding down slope through the porous surface soil at extremely high rates and that this flow generally came to the surface at the margins or banks of the small streams or gullies. Recently the writer analyzed infiltrometer runs on a deep fill of clean sand with practically no lateral slope but with a water table 5 ft to 6 ft below the surface. An analysis of these runs showed that they made possible a determination of infiltration capacity for only a few minutes after surface runoff began. Thereafter a mounded water table was flushed to the surface and the infiltrometer record showed merely the rate at which subsurface water was transmitted away from the vicinity of the plot. Both of the foregoing are unquestionably extreme cases. Many situations have been recognized in which surface runoff was produced and controlled by the infiltration capacity of the soil, but at the same time infiltration capacities continued at high values, soil-storage capacity became filled, and subsurface return flow occurred at high rates. A methodology that would recognize all of these conditions and permit the reproduction of flood flow in the streams on the basis of precipitation intensity and land factors will be much more complex than that presented by the writer. Such methodologies have been devised in connection with certain flood control surveys of the U. S. Department of Agriculture. It is hoped that some of these may be presented in detail to the engineering profession.

The remarks of Mr. Jarvis are particularly interesting. The writer is in agreement with his conclusion 3. With respect to the fourth paragraph, the writer would also anticipate that, for a watershed such as the North Concho in Texas, a revised program of land use and management might well result in a greater reduction of flood flow than was found for the Trinity basin.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2232

STRENGTH OF SLENDER BEAMS

BY GEORGE WINTER,[1] ESQ.

WITH DISCUSSION BY MESSRS. T. MCLEAN JASPER, C. M. GOODRICH, J. P. WADLECK, H. D. HUSSEY, AND GEORGE WINTER.

SYNOPSIS

Slender beams loaded in the usual manner tend to fail laterally at stresses often far below the yield point of the material. This fact is provided for in design specifications by reducing the basic allowable stress in the extreme compression fiber for beams in which the ratio of length to width is greater than a prescribed maximum. The specified reduction is based more or less on semi-empirical considerations rather than on strict analytical study.

In this paper the ultimate strength of slender beams is determined by rational methods similar to those generally accepted in column design, due consideration being given to highly probable imperfections in loading and beam shape. It is shown that these methods, whose value in column analysis is well established, can be applied with comparative ease to the fundamentally more complex beam problem.

Beam curves, similar in character to the well-known column curves, serve to illustrate the influence on the ultimate strength of such factors as slenderness ratio, shape of cross section, and degree of imperfection. In addition, a discussion is given of the influence of the character of the load, of intermediate bracing, and of elastic restraint at the end connections.

GENERAL

It has long been recognized that transversely loaded slender beams may fail by lateral buckling at stresses far below the yield point or the ultimate stress of the material (1).[2] In recent years considerable attention has been devoted to investigating the magnitude of those loads (critical loads) which cause beams of given shapes and dimensions to become unstable. These investigations apply only to very slender beams which buckle at stresses below the proportional limit. Therefore, their application to structural design is limited to very long

---

NOTE.—Published in June, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[2] Numerals in parentheses, thus: (1), refer to the corresponding items in the Bibliography (Appendix I).

and narrow beams, just as the Eüler column formula gives reliable information on the carrying capacity only of very slender compression members. The designer, however, needs to know the allowable working stresses, with a given factor of safety, for beams of any dimensions. To meet this need, in the absence of more reliable information pertaining to beams of moderate slenderness, semi-empirical design specifications have been developed in various ways (2). Thus, the compression flange of a steel beam is frequently regarded as a strut free to buckle laterally, and the supposedly safe extreme fiber stress for this flange is obtained by applying a suitable column formula to the imaginary strut. It is evident that such a procedure represents, at best, a rather crude approximation, since the effect of the joint action of the compression flange with the remainder of the section, and particularly the influence of the torsional rigidity of the beam, are neglected entirely.

The situation is comparable to that which, years ago, prevailed in column design when the Eüler formula was accepted for columns of great slenderness and different kinds of semi-empirical transition curves were recommended for columns of moderate slenderness. In the field of compression members, considerable progress toward a more rational design has been made in recent years through acceptance of the secant formula or similar approaches (3)(4), as a criterion for the design of columns of any slenderness ratio. These methods have in common that they are based on a definite degree of inaccuracy of loading or of crookedness of the column. They allow a rigorous determination of the magnitude of that compression stress, $\frac{P}{A}$, which, for the given imperfections, will stress the outer fiber to the yield point or to any other given maximum stress. Column design curves and formulas are now generally based on this $\frac{P}{A}$-value.

This paper attempts to develop a similar rational basis for the design of slender beams. Paralleling the column methods, it assumes a given amount of inaccuracy of loading and proceeds to determine the magnitude of that applied stress, $\frac{M\,c}{I}$, which will stress to the yield point or any other given maximum value that point of the section farthest removed from the centroid of the section (for example, for I-beams the edges of the flanges). This applied stress can serve as a reliable design criterion just as the $\frac{P}{A}$-value obtained from the secant formula does in column design.

An analysis of doubly symmetrical solid and hollow sections, like rectangular or box-beams, is made first. This analysis is then extended to include the more involved symmetrical I-beam sections. From the resulting data, curves are drawn for various degrees of inaccuracy of loading and for different shapes of cross sections. These curves show the relation between the magnitude of that applied stress $s = \frac{M\,c}{I}$ which results in yield point stress s_y on the most remote points of the section and the slenderness ratio $\frac{L}{b}$ (L = span, b = width). By applying a given factor of safety, these curves can be redrawn to give the allowable bending stress directly as a function of the slenderness ratio.

In addition to this part of the study, several other related factors are investigated. One of these is the influence of the character of the load. For beams stressed below the proportional limit, the critical moment M_c which causes the beam to buckle laterally depends not only on the shape and material of the beam but also on the type of loading (5). Although a complete treatment of this question is beyond the scope of this paper, a brief discussion of the magnitude of its effect is given.

The critical loads and ultimate stresses for slender beams are derived for freely supported ends, just as the Eüler and the secant formulas apply only to pin-ended columns. Most beams, however, are restrained elastically at their ends, in various degrees. The influence of such end restraints may be provided for in the same manner as for columns—namely, by introducing an equivalent reduced length instead of the actual span.

Notation.—The letter symbols in this paper are defined where they first appear (unless the definition is self-evident) and are assembled for convenience of reference in Appendix II.

STRESSES IN SLENDER BEAMS

In determining stresses in beams subject to transverse loading, generally it is assumed that the loading plane coincides with one of the principal planes of the beam. If the loading plane is inclined with respect to one of the principal axes, the moment is resolved into two components, M_x and M_y (see Fig. 1(*a*)) and the maximum stress is computed from the equation

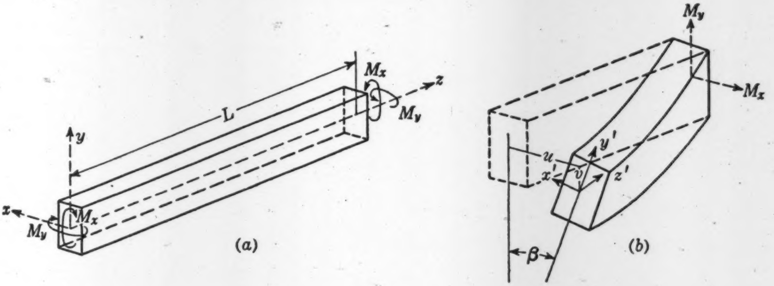


FIG. 1

$$s_m = \frac{M_x c_{x'}}{I_{x'}} + \frac{M_y c_{y'}}{I_{y'}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This formula is reasonably accurate for rather short beams with small ratios of $\frac{L}{b}$. However, for slender beams the actual maximum stress is larger than that given by Eq. 1, because a beam so loaded twists as it bends (see Fig. 1(*b*)). The principal axes x' and y' of the cross section anywhere along the span are no longer parallel to the x-axis and y-axis, respectively, the inclination between these two sets of axes changing along the length of the beam. Consequently, the bending moments about the principal axes of the section are no longer M_x and M_y, but are the projections of these moments in the x' and y' directions. Therefore, they are dependent on the angle β. It will be shown that the increase of s_m due to the twist becomes very significant for slender beams; and, in fact, it may be regarded as the principal factor in relating allowable stresses to slenderness ratios $\frac{L}{b}$.

Inaccuracies of loading or shape are inevitable in beams just as they are in columns. In view of the influence of β on s_m it is proposed to assume that the plane of loading is slightly inclined with respect to the y-axis (Fig. 1(a)) and to investigate the effect of such inaccuracy of loading on beams of any slenderness in pure bending.

To determine the stresses in a section subtending the angle β with the y-axis it is necessary to project M_x and M_y into the x' and y' directions. With u and v representing the displacements of the centroid in the x-direction and y-direction, respectively, the direction cosines between the two sets of axes (5a) are as given in Table 1. In computing the values of this table advantage was taken of the fact that u, v, and β are small quantities. For this reason terms above the first order are negligible. In addition to the values for the x'-axis and y'-axis, cosines are also given with respect to the z'-axis (see Fig. 1(b)), since these will be used subsequently.

TABLE 1.—DIRECTION COSINES BETWEEN TWO SETS OF AXES

Axes	x	y
x'	1	β
y'	$-\beta$	1
z'	$\frac{du}{dz}$	$\frac{dv}{dz}$

With the vectors of M_x and M_y as shown in Fig. 1(b) the moments about the principal axes are

$$M_{x'} = - M_x + \beta M_y \tag{2a}$$

and

$$M_{y'} = \beta M_x + M_y \tag{2b}$$

The absolute magnitude of the maximum stress is obtained from the equation

$$s_m = \frac{M_{x'} c_{x'}}{I_{x'}} + \frac{M_{y'} c_{y'}}{I_{y'}} \tag{3a}$$

in which $I_{x'}$, $I_{y'}$, $c_{x'}$, and $c_{y'}$ are, respectively, the moments of inertia about, and the distances to the outer fiber from, the x' and y' axes of the section. Substitution of Eqs. 2 in Eq. 3a gives

$$s_m = \frac{c_{x'}}{I_{x'}} (M_x - \beta M_y) + \frac{c_{y'}}{I_{y'}} (\beta M_x + M_y) \tag{3b}$$

For deep beams ($I_{x'} \gg I_{y'}$) with but slightly inclined loadings ($M_x \gg M_y$), the term $- \frac{\beta M_y c_{x'}}{I_{x'}}$ is of higher order than the remainder of the terms. Consequently, for such conditions (of primary interest) this term can be neglected.

Then

$$s_m = \frac{M_x\, c_{x'}}{I_{x'}} + \frac{M_y\, c_{y'}}{I_{y'}} + \beta\, \frac{M_x\, c_{y'}}{I_{y'}} \tag{3c}$$

A comparison of Eq. 3*c* with the usual approximate Eq. 1 shows that the two differ by the term $\frac{\beta\, M_x\, c_{y'}}{I_{y'}}$ which is due to the twist β. To determine s_m it is necessary first to compute β.

THE SECANT FORMULA FOR RECTANGULAR AND BOX-BEAMS

For small angles β, neglecting terms of higher order, the curvature $\frac{d^2u}{(dz')^2} = \frac{d^2u}{dz^2}$ and the curvature $\frac{d^2v}{(dz')^2} = \frac{d^2v}{dz^2}$. Consequently, resolution of the applied end moments M_x and M_y in the x', y', and z' directions yields the three simultaneous equations of equilibrium (6):

$$M_{y'} = \beta\, M_x + M_y = E\, I_{y'}\, \frac{d^2u}{dz^2} \tag{4a}$$

$$M_{x'} = -\, M_x + \beta\, M_y = -\, E\, I_{x'}\, \frac{d^2v}{dz^2} \tag{4b}$$

and

$$M_{z'} = -\, \frac{du}{dz}\, M_x + \frac{dv}{dz}\, M_y = G\, K\, \frac{d\beta}{dz} \tag{4c}$$

in which K is the torsional constant of the section (7). To eliminate u and v, Eq. 4*c* is differentiated and $\frac{d^2u}{dz^2}$ and $\frac{d^2v}{dz^2}$ substituted from Eqs. 4*a* and 4*b*. Thus,

$$\frac{d^2\beta}{dz^2} + \left(\frac{M^2_x}{R_{y'}} + \frac{M^2_y}{R_{x'}}\right) \beta = M_x\, M_y \left(\frac{1}{R_{x'}} - \frac{1}{R_{y'}}\right) \tag{5}$$

in which $R_{x'} = E\, G\, K\, I_{x'}$; and $R_{y'} = E\, G\, K\, I_{y'}$, E being Young's modulus and G the shearing modulus. With

$$\alpha^2 = \frac{M^2_x}{R_{y'}} + \frac{M^2_y}{R_{x'}} \tag{6a}$$

and

$$\gamma = M_x\, M_y \left(\frac{1}{R_{x'}} - \frac{1}{R_{y'}}\right) \tag{6b}$$

the general solution of Eq. 5 is

$$\beta = C_1 \cos \alpha\, z + C_2 \sin \alpha\, z + \frac{\gamma}{\alpha^2} \tag{7}$$

in which the constants C_1 and C_2 are determined from the end conditions. If the beam is "freely supported" (that is, if the ends are free to rotate about the x-axis and y-axis, while rotation about the z-axis is prevented), the end conditions are: $\beta_{z=0} = \beta_{z=L} = 0$; or—

$$\beta = \frac{\gamma}{\alpha^2}\left[1 - \left(\cos \alpha\, z + \sin \alpha\, z \tan \frac{\alpha\, L}{2}\right)\right] \tag{8a}$$

From the coefficient

$$\frac{\gamma}{\alpha^2} = \frac{M_y}{M_x} \frac{\dfrac{I_{y'}}{I_{x'}} - 1}{\left[1 + \left(\dfrac{M_y}{M_x}\right)^2 \dfrac{I_{y'}}{I_{x'}}\right]} \qquad \text{(8b)}$$

it is seen that any symmetrical beam, subject to unsymmetrical bending, twists as it bends. In fact, $\beta = 0$ only if $I_{x'} = I_{y'}$. The physical significance of this result is this: It is well known that the direction of the deflections of beams subject to unsymmetrical bending is different from that of the applied bending moment. Consequently, the normal to the plane of any cross section is not perpendicular to the vector of the applied moment. The projection of this vector in the direction of the section-normal is equal to the twisting moment at that particular section. It vanishes only if $I_{x'} = I_{y'}$, as, for example, for quadratic or circular sections, since beams of such shape deflect in the plane of the applied moment.

In practical applications the influence of β becomes important only for deep beams $\left(\text{that is, when } \dfrac{I_{y'}}{I_{x'}} \ll 1\right)$. Eqs. 6 may be rewritten in the form

$$\alpha^2 = \frac{M^2{}_x}{R_{y'}} \left[1 + \left(\frac{M_y}{M_x}\right)^2 \frac{I_{y'}}{I_{x'}}\right] \qquad \text{(9a)}$$

and

$$\gamma = \frac{M_x\, M_y}{R_{y'}} \left[\frac{I_{y'}}{I_{x'}} - 1\right] \qquad \text{(9b)}$$

If terms of the order $\dfrac{I_{y'}}{I_{x'}}$ as compared with 1 are neglected,

$$\beta = \frac{\gamma_1}{\alpha^2{}_1} \left[1 - \left(\cos \alpha_1 z + \sin \alpha_1 z \tan \frac{\alpha_1 L}{2}\right)\right] \qquad \text{(10)}$$

in which,

$$\alpha^2{}_1 = \frac{M^2{}_x}{R_{y'}} \qquad \text{(11a)}$$

and

$$\gamma_1 = -\frac{M_x\, M_y}{R_{y'}} \qquad \text{(11b)}$$

Eqs. 10 and 11 also can be obtained directly if, in Eq. 4*c*, the influence of v on β is omitted (it actually is negligible in deep beams). Then, instead of the three Eqs. 4 one has simply Eq. 4*a* and the modified expression (see Eq. 4*c*),

$$M_{z'} = -\frac{du}{dz} M_x = G\, K \frac{d\beta}{dz} \qquad \text{(12)}$$

Eq. 4*b* represents the curvature in the direction of the main bending moment and is independent of Eqs. 4*a* and 12. The simultaneous solution of Eqs. 4*a* and 12 is identical with Eq. 10. From the comparison of the approximate coefficient

$$\frac{\gamma_1}{\alpha^2_1} = -\frac{M_y}{M_x} \quad \text{.......................} (13)$$

for deep beams with the exact expression Eq. 8*b*, it is seen that in fact only terms of order $\frac{I_{y'}}{I_{x'}}$ have been neglected.

The maximum value of β occurs at $z = \frac{L}{2}$ and it is consequently at this place that the third term in Eq. 3*c* becomes most important. From Eq. 10, with $z = \frac{L}{2}$,

$$\beta_m = \frac{\gamma_1}{\alpha^2_1}\left(1 - \sec\frac{\alpha_1 L}{2}\right) \quad \text{.................} (14a)$$

or, with Eq. 13,

$$\beta_m = -\frac{M_y}{M_x}\left(1 - \sec\frac{\alpha_1 L}{2}\right) \quad \text{................} (14b)$$

Substitution of this value of β in Eq. 3*c* yields

$$s_m = \frac{M_x c_{x'}}{I_{x'}}\left(1 + \frac{M_y c_{y'} I_{x'}}{M_x c_{x'} I_{y'}} \sec\frac{\alpha_1 L}{2}\right) \quad \text{............} (15)$$

On the other hand

$$s = \frac{M_x c_{x'}}{I_{x'}} \quad \text{........................} (16)$$

is the direct bending stress for the ideally straight beam loaded exactly in the y-direction; that is, for vertically loaded beams it is the only stress usually considered by the designer. Also, with $(s)_y = \frac{M_y c_{y'}}{I_{y'}}$ —

$$k = \frac{M_y c_{y'}}{I_{y'}} \frac{I_{x'}}{M_x c_{x'}} = \frac{(s)_y}{s} \quad \text{..................} (17)$$

is a convenient measure of the inaccuracy of loading, comparable to $\frac{e\,c}{r^2}$ in column design (3)(4). Finally, from Eq. 11*a*,

$$\frac{\alpha_1 L}{2} = \frac{L}{2}\frac{M_x}{\sqrt{R_{y'}}} = \frac{L\, s\, I_{x'}}{h\sqrt{E\,G\,I_{y'}\,K}} = \frac{L}{b}\frac{s}{\sqrt{E\,G}}A' \quad \text{.........} (18)$$

in which A' is a non-dimensional characteristic of the cross section

$$A' = \frac{I_{x'}}{\sqrt{I_{y'}\,K}}\frac{b}{h} \quad \text{.......................} (19)$$

With these simplifications Eq. 15 may now be rewritten as follows:

$$s_m = s\left[1 + k\sec\left(\frac{L}{b}\frac{s}{\sqrt{E\,G}}A'\right)\right] \quad \text{..............} (20)$$

For a deep rectangular section with $I_{x'} = \frac{b\,h^3}{12}$, $I_{y'} = \frac{b^3\,h}{12}$, $K = \frac{b^3\,h}{3}$, A' has the particularly simple value

$$A' = \frac{h}{2\,b} \quad \text{(21)}$$

For other types such as box-beams, expressions for A' are obtained easily from Eq. 19.

The practical problem, then, is to use Eq. 20 for finding pairs of values s and $\frac{L}{b}$ such that the maximum stress s_m has a prescribed value. In particular, the ultimate strength of a steel beam will be reached when s_m is equal to the yield point s_y. (Actually the carrying capacity of a beam is not yet reached when the diagonally opposite edges are stressed to the yield point. The main part of the section is then still stressed to a smaller amount and the beam will sustain a further load increase (8). For this same reason the secant formula for columns gives values somewhat below the ultimate capacity of the strut, especially for small $\frac{L}{r}$-values (4). It is customary, however, to disregard this strength reserve and to consider as the limiting load that load at which yield-point stresses occur in any portion of the section. This is particularly justified for dynamic loading.) To make $s_m = s_y$, Eq. 20 is conveniently written as

$$s = \frac{s_y}{1 + k \sec\left(\frac{L}{b}\frac{s}{\sqrt{E\,G}}\,A'\right)} \quad \text{(22)}$$

which, in form, is analogous to the secant formula for columns. Thus, using Eq. 22, it is possible to plot s versus $\frac{L}{b}$ for any given yield point s_y and any degree of inaccuracy of loading k. It is seen that in addition to s_y and k, the $\left(s - \frac{L}{b}\right)$-relation also depends on the characteristic A' of the cross section. For rectangular beams, with $A' = \frac{h}{2\,b}$, the following particularly simple formula is obtained:

$$s = \frac{s_y}{1 + k \sec\left(\frac{L\,h}{2\,b^2}\frac{s}{\sqrt{E\,G}}\right)} \quad \text{(23)}$$

Just as in the case of columns, any desired factor of safety is easily applied to these secant formulas, and results in curves giving the allowable working stress as a function of $\frac{L}{b}$ for given k and A'.

THE SECANT FORMULA FOR I-BEAMS

The formulas discussed in the preceding section apply only to doubly symmetrical solid or hollow cross sections. In deriving similar equations for symmetrical I-beams account must be taken of the additional torsional resistance of such beams due to lateral bending of the flanges. Instead of Eqs. 4, one has then (6):

$$M_{y'} = \beta\, M_x + M_y = E\, I_{y'} \frac{d^2u}{dz^2} \qquad \text{(24a)}$$

$$M_{x'} = -\, M_x + \beta\, M_y = -\, E\, I_{x'} \frac{d^2v}{dz^2} \qquad \text{(24b)}$$

and

$$M_{z'} = -\frac{du}{dz} M_x + \frac{dv}{dz} M_y = G\, K \frac{d\beta}{dz} - E\, I_{y'} \frac{h^2}{4} \frac{d^3\beta}{dz^3} \qquad \text{(24c)}$$

in which the second term on the right side of Eq. 24*c* represents the contribution of the flanges to the torsional rigidity.

The end conditions for the freely supported beam are

$$\beta_{z=0} = \beta_{z=L} = \left(\frac{d^2\beta}{dz^2}\right)_{z=0} = \left(\frac{d^2\beta}{dz^2}\right)_{z=L} = 0 \qquad \text{(25)}$$

The last two conditions express the fact that the longitudinal stresses due to warping of the beam vanish at the free ends. The solution satisfying Eqs. 24 and 25, as derived jointly by D. F. Gunder and the author, is

$$\beta = q \left[p_2 \left(-\sin m_1 z \tan \frac{m_1 L}{2} - \cos m_1 z \right) + p_1 \left(\sinh m_2 z \tanh \frac{m_2 L}{2} - \cosh m_2 z \right) + 1 \right] \qquad \text{(26)}$$

in which

$$q = -\frac{\dfrac{M_x M_y}{E} \left(\dfrac{1}{I_{y'}} - \dfrac{1}{I_{x'}} \right)}{\dfrac{M^2{}_x}{E\, I_{y'}} + \dfrac{M^2{}_y}{E\, I_{x'}}} \qquad \text{(27a)}$$

$$m^2{}_1 = -\frac{2\, G\, K}{E\, I_{y'}\, h^2} + \sqrt{\frac{4\, G^2\, K^2}{E^2\, (I_{y'})^2\, h^4} + \frac{4}{h^2} \left[\frac{M^2{}_x}{E^2\, (I_{y'})^2} + \frac{M^2{}_y}{E^2\, I_{y'}\, I_{x'}} \right]} \qquad \text{(27b)}$$

$$m^2{}_2 = \frac{2\, G\, K}{E\, I_{y'}\, h^2} + \sqrt{\frac{4\, G^2\, K^2}{E^2\, (I_{y'})^2\, h^4} + \frac{4}{h^2} \left[\frac{M^2{}_x}{E^2\, (I_{y'})^2} + \frac{M^2{}_y}{E^2\, I_{y'}\, I_{x'}} \right]} \qquad \text{(27c)}$$

and

$$p_1 = \frac{m^2{}_1}{m^2{}_1 + m^2{}_2}\,; \qquad p_2 = \frac{m^2{}_2}{m^2{}_1 + m^2{}_2} \qquad \text{(27d)}$$

Since, as before, $M_x \gg M_y$ and $I_{x'} \gg I_{y'}$, the expression for q simplifies to

$$q = -\frac{M_y}{M_x} \qquad \text{(28)}$$

For the same reason, in Eqs. 27*b* and 27*c*, $\dfrac{M^2{}_y}{E^2\, I_{y'}\, I_{x'}}$ is negligible when compared with $\dfrac{M^2_x}{E^2\,(I_{y'})^2}$. Multiplying and dividing $\dfrac{4}{h^2}\dfrac{M^2_x}{E^2\,(I_{y'})^2}$ in Eqs. 27*b* and 27*c* by $h^2\,(I_{x'})^2$ and factoring,

$$m^2{}_1 = \frac{2\,G\,K}{E\,I_{y'}\,h^2}\left[\sqrt{1+\left(2\,\frac{s}{G}\,\frac{I_{x'}}{K}\right)^2} - 1\right] \ldots\ldots\ldots\ldots (29a)$$

and

$$m^2{}_2 = \frac{2\,G\,K}{E\,I_{y'}\,h^2}\left[\sqrt{1+\left(2\,\frac{s}{G}\,\frac{I_{x'}}{K}\right)^2} + 1\right] \ldots\ldots\ldots\ldots (29b)$$

in which, as before (see Eq. 16), $s = \dfrac{M_x\,h}{2\,I_{x'}}$ is the applied direct bending stress.

The maximum stress s_m is again computed from Eq. 3*c* by introducing the maximum value for β. From Eq. 26, for $z = \dfrac{L}{2}$, and with q from Eq. 28,

$$\beta_m = -\frac{M_y}{M_x}\left[1 - p_2 \sec\frac{m_1\,L}{2} - p_1 \operatorname{sech}\frac{m_2\,L}{2}\right] \ldots\ldots\ldots\ldots (30)$$

Substitution of this expression in Eq. 3*c* gives for the maximum stress

$$s_m = \frac{M_x\,c_{x'}}{I_{x'}}\left[1 + \frac{M_y\,c_{y'}\,I_{x'}}{M_x\,c_{x'}\,I_{y'}}\left(p_2 \sec\frac{m_1\,L}{2} + p_1 \operatorname{sech}\frac{m_2\,L}{2}\right)\right] \ldots (31)$$

or, with s and k from Eqs. 16 and 17

$$s_m = s\left[1 + k\left(p_2 \sec\frac{m_1\,L}{2} + p_1 \operatorname{sech}\frac{m_2\,L}{2}\right)\right] \ldots\ldots\ldots\ldots (32)$$

Convenient forms of the arguments are obtained by multiplying and dividing by b^2 the factor in front of the brackets of Eqs. 29, and by making use of the relation $G = \dfrac{E}{2\,(1+\nu)}$. Then

$$\frac{m_1\,L}{2} = \frac{L}{2\,b}\sqrt{\frac{1}{1+\nu}\,B'\left\{\sqrt{1+\left[\frac{4\,(1+\nu)\,s}{E}\,C'\right]^2} - 1\right\}} \ldots (33a)$$

and

$$\frac{m_2\,L}{2} = \frac{L}{2\,b}\sqrt{\frac{1}{1+\nu}\,B'\left\{\sqrt{1+\left[\frac{4\,(1+\nu)\,s}{E}\,C'\right]^2} + 1\right\}} \ldots (33b)$$

in which ν is Poisson's ratio and B' and C' are two non-dimensional characteristics of the cross section:

$$B' = \frac{K\,b^2}{I_{y'}\,h^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34a)$$

and

$$C' = \frac{I_{x'}}{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34b)$$

Also, using Eqs. 33 in Eq. 27*d*, the following simple expressions are obtained for p_1 and p_2:

$$p_1 = \frac{\sqrt{1 + \left[\frac{4\,(1+\nu)\,\bar{s}}{E}\,C'\right]^2} - 1}{2\sqrt{1 + \left[\frac{4\,(1+\nu)\,\bar{s}}{E}\,C'\right]^2}} \tag{35a}$$

and

$$p_2 = \frac{\sqrt{1 + \left[\frac{4\,(1+\nu)\,\bar{s}}{E}\,C'\right]^2} + 1}{2\sqrt{1 + \left[\frac{4\,(1+\nu)\,\bar{s}}{E}\,C'\right]^2}} = 1 - p_1 \tag{35b}$$

If, for abbreviation, Eqs. 33 are rewritten in the form

$$\frac{m_1 L}{2} = \frac{L}{2\,b}\,D'_1; \qquad \text{and} \qquad \frac{m_2 L}{2} = \frac{L}{2\,b}\,D'_2 \tag{36}$$

then from Eq. 32, by making $s_m = s_y$, the secant formula for I-beams is obtained:

$$\bar{s} = \frac{s_y}{1 + k\left[p_2 \sec\left(\frac{L}{2\,b}\,D'_1\right) + p_1 \operatorname{sech}\left(\frac{L}{2\,b}\,D'_2\right)\right]} \tag{37}$$

It is seen from Eq. 37 that, as in the case of rectangular or box-beams, the direct bending stress $\bar{s}$ which produces yield-point stress in the most remote fiber depends not only on k and the slenderness ratio $\frac{L}{b}$, but also on additional characteristics of the cross section, B' and C'. In fact (compare Eqs. 33 and 35), p_1, p_2, D'_1, and D'_2 all depend upon these characteristics.

Eq. 37 is not very convenient to evaluate and can be solved numerically by a cut-and-try process only. However, considerable simplification can be made in most cases. From Eqs. 33 it is seen that $\frac{m_1 L}{2} < \frac{m_2 L}{2}$. Consequently $\sec\frac{m_1 L}{2} = \sec\left(\frac{L}{2\,b}\,D'_1\right) > \operatorname{sech}\left(\frac{L}{2\,b}\,D'_2\right) = \operatorname{sech}\frac{m_2 L}{2}$. In addition, from Eqs. 35 it is seen that $p_1 < p_2$. Consequently,

$$p_2 \sec\left(\frac{L}{2\,b}\,D'_1\right) \gg p_1 \operatorname{sech}\left(\frac{L}{2\,b}\,D'_2\right) \tag{38}$$

and therefore, in most cases, the last term in the denominator of Eq. 37 is negligible. The simplified form of the secant formula for I-beams is then

$$\bar{s} = \frac{s_y}{1 + k\,p_2 \sec\left(\frac{L}{2\,b}\,D'_1\right)} \tag{39}$$

For beams of medium and large slenderness the error so introduced is entirely negligible. Only for beams of rather small slenderness does the term

$p_1 \operatorname{sech}\left(\frac{L}{2\,b}D'_2\right)$ become significant. For such short beams s approaches $\frac{s_y}{1+k}$. Numerical computations for various rolled steel I-sections showed that for $\frac{L}{b}$ corresponding to about $s = 0.9\,\frac{s_y}{1+k}$ the error caused by omitting the hyperbolic term amounts to from 2% to 5%, and increases for still larger values of s $\left(\text{smaller values of } \frac{L}{b}\right)$. Since, for such beams, the hyperbolic term is no longer negligible, as a first approximation $\frac{L}{b}$ is computed from Eq. 39. The value so obtained is somewhat too large. Therefore, a smaller value is

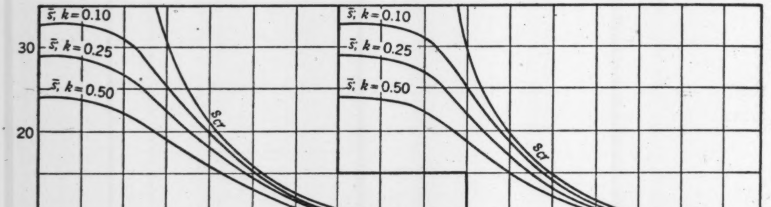


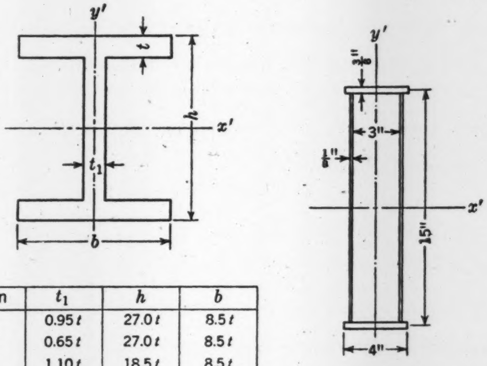
Fig. 2.—Ultimate Stress $\bar{s}$ for a Yield Point of 36,000 Lb per Sq In. and for Various Degrees of Imperfection k; and the Theoretical Buckling Stress s_c for an Ideally Perfect Beam (See Fig. 3 for Dimensions of Beams)

substituted in the accurate Eq. 37 and the process is repeated until satisfactory accuracy is obtained. This method of successive approximation requires not more than two or three trials to give $\frac{L}{b}$ correct within a fraction of 1%.

BEAM CURVES

For columns, the influence of eccentric loading on the strength is demonstrated most easily by plotting s versus $\frac{L}{r}$ for given magnitudes of eccentricity (3)(4)(5). Design curves or formulas are then established by selecting a definite eccentricity most likely to fit practical conditions and applying a given factor of safety to the curve corresponding to that eccentricity (3)(9).

To allow for a similar approach in regard to beams, curves of the same nature for structural I-beams are drawn in Fig. 2. These curves were computed by means of Eqs. 37 and 39 for a yield-point stress s_y = 36,000 lb per sq in. Since, in the case of beams, s depends not only on the slenderness ratio $\frac{L}{b}$ but also on the characteristics B' and C' of the cross section (see Eqs. 34), it is not possible to draw a single set of curves for I-beams of any shape. For this reason, curves are given for four different I-beams, the dimensions of which are indicated in Fig. 3. These dimensions were chosen to cover, as far as possible, most of the range of standard rolled sections. Thus, the dimensions of beam A are approximately those of large heavy beams like 24-in., 100-lb I-beam or 18-in., 94-lb I-beam; those of beam B correspond to the lighter sections of the same depth such as 24-in., 79.9-lb I-beam or 18-in., 54.7-lb I-beam. The dimensions of beam C are approximately those of small, heavy beams like 15-in., 75-lb I-beam or 8-in., 25.5-lb I-beam, whereas those of beam D correspond to the lighter sections of the same depth, such as 15-in., 60.8-lb I-beam or 8-in., 18.4-lb I-beam. In Fig. 2 curves are drawn for three values of the degree of imperfection—k = 0.10, 0.25, and 0.50.



Beam	t_1	h	b
A	0.95 t	27.0 t	8.5 t
B	0.65 t	27.0 t	8.5 t
C	1.10 t	18.5 t	8.5 t
D	0.65 t	18.5 t	8.5 t

FIG. 3.—DIMENSIONS OF BEAMS WHOSE $\bar{s}$-CURVES ARE GIVEN IN FIGS. 2 AND 4

In addition to the s-curves the critical buckling stresses s_c are plotted for the four beams. The stress s_c is the maximum bending stress at which an ideally straight beam bent exactly in the plane of the web buckles laterally.

This s_c is easily computed from (10*a*):

$$M_c = E\frac{\pi^2}{L^2}\sqrt{I^2_o h^2 + 2\,C\,I_o\frac{L^2}{\pi^2}} \quad \text{(40)}$$

From this equation, by simple transformation, the critical stress is found to be

$$s_c = \frac{E\,\pi^2}{2\left(\frac{L}{b}\right)^2}\left(\frac{h}{b}\right)^2\sqrt{\left(\frac{I_{y'}}{2\,I_{x'}}\right)^2 + \frac{K\,I_{y'}}{2\,(1+\nu)\,I^2_{x'}}\left(\frac{L}{\pi\,h}\right)^2} \quad \text{(41)}$$

Eq. 41 was used for plotting the $\left(s_c - \frac{L}{b}\right)$-curves in Fig. 2. From these four sets of curves the following conclusions may be drawn:

Conclusion (*A*).—The general character of the $\bar{s}$-curves is the same for all beams. They start with $\bar{s} = \frac{s_y}{1+k}$ for $\frac{L}{b} = 0$ and approach the s_c-curves asymptotically at large values of $\frac{L}{b}$.

Conclusion (*B*).—For any given values of k and $\frac{L}{b}$ the corresponding value of $\bar{s}$ depends to a considerable degree upon the dimensions of the section. Of the four beams investigated beam B is the weakest laterally, showing the smallest $\bar{s}$ for a given k and $\frac{L}{b}$. It is followed in the order of increasing lateral strength by beams *A*, *D*, and *C*:

Beam	Values of $\bar{s}$ for $k = 0.25$ and $\frac{L}{b} = 50$
B	13,300
A	14,400
D	17,700
C	19,600

Conclusion (*C*).—The $\bar{s}$-curves approach the s_c-curves faster for the laterally weaker beams (AB), so that for such beams s_c is a fair measure of strength for large values of the slenderness ratio $\frac{L}{b}$. However, for laterally stronger beams (CD) the $\bar{s}$-curves fall considerably below the s_c-curves even for large values of $\frac{L}{b}$. Therefore, for such beams, the critical loads computed in the usual way (5)(10) for ideal straightness and ideal accuracy of loading are hardly to be regarded as a reliable criterion for the strength of structural beams with their usual inevitable imperfections.

Conclusion (*D*).—The strength of beams, particularly those of small and medium slenderness, depends mainly on the degree of imperfection k. In this connection it is interesting to note that the inclinations of the planes of loading corresponding to $k = 0.1$ to 0.5 are very small indeed. From Eq. 17 it is seen that

$$k = \frac{M_y}{M_x}\frac{S_{x'}}{S_{y'}} \quad \text{(42)}$$

in which $S_{x'}$ and $S_{y'}$ are the section moduli about the x' and y' axes, respectively. Consequently, the inclination ψ of the loading plane is

$$\psi = \frac{M_y}{M_x} = k\,\frac{S_{y'}}{S_{x'}} \qquad (43)$$

Since, for rolled I-sections, in general $S_{y'} : S_{x'} = 1 : 10$, the degrees of imperfection $k = 0.10$, 0.25, and 0.50 correspond respectively to inclinations of the loading plane of $\psi = 1 : 100$, $1 : 40$, and $1 : 20$. (This compares with a tolerance of 1 : 32 for "out of square or parallel" for American standard beams (11).)

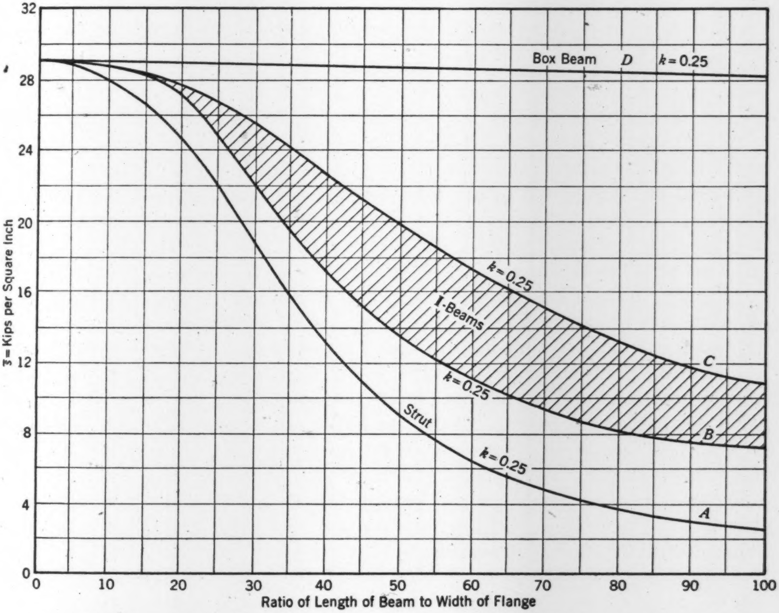


Fig. 4.—Ultimate Stress $\bar{s}$ for a Yield Point of 36,000 Lb per Sq In. and for $k = 0.25$
Curve A: Compression Flange Taken as Disconnected Compression Member Free to Buckle Laterally
Curve B: $\bar{s}$-Curve for Laterally Weak I-Beam B
Curve C: $\bar{s}$-Curve for Laterally Strong I-Beam C
Curve D: $\bar{s}$-Curve for Box-Beam (see Fig. 3 for dimensions of beams)

In Fig. 4, curves B and C for $k = 0.25$ have been drawn once more for the weakest and the strongest of the four beams. (The writer, emphatically, is not proposing to use $k = 0.25$ for design purposes. He selected this value for convenience exclusively, chiefly because design curves for columns are generally based on $\frac{e\,c}{r^2} = 0.25$. The proper degree of imperfection, k, to be used in design can be established only by extensive tests.) Additional computations showed that the curves for most of the standard I-beams and for most of those wide-flange beams which are generally used as beams rather than columns fall between these two extreme curves. In Fig. 4, therefore, the shaded strip represents the region that is of the most practical interest to the designer. (For establishing design curves it is desirable to make a more complete survey of the entire range of rolled sections than the writer was able to undertake. Also, in addition to investigating only a limited number of beams, the writer simplified the shape of the cross sections as shown in Fig. 3(*a*), and also used an approximate formula—$K = \frac{1}{3} \Sigma w t^3$—for determining K. Therefore, the shaded zone in Fig. 4 represents the range of rolled sections in an approximate way only.)

Curve A in Fig. 4 represents the secant curve $\left(\frac{e c}{r^2} = 0.25\right)$ for the compression flange as a strut free to buckle laterally. This curve is seen to fall below any of the beam curves. Since considering the compression flange as a free strut means neglecting the additional resistance to twist and to lateral deflection of the remainder of the beam it is but natural that the values of s so obtained should be much smaller than those computed for the entire beam. It is common practice with many designers to limit the stress in the compression flange according to the column formula. Fig. 4 demonstrates clearly that such a procedure imposes unnecessarily severe restrictions on the allowable compression stress in bending.

Finally, in the same Fig. 4, curve D is given for a box-beam, the dimensions of which are shown in Fig. 3. (The dimensions of this beam are such as to represent a small-scale model of a heavy box girder of the type used in crane structures.) This curve was computed by means of Eq. 22. It illustrates the far greater lateral strength of slender box-beams as compared with I-beams. This difference is caused chiefly by the large resistance to twist characteristic of such sections, or, in other words, by the large value of K as compared with $I_{x'}$. Some design specifications are so worded that the same limitations on allowable compression stress in bending apply both to box-beams and to I-beams. The curves in Fig. 4 show the much greater rigidity of box-beams which is wholly ignored in specifications of this kind. In fact, it appears from these curves that, if proper use is made of the large rigidity of box-beams, such sections are economically far more advantageous than I-sections for heavy structures of great span and considerable slenderness.

BEAMS SUBJECT TO VARIOUS TYPES OF TRANSVERSE LOADING

The foregoing conclusions were obtained by analyzing beams in pure bending; that is, subject to end moments only (see Fig. 1(*a*)). Structural beams, however, are acted upon by variously distributed transverse loads between supports rather than by end moments. It is to be asked, therefore, to what extent is an analysis of beams in pure bending applicable to transversely loaded structural members. An exact investigation of this question meets with practically insurmountable mathematical difficulties. It is easily possible, however, to explore the influence of the character of the load for the two extreme cases of very short beams on the one hand and very slender ones on the other. From such an investigation conclusions can be drawn for beams of medium slenderness.

In Eq. 3c, the moments M_x and M_y may result from any kind of transverse loading or pure bending. For given values of M_x and M_y, however, the magnitude of β is different for different types of loading. Therefore, the only term in Eq. 3c that depends on the type of loading is the one involving β. For very short beams, however, the influence of β is insignificant since, in the limit, $\beta = 0$ for $\frac{L}{b} = 0$. For this limiting case, therefore, Eq. 3c transforms into Eq. 1 which is independent of the type of loading. Consequently, for $\frac{L}{b} = 0$,

$$s = \frac{s_y}{1 + k} \qquad (44)$$

no matter how the beam is loaded. From Figs. 2 and 4 it is seen that all curves start at this value of s and all have horizontal tangents at that point. It can be stated, therefore, that the influence of the character of the load is nil for $\frac{L}{b} = 0$ and is negligibly small for beams of small slenderness ratio. If $\bar{M}_B$ represents the magnitude of M_x resulting in s_B for pure bending, and $\bar{M}_L$ the moment resulting in s_L for any particular type of transverse loading, then, for small values of $\frac{L}{b}$,

$$r = \frac{s_L}{s_B} = \frac{\bar{M}_L}{\bar{M}_B} = 1 \qquad (45a)$$

For slender beams, on the other hand, the secant curves of Fig. 2 approach asymptotically the curves for the critical buckling stress. Therefore, for such beams, the ultimate applied stresses s as obtained from the secant formula depend on the type of loading in the same way as do the critical buckling stresses, s_c. Since the buckling loads of beams subject to various types of loading have been investigated extensively (5)(10)(12), it is possible by means of available formulas to establish the ratios

$$r_c = \frac{s_{cL}}{s_{cB}} = \frac{M_{cL}}{M_{cB}} \qquad (45b)$$

in which M_{cB} is the critical moment for pure bending, M_{cL} that for any particular type of transverse loading, and s_{cB} and s_{cL} are the corresponding maximum fiber stresses. Then, because of the proximity of the s-curves and the s_c-curves for slender beams, $s_L \sim r_c\, s_B$. In this manner it is possible to investigate the influence of transverse loading for the two extreme cases of very short and of very slender beams.

To cover the most characteristic practical types of loading, the ratio r_c will be investigated for I-beams for a concentrated load at the center of the span, for uniformly distributed load, and for two equal loads applied at the quarter points.

The critical loads and moments of transversely loaded beams depend not only on the dimensions of the member and the type of loading, but also on the level, relative to the neutral axis, at which loads are applied (5)(10)(12). The ratios $\bar{r}_c$ for loads applied at the centroidal axis are independent of the configuration of the beam; for loads applied at other levels, however, $\bar{r}_c$ depends on the dimensions of the member. For this reason, loads applied at the centroid will be investigated first.

From Eq. 40, the critical moment for a symmetrical I-beam in pure bending is

$$M_{cB} = E\,\frac{\pi^2}{L^2}\sqrt{I^2_o\,h^2 + \frac{K}{1+\nu}\,I_o\,\frac{L^2}{\pi^2}} \quad \text{.............. (46)}$$

in which I_o is the moment of inertia of one flange about the axis of the web. For a concentrated load applied at the neutral axis (10*b*), the critical moment is

$$M_{cL} = \frac{P_c\,L}{4} = E\,\frac{3\,\pi^3}{L^2}\sqrt{\frac{1}{3\,(6+\pi^2)}}\sqrt{I^2_o\,h^2 + \frac{K}{1+\nu}\,I_o\,\frac{L^2}{h^2}} \quad \text{.... (47)}$$

Consequently, for a center load applied at the neutral axis

$$\bar{r}_c = \frac{M_{cL}}{M_{cB}} = \frac{3\,\pi}{\sqrt{3\,(6+\pi^2)}} = 1.36 \quad \text{.............. (48a)}$$

In the same manner it is found that (12*a*), with loads applied at the neutral axis, for uniformly distributed load,

$$\bar{r}_c = \frac{60\,\pi^2}{8\,\sqrt{30\,(\pi^4+45)}} = 1.13 \quad \text{................ (48b)}$$

and, for quarter-point loads,

$$\bar{r}_c = \frac{17.3\,\sqrt{5.63}}{4\,\pi^2} = 1.04 \quad \text{.................. (48c)}$$

For center load applied at the top flange (10*c*),

$$M_{cL} = \frac{E\,I_o}{L}\,\frac{12\,\pi^2}{6+\pi^2}$$

$$\left\{-\frac{h}{L} + \sqrt{\frac{h^2}{L^2}\left[1 + \frac{\pi^2\,(6+\pi^2)}{48}\right] + \frac{K}{(1+\nu)\,I_o}\,\frac{6+\pi^2}{48}}\right\} \quad \text{.... (49a)}$$

From this equation it is seen that M_{cL} depends in this case on $\frac{h}{L}$ and $\frac{K}{I_o}$. To obtain an estimate of the average magnitude of $\bar{r}_c$ for rolled I-beams, the mean value of $\frac{K}{I_o}$ for the four beams A to D investigated previously will be used; that is $\frac{K}{I_o} = 0.20$. For reasons stated previously, $\bar{r}_c$ is of interest only in the region where the *s*-curves are close to the s_c-curves. From Fig. 2, a reasonable value for this purpose is $\frac{L}{b} = 50$. Since, on the average, for rolled I-beams $\frac{h}{b} = 2.5$, the corresponding value for $\frac{h}{L} = \frac{1}{20}$. For convenience of computation Eq. 46 is rewritten in the form

$$M_{cB} = \frac{E\,\pi^2\,I_o}{L}\sqrt{\frac{h^2}{L^2} + \frac{K}{(1+\nu)\,\pi^2\,I_o}} \qquad (49b)$$

Substitution of $\frac{K}{I_o} = 0.20$ and $\frac{h}{L} = \frac{1}{20}$ in Eqs. 49 gives, as an average value for rolled I-beams, for center load at the top flange, $\bar{r}_c = 1.10$. In a similar manner one obtains for center load at the bottom flange $\bar{r}_c = 1.66$.

By means of the methods just used for center load (12*a*), $\bar{r}_c$ for uniform load and for quarter-point load is easily established.

The values of $\bar{r}_c$ for the three types and the three levels of loading are given in Table 2.

TABLE 2.—VALUES OF $\bar{r}_c$ (EQ. 45*b*)

Load applied at:	Type of Loading		
	Uniform	Center	Quarter point
Top flange	0.96	1.10	0.90
Centroid	1.13	1.36	1.04
Bottom flange	1.34	1.66	1.21

For the slenderness ratio $\frac{L}{b} = 50$ for which Table 2 is computed, the stress $\bar{s}_L$ can now be determined from $\bar{s}$ by means of the expression $\bar{s}_L \sim \bar{r}_c\,\bar{s}_B$.

It is seen from Table 2 that if loads are applied at the top flange the value of $\bar{r}_c$ deviates from 1 by not more than 10%. On the other hand, for short beams $\bar{r} = 1$ (see Eq. 45*a*). For reasons of continuity it is reasonably safe to state, therefore, that $\frac{\bar{s}_L}{\bar{s}_B}$ is very close to 1 for beams of any slenderness and for any type of loading applied at the top flange. (The value $\frac{L}{b} = 50$ for which Table 2 was computed is arbitrary to some extent. However, to compute $\bar{r}_c$ for smaller values of $\frac{L}{b}$ seems inadvisable because of the increasingly large deviation of the $\bar{s}$-curves from the s_c-curves. If $\frac{L}{b} > 50$ had been chosen, the values for "load at centroid" would have stayed the same and the difference between them and those for "load at the top or bottom flange" would have been smaller. Therefore, for $\frac{L}{b} > 50$ the ratios r_c for loads applied at the top flange would still be close to 1 for beams within practically possible ratios of slenderness $\frac{L}{b}$.) Since $\frac{\bar{s}_L}{\bar{s}_B}$ is very close to 1, the influence of the character of the loading on the magnitude of s is seen to be practically negligible if loads are applied at the top flange. The conclusion to be drawn from this discussion is that it appears reasonable to use the s-curves for pure bending for all types of loading rather than to complicate design procedures by taking account of the secondary influence of the character of the load.

In the rather rare cases of loads being applied at the bottom flange such a procedure would be on the conservative side. This is illustrated clearly by Table 2 which indicates the greater stability of beams loaded at the bottom flange as compared with those loaded at the top flange.

THE TORSIONAL CONSTANT K

From the preceding discussion the strength of slender beams is seen to depend on three properties of the cross section—the two principal moments of inertia $I_{x'}$ and $I_{y'}$ and the torsional constant K. Although the structural designer is fully familiar with moments of inertia, some remarks may be in order on the magnitude of K.

For determining the torsional constants of the four beams A to D investigated, the writer used the approximate formula

$$K = \tfrac{1}{3} \Sigma\, w\, t^3 \qquad (50)$$

in which w and t are, respectively, the width and the thickness of the three component rectangles of the sections. More detailed and accurate formulas for I-beams, taking account of the radii of fillets, the slope of the flanges, etc., as well as formulas for other types of cross sections have been developed and compiled by various authors (7)(13). These formulas hold only for monolithic members (rolled sections and the like).

The torsional constants K of built-up (riveted or welded) girders are smaller than those computed for the corresponding monolithic sections. This difference is due to relative slipping of the component parts of fabricated members. Since the strength of slender beams depends directly on the torsional rigidity, it is important to make proper allowance for the smaller value of K for built-up girders as compared with monolithic beams.

Elaborate experimental investigations of the magnitude of K for built-up members were conducted by I. Madsen (14) in 1941. Four I-girders were tested, two of them riveted and two welded, and four box-girders, of which three were riveted and one welded. The girders were of the type commonly used in crane construction. The results of these tests show that for riveted girders, both of box-section and I-section, the effective K amounts to from 30% to 41% of the torsional constant computed for the corresponding monolithic section. It appears, therefore, that for design purposes a value of K equal to one third of that of the monolithic section should be used for riveted girders. The welded I-beams gave effective K-values of 67% and 69% of the value computed for the solid section, whereas the percentage for the one welded box-beam tested by Mr. Madsen was 94%. For welded sections, consequently, a value of K equal to two thirds of that of the monolithic section appears to be safe for design computations.

The amount of experimental evidence on the torsional behavior of built-up members cannot yet be regarded as sufficient. A greater number of tests is desirable on solid web beams in order to provide collateral evidence to Mr. Madsen's results. In addition, to the writer's knowledge, the torsional properties of open web trusses have not yet been investigated experimentally. In view of the general tendency toward lighter and more slender steel and metal structures, the question of the torsional behavior of structural members is becoming increasingly important. Here a wide field for useful experimental work is open to the investigator.

THE INFLUENCE OF LATERAL RESTRAINTS

The findings of the preceding parts of this paper apply to beams with freely supported ends; that is, to beams the ends of which are free to rotate about the x-axis and y-axis. However, beams are generally connected to other structural members in a way that provides some measure of elastic restraint at the supports. Also, long beams are frequently braced laterally at some intermediate points between supports which, with regard to lateral deflection, make them act as beams continuous over several supports, whereas, with respect to vertical deflections, they act as simply supported beams. Such measures increase the lateral stability of beams just as end restraints or intermediate bracing raise the ultimate load of a column above that given by the Eüler formula for hinged ends. It is the purpose of this last part of the paper to indicate the method of making allowance for the influence of such lateral restraints.

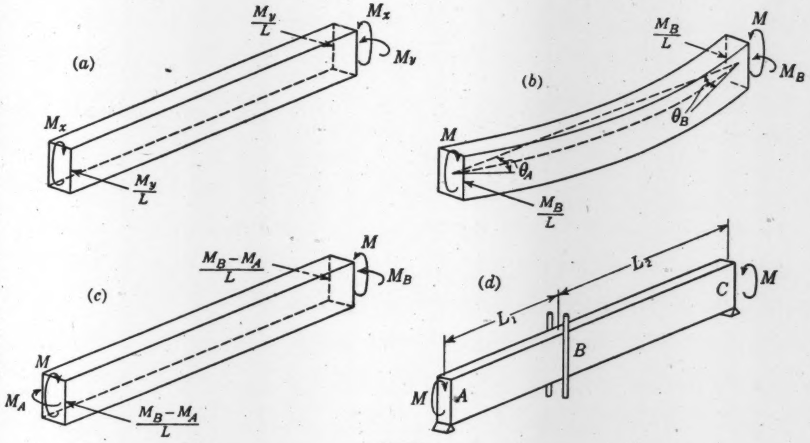


FIG. 5

This problem can be solved by first analyzing the case of a beam acted upon by end moments as shown in Fig. 5(a). With the same nomenclature as used heretofore, the differential equations of equilibrium, in this case, become

$$M_{y'} = \beta\, M_x + M_y \frac{z}{L} = E\, I_{y'} \frac{d^2u}{dz^2} \qquad (51a)$$

$$M_{x'} = -\, M_x + \beta\, M_y \frac{z}{L} = -\, E\, I_{x'} \frac{d^2v}{dz^2} \qquad (51b)$$

$$M_{z'} = -\frac{du}{dz} M_x + \frac{dv}{dz} M_y \frac{z}{L} = G\, K \frac{d\beta}{dz} \qquad (51c)$$

Elimination of u and v results in:

$$\frac{d^3\beta}{dz^3} + \frac{d\beta}{dz}\frac{1}{R_{z'}}\left(M^2{}_x + M^2{}_y \frac{z^2}{L^2}\frac{I_{y'}}{I_{x'}}\right) + \beta \frac{3\, M^2{}_y}{R_{y'}}\frac{z}{L^2}\frac{I_{y'}}{I_{x'}} = \frac{M_x\, M_y}{L\, R_{y'}}\left(\frac{2\, I_{y'}}{I_{x'}} - 1\right) \qquad (52)$$

Eq. 52 is not capable of solution in terms of simple functions. However, for the purpose of considering the stability of deep beams it is again permissible to neglect terms of order $\frac{I_{y'}}{I_{x'}}$ as compared with 1. For the same reason as in the preceding discussion (see comment on Eqs. 10 and 11), this is equivalent to neglecting the second term in Eq. 51*c*, which then becomes

$$-\frac{du}{dz} M_x = G\, K \frac{d\beta}{dz} \qquad (53)$$

Eliminating u from Eqs. 51*a* and 51*c*:

$$\frac{d^2\beta}{dz^2} + \alpha^2{}_1\, \beta = \gamma_1 \frac{z}{L} \qquad (54)$$

in which α_1 and γ_1 are given by Eqs. 11. (If Eq. 54 is differentiated with respect to z—

$$\frac{d^3\beta}{dz^3} + \frac{d\beta}{dz}\frac{M^2{}_x}{R_{y'}} = -\frac{M_x\, M_y}{L\, R_{y'}} \qquad (55)$$

A comparison of Eq. 55 with Eq. 52 shows that actually only terms of order $\frac{I_{y'}}{I_{x'}}$ have been neglected.)

If the beam is freely supported, the end conditions are

$$(\beta)_{z=0}\, (\beta)_{z=L} = 0 \qquad (56a)$$

and the solution of Eq. 54, satisfying Eqs. 51*a*, 53, and 56*a*, is

$$\beta = \frac{M_y}{M_x}\left(\frac{\sin \alpha_1 z}{\sin \alpha_1 L} - \frac{z}{L}\right) \qquad (56b)$$

For stability calculations it is convenient to use $\frac{du}{dz}$, the horizontal projection of the slope of the elastic curve. From Eqs. 53 and 56*b* this slope is found to be

$$\frac{du}{dz} = -\frac{G\,K}{L}\frac{M_y}{M^2_x}\left(\frac{\alpha_1\,L\cos\alpha_1\,z}{\sin\alpha_1\,L} - 1\right) \qquad (57a)$$

In particular, at the ends

$$\left(\frac{du}{dz}\right)_{z=0} = -\frac{G\,K}{L}\frac{M_y}{M^2_x}\left(\frac{\alpha_1\,L}{\sin\alpha_1\,L} - 1\right) \qquad (57b)$$

and

$$\left(\frac{du}{dz}\right)_{z=L} = -\frac{G\,K}{L}\frac{M_y}{M^2_x}(\alpha_1\,L\cot\alpha_1\,L - 1) \qquad (57c)$$

As an example, consider the case of a beam in pure bending built in laterally at $z = L$ (rotation about an axis parallel to y prevented at $z = L$) and freely supported at $z = 0$. In this case

$$\left(\frac{du}{dz}\right)_{z=L} = 0; \qquad \text{and} \qquad \alpha_1\,L\cot\alpha_1\,L = 1 \qquad (57d)$$

This transcendental equation is satisfied by $\alpha_1\,L = 4.49 \cdots$; and consequently, from Eq. 11*a*, the critical moment becomes

$$M_{xc} = \frac{4.49}{L}\sqrt{R_{y'}} \qquad (58)$$

It is worth noticing (5*b*) that the problem of a strut in compression fixed at one end and hinged at the other leads to the same transcendental expression as Eq. 57*d*.

Eqs. 57*b* and 57*c* make it possible to discuss the stability of elastically restrained beams in complete analogy with Professor Timoshenko's treatment of elastically restrained compression members (5).

If θ_A and θ_B represent the horizontal angles of rotation of the elastic line at the supports, positive as shown in Fig. 5(*b*), by suitable transformation Eqs. 57*b* and 57*c* may be rewritten as follows: Comparing Eq. 57*b*—

$$\theta_A = \frac{M_B\,L}{6\,E\,I_{y'}}\frac{3}{\frac{1}{2}\,\alpha_1\,L}\left(\frac{1}{\sin\alpha_1\,L} - \frac{1}{\alpha_1\,L}\right) \qquad (59a)$$

and, comparing Eq. 57*c*—

$$\theta_B = \frac{M_B\,L}{3\,E\,I_{y'}}\frac{3}{\alpha_1\,L}\left(\frac{1}{\alpha_1\,L} - \cot\alpha_1\,L\right) \qquad (59b)$$

This particular form is convenient because, from the ordinary beam theory, $\frac{M_B\,L}{6\,E\,I_{y'}}$ and $\frac{M_B\,L}{3\,E\,I_{y'}}$ are the end rotations of a beam subject to M_B only ($M_x = 0$). The terms in Eqs. 59*a* and 59*b* containing α_1, therefore, have the significance of magnification factors which represent the influence of M_x on the magnitude of the horizontal end rotation θ.

Professor Timoshenko has tabulated (5*e*) values of the functions:

$$\phi(a) = \frac{3}{a}\left(\frac{1}{\sin 2\,a} - \frac{1}{2\,a}\right) \qquad (60a)$$

and

$$\psi(a) = \frac{3}{2\,a}\left(\frac{1}{2\,a} - \cot 2\,a\right) \ldots\ldots\ldots\ldots\ldots\ldots (60b)$$

in which $2\,a = \alpha_1\,L$. Consequently

$$\theta_A = \frac{M_B\,L}{6\,E\,I_{y'}}\,\phi(a) \ldots\ldots\ldots\ldots\ldots\ldots (61a)$$

and

$$\theta_B = \frac{M_B\,L}{3\,E\,I_{y'}}\,\psi(a) \ldots\ldots\ldots\ldots\ldots\ldots (61b)$$

For a beam acted upon by moments at both ends as shown in Fig. 5(*c*), by superposition the angles of rotation are, consequently:

$$\theta_A = \frac{M_A\,L}{3\,E\,I_{y'}}\,\psi(a) + \frac{M_B\,L}{6\,E\,I_{y'}}\,\phi(a) \ldots\ldots\ldots\ldots (62a)$$

and

$$\theta_B = \frac{M_B\,L}{3\,E\,I_{y'}}\,\psi(a) + \frac{M_A\,L}{6\,E\,I_{y'}}\,\phi(a) \ldots\ldots\ldots\ldots (62b)$$

with

$$2\,a = \alpha_1\,L = \frac{M_x\,L}{\sqrt{E\,I_{y'}\,G\,K}} \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

These expressions are exactly the same as the corresponding equations for compression members (5*c*) except for the different significance of *a*. In beams horizontally restrained at the ends, M_A and M_B represent the restraining moments. Consequently, by means of the functions $\phi(a)$ and $\psi(a)$ it is possible to determine the critical moments of elastically restrained beams by exactly the same methods used to calculate the critical thrust for elastically restrained compression members.

Example (*a*).—Suppose a beam in pure bending is braced horizontally at point B as schematically indicated by the guides in Fig. 5(*d*). There are no horizontal restraining moments at the ends, $M_A = M_C = 0$; and, for reasons of equilibrium, $M_{1B} = -\,M_{2B}$. Consequently:

$$\theta_{1B} = \frac{M_B\,L_1}{3\,E\,I_{1y'}}\,\psi(a) \ldots\ldots\ldots\ldots\ldots\ldots (64a)$$

and

$$\theta_{2B} = -\,\frac{M_B\,L_2}{3\,E\,I_{2y'}}\,\psi(a_2) \ldots\ldots\ldots\ldots\ldots\ldots (64b)$$

The continuity of the beam requires that $\theta_{1B} = \theta_{2B}$, which results in

$$\psi(a_1) + \frac{L_2\,I_{1y'}}{L_1\,I_{2y'}}\,\psi(a_2) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

If, for example, the beam is of constant cross section and $L_2 = 2\,L_1$, it follows that

$$\frac{\psi(a_1)}{\psi(2\,a_1)} = -\,2 \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

From Professor Timoshenko's tables (5*e*) it is then found that $2a_1 = \alpha_1 L = 1.93$. Consequently, from $\alpha_1 = \frac{M_x}{\sqrt{E I_{y'} G K}}$:

$$M_c = \frac{1.93\sqrt{E I_{y'} G K}}{L_1} = \frac{3.86\sqrt{E I_{y'} G K}}{L_2} \quad \text{(67)}$$

The corresponding problem for a braced strut in compression is developed by Professor Timoshenko (5*d*), and the critical load for this case is

$$P_c = \frac{1.93^2 E I}{L^2_1} = \frac{3.86^2 E I}{L^2_2} \quad \text{(68)}$$

Example (*b*).—As a second example consider the beam built in laterally at both ends. In this case $M_A = M_B$ and $\theta_A = \theta_B = 0$. From Eqs. 62, $\frac{\psi(a)}{3} + \frac{\psi(a)}{6} = 0$, and $\phi(a) = -2\psi(a)$; and from Professor Timoshenko's tables (5*e*) $2a = \alpha_1 L = 2\pi$. Consequently,

$$M_c = \frac{2\pi\sqrt{E I_{y'} G K}}{L} \quad \text{(69)}$$

Eq. 69 can also be obtained directly from Eqs. 10 and 12 by making $\frac{du}{dz} = 0$ at the ends. In general, then, the critical moment for a beam in pure bending can be expressed in the form

$$M_c = \frac{\pi\sqrt{E I_{y'} G K}}{L'} \quad \text{(70a)}$$

just as the critical thrust for compression members is given by

$$P_c = \frac{\pi^2 E I}{(L')^2} \quad \text{(70b)}$$

In both cases L' is the "equivalent span" and is equal to the distance between the inflection points of the elastic curve. From the three examples computed herein (Eqs. 58, 67, and 69) it is apparent that the ratio $\frac{L}{L'}$ of the actual to the equivalent span for lateral buckling of beams is the same as the ratio $\frac{L}{L'}$ for struts in compression, with corresponding conditions of loading and restraint.

Consequently, in order to take account of any degree of lateral restraint in the design of slender beams one has only to reduce the actual span by an appropriate amount. The equivalent span L' so obtained is then used in the secant formulas or the corresponding curves instead of the actual span L.

The degree of end restraint provided by the usual types of riveted or welded end connections can hardly be determined analytically. For this reason, and also to establish the proper degree of imperfection k to be used in design, it is desirable to make systematic tests on beams under conditions simulating those in actual structures.

SUMMARY

In this paper the writer endeavors to establish a rational basis for the design of slender beams. Just as in the case of column analysis, such design should be based on the occurrence of inevitable imperfections of shape and loading. The paper contains:

(*a*) The derivation of a secant formula for rectangular and box-beams and similar shapes;

(*b*) The derivation of a secant formula for I-beams;

(*c*) Illustrative beam curves which show the influence of variations of shape and degree of imperfection on the ultimate strength of slender beams (curves of this type should serve as design criteria, rather than the buckling load of the compression flange regarded as a free strut, as is frequently done);

(*d*) A discussion of the influence of the character of the load on the ultimate strength;

(*e*) A discussion of the magnitude of the torsional constant K and particularly its value for fabricated (riveted or welded) sections; and

(*f*) A method of accounting for the influence on the ultimate strength of additional restraints such as intermediate bracing, or complete or partial end restraint.

The writer does not propose definite design curves or formulas. To establish design specifications extensive experimental work appears to be necessary to determine the degree of imperfection and the amount of end restraint under service conditions.

ACKNOWLEDGMENT

For the valuable assistance of Messrs. D. F. Gunder and Robert Lewis, Jun. Am. Soc. C. E., respectively Associate Professor and Assistant Professor of Civil Engineering at Colorado State College, Fort Collins, Colo., and at the time, respectively Resident Doctor and McMullen Research Scholar at Cornell University, Ithaca, N. Y., the writer expresses his sincere appreciation. As stated, Professor Gunder collaborated with the writer in deriving Eq. 26; and Professor Lewis undertook the numerical computations for the curves in Figs. 2 and 4.

APPENDIX I

BIBLIOGRAPHY

(1) **"Kipperscheinungen,"** Dissertation, by L. Prandtl, Nueremberg, 1899; and **"Elastic Stability of Long Beams Under Transverse Forces,"** by A. G. M. Michell, *Philosophical Magazine*, Vol. 48, 1899, p. 298.

(2) **"Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings,"** Am. Inst. of Steel Construction.

(3) **"Steel Column Research,"** Final Report of the Special Committee, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1376.

(4) **"Rational Design of Steel Columns,"** by D. H. Young, *ibid.*, Vol. 101 (1936), p. 422.

(5) **"Theory of Elastic Stability,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936. (*a*) p. 240. (*b*) p. 88, Eq. 66. (*c*) p. 13, Eq. 25. (*d*) p. 98. (*e*) p. 499. (*f*) p. 177.
(6) **"Action of Deep Beams Under Combined Vertical, Lateral and Torsional Loads,"** by C. O. Dohrenwend, *Transactions*, A. S. M. E., Vol. 63, 1941, p. A-130.
(7) **"Formulas for Stress and Strain,"** by R. J. Roark, McGraw-Hill Book Co., Inc., 1938.
(8) Discussion by George Winter of **"Theory of Limit Design,"** by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 673.
(9) **"Specifications for Steel Railway Bridges, 1938, Appendix A,"** Am. Railway Eng. Assn.
(10) **"Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending,"** by George Winter, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 247. (*a*) p. 251, Eq. 14*b*. (*b*) p. 252, Eq. 22*b*. (*c*) p.252, Eq. 22*a*.
(11) **Steel Construction Manual,** Am. Inst. of Steel Construction.
(12) Discussion by Robert K. Schrader of **"Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending,"** by George Winter, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 261. (*a*) Eqs. 55 and 62.
(13) **"Structural Beams in Torsion,"** by Inge Lyse and Bruce G. Johnston, *ibid.*, Vol. 101 (1936), p. 857.
(14) **"Report of Crane Girder Tests,"** by I. Madsen, *Iron and Steel Engineer*, November, 1941.
(15) *Bulletin No. 241*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1932.
(16) **"Buckling of Elastic Structures,"** by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.
(17) **"Strength of Steel Columns,"** by H. M. Westergaard and William R. Osgood, *Transactions*, A. S. M. E., Vol. 50, No. 17, 1928, p. 65.
(18) **"The Collapse Strength of Steel Tubes,"** by T. M. Jasper and J. W. W. Sullivan, *ibid.*, Vol. 53, 1931, p. 219.
(19) **"Importance of Compression Yield Strength in Established Setting Depths for Casing,"** by T. McLean Jasper, *Drilling and Production Practice*, Am. Petroleum Inst., 1936.
(20) *Der Eisenbau*, 1912, p. 351.
(21) **"The Strength of I-Beams in Flexure,"** *Bulletin No. 68*, Univ. of Illinois, Urbana, 1913.
(22) **"Beams Without Lateral Support,"** by S. Timoshenko, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1247.
(23) *Sitzungberichte d. bayer, Acad. d. Wiss.*, S. 295, 1921; Versuche u. d. Verdrehungsfestigkeit d. Walzeisenträger.
(24) **Structural Aluminum Handbook,** Aluminum Co. of America, 1938, p. 48.

APPENDIX II

NOTATION

The following letter symbols, adopted for use in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials,[3] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. At the same time, an effort has been made to reconcile the notation of other authors in this field, particularly Professor Timoshenko.

A = cross-section area;
A', B', C', D' are non-dimensional characteristics of the cross section (see Eqs. 19, 34, 36);
$a = \dfrac{\alpha_1 L}{2}$ (see Eq. 63) for use in the Timoshenko tables (5*e*);
B' = (see A');
b = width of beam;
C = a constant (10) $= \dfrac{K}{2\,(1 + \nu)}$; C_1 and C_2 are integration constants determined from the conditions at the ends of a beam;
C' = (see A');
c = distance from the neutral axis to the extreme fiber; $c_{x'}$ and $c_{y'}$ are the distances c measured perpendicular to the x' and y' axes, Fig. 1;
D' = (see A');
E = Young's modulus of elasticity;
e = eccentricity of a load;
G = shearing modulus of elasticity;
h = height of a beam cross section;
I = rectangular moment of inertia: $I_{x'}$ and $I_{y'}$ are the moments of inertia about the axes x' and y', Fig. 1; and I_o is the moment of inertia of one flange about the axis of the web;
K = a torsional constant;
k = degree of imperfection of loading and shape of a beam, defined in Eq. 17;
L = span length;
L' = equivalent span, or the distance between the inflection points of the elastic curve;
M = bending moment:
    M_c = critical moment;
    M_m = maximum moment;
    M_x, M_y, M_z, $M_{x'}$, $M_{y'}$, and $M_{z'}$ = the components of M on the axis denoted;
    M_B = bending moment due to pure bending;
    M_L = bending moment due to any kind of loading except pure bending;
    $\bar{M}$ = the magnitude of M that causes an ultimate applied unit stress $\bar{s}$;
m_1 and m_2 = constants first used in Eq. 26, defined in Eqs. 27 and 29;
P = a concentrated load: P_c = a critical concentrated load;
p_1 and p_2 = constants first used in Eq. 26, defined in Eqs. 27*d*;
q = a constant first used in Eq. 26, defined in Eq. 28;
$R = E\,G\,K\,I$, an abbreviation first used in Eq. 5;

---

[3] ASA—Z10*a*—1932.

r = least radius of gyration;

$\bar{r}$ = a ratio of ultimate stresses or moments: $\bar{r}_c$ = a ratio of critical stresses or moments;

s = stress per unit area:

s_c = critical unit stress;

s_m = maximum unit stress;

s_y = unit stress at the yield point;

s_B = unit stress due to pure bending;

s_L = unit stress due to any kind of loading except pure bending;

$(s)_y = \dfrac{M_y c_{y'}}{I_{y'}}$ = the applied bending stress due to M_y, for use in Eq. 17;

$\bar{s}$ = direct stress for ideally straight beam;

t = thickness; thickness of flange: t_1 = thickness of web;

u u and v (see Fig. 1) are horizontal and vertical displacement ordinates of a section of a beam after rotation through an angle β;

v = (see u);

w = width of component rectangles of an I-section;

x x, y, and z are distances measured parallel to the respective axes; x', y', and z' are the corresponding distances referred to the rotated section. As subscripts they refer a given symbol to the rotated section;

y = (see x);

z = (see x);

α = a constant first used in Eq. 7, defined in Eq. 6*a*: α_1 = an approximate value for α (Eq. 11*a*);

β = angle of twist;

γ = a constant first used in Eq. 7, defined in Eq. 6*b*: γ_1 = an approximate value for γ (Eq. 11*b*);

θ = horizontal angle of rotation of the elastic line at the supports, θ_A and θ_B referring to supports A and B, respectively;

ν = Poisson's ratio;

ϕ = "function of:" as in $\phi(a)$;

ψ = inclination of the loading plane; also, where indicated in the text, ψ = "function of:" as in $\psi(a)$.

Subscripts and Other Modifiers.—Used as subscripts, c = critical, m = maximum; A and B denote the ends of beam AB (except where B is specially defined as meaning "due to bending"); and L denotes "due to any other kind of loading except pure bending." Subscripts x, y, z, x', y', and z' refer to the corresponding axes. Primed symbols in general refer to a non-dimensional characteristic (see A') but they are also used to denote an equivalent value, as in L'. Numerical subscripts 1 and 2 serve to distinguish between two individual quantities of the same general nature, such as C_1 and C_2, m_1 and m_2, etc. Barred symbols in general refer to critical or ultimate values, except that $\bar{s}$ denotes a value for an ideally straight beam.

DISCUSSION

T. McLean Jasper,[4] M. Am. Soc. C. E.—A mathematical attempt to solve a very important problem, and one which is recognized as a necessary addition to the problems dealing with buckling, is very welcome. As a supplement to the work by Professor Winter, the writer would cite the tests on light I-beams by Jasper O. Draffin, M. Am. Soc. C. E., and Milo S. Ketchum, Jr., Assoc. M. Am. Soc. C. E., in 1932 (15) and those by Harald M. Westergaard, M. Am. Soc. C. E., in 1922 (16). The purpose of presenting the first of these references is to suggest some test results to which the author may connect his theories, and the second reference to the generalized problem of buckling which the author has not mentioned should be a helpful extension to the Bibliography.

The writer is particularly intrigued by Fig. 2 and the application of mechanics to this problem. This paper offers a method of approach that can help the designer to simplify the application of the problem. The problem is not a simple one. The physical properties of the material used influence the heavier beams greatly whereas the *E I*-values of the shape and material undoubtedly influence the lighter sections.

The other references (17)(18)(19), with their respective discussions, present very similar cases and may be useful to students of this problem in supplying test data and analysis methods that may inspire further tests which apply more directly to safe designs for slender beams.

C. M. Goodrich,[5] M. Am. Soc. C. E.—In the usual case of the loading in Fig. 6, an eyebeam ideally straight would be prevented from twisting about its longitudinal axis. In one with a slight turn, twisting would be permitted only up to the point where there was flat contact between the two pieces.

In the case of columns, two eccentricities are considered, the first due to an eccentric end load, and the second due to the maximum deviation of the axis from a straight line between the ends. Only the latter needs to be considered in the case of beams.

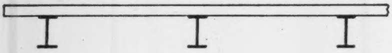


Fig. 6

If the rule for reduction of stress in beams, either in the American Institute of Steel Construction (AISC) specifications or in those of the Canadian National Building Code, is multiplied by $\frac{10}{6}$, a result fairly close to $40{,}000 - \frac{100\,L}{i}$ is obtained (assuming $i = 0.2\,b$). The assumption that $i = 0.2\,b$ was not too violent when only American standard beams were rolled. For this assumption, there is now a variation of from $i = 0.15\,b$ to $i = 0.27\,b$ for rolled beams, with grotesquely varying results if the moment of inertia is accepted as a measure of stiffness; and, of course, built girders and box-beams are not included in the rule, which leaves these cases in the air.

---

[4] Research Dept., A. O. Smith Corp., Milwaukee, Wis.

[5] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

Brik (20) presents eight tests, including those for built and box girders; H. F. Moore (21) gives forty tests, reviews many more, and recommends the formula,

$$\breve{s} = 40{,}000 - 60\,\frac{m\,L}{i} \qquad \text{(71)}$$

in which $m = 1$ for a cantilever, $m = \frac{2}{3}$ for a uniformly loaded beam, etc. In any case of two supports, the constant m is the ratio of the area of the moment diagram to its enclosing rectangle. The range of variation in the tests is stated to be ± 2,500 lb per sq in. His testing arrangement gives the test beam every opportunity to twist if so inclined. Assuming $i = 0.2\,b$, for $\frac{L}{b} = 40$ which corresponds to $\frac{L}{i} = 200$, the minimum stress would be more than 29,000 lb per sq in. for uniformly loaded beams against about 17,000 in Fig. 4. At $\frac{L}{i} = 0$, Professor Moore gives 37,500 from tests, while Fig. 4 gives 29,000. The author cautions that (see heading, "Beam Curves") "The proper degree of imperfection, k, to be used in design can only be established only by extensive tests." It is of interest, however, to note what existing tests reveal, and apparently they show that the AISC and Canadian National Building Code formulas are undesirable, while the author's mathematics yield no rule for the designer.

The factor of stiffness is taken by S. Timoshenko (22) as $\frac{A^4}{40\,(I_{xx} + I_{yy})}$, and, in his example of a 24-in., 79.9-lb I-beam, it is 3.46. This is the Saint Venant rule, which, however, was intended for sections with a chunky solid center, and not for such shapes as I-beams. For the same section the Bethlehem handbook gives 5.01 as the factor of stiffness, and $\Sigma\,\frac{1}{3}\,w\,t^3$ equals about 4, if average thickness is used in the flanges. (August Föppl's tests (23) led him to the factor of stiffness of 1.29 for this type of section, and 1.29 × 4 = say 5.1.)

In designing, when unhampered by specifications, one may perhaps use Professor Moore's rule up to $\frac{L}{i} = 200$, and, for greater values, some such formula as

$$\breve{s} = \frac{1{,}280{,}000{,}000}{\left(\frac{l}{i}\right)^2} \qquad \text{(72)}$$

for uniformly distributed loads, remembering that about 2,500 lb per sq in. is to be subtracted on account of the variation in beams.

The result, of course, is merely a probable ultimate for a condition to which any actual case is likely to be an approximation.

J. P. WADLECK,[6] ESQ.—The author should be complimented for his ability to treat the entire scope of his subject with such admirable simplicity.

---

[6] Development Engr., Niagara Machine & Tool Works, Buffalo, N. Y

The significance of his paper lies in the fact that, in so far as the writer is aware, this is the first direct attempt to rationalize the problem of lateral stability of ideal beams; that is, to find a mathematical solution to the lateral stability and the corresponding critical stress of real beams with inherent imperfections, eccentricity of load and reactions, possible slight obliquity of beam axis, etc.

The author's paralleling of the knowledge and design practice of the engineering profession on lateral stability of beams, as it is at present, with the situation existing in column design before the introduction of the secant formula, if it were true, would indeed be very fortunate.

The author undoubtedly had in mind knowledge as represented by engineering scientists and students of advanced engineering literature. The average graduate engineer, however, who, long before the existence of any of the "real column" solutions, did have or should have had a clearer understanding of the determining factors, governing stability of struts through Euler's basic column theory, finds that the mechanism of lateral beam failure and its stability theory is still beyond his grasp.

The fact that, thus far, relatively few cases of major failures, which are traceable to lateral buckling of beams, are on record is mainly due to the fact that all rolled sections were safely covered by permissible load versus span tables originating from the AISC handbook (2). It seems that, no matter how indiscriminating these tables are relative to the degree of end restraint and to type and position of loading, they apparently cover all errors with a margin on the safe side.

In the design of long, built-up I-section girders, which otherwise were not intended to resist external lateral loads, to the extent of the writer's knowledge, the designer was and still is aided only by his sense of proportion and conservative, occasionally feeble, rules of thumb, available in various specifications. How such elementary considerations proved adequate to guard against lateral failures of these built-up girders—not covered by the AISC tables—the writer can explain only with the substantial combined lateral bendings and torsional strength of the sections.

As new trends are developed, both in materials and fabricating methods, most of which will result in greatly decreased torsional rigidity of sections (due to the increasing use of welding, to the adoption of high-tensile, low-alloy steels, to the various light-weight and thin-wall formed structures in general), the problem of lateral stability of beams will rise to first-degree importance.

To meet this situation without an extensive research and test program is unthinkable. The existing meager data such as those in the AISC and the other steel handbooks are unusable, due to their limited scope.

To attain the widest validity of test results, with the least effort, it is essential that, in any test program of the future, the work be guided and the results be checked against a rigorous and rational analytical expression. In the establishment of such an approach lies the value of the author's work.

Whether the exact analytical expressions are not too involved for routine design work is another matter. If so, for the purpose of specification writing and tables, they could easily be supplemented with various empirical formulas each within the scope of the respective specification, and the use of the original analytical formula, which is in conformity with the entire range of tests, could be restricted to the treatment of special cases.

Concerning the mathematical phase of the paper the writer calls attention to some interesting possibilities by transforming Eq. 20, 22, or 23. Substituting the value of A' from Eq. 19 into Eq. 22:

$$s = \frac{s_y}{1 + k \sec\left(\frac{I_{x'}}{h}\, s\, \frac{L}{\sqrt{D\,C}}\right)} \qquad \text{(73a)}$$

in which $D = E\,I_{y'}$ = the lateral flexural rigidity; and $C = K\,G$ = the torsional rigidity of the beam. Recalling that $\frac{1}{2} \times \frac{2\,I_{x'}}{h} \times s = \frac{1}{2} M$ and $\frac{L}{\sqrt{D\,C}} = \frac{\pi}{M_c}$, thus:

$$s = \frac{s_y}{1 + k \sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)} \qquad \text{(73b)}$$

or multiplying both sides by $\frac{2\,I_{x'}}{h}$:

$$M = \frac{M_y}{1 + k \sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)} \qquad \text{(74)}$$

Inasmuch as values of $\sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)$ vary between 0 and ∞ (when $M = 0$ and $M = M_c$, respectively), it would be well worth while to explore the possibility of expressing $\sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right)$ by a closely conforming rational function, by which Eqs. 3 could be transformed into a quadratic equation.

An extremely close approximation to all values of $\sec \alpha$ between 0 and $\frac{\pi}{2}$ (which unfortunately, however, yields a cubic equation) is due to H. A. Webb:

$$\sec \alpha = \frac{1 + \frac{4\,\alpha^2}{\pi^2} \times 0.26}{1 - \frac{4\,\alpha^2}{\pi^2}} \qquad \text{(75)}$$

by which:

$$\sec\left(\frac{\pi}{2} \times \frac{M}{M_c}\right) = \frac{M^2_c + 0.26\,M^2}{M^2_c - M^2} \qquad \text{(76)}$$

If substituted into Eq. 74, Eq. 76 will lead to the expression:

$$(1 - 0.26\,k)\,M^3 - (1 + k)\,M\,M^2_c - M^2\,M_y + M^2_c\,M_y = 0 \qquad \text{(77)}$$

By letting $(1 - 0.26\,k) = \alpha$ and $(1 + k) = \beta$, and as, in general:

$$M = s\,\frac{2\,I_{x'}}{h} \qquad \text{(78)}$$

Eq. 77 can be written as:

$$\alpha \bar{s}^3 - \beta \bar{s} s^2_c - \bar{s}^2 s_y + s_y s^2_c = 0 \qquad (79)$$

which is solvable for $\bar{s}$ by either the exact or one of the approximate methods.

H. D. HUSSEY,[7] M. AM. SOC. C. E.—It is interesting to compare the beam formulas in current use with the secant formula, Eq. 37. Beam formulas from current specifications are as follows: American Railway Engineering Association and American Association of State Highway Officials specifications for bridges,

$$s_b = 18{,}000 - 5 \frac{L^2}{b^2} \qquad (80a)$$

and American Institute of Steel Construction specifications for buildings (maximum = 20,000),

$$s_b = \frac{22{,}500}{1 + \dfrac{L^2}{1{,}800\, b^2}} \qquad (80b)$$

in which s_b is the maximum allowable compressive unit stress in bending.

If the bridge formula, Eq. 80*a*, is multiplied by $\frac{28.8}{18}$, and the building formula, Eq. 80*b*, by $\frac{28.8}{20}$, ultimate stresses may be obtained which will be consistent with those presented by the author in Fig. 4. Fig. 7 shows the ultimate stresses thus derived from the bridge and building formulas when plotted in Fig. 4.

Thus the equation for curve E, Fig. 7, is:

$$\bar{s} = \frac{28.8}{18}\left(18{,}000 - 5 \frac{L^2}{b^2}\right) = 28{,}800 - 8 \frac{L^2}{b^2} \qquad (81a)$$

and, for curve F, Fig. 7 (maximum, 28,800):

$$\bar{s} = \frac{28.8}{20}\left[\frac{22{,}500}{1 + \dfrac{L^2}{1{,}800\, b^2}}\right] = \frac{32{,}400}{1 + \dfrac{L^2}{1{,}800\, b^2}} \qquad (81b)$$

For $L/b = 30$, Eqs. 81*a* and 81*b* give identical results.

Fig. 7 indicates that the present design formulas are conservative. The close correspondence between the curves representing the present design formulas and the author's results suggests that the tentative value of $k = 0.25$ may be a satisfactory assumption until the correct value can be established by tests.

The author has proved the great torsional strength of box-beams, theoretically, and has confirmed the results of tests reported by I. Madsen (14) in 1941. One of Mr. Madsen's conclusions was: "For welded (box) girders with L/b ratios up to 110, and riveted (box) girders with L/b ratios up to 80, it is not necessary to reduce the allowable compressive design stress." This statement seemed rather startling until Professor Winter gave it a theoretical background, calculated the ultimate stresses for a box-beam for varying values of L/b, and plotted them in Fig. 4. This beam which has a maximum strength of 28,800 lb per sq in. when $L/b = 0$, has a strength of more than 28,000 lb per sq in. when $L/b = 100$. Current specifications make

---

[7] Designing Engr., Am. Bridge Co., New York, N. Y.

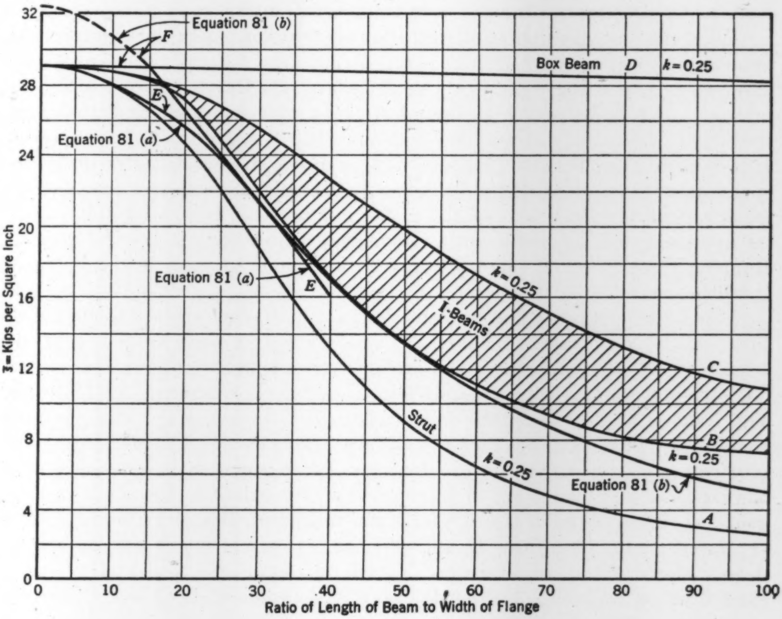


FIG. 7.—ULTIMATE STRESS $\bar{s}$ FOR A YIELD POINT OF 36,000 LB PER SQ IN. AND FOR $k = 0.25$ (COMPARE FIG. 4)

no distinction between the flanges of a box girder and an I-beam, as regards allowable flange stress. The same formula is used for both sections. Present evidence seems to justify a review of current practice in respect to allowable flange stresses in box girders.

The author is to be commended for having presented a rational solution of the strength of beams. He has demonstrated that this strength is represented by a secant formula and has thereby developed a basis for a working formula for beams. Since the secant formula cannot be solved readily, it would be necessary to use an approximation, as has been done for column formulas. Fig. 7 shows that Eqs. 81 give a good approximation of a secant curve for the ultimate stress $\bar{s}$ at any value of L/b.

Results presented in Fig. 7 indicate that (a) it is safe to use beams having an L/b ratio greater than 40, which is the currently specified limit; and (b) a formula of the Rankin-Gordon type would define a satisfactory design stress for such slender beams.

George Winter,[8] Esq.—The agreement among the discussers on the insufficiency, in some respects, of present design specifications for slender beams is gratifying to the writer. Messrs. Jasper, Wadleck, and Hussey also concur in recognizing the usefulness of this analytical treatment for establishing more rational design procedures than those used currently.

From the designer's point of view the practical conclusions drawn from the paper by Mr. Hussey, particularly Fig. 7, are most interesting. These conclusions, in which the writer fully concurs, are: (1) Current specifications do not account for the very high lateral strength of box-beams, and, therefore, lead to uneconomical design of such sections; (2) it is safe to use I-beams with an L/b-ratio larger than 40, provided that working stresses are adjusted correspondingly; (3) present design formulas for I-beams are conservative; and (4) for practical use the values of the rather cumbersome secant formula should be approximated by some more convenient expression of the Johnson or Rankine type, in much the same way as it is done in some current column specifications.

Fig. 7 demonstrates that present design stipulations are about correct or just slightly conservative for laterally weak members, such as 18-in., 54.7-lb I-beams. These specifications are extremely conservative, however, for laterally strong sections, such as 15-in., 75-lb I-beams. For example, for $L/b = 40$, Fig. 7 shows that for the latter beam a design stress is justified which is more than 30% higher than that for the former. This demonstrates the practical importance of one of the principal findings of this investigation—namely, that the ratio L/b is not sufficient to determine the strength of such beams. On the contrary, in addition to the L/b-ratio the characteristics of the cross section expressed by the quantities B' and C' (Eqs. 34) are of major importance for determining safe design stresses. Therefore, design specifications probably should not be limited to one single curve for I-beams of any shape, but a set of curves or formulas should be established to take account not only of the L/b-ratio but also of the characteristics of the cross section. This becomes even more important if the stipulations are also to apply correctly to built-up girders on the one hand and to junior I-beams on the other as observed by Mr. Wadleck. If members of this type are considered, the variations in strength for a given L/b-ratio will be found to be still larger than those for the standard I-beams in Figs. 4 and 7. These practical design formulas could well be of the Rankine type, as proposed by Mr. Hussey. A still better approximation to the secant curve can be obtained by taking a formula of the Johnson parabolic type for the lower range of L/b combined with one of the Rankine type for the higher range.

Mr. Goodrich's contention that "the author's mathematics yield no rule for the designer" is best answered by Mr. Hussey's very practical conclusions. The writer never intended to propose definite design specifications in his paper since the establishment of such codes is the task of the appropriate engineering organizations rather than of any single individual. When Eüler and others developed a rational theory of column behavior they did not, at the same time, attempt to prescribe definite design requirements; yet the total knowledge of the design of compression members would be poor indeed were it not for the fundamental investigations of these men.

---

[8] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

Mr. Goodrich cites Professor Moore's tests in 1913 and attempts to compare the results with Fig. 4 of the paper. Such a comparison is not quite appropriate for the following reasons:

(*a*) Fig. 4 is drawn for one definite yield point only, 36,000 lb per sq in. The writer could not find any reference by Professor Moore (21) to the yield points of this particular series of tests, but some other yield points reported in the course of the same investigation were as high as 40,000 lb per sq in. and more. Thus, the curves in the writer's paper, drawn for a yield-point stress of 36,000 lb per sq in., cannot coincide with Professor Moore's value of $s = 37{,}500$ to 42,500 lb per sq in. for $L/i = 0$. A further reason for this discrepancy is the well-known fact that short steel beams, with L/i close to zero, fail at loads higher than those which cause yielding in the outermost fiber (8). Since, however, incipient yielding is generally regarded as the limit of structural usefulness, this state rather than that of complete failure is analyzed by the secant formula. The difference between this limiting load and that causing complete failure decreases rapidly with increasing slenderness.

(*b*) Fig. 4 is drawn for an entirely arbitrary degree of imperfection $k = 0.25$. A comparison of Professor Moore's results with those for $k = 0.10$ would have yielded entirely different results. Probably, for the accuracy of a careful experimental investigation $k = 0.25$ is much too high.

(*c*) From a study of Professor Moore's testing arrangement complete freedom of twist and of lateral deflection does not seem to have been achieved. A rather minute restraint of lateral freedom at or near the mid-length of a column will increase the buckling strength considerably. The same is true for buckling of beams. Professor Moore's values, therefore, are likely to be rather high as compared with those that would be obtained in tests in which complete freedom of movement was provided. Such freedom in beam tests in standard testing machines is extremely difficult to achieve. To the writer's knowledge, the only satisfactory, although cumbersome, way of doing so is to suspend actual dead loads from the test beams rather than to apply the loads by means of a testing machine.

In connection with Professor Moore's tests it is interesting to cite corresponding tests on junior I-beams by Messrs. Ketchum and Draffin (15), quoted by Mr. Jasper. The formula given by these investigators,

$$s = 24{,}000 - 40\left(\frac{m\,L}{i}\right) \ldots\ldots\ldots\ldots\ldots\ldots (82)$$

gives values which are very considerably below those of Professor Moore's original formula, Eq. 71, despite the practical identity of testing arrangements. This divergence once again emphasizes the fact that neither the flange width b nor the radius of gyration i is sufficient to describe the lateral strength of beams. Empirical column formulas expressed in terms of L/i give reasonable results for the sole reason that L/i is the only shape factor on which the strength of columns depends. This is shown by all rigorous analytical investigations of column behavior. Conversely, all rigorous analytical investigations of buckling of beams (1)(5)(6)(10)(12), including the present one, show most decidedly that neither L/b nor L/i is sufficient to determine the strength of slender beams. Thus, none of the approaches proposed by Mr. Goodrich, based on L/i, will give satisfactorily accurate results. This has been recognized, for instance, in recommended design procedures for aluminum alloy products (24), in which the allowable design stress is determined by use of an "equivalent radius of gyration" whose value depends on precisely the same quantities as the writer's B' and C' and is in no way identical with Mr. Goodrich's ordinary radius of gyration i.

Mr. Goodrich's discussion of the value of the torsional constant K is somewhat extraneous to the subject of this paper. The writer gave an approximate formula, Eq. 50, and references to the most recent and complete information in this connection. Since the validity of this information is generally recognized, there seemed to be no need for a detailed discussion of this constant except as far as its value for built-up sections is concerned.

Mr. Wadleck's mathematical extension of the paper represents an interesting generalization of the writer's results. Eq. 73*b*, expressing s in terms of the ratio of the applied to the critical moment, M/M_c, agrees with general concepts on stability, in particular with the Southwell method (5*f*) of determining deflections of imperfect compression members.

Like Mr. Wadleck, the writer did attempt to express the secant formula in terms of a simpler, rational function, but met with the same difficulties as did Mr. Wadleck. However, the practical inconvenience of solving a secant formula can be overcome by approximating it with a combination of a Johnson and a Rankine type formula, as noted herein. A set of suitably chosen curves of this nature would eliminate any necessity for the designer to solve the secant formula directly. Graphs consisting of a large number of curves are widely used in reinforced concrete design where rather complicated equations are involved, like those governing the design of eccentrically loaded columns. The writer cannot see any objection to the introduction of graphs of this kind in steel design if by so doing design procedures are improved and economy is achieved.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2233

RIVETED AND PIN-CONNECTED JOINTS OF STEEL AND ALUMINUM ALLOYS

BY LEON S. MOISSEIFF,[1] M. AM. SOC. C. E., AND E. C. HARTMANN,[2] AND R. L. MOORE,[3] ASSOC. MEMBERS, AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JONATHAN JONES, AND LEON S. MOISSEIFF E. C. HARTMANN, AND R. L. MOORE.

SYNOPSIS

Development of low alloy steels and high-strength aluminum alloys, and their utilization in structures, have called the attention of engineers to the somewhat lower ductilities of these materials, as compared to carbon steel, and also to their generally higher ratios of yield strength to ultimate strength. Apprehension has been felt that these differences in physical properties would affect, unfavorably, the distribution of stress in members built of these materials, especially at connections. This belief is reflected by the fact that, in some instances, organizations responsible for design specifications, both in the United States and abroad, have set an upper limit for the ratio of yield strength to ultimate strength.

In recent years substantial advances have been made in the metallurgy of metals and in the production of numerous metallic alloys of various physical qualities. The utilizations of the metals of higher qualities have led engineers to scrutinize, more critically, the meaning and value of the information obtained from generally accepted standard tests. Along with this advance came a more thorough inquiry into the significance of the terms commonly used in characterizing and classifying metals. The old established values came to be questioned: Is there a true elastic limit? What is the true meaning of yield strength? What is ductility, and does elongation determine its value? How much ductility is needed for a given purpose, and what is its value?

In view of the practical importance of the problem and the need for experimental data concerning the behavior of these newer structural materials, it

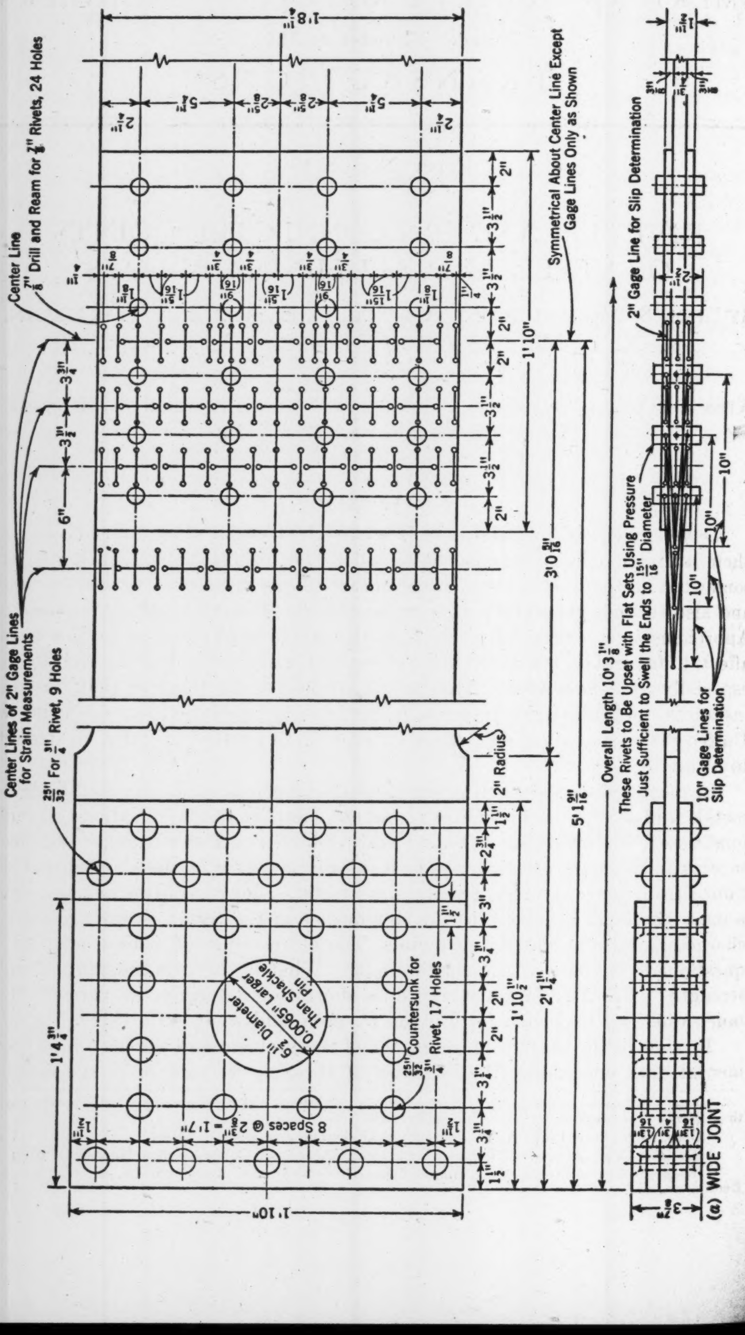
---

NOTE.—Published in January, 1943, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

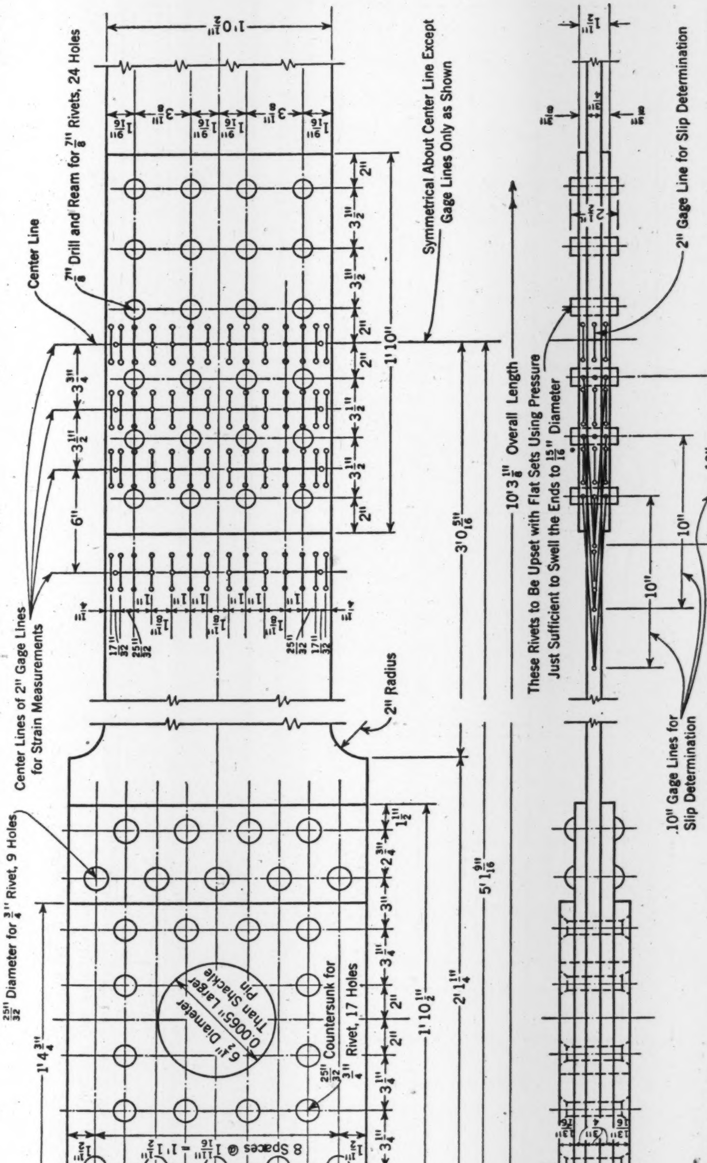
[1] Cons. Engr., New York, N. Y. (Mr. Moisseiff died on September 3, 1943.)

[2] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[3] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

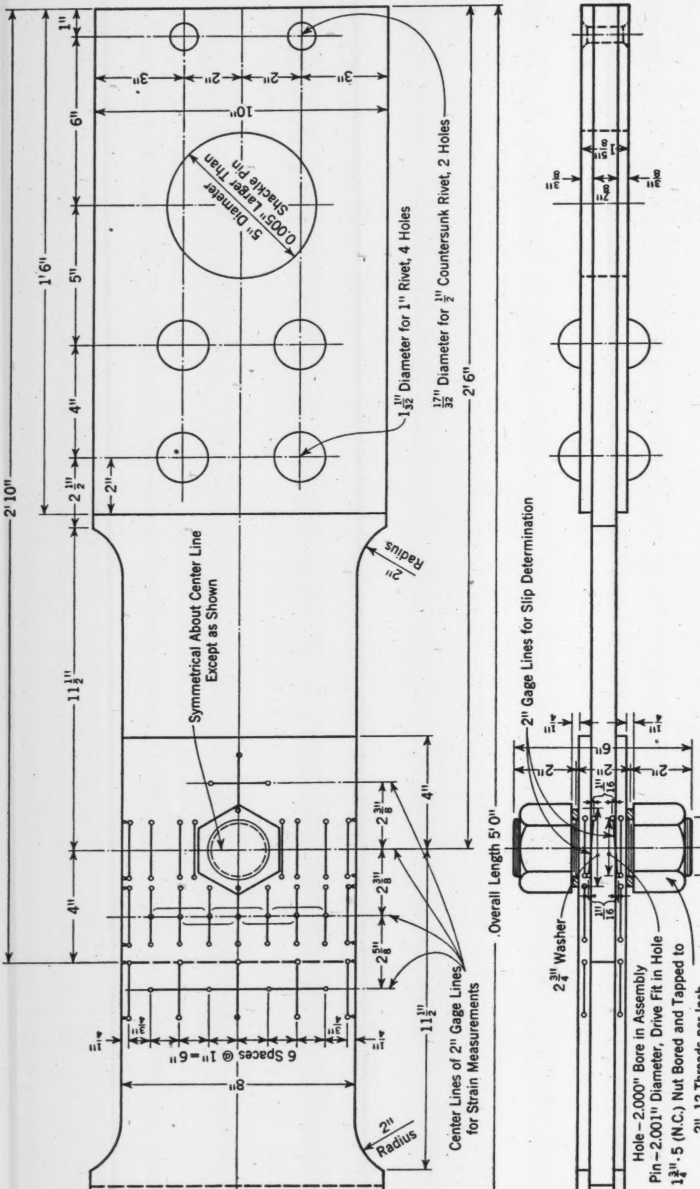

Center Line
7/8" Drill and Ream for 7/8" Rivets, 24 Holes
Center Lines of 2" Gage Lines for Strain Measurements
25/32" For 3/4" Rivet, 9 Holes
6 1/2" Diameter 0.0065" Larger Than Shackle Pin
25/32" Countersunk for 3/4" Rivet, 17 Holes
8 Spaces @ 2 3/8" = 1'7"
Symmetrical About Center Line Except Gage Lines Only as Shown
Overall Length 10' 3 1/8"
These Rivets to Be Upset with Flat Sets Using Pressure Just Sufficient to Swell the Ends to 15/16" Diameter
2" Gage Line for Slip Determination
10" Gage Lines for Slip Determination
2" Radius
1'8 1/4"
1'10"
3'0 5/16"
5'1 9/16"
2'1 1/4"
1'10 1/2"
1'4 3/4"
1'10"
10"
3 3/4"
3 1/2"
6"
2"
2 1/4"
3"
3 1/4"
1 1/2"
5 1/4"
2 5/8"
1 1/4"
7/8"
1 15/16"
1 9/16"
3/8"
3 5/8"

(a) WIDE JOINT

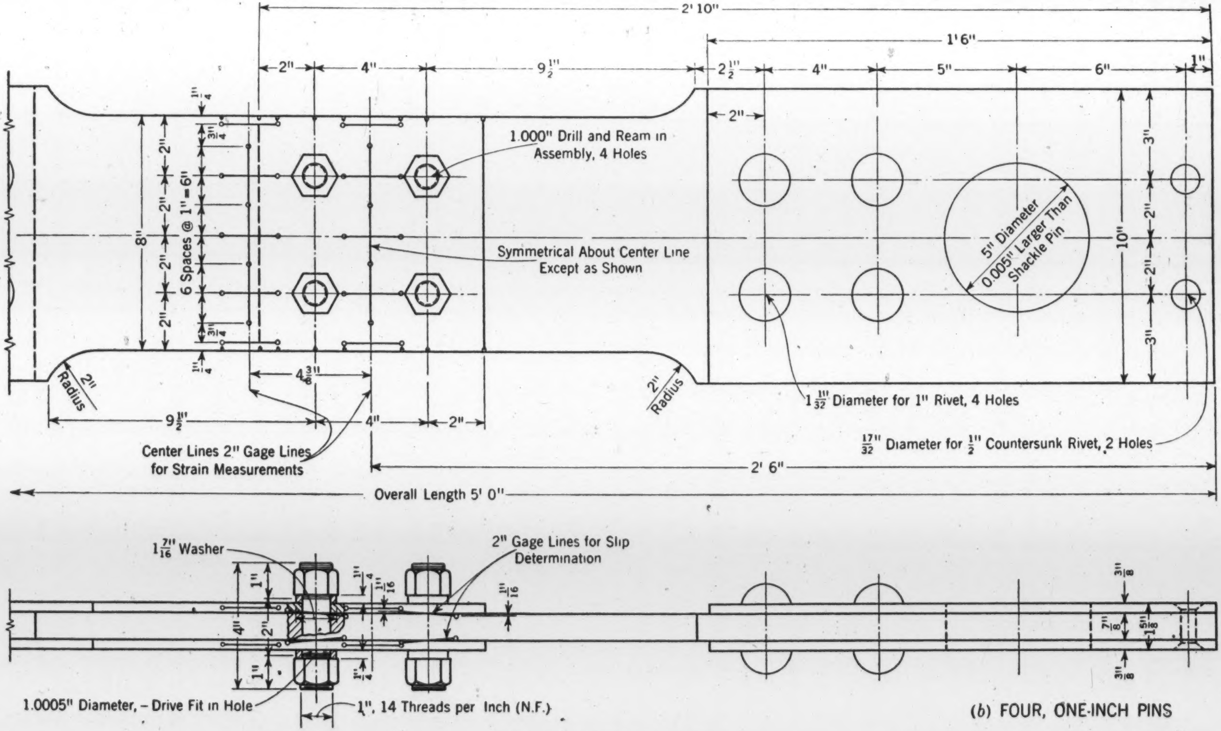

Center Line
7/8" Drill and Ream for 7/8" Rivets, 24 Holes
Symmetrical About Center Line Except Gage Lines Only as Shown
Center Lines of 2" Gage Lines for Strain Measurements
25/32" Diameter for 3/4" Rivet, 9 Holes
2" Radius
These Rivets to Be Upset with Flat Sets Using Pressure Just Sufficient to Swell the Ends to 15/16" Diameter
2" Gage Line for Slip Determination
10" Gage Lines for Slip Determination
6 1/2" Diameter 0.0065" Larger Than Shackle Pin
25/32" Countersunk for 3/4" Rivet, 17 Holes
8 Spaces @ 1 11/16" = 1' 1 1/2"
1' 0 3/4"
1' 4 3/4"
1' 4 3/4"
1' 10"
1' 10 1/2"
2' 1 1/4"
3' 0 5/16"
5' 1 9/16"
10' 3 1/8" Overall Length
10"
10"
10"
2"
3 1/2"
3 1/2"
3 1/4"
6"
3 1/8"
1 9/16"
1 1/2"
2 3/4"
3"
3 1/4"
3 3/8"

(b) NARROW JOINT

FIG. 1.—TEST JOINTS WITH TWENTY-FOUR, ⅞-IN. RIVETS


1"
6"
5"
4"
2½"
2"
1'6"
2'10"
11½"
4"
4"
3"
2"
2"
3"
10"
5" Diameter 0.005" Larger Than Shackle Pin
1$\frac{1}{32}$" Diameter for 1" Rivet, 4 Holes
$\frac{17}{32}$" Diameter for ½" Countersunk Rivet, 2 Holes
2'6"
2" Radius
Symmetrical About Center Line Except as Shown
2⅜"
2⅜"
2⅝"
4"
11½"
½"
¾"
¾"
½"
6 Spaces @ 1" = 6"
8"
2" Radius
Center Lines of 2" Gage Lines for Strain Measurements
Overall Length 5'0"
⅜"
⅞"
⅜"
1⅝"
2" Gage Lines for Slip Determination
½"
½"
2"
2"
6"
2"
$\frac{1}{16}$"
$\frac{1}{16}$"
2", 12 Threads per Inch (N.S.)
2¾" Washer
Hole – 2.000" Bore in Assembly
Pin – 2.001" Diameter, Drive Fit in Hole
1¾".5 (N.C.) Nut Bored and Tapped to 2", 12 Threads per Inch

(a) TWO-INCH PIN


2' 10"
1' 6"
2"
4"
9½"
2½"
4"
5"
6"
1"
2"
1.000" Drill and Ream in Assembly, 4 Holes
Symmetrical About Center Line Except as Shown
5" Diameter 0.005" Larger Than Shackle Pin
10"
3"
2"
2"
3"
¼"
¾"
8"
6 Spaces @ 1" = 6"
4⅜"
2" Radius
9½"
4"
2"
1 11/32" Diameter for 1" Rivet, 4 Holes
17/32" Diameter for ½" Countersunk Rivet, 2 Holes
Center Lines 2" Gage Lines for Strain Measurements
2' 6"
Overall Length 5' 0"
1 7/16" Washer
2" Gage Lines for Slip Determination
1"
4"
2"
1/16"
3/8"
7/8"
1⅝"
1.0005" Diameter, - Drive Fit in Hole
1", 14 Threads per Inch (N.F.)

(*b*) FOUR, ONE-INCH PINS

FIG. 2.—PIN-CONNECTED JOINTS

was thought best to undertake a program of tests of riveted and pin-connected joints. Five metals—two steels and three aluminum alloys—having a fairly wide range of ductilities and ratios of yield strength to ultimate strength were included. All the joints tested behaved in a manner consistent with the basic assumptions of design and with the properties of the materials. Furthermore, the tests showed that the plastic yielding necessary to effect a uniform distribution of load among the rivets must be relatively small, and that differences in ratios of yield strength to ultimate strength, elongation values, and moduli of elasticity were not significant factors as far as static ultimate strengths were concerned.

INTRODUCTION

It is a fact well known to engineers and demonstrated by numerous investigators[4] that within the elastic range the load on a multiple-riveted joint is not uniformly distributed between the rivets. The basis for the usual assumption of equal shear per rivet in design is that, in ductile materials, the yielding which occurs at the points of highest stress effects a redistribution of load which becomes essentially uniform. Numerous tests on riveted joints have demonstrated the validity of this assumption as far as ultimate static strengths are concerned.[5,6]

The increasing use of alloy steels and high-strength aluminum alloys in the structural field has raised a question regarding the possible effect of the reduced ductility of these materials upon the load distribution and the strength of multiple-riveted joints. Ductility is an essential requirement of a structural material, but the limits within which it may be varied without appreciably influencing the behavior of large riveted joints are not known. In the case of the aluminum alloys, the belief has been expressed by some that the effect of reduced ductility may be offset somewhat by the low modulus of elasticity.

The tests described in this paper were undertaken in 1939, in an effort to obtain information regarding load distribution and the strength and general behavior of multiple-riveted joints involving various combinations of steel and aluminum alloys. An attempt was made to keep the specimen proportions within the limits of good design practice, and at the same time to provide a fairly wide range of strength, stiffness, and ductility. In addition to the multiple-riveted joints, a series of pin-connected joints of the same materials was also included. The scope of the program limited the tests to three rows of rivets on each side of the joint.

DESCRIPTION OF JOINTS

Five different types of joints were fabricated from each of the combinations of material investigated. Fig. 1(*a*) shows the dimensions of the wide double-

---

[4] "Work of Rivets in Riveted Joints," by A. Hrennikoff, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 437–489, especially pp. 461–469.

[5] "Tests of Joints in Wide Plates," by W. M. Wilson, James Mather, and C. O. Harris, *Bulletin No. 239*, Univ. of Illinois Eng. Experiment Station, 1931.

[6] "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 1193–1299.

TABLE 1.—ELEMENTS OF TEST JOINTS

No.	Description	Plate Material[a]				
		Silicon	Carbon	27S-T	17S-T	53S-T
1	Rivet or Pin Material	Carbon	Carbon	Carbon	17S-T	53S-W
	(*a*) JOINT DESIGNED TO FAIL IN SHEAR (SEE FIG. 1(*a*)); SPECIMEN 20¼ IN. WIDE, WITH 24, ⅞-IN. RIVETS; SHEAR AREA, 14.43 SQ IN.[b]					
	Thickness, in Inches.					
2	Main plates	0.750	0.773	0.740	0.780	0.760
3	Cover plates	0.380	0.380	0.389	0.373	0.368
	Mimimum Areas, in Square Inches:					
4	Gross tension	15.20	15.38	14.98	15.10	14.90
5	Net tension[b]	12.55	12.72	12.40	12.50	12.32
6	Bearing[b]	7.88	7.98	7.77	7.83	7.72
	(*b*) JOINT DESIGNED TO FAIL IN TENSION (SEE FIG. 1(*b*)); SPECIMEN 12½ IN. WIDE, WITH 24, ⅞-IN. RIVETS; SHEAR AREA, 14.43 SQ IN.[b]					
	Thickness, in Inches:					
7	Main plates	0.750	0.755	0.740	0.780	0.760
8	Cover plates	0.380	0.380	0.389	0.373	0.368
	Minimum Areas, in Square Inches:					
9	Gross tension	9.37	9.44	9.25	9.32	9.20
10	Net tension[b]	6.75	6.80	6.66	6.71	6.62
11	Bearing[b]	7.88	7.93	7.77	7.84	7.73
	(*c*) AUXILIARY CONTROL SPECIMEN DESIGNED TO FAIL IN SHEAR; SPECIMEN 5 IN. WIDE, WITH 4, ⅞-IN. RIVETS[c]; SHEAR AREA, 2.41 SQ IN.[b]					
	Thickness, in Inches:					
12	Main plates	0.750	0.750	0.750	0.750	0.750
13	Cover plates	0.375	0.375	0.375	0.375	0.375
	Minimum Areas, in Square Inches:					
14	Gross tension	3.75	3.75	3.75	3.75	3.75
15	Net tension[b]	3.09	3.09	3.09	3.09	3.09
16	Bearing[b]	1.31	1.31	1.31	1.31	1.31
	(*d*) JOINT DESIGNED TO FAIL IN BEARING (SEE FIG. 2(*a*)); SPECIMEN 8 IN. WIDE, WITH ONE, 2-IN. PIN; SHEAR AREA, 6.28 SQ IN.[b]					
	Thickness, in Inches:					
17	Main plates	0.875	0.875	0.875	0.875	0.875
18	Cover plates	0.379	0.382	0.382	0.369	0.368
	Minimum Areas, in Square Inches:					
19	Gross tension	6.06	6.11	6.11	5.90	5.89
20	Net tension[b]	4.54	4.58	4.58	4.42	4.42
21	Bearing[b]	1.52	1.53	1.53	1.48	1.47
	(*e*) JOINT DESIGNED TO FAIL IN SHEAR (SEE FIG. 2(*b*)); SPECIMEN 8 IN. WIDE, WITH FOUR, 1-IN. PINS[d]; SHEAR AREA, 6.28 SQ IN. (EXCEPT 53S-T, WHICH WAS 7.09 SQ IN.)[b]					
	Thickness, in Inches:					
22	Main plates	0.875	0.875	0.875	0.875	0.875
23	Cover plates	0.379	0.382	0.382	0.369	0.368
	Minimum Areas, in Square Inches:					
24	Gross tension	6.06	6.11	6.11	5.90	5.89
25	Net tension[b]	4.54	4.58	4.58	4.42	4.32
26	Bearing[b]	3.03	3.06	3.06	2.95	3.13

[a] 27S-T, 17S-T, and 53S-T are aluminum alloy and temper designations.

[b] Areas indicated are based on the nominal hole diameters shown in Figs. 1 and 2.

[c] Specimens of each alloy in this group were also fabricated with two, ⅞-in. rivets.

[d] The diameter of the 53S-W pins in Table 1(*e*) is 1 1/16 in.

strap butt joints having twenty-four $\frac{7}{8}$-in. rivets designed to fail in shear. The longitudinal spacing of the rivets was governed largely by the space required for the operation of a 2-in. Berry strain gage between rivet rows, although it is also consistent with design practice for the size of rivets and thicknesses of plate involved. Fig. 1(*b*) shows the dimensions of the narrow double-strap butt joints, also having twenty-four $\frac{7}{8}$-in. rivets, but designed to fail by tension in the plates. These joints were similar to the wide joints shown in Fig. 1(*a*) except for the over-all width and transverse spacing of rivets.

Small auxiliary double-strap butt joints, 5 in. wide and having two or four $\frac{7}{8}$-in. rivets, were designed to serve as control specimens for determining the shear strength of the driven rivets. The plate thicknesses were the same as shown in Fig. 1(*a*). Three of these specimens were provided for each combination of material, two being fabricated with four rivets (two in each half), whereas the third contained only two rivets.

Fig. 2(*a*) shows the dimensions of the joints having a single 2-in. pin, designed to fail by bearing in the outside cover plates. Fig. 2(*b*) shows the dimensions of the joints having four 1-in. pins designed to fail in shear. In one instance, it was necessary to make two sets of pins in order to obtain the close fit desired, and pins of $1\frac{1}{16}$-in. diameter were finally used.

The rivets used in the joints shown in Fig. 1 were not conventional ones, but consisted of $2\frac{1}{2}$-in. lengths of $\frac{7}{8}$-in. rod which were pressed into $\frac{7}{8}$-in. drilled and reamed holes and then upset cold in an 80-ton yoke-type riveter to approximately $\frac{15}{16}$ in. at the ends. This type of rivet was used to permit an inspection of the plate material adjacent to the rivets during the test, to provide additional clearance for taking strain readings between rivets, and to minimize the uncertain effect of friction.

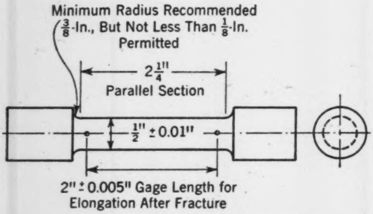

Minimum Radius Recommended
$\frac{3}{8}$-In., But Not Less Than $\frac{1}{8}$-In.
Permitted
$2\frac{1}{4}$"
Parallel Section
$\frac{1}{2}$" ± 0.01"
2" ± 0.005" Gage Length for
Elongation After Fracture

FIG. 3.—STANDARD ROUND TENSION TEST SPECIMEN WITH 2-IN. GAGE LENGTH

The pins used in the joints shown in Fig. 2 were turned oversize by approximately 0.0005 in. per in. diameter to provide a press fit in the joints. All holes in both riveted and pin-connected joints were drilled and reamed in assembly. Table 1 gives a summary of the dimensions of all the joints as well as the areas used in computing shear, bearing, and tensile stresses.

Materials.—The following combinations of material,[7] providing a fairly wide range of ductilities and ratios of yield strength to ultimate strength, were investigated in each of the foregoing types of joints:

1. Silicon steel plates with carbon steel rivets or pins;
2. Carbon steel plates with carbon steel rivets or pins;
3. Aluminum alloy 27S-T plates with carbon steel rivets or pins;
4. Aluminum alloy 17S-T plates with 17S-T rivets or pins; and
5. Aluminum alloy 53S-T plates with 53S-W rivets or pins.

---

[7] "Structural Aluminum Handbook," Aluminum Co. of America, Pittsburgh, Pa., 1940.

In the foregoing list, all specimens were proportional to that shown in Fig. 3.[8] Table 2 gives a summary of the tensile properties of the plate material. As will be noted, there is some variation between the properties of different

TABLE 2.—TENSILE PROPERTIES OF PLATE MATERIAL

No.	Material	Nominal thickness (in.)	Diameter of specimen (in.)	Orientation of test specimen[a]	Pounds per Square Inch		Ratio: Col. 5 / Col. 6	Percentages	
					Yield strength[b]	Ultimate strength		Elongation in four diameters	Reduction of area
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Silicon Steel	⅜	0.250	Normal	64,000	94,700	0.68	24.0	43
2		⅜	0.250	Parallel	64,100	95,100	0.67	29.0	57
3		¾	0.500	Normal	45,900	84,400	0.54	24.5	43
4		¾	0.500	Parallel	49,900	88,800	0.56	26.5	53
5	Carbon Steel	⅜	0.250	Normal	48,100	62,000	0.78	34.0	57
6		⅜	0.250	Parallel	43,300	62,500	0.69	39.0	64
7		¾	0.500	Normal	35,700	62,000	0.58	33.0	56
8		¾	0.500	Parallel	33,500	60,200	0.56	37.0	64
9	27S-T	⅜	0.188	Normal	46,300	62,600	0.74	14.7	24
10		⅜	0.188	Parallel	47,800	61,900	0.77	17.3	32
11		¾	0.500	Normal	45,700	59,300	0.77	11.0	20
12		¾	0.500	Parallel	46,000	58,400	0.79	12.5	30
13	17S-T	⅜	0.188	Normal	36,800	63,000	0.58	25.3	32
14		⅜	0.188	Parallel	41,600	63,300	0.65	24.0	38
15		¾	0.500	Normal	42,600	66,400	0.64	18.0	22
16		¾	0.500	Parallel	46,800	65,800	0.71	19.0	27
17	53S-T	⅜	0.188	Normal	30,500	39,100	0.78	24.0	48
18		⅜	0.188	Parallel	31,500	38,600	0.82	24.0	53
19		¾	0.500	Normal	36,100	40,700	0.89	14.0	35
20		¾	0.500	Parallel	35,700	39,000	0.92	17.5	50

[a] "Normal" denotes that the specimen was normal to the direction of rolling; and "parallel" denotes "parallel to the direction of rolling."

[b] For aluminum, the yield strength is measured by the stress corresponding to an offset of 0.2% from the initial modulus line; and, for steel, it is the stress corresponding to the drop-of-the-beam yield point.

thicknesses of the same materials, particularly in the case of yield strengths, and there is also some evidence of directional properties. Ratios of tensile yield strength to ultimate strength ranged from a minimum of 0.54 for the silicon steel to a maximum of 0.92 for the aluminum alloy 53S-T. Corresponding ratios for the carbon steel plates exceeded those for the silicon steel, the highest value in any case being 0.78, which is considerably above that generally assumed for such material.

The elongation values given in Table 2 range from a minimum of 11% for the aluminum alloy 27S-T to a maximum of 39% for the carbon steel. It should be emphasized that elongation values do not necessarily bear a definite relationship to ratios of tensile yield strength to ultimate strength; nor do they always give a complete picture of the ductility of a metal. For example, the elongations found for the aluminum alloy 53S-T plates exceeded those for the aluminum alloy 27S-T; yet the ratios of yield strength to ultimate strength for the 53S-T alloy were considerably higher. The elongations for the aluminum alloy 17S-T plates were slightly higher than for the aluminum alloy 53S-T plates and the yield-ultimate strength ratios were lower; yet in general 17S-T alloy is not as satisfactory for forming operations as 53S-T alloy. The explanation for this behavior is indicated, at least qualitatively, by the reduction in area values included in Table 2. The values for the 53S-T alloy plates were higher than those for either of the other aluminum alloys, being more nearly comparable with those for the silicon steel. The reductions in area for the carbon steel plates were the highest observed.

---

[8] "Tentative Methods of Tension Testing of Metallic Materials," Supplement to ASTM Standards, Pt. 1, 1940, E8-40T, Fig. 3, p. 454.

No attempt was made to determine moduli of elasticity, the nominal values of 29,500,000 and 10,300,000 lb per sq in. being assumed satisfactory for interpreting the measured strains in the steel and aluminum alloys, respectively. The corresponding values of Poisson's ratio were assumed to be ¼ and ⅓.

Testing Procedure

All tests, except those on the small auxiliary shear specimens, were made in the Templin precision metal-working machine having a 1,000,000-lb maximum capacity tensile range (named for R. L. Templin, M. Am. Soc. C. E.).[9] Load was applied to the large joints of the type shown in Fig. 1 through 6.5-in. pins fitted in special tension shackles. The smaller pin-connected joints shown in Fig. 2 were loaded through 5-in. pins equipped with bushings to fit in the same tension shackles. Figs. 4 and 5 are close-up views of different types of joints. The small auxiliary shear specimens were tested in a machine having a 300,000-lb maximum capacity range.

The test procedure followed on all the large joints included one or more types of slip determination, using a 10-in. Whittemore strain gage or dial indicators on 2-in. and 48-in. gage lengths; the measurement of strains by means of 2-in. Berry strain gages; and the determination of ultimate strengths. The slip readings obtained by means of the Whittemore strain gage involved establishing 10-in. gage lengths on the edges of the main plates of the joints with one gage hole at each rivet section where slip was to be determined. Single gage holes were then located on the adjacent cover plates at these same rivet sections so that readings could be taken on three lines at each section using a common gage hole at one end. The difference in readings obtained on the main and cover plates gave the slip directly, irrespective of the strain that may have occurred over the gage lengths. The angle between the gage lines was so slight that the error introduced by the difference in slopes was on the order of one part in a thousand. Check readings were held to within a tolerance of one division of the dial, which corresponded to a change in gage length of 0.0001 in. Fig. 1 shows the location of the 10-in. slip gage lines.

A dial indicator, graduated in thousandths of an inch and having a 1-in. travel, was used for the combined slip and strain determinations made on the 2-in. and 48-in. gage lengths. For the short gage length, two points were attached to the indicator—one a fixed point attached to the back of the dial and one a movable point attached to the stem so that it could be operated over a considerable range in the vicinity of 2 in. Readings estimated to the nearest 0.0001 in. were taken with this instrument across the gap between the main plates at the center of all the double-strap butt joints and also diagonally across the main and cover plates in the pin-connected joints at the sections through the pins. Figs. 1 and 2 show the location of these slip gage lines. For the 48-in. gage lengths (which, of course, were limited to the large joints), a dial indicator was attached to an adjustable measuring rod and readings were taken between small pins screwed into tapped holes in the edges of the main plates.

---

[9] *Civil Engineering*, May, 1940, p. 323.

Figs. 1 and 2 show the location of the 2-in. longitudinal and transverse gage lines on which strains were investigated by Berry strain gages. It was appreciated that a complete picture of the highly localized stress distribution in the vicinity of the rivets and pins could not be obtained from the layout of the gage lines shown; yet this procedure seemed satisfactory as far as indicating general characteristics of behavior for different materials was concerned. Check strain readings were held to within a tolerance of four-tenths of one division on the dial, which corresponds to a stress of approximately 1,200 lb per sq in. in steel and 400 lb per sq in. in aluminum.

The measurement of both longitudinal and transverse strains at a number of points was necessary to determine the state of stress existing and to indicate the manner in which the strains should be interpreted. The consistent tendency for the measured transverse strains to range from ¼ to ⅓ the longitudinal values, and to be of opposite sign, provided ample justification for the assumption of a unidirectional state of stress on the sections investigated. Accordingly in all tests, measured longitudinal stresses (in the direction of loading) were obtained by multiplying the unit longitudinal strains by the modulus of elasticity.

Because of the fact that the tests on a number of the joints extended over a period of several days and it was not always possible to maintain a uniform temperature, particular attention was paid to the matter of providing standard bars for all the slip and strain measuring instruments used. These reference bars, of both steel and aluminum, were kept in contact with, or near, the joints being tested at all times.

All of the joints were subjected to a number of increments of load leading to ultimate failure. The general procedure followed was to select increments of load small enough that from six to eight sets of slip determinations could be obtained below the design load. In most cases these same increments were used as long as any slip measurements were taken. Permanent sets or slips were measured for the design loads and for several increments exceeding this value.

Strain measurements were usually limited to from four to six increments of load within the elastic range of the plate material or within the limits of strain that could be covered by the gages. In most cases an effort was made to obtain a complete set of strains at loads equal to approximately 1, 1½, 2, and 2½ times the design load. Intermediate as well as higher loads were used in a

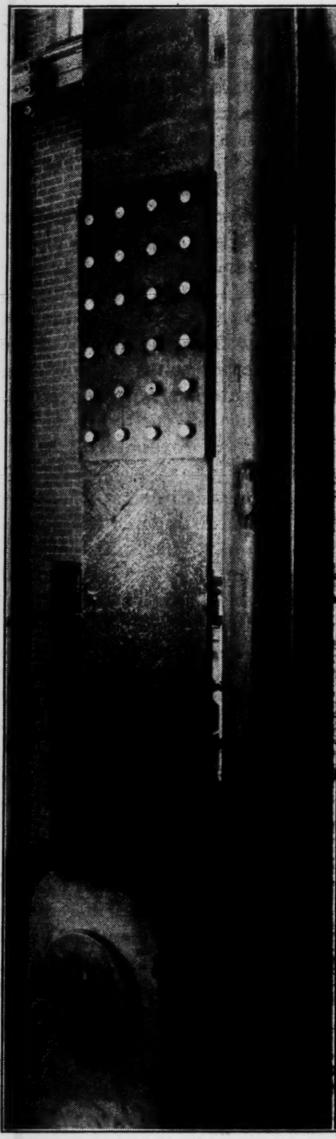


FIG. 4.—CARBON STEEL JOINTS, 12½ IN. WIDE

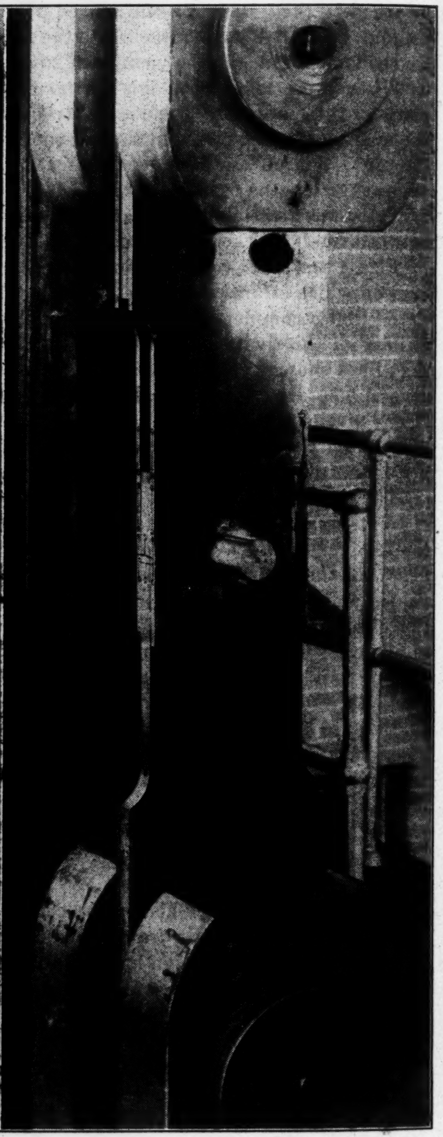


FIG. 5.—PIN-CONNECTED JOINT, 8 IN. WIDE; 2-IN. PIN

number of cases, however, particularly on the aluminum joints. All the joints of any one type were subjected to at least one common load so that a direct comparison of strains could be made. Design stresses used as a basis for selecting increments of test load are given in Table 3.

TABLE 3.—DESIGN STRESSES (KIPS[a] PER SQUARE INCH) USED AS A BASIS FOR SELECTING TEST LOAD INCREMENTS

Type of stress	Silicon steel	Carbon steel	27S-T	17S-T	53S-T[b]
Shear........	..	13.5	..	10	7[b]
Tension.......	24	18.0	21	16	12
Bearing.......	32	27.0	30	26	18

[a] 1 kip = 1 "kilo-pound" = 1,000 lb. [b] The rivets and pins in this group were 53S-W.

RIVETED JOINTS DESIGNED TO FAIL IN SHEAR

Fig. 1(*a*) shows the dimensions of the large joints included in this series of tests. There are four points of interest to be considered: The distribution of load between the rivet rows; the load at which the first appreciable yielding or permanent slip between plates was observed; the distribution of measured strain in the plates; and the ultimate shear strength of the joints.

Table 4 presents data for plotting the load-slip relations for different rivet sections in the small auxiliary control specimens, having two and four rivets, and in the joints, 20¼ in. wide, having twenty-four rivets. By reducing all loads to an average value per rivet, it was possible to plot these relations for all joints of the same material in one figure, thereby facilitating direct comparisons (see Fig. 6). In the case of the auxiliary specimens having only two rivets, of course, there was no question as to the distribution of load. In the auxiliary specimens having four rivets the load should have been equally distributed between the rivets from conditions of symmetry, and the load-slip relations shown for these specimens support this contention. In the large joints having twenty-four rivets, however, there is a consistent difference between the slip obtained at sections A and C, through the end rows of rivets, and that obtained at section B through the intermediate rows. If the load on each row of rivets was proportional to the observed slip between the plates, then rows A and C were more effective than row B, which is in accordance with the theory for the behavior of such joints.[4] The fact that slip was observed in all joints beginning with the application of the first load indicates that there was no appreciable friction between the plates, which result is contrary to that reported by recent investigators using riveted joints of a more conventional type.[6,10] The relations between load and slip were not linear, even in the early stages of the tests, which is also contrary to the behavior usually assumed for riveted joints within the elastic range. There was no evidence of any slip necessary to bring the rivets into bearing, probably because the rivets in these joints were driven cold, in holes which had been drilled and reamed in assembly.

Table 5 gives a summary of the measured slips obtained on the auxiliary shear specimens with corresponding ratios of load to slip, or so-called "*K*-values." Since linear load-slip relations were not obtained for any of the joints, *K*-values have been given for the shear design loads only. Values of

---

[10] "Fatigue Tests of Riveted Joints," by W. M. Wilson and F. P. Thomas, *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, Urbana, 1938.

TABLE 4.—AVERAGE PLATE SLIP, IN TEN THOUSANDTHS OF AN INCH, AS OBSERVED ON JOINTS DESIGNED TO FAIL IN SHEAR

No.	Joint[a]	Average Load per Rivet, in Kips:											
		2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0
	(a) Silicon Steel Plates; Carbon Steel Rivets												
	Two-Rivet:												
1	Section A	2	2	4	6	8	10	13	18	36	90	138	195
	Four-Rivet:												
2	Section A	2	3	5	8	9	12	15	23	37	81	129	189
3	Section B	2	4	6	7	10	13	16	22	39	86	...	196
	Twenty-Four Rivet:												
4	Section A	3	4	6	9	12	16	19	26	51	95	132	195
5	Section B	1	2	2	5	7	8	11	17	42	86	...	186
6	Section C	1	3	6	9	12	14	18	25	49	93	133	195
	(b) Carbon Steel Plates; Carbon Steel Rivets												
	Two-Rivet:												
7	Section A	1	3	5	7	9	12	15	18	26	94	145	204
	Four-Rivet:												
8	Section A	1	3	4	6	9	11	14	18	31	91	136	195
9	Section B	1	3	5	7	10	12	16	20	33	93	141	200
	Twenty-Four Rivet:												
10	Section A	2	5	7	10	13	17	20	27	56	100	155	225
11	Section B	1	2	4	5	7	10	13	19	47	91	146	217
12	Section C	2	3	5	7	10	13	17	23	51	91	153	226
	(c) 27S-T Plates; Carbon Steel Rivets (See Fig. 6)												
	Two-Rivet:												
13	Section A	3	7	11	16	21	28	35	45	69	107	158	216
	Four-Rivet:												
14	Section A	3	7	12	17	23	30	37	47	72	111	162	229
15	Section B	3	7	10	15	19	25	31	42	67	108	160	228
	Twenty-Four Rivet:												
16	Section A	4	9	15	21	27	35	45	62	92	142	207	286
17	Section B	2	4	7	11	15	20	26	37	69	120	182	260
18	Section C	4	8	14	19	26	33	43	60	91	141	210	...
	(d) 17S-T Plates; 17S-T Rivets												
	Two-Rivet:												
19	Section A	5	11	17	23	31	40	50	61	77	96	121	162
	Four-Rivet:												
20	Section A	5	11	17	23	31	40	50	62	77	96	122	164
21	Section B	5	11	17	24	32	40	50	61	76	95	121	168
	Twenty-Four Rivet:												
22	Section A	5	11	18	26	36	47	62	78	98	126	165	226
23	Section B	3	8	13	19	26	35	45	59	77	101	138	197
24	Section C	6	13	20	28	38	49	63	80	100	128	170	228
	(e) 53S-T Plates; 53S-W Rivets												
	Two-Rivet:												
25	Section A	4	10	18	28	42	64	126	...	...	...	...	...
	Four-Rivet:												
26	Section A	5	11	18	28	43	70	130	...	...	...	...	...
27	Section B	5	11	19	30	45	72	134	...	...	...	...	...
	Twenty-Four Rivet:												
28	Section A	6	13	23	35	52	85	160	...	...	...	...	...
29	Section B	3	7	13	24	40	68	145	...	...	...	...	...
30	Section C	5	11	20	31	49	82	155	...	...	...	...	...

[a] For diagrams of joints, location of sections, and typical plotting of data, see Fig. 6.

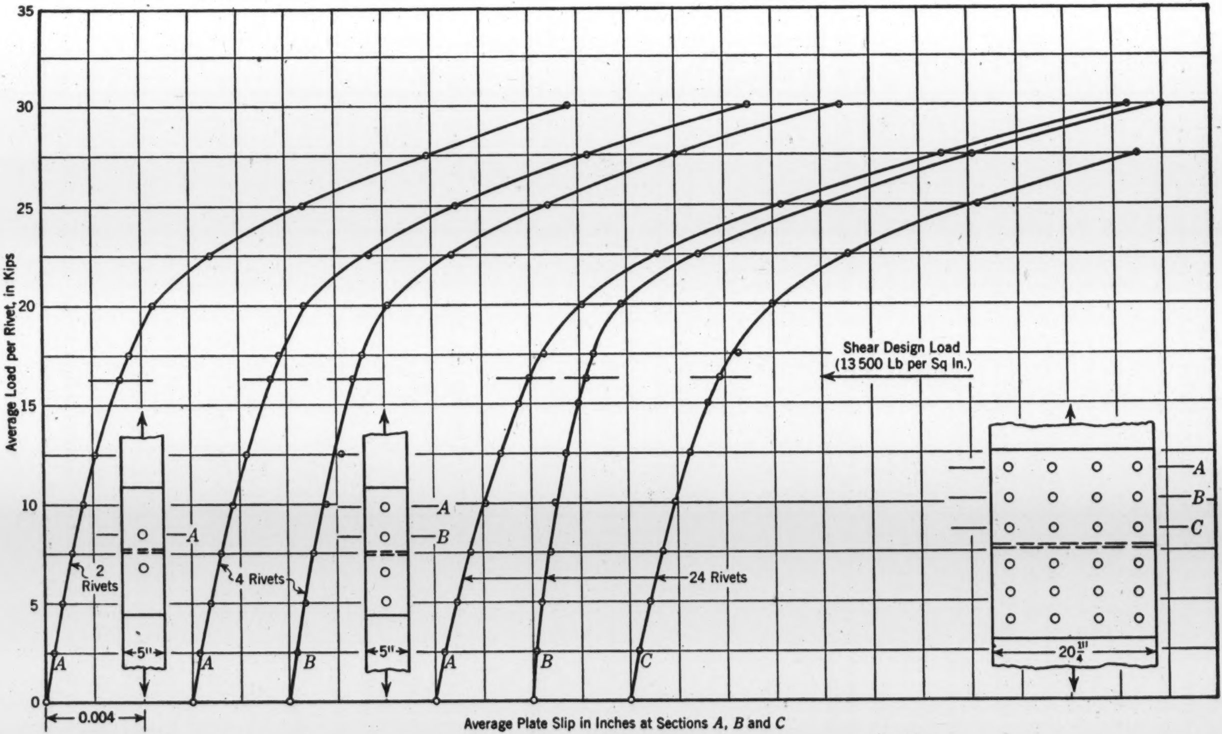


FIG. 6.—LOAD-SLIP CURVES FOR JOINTS OF 27S-T PLATES AND CARBON STEEL RIVETS

the latter are indicated on the curves in Fig. 6 and by the data in Table 5. There are several points of interest to be noted from the data given in Table 5. The measured slips for section A of the joints having only two rivets were practically the same as those measured on sections A and B of the joints having four rivets; the *K*-values for the joints having aluminum plates and rivets were about one third of those obtained for the all-steel specimens, which corresponds approximately to the ratio of the moduli of elasticity for these two materials; and the *K*-values for the joints having aluminum alloy 27S-T plates and steel rivets were about 43% of those obtained for the all-steel specimens, indicating that deformation in the aluminum plates rather than detrusion of the rivets was largely responsible for the magnitude of the slips observed.

TABLE 5.—MEASURED PLATE SLIP IN AUXILIARY SHEAR SPECIMENS

MATERIAL		Shear design stress (lb per sq in.)	Corresponding design load per rivet (lb)	MEASURED SLIP, IN INCHES[a]				Slip modulus or *K*-value[b]
Plates	Rivets			Two-rivet specimen, section A	Four-rivet specimen		Average	
					Section A	Section B		
Silicon	Carbon	13,500	16,250	0.0012	0.0013	0.0014	0.0013	12,500,000
Carbon	Carbon	13,500	16,250	0.0013	0.0013	0.0014	0.0013	12,500,000
27S-T	Carbon	13,500	16,250	0.0031	0.0033	0.0027	0.0030	5,420,000
17S-T	17S-T	10,000	12,000	0.0029	0.0028	0.0030	0.0029	4,140,000
53S-T	53S-W	7,000	8,400	0.0021	0.0021	0.0022	0.0021	4,000,000

[a] Determined from load-slip curves shown in Fig. 6 and Table 4. [b] Ratio of design load per rivet to average slip.

The distribution of load between sections A, B, and C of the large joints may be determined from the ratio of the measured slips on these sections without regard to the results obtained on the auxiliary specimens where the load distribution was known. Table 6 gives a summary of the measured slips on sections A, B, and C for the shear design loads and gives the corresponding percentages of load carried by each rivet row. These results are consistent

TABLE 6.—MEASURED PLATE SLIP AND CORRESPONDING LOAD DISTRIBUTION IN RIVETED JOINTS 20¼ IN. WIDE DESIGNED TO FAIL IN SHEAR

MATERIAL		Design load per rivet (lb)	MEASURED SLIP IN INCHES[a]; SECTIONS:			PERCENTAGE OF TOTAL LOAD CARRIED BY SECTIONS:[b]			THEORETICAL LOAD DISTRIBUTION (PERCENTAGES)[c] AT SECTIONS:		
Plates	Rivets		A	B	C	A	B	C	A	B	C
Silicon	Carbon	16,250	0.0018	0.0009	0.0015	43	21	36	36.8	26.4	36.8
Carbon	Carbon	16,250	0.0018	0.0010	0.0014	43	24	33	36.8	26.4	36.8
27S-T	Carbon	16,250	0.0040	0.0023	0.0037	40	23	37	37.4	25.2	37.4
17S-T	17S-T	12,000	0.0034	0.0024	0.0036	36	26	38	36.7	26.6	36.7
53S-T	53S-W	8,400	0.0027	0.0016	0.0024	40	24	36	36.6	26.8	36.6

[a] Determined from load-slip curves shown in Fig. 6 and Table 4. [b] Determined from relative magnitudes of measured slips on sections *A*, *B*, and *C*. [c] Based on *K*-values given in Table 5.

in indicating that the two end rows in each half of the joints carry more load than the intermediate rows. There are differences between the percentage values for sections A and C, however, which cannot be explained, other than by stating that they probably resulted from minor inequalities in fabrication. In four out of five cases the rivets on section A apparently carried from 3% to 10% more of the total load than the rivets on section C which theoretically should have been equally effective. Table 6 also gives, for comparative purposes, the theoretical distribution of load for the large joints based upon the *K*-values obtained from the auxiliary shear specimens. The agreement between these theoretical values and those determined from the slip measurements is not particularly good, although in both cases the relative ineffectiveness of rivet row B, with respect to rows A and C, is clearly indicated. On the basis of the slip measurements it appears that the differences between the ordinary assumption of uniform load distribution and that actually obtained may be greater than indicated by theoretical considerations. This observation was also made by Raymond E. Davis and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E., who reported that inequalities arising from fabrication may cause an erratic and irregular load partition in large riveted joints which is great in comparison with the differences in load partition called for by the elastic theory.[6]

Table 7 gives a summary of loads for sections A, B, and C of the wide joints computed from the measured slips for the design loads and the *K*-values obtained from the small auxiliary specimens. The fact that the differences between the total loads obtained for the wide joints in this manner and those actually applied ranged from only 6% to 11% for all joints indicates a satisfactory correlation between the results of these related tests.

TABLE 7.—COMPARISON BETWEEN DESIGN LOADS AND CORRESPONDING LOADS BASED ON MEASURED SLIPS FOR RIVETED JOINTS $20\frac{1}{2}$ IN. WIDE DESIGNED TO FAIL IN SHEAR

MATERIAL		Shear design stress (lb per sq in.)	Corresponding design load for joint (lb)	LOAD CORRESPONDING TO MEASURED SLIP[a] (LB)				Ratio:
Plates	Rivets			Section A	Section B	Section C	Total	$\frac{\text{Load from slip}}{\text{Design load}}$
Silicon	Carbon	13,500	195,000	90,000	45,000	75,000	210,000	1.08
Carbon	Carbon	13,500	195,000	90,000	50,000	70,000	210,000	1.08
27S-T	Carbon	13,500	195,000	86,700	49,800	80,200	216,700	1.11
17S-T	17S-T	10,000	144,000	56,300	39,700	59,600	155,600	1.08
53S-T	53S-W	7,000	101,000	43,200	25,600	38,400	107,200	1.06

[a] Based on *K*-values for auxiliary shear specimens given in Table 5 and measured slips for sections A, B, and C of wide joints given in Table 6.

From the load-slips curves shown in Fig. 6 and from data in Table 4, it is not possible to select a well-defined point of first yielding although the "knees" in the curves for the steel joints are somewhat more pronounced than in those for the aluminum joints, as would be expected from the fundamental difference in the stress-strain characteristics of these materials. One observation which may be made from these data, however, is that the slip showed a marked increase at loads only slightly greater than the shear design values for the rivets. Slip measurements on 2-in. gage lengths across the gap between the main plates indicated the same load-slip characteristics as far as first yielding was concerned. Fig. 7 shows a typical set of load-slip curves of this type, as well as permanent slip data for the aluminum alloy 27S-T joints with carbon steel rivets. In every case, permanent slips corresponding to the design loads were on the order of only a few ten thousandths of an inch.

To determine the load-strain curves for the wide joints of this series, strains, combined with slip, were measured over 48-in. gage lengths. Except for the fact that these curves indicated a more definite elastic range or linear relationship between load and strain, their shape was similar to those in Fig. 7. Within the elastic range the total measured elongations varied from about 6% to 10% less than those computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500,000 and 10,300,000 lb per sq in. for steel and aluminum, respectively. This result is indicative of the tightness of the joints, where the effect of slip was more than offset by the effect of the extra plate thickness at the joint splices.

Fig. 8 shows the measured distribution of longitudinal stress in the wide joints for two increments of load, one within the range before any appreciable permanent slip had occurred and the other well above the range of first yielding. The values given are the measured unit strains multiplied by the moduli of elasticity—for steel, 29,500 kips per sq in., and for aluminum, 10,300 kips per sq in. An examination of the data indicates that essentially the same behavior was obtained for all the different combinations of materials investigated. On section A in the main plates a reasonably uniform distribution of stress was observed, as would be expected, and the averages of the measured values for all gage lines checked the average computed stresses within a few per cent. On sections B and C, between rivet rows on the cover plates, a fairly uniform distribution of stress was observed for the smaller load, but wide variations were found for the higher loads. In fact, compressive rather than tensile strains were measured in some cases on the longitudinal gage lines in line with the rivets.

Fig. 8 also shows the percentages of total applied load accounted for by the average measured cover-plate stresses on sections B and C. As will be noted from a comparison with the percentages given in Table 6, these values are not in very good agreement with the load distribution based on the slip measurements. The uncertainties apparent regarding the actual stress distribution produced in the cover plates suggest that the strain measurements as obtained were probably not very sensitive to the load distribution between rivet rows. There is no question, of course, as to the amount of load carried on section D at the middle of the joints. The distribution of stress measured at this section was quite varied, although the average values in most cases were within a few per cent of the averages computed.

It should be stated that the average measured stresses referred to herein, and throughout the remainder of this paper, are simply the arithmetic average of all the values found on the gage lines of any one section. No effort was

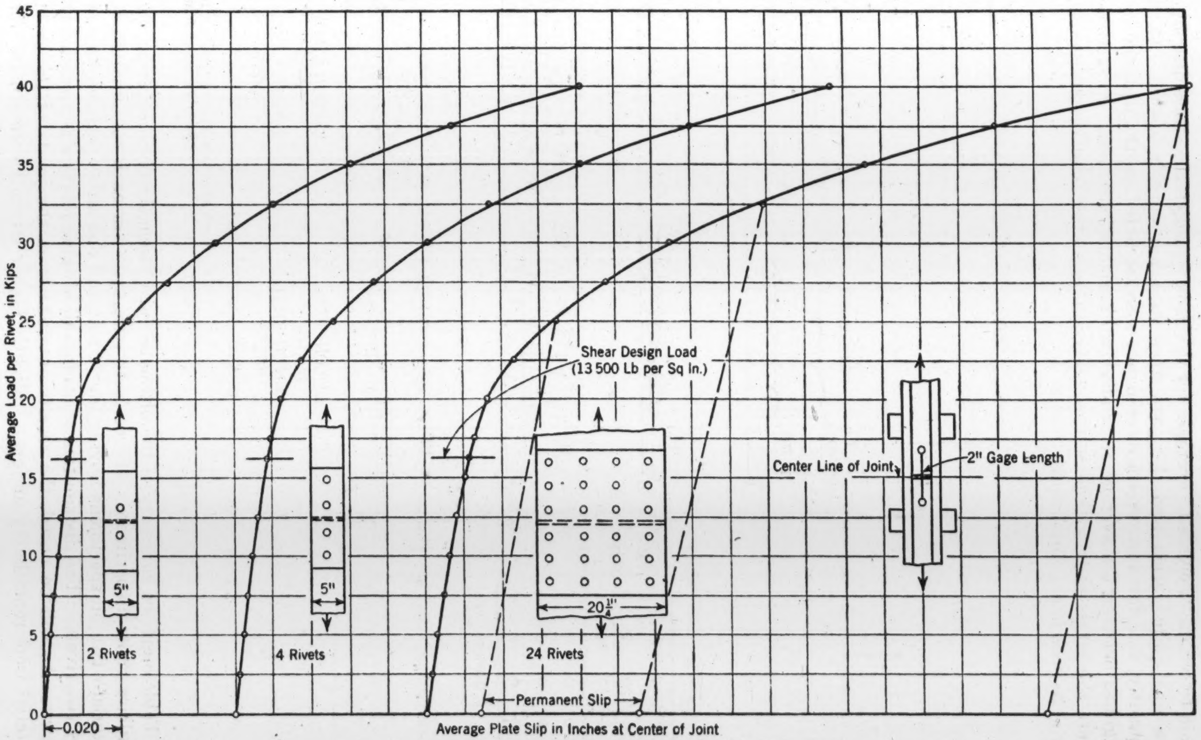


Fig. 7.—Slip Determined by a Dial Indicator on 2-In. Gage Lengths; Joints of 27S-T Plates and Carbon Steel Rivets, Designed to Fail in Shear

made to obtain "weighted" averages for the cases in which the stress distribution was quite varied and the gage lines were not equally spaced. Consequently, in a few instances a discrepancy may be noted on the distribution diagrams between the average and individual measured stresses. On sections where the stress distribution is reasonably uniform, the relative position of the gage lines is not important.

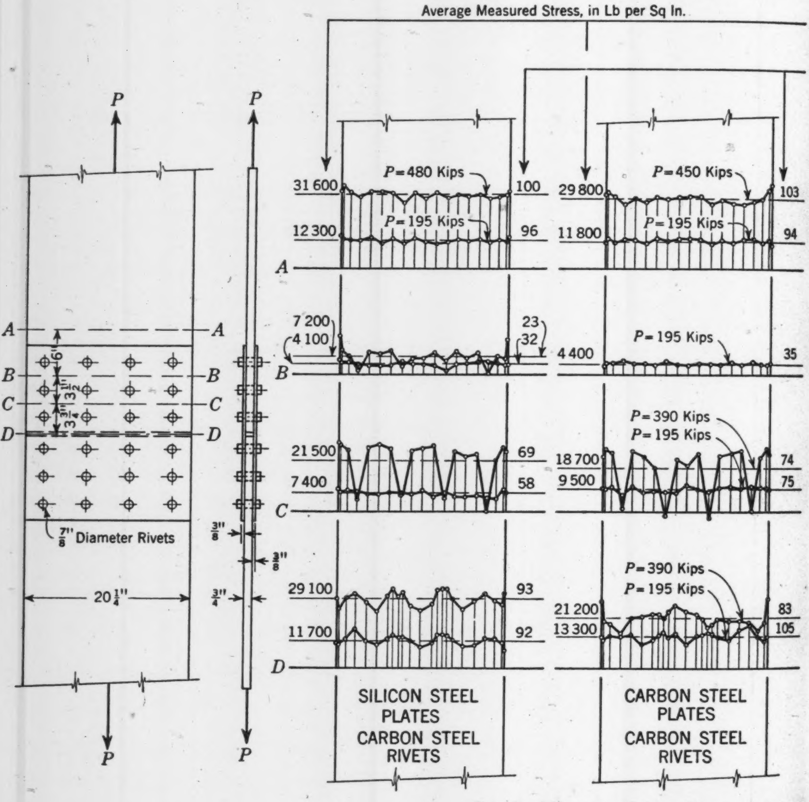
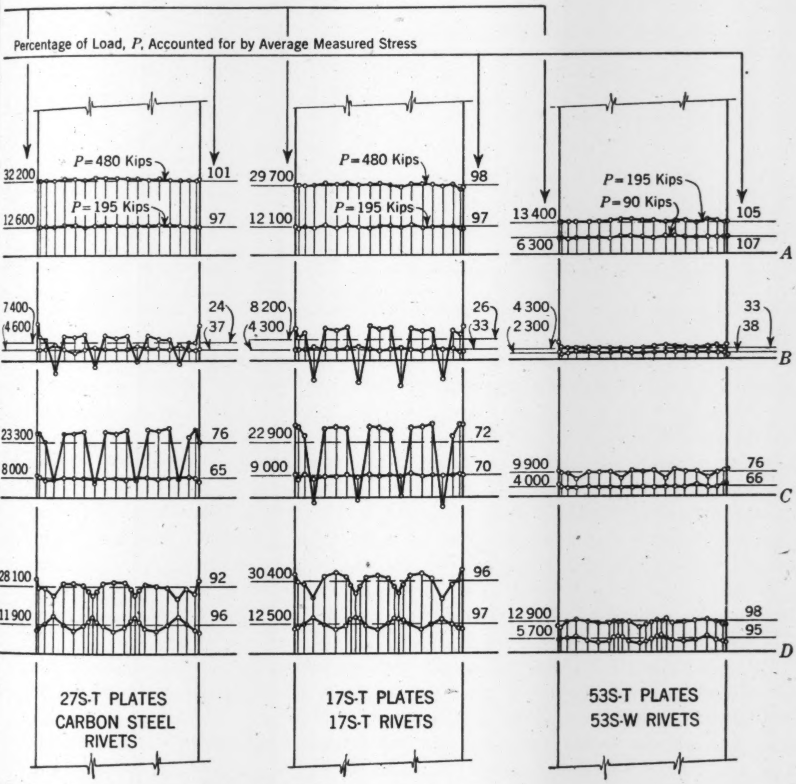


FIG. 8.—MEASURED STRESS DISTRIBUTION IN
(Where Values of *P* Are Not

Table 8 gives a summary of the ultimate shear strengths obtained from the auxiliary control specimens. Three tests were made for each combination of material—two on specimens having four rivets and one on a specimen having only two rivets. The distribution of load in all of these joints was presumably known. As will be noted, the results obtained from the joints of the same material were remarkably uniform. Slightly higher strengths were found for the steel rivets in 27S-T plates than for the same kind of rivets in steel plates, although the difference was too small to appear significant. It is of interest to point out in connection with the shear strengths given for the steel rivets that tests on this material in an Amsler shear tool using a specimen ½ in. in diameter machined from the ⅞-in. rivet stock indicated an average strength of approximately 44,000 lb per sq in. From this result it appears that the full



Wide Joints Designed to Fail in Shear
Given, Refer to Curves Above)

shear strength of the material was not obtained in the type of rivet used in the joints. Considerable bending was observed in the rivets in all cases and this behavior was aggravated, of course, by the fact that the cover plates pulled away from the main plates as the ultimate strength was approached. The use of rivets with full-size heads would undoubtedly have resulted in higher shear strengths.

The maximum measured plate slips included in Table 8 give an idea of the deformations produced in the joints before failure occurred. There was no marked difference between the behavior of the steel and aluminum joints, considering the loads involved. These slip determinations seem reasonably consistent with those of Messrs. Davis, Woodruff, and Davis,[6] who reported that shear failures occurred in $\frac{7}{8}$-in. rivets at a detrusion of about 0.3 in.

TABLE 8.—SHEAR STRENGTH OF

No.	MATERIAL		ULTIMATE LOAD, IN KIPS			LOAD PER RIVET, IN KIPS		
	Plates	Rivets						
....	Specimen....................		A	B	C	A	B	C
1	Silicon	Carbon	96.5	97.3	49.0	48.25	48.65	49.0
2	Carbon	Carbon	99.0	98.6	49.6	49.50	49.30	49.6
3	27S-T	Carbon	100.0	99.7	50.8	50.00	49.85	50.8
4	17S-T	17S-T	90.0	89.2	44.7	45.00	44.60	44.7
5	53S-T	53S-W	53.3	52.8	27.5	26.65	26.40	27.5

[a] Readings were taken across the gap between the main

Table 9(*a*) gives a summary of the ultimate loads obtained on the wide joints with the corresponding average shear, tensile, and bearing stresses, maximum measured plate slips and predicted loads. Shear failures were obtained in all cases at loads averaging within about 2% of the predicted values, the latter based on the shear strengths obtained from the auxiliary control specimens. The obvious conclusion to be drawn from this result is that the nonuniform distribution of load observed within the elastic range was not a significant factor as far as ultimate strengths were concerned. Differences in ratios of yield strength to ultimate strength, in elongation values, or in moduli of elasticity, likewise appeared to have no bearing upon the results obtained. Factors of safety on the shear design loads against ultimate failure ranged from 3.0 for the steel rivets to 3.6 for the 17S-T rivets.

In connection with the uniformity of the shear strengths obtained in these tests in which one to twelve rivets were sheared simultaneously, a similar result has been found in tests of hull plate splices.[11] Joints containing both carbon steel and 53S-W rivets, in which nine to sixteen rivets were sheared simultaneously, gave average shearing strengths ranging from 92% to 100% of the strengths obtained for similar rivets tested in auxiliary control specimens. Considerable bending was observed in the rivets and this action naturally resulted in the cover plates pulling away from the main plates. The rivets themselves sheared on only one plane and were pulled through the main plates by the bending in the opposite cover plates. In the steel joints there was the customary scaling of the plates around the rivets, although it should be emphasized that this action was not observed until some time after it was evident from the slip measurements that yielding was taking place.

---

[11] "Static and Repeated Load Tests of Aluminum Alloy and Steel Riveted Hull Plate Splices," by R. L. Templin and E. C. Hartmann, *Technical Paper No. 5*, Aluminum Research Laboratories, 1941.

RIVETED JOINTS DESIGNED TO FAIL IN TENSION

Fig. 1(*b*) shows the dimensions of the joints included in this series of tests. The joints were similar to those designed for shear (Fig. 1(*a*)), except for the width of plates and the transverse spacing of rivets. The principal points of interest are, as before—the distribution of load between the multiple rows of rivets; the load at which first yielding or permanent slip occurred; the measured distribution of strain in the plates; and the ultimate strengths.

AUXILIARY CONTROL SPECIMENS

CORRESPONDING AVERAGE SHEAR STRESS, IN KIPS PER SQUARE INCH				MAXIMUM MEASURED PLATE SLIP,* IN INCHES			LOAD PER RIVET, IN KIPS, CORRESPONDING TO SLIP		
A	B	C	Average	A	B	C	A	B	C
40.1	40.4	40.7	40.4	0.21	0.21	0.20	45.0	45.0	45.0
41.2	41.0	41 2	41.1	0.29	0.23	0.21	47.5	47.5	45.0
41.6	41.4	42.2	41.7	0.21	0.25	0 24	42.5	45.0	45.0
37.4	37.1	37 2	37.2	0.19	0.10	0.17	42.0	42.0	42.0
22.2	22.0	22.9	22.4	0.18	0.18	0.17	25.0	25.0	25.0

plates at the center of specimens, as indicated in Fig. 7.

Fig. 9 shows the load-slip curves for the three rivet sections investigated in each joint. The characteristics of these curves are similar to those in Fig. 6 and Table 4 for the wide joints designed for shear. The fact that the measured slip for the end rivet sections A and C was greater than that observed for the intermediate section B again emphasizes the difference in the effectiveness of these three rivet rows. As before, the load-slip relationships shown are not linear and there is no definite knee or break in the curves to indicate a point of sudden yielding or slip. It will be noted, however, that the increase in yielding in the steel joints is somewhat more pronounced than in those of aluminum. Although a definite yield point cannot be selected from these curves, the margin between the design loads, indicated on the figures, and the loads producing a considerable increase in slip appeared somewhat greater than was found for the joints designed for shear.

Table 10 gives a summary of the measured slips for sections A, B, and C, and the corresponding load distribution based on the relative magnitudes of the slip observed at the tensile design loads. The percentage of the total load carried by the rivets on each of the end sections A and C ranged from 37% to 42% of the total, and the loads on the intermediate section B ranged from 16% to 21% of the total. The differences between the supposedly like sections A and C are less than those found for the wide shear joints. The results are consistent also in indicating that the rivets in row B are less effective than in the wide shear joints.

Fig. 10 shows the load-slip curves for all joints obtained from measurements taken across the gap between the main plates at the center of the joints. The yielding of the joint composed of carbon steel plates is in marked contrast to that shown for the other materials. Fig. 11 shows the load-strain curves obtained from the measurements of combined strain and slip on the 48-in. gage lengths. Since these curves are more indicative of strain in the plates than slip between the plates, a fairly definite elastic range was observed. The total measured elongations within the elastic range were from 9% to 12% less than computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500 and 10,300 kips per sq in. for steel and aluminum, respectively.

TABLE 9.—STRENGTH OF WIDE-RIVETED AND PIN-CONNECTED JOINTS

MATERIAL		Ultimate load (lb)	STRESSES (LB PER SQ IN.) CORRESPONDING TO ULTIMATE LOAD			Plate slip (in.)	Load[b] (lb)	Type of failure	Predicted ultimate load (lb)	Ratio: Col. 10 / Col. 3
Plates	Rivets		Shear	Tension[a]	Bearing					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*a*) DESIGNED TO FAIL IN SHEAR; 20½ IN. WIDE, RIVETED										
Silicon	Carbon	585,400	40,500	46,600	74,200	0.22[c]	540,000	Shear	583,000[d]	0.997
Carbon	Carbon	586,900	40,600	46,100	73,500	0.24[c]	480,000	Shear	596,000[d]	1.015
27S-T	Carbon	585,000	40,500	47,200	75,300	0.20[c]	480,000	Shear	602,000[d]	1.028
17S-T	17S-T	512,000	35,500	41,000	65,400	0.20[c]	480,000	Shear	537,000[d]	1.050
53S-T	53S-W	324,000	22,400	26,300	42,000	0.15[c]	285,000	Shear	323,000[d]	0.998
(*b*) DESIGNED TO FAIL IN TENSION; 12½ IN. WIDE, RIVETED										
Silicon	Carbon	563,200	39,000	83,400	71,400	0.32[c]	510,000	Shear	583,000[e]	1.035
Carbon	Carbon	437,200	30,300	64,300	55,100	0.64[c]	437,000	Tension[f]	409,500[e]	0.938
27S-T	Carbon	405,900	28,100	61,000	52,200	0.13[c]	390,000	Tension[f]	389,000[e]	0.958
17S-T	17S-T	417,900	28,900	62,200	53,300	0.12[c]	340,000	Tension[g]	425,000[e]	1.016
53S-T	53S-W	261,900	18,200	39,600	33,900	0.08[c]	220,000	Tension[g]	255,700[e]	0.976
(*c*) DESIGNED TO FAIL IN BEARING; PIN-CONNECTED										
Silicon	Carbon	200,800	31,900	44,200	132,000	0.35[i]	170,000	Shear[h]	....	....
Carbon	Carbon	201,200	32,000	43,900	131,500	0.36[i]	140,000	Bearing	....	....
27S-T	Carbon	153,200	24,400	33,400	100,100	0.16[i]	120,000	Bearing	....	....
17S-T	17S-T	145,600	23,200	32,900	98,500	0.22[i]	120,000	Bearing	....	....
53S-T	53S-W	101,500	16,200	23,000	69,000	0.13[i]	70,000	Bearing	....	....
(*d*) DESIGNED TO FAIL IN SHEAR; PIN-CONNECTED										
Silicon	Carbon	248,300	39,500	54,600	82,000	0.13[i]	200,000	Shear	....	....
Carbon	Carbon	258,900	41,200	56,600	84,600	0.10[i]	180,000	Shear	....	....
27S-T	Carbon	260,700	41,500	56,900	85,200	0.11[i]	180,000	Shear	....	....
17S-T	17S-T	250,700	40,000	56,800	85,000	0.06[i]	170,000	Shear	....	....
53S-T	53S-W	152,700	21,600	35,300	48,800	0.08[i]	110,000	Shear	....	....

[a] Based on the net areas given in Table 1. [b] The load corresponding to the plate slip given in Col. 7. [c] Readings for maximum measured plate slip were taken across the gap between the main plates, at the center of the joints, as indicated in Fig. 7. [d] Based on the average shear strengths obtained from the auxiliary specimens given in Table 8. [e] Based on the tensile strengths of parallel specimens given in Table 2 and shear strengths in Table 8. [f] ¾-in. plate. [g] ⅝-in plates. [h] Shear and bending of pin. [i] Readings for maximum measured plate slip were taken diagonally across the main plates and cover plates, at sections through the pins.

Fig. 12 shows the measured distribution of longitudinal stress in all the joints of this series for two increments of load and gives the percentages of total applied load accounted for by the average measured stress on each of the sections investigated. Similarly to Fig. 8, the values indicated are the average measured unit strains multiplied by the moduli of elasticity (29,500 and 10,300 kips per sq in., respectively). Values of load *P* for sections B, C, and D are the same as for section A. On section A in the main plates the distribution was reasonably uniform (as would be expected) and the average values of stress were within a few per cent of those computed. On sections B, C, and D, however, the distribution was quite varied. The percentages of load accounted for by the average measured stresses on sections B and C are not as consistent, nor can they be considered as reliable, as the distribution of load obtained from the slip measurements. On section D, the average measured stresses were, in most cases, in fair agreement with the corresponding computed values. Perhaps the most significant point to be emphasized is that the strain measurements indicated essentially the same characteristics of behavior for all combinations of materials.

TABLE 10.—MEASURED PLATE SLIP AND CORRESPONDING LOAD DISTRIBUTION IN RIVETED JOINTS 12½ IN. WIDE, DESIGNED TO FAIL IN TENSION

MATERIAL		Tensile design stress (lb per sq in.)	Corresponding design load[a] (lb)	MEASURED SLIP[b] (IN.)			PERCENTAGE OF TOTAL LOAD CARRIED BY EACH SECTION[c]		
Plates	Rivets			Section A	Section B	Section C	Section A	Section B	Section C
Silicon	Carbon	24,000	162,000	0.0015	0.0007	0.0014	42	19	39
Carbon	Carbon	18,000	122,000	0.0010	0.0005	0.0009	42	21	37
27S-T	Carbon	21,000	140,000	0.0025	0.0010	0.0025	42	16	42
17S-T	17S-T	16,000	107,500	0.0026	0.0013	0.0027	39	20	41
53S-T	53S-W	12,000	79,400	0.0020	0.0010	0.0022	39	19	42

[a] Based on net areas given in Table 1. [b] Determined from load-slip curves shown in Fig. 9. [c] Determined from relative magnitude of measured slips on sections A, B, and C.

Table 9(*b*) gives the ultimate strengths for all the joints of this series, with the corresponding shear, tensile, and bearing stresses, maximum measured slips, and predicted loads. Tensile fractures, either in the main plates or the cover plates, were obtained in all joints except the one composed of silicon steel plates and carbon steel rivets, where a shear failure was obtained. The latter result was expected, however, after the results of the auxiliary shear tests indicated rivet strengths somewhat less than those relied upon in the original design. The predicted shear load was within about 3% of that actually obtained. The predicted tensile loads, based on the properties of the plate material given in Table 2, were within 2% to 6% of the actual values.

Of the four cases of tensile fractures listed in Table 9(*b*), three show plate tensile strengths which were higher than the tensile strengths of the material determined on the control specimens. Recent tests by other investigators[9,10] show this same trend for both silicon steel and carbon steel riveted joints. In comparing the results of these tests with those reported by the other investigators, however, an important difference in the procedure for computing strengths should be noted. In the investigations by Wilbur M. Wilson, M. Am. Soc. C. E., and F. P. Thomas,[10] and by Messrs. Davis, Woodruff, and Davis,[6] the net section areas were based on hole diameters arbitrarily taken as $\frac{1}{8}$ in. greater than the nominal rivet diameters, a procedure commonly required by design specifications.[12,13] In these tests, however, the net section areas were based on the nominal rivet hole diameters, a procedure believed to be entirely justified for investigational purposes. Had the other procedure been used in these tests, the tensile strengths on the net sections would have been increased about 6% more than the values given in Table 9(*b*). Although it has not been possible to show a correlation between the tensile design loads and the loads producing the first appreciable permanent slip, the factors of safety against ultimate tensile failure ranged from 2.9 for the 27S-T joint to 3.9 for the 17S-T joint.

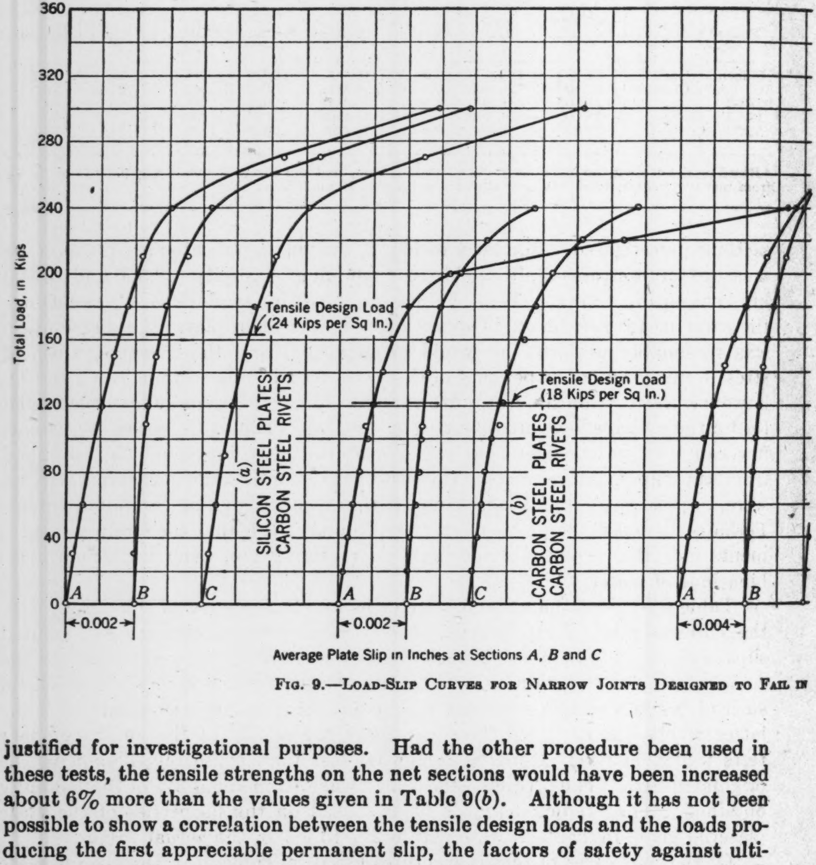


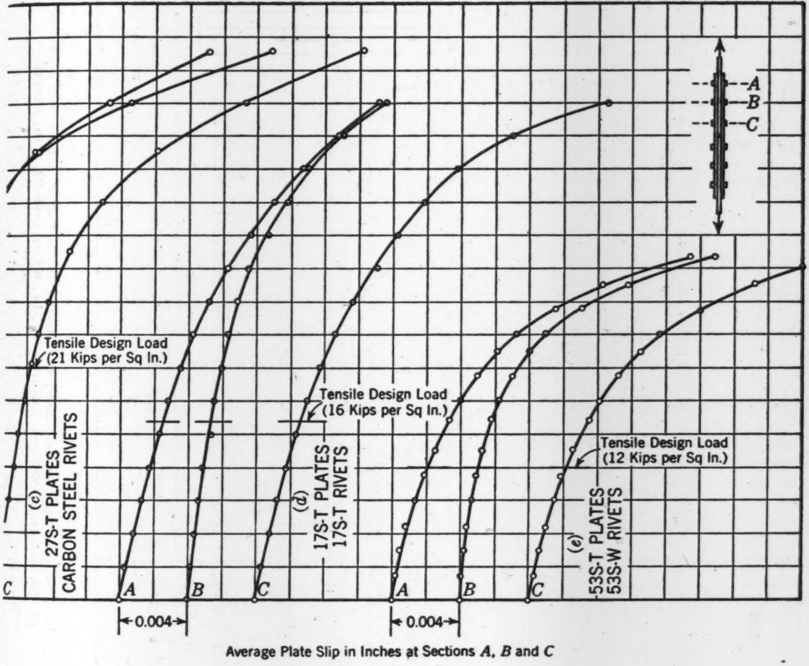
FIG. 9.—LOAD-SLIP CURVES FOR NARROW JOINTS DESIGNED TO FAIL IN

---

[12] "Design Specifications for Bridges and Structures of Aluminum Alloy 27S-T," prepared by Leon S. Moisseiff for Aluminum Co. of America (1940), Article 205.

[13] "Specifications for Steel Railway Bridges," A.R.E.A. (1938), Article 409.

Fig. 13 shows typical examples of the tensile fractures obtained. In the case of the joint composed of carbon steel plates, the slip lines, elongation of the rivet holes, and lateral contraction of both main and cover plates at the most highly stressed rivet sections are clearly indicated. The first evidence of slip lines on the surface of the plates in this joint was obtained at a load of approximately 240 kips, or only a little more than one half the breaking load. The gap opening between the main plates at the center of the joint (originally closed) was 0.64 in., and this evidence of ductility is clearly indicated in the photograph.



TENSION (12½ IN. WIDE, WITH 24, ¾-IN. RIVETS; 10-IN. STRAIN GAGE)

Fig. 13(*b*) shows that failure in the 27S-T joint was accompanied by almost no elongation of the rivet holes, lateral contraction of the plates, or opening of the gap at the center of the joint. The opening between the main plates, determined after fracture, was only 0.08 in., as compared to the opening of 0.64 in. observed for the carbon steel joint. Despite this apparent lack of ductility it should be emphasized that the full strength of the 27S-T plate material was developed. Fig. 13(*c*), which shows the failure of the 17S-T joint, indicates a behavior more like that obtained for the carbon steel joint.

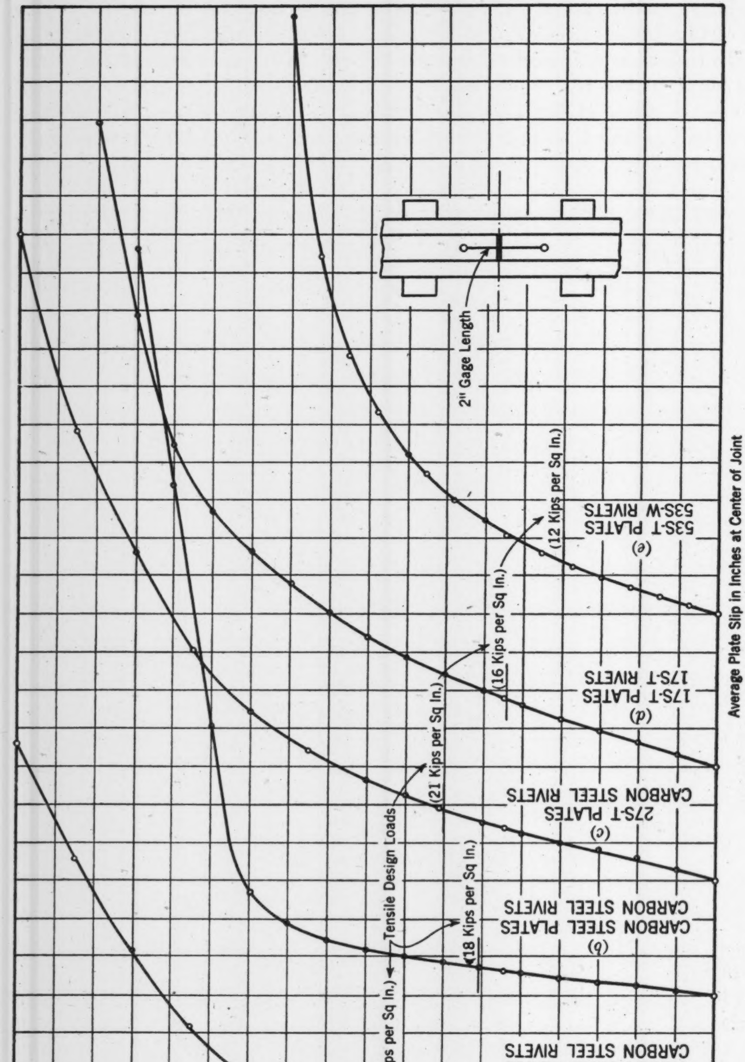


FIG. 10.—LOAD-SLIP CURVES FOR NARROW JOINTS DESIGNED TO FAIL IN TENSION; MEASUREMENTS TAKEN ACROSS THE GAP BETWEEN THE MAIN PLATES AT THE CENTER OF THE JOINTS; 2-IN. GAGE LENGTHS (12¼-IN. WIDE, WITH 24, ⅞-IN. RIVETS)

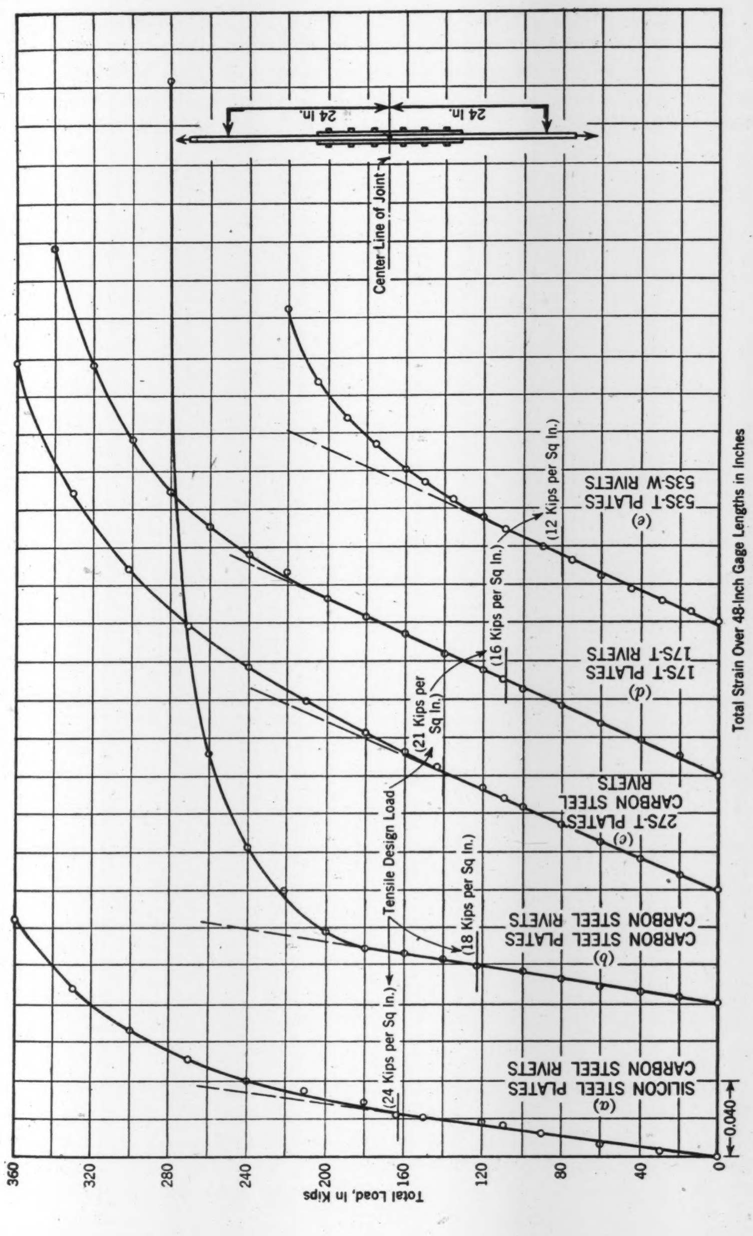


FIG. 11.—LOAD-STRAIN CURVES FOR NARROW JOINTS DESIGNED TO FAIL IN TENSION; STRAIN MEASURED BY DIAL INDICATOR ON 48-IN. GAGE LENGTHS (12½-IN. WIDE, WITH 24, ⅞-IN. RIVETS)

PIN-CONNECTED JOINTS DESIGNED TO FAIL IN BEARING

Fig. 2(*a*) gives the dimensions of the joints included in this series. All strain measurements were made with the nuts and washers removed from the pins, to permit access to the strain gage lines. At the conclusion of these measurements the nuts were replaced and the tests carried to failure.

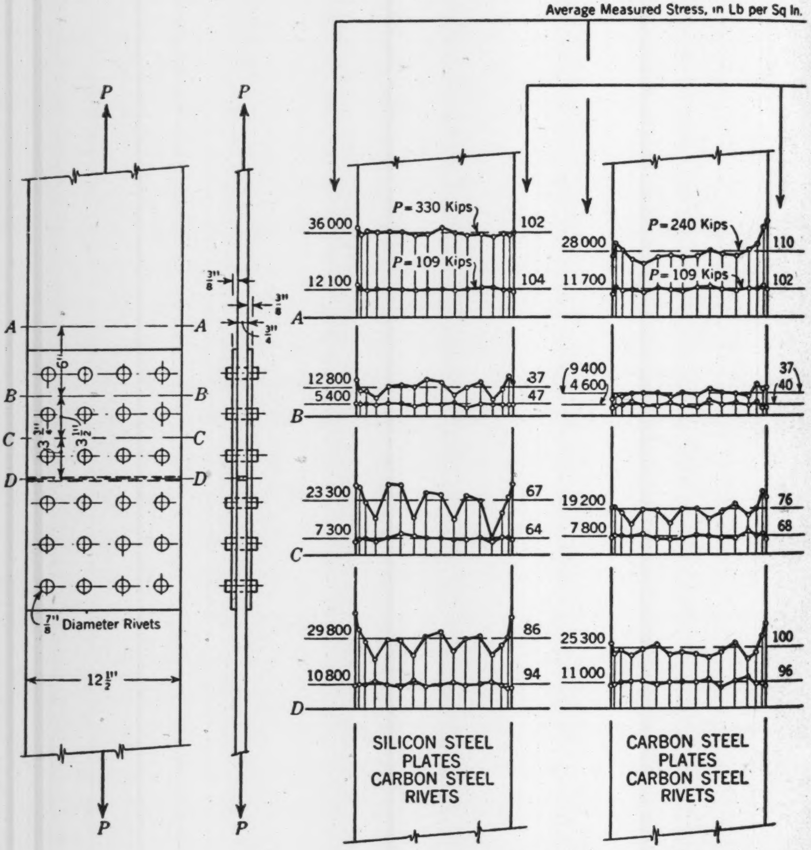
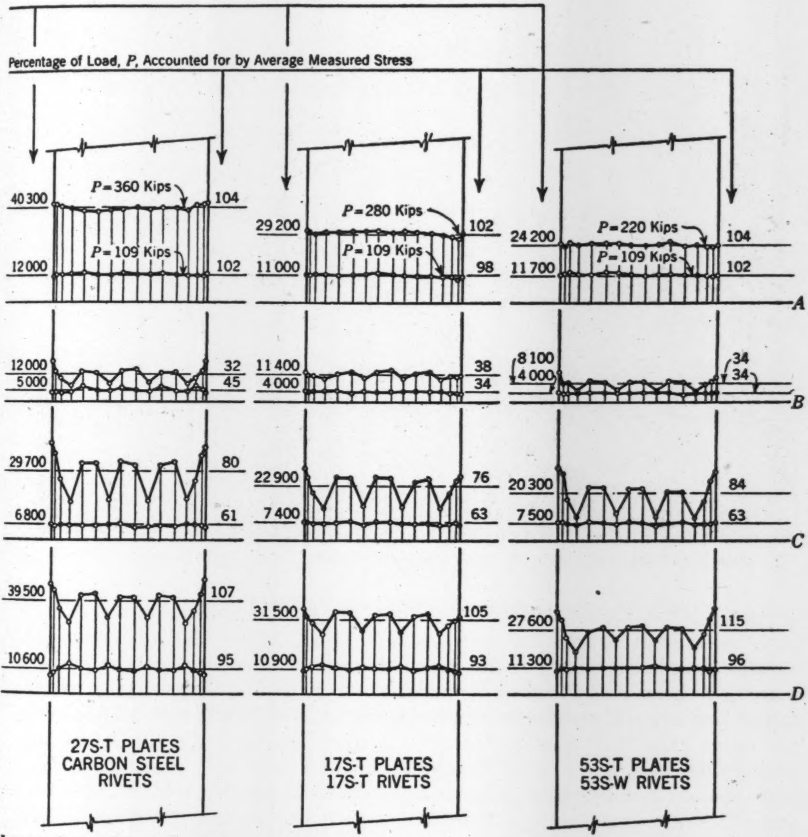

Average Measured Stress, in Lb per Sq In.
P
P
A
B
C
D
6"
3 2"
3 1/2"
3/8"
3/8"
3/4"
7/8" Diameter Rivets
12 1/2"
P = 330 Kips
P = 109 Kips
36 000
12 100
102
104
12 800
5 400
37
47
23 300
7 300
67
64
29 800
10 800
86
94
P = 240 Kips
P = 109 Kips
28 000
11 700
110
102
9 400
4 600
37
40
19 200
7 800
76
68
25 300
11 000
100
96
SILICON STEEL PLATES CARBON STEEL RIVETS
CARBON STEEL PLATES CARBON STEEL RIVETS

FIG. 12.—MEASURED STRESS DISTRIBUTION IN
(Where Values of *P* Are Not

As was the case in all the previous tests described, the relationships observed between load and slip were not linear and there was no definite point of first yielding. The margin between the loads corresponding to the bearing design stresses for the plate materials and the loads for which appreciable yielding was found appeared to be somewhat greater than that shown in Fig. 6

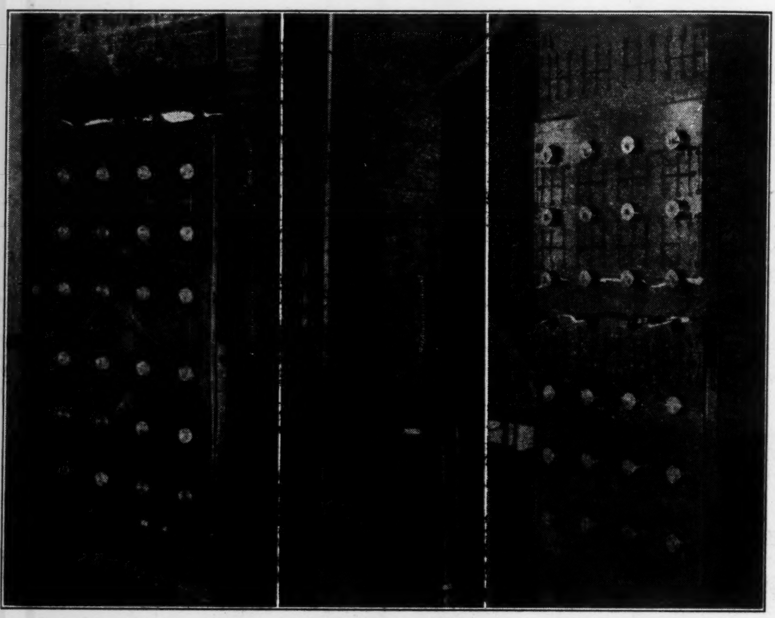
(or by the data in Table 4) for the wide joints designed for shear. Load-slip curves for the pin-connected joints have not been included herein because of their similarity to the numerous curves of this type already shown.

Fig. 14 shows the measured distribution of stress in these joints. Again (see Fig. 8), the values indicated are average measured unit strains multiplied by the moduli of elasticity of steel and aluminum (29,500 and 10,300 kips per sq in., respectively). As may be noted, essentially the same behavior was observed for all materials. On section A, which was considerably removed from the influence of the pin, a reasonably uniform distribution of stress was observed and the average measured values were within a few per cent of those



NARROW JOINTS DESIGNED TO FAIL IN TENSION
Given, Refer to Curves Above)

computed. On section B the effect of the pin was more pronounced although, in most cases, the averages of the measured stresses were within a few per cent of those computed. On section C, through the center of the pins, the range of stresses observed is clearly indicative of the high stress concentrations which must have been present at the boundaries of the pin openings. In view of the limited scope of the strain measurements and the fact that the strains on the inside gage lines exceeded the elastic limit for relatively low loads, no attempt has been made to make any comparisons between average measured stresses and the corresponding computed values. According to previous tests and photoelastic measurements[14] the stress concentration factor at the edge of a pin opening in plates of the proportions used should be approximately 4.7. Although it has not been possible to obtain a check on this concentration factor, it is of interest to point out that if such a stress concentration is assumed, the loads computed for first local yielding would correspond approximately to the design values for bearing. These computed loads were in every case below



(*a*) **Carbon Steel Plates and Rivets** (*b*) **27S-T Plates and Carbon Steel Rivets** (*c*) **17S-T Plates and Rivets**

FIG. 13.—TENSILE FRACTURE OF A NARROW JOINT (12½-IN. WIDE)

the points on the load-slip curves at which some permanent yielding seemed to have occurred.

---

[14] "Stress-Concentration Factors Around a Central Circular Hole in a Plate Loaded through Pin in the Hole," by M. M. Frocht and H. N. Hill, *Journal of Applied Mechanics, Transactions*, A.S.M.E., March, 1940, pp. A5–A9.

The strain measurements taken on section D, Table 11, represent the only attempt to measure bearing stresses directly. Even though it was obviously not possible to obtain maximum values from measurements over 2-in. gage lengths, centered 1½ in. from the edge of the pins, the stresses corresponding to the strains measured were in all cases greater than the maximum values found on section C for the same loads. Table 11 gives the longitudinal and transverse stresses for some of the same loads for which the stress distribution on section C was shown in Fig. 14. Values for the maximum loads have not been included because the corresponding strains were outside the range of the measuring instruments used. Since there was some tendency for bending in the ends of the cover plates, this action may have been partly responsible for the large strains observed.

TABLE 11.—MEASURED STRESS DISTRIBUTION AT SECTION D (FIG. 14) IN JOINTS DESIGNED TO FAIL BY BEARING

(Stresses Are in Kips per Square Inch)

Description	PLATE MATERIAL									
	Silicon steel		Carbon steel		27S-T		17S-T		53S-T	
Material in pins	Carbon steel		Carbon steel		Carbon steel		17S-T		53S-W	
Loads, in kips	44	110	44	85	44	85	44	85	44	63
Measured Stresses:[a]										
Longitudinal........	−4.4	−41.6	−3.4	−38.4	−6.6	−20.7	−8.2	−48.9	−11.9	−45.1
Transverse..........	+4.4	+13.0	+3.8	+ 5.3	+3.4	+ 6.8	+4.3	+ 2.0	+ 3.0	− 1.3

[a] Based on longitudinal and transverse strains assuming elastic action.

Table 9(*c*) gives a summary of the ultimate loads developed by these single-pin joints, with the corresponding shear, tensile, and bearing stresses and the maximum measured slips. So-called bearing failures were obtained for all but the joint composed of silicon steel plates, in which case the pin was badly bent and finally sheared off at a stress considerably below the nominal shear value for steel.

Fig. 15 shows the different types of failures obtained. A typical bearing failure involved a combination of elongation of the hole, bending of the pin, and buckling of the free ends of the cover plates. The ultimate bearing stresses obtained from such a test are so dependent upon the relative proportions of the plates and pin and the amount of lateral restraint provided against the bulging of the plates that they can scarcely be considered as representing definite properties of the materials. It may be pointed out, however, that the ultimate bearing strengths obtained for the silicon and carbon steels were approximately 1.4 and 2.1 times the tensile strengths, whereas the corresponding ratios for the aluminum alloys ranged from about 1.6 to 1.8. The factors of safety on the bearing design stresses used ranged from a minimum of 3.3 for the 27S-T to 4.9 for the carbon steel.

PIN-CONNECTED JOINTS DESIGNED TO FAIL IN SHEAR

Fig. 2(*b*) shows the dimensions of this series of joints. The tensile and shear areas were nominally the same as in the single-pin joints designed to fail in bearing but the bearing areas were 100% greater. The first loadings were made with the nuts and washers removed, but these were replaced before completing the tests to failure.

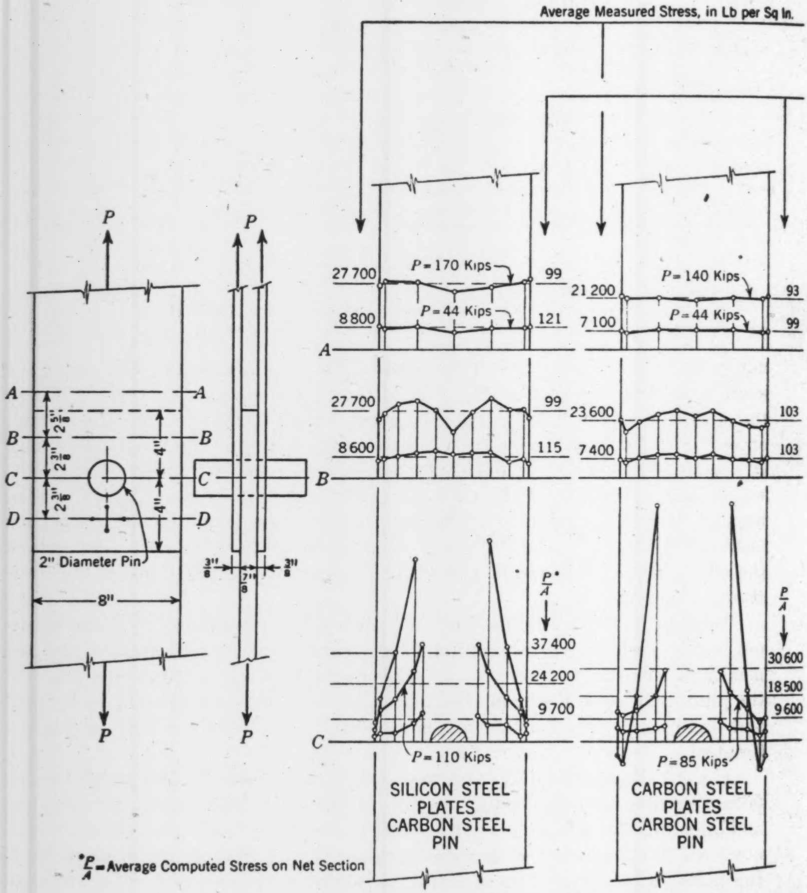
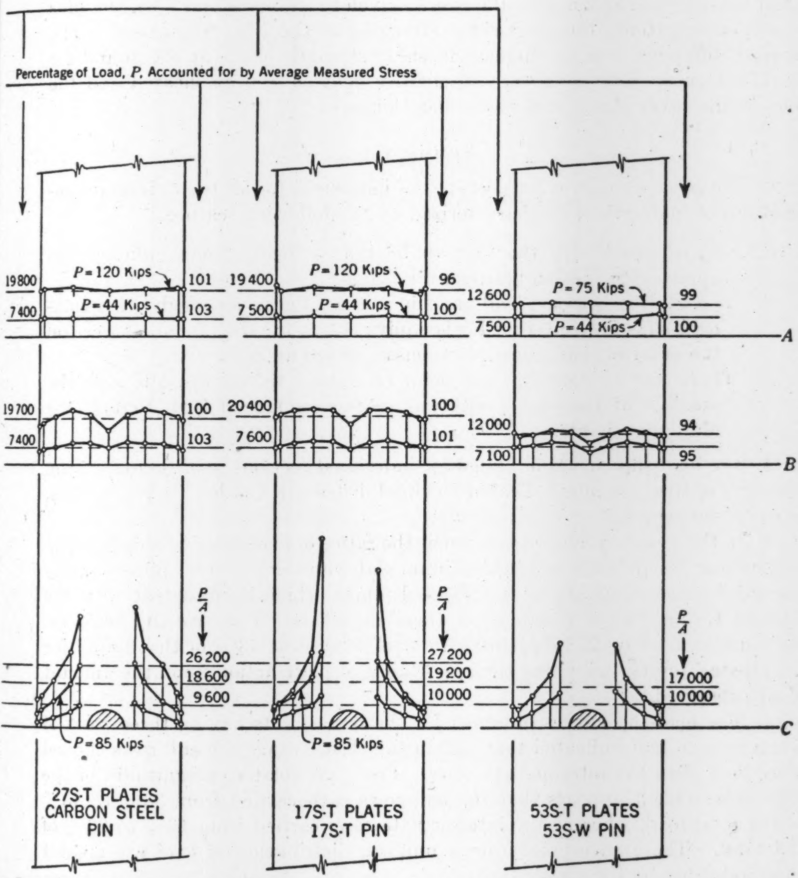

Average Measured Stress, in Lb per Sq In.
P
A
B
C
D
$2\frac{5}{8}''$
$2\frac{3}{8}''$
4"
2" Diameter Pin
8"
$\frac{3}{8}''$
$\frac{7}{8}''$
P = 170 Kips
P = 44 Kips
27 700
8 800
99
121
27 700
8 600
99
115
P = 140 Kips
P = 44 Kips
21 200
7 100
93
99
23 600
7 400
103
103
P/A*
37 400
24 200
9 700
P/A
30 600
18 500
9 600
P = 110 Kips
P = 85 Kips
SILICON STEEL PLATES CARBON STEEL PIN
CARBON STEEL PLATES CARBON STEEL PIN
*P/A = Average Computed Stress on Net Section

FIG. 14.—MEASURED STRESS DISTRIBUTION
(Where Values of *P* Are Not

Since the load-slip curves for these joints exhibited the same general characteristics of behavior as were found for the other types of joints, none of them have been included here. The yielding in the joints with the steel pins was somewhat more pronounced than in the other cases, but the margin between the shear design loads and this yielding was about the same. The measured slips indicated an equal distribution of load between the two rows of pins. As was the case for all the different types of joints previously considered, essentially the same behavior was found for all combinations of material. On the sections outside the influence of the pins, the distribution of stress was reason-


Percentage of Load, P, Accounted for by Average Measured Stress
19800
7400
P = 120 Kips
P = 44 Kips
101
103
19 400
7 500
P = 120 Kips
P = 44 Kips
96
100
12 600
7 500
P = 75 Kips
P = 44 Kips
99
100
A
19 700
7400
100
103
20 400
7 600
100
101
12 000
7 100
94
95
B
P/A
26 200
18 600
9 600
P = 85 Kips
P/A
27 200
19 200
10 000
P = 85 Kips
P/A
17 000
10 000
C
27S-T PLATES
CARBON STEEL
PIN
17S-T PLATES
17S-T PIN
53S-T PLATES
53S-W PIN

in Joints Designed to Fail by Bearing
Given, Refer to Curves Above)

ably uniform, particularly for the smaller loads, and the average measured stresses were in fair agreement with the corresponding computed values. On the sections midway between pins, the distribution of stress was extremely varied and the stress picture obtained was not complete enough to give a very satisfactory comparison of average measured and computed values.

Table 9(*d*) gives a summary of the ultimate loads with the corresponding shear, tensile, and bearing stresses and maximum measured slips. As may be noted from a comparison with the results given in Tables 8 and 9(*a*), the pins developed essentially the same shear strengths as the $\frac{7}{8}$-in. rivets used. The greatest difference was an increase in shear strength of about 8% found for the 17S-T pins. The pins were all slightly bent as well as sheared and the holes in the cover plates were somewhat elongated.

SUMMARY

1. The load-slip curves for all types of joints and for all the different combinations of materials were characterized by the following features:

(*a*) Slip occurred with the first application of load, which indicated no appreciable friction between the plates.

(*b*) The relation between load and slip was not linear, although the permanent slips measured for loads equivalent to the design values were on the order of only a few ten thousandths of an inch.

(*c*) There was no definite yield point on any of the curves, although the yielding of the steel joints was more pronounced than that of the aluminum joints.

2. The load-slip curves obtained for individual sections in all joints having two rows of rivets or pins indicated an equal distribution of load between rows, as would be expected.

3. In the auxiliary control specimens the ratios of load-to-slip corresponding to the shear design loads for the aluminum plates and rivets were approximately one third those obtained for the all-steel joints, which is consistent with the ratio of the moduli of elasticity of these materials. The load-slip ratio for the aluminum alloy, 27S-T, joint with steel rivets was 43% of that found for the all-steel joints, indicating the importance of plate action upon the amount of slip observed.

4. The load-slip curves obtained for the large joints having three rows of rivets in each half indicated that within the elastic range the end rows carried more load than the intermediate rows. From the relative magnitudes of the slips measured, it appears that the end rows each carried from 33% to 43% of the total load, whereas the intermediate rows carried from 16% to 26% of the total. These variations from a uniform distribution of load are greater than indicated by the elastic theory.

5. Measurements of combined slip and strain made on the large joints over 48-in. gage lengths indicated total elongations approximately 10% less than computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500 and 10,300 kips per sq in. for the steel and aluminum alloys, respectively.

6. The measured longitudinal strains agreed in general with the type of behavior expected on the sections investigated. The interpretation of these strains on the basis of a unidirectional state of stress gave average stresses which, in most cases, were in reasonably good agreement with the averages computed.

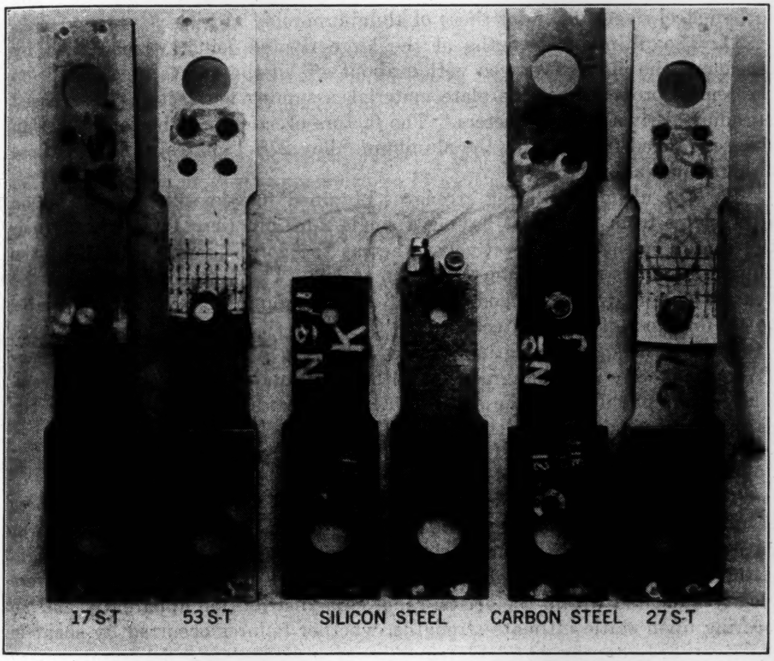


FIG. 15.—FAILURES IN NARROW JOINTS; 8 IN. WIDE, WITH A PIN 2 IN. IN DIAMETER

7. The maximum variations from a uniform distribution of measured stress in the riveted joints occurred in the cover plates on the gage lines in line with the rivets. For average tensile stresses well within the yield strength of the material, the individual stresses on these gage lines were in many cases found to be in compression.

8. The strains measured on the sections through the center of the 2-in. pins were clearly indicative of the stress concentrations which must have occurred at the edges of the pin holes, although no quantitative measure of the magnitude of these stress concentrations was obtained.

9. The loads at which the maximum stresses at the hole boundaries for the 2-in. pins were computed to be equal to the yield strength of the material, assuming a stress concentration factor of 4.7 as indicated by other tests, were below those at which first yielding was apparent from the load-slip curves.

10. The ultimate shear strengths obtained for the large joints having twenty-four rivets averaged within about 2% of the values predicted from the

tests on the auxiliary control specimens, having two and four rivets. From this result it appears that the inequality of the load distribution found within the elastic range had no bearing upon the ultimate shear strengths. Differences in elongations, moduli of elasticity, and ratios of yield to ultimate strength likewise appeared to have no bearing upon the strengths of the joints. The factors of safety on the shearing design stresses ranged from 3.0 for the carbon steel rivets to 3.6 for those of aluminum alloy 17S-T.

11. The ultimate strengths of the large riveted joints which failed by tension in the plates averaged within about 4% of the values predicted from the tensile properties of the plate material, assuming net section areas based on nominal rivet hole diameters. The factors of safety on the tensile design stresses ranged from 2.9 for the aluminum alloy 27S-T plates to 3.9 for those of aluminum alloy 17S-T.

12. The ultimate bearing strengths obtained for the silicon and carbon steel plates from the joints having a single 2-in. pin were 1.4 and 2.1 times the tensile strengths, respectively, whereas the corresponding ratios for the aluminum alloys ranged from about 1.6 to 1.8. The factors of safety on the bearing design stresses ranged from 3.3 for the aluminum alloy 27S-T plates to 4.9 for those of carbon steel.

13. The shearing strengths obtained from the joints having four 1-in. or $1\frac{1}{16}$-in. pins were in good agreement with those obtained from the $\frac{7}{8}$-in. rivets. The factors of safety on the shearing design stresses ranged from 2.9 for the carbon steel pins to 4.0 for those of 17S-T.

CONCLUSIONS

The most significant conclusion to be derived from the results of these tests is that the behavior of all the large riveted and pin-connected joints was entirely consistent with the basic assumptions generally made in design. Although inequalities in load distribution between multiple rivet rows were clearly indicated within the elastic range, this distribution apparently had no bearing upon static ultimate strengths, whether failures occurred by shear in the rivets or by tension in the plates. The amount of plastic yielding necessary to level off high stress concentrations and to effect a uniform distribution of load among the rivets must be relatively small. Essentially the same behavior was obtained for all the different combinations of the materials investigated. Elongations, moduli of elasticity, and ratios of yield to ultimate strengths were apparently not significant factors as far as the static strengths of these joints were concerned.

ACKNOWLEDGMENT

This paper was prepared under the auspices of the Committee of the Structural Division on Design in Lightweight Structural Alloys. The tests were made in 1939 at the Aluminum Research Laboratories, Aluminum Company of America, in New Kensington, Pa.

The tests described were made under the direction of Mr. Templin, chief engineer of tests, Aluminum Company of America. The writers wish to acknowledge the assistance of Mr. Templin and various members of his staff in planning and conducting these tests.

DISCUSSION

Jonathan Jones,[15] **M. Am. Soc. C. E.**—The only question the writer would ask with respect to the program as a whole is: If the objective was (as stated in the "Introduction") to investigate longitudinal distribution of rivet loads, would it not have been more convincing to arrange the twelve rivets on each side of a joint in two lines of six each or three lines of four each, instead of in four lines of three each, the plates to be correspondingly narrower and thicker? All further comment herein is directed solely to the tests of carbon steel plates.

The reported shearing strength of rivets (41,100 lb per sq in. in Table 8 and 40,600 in Table 9(*a*)) appears to be lower than may be expected in hot-riveted work. It should be noted that these rivets were not hot-driven, and their strength therefore was not enhanced by the usual driving "quench."[16] Neither was their strength enhanced by cold work, as in standard cold driving, since they had a press fit in the holes and the work of cold upsetting must have been substantially confined to the heads.

One of the most important findings to record from a test of a riveted joint is the ratio between the shear strength of the driven rivet and the tensile strength of the original rivet bar.[17,18] In the paper the latter value is not given, so it cannot be determined whether, as the writer has just intimated, this important ratio was low. If any of the original materials are still at hand it would be very interesting if the ratio in question could be recorded for (*a*) the rivets as driven and recorded in the paper, (*b*) rivets driven hot, and (*c*) rivets driven by the usual technique of cold driving in which the shank is $\frac{1}{16}$-in. slack in the hole before upsetting.

In sum, it is felt that the results reported in this paper should not influence the rivet shear allowance in specifications based on standard structural practices.

In the test designed to fail in bearing, the actual failure (see Fig. 15) was not a bearing failure, which implies collapse on the contact surface between pin and plate, but a bulging out behind the pin through a splitting or shearing action, accompanied by a transverse tension tending to rupture the free end of the plate. Evidently the greater the length of plate behind the pin, the more resistance there would be to this effect, and the higher the true bearing pressure could be raised. Had the length behind the pin in this test been about 4 in. instead of 3 in., it is probable, judging by other tests, that the load could have been increased and that there then would have been about an equal chance of failure of the type that did occur, and of failure in tension across the pinhole. A true bearing failure, meaning a crumpling of plate material at point of contact with the pin, is probably not attainable under ordinary relationships between thicknesses of plate and diameter of pin or rivet, although some thickening of the plate will be observed.

---

[15] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[16] "Static Tests of Riveted Joints," by Jonathan Jones, *Civil Engineering*, May, 1940 (see heading, "Strength of Rivets in Shear"), p. 286.

[17] *Ibid.*, p. 287.

[18] "Physical Properties of Driven and Undriven Rivets of High-Strength Structural Steels," Progress Report of the Committee of the Structural Division on Structural Alloys, *Proceedings*, Am. Soc. C. E., May, 1942, p. 769, Table 13, Col. 26.

In the tests reported[17] by the writer in 1940 the rivet bearing reached 165,000 lb per sq in. before joint yield was observed, and 204,000 lb per sq in. before failure occurred. The authors' test evidently did not exhaust the bearing capacity of the steel plate, despite the fact that the unit bearing stress at failure was 3.3 times the American Institute of Steel Construction (A.I.S.C.) allowance for enclosed rivet bearing and 4.1 times the A.I.S.C. allowance for nonenclosed rivet bearing or for bearing on pins.

In all pin-connected or riveted construction, due attention should be paid to the length of plate behind the pin or rivet. Tests have indicated that if a pin-connected flat plate of width *b* has a net width across the pin of 1.30 *b* and a net length behind the pin of 0.85 *b*, failure will occur in the bar and not in the head.

LEON S. MOISSEIFF,[19] M. AM. SOC. C. E., AND E. C. HARTMANN,[20] AND R. L. MOORE,[21] ASSOC. MEMBERS, AM. SOC. C. E.—If the sole object of this investigation had been to determine the load distribution on rivets, the arrangement of the twelve rivets in each half of the joints in three lines of four rivets or two lines of six rivets might have been a more desirable pattern, as Mr. Jones suggests. As the "Introduction" states, however, these tests were undertaken in an effort to obtain information on the strength and general behavior of large multiple-riveted joints involving different combinations of steel and aluminum alloys, as well as to obtain information on load distribution. The arrangement of the rivets in four lines of three each was chosen because it represented a more frequently used pattern of joint and appeared well suited for the over-all purposes of these tests. The results of tests of riveted joints involving combinations of steel and aluminum plates with five rivets in line on either side of the joints have been reported elsewhere.[22]

The average shear strengths of 40,600 lb per sq in. and 41,100 lb per sq in. obtained from the tests of carbon steel rivets in carbon steel plates (Tables 8 and 9(*a*)) were admittedly not as high as would be expected from conventional hot-driven rivets. These low-strength values, however, may be attributed principally to the type of rivet used rather than to deficiencies in the quality of the material. Shear strengths obtained from tests of ½-in. diameter specimens machined from the rivet bar, made in a double-shear tool, averaged slightly more than 44,000 lb per sq in. It should be emphasized that the rivets used in these joints were not conventional ones, but consisted of 2½-in. lengths of ⅞-in. rod which were pressed in the ⅞-in. drilled and reamed holes and then upset cold in a yoke type riveter to approximately 1⅛-in. diameter at the ends. This type of rivet was selected for test purposes to permit an inspection of the plate material adjacent to the rivets, to provide additional clearance for strain readings between rivets, and to minimize the uncertain effects of friction. The writers share Mr. Jones' opinion that the results of these shear tests should not be considered as having any effect upon rivet shear allowances in specifications for carbon steel.

---

[19] Cons. Engr., New York, N. Y. (Mr. Moisseiff died on September 3, 1943.)

[20] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[21] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[22] Discussion by H. N. Hill and Marshall Holt of paper entitled "Work of Rivets in Riveted Joints," by A. Hrennikoff, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 464.

Mr. Jones states that Fig. 15 does not show a bearing failure for the carbon steel pin-connected joint. The writers agree that a true bearing failure, which involves a crushing of the material at the contact surfaces between pin and plates, is probably not attainable in a ductile metal any more than is a definite value of the ultimate compressive strength of such a material. Fig. 15 merely illustrates the types of failure obtained in tests in which bearing stresses were of primary importance.

An increase in the marginal distance between the pin and the edge of the plate in the bearing tests would undoubtedly have increased the ultimate bearing strength of the carbon steel joint. The increase would not be as large, however, as would be obtained in specimens having a lower ratio of pin diameter to plate thickness (D/t). The failure to develop a bearing strength in the test of the carbon steel joint comparable to strengths developed in certain tests described by Mr. Jones,[16] probably may be attributed to differences between the D/t-ratios of the specimens used. Tests to determine allowable bearing values (unenclosed bearing) for aircraft structural design, for example, have shown that, for a D/t-ratio of 8, increases in edge distance beyond approximately 2 diameters, measured from the center of the hole, result in no appreciable increase in ultimate bearing strength. For D/t-ratios of 4 or less, however, bearing strengths increase with edge distance for values of the latter ranging from 2 to 4 diameters.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2234

PENDLETON LEVEE FAILURE

By Kenneth E. Fields,[1] and William L. Wells,[2]
Assoc. Members, Am. Soc. C. E.

With Discussion by Messrs. Ralph B. Peck, K. Terzaghi, T. A. Middlebrooks, D. P. Krynine, and Kenneth E. Fields and William L. Wells.

Synopsis

In the fall of 1939 the Vicksburg Engineer District was forced to construct a setback levee along the south bank of the Arkansas River near Pendleton, Ark. At one point the proposed levee, known as the Pendleton New Levee, crosses Lake Lenox, a shallow body of water which was formerly the channel of the Arkansas River. Foundation investigations revealed that the levee would be underlain by extremely soft clay at the lake crossing. Calculations of sections required to insure stability showed that the most economical procedure would be to displace the clay. In view of this fact, construction of the levee presented an excellent opportunity for securing data concerning a foundation failure.

Investigations of the foundation were made to determine soil conditions and characteristics. Various devices were installed in the foundation to observe movements, and hydrostatic and earth pressures. By means of such observations, it was thought that valuable and much-needed data concerning the physics of the failure would be obtained. The data obtained are given herein along with several analyses of the failure employing these data.

Field and Laboratory Data

The results of the foundation explorations are shown in Fig. 1. As can be noted, foundation conditions were found to be as follows:

(*a*) A thin, noncontinuous surface stratum of clay (not shown);
(*b*) A stratum of fine sand, from 4 ft to 9 ft thick;
(*c*) A stratum of extremely soft clay, averaging about 12 ft thick; and
(*d*) A relatively thick stratum of fine to coarse sand, underlying (*c*).

The usual physical soil tests were performed on undisturbed samples of the soft clay. From consideration of the age of the deposit and the time-consolida-

---

Note.—Published in December, 1942, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Lt. Col., Corps of Engrs., U. S. Army, Camp Polk, La.

[2] Pres., Vicksburg Bridge Co., Vicksburg, Miss.

tion characteristics of the material, it was found that the foundation material was fully consolidated under its present overburden pressure. From the results of conventional direct-shear tests of the consolidated quick type, a value of cohesion of 0.04 ton per sq ft was obtained, as well as an angle of internal friction of 19° (see Fig. 2). With these data, stability computations were made to determine the cross section that would produce failure for a height of approximately 30 ft.

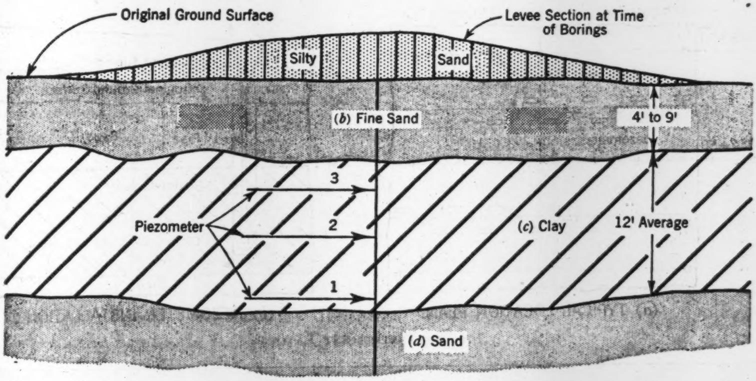


Fig. 1.—Generalized Foundation Cross Section

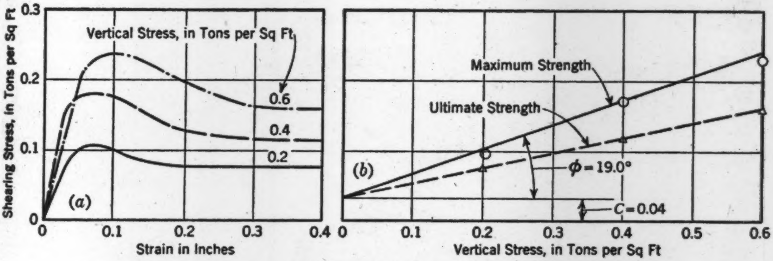


Fig. 2.—Shearing-Strength Curves; Sample from Clay Stratum

Provisions for Physical Measurements

Provisions for physical measurements consisted of settlement plates, earth and hydrostatic pressure cells, and cross-section surveys. Settlement plates were of the usual type (see Fig. 3(*a*)). Because of the short time available, the only earth pressure cells which could be obtained were six Wilson cells. These are similar to the Goldbeck cell in that internal air pressure is increased until a balance with the external pressure is reached. Further details of the installation are of no importance inasmuch as the cells failed to function.

As a means of observing pore-water pressures in the foundation during construction, piezometers were installed in the underlying clay stratum at one cross section. Details of the apparatus are shown in Fig. 3(*b*).

CONSTRUCTION AND FAILURE

Construction consisted first of the formation of the base of the embankment, the material for which was hauled in by tractors and wagons and spread by bulldozers. After the completion of the base, on January 15, 1939, a triangular section having a base width of about 320 ft and a total height of about 18 ft was then built. From this section a triangular section was built in 3-ft lifts as rapidly as possible.

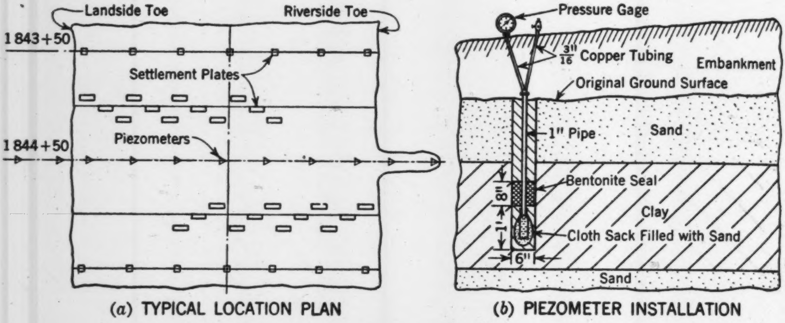

Landside Toe
Riverside Toe
1843+50
Settlement Plates
Piezometers
1844+50
Pressure Gage
$\frac{3}{16}$" Copper Tubing
Embankment
Original Ground Surface
1" Pipe
Sand
Bentonite Seal
8"
1'
Clay
Cloth Sack Filled with Sand
6"
Sand
(a) TYPICAL LOCATION PLAN
(b) PIEZOMETER INSTALLATION

FIG. 3.—INSTRUMENT LAYOUT

Riverside
Sta. 1844+50
Landside
Center Line
Sta. 1843+50

FIG. 4.—GENERAL VIEW OF FAILURE

On the night of February 14, 1940, when the section had reached a height of about 32 ft, the foundation failed. This failure was confined to the land-side half of the embankment and extended longitudinally for about 500 ft. Fig. 4 shows a part of the embankment after failure. As can be noted from this photograph and from Fig. 5, the following phenomena were characteristic of the failure:

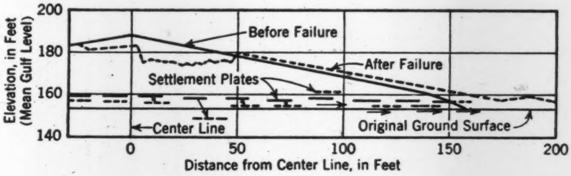


FIG. 5.—MOVEMENTS OF SETTLEMENT PLATES; LAND SIDE OF LEVEE

(*a*) A nearly vertical subsidence of 6 to 10 ft of that part of the embankment between the center line and points 50 to 70 ft land side of the center line (this subsidence left a trough-shaped depression in the embankment, in plan curving toward the land-side toe near the ends of the failure zone);

(*b*) A horizontal movement outward of the remainder of the land-side part of the embankment (approximately 10 ft); and

(*c*) An upheaval, 4 to 5 ft high, just beyond the land-side toe.

The most significant physical data obtained were those concerning pore pressures. Readings of the piezometer gages to the nearest 0.25 lb per sq in. were taken daily during the period of construction and to failure and beyond, except on days when the site was inaccessible due to rains. Fig. 6 shows plots of several typical sets of pore pressure data and corresponding loading diagrams. To be noted is the correlation between hydrostatic excess pressure and structure pressure near the center line and the building up of hydrostatic excess pressure to values greater than the pressure imposed by the structure near the toes (175 ft from the center line). Fig. 7 contains hydrostatic excess pressure data for all stations on the several dates indicated.

NOTATION

The letter symbols in this paper conform essentially with *Manual of Engineering Practice No. 22.*

ANALYSES OF FAILURE

In the study of the failure, plots, such as the one in Fig. 8, were made for each piezometer station. Thus, at Station 1844 + 50, 100 ft to the land side, the original effective vertical stress (the overburden load) was apparently decreased at piezometers 1 and 2. The word "apparently" is used purposely because the total stresses were estimated by use of the elastic theory and actual values may be different. If these effective stresses are used in the following well-known shearing strength formula:

$$s = c + (p - u) \tan \phi \qquad (1)$$

a loss of strength is indicated at all such locations where effective stresses decreased. Such decrease occurred at all stations near the toes of the structure and particularly near the bottom of the clay stratum. (In Eq. 1: s = unit shearing strength; c = cohesion per unit area; p = total normal stress on the plane of failure; u = unit hydrostatic or pore water pressure; and ϕ = true angle of internal friction of the soil.) Fig. 9 shows plots of shearing strength along the bottom of the clay stratum, estimated by use of Eq. 1. (The apparent increase in strength on the river side between February 5 and February 14 was caused by an unexplained decrease in the reading of piezometer 1, 100 ft to the river side at this station.)

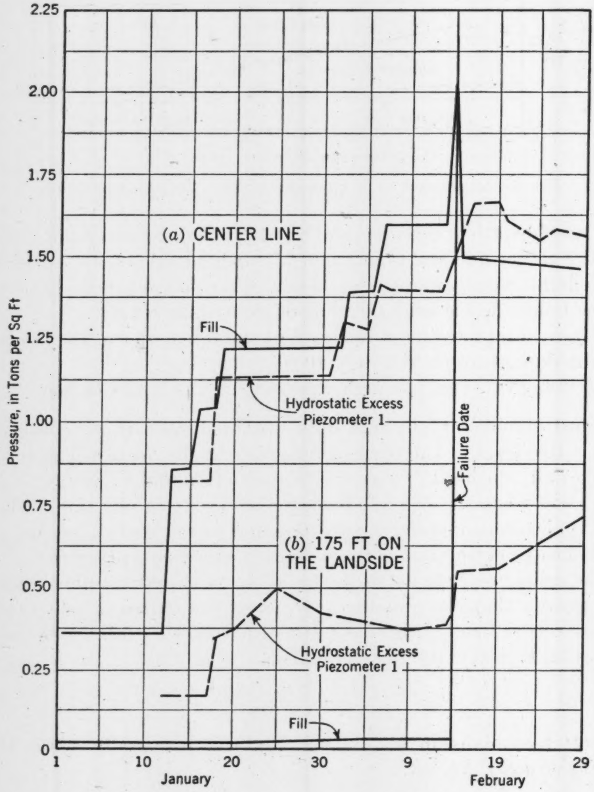


FIG. 6.—HYDROSTATIC EXCESS AND FILL PRESSURES, STATION 1844 + 50, DURING 1940

To extend the analysis of the failure still further in the foregoing manner, wherein shearing strength is expressed by Eq. 1, stability analyses by a modified circular arc method were made of the land side of the levee—the side that failed.

(See the Appendix for a description of the modified circular arc method.) These analyses at several cross sections in the failure zone gave factors of safety on the day of failure varying from 0.90 to 1.07. The factor of safety at one lift

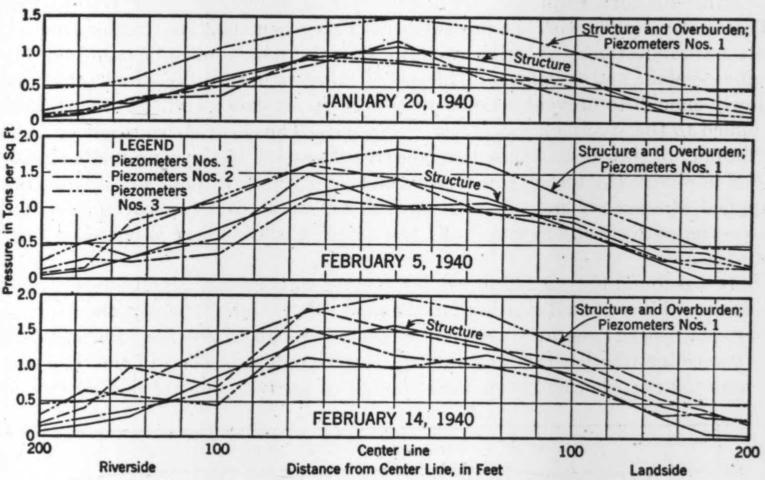


FIG. 7.—STRUCTURE AND EXCESS HYDROSTATIC PRESSURES, STATION 1844 + 50

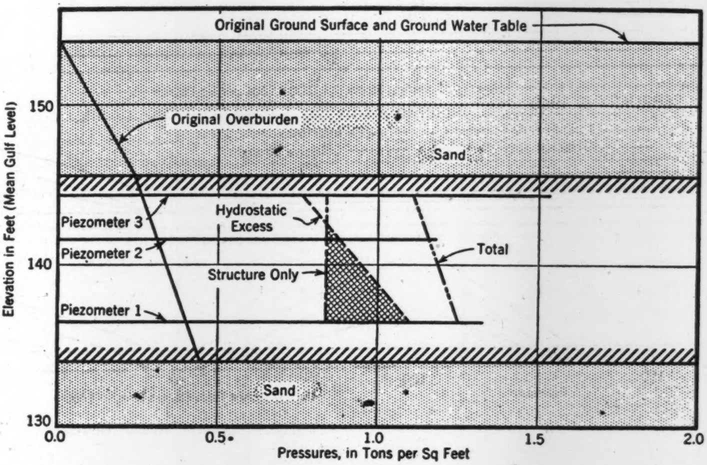


FIG. 8.—STRUCTURE AND HYDROSTATIC EXCESS PRESSURES, 100 FT ON THE LAND SIDE AT STATION 1844 + 50, FEBRUARY 14, 1940

before failure at Station 1843 + 50 was 1.31; just prior to failure, 0.90; and the day after failure, 0.98. Thus this method appears reliable when the failure surface and the hydrostatic excess pressures are employed in the stability analyses together with values of shearing strength, as expressed by Eq. 1 and values of c and ϕ, as determined by consolidated quick tests.

The peculiarity of the foregoing method of computation is that the values of shearing strength employed are based upon consolidated quick test results. This is at variance with the present belief that when effective stresses are used in the computation of shearing strength, slow test results must be used to determine c-values and ϕ-values. This belief is held because the slow test is run slowly enough to prevent a build-up of pore-water stress, and thus the stresses applied to the specimen are effective stresses. The shear curve resulting from a number of slow tests is based on effective stresses. In the consolidated quick test, however, the test is run rapidly and pore pressures built up. Thus, the applied stresses are not effective stresses but total stresses—part pore-water pressure, part effective stress—and the resulting shear curve is based on total stresses.

It was found that, if estimated in the same manner as the land side, the river side should have failed sooner than the land side. Unfortunately, the exploration and testing programs were not extensive enough to provide sufficient data to determine whether this was due to greater strength in parts of the river-side foundation; or whether the method of analysis used on the land side resulted in a factor of safety of approximately 1.0 only by accident—and that actually the method is not correct. This doubt, together with the fact that c-values and ϕ-values were determined from consolidated quick tests rather than slow tests, suggests caution in other cases; and also the great need for continued study of other failures and of the factors determining shearing strength.

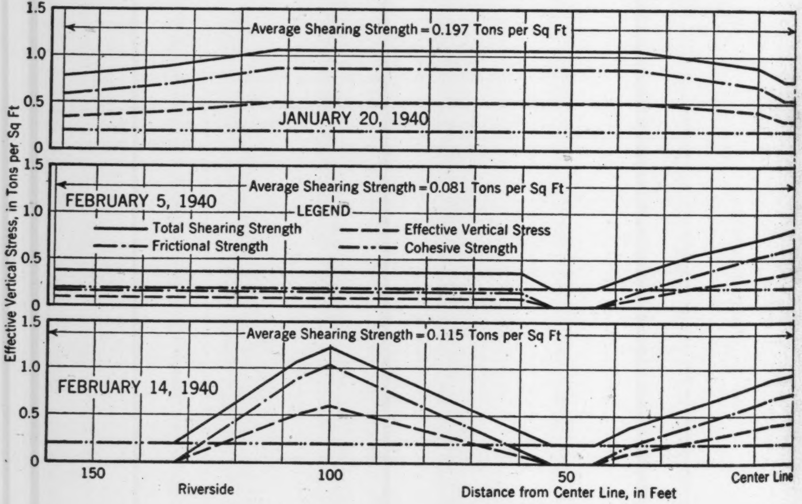
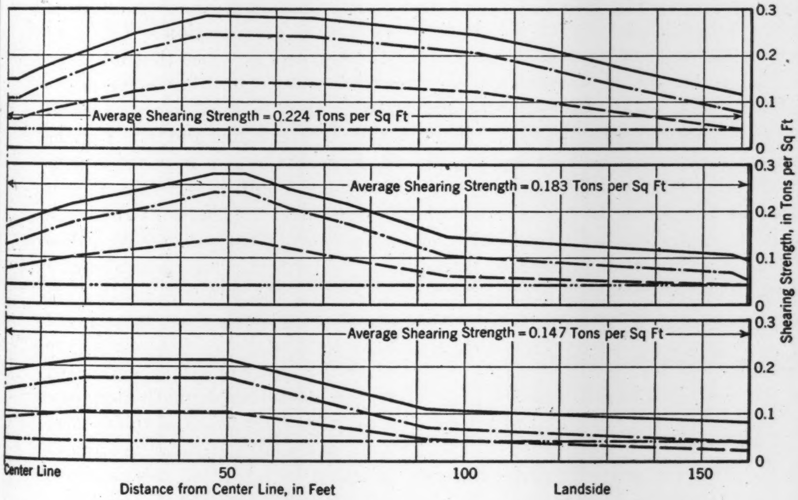


FIG. 9.—SHEARING STRENGTH ALONG THE BOTTOM

A more recent interpretation of the data obtained on the failure has been one in which the pore pressure data have been used to indicate the change in distribution of total stresses rather than as a means of estimating effective stresses.

The analysis is based on the premise that the excess pore pressures were built up primarily by the increased stress or deformation caused by the applied load. As the failure data approached, some of the excess pore pressures were no doubt caused by the remolding of the clay in areas where shearing stress exceeded strength. However, disregarding such a factor for the moment, it is of interest to consider the pore pressures only as an indication of the change in stress distribution which occurred; that is, from them to estimate total stresses on the day of failure. If this is done then (see Figs. 10 and 11) at Station 1844 + 50, on the 175-ft land side, where upheaval occurred, the total stresses on the cross-hatched, rectangular section (Fig. 11) of the clay stratum prior to construction were, for practical purposes: σ_y (average) = 0.38 ton per sq ft; and σ_x (average) = 0.34 ton per sq ft. (The assumption is made that $\sigma_x = 0.90\ \sigma_y$ for pressure at rest.) Values in Fig. 11 were computed from data in Fig. 10. From the pore pressure data the increase in pore pressure was 0.35 ton per sq ft (average). Considering this to mean an equal increase in σ_x, and that σ_y remained unchanged, since there is sand above and below the clay stratum, the total stresses on the section on the day of failure were: $\sigma_y = 0.41$ ton per sq ft (there was an increase of applied load of 0.03 ton per sq ft); and $\sigma_x = 0.37 + 0.35 = 0.72$ ton per sq ft. The maximum shearing stress, τ (max), for these principal stresses is then: $\tau\ (\text{max}) = \dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{(0.37 + 0.35) - 0.41}{2} = 0.16$ ton per



OF THE CLAY STRATUM; STATION 1844 + 50

sq ft. This shearing stress at failure checks quite well with the shearing strength as indicated by the consolidated quick test data, since, for a normal pressure of $\sigma_y = 0.38$ (the original average overburden pressure for the section), the shearing strength is 0.17 ton per sq ft (see Fig. 2). Similar computations for the river side also indicate a fairly close agreement of stress and strength.

Another interesting interpretation of the failure is that it was caused by the building up of pore pressure in the clay stratum near the toes of the structure to values equal to or greater than the overburden pressure at that point. Thus the upper piezometer located 175 ft on land side of the center line (Fig. 10), where upheaval occurred, indicated a hydrostatic excess pressure of 0.36 ton per sq ft on the day of failure. Computed vertical pressure, due to the natural overburden and structure, at the same point and on the same date was 0.35 ton per sq ft. It is pertinent to note that similar comparisons for the river side show that the hydrostatic excess pressure exceeded the computed vertical stress prior to the date of actual failure.

The natural question is how to proceed in subsequent problems. In an effort to indicate how an engineer would proceed in other problems, a stability analysis by the modified circular arc method for the slope and height attained was made, using the values of strength corresponding to original overburden pressures and the consolidated quick test curve. Although this computation gave a factor of safety of 1.3, failure occurred for that cross section. This may be due to the fact that in such an analysis the engineer assumes that all along the failure arc the maximum shearing strength is mobilized. Since the materials vary in strength-deformation characteristics, the strain necessary to mobilize maximum strength will vary. Failure will be progressive and in the upper sections such as segments 1A and 2A, Fig. 14, Appendix, cracks may occur prior to complete failure. These decrease the shearing strength there. Hence it is normally impossible to mobilize maximum strength at all points at the same time. By referring to Fig. 12(*c*) (Appendix), which shows the forces on a slice of the embankment, it can be seen that, to maintain equilibrium, either shearing strength on the bottom of the slice or normal force on the side of the slice must be mobilized. For equilibrium a decrease or deficiency in one must be balanced by an increase or sufficiency in the other. At Pendleton the σ_x-forces at the toes were built up to their limiting values when the factor of safety, estimated on the original strength, equaled 1.3. Thus, the maximum strength along the failure arc was not mobilized at the time of failure. How much of this lack of maximum strength is due to insufficient strain, or to failure and remolding of the clay, cannot be determined from the available data. All that can be said is that such lack of maximum strength apparently did exist and that it is probably common in every failure.

The writers believe that this latter picture of the physics of the failure is the correct one. Thus, the visualization of the problem is one having two aspects. The first, in design, is the use of strengths from quick tests and a factor of safety which will insure that sufficient shearing strength is mobilized to prevent pressures at the toe sufficient to cause upheaval. The factor of safety to use would depend upon the materials involved, and their characteristics. The second, in control of construction, is the measurement of total pressures and pore pressures to prevent upheaval at the toes.

Of course, more definite recommendations for future cases are desirable; but the shearing strength of clays is a subject on which there is much more to learn. The trend of all thought has been toward the belief that effective stresses are a prime factor. To date, however, there have been few, if any, cases wherein the effect of effective stresses on shearing strength has been isolated. Measurements of both pore pressures and total pressures must be accomplished to do this. It is believed that the Pendleton experiences will assist, in some measure, the future studies of this most difficult question.

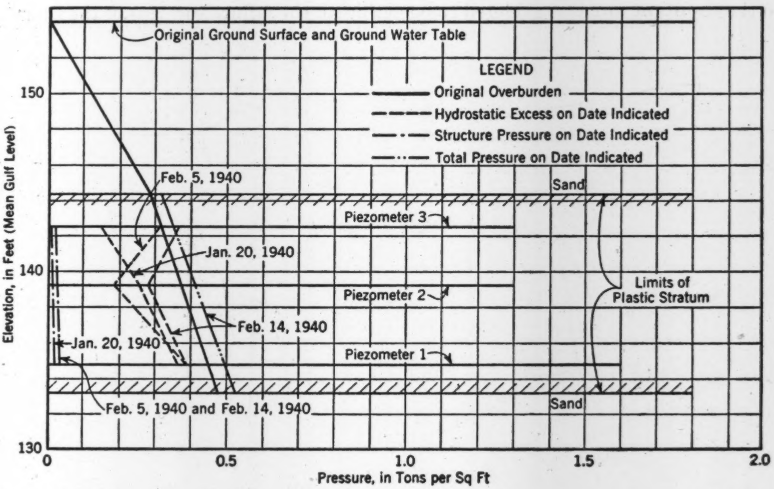


FIG. 10.—STRUCTURE AND HYDROSTATIC PRESSURES, 175 FT ON THE LAND SIDE, AT STATION 1844 + 50

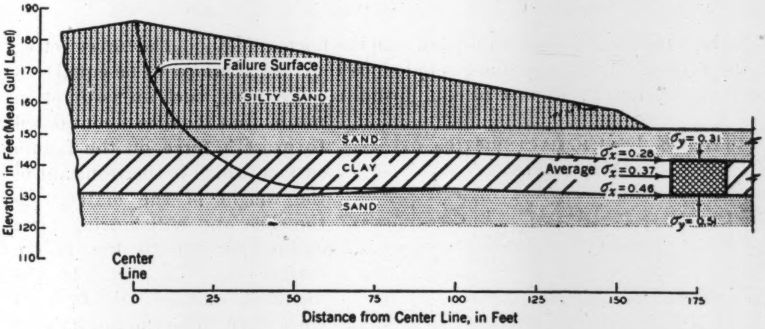
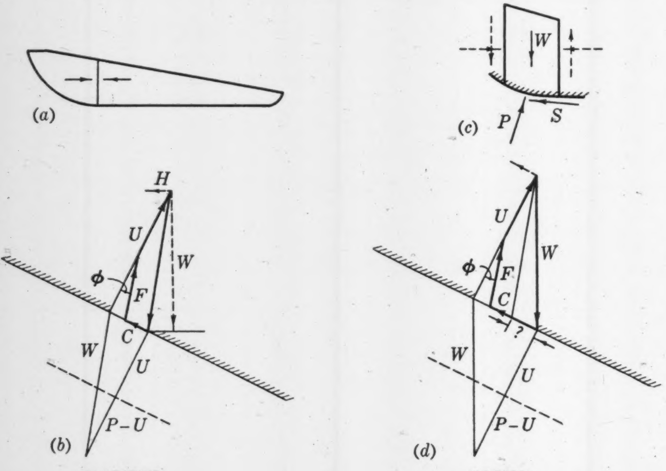

Failure Surface
SILTY SAND
SAND
CLAY
SAND
$\sigma_y = 0.31$
$\sigma_x = 0.28$
Average $\sigma_x = 0.37$
$\sigma_x = 0.46$
$\sigma_y = 0.51$
Elevation in Feet (Mean Gulf Level)
190
180
170
160
150
140
130
120
110
Center Line
0
25
50
75
100
125
150
175
Distance from Center Line, in Feet

FIG. 11.—STABILITY OF THE TOE SECTION, 175 FT ON THE LAND SIDE, AT STATION 1844 + 50, FEBRUARY 14, 1940
(All Stresses in Tons Per Square Foot)

APPENDIX

Modified Circular Arc Method

The modified circular arc method differs from the normal method in regard to the resultant forces considered to be acting on the failure surfaces of the slices into which the sliding mass is divided. In the normal method, the forces acting on the sides of the slices are neglected since they are assumed to be equal and opposite. Actually, because of equilibrium requirements, such an assumption means that the slice is considered to exert a force on the failure surface which is the resultant of the weight of the material in the slice and a balancing force acting parallel to the failure surface (see Fig. 12). In the modified method the force is considered to be the resultant of the weight of the material in the slice and a balancing force acting horizontally. Fig. 12(a) shows that in the Pendleton failure the movement of the greater part of the sliding mass was horizontal.



MODIFIED NORMAL

Fig. 12.—Comparison of Modified and Normal Circular Arc Methods

TABLE 1.—Characteristics of Materials

Material (see Fig. 13)	Material	Weight (lb per cu ft)	Cohesion (tons per sq ft)	Angle of internal friction ϕ
A	Embankment material...	112	0.10	32.5°
B	Foundation sand........	120	0.0	32.5°
C	Foundation clay........	96	0.04	19.0°

The modified circular arc method is shown in greater detail in Fig. 13 with Tables 1, 2, and 3. The forces, shown in Fig. 14, are:

(a) The total weight of the segment, W (soil plus water), known in magnitude and direction;

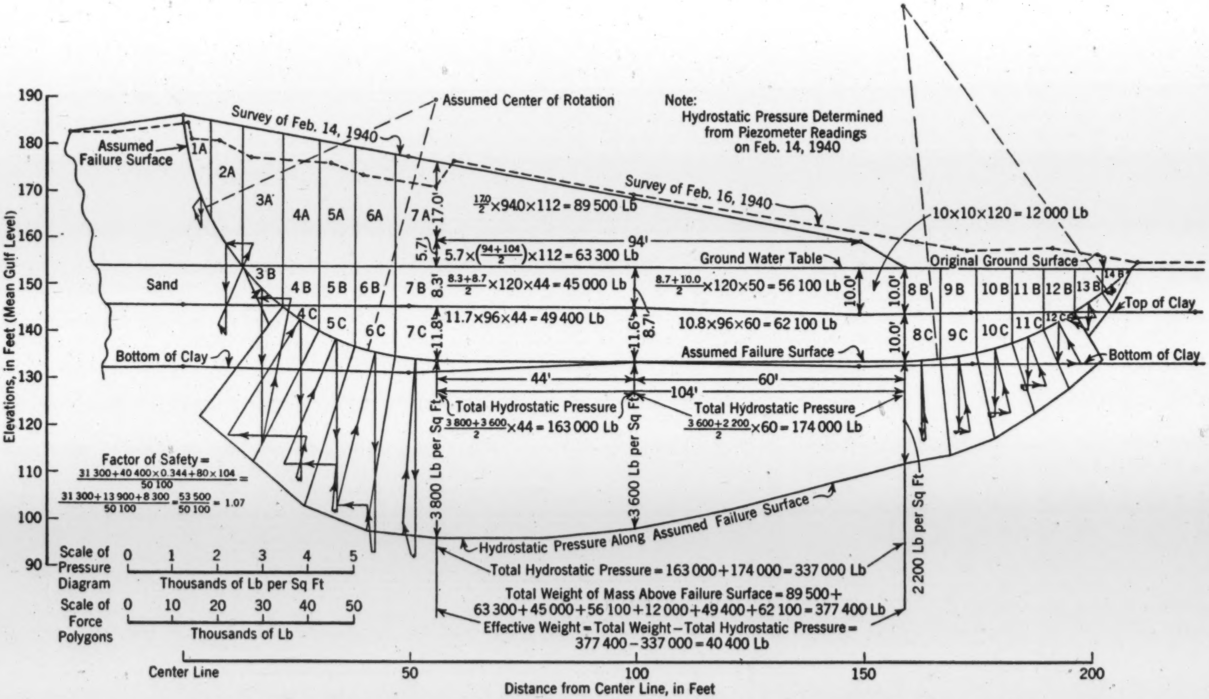

Assumed Center of Rotation
Note:
Hydrostatic Pressure Determined
from Piezometer Readings
on Feb. 14, 1940
Survey of Feb. 14, 1940
Survey of Feb. 16, 1940
Assumed Failure Surface
1A
2A
3A
4A
5A
6A
7A
3 B
4 B
5 B
6 B
7 B
8 B
9 B
10 B
11 B
12 B
13 B
14 B
4 C
5 C
6 C
7 C
8 C
9 C
10 C
11 C
12 C
Sand
Bottom of Clay
Ground Water Table
Original Ground Surface
Top of Clay
Assumed Failure Surface
Bottom of Clay
17.0'
5.7'
8.3'
11.8'
11.6'
8.7'
10.0'
10.0'
10.0'
94'
44'
60'
104'
$\frac{17.0}{2}$×940×112 = 89 500 Lb
5.7×($\frac{94+104}{2}$)×112 = 63 300 Lb
$\frac{8.3+8.7}{2}$×120×44 = 45 000 Lb
11.7×96×44 = 49 400 Lb
$\frac{8.7+10.0}{2}$×120×50 = 56 100 Lb
10.8×96×60 = 62 100 Lb
10×10×120 = 12 000 Lb
Total Hydrostatic Pressure
$\frac{3\,800+3\,600}{2}$×44 = 163 000 Lb
Total Hydrostatic Pressure
$\frac{3\,600+2\,200}{2}$×60 = 174 000 Lb
3 800 Lb per Sq Ft
3 600 Lb per Sq Ft
2 200 Lb per Sq Ft
Hydrostatic Pressure Along Assumed Failure Surface
Total Hydrostatic Pressure = 163 000 + 174 000 = 337 000 Lb
Total Weight of Mass Above Failure Surface = 89 500 + 63 300 + 45 000 + 56 100 + 12 000 + 49 400 + 62 100 = 377 400 Lb
Effective Weight = Total Weight − Total Hydrostatic Pressure = 377 400 − 337 000 = 40 400 Lb
Factor of Safety = $\frac{31\,300+40\,400\times0.344+80\times104}{50\,100} = \frac{31\,300+13\,900+8\,300}{50\,100} = \frac{53\,500}{50\,100} = 1.07$
Scale of Pressure Diagram
0
1
2
3
4
5
Thousands of Lb per Sq Ft
Scale of Force Polygons
0
10
20
30
40
50
Thousands of Lb
Elevations, in Feet (Mean Gulf Level)
190
180
170
160
150
140
130
120
110
100
90
Center Line
50
100
150
200
Distance from Center Line, in Feet

FIG. 13.—STABILITY ANALYSIS OF THE PENDLETON NEW LEVEE (LAND SIDE), BY THE MODIFIED CIRCULAR ARC METHOD

TABLE 2.—VALUES OF *u* AND *c*

Segment No. (see Fig. 13)	(*a*) HYDROSTATIC PRESSURES					(*b*) TOTAL COHESION		
	Left	Right	Average	Length of chord (ft)	Total force (lb)	Length of arc (ft)	Cohesion	
							Tons	Lb
1	....	....	....	....	....	22.4	2.24	4.480
2	....	....	....	....	....	13.0	1.30	2,600
3	0.0	520	260	12.4	3.220	12.6	....	....
4	3,000	3,200	3,100	9.6	29,800	9.7	0.39	780
5	3.200	3,600	3.400	8.6	29.300	8.7	0.35	700
6	3.600	3,800	3,700	9.2	34.000	9 2	0.37	740
7	3,800	3,800	3,800	9.2	35.000	9.2	0.37	740
8	2.200	2,100	2,150	8.0	17.200	8.0	0.32	640
9	2,100	1,900	2.000	8.2	16.400	8.2	0.33	660
10	1,900	1,600	1,750	8.2	14.400	8.3	0.33	660
11	1,600	1,400	1,500	7.5	11,300	7.7	0.31	620
12	1,400	1,200	1,300	8.0	10.400	8.0	0.32	640
13	600	350	475	7.3	3.170	7.3	....	....
14	350	0	175	8.7	1,520	8.8	....	....

TABLE 3.—COMPUTATION OF WEIGHTS OF SEGMENTS

Segment No.[a] (Fig. 13)	Width (ft)	HEIGHT, IN FEET			Area (sq ft)	Weight (lb)	*H* (lb)
		Left	Right	Average			
1A	6.0	0.0	20.3	10.1	60.6	6.800	0
2A	7.0	20.3	30.0	25.1	175.8	19.700	6,200
3A	9.0	30.0	28.2	29.1	26.2	29,400	....
3B	9.0	0.0	8.4	4.2	37.8	4.540	....
Total						33,910	8,400
4A	8.0	28.2	26.8	27.5	220.0	24,600	....
4B	8.0	8.4	8.4	8.4	67.2	8,050	....
4C	8.0	0.0	5.2	2.6	20.8	2,000	....
Total						34,650	17,200
5A	8.0	26.8	25.6	26.2	210 0	23,500	....
5B	8.0	8.4	8.4	8.4	67 2	8,050	....
5C	8.0	5.2	9.7	7.4	59.2	5 690	....
Total						37.240	11,800
6A	9.0	25.6	24.2	24.9	224.0	25.100	....
6B	9.0	8.4	8.4	8.4	75.5	9 050	....
6C	9.0	9.7	10.8	10.2	91.6	8,800	....
Total						42,950	6,500
7A	9.0	24.2	22.7	23.4	211.0	23,600	....
7B	9.0	8.4	8.3	8.3	74.7	8.950	....
7C	9.0	10.8	11.8	11.3	102.0	9.800	....
Total						42,350	0
8B	8.0	10.0	9.8	9 9	79.2	9.500	....
8C	8.0	10.0	9.7	9.9	79.2	7.600	....
Total						17 100	−1,500
9B	8.0	9.8	9.8	9.8	78.3	9,390	....
9C	8.0	9.7	8.3	9.0	72.0	6,910	....
Total						16,300	−3,300
10B	8.0	9.8	9.7	9.7	77.6	9,300	....
10C	8.0	8.3	6.2	7.2	57.6	5,540	....
Total						14,840	−5,200
11B	7.0	9.7	9.7	9.7	67.9	8,140	....
11C	7.0	6.2	3.6	4.9	34.4	3.310	....
Total						11,450	−5,900
12B	7.0	9.7	9.7	9.7	68.0	8,150	....
12C	7.0	3.6	0.0	1.8	12.6	1.210	....
Total						9,360	−5,800
13B	6.0	9.7	5.5	7.6	45.6	5,460	−7,900
14B	6.7	5.5	0.0	2.7	18.1	1,740	−2,200

Total positive..............50,100
Total negative..............31,300

[a] A signifies embankment material; B signifies foundation sand; and C signifies clay.

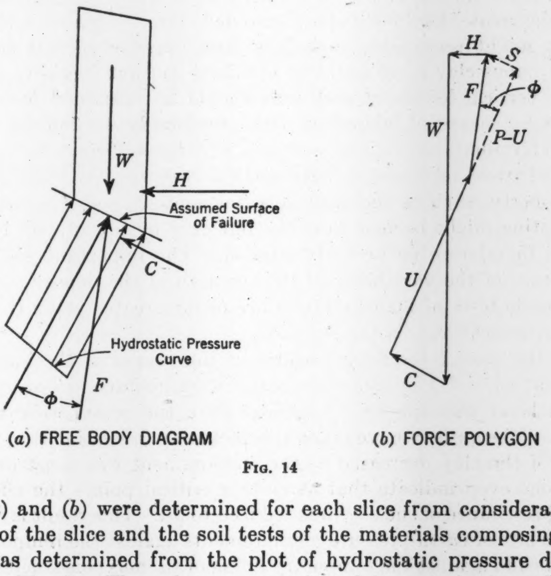
(*b*) The total cohesion, C, acting along the base of the slice, known in magnitude and direction;

(*c*) The total hydrostatic pressure, U, acting on the base of the slice (the direction of this force is normal to the assumed failure surface and its magnitude is determined from the hydrostatic pressure observations);

(*d*) The intergranular soil reaction, F, acting on the base of the slice (the direction of this force is at an obliquity of ϕ degrees with the normal to the assumed failure surface);

(*e*) The total normal stress, P; and

(*f*) The horizontal force, H, required either to maintain the slice in equilibrium or to overcome the resistance of the slice to movement (its direction is assumed to be horizontal, as movement of the greater portion of the sliding mass was horizontal).



(*a*) FREE BODY DIAGRAM (*b*) FORCE POLYGON

FIG. 14

Items (*a*) and (*b*) were determined for each slice from consideration of the dimensions of the slice and the soil tests of the materials composing the slice. Force (*c*) was determined from the plot of hydrostatic pressure distribution contained in Fig. 7. These forces were plotted in magnitude and direction, as shown in Fig. 14. The direction of the soil reaction $(p - u)$, was then plotted on the force diagrams, and the magnitude of the horizontal force to close each diagram was determined. Horizontal forces acting in the direction of motion were considered positive, whereas those acting to oppose motion were considered as negative.

The factor of safety is then defined as the ratio of the horizontal forces opposing motion to the unbalanced forces tending to cause motion. This is not comparable to the factor of safety as ordinarily defined (that is, the ratio of total resisting forces to total driving forces), but is a more sensitive index of stability as unbalanced forces only are considered.

DISCUSSION

Ralph B. Peck,[3] **Jun. Am. Soc. C. E.**—The results of a type of field test whose importance cannot be overestimated in the field of soil mechanics are presented in this paper. The ability to recognize the possibility for valuable observations in the circumstances surrounding the construction of the new levee, and to make the observations, is to be commended greatly.

A feature that would have greatly enhanced the value of the investigation is a more adequate foundation exploration. Inasmuch as the failure occurred along an essentially horizontal surface, the presence of a single lens or stratum of weak material, even if only a few inches thick, might have conditioned the entire failure. Such a foundation detail could easily have been responsible for the land-side failure of the structure when water-pressure measurements suggested the greater likelihood of a river-side failure. An adequate foundation exploration might reasonably include a large number of test borings from which continuous clay cores could be obtained without excessive disturbance. The entire vertical length of each core should be examined for the possible weak zones by means of laboratory tests, preferably unconfined compressive strength determinations on every sample. After sufficient tests had established a statistical relationship between the compressive strength and some simpler property, such as the natural water content, some reduction in compression testing might become possible, but only by a thorough investigation can such a foundation be properly studied. The important goal is the statistical picture of the variability of the strength of the deposit. In this connection, simple tests of many samples are of far greater value than the most elaborate investigation of a few samples.

One of the most interesting features of the results of the investigation is the fact that the total measured increase in hydrostatic pressure in the clay foundation layer was appreciably greater than the added weight of the embankment. The authors have taken this fact as an indication that the shearing resistance of the clay decreased as the embankment was constructed. Their computations even indicate that at certain critical points the effective angle of shearing resistance of the clay was reduced to 0°. This result is in agreement with the results of earth-pressure measurements against the temporary bracing of open cuts and against the temporary lining of tunnels constructed in Chicago (Ill.) clays for the subway. In these cases the field evidence for an inconsequential angle of shearing resistance is very strong. The shearing resistance in the field was in close agreement with the laboratory results on simple, unconfined compression tests. Furthermore, the mobilization of that part of the shearing resistance which was effective, and the simultaneous loss of frictional resistance, occurred at a relatively small strain. This amount of strain appeared to be so small in the case of the Chicago open cuts that it may be considered to be the minimum strain inevitably associated with the development of the shearing resistance. It is possible that the term

---

[3] Research Asst. Prof., Soil Mechanics, Univ. of Illinois, Urbana, Ill.

"remolding" is too extreme to describe this phenomenon. The best so-called "undisturbed" samples have probably experienced greater strains in the sampling process.

In spite of the aforementioned field evidence, it has not been possible in the laboratory to subject samples to shear tests which appear to be analogous to the conditions in the field and to duplicate the result of a small angle of shearing resistance. The consolidated, quick type of test produces an apparent angle of shearing resistance of approximately 17° for almost any clay. Whether the small disturbance caused by the sampling is sufficient to overshadow subsequent laboratory procedure is unknown. The fact remains that, so far, a discrepancy exists between the results of field and laboratory that constitutes an important gap in current knowledge. Only the unconfined compression test, free from lateral pressure or surcharge, seems to give values of shearing resistance in accord with the field measurements.

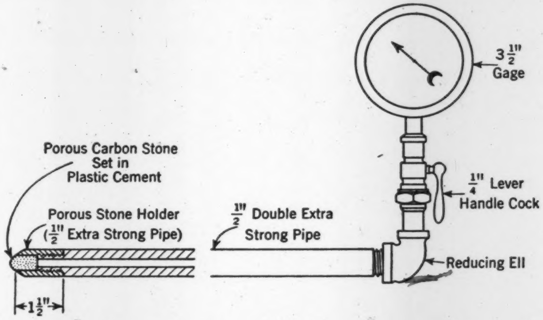

$3\frac{1}{2}$" Gage
Porous Carbon Stone Set in Plastic Cement
$\frac{1}{4}$" Lever Handle Cock
Porous Stone Holder ($\frac{1}{2}$" Extra Strong Pipe)
$\frac{1}{2}$" Double Extra Strong Pipe
Reducing Ell
$1\frac{1}{2}$"

FIG. 15.—HYDROSTATIC PRESSURE MEASURING DEVICE, CHICAGO SUBWAY PROJECT

Measurements of earth pressures and of accompanying hydrostatic pressures in regions where large shearing stresses are anticipated may constitute an important source of future knowledge. The water-pressure measurements described in this paper appear to have been satisfactory. Other types of installations may be desirable for attacking the same problem under different circumstances. For measuring water pressures in Chicago clays, for example, the device shown in Fig. 15 was developed. It consists essentially of an ordinary Bourdon gage, a length of pipe, and a porous tip, all assembled and completely filled with water before installation. Because of the low permeability of clays, it is extremely important that no leakage should occur in the measuring system. The Chicago devices possess the advantage that the porous tip can be replaced in the laboratory by a pipe cap and the entire assembly subjected to a large hydrostatic pressure for a sufficient length of time to detect any leaks. The porous stone is then substituted for the pipe cap. The assembly does not require the drilling of a hole in advance. It is merely jacked into the clay. It may be used in any position as long as the pressure is positive at the elevation of the pressure gage, a restriction which also applies to the Pendleton devices. In the Chicago clays, which have a permeability of approximately $5 \times (10)^{-8}$ cm per sec, the time necessary to reach equilibrium was between 12 hr and 24 hr.

The Chicago devices have proved satisfactory in measuring hydrostatic pressures around the subway tunnels. An example of the results is shown in Fig. 16, where a decrease in water pressure around the tunnel tubes, due to their action as drains, was measured, in spite of the fact that the free water surface remained constant over the tunnels because of the presence of a pervious silt layer in which it lay. Similar installations could provide valuable information concerning the excess water pressure along the theoretical failure surfaces adjacent to open cuts. Although the water devices were not developed to a satisfactory point in time to perform such experiments on the initial system of Chicago subways, it is hoped that the success of the water-pressure measurements described in this paper, and those elsewhere, will encourage additional measurements to obtain evidence on this extremely important aspect of the shearing resistance of natural clay soil.

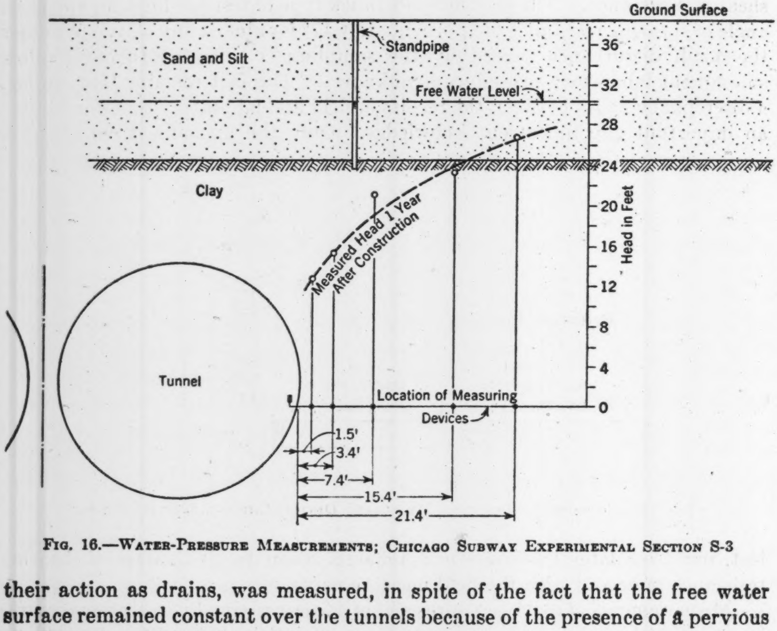

Ground Surface
Standpipe
Sand and Silt
Free Water Level
Clay
Measured Head 1 Year After Construction
Tunnel
Location of Measuring Devices
1.5'
3.4'
7.4'
15.4'
21.4'
36
32
28
24
20
16
12
8
4
0
Head in Feet

FIG. 16.—WATER-PRESSURE MEASUREMENTS; CHICAGO SUBWAY EXPERIMENTAL SECTION S-3

K. TERZAGHI,[4] M. AM. SOC. C. E.—The Pendleton Levee failed along a surface of sliding that was mostly within a fairly horizontal stratum of clay. During the fifteen years since 1928 several major failures of a similar nature have occurred. The failure of the Lafayette Dam in California in September, 1928,[5] was followed by the failure of the embankment for the water supply reservoir, Chingford II, north of London, in July, 1937,[6] the failure of the Marshall Creek Dam in Kansas in September of the same year,[7] and the failure of a flood-protection dike within the city limits of Hartford, Conn., in August, 1941.[8] The loss of capital due to these failures amounted to several million dollars and the frequency of the failures of fills above horizontal clay strata does not yet show any tendency to decrease. Therefore the subject of the paper by Messrs. Fields and Wells deserves the attention of every engineer who deals with the construction of earth dams or embankments.

---

[4] Winchester, Mass.

[5] "Reconstruction of Lafayette Dam Advised," *Engineering News-Record*, Vol. 102, 1929, pp. 190–192.

The paper is divided into three parts: (1) The results of the soil tests and field observations; (2) an analysis of the failure; and (3) the description of a graphical method for computing the stability of fills on horizontal clay strata.

The graphical stability computation consists essentially in replacing the true surface of sliding by one plane and one or two cylindrical surfaces. Such ideal surfaces are known as composite surfaces of sliding. The method seems to have originated before 1933 in the Preussische Versuchsaustalt für Wasserbau aud Schiffbau in Berlin, Germany, and it has been used repeatedly with minor modifications both in the analysis of failures and in the design of new dams. Thus, for instance, the failure of the Chingford embankment was analyzed by the writer in 1938 by means of this method. The method is accurate enough for any practical purpose, provided fairly accurate values have been chosen for the soil constants. Therefore, only the method for determining the soil constants is properly a topic for discussion.

The failure analysis in the paper under discussion is based on the tacit assumption that the clay stratum does not contain any layers of cohesionless material. The c-value of the material within the stratum is equal everywhere to 0.04 ton per sq ft, and the ϕ-value is equal to 19°. The value of ϕ has been determined by means of quick shear tests. On the basis of these data, the authors found that the factor of safety of the land side of the embankment was roughly equal to unity at the instant of failure. However, according to the results of a similar computation based on the same values, the factor of safety of the river side where no failure occurred was even less than unity. This fact suggests that the correct result which was obtained for the land side was merely due to chance. Further, it should be noted that the computation of the shearing resistance was made by means of a formula (Eq. 1) which contains the pore water pressure u. The value ϕ which appears in such an equation should be determined by means of slow, and not of quick, shear tests. The slow tests yield an angle of shearing resistance ϕ_1, a much higher value than the quick tests. The computation should have been made on the basis of the higher value, ϕ_1. Under this condition it would have led most likely to the conclusions that the factor of safety of the land side, on the day of the failure, was equal to about 1.4 and that of the river side was equal to about 1.2 (writer's guess). In other words, the computation would not account for the failure of the land side of the levee.

---

[6] "The Analysis of the Failure of an Earth Dam During Construction," by L. F. Cooling and H. Q. Golder, *Journal*, Institution of Civ. Engrs., No. 1, November, 1942, pp. 38–55.

[7] "Foundation of Earth Dam Fails," *Engineering News-Record*, Vol. 119, 1937, p. 532.

[8] "Foundation Failure Causes Slump in Big Dike at Hartford, Conn.," *ibid.*, Vol. 127, 1941, p. 142.

This conclusion leads to what appears to be a weak point of the present attitude of engineers toward stability computations in general. Since about 1928 the attention of research men has been concentrated almost exclusively on the mathematics of stability computations and on refining the technique of sampling and testing. The results of these efforts were very useful and illuminating. At the same time they created a rather indiscriminate confidence in laboratory test results. In some instances confidence is warranted, but, in many others, it can lead to erroneous conclusions, because the attention of the investigator is diverted from what is really essential. The failure of the Pendleton Levee demonstrates that the essentials in the stability conditions are by no means always within the reach of the laboratory eye and of the slide rule.

When examining Fig. 5 the reader will notice that a strip of the fill, about 50 ft wide, on the land side of the center line subsided. The remainder of the fill, beneath the lower part of the land-side slope, moved bodily forward, and the slope angle remained almost unchanged. It should be noted further that the settlement plate beneath the area of subsidence descended to a depth of more than 5 ft below the level of the adjoining settlement plates.

These observations suggest that the failure occurred as indicated in Fig. 17. The fill beneath line ab settled into a trough-like depression. The block of fill, sand, and clay beneath the slope section bc moved bodily on a plane ef through a distance of about 6 ft to the right. The horizontal pressure exerted by the moving mass bceg caused a horizontal shortening and an upward bulge of the block cdgf whose presence interfered with the movement. The absence of any appreciable deformation within the mass bceg indicates that the resistance against sliding on the base ef of this mass was practically equal to zero. If the weight of the overburden above a horizontal section through a mass of homogeneous clay is carried by the pore-water pressure, the shearing resistance along this section is equal to the cohesion of the clay. Therefore, the type of failure illustrated by Fig. 17 suggests that the surface of sliding ef was not within the clay but within a fairly continuous layer of sand or silt which separated the lower stationary part of the clay stratum from the upper, moving one. Since the piezometer readings showed that the pore-water pressure was equal to, or greater than, the overburden pressure over the entire area ef, the frictional resistance in the layer was equal to zero. The presence of fairly continuous layers of highly permeable material in the central part of the clay stratum is demonstrated by Fig. 10. In this figure the dashed lines marked February 5, 1940, and February 14, 1940, indicate a hydraulic gradient

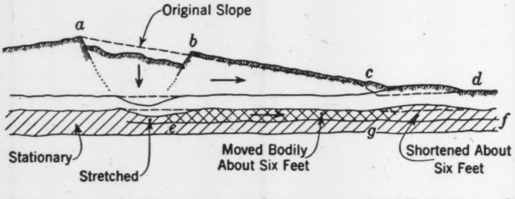


FIG. 17.—MASS MOVEMENTS AND DEFORMATIONS ASSOCIATED WITH FAILURE OF PENDLETON LEVEE

which was directed from the boundaries of the clay stratum toward the central part of the stratum. Hence, at the points where the piezometers were located, the excess water drained toward the central part of the stratum. No such flow possibly could have occurred unless the central part of the stratum contained some layers through which the water could escape in horizontal directions.

The only point that requires any further discussion is the origin of the excess hydrostatic pressure in the clay above and below section ef, Fig. 17. Under the heading, "Analyses of Failure," the authors state that "* * * some of the excess pore pressures were no doubt caused by the remolding of the clay in areas where shearing stress exceeded strength * * *." In this connection it should be noted that the deformation of the clay on both sides of section ef of the surface of sliding remained insignificant until the failure occurred. Hence, if any of the excess pore-water pressure were really due to remolding, these pressures must be considered not a cause but a consequence of the failure. Yet, no such hypothesis is required, because the excess pore-water pressures in the clay beyond the toes of the fill can be explained adequately by the sedimentary origin of the clay. The permeability of clays that were placed in horizontal layers is much greater in horizontal directions than in vertical ones. Therefore, the excess pore-water pressure produced by a local surcharge spreads from the area covered by the surcharge in both directions. The mechanics of this process are illustrated by Fig. 18, which shows a horizontal clay stratum between two sand strata. The surface of the upper sand stratum carries a fill with inclined sides. If the clay stratum is perfectly homogeneous, the excess water drains out of the clay in an almost vertical direction and the distribution of the excess pore-water pressure over a horizontal section through the clay is as shown in Fig. 18(*a*). The excess pore-water pressure, represented by the vertical distance between the water table and the dashed line, decreases from a maximum beneath the crest of the fill to almost zero beneath the toes. On the other hand, if the clay stratum contains fairly continuous layers of

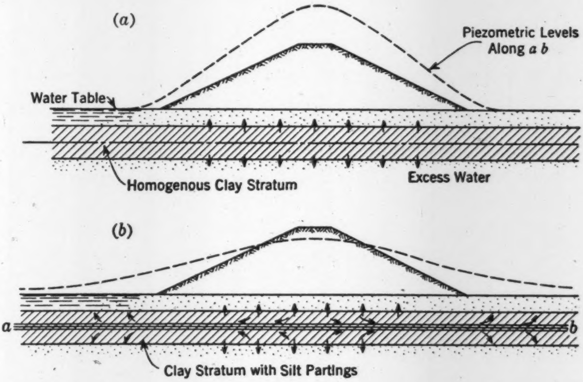


FIG. 18.—INFLUENCE OF SILT PARTINGS IN A CLAY STRATUM ON THE DISTRIBUTION OF EXCESS HYDROSTATIC PRESSURE AT THE MIDHEIGHT OF THE STRATUM

coarse silt, part of the excess water drains out of the clay into the silt, leaving the silt beyond the area which is occupied by the fill. The clay stratum beneath the Pendleton Levee contained such layers, because, according to Fig. 10, part of the excess water drained out of the clay toward a plane not far from midheight of the stratum. The distribution of the excess pore-water pressure over a horizontal section through the layer of silt is shown by the dashed line in Fig. 18(*b*). The presence of the layer reduces the excess pore-water pressure beneath the central part of the fill and increases it within the belts beneath the toes.

The preceding failure analysis leaves no doubt that the clay stratum contained fairly continuous silt or sand partings and that these partings played a decisive rôle in the mechanics of the failure. On account of the presence of these partings, the resistance against sliding was equal to zero over two broad areas in silt or sand partings in the clay, beneath the toes of the fill. The safety of the fill with respect to sliding depended primarily on the position of the inner boundaries of these areas with reference to the center line of the fill; yet the position of these boundaries depended on minor geological details that could not be ascertained by any practicable means of exploration or computation. Therefore, one cannot predict which side of the fill will fail first, unless the stability conditions on both sides of the center line are conspicuously different.

At the instant of failure the mass of soil above one of the two frictionless areas moved out bodily. The outward movement was resisted only by the passive earth pressure of the mass of soil above the frictionless area, beyond the toe of the fill. Within the clay stratum the passive resistance of the clay is equal, roughly, to the average unconfined compressive strength of the clay.

The inevitable uncertainties involved in estimating the distribution of the excess pore-water pressure on horizontal sections within sand or silt partings eliminate the usefulness of refined theoretical or experimental investigations in connection with the design of fills above horizontal clay strata of sedimentary origin. This is demonstrated by the disappointing results of the theoretical investigations contained in the paper. The most important requirement for adequate design is a thorough knowledge of the type and character of the stratification of the clay stratum, of the initial shearing resistance of the weakest layer, and of the average unconfined compressive strength of the entire layer at different points.

The safest way to secure information concerning the stratification is to inspect the sides of amply large test shafts closely. If the excavation of test shafts is impracticable, a careful study should be made of continuous cores of the stratum. To estimate the initial shearing resistance of the weakest layers of the clay and the average unconfined compressive strength, compressive-strength diagrams are needed which show the variation of the unconfined compressive strength in a vertical direction in different parts of the clay stratum. The samples can be obtained by means of 2-in. seamless tubes. One compressive-strength value should be secured for every 4-in. section of the cores. One compression test requires about 10 min. However, after an experienced experimenter has made some forty or fifty tests on very different samples from the same stratum, he is able to estimate the compressive strength of any sample from this stratum. Once he has acquired this capacity he needs to test only the softest and the stiffest samples. The strength of the intermediate ones can be estimated. The compressive-strength profiles should be supplemented by water-content diagrams, because these diagrams facilitate the identification of individual layers encountered in different drill holes. Consolidation tests and Atterberg limit tests should be made on representative samples, primarily for the purpose of securing general information concerning the clay. The test results can be used for computing the rate of consolidation, only if the ratio between the average coefficient of permeability of the clay in a horizontal and in a vertical direction is known.

In conclusion the writer wishes to emphasize once more the outstanding practical value of the observations made on the Pendleton Levee. The published data give a clear picture of the processes that precede the failure of a fill on a horizontal clay stratum, and they illustrate impressively the uncertainties involved in any attempt to predict the failure conditions. In connection with the design of foundations, a clear realization of the inevitable gaps in current knowledge is the beginning of wisdom, and these uncertainties can be realized only on the basis of comprehensive field observations such as those made by the authors. It is hoped that Messrs. Fields and Wells will supplement the information contained in their paper by further information concerning the structure and stratification of the clay stratum. It is also hoped that, some day, they will be able to supplement their record of the failure by several water-content profiles and compressive-strength profiles of the clay. The record is by far too valuable to remain incomplete.

In connection with the technique of the field observations, many readers, including the writer, would appreciate information concerning the technique of sealing the drill holes for the piezometric tubes. The paper does not mention the grain size of the commercial bentonite product used. If the product is very fine-grained, the seal may be imperfect, and, if it is very coarse-grained, the swelling pressure is likely to create a local disturbance of the hydrostatic pressure conditions in the clay. The data in the paper suggest that some of the measured pore-water pressures are too high, because, beyond the toes of the fill, the pore-water pressure cannot become greater than the total weight of the material above the seat of the pressure. As soon as the pore-water pressure in a sand or silt parting becomes equal to the weight of the overburden, the overburden is lifted gradually by the accumulating water; but the pore-water pressure remains constant.

T. A. MIDDLEBROOKS,[9] ASSOC. M. AM. SOC. C. E.—The levee failure in the Vicksburg Engineer District is an interesting experiment which the writer has followed from its inception. The authors are to be complimented on their paper. Installation of the hydrostatic pressure points was part of a general program to obtain information on the development and distribution of hydrostatic pressure in unconsolidated silt and clay strata when subjected to a superimposed load.

---

[9] Prin. Engr., Corps of Engrs., War Dept., Soil Mechanics Unit, Washington, D. C.

Evidence, accumulated over a period of years, has convinced the writer that excess hydrostatic pressure is a major factor in the design of embankment over unconsolidated foundations. During the Mississippi levee construction program after the 1927 flood, the writer had the opportunity of observing numerous foundation failures over soft clay foundation. After developing an empirical method of designing levees over such a foundation and using it satisfactorily for levees constructed at a normal rate, the writer was surprised to find that it was not satisfactory for a levee constructed at top speed. On a levee which had to be completed before a flood, the width of the berm, or total base width of the levee, had to be more than doubled to provide a stable section for such rapid construction.

Hydrostatic pressure observations are considered essential as a check on design and control during construction. The rate of consolidation, which is one of the most important variables in design, can be checked closely during construction and, if excessively high pressures are developed, the rate of construction can be decreased or the slopes flattened to insure a stable structure. Such observations are inexpensive insurance against possible trouble. In most cases, there is no need for elaborate installation. These observations can be made by a simple installation of small, open-end pipes set in the clay at the desired elevation.

As construction control measures and for research purposes, the War Department has made numerous installations for observing hydrostatic pressure under earth dams and levees in the past few years. Two recent installations under large earth dams—one in a silt stratum and one in a micaceous clay stratum—confirmed the conclusions indicated by laboratory tests that these strata would consolidate rapidly during construction. In both of these cases, the building up of excess hydrostatic pressure was small and decreased rapidly when construction of the embankment was delayed due to weather or other reasons.

The writer cannot agree with the authors' use of the "consolidated-quick" test for determining the strength of the clay where the excess hydrostatic pressure readings are available. In this type of test, there is a building up of hydrostatic pressure in the sample due to the rapid rate at which the sample is sheared. In using these test results, the effect of excess hydrostatic pressure is doubly accounted for: First, in the quick shear strength value; and, second, in the actual hydrostatic pressure observed in the clay. The result is that a lower factor of safety is obtained than if the net (grain-grain) strength had been determined. The writer also disagrees with the use of the maximum strength value in place of the ultimate or minimum strength value. The result, in this case, is that a higher factor of safety is obtained. Under these conditions, it would appear that these two factors were more or less compensating, thus accounting for the close check obtained.

The stability analysis in such cases should be based on the shearing strength determined from a "consolidated-slow" test, where excess hydrostatic pressure is not allowed to develop in the sample, and the net strength is determined. This would appear to be elementary, since in the analysis the acting forces are divided into two distinct parts: First, the measured hydrostatic pressure;

and, second, the net force, determined by subtracting the hydrostatic force from the total acting force. Then the strength to be used would be the net strength, determined from the "consolidated-slow" test. In addition, the minimum strength should be used, since in all failures of this type there is sufficient progressive movement before the full failure plane is developed. Thus the minimum strength is existing all along the so-called failure plane or zone. The authors recognized this fact but failed to employ it in their analysis.

In 1934 and 1935, the writer had an opportunity to investigate the deformation of clays in connection with the design of the Fort Peck Dam on a deep clay formation.[10] His conclusions were borne out by the satisfactory performance of the dam over this clay strata. Model experiments in the laboratory, combined with previous experience, convinced the writer that the "yield point" strength (about equal to the minimum strength) should be used instead of the maximum strength. Fig. 19 shows typical results of these model experiments. It will be noted that the clay did not develop a true failure plane even though the deformation was sufficient to term the clay mass "failed."

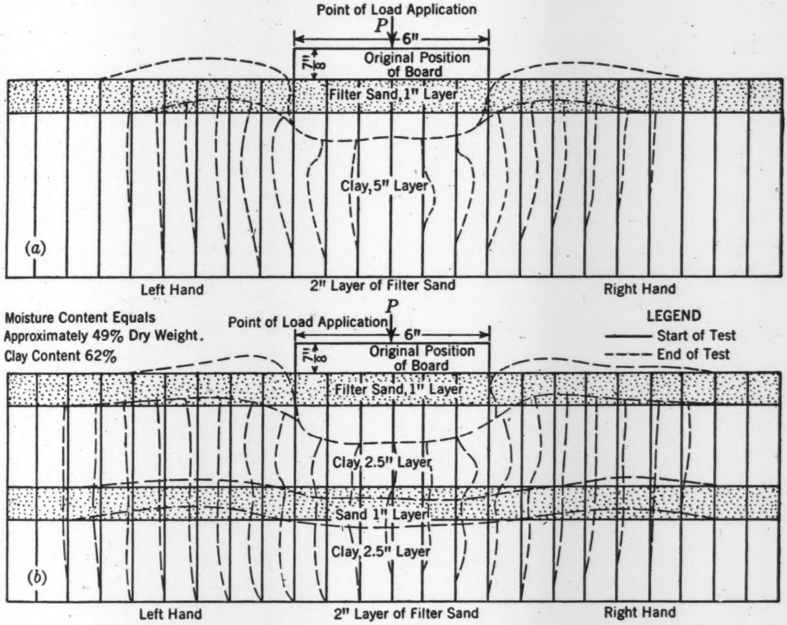

Point of Load Application
P
6"
7"/8
Original Position of Board
Filter Sand, 1" Layer
Clay, 5" Layer
(a)
Left Hand
2" Layer of Filter Sand
Right Hand
Moisture Content Equals Approximately 49% Dry Weight.
Clay Content 62%
P
Point of Load Application
6"
7"/8
Original Position of Board
LEGEND
Start of Test
End of Test
Filter Sand, 1" Layer
Clay, 2.5" Layer
Sand 1" Layer
Clay, 2.5" Layer
(b)
Left Hand
2" Layer of Filter Sand
Right Hand

FIG. 19

The circular slide method appears to give results that are satisfactory in this case, and it is generally considered accurate enough for all practical purposes. However, a better understanding of the factors involved in problems of this nature can be obtained by the elastic-theory method as outlined by the writer in the previously mentioned paper[10] and in a subsequent one on the failure of the Fort Peck Dam.[11]

---

[10] "Foundation Investigation of Fort Peck Dam Closure Section," by T. A. Middlebrooks, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936.

It is hoped that, in their closing discussion, the authors will present data on "consolidated-slow" tests and, if possible, a stability analysis using the minimum strength determined by this method.

D. P. KRYNINE,[12] M. AM. SOC. C. E.—Because of a favorable coincidence, two analogous papers, both very interesting, have been published simultaneously: The paper by Messrs. Fields and Wells, and that on the Chingford, Essex (England), earth dam failure.[6] At the time of failure both structures were approximately of the same height (32 ft at Pendleton and 26 ft at Chingford). The soil conditions are also very much alike. Those at Pendleton are shown in Fig. 1 and those at Chingford are: Soft clay varying between 0 and 12 ft, usually about 3 ft thick; a layer of "river ballast"; and finally the so-called "London clay." The total thickness of both the "river ballast" layer and the upper soft clay layer is from 10 to 30 ft. There was a puddle core at Chingford and no core at Pendleton. In both cases the failure occurred in the same way. A large part of the failure line (shear surface) is practically horizontal and is within the clay layer underlying the structure (Figs. 11 and 20[13]).

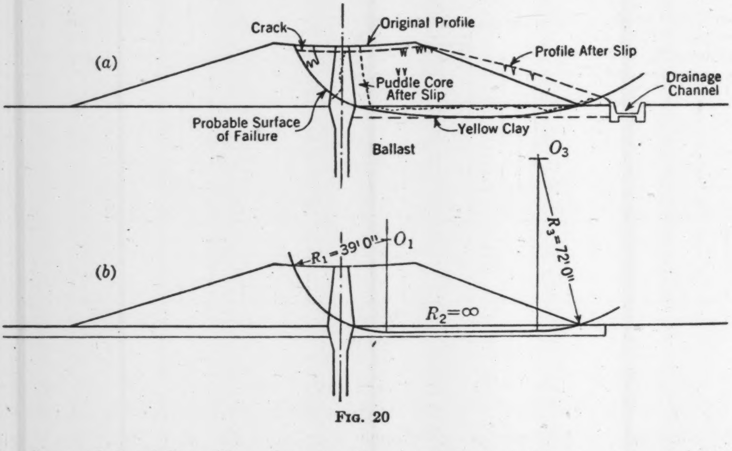


FIG. 20

The slope of the failure line close to the center line of the structure is rather steep, and there is a sinking of the crown in both cases. It may be concluded from the movement of the settlement plates at Pendleton (Figs. 3(*a*) and 5) that the clay compressed between two sand layers was squeezed out toward the toes. Apparently, in Chingford there was an analogous situation.

---

[11] "Fort Peck Slide," by T. A. Middlebrooks, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 723.

[12] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[13] "The Analysis of the Failure of an Earth Dam During Construction," by L. F. Cooling and H. Q. Golder, *Journal*, Institution of Civ. Engrs., No. 1, November, 1942, Figs. 4 and 5*b*.

Horizontal Movement of Pore Moisture.—Fig. 21 represents a hypothetical clay layer between two sand layers. Points A and A′ are on a vertical line very close to the center line of the dam, whereas points A, A″ and A‴ are on a horizontal line. During the process of consolidation, the pore-moisture pressure at point A is greater than at point A′; and the pressure at point A is greater than at point A″ because of the difference in the heights of the dam. In its turn, the pressure at A″ is greater than at A‴. Hence at point A there is a hydraulic gradient in the vertical direction driving moisture from the clay into the sand; and, at point A, there is a hydraulic gradient in the horizontal direction driving moisture from the center of the dam toward the toe. In reality, moisture driven out of the clay layer follows curvilinear paths. For the sake of simplicity, however, vertical and horizontal components of this movement will be considered.

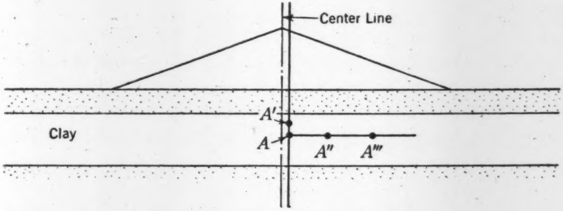


FIG. 21

The behavior of that moisture which moves horizontally depends mostly on the capacity of the sand layers above and below the clay to take up moisture from the clay, especially close to the toes of the structure. If drainage at the toe is poor (saturated sand, or rainy weather, or both), moisture will accumulate close to the toes in an adiabatic way (that is, without leaving the mass).

Difference in Vertical and Horizontal Permeability.—If the coefficient of horizontal permeability in the clay layer (Fig. 21) is considerably larger than the coefficient of vertical permeability, moisture arriving horizontally at the toes will not be satisfactorily drained vertically in the clay layer itself. This can be proved easily by computing the discharge at the toes, in both horizontal and vertical directions, using the Darcy formula with two different coefficients of permeability. All measures tending to relieve pore pressure at the toes—particularly the drainage of the clay mass by vertical sand wells—contribute to the stability of the structure.

Character of the Failure.—Adiabatic pore moisture accumulated at the toe of the dam makes the clay swell. The process of swelling and consequent growth of moisture films gradually relieves original cohesion ($c = 0$ in Eq. 1). At the same time the growing value of the pore-moisture pressure ("neutral stress," u) gradually decreases the second member at the right side of Eq. 1. As soon as the values of u and p are balanced, the clay mass under the toes of the dam becomes an exceedingly soft plastic mass (shearing strength $s = 0$) under pressure. At this stage it breaks through the earth mass to reach the earth surface; and, in this connection, it must overcome what may be called "passive pressure" or "passive resistance" of the earth mass outside the dam. In other words, a shearing surface (failure line), to the right of the toe (Fig. 13), is formed, and a part of the clay material is thrown out to form the "upheaval" (or "mud wave"). As soon as this quasi-liquid mass reaches the earth surface, its pore pressure is lost (in the same way as pressure in the water running from a faucet). The clay gradually hardens; and sometimes one is able to walk on these "mud waves" a few days after failure.

The shape of the failure line (shearing surface) close to the middle of the dam may be circular in a general case (Fig. 13), but it is very possible that, under the given geological conditions, it is much steeper (Fig. 5). In the opinion of the writer (which he does not advance as a definite one, however), the separation of the wedge from the remainder of the dam in the given case is due mostly to tensile stresses.

All that has been said in this discussion about the character of the failure is nothing more than a proof and clarification of an interpretation presented by Messrs. Fields and Wells (see heading, "Analyses of Failure")—"the failure * * * was caused by the building up of pore pressure in the clay stratum near the toes of the structure to values equal to or greater than the overburden pressure at that point." The writer agrees completely with this clear-cut statement since he considers it accurate enough for all practical purposes.

Summary.—Considerable work has been done in this field both in the United States and in England. The writer believes that research along these lines should be continued and that, in this connection, more attention should be paid to quantitative studies of soil permeability and drainage conditions under a dam. This remark is not intended to detract anything from the value of the papers discussed. In the last sentence of their paper, Messrs. Fields and Wells hope that "* * * the Pendleton experiences will assist, in some measure, the future studies of this most difficult question." The writer shares this hope and wishes to extend this statement to the British counterpart of the Pendleton Levee.[6]

KENNETH E. FIELDS,[14] AND WILLIAM L. WELLS,[15] ASSOC. MEMBERS, AM. SOC. C. E.—Professors Peck and Terzaghi suggest that the failure may have been conditioned by the presence of a thin horizontal layer of weak material not disclosed by the borings. It is hoped that it will be possible to obtain continuous cores of the clay stratum at the site at some future date when the demands of Mars have been met.

Professor Peck describes a water-pressure measuring device used in connection with the Chicago Subway project which seems more simple and easy to install than the Pendleton devices.

Professor Terzaghi is of the opinion that failure occurred along a fairly continuous horizontal layer of sand or silt near the middle of the clay stratum. The writers agree that the failure occurred along a horizontal plane within the clay stratum; but all the data obtained indicate that the plane of failure was near the bottom of that stratum. Pore-water pressure was a maximum and effective pressure a minimum near the bottom of the clay stratum on the day of failure at all piezometer locations except those beyond the embankment toes. The pore-water pressure distribution shown in Fig. 8 is typical for all piezometer locations beneath the embankment on the land side. Pore-water pressures were not equal to or greater than total overburden pressures except in zones immediately beneath and beyond the embankment toes. Thus, with strength a minimum near the bottom (assuming homogeneity) and shearing stress a maximum, it is logical to assume that failure occurred along or near the bottom of the stratum.

---

[14] Lt.-Col., Corps of Engrs., U. S. Army, Fort Benning, Ga.

[15] Capt., Corps of Engrs., U. S. Army, Vicksburg, Miss.

Professor Terzaghi bases his argument for the presence of fairly continuous layers of highly permeable material in the central part of the clay stratum upon Fig. 10 which contains hydrostatic pressure distributions at a point 175 ft land side of the center line, or about 15 ft beyond the land-side toe of the embankment. This value indicates a hydraulic gradient directed toward the central portion of the stratum. Such a distribution of pore-water pressures was exceptional rather than general within the land-side portion of the foundation. Further, at and beyond the toes of the structure, flow would be directed away from the silt or sand partings as illustrated by Fig. 18(*b*) of Professor Terzaghi's discussion, should such partings be continuous beneath the embankment.

Professor Terzaghi then states that the excess pore-water pressures in the clay beyond the toes of the fill can be explained adequately by the sedimentary origin of the clay—that is, by the fact that the permeability of the clay in a horizontal direction is much greater than that in a vertical direction. In the original report[16] on the subject the following three theories were considered as possible explanations of the pore-water pressure distributions observed:

(*a*) Arching in the fill resulting in a spreading of the load laterally;
(*b*) A fluid-like transmission of pressure laterally in the clay stratum;
(*c*) Lateral drainage away from the zone of maximum pressure.

In connection with theory (*c*), Fig. 22 illustrates the stratified nature of some parts of the clay stratum. Some of the other samples taken were much less distinctly stratified.

All these theories were rejected as primary causes of the pore-water pressure distributions noted since they do not explain the large variations of the hydrostatic excess pressure with depth (Fig. 8) and the only theory which apparently explains the phenomenon adequately is that a transference of intergranular pressure to hydrostatic or pore-water pressure occurred within zones of maximum shearing stresses. This phenomenon was called "remolding" by the writers, a term which may be too extreme, as suggested by Professor Peck, since it may occur at relatively small strains. In the Pendleton case, the strain within the clay may not have been so small since the base of the embankment near its center settled a total of about 1 ft up to February 12, 1940, two days prior to failure, and one settlement plate 150 ft land side, near the toe of the embankment, indicated an apparent uplift of about 1 ft as early as January 12.

---

[16] *Technical Memorandum No. 178–1*, U. S. Waterways Experiment Station, Vicksburg, Miss., January 4, 1941.

The abrupt rise in the readings of all piezometers located between points 50 ft river side and 175 ft land side, which commenced one day before failure and continued for one to two days thereafter, appears to be further evidence that, prior to and during failure, a transference of intergranular pressure to pore-water pressure occurred.

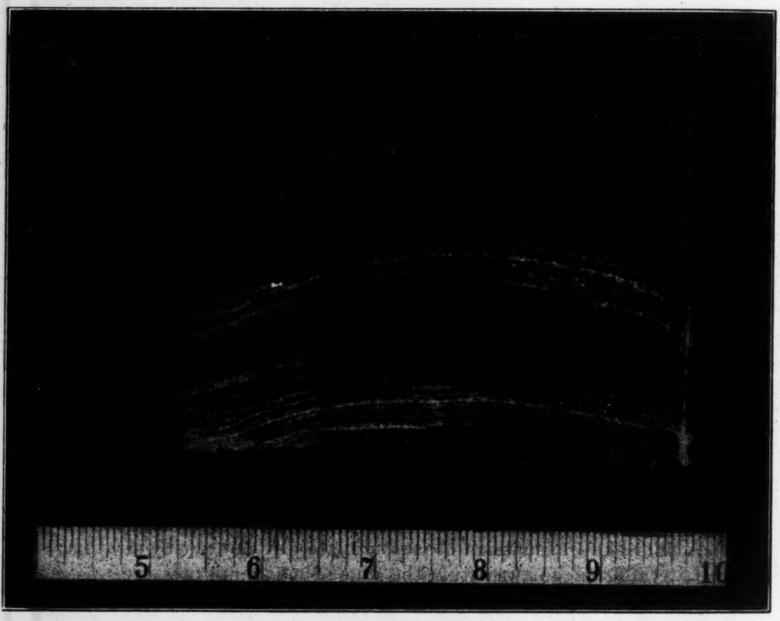


FIG. 22.—CROSS SECTION OF AN "UNDISTURBED" SAMPLE OF FOUNDATION CLAY

Professor Terzaghi inquires as to the type of bentonite used for sealing the drill holes used for the piezometers. The bentonite used was a standard commercial grade having a grain size of 0.3 to 0.5 mm. This material exerts some pressure when confined and supplied with water. In the present case, however, the situation is somewhat different since any water which the bentonite absorbed must be supplied by the pore water within the clay. Six of the piezometers installed were located 225 ft from the center line of the embankment, or about 65 ft beyond the toes. Three of these piezometers, on the river side, did not indicate hydrostatic pressure excesses at any time. The other three, on the land side, indicated only normal hydrostatic pressure up to January 24, at which time they had been in place about one month. This indicates that swelling of the bentonite seal did not create excess pore pressures in the clay. It seems entirely possible for pore-water pressures to become somewhat greater than total overburden pressures over small localized areas, especially in the case where the clay stratum is overlain by sand of appreciable thickness. The upward movement of the column of sand overlying the pressure concentration would be resisted not only by the weight of the column but also by friction along its sides.

Both Professor Terzaghi and Mr. Middlebrooks think that the computations of shearing resistance should have been based upon the results of fully consolidated shear tests rather than "consolidated-quick" tests. The writers are inclined to agree with this viewpoint. Although no such tests on the Pendleton clay are available, slow triaxal shear tests on other clays have shown that the "failure envelope" is a straight line passing through the origin (cohesive strength of zero) and having an inclination of about 30°. If cohesive strength of zero and a true angle of internal friction of 30° is assumed for the Pendleton clay, the computed factors of safety of the land-side portion of the foundation remain practically unchanged; but those for the river side become smaller. The writers hope to investigate this matter more thoroughly at the earliest opportunity.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2235

CHART FOR THE FUTURE

ADDRESS PREPARED FOR THE ANNUAL CONVENTION, CLEVELAND, OHIO, JULY 19, 1944

By Malcolm Pirnie,[1] President, Am. Soc. C. E.

Present recording of the experiences of the past is the tried-and-proved method of making a chart upon which a successful voyage into the future may be plotted. Eternally colorful words of Patrick Henry forcefully express this fact:

> "I have but one lamp by which my feet are guided, and that is the lamp of experience. I know of no way of judging the future but by the past."

Liberty-loving peoples of the earth are now engaged in the greatest of recurrent revulsions against unbearable usurpation of the unalienable rights of all men by powerful human cliques. They were so engaged in the lifetime of Patrick Henry when that New World crisis brought forth enunciation of a great principle, upon which this nation was founded.

The concordant discoveries of the world's most revered scholars and teachers were selected by Thomas Jefferson to guide his task of wording the Declaration of Independence. He said of his work,

> "I did not consider it as any part of my charge to invent new ideas altogether and to offer no sentiment which had ever been expressed before."

Thus, resistance to invasion was cemented in the heat of battle just 168 years ago by defining the cause of the people clearly in a simple 112-word declaration of unalienable rights of men and a recitation of the usurpation of many rights which had been suffered. The representatives of the first thirteen states pledged to each other their lives, fortunes, and sacred honor in signing that Declaration. Victory was finally won by men willing to endure great hardship for a common cause, so clearly defined in the light of their experiences that they were willing to die to establish their declaration as the birthright of their descendants.

---

[1] Cons. Engr., New York, N. Y.

When every effort was being exerted to define the purpose of our Revolutionary War and to support the armed forces from pitifully limited resources, comparable labors were undertaken at the same time to define and implement the peace. A committee to devise a plan for a union of the states was appointed by the Continental Congress on the same day that the committee was appointed to formulate a Declaration of Independence. The resulting Articles of Confederation were reported to Congress eight days after the first Independence Day. Inevitably, the jealous retention by the states of the newly proclaimed rights of their citizens produced a national government with insufficient powers even to give its armed forces the support they so desperately needed. A decade of bitter internal and external experience ultimately resulted in Congress's calling the first Constitutional Convention. Presiding over the deliberations of that convention of distinguished men, representative of their respective states, was George Washington, engineer, military leader, and statesman. In 1791, when the first ten amendments were adopted after the last of the thirteen states ratified the Constitution, a sovereign people had instituted and endowed with just powers the Government of the United States of America to secure the rights of the people.

The Constitution of the United States is figuratively a chart recording the depths, the shallows, and known currents of the age-old struggle for individual freedom among men. Ventures into the future have been plotted on this chart in vast numbers during the century and a half of its existence. They are the voyages of free enterprise into the future, with courses set from known positions on the chart. Promise of security thus made available has been sufficient to justify risk of life and property in the search for new products and services for the use and convenience of man. The unleashed capacity of a people to make effective use of its rights raised living standards within this brief period of time to more than double the standards of living attained during all the preceding centuries.

A veritable avalanche of new conveniences and time-consuming pleasures aroused desires to possess them. This multiplied incentive for constantly increasing production. In the scramble to produce and enjoy such mounting abundance, there was diminishing time given to thought of the source of such freedom and its protection. Developments in education lagged far behind knowledge needed to convert the products of free enterprise into blessings of permanent benefit. So engrossed were the people in the problems of their separate fields of endeavor that the clouds of a powerful reactionary storm to usurp the unalienable rights of all men broke with inhuman force over a large part of the world before our nation began to awake to imminence of its peril.

Not until England's Dunkirk did the flow of outmoded material and equipment start across the Atlantic in relatively insignificant aid of her heroic resistance. We took in exchange grants of sites for defensive bases and proceeded to construct them in feverish haste. In November, 1940, our voters recorded their approval of the proposition that their Army and Navy would not be called upon to fight beyond our national boundaries. Then, thirteen months later, Japanese carrier-borne planes dove out of the clouds on a Sunday morning and destroyed the greater part of our Pacific Fleet snugly tied up in Pearl

Harbor. This assault was patterned on maneuvers of the winning squadron in an earlier American fleet war game. A few hours later our air force was destroyed on the ground in the Philippines. These tragedies had to happen before sound-reasoning common sense could replace comfortable wishful thinking and willingness to let the incompetent do what thinking was done.

There has followed in the succeeding two and a half years an astounding demonstration of the productive capacity of free enterprise energized by almost complete unity of purpose. There was planning and action which vastly accelerated development of transportation and communications, which provided the facilities required to create and implement huge armed forces and a Navy with auxiliary ships strong enough to carry the war to the enemy and to deliver needed supplies to our embattled allies. These developments have conclusively demonstrated that the natural barriers behind which some nations formerly were able to isolate themselves no longer exist.

Now that communications can be instantaneous and commerce can span distances in hours instead of days or weeks, ventures into the future must be planned on this latest radical revision of the chart recording our experiences. The peoples of this new earth must learn to work harmoniously for the common good. Otherwise, they will destroy potential progress and each other. Present speed with which implements of destruction are produced and applied over land, mountains, and oceans warns nations of the future horrible consequences of their failure to live in harmony with their neighbors. The United States of America and allied nations, whose citizens have experienced freedom to use many of their unalienable rights, are now well on the road to force unconditional surrender of the Axis Powers, whose people gave all their rights to government in exchange for its vain promise to make them lords over a subservient world.

There is too little time left before victory for our country to plan adequately to solve its internal problems of reconversion from a wartime to a peacetime basis. There is too little time left to plan with care our foreign relations. The most that can be done within the limited time available is to take the available and demonstrated truths from the chart of experience and express them so clearly that the vast majority of our people can understand the destination of what should be a proud venture into the future and will determine to set forth in that direction. Devious courses undoubtedly will be undertaken, but with the aid of instantaneous communications and interpretations by representatives of the people selected from those best qualified for their respective tasks, our nation can be kept advised of positions reached which are definitely off course and of those most likely to reach our destination.

As great a need for inspired leadership, universal discussion, and prayerful thought by all citizens exists in our nation today as that which led to creation of our birthright in the days of the Continental Congress. We must here and now resolve to give a substantial part of our time for considered thought to rebuild the protections of our rights and reach beyond our borders to offer the aiding hand of true friendship to peoples less fortunate than our inheritance has made us. Of necessity, this must be a continuing process depending upon the development of capacities of peoples to use effectively such rights as they may from time to time possess. Here in this nation we can neither stand still nor regress. We must progress to higher standards of lifetime service to justify to our people our rich inheritance and to other peoples of the earth their sufferance of our existence.

During the past decade, many small units of free enterprise have been forced to go out of existence or to seek protection in affiliation with large units. New obstructions on the chart of recorded experience have appeared to cause this trend. A threat resulting in such demonstrated discouragement of ventures into the future of a formerly greater number of small units of free enterprise also presents dangerous hazards to be navigated by the surviving larger units. The most important immediate assignment for the best qualified among our leaders is to remove these obstructions as fast as possible to re-establish incentive for development of free enterprise.

Where does the engineer stand in the present call for best qualified leaders to interpret the chart of experience, to define the destination of ventures into the future so clearly that the vast majority of his fellow citizens will plot their courses in that direction, and to offer suggestions of courses he believes may be most likely to prove successful? The engineer must admit that he is largely responsible for planning the phenomenal advances made in harnessing power to aid men in the development of mass production, in the acceleration of transportation, and in the accomplishment of instantaneous communications. To accomplish so much in so short a time, he has concentrated his effort almost to the extent of his full capacity for thought within his chosen limited field of applied science.

Taught to plan his creations with due regard to their usefulness related to their costs, he has made more and more material things available to his fellow men in exchange for less and less effort required of them. The engineer has also aided in the development of entertainments to absorb the increasing free time of the people. Recreation time, formerly available for physical exercise, study, thought, and discussion, is competed for by absorbing programs broadcast into every home and by tempting amusements that can be reached by a drive in the family car. The engineer has given too little thought to the social effects of the results of his enterprise. The engineering profession is inarticulate because it is separated into a number of limited specialties. Within each special field, engineers discuss their problems in language which is foreign even to many engineers in other fields and totally uncomprehensible to the general public. On the chart of experience the present position of the engineering profession is a galaxy of isolated spots, only a few of which are on the proper course to a desirable future destination.

Today, standing on the threshold of the greatest opportunity for service that the world has yet offered them, engineers have a grave responsibility in the development of a sound national and international future program. To begin this assignment, engineers must join hands across the boundaries of their specialties and work together to remove unnecessary obstructions to future ventures of free enterprise and to chart a safe course toward lasting peace with organized justice in a free world. They must be adequately represented in the departments of government, on committees of business men advisory to government, and among the delegates who will negotiate a lasting peace. To be entrusted with such responsibilities of representation of their fellow citizens, a majority of these must first approve the principles and propositions for applying them offered by the engineers. It will not suffice to know that an engineer designed the best airplane engine or a bridge to span a great river. The people will always take such qualifications for granted but will need assurance that the engineer understands many of the present problems confronting the nation as a whole, which are in fact engineering or closely related problems. Concrete evidence that engineers, representative of the entire profession, recognize the grave responsibilities of their greatest opportunity for service in world development toward lasting peace, must be formulated now and submitted in comprehendible form for criticism, revision, and majority approval by their fellow citizens.

To this end, let us consider the following four propositions. It is necessary that they should receive considered thought and criticism conducive to clearer expression of principle in each proposition. The development of other propositions equally important is omitted here in the desire to focus initial attention on an obviously important few.

PROPOSITION 1

The Declaration of Independence states in part:

> "We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable rights, that among these are life, liberty and the pursuit of happiness. That to secure these rights, Governments are instituted among men, deriving their just powers from the consent of the governed. That whenever any form of Government becomes destructive of these ends, it is the right of the people to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their safety and happiness."

With the authors of the Declaration of Independence we recognize that the statement quoted above is restricted to rights and does not imply present equality in capacity and will of individuals among men to use such rights effectively and for the common good. Rights are equal and unalienable at birth. Use made of such rights by the individual is accountable by him to his Creator and to his fellow men only to the extent that the rights of others are transgressed.

PROPOSITION 2

The American way of life can be continued and improved in an increasing complexity of activities and new developments. This is possible only if a substantial majority of the people believe and will aggressively assert the principle simply phrased by Walter Lippmann:

> "In a free society the State does not administer the affairs of men. It administers justice among men who conduct their own affairs."

Constitutional checks upon natural human tendency to exercise power beyond the boundaries of authority granted by the people must be made equally mandatory upon all agencies established by Congress to direct details of administration of the collective needs of free enterprise within well-defined boundaries of the delegated authorizations.

PROPOSITION 3

Leaders of opinion at the so-called Mackinac Conference (1943) subscribed to the following statement, which Sumner Welles states is unexceptionable and in accordance with the spirit and the letter of the announced plan of President Franklin D. Roosevelt:

> "We favor responsible participation by the United States in post-war cooperative organization among sovereign nations to prevent military aggression and to attain permanent peace with organized justice in a free world."

To start implementing these principles, an international congress representative of the Allied Nations should be inaugurated at the earliest possible date.

The Allied Congress should be directed:

1. To make a declaration of principle.
2. To prepare articles of joint responsibility in undertakings to prevent military aggression.
3. To draft a charter or constitution under which representatives of free peoples can organize justice and support its development to gain majority approval and respect to the end that permanent peace may ultimately be attained.

PROPOSITION 4

A people endowed with reverence for good should be urged to adopt universal education to develop capacity to produce good works.

Facilities for education designed to increase incentive to develop good citizenship and acquire knowledge should be perfected in competitive balance with the productions of the people's enterprise. Incentive to expand production will increase in proportion to unalienable rights entrusted to the people.

In the United States, incentive to acquire good citizenship and knowledge is insufficient to compete with development of time and labor-saving conveniences and time-consuming diversions. Education designed for the tempo of average capacity is productive of lazy mental and disruptive social habits for half of the pupils endowed with greater-than-average capacity to develop qualities of good citizenship and to acquire knowledge.

Free debate upon these four propositions will clarify their wording to convey meanings acceptable to a substantial majority of free citizens. Principles involved are fundamental and must be understood as the foundations upon which millions of special fields for free enterprise developments can be established. Government pronouncement through agencies of new rules that transgress justice, which has earned majority respect and support of a people through past ages, is productive of uncertainty to the point of chaos in the administration of justice.

Never before in history has so dependable a chart been created as that recording the scant two centuries of experience of our nation. It must be a bright lamp by which our feet are to be guided if we are to form sound judgment in directing our future course. Engineers must join in charting a course secured by references to those established landmarks which have assisted in guiding our nation to its present greatness. We must never depart from the course so clearly set forth in the most inspired of all state documents, the Constitution of the United States.

All citizens entrusted with public office and accepted into the armed forces have sworn to support and defend the Constitution. We are united with our allies to defeat the demonstrated end-product of National Socialism. On the home front we must voluntarily swear to understand, defend, and perpetuate the national institutions responsible for the American way of life to the end that the sacrifices we are demanding of our armed forces shall not be in vain.

MEMOIRS OF DECEASED MEMBERS

GEORGE STEWART DAVISON, Past-President and Hon. M. Am. Soc. C. E.[1]

Died October 3, 1942

George Stewart Davison was born on September 21, 1856, in the borough of Lawrenceville, Pa. (absorbed by Pittsburgh, Pa., in 1868). His father, Edward Davison, came to America from England in 1816; his mother, Isabella Kennedy, born in Scotland, came to Pittsburgh from Canada in 1832. The family of Edward and Isabella (Kennedy) Davison consisted of seven sons and three daughters. With the death of his younger brother Charles in 1937, George Stewart Davison was the sole surviving member of that large family.

Completing his preparatory education in the schools of his birthplace, Mr. Davison entered Rensselaer Polytechnic Institute, at Troy, N. Y., in 1874, being graduated in 1878 with the degree of Civil Engineer. From the day of his graduation throughout the remaining sixty-four years of his life he was deeply interested in the "Institute" and served as one of its Trustees from February, 1909, until his death. In 1926 Rensselaer Polytechnic Institute conferred upon him the degree of Doctor of Engineering. In the same year the University of Pittsburgh honored him with the degree of Doctor of Science.

In spite of the pressure of business and the cares that accompany responsibilities, Mr. Davison found time to take an active part in movements that would benefit his fellow man and his community. Seldom is seen such devoted service as he gave to the institutions on whose boards he served. No one person could tell a complete story of his generosity. At the beautiful service held in the Edgewood Presbyterian Church in Pittsburgh on October 5, 1942, every station in the community's life was represented. All were there because of a profound sense of loss.

In character George Stewart Davison was everything that was fine and noble. He impressed everyone with his sincerity and his absolute integrity. He was a man of the highest moral standards—generous, kind, friendly, truly a Christian gentleman. His death on October 3, 1942, coming most unexpectedly, inflicts upon his family, his friends, and his community no ordinary loss. Death has made vacant a place that cannot be filled.

His civic and humanitarian interests were many. In 1893 he was a member of a committee to study the ravages of typhoid fever, and, as a result of the findings of that committee, the City of Pittsburgh built its sand filtration plant which practically eliminated that dread disease. Mr. Davison was an active member of the Chamber of Commerce of Pittsburgh for twenty-seven years; a member of the Pittsburgh Flood Commission for thirty years, becoming its president following the disastrous flood of 1907. He was chairman of the City Transit Commission for ten years, of the City Planning Commission for three years, and of the Citizens Committee on City Plan for twelve years. He was the originator of the Metropolitan Plan for Allegheny County (Pennsylvania). This plan was approved by 70% of the votes cast, but the will of the voters was thwarted when a few members of the Legislature tampered with the Enabling Act which provided for the election.

---

[1] Memoir prepared by J. N. Chester, M. Am. Soc. C. E., and Allen S. Davison, Assoc. M. Am. Soc. C. E.

In 1927, with twenty-three other leading industrialists, he shared the authorship of a book entitled "Pittsburgh and the Pittsburgh Spirit," published by the Chamber of Commerce. The title of Mr. Davison's contribution to that work was "Petroleum from the Earliest Days."

For almost a quarter of a century Mr. Davison served as a director of The Western Pennsylvania Hospital, one of the largest hospitals in Pittsburgh. He was elected a vice-president in 1933 and president in 1940. To the day of his death this institution was the object of his untiring devotion.

Those who knew him well were aware of his amazing memory and his deep sentiment. He was unashamed of the tears that came to his eyes as he recounted his rare experiences or those of his family, and he had the faculty of relating in a fascinating way his seemingly inexhaustible fund of stories. Measured by years, his life was a long one. Measured by achievements, he might have been regarded as a patriarch. He lived to realize his oft expressed hope—that the world might be bettered because of his living in it.

Mr. Davison entered the practice of engineering on July 1, 1878. Like so many prominent engineers of his day, he began his career in railroad engineering. No attempt will be made to dwell upon his business and professional achievements; they are recorded quite fully in *Civil Engineering*[2] upon the occasion of his election to Honorary Membership. On January 1, 1890, Mr. Davison joined the late William Glyde Wilkins,[3] M. Am. Soc. C. E., in his already established general engineering practice. That partnership came to an end in 1900 when he entered into a contract with A. W. Mellon and R. B. Mellon to serve as manager and executive of corporations and other interests under their control. The greatest of these corporations later became the Gulf Oil Corporation. When Mr. Davison retired from the oil industry in 1929, he was vice-president of the Gulf Oil Corporation and president of the Gulf Refining Company and its eight or ten affiliated companies which engage in every phase of the oil business from production to marketing, including the intermediate transporting and refining. In the latter part of 1906, Mr. Davison initiated locations for an oil pipe line extending from Tulsa, Okla., to the Gulf coast at Port Arthur, Tex., a distance of five hundred miles, designing and constructing its pump stations. In this connection Mr. Davison is believed to have played the rôle of pioneer in the use of large internal combustion engines as the power source for pumping oil through pipe lines. In 1910, he designed and had direct charge of the construction of the one-and-one-half mile extension of the Port Arthur Ship Canal to the refinery of the Gulf Refining Company at Port Arthur, which improvement included a concrete wharf 1,000 ft long. In September, 1941, Mr. Davison made the round trip by plane from Pittsburgh to Beaumont, Tex., to witness the unveiling of a monument to the Lucas Well in Spindle Top Field which came in on January 10, 1901, the largest producer in this continent, reported as 100,000 bbl a day. The inability of the original owners to control and store this tremendous volume of oil caused their bankruptcy and out of this situation was born a few years later the present Gulf Oil Corporation. On September 30, 1929, Mr. Davison's contract with the Mellon interests was terminated at his request, although he continued to manage the Pennsylvania Water Company of which he became executive vice-president in 1902 and president in 1933 upon the death of R. B. Mellon.

---

[2] *Civil Engineering*, January, 1938, p. 54.

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1742.

In 1880 Mr. Davison became a member of the Engineers' Society of Western Pennsylvania, a few months after its organization, and for the first twenty years of his membership he left no stone unturned to advance its interests. He was a director in 1891–1892, vice-president in 1896–1897, and president in 1898. On May 23, 1942, this society conferred honorary membership upon him for his "outstanding contributions to the engineering profession, to the community, and to the society."

On April 2, 1890, Mr. Davison was elected a Member of the American Society of Civil Engineers. To quote from a statement he made in 1920 before a meeting of the Pittsburgh Section, "I made application for membership with the selfish idea that my connection with the Society would pay me large dividends. In this I have not been disappointed. In fact, there came a time when I became convinced that the rate upon which my dividends were being received entitled me to a place in the profiteering class." In 1903–1905 he served the Society as a Director. He was elected Vice-President in 1923–1924; became a Life Member in 1925; was elected President in 1926; and, on October 4, 1937, was elected to Honorary Membership. He represented the Society as a delegate at the dedication, on July 4, 1928, of the $80,000 memorial carillon, given by the engineers of America, in the new fire-proof library of the University of Louvain, at Louvain, Belguim, and was its official delegate at the World's Engineering Congress in Japan in 1929. In the years that intervened between 1905 and 1923, Mr. Davison's business activities and civic interests occupied his time, but he never lost sight of his obligations to the Society. He was proud of his profession, and, whether in the conduct of his employers' affairs or of his own, he constantly kept in mind the thought that he was a member of an honorable profession, that he should do nothing that would bring disgrace to himself or discredit to that profession, and on occasions he vigorously protested instances of their misconduct with violators of professional ethics.

In 1926 Mr. Davison became a member of the American Institute of Consulting Engineers and was always active in promoting its interests. Mr. Davison was a member of Delta Phi and Sigma Xi fraternities. He was also a member of The Historical Society of Western Pennsylvania, the Duquesne Club of Pittsburgh, and the Pittsburgh Athletic Association.

In World War I, Mr. Davison was engaged in war work one week before the United States declared war on Germany, and for two years he devoted practically one third of his time to the problems of the war. One of these problems was to supply gasoline and fuel oil to the Allied armies and navies—the duty of the National Petroleum War Service Committee which was composed of presidents of oil companies.

In religion, Mr. Davison was a Presbyterian and, at the time of his death, was a member of the Edgewood Presbyterian Church, which he served as a Trustee from 1910 to 1926. He was elected president of that Board in 1911 and continued in that office until his resignation in 1926.

Politically, he was a staunch Republican.

On May 19, 1881, he was married at Melrose, N. Y., to Clara Elizabeth Lape, daughter of Pamelia Simmons and Frederick Allen Lape, of Troy. Mrs. Davison died in Pittsburgh on September 3, 1916. They had one son, Allen Stewart Davison, who, with six grandchildren and two great-grandchildren, survives. To them George Stewart Davison has left a wonderful heritage.

As previously stated, Mr. Davison was elected a Member of the American Society of Civil Engineers on April 2, 1890.

JOHN FRANK STEVENS, Past-President and Hon. M. Am. Soc. C. E.[1]

DIED JUNE 2, 1943

John F. Stevens lived thirty-eight days beyond the age of 90. His professional career, too, was a long and varied one. It extended from 1873 to 1931—fifty-eight busy years; and it included distinguished achievements in three well-defined fields: First, that of railroading, which he never gave up and in which his greatest contributions probably were made during the decade 1889 to 1899; second, his work in Panama from 1905 to 1907; and third, his Russian service from 1917 to 1923.

John Stevens was born in West Gardiner, Me., on April 25, 1853, the son of John and Harriet (French) Stevens. He attended the Maine State Normal School, and this was the end of his formal education. In his address at Boulder, Colo., on May 22, 1925, Mr. Stevens had this to say regarding his training:

> "So being endowed by Nature with a bull dog tenacity of purpose, I started on the acquisition of engineering knowledge, at the same time I was working for a bare livelihood for my family and myself. In this sense I became, in a measure, self educated, as I never attended a day at a technical school, or even listened to a technical lecture in my younger days."

After leaving normal school, he went west. He served as assistant city engineer of Minneapolis, Minn., for two years, and then went into railroad work in which he was remarkably successful. Eventually he became System Operating Officer of such large railways as the Great Northern Railway Company; Chicago, Rock Island and Pacific Railway Company; and the New York, New Haven and Hartford Railroad Company. He was President of the Spokane, Portland and Seattle Railway Company, and the Oregon Trunk Railway, subsidiaries of the Great Northern and the Northern Pacific railway companies when they were rounding out the so-called Hill Lines in the Pacific Northwest. Under Stevens they reached Portland, Ore., by an advantageous route down the Columbia River and entered central Oregon, the gateway to California, by building up along the Deschutes River. His administration of these high offices was notably efficient, so that, in addition to the distinction he attained by other more unique accomplishments, he is rightfully accorded a place in railway history as a successful major executive of important railways.

---

[1] Memoir prepared by Ralph Budd, Hon. M. Am. Soc. C. E.

Stevens' most valuable contribution to the railways of America, however, was as a civil engineer, more particularly in reconnaissance, location, and construction work. He was fortunate, and so was the industry and indeed the public generally, in that the period of most rapid railway expansion in the United States and Canada coincided with his young manhood when he was at that stage of his developing experience and ability which made railway reconnaissance and location especially attractive to him. From 1876, when he was 23, until 1899 when he reached 46 years of age, 140,000 miles of railway were built in the United States. He was in the thick of that work, more or less in the following order: The Sabine Pass and Northwestern Railroad Company; Denver and Rio Grande Railway Company; Chicago, Milwaukee & St. Paul Railway Company; Duluth, South Shore & Atlantic Railway Company; Canadian Pacific Railway Company; and Great Northern Railway Company.

There were many locating engineers in those days, and it would be pleasant to state that all railway lines were well and properly laid out. They were not, however, and the cost of operating many poorly located routes, as well as the cost of extensive rebuilding, ran into millions of dollars. There were, of course, numerous instances in which the cost of the lowest grades and easiest alinement would have been prohibitive, and where the first track built was known to be temporary, a permanent route being selected for later development.

The principles of railway location were thoroughly understood by some railway engineers from the early '70's, although the great work by Arthur Mellen Wellington,[2] M. Am. Soc. C. E., was not published until 1877. Every engineer well knows, however, that the best of formulas requires the exercise of good judgment to yield the best results. It would be difficult to find a better illustration of this truth than that found in the choice of ruling grades and other physical standards when laying out a railway. Assumptions as to the future volume of traffic, the speed requirements from the standpoint of adequate service and competition, the cost of maintaining the line in alternative locations, possible floods, slides, snow troubles, etc., are some of the factors that must be based upon judgment. The commonest error was failure to realize the extent to which railroad cars and locomotives would increase in weight and the high train speeds which would be attained; another was to underestimate the heavy future traffic on some of the lines.

---

[2] For memoir, see *Proceedings*, Am. Soc. C. E., Vol. XXI (1895), p. 199.

James J. Hill,[3] F. Am. Soc. C. E., and John F. Stevens were notable collaborators. Hill had such faith in the future and knew so well the importance of favorable physical characteristics that he was willing to spend large amounts to secure the best route, and, when the cost of the ultimate was prohibitive, he approved of a makeshift line that would tie in to the future location with least waste. The inestimable value of proper location of a railway was admirably summarized by Justice Pierce Butler at the unveiling of the statue of John F. Stevens at Marias Pass in July, 1925,[4] when he said:

> "Who can appraise the thing accomplished? It is impossible to estimate the value of such a pass for the transportation of transcontinental commerce in ever-increasing volume. The saving of power, labor, and equipment results directly to the great benefit of the company and of the public served. The shortest line fixes the distance, and the best grades establish the standard of efficiency that competitors must meet. So, indirectly, the use of this pass results to the advantage of all interested in transcontinental transportation by whatever route."

The science of railroading was advancing very rapidly during the period under discussion. In the progressively higher positions he held, Stevens exerted a great and growing influence upon the improving standards of construction and operation.

When in 1905, succeeding John F. Wallace,[5] M. Am. Soc. C. E., Stevens became chief engineer of the Panama Canal, he was undoubtedly the world's foremost railway civil engineer. This was true in spite of the fact that he was then not in engineering but in executive work, having been advanced to the position of vice-president of the Chicago, Rock Island and Pacific Railway Company, following his general managership of the Great Northern. It is significant that in the professional, in distinction to the administrative, field, the first and most important question which had to be decided after he took charge was the type of canal to be built. The problem was elementary. The considerations for and against the sea level and the lock types of canal were essentially the same as those which a railway locating engineer must weigh over and over again in the course of his duties. The difference in the case of the canal problem was in its magnitude.

A Board of fourteen noted consulting engineers decided 9 to 5 for the sea-level route. The minority five were Alfred Noble,[6] Past-President, Am. Soc. C. E., Henry L. Abbott, Frederic P. Stearns,[7] Past-President, Am. Soc. C. E., Joseph Ripley, M. Am. Soc. C. E., and Isham Randolph,[8] M. Am. Soc.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI, December (1917), p. 1829.

[4] The story of Stevens' discovery of Marias Pass has been fully covered in the following references: *Proceedings*, Am. Soc. C. E., May, 1925, Vol. LI, p. 220, and October, 1926, Vol. LII, p. 445; "The Blazed Trail of the Old Frontier," by Agnes Laut, McBride, 1926; "Story of Marias Pass," by Grace Flandrau, Great Northern Railway, 1926; "John Fritz Medal Booklet," Four National Engineering Societies, 1925; "An Engineer's Recollections," by John F. Stevens, *Engineering News-Record*, March 21, and November 28, 1935; also published by McGraw-Hill Book Co., Inc., 1936.

[5] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1635.

[6] For memoir, *ibid.*, Vol. LXXIX (1915), p. 1352.

[7] For memoir, *ibid.*, Vol. LXXXIII (1919–1920), p. 2132.

[8] For memoir, *ibid.*, Vol. LXXXIV (1921), p. 902.

C. E. Stevens agreed with them in favor of a lock canal, and through his influence Congress adopted that plan on June 29, 1906.[9] His letter of January 26, 1906, to the Isthmian Canal Commission was characteristic:

> "Prior to the receipt or inspection by myself of any report from the Consulting Board, my judgment was and is yet in favor of a high-level canal, and it has only been strengthened by the very able presentation of the facts and deductions made therefrom in the minority report.
>
> "I can therefore conscientiously and fully approve the adoption of the high-level type, and strongly recommend that the Commission give its official voice in favor of such a type as is described in the minority report, and there seems to be nothing that I can add to such report, in view of its clearness and completeness, more than that I am heartily in favor of its adoption."

This decision was basic, just as is the choice of a route for a railway when it is to be constructed. The government of the United States never gave John F. Stevens any special credit or any reward for this momentous decision, but if he had joined the majority for a sea-level canal there scarcely can be any doubt of the criticism which would have come his way when the mistake was realized. When the lock level had been chosen and fixed at 85 ft above the sea, the organization of forces and prosecution of the work followed with great vigor. The American public was clamoring for progress and speed and soon had reason to be satisfied.

Two general types of engineering work were involved: (1) The digging of Culebra Cut and other dry excavation between the Gatun Locks and those at Miraflores, and (2) the building of the locks and dams and approaches thereto from tidewater. The first of these undertakings was like a huge railroad construction project—nothing else. The Culebra Cut was 8 miles long, more than 300 ft deep on the slopes, 300 ft minimum width at the bottom, and more than half a mile across the top, but digging it out was essentially a matter of steam-shovel operation at various levels and hauling the excavated material away by train.

There was a job in the United States which was and still is much like Culebra Cut (later called Gaillard Cut) in many respects. That is the stripping and mining of iron ore at the Mahoning and Hull Rust open pits on the Mesabi Range in Minnesota. There are, of course, several other similar operations. John F. Stevens once protested the assertion that the canal excavation was patterned to some extent on what he had seen and had a part in doing on the Mesabi Range, but it is difficult to believe that Culebra could have been organized so promptly and perfectly without that experience. The locks, dams, and sea-level approaches were of course the kinds of projects that the army engineers and their civilian associates were more familiar with, except for the unprecedented magnitude at Panama. These were placed in the hands of those engineers who had had wide experience in such construction.

There were literally all manner of collateral organizations and engineering activities involved in carrying on such an extensive undertaking as the Canal so far from sources of supplies. Most vital was the matter of housing and caring for the many employees of various nationalities, including especially that of making it safe to live there. Ridding the Canal Zone of yellow fever, reducing malaria to the minimum, and cleaning up the area were given preference by Stevens. This pioneer project in sanitation not only was necessary to the successful production of the canal but was an humanitarian act of first magnitude. Its brilliant accomplishment was under the direction of Col. William C. Gorgas, who enjoyed Stevens' full confidence and support.[10]

---

[9] "Report of the Board of Consulting Engineers for the Panama Canal," Government Doc., 1906; also "Best Type of Canal for Panama," *H. R. Doc. No. 10,* March 26, 1909.

The record at Panama is that by April, 1907, when Stevens turned the job over to Col. George W. Goethals,[11] M. Am. Soc. C. E. (later General), the entire work was planned and organized; construction was under full headway and proceeding at a rapidly accelerating rate. After Stevens left there were no substantial changes and his plans were carried forward to successful conclusion and on his time schedule.

Upon returning to the States from the Canal Zone in 1907, Stevens spent four years in active railroad administration and then six years as a consulting engineer. During this period he served two years as vice-president of the New Haven and two as president of the Hill Lines' subsidiaries in the Pacific Northwest.

It is a notable feature of Stevens' life story that he never remained long in one place or with one concern. The nature of his work would account for this until he became so firmly entrenched with Mr. Hill after 1889. Then for ten years he handled the Great Northern's expanding construction as chief engineer and in 1901 became general manager as well. He went to the Rock Island in 1903 and stayed there two years, when he assumed charge of the Panama Canal construction.

Stevens' frequent changes after leaving the Great Northern were not because he was ambitious to better his position, or because he did not make good in the high places he held, but rather were due to some other traits of his character. He was possessed of a New England conscience, an unfailing ability to detect sham or pretense, and was frank in showing his feelings about such things, especially in the case of those holding higher positions. He was a debunker by nature and environment. Handsome, prepossessing, and impressive in every way, Stevens was at the same time modest almost to the point of being shy. His years of struggle and lack of privileges in his youth might account for this. Yet another characteristic was his sensitiveness to criticism. Possibly it reflected a mindfulness of his lack of formal engineering education, although he must have known that any such deficiency had been more than compensated by his study and his wide experience in the field.

Mr. Stevens had a good sense of humor which manifested itself in dry Yankee fashion, never in the form of practical jokes. His response on the unveiling of his statue at Marias Pass was considered by many a model for such an occasion and he livened the occasion by saying:

---

[10] "William Crawford Gorgas, His Life and Work," by Marie Cook Gorgas and B. J. Hendrick, Doubleday, Page and Co., New York, N. Y., 1924.

[11] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1813.

> "As these proceedings have been a little bit sober, to say the least, I want to illustrate, if you will pardon my levity, why I might not properly express my appreciation by a lengthy address. Many years ago, probably longer back than many of you remember, there was an eminent Irish author and actor and one of his famous characters was Con Carney. Con Carney was a worthless, happy-go-lucky fellow. He was supposed to die and at the funeral there was, of course, lots of whiskey and lots of fighting. When his friends went to pay their final respects to Con, to their surprise he rose up in his coffin and they exclaimed: 'Are you alive—why didn't yez spake?' and Con said: 'Spake! Do yez think I would spile a good wake?'
>
> "So perhaps, while this is not a wake, it is better that I do not 'spake.' "

Outstanding in Stevens' relationship with other men was the universal affection in which he was held by subordinates and the loyalty of those who worked under his direction. He was most considerate of them, while they thoroughly believed in him and gave unstintingly their fullest and best efforts. His unlimited respect and admiration for Mr. Hill endured through the years and it is only fair to state that the regard was mutual. It was Mr. Hill who convinced William H. Taft, then Secretary of War, that Stevens was the best man to build the Panama Canal. As has been implied, Stevens was not so happy in his relationships with men holding coordinate positions, nor always with his superiors. This was recognized among some of his closest friends, including Mr. Hill, and it may have interfered with his advancement to the presidency of one of the larger railroads. Stevens felt that he had reason to expect such promotion about 1903 and again in 1911, and its failure to materialize hurt him deeply.

The married life of John F. Stevens and his devoted wife (Harriet T. O'Brien Stevens) was ideal. Mrs. Stevens enjoyed having people around her, and she had many friends wherever she went. Mr. Stevens had great affection for her and left to her care and judgment everything pertaining to the home. Her death in 1917, ending forty years of happy married life, left him disconsolate. Surviving are three sons, Donald French, John F., Jr., and Eugene C.

Although his sons made him welcome at all times and were a great comfort he was never again to enjoy a real home of his own where he could mingle extensively with his friends.

All personal ambitions and any disappointments concerning them had no doubt been completely outlived before Stevens went to Russia[12] during World War I in 1917. He was 64 years of age then, and possessed neither political nor personal aspirations. His years of experience in many fields had made him cosmopolitan and philosophical. The service he was to render in Russia was wholly disinterested and devoid of guile either on the part of his country or himself. He represented a friendly nation, desirous only of assisting a people who were traditionally our allies at a time when they had become helpless through the ravages of civil strife.

The Russian government under Alexander Feodorovitch Kerensky constituted an attempt to reorganize a weary populace and restore order out of the chaos following collapse of the Czar's regime. Railway service was essential to success, and the United States sent a Transport Board to assist the Russian officials in their efforts. The Kerensky government fell and then for nearly two years Russia was torn with the struggles between the Bolshevists under Nikolai Lenin and Leon Trotsky and the White Army under Alexander Kolchak and other leaders.

---

[12] "America's Siberian Adventure," by William S. Graves, Jonathon Cape, and Harrison Smith, 1931.

The United States and its allies were concerned with keeping open the Trans-Siberian Railway as an artery for necessary travel and the movement of vital supplies from one part of Russia to another. The policy of the United States was to remain strictly neutral concerning internal affairs of Russia. To carry out these good intentions there was organized the Inter-Allied Technical Board of eight nations: China, Great Britain, France, Italy, Czechslovakia, Japan, Russia, and the United States. John F. Stevens was the United States member and Chairman. At least so far as the conduct of the United States was concerned, there cannot be any doubt that the principles laid down were strictly observed.

To assure compliance with the Board's policies the property and operations of the Trans-Siberian Railway were safeguarded by the military of several nations for about two years, Maj.-Gen. William S. Graves being in charge of the United States troops. Stevens also had a carefully selected group of nearly three hundred railway operating men, with northern United States railway experience. These men were under the leadership of Col. George H. Emerson, general manager of the Great Northern. They actively supervised the railway operations from Vladivostok as far west as Omsk. The property was kept intact and service maintained, although not always with much regularity. Some critical raw materials were brought out, and much needed supplies taken in. The greatest and most spectacular movement over the Trans-Siberian during this period was that of the Czech Army. There were several Divisions of these soldiers who found themselves so far east of the eastern war front that they could not return as they had come and were taken over the Russian railways all the way east to Vladivostok, thence home by sea.

About as much tact was required to handle the Inter-Allied Board as to keep in hand the situation along the Siberian Railway; and the successful administration of this assignment no doubt was the most difficult undertaking of Stevens' whole life. He achieved real greatness in its discharge, and such men as William H. Taft, Elihu Root, and Newton D. Baker were enthusiastic in their praise. Both the military protection and the railway contingent withdrew in 1921, after which Stevens, with only one or two assistants, continued in an advisory capacity until 1923.

In 1935, Stevens made a noteworthy contribution to engineering literature by writing about his life and work, omitting, however, any reference to the years he spent in Russia. A final chapter of this series of "recollections,"[13] as he called them, is addressed "To The Young Engineers Who Must Carry On." This chapter of counsel and advice doubtless reflects his final judgment of values at the close of his long and brilliant career. It contains not only admirable counsel for engineers of all ages, but affords a revealing glimpse into the character of the author. It is a declaration of ideals and of faith in the future with a closing word on rewards—

---

[13] "An Engineer's Recollections," by John F. Stevens, *Engineering News-Record,* March 21 and November 28, 1935; also published by McGraw-Hill Book Co., Inc., 1936.

> "While the rewards are not always in the shape of great material wealth as the world regards wealth, they will come in the knowledge that we can leave a good name to our posterity, and that we have contributed something to the comfort and happiness of succeeding generations."

Stevens received the following governmental honors: Distinguished Service Medal, United States; Officer of the Legion of Honor of France; Second Class Order of the Rising Sun, Japan; Order of Chia Ho (Golden Grain), China, highest civil decoration; Order of Wen Hu (Striped Tiger), China, highest military decoration; and the Czechoslovak Military Cross. He was an honorary member of the Association of Chinese and American Engineers and the Northwestern Society of Engineers. In 1922, he received the degree of Doctor of Laws from Bates College, at Lewiston, Me. He served as chairman of the U. S. Commission of Railway Experts to Russia, and as President of the Inter-Allied Technical Board. He was elected an Honorary Member of the Western Society of Engineers in 1935.

He received the John Fritz Gold Medal on March 23, 1925, for "great achievements as a Civil Engineer, particularly in planning and organizing for the construction of the Panama Canal; as a builder of railroads, and as administrator of the Chinese Eastern and Siberian Railways." The Hoover Gold Medal was given to him in 1938.

Mr. Stevens was elected a Member of the American Society of Civil Engineers on June 6, 1888, and an Honorary Member on June 19, 1922. In 1927 he served as President of the Society.

JOHN WATSON ALVORD, Hon. M. Am. Soc. C. E.[1]

Died July 31, 1943

John Watson Alvord, son of the Rev. John Watson and Myrtilla Mead (Peck) Alvord, was born at Newton Center, Mass. (later a section of Boston, Mass.), on January 25, 1861, from pioneer Colonial New England stock on both sides of the family.

He was educated in private schools, including Howard University Preparatory School in Washington, D. C., where the family moved when he was a young child. At the age of 17, he struck out for himself, and, after a short experience as a law clerk in New York, N. Y., he went west and entered the employ of the Denver Pacific Railway and Telegraph Company as a brakeman and fireman running between Denver, Colo., and Cheyenne, Wyo., in the mining boom days. A framed letter hanging on the wall of his office certifies his fitness as a railroad man. In 1879 he went to Chicago, Ill., and entered the employ of his brother-in-law, the late John A. Cole,[3] M. Am. Soc. C. E., at that time a practicing civil engineer engaged on the municipal improvements of the suburban towns immediately surrounding rapidly growing Chicago. He studied surveying, became a good draftsman, learned to design under direction, to supervise the construction of water works and sewers, and to lay out the villages and subdivisions later to become part of Chicago. This work included the construction of the water works system for the Village of Hyde Park, including an inlet lake tunnel under Lake Michigan on which he was assistant engineer. In 1883 his employer admitted him to partnership under the firm name of Cole and Alvord. During this year he was principally engaged in the design of a sewer system for the town of Lake View (a suburban area later part of Chicago).

---

[1] Memoir prepared by Charles B. Burdick and Louis R. Howson, Members, Am. Soc. C. E.

From 1884 to 1887 Mr. Alvord served as city engineer of Lake View, designing and executing municipal work—water supply, sewer extensions, street improvements, and lake shore protection—costing $3,500,000. During this period at Lake View he profited greatly by meeting Benezette Williams, an able consulting engineer who was employed to advise on the Lake View improvements. His association with Mr. Williams so inspired Mr. Alvord's desire for knowledge that he spent the greater part of the year 1888 on an engineering inspection trip to the eastern United States and Europe, studying hydraulic and sanitary engineering works in England, France, Germany, Austria, and Italy—frequently traveling from town to town on foot.

Returning to the United States, he resumed his engineering connection with Mr. Cole, under the firm name of Cole, Alvord and Shields and for a year was engaged in additional suburban work including an adventure as a contractor on two Lake Michigan water works intakes involving considerable hazard. In 1890 he became town engineer for Cicero, Ill., and designed the sewer system for 17 square miles of territory (part of which now is within the City of Chicago).

Late in 1890 Mr. Alvord accepted a position with the Worlds Columbian Exposition (Chicago) heading the Department of Surveys and Grades which employed a large force of engineers on the layout of the Exposition, the lake shore protection, and the system of lagoons that was an important feature of the landscaping operations. This was a valuable and inspiring experience during which he had intimate contact with many experts in all branches of architecture, art, and engineering including close association with Daniel H. Burnham, the director of works.

When his work at the Exposition was completed in 1894, Mr. Alvord again visited England and France for further study of engineering works. Upon his return to Chicago, he opened an office as consulting engineer which continued without interruption until his death, although his responsibilities were minor for the last twenty years of his life. For about two years following 1896, he operated in partnership with William S. Shields who had been an associate during his early experience. In 1902 he formed the partnership of Alvord and Burdick, which in 1922 became the firm of Alvord, Burdick and

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1634.

Howson. During this time, the engineering work of the firm extended widely throughout the United States.

Mr. Alvord was particularly interested in the subject of sewage treatment, designing several of the early plants utilizing sand filtration after preliminary treatment in septic tanks then coming into use. In 1897 he was employed by the City of Columbus, Ohio, and made a study and report on all methods of treatment available at that time. He early recognized the disadvantages of the septic tank and in 1913 designed one of the earliest plants for sedimentation with sludge digestion in separate tanks, which shortly before had come into use in England and which were the antecedents of separate digestion effected through mechanical equipment in modern plants. In 1907 Mr. Alvord was employed by the U. S. Steel Company to design the water works and sewers for the new steel town of Gary, Ind., planning the works for a population of 100,000 people, including a water works tunnel and intake from Lake Michigan, supervising the original construction and planning numerous additions, including sewage treatment, to these public works in subsequent years. He also designed and supervised similar works for the new but smaller coal town of Kincaid, Ill.

In 1910 he became a member of an engineering commission which reported on sewage disposal for Milwaukee, Wis., the beginning of the treatment works later completed. He was also a member of the engineering commission which outlined plans for sewage collection and treatment of the North Shore Sanitary District prior to the annexation of this territory to the Chicago Sanitary District. Through his connection with the Chicago Real Estate Board, he took an active interest in sewage treatment for Chicago. He was instrumental in an early study of this problem by independent experts in a privately financed investigation, and later in 1925 he thoroughly studied and reported to the Secretary of War on the disposal of the sewage of the Sanitary District of Chicago, reviewing the possibilities of sewage treatment and the subject of sewage dilution.

Mr. Alvord also specialized in flood protection methods. In 1913 he became a member of the Consulting Engineering Board to devise flood protection for Dayton, Ohio, and its vicinity following the great 1913 flood. In the same year he and his organization were appointed to do similar work by the Franklin County Conservancy District at Columbus, Ohio, and during the ensuing two years made a thorough study and report for protective works on the Scioto River (only partly finished). At about this time, he was also engaged on numerous other investigations for relief from floods on the Illinois River for the State of Illinois and elsewhere. In addition, he was employed by numerous railroads and other organizations on matters relating to floods. He was coauthor of a book on this subject published in 1918.

During World War I, Mr. Alvord served at Camp Grant, Rockford, Ill., as supervising engineer for the Quartermaster in charge of water supply, sewers, and roads. Upon the completion of that work, he became chief engineer of the U. S. Housing Corporation, U. S. Department of Labor, Washington, D. C., heading a force of engineers, until the Department was liquidated after the Armistice. He then returned to Chicago and resumed his practice.

In the later decades of Mr. Alvord's active work, he devoted much attention to the valuation of water works properties. In 1899 he was employed by the Dubuque Water Company to inventory their property and to be a witness in the valuation proceedings conducted by a layman-lawyer-engineer commission. In this case, many of the procedures were inaugurated which have been followed on later work. He was subsequently a member of many commissions and boards which transferred the ownership of such property. In numerous cases, on account of a reputation for judgment and fairness, he was selected by appraisers as third man. He is the author of several engineering papers on the subject of valuation, depreciation, and going value.

Although Mr. Alvord never held public office, he frequently gave his services to boards and commissions serving the public. In 1910 he became a member of the Chicago Commission on Public Expenditures, a privately financed organization which operated for several years with a staff of assistants. In 1914 he served as a member of the Citizens Terminal Plan Commission. For many years, he was a member of the Chicago Real Estate Board, serving as chairman of the Committee on Rivers and Harbors and on various other committees and commissions dealing with health, taxation, and other local problems. In 1914 he became a member of the State Board of Natural Resources and Conservation, an Illinois state advisory and administrative commission on which he served for many years.

Mr. Alvord always felt keenly his lack of a technical university education. It made him a student all his life, and he had a special aptitude for study. His library was extensive and included papers and reports on the engineering subjects in which he became interested. He had a high regard for the help of others and was quick to appreciate the services of experts on the special problems. Nearly every one with whom he came in close contact became a valuable friend. He had the habit of commending good work wherever he saw it by an appropriate letter which made friends, with some of whom he maintained occasional correspondence for many years. This practice ultimately led in 1916 to his founding and endowing the Washington Award named in honor of George Washington who was one of the earliest American engineers, an honor administered by the Western Society of Engineers in cooperation with the four national Founder Societies and conferred annually upon some engineer in recognition of preeminent services in promoting the public welfare. Twenty engineers in widely different lines of endeavor have been thus honored at dinner meetings in Chicago.

Mr. Alvord was granted the degree of Civil Engineer by the University of Wisconsin, at Madison, in 1913. He was an honorary member and past-president of the Western Society of Engineers, American Water Works Association, and the Illinois Society of Engineers; a fellow of the American Society for the Advancement of Science; and a member of the Institution of Civil Engineers of Great Britain, American Institute of Consulting Engineers, New England Water Works Association, Sons of the American Revolution, Society of Mayflower Descendants, Chicago Engineers Club, Union League Club of Chicago, and University Club of Winter Park, Fla.

Mr. Alvord was married to Helen C. Cornell of Chicago on September 4, 1889. She died in 1926. In 1927, he was married to Lucy Bicknell Pitkin of Oak Park, Ill., who survives him.

Mr. Alvord was elected a Member of the American Society of Civil Engineers on January 4, 1893, and an Honorary Member on October 14, 1935. He served as a Director in 1917–1918 and in 1919–1921.

JOHN DEBO GALLOWAY, Hon. M. Am. Soc. C. E.[1]

DIED MARCH 10, 1943

John Debo Galloway, the son of James and Emily Meyers (Hoover) Galloway, was born on October 13, 1869, at San Jose, Calif. His ancestors were residents of Maryland, New Jersey, and Pennsylvania prior to the American Revolution. His parents died when he was still quite young. Left to his own resources, one of his dominant characteristics, self-reliance, early came to the fore. His boyhood experiences, some of which were acquired at Virginia City, Nev., when that city was the center of the great mining activity incident to the discovery and development of the famous Comstock Lode, made a deep impression upon him. At the age of eight he was taken to live with friends in Napa Valley, California.

His technical education was gained, not without some financial struggles, at Rose Polytechnic Institute, Terre Haute, Ind. There originated lifelong friendships with others who, like himself, later proved to be leaders in the Society. Mr. Galloway was graduated from Rose Polytechnic Institute in 1889. His attendance there had been inspired by his brother Mason Galloway, who was graduated in 1890.

Next, he was employed in railroad work in the Pacific Northwest. Recalling these days, he later enjoyed telling the late John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., of a good turn he received from him. Although Mr. Stevens had not met Mr. Galloway, through a consolidation of railroad interests, Mr. Stevens, directing the affairs of his company, released Mr. Galloway to (it later proved) better and more congenial employment. After a short stay in the Pacific Northwest, Mr. Galloway returned to California and throughout the remainder of his career maintained his headquarters in San Francisco although his practice often took him far afield.

From 1892 to 1896 he was chief engineer for Healy-Tibbitts Construction Company of San Francisco. In this capacity he personally directed the construction of important bridges, harbor works, sewers, and difficult foundation work. From June, 1896, to September 1897, he was employed in the engineering department of Pacific Rolling Mill Company of San Francisco, on designing and detailing steel building frames.

---

[1] Memoir prepared by a Committee consisting of Walter L. Huber, *Chairman,* Henry D. Dewell and A. Kempkey, Members, Am. Soc. C. E.

From September, 1897, to October, 1899, he was an instructor in drawing and mechanics at California School of Mechanical Arts in San Francisco. In this capacity he made a deep impression on his students, many of whom became leaders of the engineering profession. Among them was the late Arthur H. Markwart,[2] M. Am. Soc. C. E., who later became Mr. Galloway's partner. From October, 1899, to May, 1900, Mr. Galloway was chief engineer for Colusa Stone Company. After May, 1900, he maintained an independent practice as consulting engineer, although on two occasions, covering several years, he was associated in partnerships.

For the California Gas and Electric Company, predecessor of the Pacific Gas and Electric Company, he directed the design and construction of the cable span across the Straits of Carquinez. This span, which was a part of the world's then longest transmission line, was also the then longest cable span. It is still in service as an important part of this company's great system.

From 1900 to 1906, the period just prior to the San Francisco earthquake and fire of April 18, 1906, he designed and supervised hydraulic and structural construction for several hydroelectric plants which were later absorbed in the system of the Pacific Gas and Electric Company. He also designed the hydraulic and structural features for the first two hydroelectric plants of the system later owned by California Electric Power Company; and the structural elements of some of the city's principal buildings including the Mutual Savings Bank Building, the St. Francis Hotel, the Shreve Buildings, the Schroth Building, the Syndicate Building of Oakland, etc. He was one of the earliest advocates of earthquake-resistant design, and the merit of his reasoning was fully demonstrated by structures designed by him, when they were subjected to the effects of the great earthquake.

Immediately after the earthquake he formed a partnership with John Galen Howard, as Howard and Galloway, Architects and Engineers. William C. Hays and Mr. Markwart were junior partners. This firm took a leading part in the design of San Francisco's reconstructed commercial district. Many of the city's most important office and commercial buildings were designed under Mr. Galloway's direction. In spite of the exacting demands of his professional work at this critical time, he devoted much personal effort to the revision of the city's building ordinance and to relief work. His firm also designed and supervised the construction of buildings and grounds for the Alaska-Yukon-Pacific Exposition at Seattle, Wash., and the auditorium, chemistry and engineering buildings, and a steam-electric power plant for the University of Washington at Seattle.

At this time he served with Charles D. Marx,[3] Past-President and Hon. M. Am. Soc. C. E., and Wynne Meredith, M. Am. Soc. C. E., on the Board of Advisory Engineers on the design and construction of the Stanislaus Hydro-Electric Power Plant, California.

From 1908 to 1920 he was associated with Mr. Markwart on consulting work. However, this partnership had a number of intermissions. From

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1877.

[3] For memoir, *ibid.*, p. 1785.

July, 1912, to April, 1915, Mr. Markwart was absent while in charge of the design and construction of the Panama Pacific Exposition at San Francisco. From November, 1917, to February, 1919, Mr. Galloway served as an officer with the American Expeditionary Force in France. In November, 1920, Mr. Markwart became head of the engineering department of Pacific Gas and Electric Company and the partnership was then dissolved. After this Mr. Galloway continued in independent practice.

With Mr. Markwart, he designed and built several railway and combined highway and railway bridges in the Sacramento Valley, California, a hydroelectric power plant in Yosemite National Park, and a number of important buildings. The firm made many investigations and reports upon utility properties, including those of the Oro Electric Corporation, Butte and Tehama Power Company, City Electric Company, Yolo Water and Power Company, and Santa Barbara Gas and Electric Company. Mr. Galloway investigated and reported on the steam and hydroelectric plants of Treadwell Mines in Alaska; on proposed power development on the Potomac River and Shenandoah River at Harpers Ferry, W. Va.; on existing and possible additional power developments on the Menominee River, Michigan and Wisconsin; on power possibilities on Deschutes River in Oregon; on power developments on Feather River and Yuba River in California; on development of hydroelectric power on Vancouver Island, B. C., Canada; and on power projects on Pit River, California. He also conducted extensive examinations and reports, in many instances including appraisals of irrigation systems. Among the properties concerned were those of Turloch Irrigation District and Modesto Irrigation District; Fresno Canal and Land Corporation; James Irrigation District; Honcut-Yuba Irrigation District; Mendota Irrigation District; Tulare Lake Basin Water Storage District; and Sutter Butte Canal Company. With Mr. Markwart, he made surveys and prepared designs for the irrigation system which, with some modifications, was built by Merced Irrigation District at a cost of approximately $16,500,000. With the late J. B. Lippincott,[4] Hon. M. Am. Soc. C. E., B. A. Etcheverry, F. C. Herrmann, W. L. Huber, and the late F. H. Tibbetts,[5] Members, Am. Soc. C. E., he was a member of the Board of Advisory Engineers on the Sacramento Valley Division of the Central Valley Project in California. This board made studies of dams on the larger rivers giving special consideration to the Shasta (Kennet) Dam and the use of water in the Sacramento Valley.

Mr. Galloway rendered extensive service to Great Western Power Company: First in connection with the design and construction of extensions and additions to its Las Plumas Plant on Feather River; and later in connection with the design and construction of its Caribou Plant on the same stream. As consulting engineer, he represented this company on the construction of the Bucks Creek hydroelectric power plant, a plant with 2,660-ft head.

With Frank G. Baum,[6] M. Am. Soc. C. E., and W. F. Durand, Mr. Galloway was a member of the Board of Consulting Engineers on the location and design of the Moccasin Creek hydroelectric plant on the Hetch Hetchy Aqueduct of the City of San Francisco. For Pacific Gas and Electric Company he served as consulting engineer in connection with the concrete masonry dams for Pit No. 3 and Pit No. 4 power plants on Pit River, the enlargement of Fordyce rock-fill dam on Yuba River, and the design and construction of the 328-ft Salt Springs rock-fill dam on Mokelumne River. With Robert Ridgway[7] and Arthur N. Talbot,[8] Past-Presidents and Honorary Members, Am. Soc. C. E., and M. M. O'Shaughnessy,[9] M. Am. Soc. C. E., Mr. Galloway was a member of the first Commission of San Francisco on the location of the San Francisco-Oakland Bay Bridge. The foregoing list of his professional engagements is necessarily incomplete. As one of his colleagues recently stated, he had a part in practically every major engineering project in northern California during a period of forty years.

---

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1543.

[5] For memoir, *ibid.*, Vol. 105 (1940), p. 1924.

[6] For memoir, *ibid.*, Vol. 99 (1934), p. 1407.

As previously noted, his professional career was interrupted by World War I. In 1917 he was chairman of a temporary organization of about 750 engineers for the study of war work and for supplying information preparatory to army service. In November, 1917, he was called to active duty and ordered to France, arriving in December, 1917. He served as Major of Engineers in G-2 Section of the General Staff at Headquarters of the American Experitionary Force at Chaumont, France.

In 1942, during World War II, he was board member of the Price Adjustment Section for the Pacific Division, Corps of Engineers, U. S. Army. He was Director of Defense Transportation (Office of Civilian Defense) for Berkeley, Calif., until the time of his death.

He was married on April 19, 1894, to Nettie Mabel Barnes whom he had known since early childhood in Napa Valley. Mrs. Galloway died on June 14, 1941. Two daughters, Mrs. E. Ronald Foster and Mrs. Alice Wilder; three grandchildren, John D., Margaret Ellen, and Rosemary Foster; and a brother, Mason Galloway, survive him.

Mr. Galloway was a life member and past-president of the Astronomical Society of the Pacific, a member of the Society of Military Engineers, a member of the Seismological Society of America, a member of the California Historical Society, a charter member and life member of the Commonwealth Club of California; and a director of California School of Mechanical Arts and past-president of its Board of Directors. San Francisco's noted Bohemian Club, a group of professional and civic leaders, learned to appreciate his ability as he was charged with upkeep of its summer retreat in the Redwood Country—the locale of events famous for their originality and convivial atmosphere. Mr. Galloway was particularly proud of the fire-protection system which was installed there under his direction.

His was a rich life filled with interests beyond his professional practice. He had a passionate fondness for historical research, particularly when it related to the early development of the West such as the Pacific railroads, the utilities which served the western mining regions, and early explorations in general. He made a fine collection of rare prints. Gardening gave much pleasure to him and his wife.

---

[7] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1527.

[8] For memoir, *ibid.*, Vol. 108 (1943), p. 1530.

[9] For memoir, *ibid.*, Vol. 100 (1935), p. 1710.

He wrote many authoritative articles, especially for the Society. For his paper on "The Design of Rock-Fill Dams,[10] he received the Thomas Fitch Rowland Prize in 1941, and, at the same Society meeting, he was awarded Honorary Membership. The dual honor was a significant tribute to the depth and catholicity of his interests. His splendid article, "Theodore Dehone Judah—Railroad Pioneer," [11] is, so far, the only printed evidence of the great amount of study and research which he devoted to the subject of the building of the Central Pacific Railroad. It is hoped that some of his manuscripts on this subject will yet be printed.

Among many important assignments which he filled for the Society one stands out in bold relief—his chairmanship of the Society's Earthquake Committee which was instituted following the Japanese disaster of 1923. After years of study this committee produced a monumental report, which is still consulted for authentic information on a wide variety of earthquake problems.

Besides the papers mentioned he was the author of many other papers and discussions, among them: "Report of Committee on Fire and Earthquake Damage to Buildings" [12] (chairman of committee), "The Design of Hydro-Electric Power Plants," [13] "Hydro-Electric Developments on the Pacific Coast," [14] and "Earthquakes and Structures," [15] in collaboration with L. M. Hoskins. He contributed valuable discussions to a great many papers published by the Society. He was the author of "Hydraulic Power Development and Use" presented at the International Engineering Congress in San Francisco in 1915. He also contributed a discussion of a paper entitled "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon, Australia" to the *Journal* of the Institution of Civil Engineers, London. Among his numerous papers presented to the Commonwealth Club of California may be mentioned "The Hetch Hetchy Water Supply for San Francisco," "The Bay Cities Water Problems," "Reception to Members from the Front," "Water Power and Irrigation in California," "California Water and Power Act," "Hetch Hetchy Power," and "The Swing Johnson Bill."

Mr. Galloway had a capacity for genuine friendship but when occasion demanded he could be a frank critic. Whatever decision his logical mind led him to take, he defended it strongly. This independence combined with his great kindness made him greatly admired and loved. As noted in *Civil Engineering,*[16] "No civil engineer on the Pacific Coast has enjoyed a more secure place in the esteem and affection of his fellows." One of his loyal friends of many years recently made a statement which is offered here because it describes Mr. Galloway's personality so truly.

---

[10] *Transactions,* Am. Soc. C. E., Vol. 104 (1939), p. 1.

[11] *Civil Engineering,* October, 1941, p. 586; November, 1941, p. 648.

[12] *Transactions,* Am. Soc. C. E., Vol. LIX (1907), p. 223.

[13] *Ibid.,* Vol. LXXIX (1915), p. 1000.

[14] *Ibid.,* Vol. LXXXVI (1923), p. 803.

[15] *Ibid.,* Vol. 105 (1940), p. 1269.

[16] ***Civil Engineering,*** **April, 1943, p. 190.**

> "Personally, I not only respected but also appreciated 'J. D.' both as a student and as a scholar, and I certainly did look up to him as a genuine engineer—sound as to judgment and eminently practical as regards the application of engineering knowledge and skill.
>
> "My very deep affection for him, however, was founded upon his humanitarian traits, for after all he was a man both human and humane.
>
> "I did so enjoy his little 'bark' which never could become anything even remotely resembling a 'growl' because of the smile that at such times always lingered at the corners of his mouth, and the ever present bright twinkle in his eye.
>
> "The structures which he designed and the projects in which his influence is so clearly shown will endure as physical mementos to his worth as an engineer; but as a man and as a friend, his memory ever bright will live always in my heart."

Mr. Galloway was elected a Member of the American Society of Civil Engineers on December 6, 1905, and an Honorary Member on October 14, 1940. He became a Life Member on January 1, 1940. He was a Charter Member and Past-President of the San Francisco Section of the Society.

JOSEPH RUGGLES WORCESTER, Hon. M. Am. Soc. C. E.[1]

DIED MAY 9, 1943

Joseph Ruggles Worcester, whose membership in the Society dates from the year 1895, and who was elected to Honorary Membership in 1937, passed away on his eighty-third birthday, May 9, 1943, at his home in Waltham, Mass. Waltham was the city of his birth and he resided there throughout his life.

He was born on May 9, 1860, the son of Benjamin and Mary Clapp (Ruggles) Worcester. His early educational and preparatory training was acquired at what was then called the Waltham New Church School (later known as Chapel Hill School), a private institution founded originally in 1860 by his father to serve a neighborhood group, and later opened to all suitable applicants. As a result of the splendid instruction received at this school, Mr. Worcester was enabled to enter Harvard University in Cambridge, Mass., in 1878 with advanced standing in mathematics and the classics.

In 1882 Mr. Worcester was graduated from Harvard University with the degree of Bachelor of Arts, and immediately entered the employ of the Boston Bridge Works, engineering department. At the end of two years of service with that company, he had become its chief engineer, in which capacity he soon acquired a widespread and favorable reputation as an outstanding authority on the subject of structural design.

The persistent demand for his services as consultant on important engineering projects, both public and private, led him to resign his position with the bridge company in 1894 and to open an office in Boston for the practice of consulting engineering exclusively. Twelve years later, he organized the business under the firm name of J. R. Worcester and Company, and assumed the position of senior partner. He continued his association with this concern until his death, although for the last seventeen years of his life his contribution was largely of an advisory character.

---

[1] Memoir prepared by Charles R. Gow and John P. Wentworth, Members, Am. Soc. C. E.

In the course of his experience as a consulting engineer, he was called upon to prepare the designs for many important engineering projects, among them the original Boston subway, together with various features involved in its later extensions; the elevated structure of the Boston Elevated Railway; the East Boston and Cambridge tunnels; the South Terminal Station train shed; and the Cambridge Viaduct. He likewise designed and supervised the erection of the steel arch bridge over the Connecticut River at Bellows Falls, Vt., the Harvard University Stadium, and the reinforced concrete bridge connecting the municipalities of Portland and South Portland, Me. He similarly collaborated with the late John R. Freeman,[2] Past-President and Hon. M. Am. Soc. C. E., in designing and constructing the Aluminum Reduction Plant at Niagara Falls, N. Y. These undertakings constituted but a few of the numerous projects of note with which he identified himself in the course of his long and active professional career.

Mr. Worcester was an early pioneer in the United States in developing the technique and principles of reinforced concrete design, and for a period of several years after its introduction he was recognized as a leading authority in that branch of engineering science. Most of the earlier structures of this character which were built throughout the New England territory were the product of his office.

At an early period of the present century, he acquired a reputation as a leading authority on the subject of foundation design, and his advice in this respect was widely sought and accepted throughout his entire engineering career. His elaborate paper on "Boston Foundations" published in 1914[3] has been considered a classic in that field and is still regarded as a most authoritative treatise on the subject. In recognition of its superior merit, the Desmond Fitzgerald Medal was awarded to its author by the Boston Society of Civil Engineers.

In the course of his active participation in the engineering field, Mr. Worcester was a prolific contributor of papers and discussions before the many technical organizations in which he held membership. The 1867 to 1920 "Index" of the Society alone lists twenty-one instances of his participation. He likewise contributed liberally of his time and knowledge in the delivery of student lectures covering the various phases of engineering science and practice upon which he was competent to speak with authority.

From 1904 to 1917, Mr. Worcester served as a member of the Society's Joint Committee on Concrete and Reinforced Concrete, and for several years as its chairman. He was likewise a member of the Society's Special Committee on Structural Columns and Struts. He similarly placed his exceptional talents at the disposal of many other professional organizations of which he was a member, and devoted himself generously to the propagation of sound engineering knowledge for the benefit of his contemporaries.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1471.

[3] *Journal*, Boston Soc. of Civ. Engrs., Vol. 1, No. 1, January, 1914.

His contribution to the public service was also noteworthy. In 1921, Herbert Hoover, Hon. M. Am. Soc. C. E., then serving as Secretary of Commerce, selected Mr. Worcester as a member of the Federal Building Code Committee to which he rendered a conspicuous and invaluable contribution. Upon another occasion, the Governor of Massachusetts selected him as one of three citizens of the Commonwealth to serve as a Special Commission for constructing the six-million-dollar Memorial Bridge spanning the Connecticut River and joining the municipalities of Springfield and West Springfield in that state. His loyalty and devotion to his native city of Waltham were manifested on innumerable occasions by the advice and counsel which he frequently gave in connection with its various problems of an engineering or economic nature. He was a trustee and vice-president for many years of the Waltham Savings Bank, as well as president and trustee of the Waltham Hospital, to both of which institutions he devoted himself and his talents without stint. In addition, he served as a member of the City Hall and Library Commission of his native city. He was a member of the Piety Corner Club of Waltham, the Choral Society, and the Boston Handel and Haydn Society. He served as clerk of the Waltham Society of the Church of the New Jerusalem from 1883 until his death, a period of sixty years.

Because of his highly developed sense of fairness, his pronounced intellectual honesty, and his faculty of clear thinking and ready grasp of essentials, he was in constant demand for service as an expert court witness, as well as for arbitration proceedings. Few men of his time commanded greater respect and confidence than that which attended his statements and decisions in those capacities.

He was an inspiration to the younger members of his profession who constantly sought his opinion on problems entrusted to their care. He invariably took a kindly interest in their welfare and endeavored always to guide them along lines of sound and conservative engineering practice. He was extremely proud of his profession and devoted himself wholeheartedly to its advancement.

Throughout his lifetime, he retained a deep interest in the school which his father had founded and from which he had derived his early education. For a period of forty-three years, he served as a member of the governing board of the New Church Institute of Education. He was clerk of the Institute for twenty-five years and gave courses in the subject of astronomy in which he was deeply interested. His sound judgment and unusual foresight were continuously applied to guide the fortunes of the school through both good times and bad.

Mr. Worcester was extremely fond of outdoor life, particularly mountain climbing, bicycling, swimming, and sailing. An item in his mother's diary relates the fact that he climbed a mason's ladder to the piazza roof on his third birthday.

From early childhood, he always evinced a deep interest in some incidental activity. In the course of his younger school experience, he and some other boys built sextants for their own amusement and later on acquired a printing press from which he developed a hobby that he retained throughout his lifetime. To every such interest he devoted himself assiduously, never undertaking a task without completing it.

Among the pupils who attended the Waltham New Church School in his time was Alice Jeannette Wheeler, of Lincoln, Mass., who became Mr. Worcester's wife on January 2, 1889. Mrs. Worcester died on November 24, 1934. Five children were born to them, four of whom survive their parents. They are Alice Martha (Mrs. Clarence D. Howe), Barbara (Mrs. Charles T. Porter), Thomas, and Ruth Hunt.

To his many friends who had learned to love and admire his splendid personal qualities, Mr. Worcester's loss will be irreparable. His was a soul that knew no vanity, nor was he ever captivated by any sense of self-importance. He was inherently gracious, generous, and just, gifted with indomitable consistency and courage and absolutely unswerving and unselfish in his fidelity to his honest convictions. He worshipped at the altar of impersonal loyalty of principle and ever spoke the promptings of a heart to which guile and hypocrisy were total strangers.

His body, mind, and soul were unceasingly devoted to the service of others whether poor or rich, grand or obscure. With even temper, humility, and good nature he sought always to lighten the burden and trials of those about him. His aptitude of comprehension, his richness of imagination, and the rounded measure of his many splendid faculties were freely and lavishly bestowed upon all who sought his aid and counsel.

He stood uncompromisingly for independence, for courage, and above all, for absolute integrity. With unfaltering lip and heart, with a soul aflame for truth and righteousness, and yet entirely free from any trace of bitterness, he proved himself worthy of every confidence reposed in him by those who sought his guidance. His entire life, rich as it was in unselfish service, untinged by any prejudice and unconscious of a single unworthy motive, is entitled to wide emulation by those who follow after him.

Those who were privileged to benefit from the influence of his genial nature and to enjoy the blessings of the peaceful atmosphere of good-will which he invariably created will acknowledge freely the unusual debt of gratitude which they owe to one who contributed so much to their welfare in helping to make this world a brighter and better place in which to live, to labor, and to learn.

Joseph Worcester was elected a Member of the American Society of Civil Engineers on January 2, 1895, and an Honorary Member on October 4, 1937.

THOMAS DAVID ALLIN, M. Am. Soc. C. E.[1]

DIED OCTOBER 17, 1942

Thomas David Allin was born near West Branch, Iowa, on January 13, 1864, the son of John and Jamima (Townsend) Allin. Both his parents were of English ancestry, his mother's family having come to America prior to the Revolutionary War. The Allin family remained in Iowa until 1882, then moved to Southern California and settled in the new community which later was to become Pasadena, Calif.

---

[1] Memoir prepared by Morris S. Jones, M. Am. Soc. C. E.

Mr. Allin received his early education at the Iowa City Academy, Iowa City, Iowa. After arriving in Southern California he attended the Columbia Hill Academy in Pasadena for two years and in 1886 entered the University of California at Berkeley, the first boy to attend the State University from the Pasadena area. At Berkeley he completed a special course in civil engineering in two-and-one-half years. He took an active part in University affairs and was one of the charter members of the University of California Chapter of Phi Gamma Delta fraternity.

After leaving college in 1889 Mr. Allin spent several years in irrigation work as assistant engineer on the Elsinore Irrigation District Project in Riverside County, California, and later was assistant engineer on the Bear Valley Irrigation Company's development in San Bernardino County, California.

In 1891 he returned to Pasadena to enter private engineering practice and for the next fifty years was closely identified with the development of Pasadena and the neighboring communities of South Pasadena, Calif., and Alhambra, Calif. At one time or another, he served all three communities as city engineer.

In his professional work, Mr. Allin conducted a general engineering practice, specializing in land subdivision. He also was in charge of most of the water development for private water companies which supplied the Pasadena area during this period. As engineer for the Pasadena Lake Vineyard Land and Water Company, principal company serving water in the city, he laid out its pipe distribution system and built two large reservoirs of 19,000,000-gal and 16,000,000-gal capacities. He also directed its Devils Gate water development, which included an intercepting tunnel one-and-one-quarter miles long, which for many years was the major water supply of the city. He also was retained from time to time by the Pasadena Land and Water Company, the North Pasadena Land and Water Company, and many of the other smaller companies operating in Pasadena and adjacent territory.

Mr. Allin was city engineer of Pasadena from 1901 to 1905. During this time he laid out and supervised the enlargement of a sewage system, paved many miles of streets, and planned a city-wide system of drains to carry away storm waters. From the engineering viewpoint, this work helped to place Pasadena in the category of a modern city.

After leaving the city engineer's office, Mr. Allin returned to private practice, opening an office in partnership with his brother, Charles. In addition to his private work, he was consultant for the city on the planning and construction of the storm-drain system. He also reported on the type of sewage treatment plant best suited to the city and was instrumental in the selection of the activated sludge method of disposal. This plant was built at a cost of $500,000 and is used by the four cities, Pasadena, South Pasadena, Alhambra, and San Marino, Calif.

In 1913 he was elected commissioner of public works under the commission form of government and served until 1919. He was vice-chairman of the commission for the last four years of this period. Subsequently, from 1922 to 1938, he was a member of the Pasadena City Planning Commission, for four of those years as chairman.

Interested in many other civic, community, and welfare activities in Pasadena, he was honored by his fellow citizens by being given the Arthur Noble Award, an annual recognition granted to a Pasadena citizen for outstanding service to the city.

Mr. Allin had a gift of real friendship. He knew thousands of people by name, and they in turn called him "T. D." or "Tom." With pride and love, almost of a father, Mr. Allin saw Pasadena, the community of his choice, grow from a dusty settlement into a beautiful city. He helped in large measure to shape the city's destiny, yet his thoughts were never of any personal achievement but rather of wholehearted affection for his home town.

He was a loyal worker in the Methodist Church. He was a Thirty-Second Degree Mason and a Shriner. He was a member of the Pasadena Pioneers Association, the Pasadena Historical Society, New Century Club, and the Pasadena Chamber of Commerce.

On October 18, 1892, Mr. Allin was married to Jessie E. Patterson in Pasadena. Mrs. Allin survived her husband by only a few months. Their two children are Hazel (Mrs. Bert Rolfe) and Ralph Walter Allin.

Mr. Allin was elected a Member of the American Society of Civil Engineers on January 3, 1906.

THOMAS WARD BAILEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 18, 1942

Thomas Ward Bailey was born in Kingston, Mass., on February 19, 1874, the son of sea captain Frederick Coggswell Bailey and Helen May (Peterson) Bailey, of Kingston. He was born in the house built by David Bradford, grandson of William Bradford, an early governor of Massachusetts Bay Colony, and purchased in 1832 by Thomas Bailey, the grandfather of Thomas Ward Bailey.

At the early age of six years young Bailey enrolled in the Stony Brook District School of Kingston. There was one departure from routine during his grammar school years—at the age of nine he went on a sea voyage with his mother and father in the American bark *Nehemiah Gibson,* of which his father was captain. During the voyage, which lasted many months, the vessel touched at Batavia, Java, and Hong Kong, China. In later life Mr. Bailey recalled many interesting pleasures and educational features of this trip. He always regretted that he had not been older so that the voyage would have been even more interesting to him.

---

[1] Memoir prepared by Gibb Gilchrist, M. Am. Soc. C. E.

Upon finishing grammar school, Mr. Bailey entered Kingston High School, being graduated with honors as valedictorian of his class at the age of seventeen. He had not reached the minimum age requirement of his chosen college, the Massachusetts Institute of Technology at Boston, and as a result he spent a year in home study and in tutoring other less fortunate boys who desired to enter college with him. In September, 1892, Mr. Bailey entered Massachusetts Institute of Technology for the course in civil engineering, and was graduated with the degree of Bachelor of Science in June, 1896. Throughout his life Mr. Bailey improved himself by reading and study. In his mature years he was well known not only as a successful engineer, but as one who by much study and self-improvement had achieved for himself a well-rounded education in cultural subjects, the possession of which made him a more valuable engineer.

Immediately after graduation for two years, Mr. Bailey was with the Boston Transit Commission as rodman, instrumentman, and inspector, gaining much valuable experience in excavation, concrete, sewers, pile-driving, waterproofing, stonework, brickwork, and steel construction.

In 1898 he worked for the United States Government Engineers in connection with the fortification work in Boston Harbor during the Spanish-American War. From 1898 through 1904 Mr. Bailey was employed in various capacities, advancing to assistant engineer with the Massachusetts Harbor and Land Commissioners. During this period his experience in marine matters was enhanced, and, in addition, he became experienced in water terminal construction and operation which was an asset to him throughout his life. His last specific employment in connection with Boston Harbor, as superintendent of the dredging company, was terminated upon the completion of important dredging work in the main channel with the disposition of the dredge material at sea.

From 1904 through 1917 Mr. Bailey was in business as a civil engineer in Plymouth, Mass. In addition to the consulting service, he directed surveying and mapping crews on various civil engineering projects in that vicinity. This work involved investigation and reports on agricultural, power, timber, irrigation, and reclamation projects; building electric transmission lines; construction of municipal works including bridges, sewers, and water works; and evaluations, as well as research on land titles and subdivisions.

Mr. Bailey served on the Mexican border in 1916 as a non-commissioned officer in the Second Brigade, Massachusetts Voluntary Militia.

Immediately upon the declaration of World War I in 1917, Mr. Bailey entered the first officer training camp at Plattsburg, N. Y. He was ordered to active duty as First Lieutenant in the Corps of Engineers on July 13, 1917, and went to France with the 101st Regiment of Engineers, 26th Division, in September, 1917. He served with his regiment in the combat areas until August, 1918, when he was ordered back to the United States pursuant to an order sent by the Adjutant General to Gen. John J. Pershing, from which the following is quoted:

> "In order to provide officers and noncommissioned officers for new divisions about to be organized it is necessary to have some of the officers and noncommissioned officers selected from your forces who are efficient, who have had training in France in the latest methods of warfare, experience in the face of the enemy and who are deserving of promotion."

When he returned to the United States, Lieutenant Bailey was commissioned a Captain of Engineers on August 20, 1918. From that time until January, 1919, he served with the 7th Engineer Training Regiment at Camp Humphreys, Virginia, and, from January, 1919, until August, 1919, he was with the Statistics Branch of the General Staff at Washington, D. C.

Upon leaving the army Captain Bailey retained his reserve commission. It is not known when this was terminated, if at all, but he was on reserve status with the United States Army at least through 1934 and perhaps later. In person and in official correspondence he was designated by his military title; always it seemed the right and proper designation for him.

On November 4, 1910, Thomas Ward Bailey was married to Mary I. Perkins, of Plymouth. He is survived by his widow and one son, Harold Thomas Bailey, who was graduated from The Agricultural and Mechanical College of Texas, at College Station, in 1935. Kingston and Plymouth are only a few miles apart, and both Captain Bailey's antecedents and those of his wife trace back to the Pilgrim fathers who first landed at Plymouth in 1620. The staunch integrity of these pioneer Americans was in evidence in the character of Captain Bailey throughout his life wherever he lived and also in the character of his wife and son.

In 1921 through 1924 Captain Bailey was with the Holway Engineering Company on the Spavinaw Water Project at Tulsa, Okla. He served throughout this project, first on the preliminary survey and finally as division engineer on the construction of the 54-mile conduit, costing $4,000,000. From 1924 through 1926 he was with the Oklahoma State Highway Commission as resident engineer on state highway projects in Rogers County, with headquarters at Claremore, Okla. During this period he was in charge of highway location and design, as well as construction and maintenance. In November, 1926, he was appointed division engineer of the Oklahoma State Highway Department, with headquarters at Tulsa. In this capacity, he supervised construction and maintenance in seven Oklahoma counties.

In September, 1927, Captain Bailey accepted a position as division engineer for the Texas State Highway Department, Division No. 16, with headquarters at Corpus Christi, Tex. At that time the division comprised twenty-one counties. He continued in this position until the latter part of 1938, when he was transferred to Del Rio, Tex., as division engineer of District No. 22. At Corpus Christi, Captain Bailey was close to the sea, and to the end of his life he loved the water and all that pertained to marine activity. In 1933 a coastal storm washed away the deck of the 2-mile wooden trestle across Corpus Christi Bay, linking his headquarters city with the coastal towns to the east. Because of his knowledge of coastal storms and the inflow and outflow of storm tides, Captain Bailey was able to recover 90% of the timber decking, much of which had floated back to sea after subsidence of the storm, and in 60 days the bridge was reconstructed and in use.

After his family, his country, and his profession, it is evident that Captain Bailey's chief interest was in the sea. He was familiar with every nook and cranny along the Texas coast from Rockport to Boca Chica. He always knew where the best duck hunting and fishing might be found; he always owned a boat; and he was never happier than when he could be on or near the water.

As a professional man, Captain Bailey was respected for his ability. His knowledge and training were such that he could be depended upon to solve any problem, major or minor. His work in south Texas exemplifies this ability. He found time to make friends with almost every one in his district, including both those who worked under his direction and those for whom he toiled.

Thomas Ward Bailey was elected a Member of the American Society of Civil Engineers on July 15, 1929.

BERNARD BENFIELD, M. Am. Soc. C. E.[1]

Died March 4, 1943

Bernard Benfield, the son of Elias and Bertha (Kunreuther) Benfield, was born in New York, N. Y., on December 11, 1861. In 1862, his family moved to California. His early education was obtained at the San Francisco (Calif.) Boys' High School, from which he was graduated in 1878. He attended the University of California, at Berkeley, and was graduated with the degree of Bachelor of Science in Civil Engineering in 1882.

Immediately after graduation, Mr. Benfield became a rodman for the Oregon Railway and Navigation Railroad Line and also assistant engineer on construction and in the office. During 1883 he was engaged on irrigation canal work in southern California and later was office man and levelman on railroad location with the Southern Pacific Railroad Company from Redding, Calif., north. During this period the so-called "Hood Taper" or transition curve for railroads, was developed, and he collaborated with the late William Hood,[2] M. Am. Soc. C. E., of the Southern Pacific Railroad Company, in preparing the tables. In 1884 as assistant engineer with the Southern Pacific Railroad Company, he had charge of railroad reconstruction work in western Texas.

From 1885 to 1890 Mr. Benfield was on independent work in California. In 1890 he returned to the Southern Pacific Railroad Company as assistant engineer on railroad construction in California and Arizona. In 1893 he became associated, as assistant engineer, with the Pacific Improvement Company, a branch of the Southern Pacific Railroad Company. While holding this position, he discovered and developed an extensive vein of asphalt and introduced and actively promoted the use of California asphalt throughout the United States. He managed the business of the Standard Asphalt Company, a subsidiary of the Pacific Improvement Company, for many years and was president of the former company in 1900.

---

[1] Memoir prepared by a Committee of the San Francisco Section consisting of Walter H. Kirkbride, Chairman, E. T. Thurston, and Charles B. Wing, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1096.

In 1901 Mr. Benfield was appointed on a commission of engineers to appraise the Kern River Oil Fields in California. At the same time, he was engaged as geologist by the Associated Oil Company, in which capacity he visited and reported on the oil fields of Borneo and other islands of the Dutch East Indies. Thenceforth, his professional work covered a wide field, including chemistry and geology as well as civil engineering.

Mr. Benfield maintained a constant and active interest in the welfare of the community in which he lived, and belonged to the Commonwealth Club of California and other organizations for civic betterment.

One of Mr. Benfield's particular hobbies was astronomy. Becoming an affiliate of the Astronomical Society of the Pacific in 1916, he at once became interested in the affairs of that society and served it in many ways—as a member of its board of directors, as vice-president for several years, and as president in 1925. At his suggestion, the Astronomical Society of the Pacific began the publication of *Leaflets,* and he personally financed and helped to edit the earlier numbers. Throughout the years he contributed generously to the support of lectures sponsored by this society. The directors named him a "Patron of the Society." He was also a "Fellow" of the Royal Astronomical Society of England.

Mr. Benfield was never married; he was a kindly man, and a devoted son and brother.

Mr. Benfield was elected a Member of the American Society of Civil Engineers on May 6, 1903.

REGINALD PELHAM BOLTON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 18, 1942

Reginald Pelham Bolton, a member of the Society for more than forty years, who died on February 18, 1942, at his home in the Washington Heights section of the Borough of Manhattan, New York, N. Y., led a life of great usefulness not only as an engineer but as an American citizen interested in the social and political life of the community.

Although he was born in England, October 5, 1856, he had a line of American ancestors extending through five generations from Robert Bolton, a native of Yorkshire, England, who emigrated to America in 1718, settling first in Philadelphia, Pa., and later at Savannah, Ga. Reginald's father was the Rev. James Bolton, connected for a time with the Pelham Priory in Westchester

---

[1] Memoir prepared by Rudolph P. Miller, M. Am. Soc. C. E.

County, New York. His mother was Lydia Louisa Pym of Hertfordshire, England. The ancestry of the Bolton family has been traced to Oughtred de Bolton (1100–1135) among whose descendants was the late Channing Moore Bolton,[2] M. Am. Soc. C. E.

Mr. Bolton's early education was obtained in the private schools of Wimbledon, England, referred to as "ordinary home schools" in his application for membership in the American Society of Mechanical Engineers. His engineering education was begun at the age of seventeen at the shops of James Waite in Epsom, Surrey, England. At nineteen he was apprenticed to Messrs. Powis, James and Company of Lambeth, London, England, where he worked in the designing department. In 1878 he was a draftsman with Davey, Paxman and Company of Colchester, England, a firm of agricultural and mining engineers. In December, 1879, a desire for wider experience brought him to the United States, where he found employment as assistant engineer in the Edgemoor Iron Works at Wilmington, Del. About two years later he was assistant to the late Erasmus D. Leavitt,[3] M. Am. Soc. C. E., of Boston, Mass., consulting engineer to the Calumet and Hecla Mining Company on the design of engines and mining machinery. In November, 1881, ill health caused Mr. Bolton to return to England, where he established his own business as a consulting engineer. In 1888 this was interrupted for about a year when Mr. Bolton became general manager for Appleby Brothers Limited of Greenwich, London, England, a firm of prominent crane engineers.

In connection with his professional practice he traveled extensively in England and on the European continent, designing and supervising the erection of machinery and special machine tools. At the Elmore Company's works at Leeds, England, he introduced electric hoisting machinery. He designed and installed the first mining plant for the Gordo sulfur mines at Caltanissetta, Sicily. Other works designed and installed by him included a dredging plant at Taranto, Italy; a mining plant for contractors at Naples, Italy; electric light plants in the Asturias, Spain; and a large gantry crane at Rotterdam, Holland. For the Italian Transport Commission he designed marine engines, and for the governments of Spain and Mexico he designed stern-wheelers and other boats.

Mr. Bolton had a number of inventions to his credit. His earlier products, developed before he came to the United States, included a hydraulic governor (patented in the United States), the Bolton and Hartley air compressor (1882), an electric crane gearing (1890), and electrical rock-boring and hammering apparatus (1891). He devised and introduced into the tall office buildings of New York a system of vertical wiring, as well as a system of multiple water supply for fire and house service.

When he came to the United States in 1894 to settle permanently, the character of his work changed. This was the period of the development of the skyscraper, the equipment of which opened up a new field for the engineer. Mr. Bolton became a leading engineer on the design of heating and ventilating methods, electric lighting and power plants, elevator installations, and plumbing and drainage systems. Among the buildings in New York City equipped by him or for which he acted as consultant were the Bowling Green Building, Syndicate Building, Lord's Court, Silk Exchange, Central Bank Building, R. G. Dun Building, La Rochelle Hotel, Grand Central Terminal, Ansonia Hotel, and Ridgwood Pumping Station of the New York Department of Water Supply. The Narragansett Electric Light and Power Company, New York Edison Company, and R. H. Macy and Company were a few of his clients.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1633.

[3] For memoir, *ibid.*, Vol. LXXXVII (1924), p. 1376.

The active part that Mr. Bolton took in the development of the equipment of the modern skyscraper, without which the high office building could never have attained the success it enjoyed for so long a period, is attested not only by the many installations he designed but, more especially, by his contributions to technical literature and by his activity in the promotion of reasonable regulations, both governmental and otherwise, in relation to safety and good practice. In this connection he made the following contributions: In 1900 at the annual convention in London, England, joining a general discussion on the "Height of Buildings";[4] in 1901 commenting on the article by Charles G. Darrach on the "Mechanical Installation in the Modern Office Building";[5] and in 1904, discussing by letter a comprehensive paper by Thomas E. Brown on "Passenger Elevators"[6] at the International Engineering Congress at St. Louis, Mo. His paper on "The Operation of Passenger Elevators"[7] presented a subject to which he had given much study and involving considerable research. He also wrote several articles for the American Society of Mechanical Engineers on the following topics: "The Equipment of Tall Office Buildings in New York City,"[8] "Test of a Hydraulic Elevator System,"[9] "The Power Plant of Tall Office Buildings,"[10] and "Operating Conditions of Passenger Elevators."[11] Ten papers by him, between 1897 and 1915, dealing generally with operating conditions in steam heating systems, are included in the *Transactions* of the American Society of Heating and Ventilating Engineers.[12]

In his treatment of the subject of the equipment of high buildings Mr. Bolton always stressed the elevator installations as a most important element. In his discussion of the paper by Mr. Darrach he held that "without transportation high buildings would be folly." At the time he came to the United States, although tall buildings (some as high as twelve stories) were not unknown, the development of the real skyscraper, and with it the elevator installation, was just starting. Mr. Bolton took a prominent part in that movement. He was generally present at public discussions and ready to help arrive at safe and reasonable standards, especially when they were under consideration as governmental regulations. His influence was always directed toward public safety. One of the active topics in this connection was the necessity or desirability for door and car interlocks. The relative merits of hydraulic and electric elevators, more particularly from the standpoint of economy of operation, was another open question. The practicability and limitations of the plunger elevator were much discussed. On all these matters he freely expressed his opinions based on a wide experience.

---

[4] *Transactions*, Am. Soc. C. E., Vol. XLIV (1900), p. 454.

[5] *Ibid.*, Vol. XLVIII (1902), p. 29.

[6] *Ibid.*, Vol. LIV (1905), Pt. B, p. 187.

[7] *Ibid.*, Vol. LXIV (1909), p. 231.

[8] *Transactions*, A. S. M. E., Vol. XX, 1899, p. 873.

[9] *Ibid.*, Vol. XXIV, 1903, p. 933.

[10] *Ibid.*, Vol. XXV, 1904, p. 1011.

[11] *Ibid.*, Vol. 32, 1910, p. 1295.

[12] "Circulation of Steam for Heating Purposes at or Below the Pressure of the Atmosphere," *Transactions*, Am. Soc. of Heating and Ventilating Engrs., Vol. III, 1897, p. 155; "The Element of Time in Contracts," *ibid.*, Vol. VII, 1901, p. 43; "Depreciation, Maintenance and Interest Charges," *ibid.*, Vol. VIII, 1902, p. 82; "Smoke and Gas-Flue System in the Ansonia Apartment Hotel, New York," *ibid.*, Vol. IX, 1903, p. 144; "Steam-Heating in Connection with Condensing Engines," *ibid.*, Vol. XI, 1905, p. 64; "The Use and Abuse of Fuel," *ibid.*, Vol. XVIII, 1912, p. 57; "Definition of the Unit of Heat," *ibid.*, p. 299; "Hydro-Electric Power Compared with Steam," *ibid.*, Vol. XX, 1914, p. 374; "The Problem of City Dust," *ibid.*, Vol. XXI, 1915, p. 225; and "The Establishment of a Standard for Transmission Losses from Buildings of All Constructions," *ibid.*, p. 552.

His contributions to technical literature were not limited to the papers and discussions appearing in the publications of the several societies of which he was a member. He often addressed organizations, such as the National Council for Industrial Safety, the annual convention of Building Managers and Owners, and others, in the interest of public safety. Some of his more pretentious technical works were on the following subjects: "Motive Powers and Their Practical Selection" (1895); "Elevator Service: Operating Conditions and Proportions, with Diagrams, Formulas and Tables for Passenger Travel" (1908); "Building for Profit: Principles Governing the Economic Improvement of Real Estate" (1911); "An Expensive Experiment: The Hydro-Electric Power Commission of Ontario" (1913); "Power for Profit: Principles Governing the Use of Machinery and Labor in Modern Buildings" (1915); "Results of the Test of the Steam and Electric Plant at the Hall of Records, New York City" (1916); and "A Municipal Experiment: Or the Hall of Records Power Plant" (1917).

Mr. Bolton was most interested in professional activities. In addition to his membership in the Society, he was a charter member of the American Institute of Consulting Engineers, from 1899 a member of the American Society of Mechanical Engineers, from 1902 to 1907 a member of the American Society of Naval Engineers, and from 1918 a member of the New York Electrical Society. In 1897 he joined the American Society of Heating and Ventilating Engineers of which he was president in 1911, was elected to life membership in 1929, and was made an honorary member in 1936 "In recognition of the eminent place occupied as a consulting engineer, author and historian." In these several technical organizations, except the Naval Engineers, he retained his membership up to the time of his death. Before Mr. Bolton came to the United States he had been inducted as associate member in the Institution of Civil Engineers of London, England. In 1902 this organization awarded him the Telford (gold) Medal for his paper on "The High Office Buildings of New York."[13] His connection with the English body lasted slightly over half a century.

Mr. Bolton, a very active man in his own chosen profession, had wider interests than engineering. It was nearly half a century before his death that he made his home in Washington Heights, then a sparsely settled section of Manhattan. Relics of early Indian habitations in which this region abounded aroused his interest. His studies of these remains and research into the life and living of the colonial period of his neighborhood and of the surrounding country was so thorough that he became an authority on the history of New York and Westchester counties. His advice was frequently sought regarding the significance of artifacts or human bones that were from time to time unearthed when excavations were being made for new buildings or public works. Many of his findings have been recorded in pamphlets published by himself or in publications of the Museum of the American Indian (Heye Foundation), the Empire State Society of the Sons of the American Revolution, the Dyckman Institute, and the American Scenic and Historic Preservation Society. Because of his activities in this field, he received an honorary life membership in the New York Historical Society. He also belonged to the City History Club, The American Folk Lore Society, and the American Scenic and Historic Preservation Society (at one time, trustee).

---

[13] *Proceedings*, Institution of Civ. Engrs., Vol. CXLIII, London, 1901, p. 213.

He was a leading spirit in the efforts made to preserve for posterity historic landmarks of New York City. Among these landmarks were the Jumel Mansion on Washington Heights, not far from his home, built in 1758 and serving in the early days of the Revolutionary War as Washington's Headquarters; Poe's Cottage in The Bronx, once the home of the poet; and the Dyckman Museum in Manhattan, built in 1748, the only eighteenth-century farmhouse still existing on Manhattan. In 1931 he was enthusiastically active in the movement to make the New York home of the naturalist, James J. Audubon, a national shrine.

Busy as his professional work and the activities of historic investigation must have kept him, Mr. Bolton found time to take part in civic affairs. His devotion to such matters caused him to be known in his locality as the "Number One Citizen of Washington Heights." He took an active part in the local taxpayers association of which he was secretary for more than twenty years, and, at the time of his death, its honorary president. His membership in the Municipal Art League was another instance of his interest in civic affairs.

Like most of his forebears he was closely identified with the Protestant Episcopal Church. In his latter years he attended the Chapel of the Intercession, a part of Trinity Parish of New York City. He one time boasted of his numerous relations connected with the Church. His father, ordained in England as a priest in 1849, "as a preacher to children was never surpassed." His two grandfathers, two great-grandfathers, two great-great-grandfathers, two brothers, and six uncles were in the priesthood.

Reginald Pelham Bolton was married at London, England, in 1878, to Kate Alice Behenna, youngest daughter of Capt. Henry Behenna, R. A. She died in 1891 before he came to this country to live, leaving him two children—a daughter, Ivy May Bolton, who later entered the Protestant Episcopal Community of St. Mary at Peekskill, N. Y.; and a son, St. George Guy Reginald Bolton, a playwright associated with W. Somerset Maugham in the production of "Theatre." Another son died in infancy in England. He was married for the second time at Pelham, N. Y., in 1892, to Ethelind Huyck, daughter of Leonard Huyck. Mrs. Bolton survives him.

It would seem appropriate that this memoir close with a few lines of a poem "The Worm" from "Vagabond Verse" published in 1937. In this poem Reginald Pelham Bolton is reflecting on the futility of earthly possession when death comes, and he continues:

"But in their graves are never hid
The good and generous things they did
Nor in the quiet earthy bed
Are laid the kindly words they said.

"Above the sods these all remain
And spring to life in Man again
These are sweet things Death never parts
That live and grow in human hearts."

Mr. Bolton was elected a Member of the American Society of Civil Engineers on September 6, 1899.

CHARLES EDWARD BOWRON, M. Am. Soc. C. E.[1]

DIED JUNE 17, 1942

Charles Edward Bowron, son of James and Ada Louisa (Barrett) Bowron, was born in Redcar, Yorkshire, England, on May 25, 1871. His parents came to the United States when he was a child.

He attended public schools in Nashville, Tenn., and Vanderbilt University in Nashville, from which he was graduated with the degree of Bachelor of Engineering in 1891.

His first engineering experience was with the Tennessee Coal, Iron and Railroad Company of Pratt City, Ala., as rodman in coal mines. Following this, he was resident engineer for four years at Tracy City, Tenn.; for one year at ore mines in the Birmingham (Ala.) district; and for one year at the Ensley (Ala.) Steel Plant of Tennessee Coal, Iron and Railroad Company. He then served as purchasing agent and acting chief engineer for Alabama Consolidated Coal and Iron Company (later Sloss, Sheffield Steel and Iron Company) in Birmingham, operating coal and ore mines, coke ovens, and blast furnaces. In 1901 he became general superintendent of Lookout Mountain Iron Company in Battelle, Ala., and supervised the construction of blast furnace, coke ovens, village improvements, water works, and the opening of coal and ore mines. Then, as consulting engineer, he designed various industrial plants in the Birmingham district.

---

[1] Memoir prepared by A. Clinton Decker and Carlton A. Wilmore, Members, Am. Soc. C. E.

In 1906, Mr. Bowron went to California as construction superintendent for the Pacific Fruit Express (Southern Pacific Railroad Company) on refrigerating plants at Colton, Calif., and Roseville, Calif.; an icing plant at Carlin, Nev.; and a reinforced concrete dam at Truckee, Calif. His next tour of duty was with International Vacuum Precooling Company in Chicago, Ill., as engineer and superintendent of construction on railroad precooling plants at San Benito, Tex., and Charleston, S. C. During this same period, he supervised construction of ice plants at Harlingen, Tex., and Bay City, Tex.

In 1915 he became chief engineer for Gulf States Steel Company in Birmingham, remaining in that position through 1937. Work under his direction included blast furnaces, open-hearth furnaces, coke ovens, blooming mill, annealing plant, plate and sheet mills, copperas plant, the electrification of rod mill, power plant, shops, coal and ore mines, quarry, mine haulages, tipples, coal washer, water works, office, stores, construction of and improvements in villages, and general engineering supervision of all operation.

In 1937 Mr. Bowron opened consulting offices in Birmingham. However, in 1940, he was appointed chief engineer on building construction for the State of Alabama, continuing in this capacity until he died.

He served as chairman of Suggestion Committee, U. S. Patent Office, and was author of the book *Dimensions of Bars* for bar dimensions.

The scope of the construction projects on which Mr. Bowron was engaged is indicative of his broad knowledge in engineering and executive fields. He was a man with a keen analytic mind and a great student, industrious and persevering, but withal so modest and retiring, that one had to know his works in order to appreciate his true worth. He possessed a keen dry wit which was known to and appreciated only by his intimates. A great lover of home and family, his home life was as nearly perfect as possible.

Mr. Bowron was, throughout his adult life, a member of the South Highlands Presbyterian Church of which he was a deacon and treasurer for many years. He often referred to this service as his "one real hobby."

He was a Registered Professional Engineer in the State of Alabama. He was active in fraternal work and was a member of Birmingham Temple Lodge 636 A. F. and A. M.; the Alabama Consistory, Scottish Rite; and Zamora Temple Shrine.

His long and meritorious professional service, forthright honesty, loyalty to associates and friends, and his many civic and religious activities are an eloquent tribute to a distinguished engineer, a courteous gentleman, and a fine citizen.

On May 25, 1897, he was married to Adelaide Reilly at South Pittsburg, Tenn. He is survived by his widow; two daughters, Dorothy (Mrs. John B. Collins) and Lillian (Mrs. Fred W. Streit, Jr.); one sister; and eight brothers.

Mr. Bowron was elected a Member of the American Society of Civil Engineers on March 14, 1938.

SYLVESTER QUAYLE CANNON, M. Am. Soc. C. E.[1]

DIED MAY 29, 1943

Sylvester Q. Cannon was born in Salt Lake City, Utah, on June 10, 1877. He was the son of George Q. and Elizabeth (Hoagland) Cannon, both 1847 pioneers of Utah. His father was an editor, a delegate to Congress, a director of the Union Pacific Railroad Company, and a member of the First Presidency of the Church of Jesus Christ of Latter-Day Saints.

After work at the Latter-Day Saints (L.D.S.) College and University of Utah, both located in Salt Lake City, Mr. Cannon attended the Massachusetts Institute of Technology at Boston for four years and was graduated with the degree of Bachelor of Science in Mining Engineering in 1899. Immediately after he was graduated he was called by his church to serve as an unpaid missionary to the Netherlands and Belgium. He became a fluent speaker of Dutch and French and presided over the group of elders in that mission during the latter part of his three-year stay. During this time he made a journey to Palestine.

Soon after his return to the United States he was married to Winnifred Saville. They had four sons, Julian, Winfield, Lawrence, and Donald James; and three daughters, Elinor, Sylvia, and Lucile. The two youngest sons are in the service of the United States.

In 1902 Mr. Cannon began his active practice of engineering, devoting himself to mining, a business in which his family was extensively interested. However, his inclinations, which were conservative, gradually led him to other fields. From 1905 to 1907 he was in charge of hydrographic and irrigation surveys of the Weber River System in Utah, under the direction of the state engineer.

For two years, 1907 to 1909, with his wife and two young children he again went to Europe, being called to serve a second time by his church as president of the Netherlands-Belgium mission. On his return he went into hydraulic and irrigation engineering practice and in 1912 was employed by Salt Lake City to look after its complicated water supply. A year later in April, 1913, he was chosen city engineer, serving until August, 1925. In this capacity, Mr. Cannon was in charge of all engineering and construction, including water supply and water system, sewer construction and maintenance, 60 miles of street pavement, construction of sidewalks, smoke abatement work, and city planning and zoning. Some $11,000,000 was spent under his direction.

During his employment by Salt Lake City, Mr. Cannon arranged many water exchanges with farmers. The farmers, inheriting rights to the mountain streams, which were so well suited to culinary purposes, accepted in exchange water from Utah Lake which was equally suitable for irrigation. The city had acquired the lake water during the days of Brigham Young to provide means of transporting stone through a canal from the mountain quarry for the construction of the Salt Lake Temple. The coming of the railroad made the canal unnecessary, but the water rights acquired were very valuable to the city for exchange purposes.

---

[1] Memoir prepared by a Committee of the Utah Section consisting of Richard R. Lyman, Chairman, R. A. Hart, and R. K. Brown, Members, Am. Soc. C. E.

As city engineer, Mr. Cannon approved the design and supervised the construction of the Mountain Dell Multiple Arch Dam for the reservoir in Parley's Canyon (reinforced concrete, 140 ft high, 610 ft long at the top) and designed and supervised the construction of the Lake Phoebe-Mary Dam (60 ft high and 330 ft long) and the Twin Lakes Dam (72 ft high and 650 ft long)—all for the storage of culinary water for the city.

In 1933 he was one of the three members of the advisory board set up by the federal government through the Public Works Administration to scrutinize and pass upon all state, county, and municipal projects in Utah for inclusion in the $3,300,000,000 federal fund for public works.

Mr. Cannon was a member of the Sons of the American Revolution and of the Timpanogos Club. In 1914–1915 he was president of the Utah Society of Professional Engineers. He had been a member of the Chamber of Commerce and devoted much time to city planning.

From May, 1913, to October, 1925, he was chairman of the Board of Canal Presidents, supervising and directing the Utah Lake Pumping Plant and distributing water to the five main canal companies that supply the irrigation water for Salt Lake Valley. From July to September, 1923, Mr. Cannon acted as consulting engineer for the United States Reclamation Service on public improvements for the new town of American Falls, Idaho.

In June, 1925, an important change occurred in his life when he was chosen by his church to be its Presiding Bishop. (This officer has large responsibilities in the temporal affairs of the institution and is one of its General Authorities.) Mr. Cannon's engineering, as well as his missionary and other spiritual, background was an asset to him in his work. Supervision of buildings, their construction, and other obligations took Mr. Cannon through most of the western United States. Administrative work brought him in close contact with highly varied activities.

Mr. Cannon was president of the board of trustees of the Dr. Groves L.D.S. Hospital at Salt Lake City; the Dee Hospital at Ogden, Utah; and the Idaho Falls Hospital at Idaho Falls, Idaho—all church owned. He was also president of the L.D.S. Business College, Zion's Aid Society, Deseret Gymnasium, McCune School of Music and Art, Radio Service Corporation of Utah (KSL), and Deseret News. He was treasurer of the Uintah Basin Construction Company, supervisor of Salt Lake County Drainage District No. 2, director of Zion's Cooperative Mercantile Institution, Zion's Securities Corporation, Utah Hotel Company, Amalgamated Sugar Company, and U. S. Fuel Company. By appointment of the Governor of Utah, he was chairman of the Special State Flood Commission (September, 1930, to February, 1931) and of the State Advisory Council for Unemployment (August, 1931, to January, 1932).

In April, 1938, Mr. Cannon was chosen assistant to the Council of Twelve Apostles and in October, 1939, one of the members of that council. In discharge of his new duties he traveled almost constantly throughout the United States and also in Hawaii and in Europe.

Mr. Cannon's blameless life, his calmness, and pleasant demeanor gave him a place in the hearts of a great circle of friends. His honesty, tolerance, and thoroughness fitted him for his responsibilities in business, civic, and church positions. He was a tall man in stature and character, and he walked with dignity through life.

Mr. Cannon was elected a Member of the American Society of Civil Engineers on January 15, 1917.

WOOLSEY MOORMAN CAYE, M. Am. Soc. C. E.[1]

Died July 28, 1942

Woolsey Moorman Caye was born at Louisville, Ky., on January 27, 1885, the son of Dr. William Cassius Caye, Sr., and Priscilla (Moorman) Caye.

Mr. Caye was educated in the public schools of Louisville. At du Pont Manual Training High School, which he attended from 1900 to 1904, he is remembered as captain of the famous 1903 football team which defeated the rival team of Louisville Male High School 17–7. While in high school, Mr. Caye gave evidence of his great ability as a designer and draftsman. He produced some freehand drawings of ships and other subjects which were considered by some people to be masterpieces.

Although his father and others urged him to continue his education at college, Mr. Caye was anxious to start active work, and, therefore, secured a job on railroad construction in Mississippi in 1904. He remained in the South for about one year, working on railroad, dock, and harbor construction projects in Mississippi and Alabama. In June, 1905, he returned to Louisville and accepted a position as draftsman with the municipally owned Louisville Water Company in connection with the construction of the Crescent Hill Filtration Plant and additions to the city's water-distribution system.

In December, 1908, he left the Louisville Water Company and entered the employ of the Commissioners of Sewerage of Louisville, a separate nonpartisan municipal corporation established by an act of the Kentucky State Legislature for the purpose of handling the proceeds of a $4,000,000 sewer bond issue of the City of Louisville. Mr. Caye was employed as inspector on construction work and, later, as draftsman in the office. The high quality of the work done by the Commissioners of Sewerage and the training he received (at this time and later) under the chief engineer, J. B. F. Breed, and the designing engineer, J. H. Kimball, M. Am. Soc. C. E., had a permanent influence on Mr. Caye's future career. Mr. Kimball, particularly, took pains to instruct him in design work. In March, 1912, when the Commissioners of

---

[1] Memoir prepared by C. Frank Johnson, M. Am. Soc. C. E.

Sewerage completed this work and dissolved, Mr. Caye went with the Board of Public Works of the City of Louisville, as assistant engineer in a special engineering department on the design and preparation of contract plans and specifications for certain sewerage, drainage, and similar work.

From February, 1915, when he left the Board of Public Works of the City of Louisville, until May, 1916, he was computer with the Miami Conservancy District of Dayton, Ohio, on the design of flood-protection, sewerage, and allied projects for the Miami River Valley—a $35,000,000 undertaking. On this job he again worked under Mr. Kimball, his former supervisor in Louisville.

When the First Kentucky Infantry of the Kentucky National Guard was ordered into active service, in May, 1916, because of the Mexican border trouble, Mr. Caye resigned his position and enlisted in this regiment. After a period of training at Hattiesburg, Miss., he served on the Mexican border, as First Lieutenant of infantry. Later he took artillery instruction at Fort Sill, Oka., and was promoted to Captain in the field artillery. He then went to France, where he served with the 138th Field Artillery in the American Expeditionary Force.

Upon his return to the United States, in May, 1919, he rejoined the organization of the Miami Conservancy District in Dayton as assistant engineer on the design and preparation of contract plans for flood-protection works.

In November, 1920, a second Commissioners of Sewerage of Louisville was established, and Mr. Caye returned to Louisville to serve this organization as designing engineer, in charge of all design. He continued in this position until the death of Mr. Breed, chief engineer, in January, 1928, when he was placed in charge of the entire engineering department, with the title of "consulting technical engineer." In this capacity he performed the duties, and had the responsibility, of a chief engineer, and he continued in this position until his death.

During Mr. Caye's service as designing engineer for the commission (1920–1928), sewerage and drainage work to the value of $7,000,000 was designed, put under contract, and constructed, including a large creek-improvement project with a concrete channel and several tunnel projects, as well as many trunk and main sewers of conventional design. Practically all of this construction required reinforced-concrete sewers of the larger sizes.

During the period (1928–1942) when Mr. Caye was in full charge, sewerage and drainage work costing $11,000,000 was completed by the Commissioners of Sewerage of Louisville. Perhaps the outstanding achievement of this period was the construction of the Southwestern Outfall, a combined sewer serving a large part of the city. This sewer is 4.2 miles long, cost slightly more than $3,855,000, and has an inverted egg-shaped section ranging from 15 ft 8 in. wide by 23 ft 6 in. high, to 18 ft 4 in. wide by 27 ft 6 in. high, interior dimensions. This is probably one of the largest sewerage projects ever completed in the United States. The sewer was constructed through water-bearing material, the ground water varying from 10 ft to 27 ft above the elevation of the inside bottom of the sewer. After completion of this project, the ground-water infiltration into the sewer was measured and found to be practically nil—evidence of first-class construction under difficult conditions.

Besides supervising the design and construction of sewerage and drainage projects, Mr. Caye initiated and directed for the commission many special studies of problems relating to sewerage work. Among these was the study made in 1940 of the sewage-treatment problem of Louisville. Also, in connection with the unprecedented 1937 flood of the Ohio River at Louisville and the resulting proposed flood-protection work, he made several comprehensive investigations and studies of sanitary and hydraulic problems affecting the city sewerage system directly or indirectly.

Mr. Caye took a leading part in the development of Louisville's modern sewerage system. As a result of his insistence and that of his predecessor, Mr. Breed, upon the adoption of high standards, the quality of the engineering and construction work at Louisville probably is exceeded by that of very few cities. The commission's work has always had the reputation of being "the last word" in the field of sanitary and hydraulic engineering. In supervising the design, Mr. Caye utilized the latest and most scientific methods known to him. He was responsible for the introduction of many innovations in Louisville's sewerage system.

Mr. Caye was interested in seeing that the city received a dollar of value for every dollar spent. He insisted on the scrupulous performance by the contractors of their engagements and, to this end, imposed a rigid inspection, which applied impartially to all contractors. Similarly, he insisted that the engineering employees of the commission observe the highest standards in their work. No careless or make-shift work was acceptable. The merit system strictly prevailed, in employing and retaining personnel. No political or other similar considerations ever influenced in any degree the hiring or retention of the employees of the engineering department, and such considerations did not influence any work under Mr. Caye's direction.

Mr. Caye worked energetically and enthusiastically on any project with which he was concerned. One of the members of the sewerage commission by which he was employed stated that Mr. Caye was the "most intense man" he had ever known. This very intenseness may have contributed to his death at the comparatively early age of fifty eight. About a year before his death he had suffered a heart attack from which he appeared to have recovered fully. However, he was under his physician's order to take better care of his heart by not working so hard. This proved to be difficult for him to do. His death came suddenly at lunch one day, during a period when he was working just as hard as, if not harder than, ever.

Mr. Caye took an active part in professional affairs. He was a member of the Engineers and Architects Club of Louisville, and its president in 1931. He was a charter member of the Kentucky Society of Professional Engineers, having served as vice-president in 1935 and 1936, as president in 1939, and as a member of the board of direction in 1934, 1937, 1938, 1940, and 1941. He assisted in promoting the adoption by the State of Kentucky of a registration law for engineers, as well as helping with other work designed to promote the well-being of the engineering profession. He wrote several articles which were published in technical journals.

His hobby was the dual one of shooting and hunting. An expert with gun and pistol, he had won many trophies for his marksmanship at both local and national matches. Like all experts, he made his own cartridges for use in the shooting matches. He had quite an extensive collection of old guns and pistols of all kinds, and belonged to local and national rifle clubs. During the hunting season Mr. Caye spent much time in the field, making good use of his shooting skill. His vacations were usually spent in the woods and wilds of Kentucky, Florida, Canada, or elsewhere, hunting deer and other wild game. He was a member of the Juniper Club—Louisville sportsmen who hunt and fish every year on their own property in Florida.

On April 3, 1918, Mr. Caye was married to Ernestine Wentworth, of Brookhaven, Miss. Besides his widow, he is survived by three daughters, Margaret (Mrs. Paul Campbell), Juanita (Mrs. Cyril Kinsey), and Priscilla; a son, Woolsey M., Jr.; two brothers, L. Chase and William C.; and a sister, Artelia (Mrs. George B. Perry).

Mr. Caye was elected a Member of the American Society of Civil Engineers on December 14, 1925.

EDWARD CARTWRIGHT CONSTANCE, M. Am. Soc. C. E.[1]

DIED APRIL 29, 1943

Edward Cartwright Constance, the son of William Edwin and Josephine (Florence) Constance, was born at High Hill, Mo., on December 27, 1874. His boyhood was spent on his father's farm.

He attended the public schools in High Hill and Jonesburg, Mo., and took more advanced courses in the Central Wesleyan College at Warrenton, Mo., and the State Normal School at Kirksville, Mo. After teaching school for several years, he entered the University of Missouri at Columbia, in 1900, being graduated in 1904 with the degree of Bachelor of Science in Civil Engineering. In 1910 he received the advanced degree of Civil Engineer from the same institution.

In August, 1904, Mr. Constance entered the employ of the United States Government, in the U. S. Engineer Department, St. Louis (Mo.) District, engaged on the improvement of the Mississippi River. Except for a transfer to the Kansas City (Mo.) District, from August, 1916, to December, 1922, he remained in the service of the St. Louis District, without intermission, until his death.

During his long service, his experience embraced a wide range of subjects. He built thousands of feet of pile dikes and bank revetments, and made extended surveys and computations. He supervised the design, construction, and maintenance of large towboats, barges, dredges, and other floating plant.

---

[1] Memoir prepared by James W. Skelly, M. Am. Soc. C. E., in collaboration with Joseph M. Chandlee, Electrical Engr., Webster Groves, Mo.

For several years he was in charge of removal by blasting of an extensive submerged rock which was considered an obstruction to navigation. While engaged in this work, a premature explosion broke both his ankles and impaired his movements throughout the remainder of his life.

He was a man of splendid physique, hale in appearance, and generally enjoyed good health. Early in April, 1943, however, he was stricken with a somewhat rare malady, a deficiency in blood corpuscles, for which there appeared to be no remedy, and on April 29 he died. He was buried in Mount Pleasant Cemetery, in the neighborhood near High Hill, where he had grown up.

Mr. Constance's entire service was characterized by loyalty and devotion to the department which he served so well. He was trained under several engineers and gentlemen of the old school, in the days when government appropriations were granted sparingly, and results were scanned critically.

On December 23, 1905, he was married in Litchfield, Ill., to Ollie Burton, who died on August 31, 1930. Two sons were born to them, Roger Edward and William Burton. On March 31, 1934, Mr. Constance was married in Decatur, Ill., to Mrs. Gladys Mowery Hamman, who survives him.

He was a member of the Masonic fraternity, the Modern Woodmen of America, and was an Elder in the Presbyterian Church.

Mr. Constance was elected a Junior of the American Society of Civil Engineers on December 6, 1904; an Affiliate on February 6, 1907; and a Member on January 20, 1922.

PAUL DARWIN COOK, M. Am. Soc. C. E.[1]

DIED FEBRUARY 26, 1943

Paul Darwin Cook was born on November 13, 1883, at Wysox, Pa., the son of the Rev. Milton Lewis Cook and Mary (Thomas) Cook. He was the third of eight children, five girls and three boys, all of whom were graduated from either Mount Holyoke College at South Hadley, Mass., or Lafayette College at Easton, Pa. His father and two grandfathers were Presbyterian ministers, from whom he inherited a sturdy character and an independent mind.

From the Susquehanna Collegiate Institute at Towanda, Pa., Mr. Cook entered Lafayette College. He was graduated in Mining Engineering in 1905. He began his professional career as draftsman for the Gregg Company, Limited, at Newburgh, N. Y. Later, he went to Chicago, Ill., as resident engineer on construction of yards and railway terminal buildings for the Manufacturers Junction Railway Company, a subsidiary of Western Electric Company. From that time Mr. Cook had the following varied and interesting career:

---

[1] Memoir prepared by a Committee of the Iowa Section consisting of Ross M. Coomer and Almon H. Fuller, Members, Am. Soc. C. E., with the assistance of William L. Steele, member of the firm, Kimball, Steele & Sandham, Omaha, Nebr.

March, 1907, to July, 1909—Chicago, Milwaukee and St. Paul Railway Company, as draftsman and structural engineer, designing bridges and other railway structures on the original system and on the Puget Sound extension.

July, 1909, to September, 1910—Engineer of construction on a $500,000 grade separation viaduct for five interested railroads at Sioux City, Iowa, reporting to the chief engineer of the Chicago, St. Paul, Minneapolis and Omaha Railway Company.

September, 1910, to June, 1912—Engineer for Lewis and Leeder, Municipal Contractors of Sioux City.

June, 1912, to January, 1913—Superintendent of construction for the Davidson Realty Company in charge of construction of a $600,000 fireproof office building in Sioux City.

January, 1913, to May, 1917—Consulting engineer in Sioux City, designing bridges, water works, office buildings, hospitals, churches, schools, warehouses, and viaducts. Among the structures on which he was engaged were the $700,000 Woodbury County Court House and the $180,000 St. Vincent's Hospital.

From the fall of 1916 until the spring of 1919 Mr. Cook was engaged actively in the problems of war. In 1916 the Society suggested to its members that they offer their services to the government. As a result, Mr. Cook, with numerous other members of the Society, was invited by the U. S. War Department to present himself for examinations for a commission:

May, 1917, to September, 1917—First Officers Training Camp at Fort Snelling, Minnesota, and at Fort Leavenworth, Kansas; commissioned Captain of Engineers.

September, 1917, to October, 1918—Captain, commanding Company C, 109th Engineers.

October, 1918, to March, 1919—Major, commanding 1st Detached Battalion, 138th Engineers.

In 1919 Mr. Cook resumed the private practice interrupted in 1917 by the insistent obligations of war:

March, 1919, to June, 1923—Consulting engineer in Sioux City on buildings, bridges, and water works. In this period (winter of 1921–1922) a disability incurred in army service developed which required him to live in a warmer climate. He went to New Orleans, La., where he worked as a draftsman for Favrot and Livaudais, Limited, on structural design.

June, 1923, to February, 1924—District engineer for the U. S. Fidelity and Guaranty Company at Des Moines, Iowa, adjusting surety bonds and completing defaulted contracts in the states of Iowa and Nebraska.

February, 1924, to December, 1934—With Favrot and Livaudais, in responsible charge of structural design and field supervision. He also supervised the New Orleans Municipal Auditorium and New Orleans Public Service Building (each costing about $1,500,000), the Roosevelt Hotel Addition, and Baptist Hospital in New Orleans; and the Botton High School at Alexandria, La.

December, 1934, to April, 1936—Resident engineer inspector for the Public Works Administration (PWA) on an $800,000 high school building at Sioux Falls, S. Dak.

April, 1936, to April, 1938—City engineer of Sioux City.

July, 1938, to September, 1940—Examining engineer for PWA at Regional Headquarters, Omaha and Chicago.

September, 1940, to February, 1941—Airport manager for the municipal airport at Sioux City.

February, 1941, to April, 1942—Consulting engineer on structural design of municipal improvements.

April, 1942, to February, 1943—City engineer of Sioux City.

Paul Cook was a student all his years, and had an extraordinary memory for facts and figures—in fact, an encyclopedic mind. He was well informed not only on engineering topics but also on current events, economics, and things in general. He was interested in reading, but much more so in men. He had a shrewd insight into personal character and hated humbugs and "stuffed shirts." His thought was modestly expressed, without any assumption of authority or superior mental equipment, although he could have laid claim to both. When he spoke, what he said was logical and tempered with common sense. His professional ideals were high, and in his interpretations of professional ethics he was keenly aware of the spirit rather than of the letter. He was modest, unassuming, generous, patient, and always a gentleman. He was a good companion and possessed a keen and sympathetic sense of humor. He was never a "show-off." He did not care who got the credit for a job, if it was done successfully.

A leading citizen of Sioux City, Mr. Cook was active in the Rotary Club from 1913 until he left Sioux City in 1923; he was the executive member of the committee which produced Sioux City's first building code (in fact, he practically wrote most of it); and he was a splendid superintendent of construction, for he not only "knew his stuff," but was a keen and active observer, fair-minded, impartial, and just. Paul Cook inspired confidence and never abused it. As a designer, he studied his problems thoroughly, and his solutions were correct, economical, and often brilliant. He was an ideal consultant on architectural work because any differences of opinion were accompanied by frank arguments and discussions, without any real conflict between architect and engineer. He believed in teamwork and considered it necessary for successful building.

On October 16, 1912, Mr. Cook was married to Hazel E. Miller of Sioux City. He is survived by his widow and two sons, Henry L. and Thomas D.

Mr. Cook was elected an Associate Member of the American Society of Civil Engineers on January 31, 1911, and a Member on June 24, 1916.

WILLIAM OTTO GALBREATH, M. Am. Soc. C. E.[1]

Died March 14, 1943

William Otto Galbreath was born on a farm near Hebron, Ind., on August 24, 1871, the son of Martin Van Buren and Emily (Turner) Galbreath. In 1877 his parents moved to Carrollton, Mo., and, in 1881, to a farm near Utica, Mo.

William Galbreath attended the public schools in Carrollton and Utica and was graduated from the Utica High School in 1891. In 1892 he entered the University of Kansas, at Lawrence, and was graduated with the degree of Bachelor of Science in Civil Engineering in 1898.

In the spring of 1899 he entered the service of the Mexican Central Railroad Company and continued with that company until September, 1906, at which time he joined the engineering staff of the Mexican National Railroad Company (later consolidated with others to form the National Railways of Mexico). He remained with that organization until the spring of 1912. While in Mexico, Mr. Galbreath was promoted from instrumentman on location to division engineer on construction, then to locating engineer, and finally to district engineer in charge of maintenance and construction. In these positions he gained broad experience in all branches of railroad engineering, and, during a part of the time, was in direct charge of the construction of approximately 160 km of railroad including bridges, depots, yards, water stations, etc. However, as a result of the revolution which caused very unsettled conditions and made it unsafe for his family, he left Mexico in the spring of 1912 and returned to the United States. He immediately joined the forces of the Missouri and North Arkansas Railroad Company as engineer of maintenance of way, remaining for four years. During this time, he was in charge of all engineering work including the design and construction of a general shop and yard at Harrison, Ark.

Mr. Galbreath resigned from the Missouri and North Arkansas Railroad Company in the spring of 1916, and was employed at once by the City of Harrison to design and build a water distribution system for the city. After designing the system and letting the contract for its construction, he accepted a position in the office of the chief engineer of the Chicago Great Western Railroad Company in Chicago, Ill., and, after about a year, was made division engineer of the Southern Division with headquarters at Des Moines, Iowa.

In the fall of 1917, he left the employ of the Chicago Great Western Railroad Company to become chief engineer of the Kansas, Oklahoma and Gulf Railway Company at Muskogee, Okla. Thus, he realized the ambition of all railroad engineers. He held this position for three years, until November, 1920.

---

[1] Memoir prepared by Allston T. Cushing, M. Am. Soc. C. E.

After leaving the Kansas, Oklahoma and Gulf Railway Company, Mr. Galbreath was employed by the Kansas City Southern Railway Company and given special assignments including the location of a 0.5% grade line from a point on this company's tracks south of Grandview, Mo., to a connection with the Missouri Pacific Railroad Company, south of Martin City, Mo. He continued with the Kansas City Southern Railway Company until the fall of 1921 when he resigned to become assistant engineer on the plans and construction of the double tracking of the Missouri Pacific Railroad Company west of St. Louis, Mo., remaining for approximately a year.

Early in 1923, Mr. Galbreath moved to Chillicothe, Mo., where he engaged in contracting work on street paving and sewer construction. In August, 1924, he accepted a position with the U. S. Department of Agriculture as valuation engineer, on appraisals of stockyards for use in the establishment of fair rates. He remained in the service of the federal government until his retirement on August 31, 1941—a period of seventeen years, the last five years of which he was in charge of the Kansas City (Mo.) office. At the time of his retirement he held a rating as senior valuation engineer.

Mr. Galbreath's entire career was one of accomplishment. Several of the positions he held carried great responsibility and from them he acquired the ability to grasp a situation quickly, think it through, and then act. The great diversity of the various projects which he designed, built, and appraised shows a large degree of versatility in the use of basic engineering training.

Always of a modest disposition, Mr. Galbreath was calm and quiet under all circumstances. Sincerely enthusiastic about his work, he made such good use of his powers that accomplishment stands out as the main feature of his professional life. He never made a speech or presented a paper before an assembly, but his work spoke for him.

After Mr. Galbreath retired, he lived a quiet life with his family at his home in Kansas City. He made several extended motor trips, visiting his brothers in Michigan and Indiana, and spent his summers in Colorado in the mountains he loved so well. Mr. Galbreath was very active until shortly before his death on March 14, 1943.

William Galbreath was married in Utica, on October 14, 1903, to Blanche Stone, daughter of Ashford and Mary Stone of Utica. He is survived by his widow; two sons, Ashford M. and Thomas S. Galbreath; a daughter, Anita Blanche Galbreath; three grandchildren; five brothers; and one sister. He was a member of the Masonic Lodge.

Mr. Galbreath was elected an Associate Member of the American Society of Civil Engineers on April 6, 1904, and a Member on April 7, 1930. He became a Life Member in 1939.

JOHN PHILIP HALLIHAN, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1944

John Philip Hallihan, the son of Timothy and Harriet (White) Hallihan, was born on April 27, 1867, in Fond du Lac, Wis. After being graduated from high school at the age of nineteen, he began his engineering career on location and construction work for the Union Pacific and several other western railroad companies. From 1891 to 1897 he served on irrigation, railroad, and mining projects in New Mexico and Colorado. From 1898 to 1901 he was assistant engineer for the El Paso and Northeastern Railroad Company. From 1901 to 1904 he was employed as resident engineer and superintendent of the railways for the Cananea Consolidated Copper Company of Cananea, Mexico. From 1904 to 1907 he was assistant to the president and chief engineer of the Rio Grande, Sierra Madre and Pacific Railroad, locating 150 miles with a 1% grade over the continental divide, and making a reconnaissance of 400 miles for a railroad to the Pacific. He was also in charge of operation on 150 miles of road.

Mr. Hallihan was in South America from 1907 to 1909 as general manager of the South American Construction Company, of which the writer was president. He constructed 125 miles of railroad in Bolivia from Viacha to Oruro, all on a plateau 12,000 ft above sea level. This project was completed within the time limit and at a cost 5% less than that estimated by the chief engineer of the Bolivia Railway Company, for which the work was done.

In 1910 Mr. Hallihan was again associated with the writer, making a reconnaissance of 700 miles for a proposed railroad in Spain, the detailed surveys of which were made by Spanish engineers. On this trip were representatives of the National City Bank of New York and Hallgarten and Company who investigated the financial aspects of the proposed railroad. In Madrid, Spain, the writer and Mr. Hallihan met Señor Canalejas, then Prime Minister, noted for his progressive attitude, who was much interested in having American engineers and contractors develop railroads and other public works in Spain. Unfortunately the prime minister was assassinated the following year, and the project was dropped as far as Americans were concerned.

In succeeding years Mr. Hallihan continued his work in Latin-American countries—railroad surveys in Peru, railroad construction in Haiti, and operation of 3,500 miles of railroad in Brazil.

In 1918 he resigned from his position in Brazil to enter the United States Army, first as commanding officer, Seventieth Engineers, and later as deputy officer in charge of Engineer Supplies Section, American Expeditionary Force. After the armistice he was assigned to the engineering department of the Peace Commission and reported on damages to French railroads. After retiring from the army in 1919, he was associated for several years with the late Frederic A. Molitor,[2] M. Am. Soc. C. E., examining the deferred maintenance situation on various railroads in the United States preparatory to presenting briefs to the Interstate Commerce Commission.

---

[1] Memoir prepared by Philip W. Henry, M. Am. Soc. C. E.

From 1923 to 1935 he was with the late Daniel L. Turner, M. Am. Soc. C. E., on the design of a rapid transit system for Detroit, Mich., and was executive engineer for the Rapid Transit Commission of that city.

In 1935 Major Hallihan was put in charge of the Municipal Section of the Work Projects Administration (then called Works Progress Administration). His broad experience was of great value in working out many of the complications of administration and engineering for WPA. While he was with WPA he was loaned to the Army Specialist Corps where he handled many of their important engineering problems. He remained with WPA until 1943 and had charge of the clearing of the final reports of that organization.

In March, 1943, he was made Chief of Transportation and Engineering Division, Engineering Staff, Railroads, of the Foreign Economic Administration (then known as the Office of Economic Warfare). Here again his wide experience fitted into the many problems that the Foreign Economic Administration had before it.

Major Hallihan, because of his experience in Latin America and his desire always to do his best, spoke Spanish with unusual correctness and fluency. Only a short time before his death he spent several months in Ecuador and Venezuela, investigating certain engineering and construction projects for a well-known contracting firm of New York, N. Y. The author of articles for various engineering publications, his opinions were highly regarded. An active member of the Society, he was President of the Detroit Section in 1926. Major Hallihan also belonged to American Railway Engineering Association, American Transit Association, Detroit Engineering Society, American Institute of Consulting Engineers (member of the Council, 1932–1934), and Tiger Post of the American Legion in New York.

He is survived by his widow, Helen Britt Hallihan, to whom he was married in Leadville, Colo., on June 25, 1897, and by a son, Edward B.

Major Hallihan was elected an Associate Member of the American Society of Civil Engineers on April 4, 1900, and a Member on June 5, 1906.

GEORGE PRINCE HAWLEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 26, 1941

George Prince Hawley was born at Niagara Falls, N. Y., on August 22, 1872, the son of Royal A. and Helen (Day) Hawley. When Mr. Hawley was six years old, the family moved to Denver, Colo., but three years later they went to Round Grove, Ill., which Mr. Hawley always regarded as his early home. He attended the local grade school, Dixon Business College, and spent two years at Deveaux Military Academy at Niagara Falls, making an outstanding record in debate work.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1838.

[1] Memoir prepared by James H. Brace, M. Am. Soc. C. E.

In 1893 Mr. Hawley entered the University of Wisconsin at Madison as a special student in civil engineering, continuing in this field for the greater part of his college years until 1896. While at the university, Mr. Hawley was employed during the summer vacations on the construction of the Hennipin Canal near Wyanet, Ill., and on various railroad jobs.

In 1896 Mr. Hawley left the university to become resident engineer on the double-track construction of the Chicago and North Western Railway Company between Madison and Baraboo, Wis. In this work he was associated with the late Walter H. Gahagan,[2] M. Am. Soc. C. E., his lifelong friend.

Next he joined the staff of the late Wallace C. Johnson,[3] M. Am. Soc. C. E., of Niagara Falls on some work at the Cave of the Winds and later at Shawinigan Falls, Quebec, Canada. Although he did not continue long with Mr. Johnson, he first met the fine group of hydraulic and electrical engineers (assembled by Mr. Johnson) who, directed by the late Julian C. Smith,[4] M. Am. Soc. C. E., have supervised the great power developments on the St. Maurice River in Quebec, and assisted in those on the St. Lawrence near Montreal, Canada.

From 1905 until 1911 Mr. Hawley was at De Pere, Wis., first as resident engineer for his college mate and friend, W. G. Kirchoffer, of Madison, on the rebuilding and extension of the city water works and then as city engineer with a private practice.

In 1911 Mr. Hawley again became associated with the St. Maurice-St. Lawrence power developments as resident engineer for the Shawinigan Water and Power Company, the Cedars Rapids Manufacturing and Power Company, and the Montreal Island Power Company. He had a responsible part in the surveys, preparation of plans, and construction of the larger hydroelectric plants and transmission lines of these associated companies which together constitute one of the largest and most economically successful power systems in the world. He played a prominent part in the study and execution of the plans that have met with marked success in minimizing the trouble from frazil ice, that bugbear of the operators of all hydraulic power stations in the north.

Through his integrity, sound engineering knowledge, and good common sense, Mr. Hawley had the confidence of a group of executives and engineers, who have had no peers and who have directed this system through to an outstanding success.

Mr. Hawley was married in 1905 to Nettie Mills of Lodi, Wis. They had no children. A very devoted and congenial couple, they lived much of the time in comparatively small and isolated communities, taking a keen interest in the welfare of the neighborhood and endeavoring to improve educational and religious conditions. They were both much interested in gardening and poultry raising. After Mr. Hawley's retirement at the end of 1937, the Hawleys planned to find some pleasant spot where they could devote themselves to the development of unusual plants, both flowers and vegetables, but, before their plan was realized, they both died, Mrs. Hawley preceding her husband in death by less than three months. In the communities where the Hawleys lived, they were the ones to whom those in trouble could freely appeal—in other words, the best of citizens and friends. Can we say more.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1545.

[3] For memoir, *ibid.*, Vol. LVIII (1907), p. 538.

[4] For memoir, *ibid.*, Vol. 105 (1940), p. 1916.

Mr. Hawley was elected an Associate Member of the American Society of Civil Engineers on February 6, 1901, and a Member on June 24, 1914.

JOHN COFFEE HAYS, M. Am. Soc. C. E.[1]

DIED JULY 21, 1942

John Coffee Hays was born on January 5, 1888, in Tulare County, California, of parents whose forebears were prominent in the development of the West. His paternal grandfather, also John Coffee Hays named for Gen. John Coffee, was the famous Captain (later Colonel) Jack Hays, first commander of the Texas Rangers when they were organized in 1840. The exploits of the Rangers under the daring leadership of Jack Hays are among the most thrilling accounts of the early development of the West. Colonel Hays went to California in 1849; became the first sheriff of San Francisco County; and was subsequently appointed Surveyor General for California by Franklin Pierce, the fourteenth president of the United States. (His early training was in surveying, a profession which he also practiced in Texas when not engaged in Ranger service.) Jack Hays' father, Harmon Hays, and grandfather, John Hays, whose residence was the Hermitage, in Tennessee (later the home of Andrew Jackson, seventh president of the United States), served as officers under General Jackson. It was John Hays who built Fort Maysboro in Tennessee. Aside from daring and courage, Col. Jack Hays was known for his extraordinary modesty, as was his grandson although the latter's important accomplishments were principally those of peacetime. However, John Coffee Hays participated in World War I, receiving the title of Major by which he was subsequently known to his many friends.

Susan Calvert of the Alabama family of that name was the paternal grandmother of John Coffee Hays. She made the journey from Alabama with her father by wagon to Seguin, Tex., where she became engaged to Col. Jack Hays and was married to him when he returned from the Mexican War. It was she, with her accounts of the many adventures of her early life, who made a profound impression on her young grandson.

The maternal grandfather of John Coffee Hays was John McMullin, Captain in the Texas Rangers, who came to California with his old friend Colonel

---

[1] Memoir prepared by H. L. Lowe, Stone & Webster Engineering Corp., New York, N. Y.

Hays. He went east to Washington, D. C.; was married to Eliza Morgan; and returned with her to live in the San Joaquin Valley in California where he had purchased land. The two families lived in the closest friendship and his eldest child, Anna, was married to Colonel Hays' eldest son John. Anna and John Hays were the parents of John Coffee Hays.

The childhood and early manhood of John Coffee Hays were spent in California at Fernwood, his grandfather's famous ranch, where were held brilliant parties attended by prominent people from many parts of the United States. He attended private schools in Oakland, Calif., and the Berkeley-Owens Academy, Berkeley, Calif.

At the age of sixteen he entered the employ of the Mount Whitney Power Company and received practical training as lineman, wireman, and station operator, as well as experience in electrical construction. He became chief operator of power house No. 1, a hydroelectric plant of the Mount Whitney Power Company, but left this position to gain further experience on power plant construction with the Henshaw-Bulkley Company of San Francisco, Calif. In order to expand his engineering training he took up experimental engineering at the Pittsfield (Mass.) works of the Stanley Electric and Manufacturing Company (later acquired by the General Electric Company). From 1905 to 1907 he was a member of the engineering staff of the late L. B. Stillwell, M. Am. Soc. C. E., consulting electrical engineer, New York, N. Y., and had charge of all construction work on the New York and Queens County Railway, Long Island Electric Railway, and New York and Long Island Traction Company. He also acted in an advisory capacity on operation and made special investigations and reports of a technological, economic, and business nature.

Upon completion of his work with Mr. Stillwell, Mr. Hays returned to California to make an investigation of the property and business of the Mount Whitney Power Company where he had been employed in the early part of his engineering career. As a result of his studies, he recommended to the board of directors a program for the development of additional hydroelectric power sites and the simultaneous development of market for the power. He clearly foresaw the course which the electric industry would follow in this area and the policy which should be formulated by the company in rendering a service which would be of ultimate maximum benefit to agriculture, the principal industry of this area. In 1907 Mr. Hays became president, general manager, and consulting engineer of the Mount Whitney Power Company—the first company to develop commercially the use of electric power for pumping water for irrigation for which Mr. Hays subsequently was given full credit publicly.

The program of development of the company's property and business recommended in his report was carried out under his direction. The engineering design and construction work was done entirely by the company's forces under his personal supervision. Two new hydroelectric developments, trebling the company's installed hydroelectric generating capacity, were built which embodied modern features of design and construction; and surveys were completed and plans prepared for another development having even greater capacity than the two new ones combined. The capacity of the steam relay plant was trebled and synchronous condenser capacity installed. The transmission and distribution systems of the company were extended greatly and modern substations installed at strategic points. These extensive improvements were undertaken boldly by Mr. Hays in anticipation of a large demand for power which he was confident would result from the existence of an evident ample supply. Thus, the company was in a position to serve a valley agriculturally rich when irrigated as the valley was underlain by a vast subterranean reservoir so near the surface that deep-rooted trees flourished through the long dry season. Mr. Hays contributed to the development of California by the application of his knowledge of engineering and economics to the problem of irrigating a rich valley with subsurface water, at a cost within the reach of the agriculturist. His vision was fulfilled.

Coincident with this work, he was engaged in the formation of several organizations for the development of large hydroelectric projects in California and held the position of president and consulting engineer of the Yosemite Power Company, Mount Shasta Power Company, Tuolumino Power and Light Company, and LaGrange Water and Power Company.

When the United States entered World War I in 1917, he followed the tradition of his family and joined the armed forces, his extensive engineering and construction experience naturally placing him in the Construction Division of the U. S. Army. He served as Construction Quartermaster and Camp Utilities Officer at Camp Lewis in Washington, and Supervising Utilities Officer for Camp Fremont and Camp Kearney in California, with the rank of Major.

When released from the Army in 1919, Major Hays joined the engineering staff of Stone and Webster, Inc. (later Stone and Webster Engineering Corporation) in Boston, Mass. In this capacity, he was consulted on numerous hydroelectric developments and irrigation projects designed and constructed or investigated by that firm. Subsequently, he was placed in charge of engineering reports and, still later, of appraisals made by the organization, in order that his engineering and construction experience, broad understanding of economics, and business acumen could be made fully available to clients. During this period the volume of report and appraisal work done by the corporation greatly increased as did the proportion of work done in the rapidly-growing industrial field. The advice of Major Hays was sought on industrial problems because he was able to apply with accuracy the fundamental principles of engineering and economics which he had learned so thoroughly in the public utility field. During the long years of depression, at its depth in 1932, the corporation advised industries in their attempt to weather the economic storm, and Major Hays' sound business counsel was valued highly. In 1934 the Bridgeport (Conn.) Brass Company, then facing many business and engineering problems, asked him to serve first on the board of directors and then on its executive committee. When the business clouds showed signs of lifting, this company began a program of improvement and expansion of its manufacturing facilities, in which Major Hays played a prominent part. Thus, this company was able to take full advantage of the revival of business activity and was ready to respond to the demand for production of war materials with the beginning of World War II.

In 1929, Major Hays was appointed vice-president of Stone and Webster Engineering Corporation and in 1938 executive vice-president. He was a member of the American Society of Mechanical Engineers, American Institute of Electrical Engineers, Military Order of the World War, Bankers Club of America, Engineers Club of New York, Exchange Club of Boston, and San Francisco Golf and Country Club. He was an enthusiastic yachtsman and was a member of the Columbia Yacht Club of Port Washington, N. Y., and the Dauntless Club of Essex, Conn.

Major Hays is survived by his widow, the former Eva L. Harwood, to whom he was married in New York on December 2, 1908, and by his brother, Harry T. Hays.

Major Hays was elected an Associate Member of the American Society of Civil Engineers on May 4, 1909, and a Member on June 4, 1913.

WILLIAM HERZBERG, M. Am. Soc. C. E.[1]

Died June 4, 1943

William Herzberg was born at Gadsden, Ala., on January 10, 1890, the son of Annie Laurie (Denson) and William Herzberg. When he was nine years old, his family moved to Birmingham, Ala., where he finished grade and high schools. He attended the Sewanee Grammar School (later known as the Sewanee Military Academy) at Sewanee, Tenn., from 1907 to 1910.

After completing his studies at the university, Mr. Herzberg returned to Birmingham in the fall of 1910 and went to work for the Corey Land Company as a rodman on the development of Corey—a model village, later Fairfield, Ala., an industrial suburb of Birmingham. He continued on construction work with this firm and rose to the position of instrumentman. In June, 1911, he became associated with the firm of Meade and Huey, consulting engineers of Birmingham, in the capacity of chief of party on general engineering work in and around Birmingham.

From May, 1912, through June, 1915, Mr. Herzberg was county engineer of Noxubee County, Mississippi, building a county system of gravel highways. He returned to Birmingham in June, 1915, handling a number of Birmingham's earlier subdivisions for the Wheelock Engineering Company. He was connected with this firm through 1917.

Early in 1918, Mr. Herzberg was employed by the Tennessee Coal, Iron and Railroad Company as chief of party and construction engineer on the construction of the Westfield Plant, near Birmingham, on railroad and storm sewer construction. When the Westfield Plant was completed after World War I late in 1919, Mr. Herzberg became party chief on location in heavy country for the Georgia State Highway Department and later office engineer of the Fourth Division at Gainesville, Ga. He remained with the Georgia Highway Department until early in 1921.

---

[1] Memoir prepared by a Committee of the Alabama Section consisting of J. H. Mayer, M. Am. Soc. C. E., Chairman, C. J. Rogers, M. Am. Soc. C. E., and Theo Massey, Assoc. M. Am. Soc. C. E.

On April 1, 1921, he entered the employ of the Second Division of the Alabama Highway Department as resident engineer. He advanced steadily, from assistant division engineer to division engineer, then to associate highway commissioner, and finally to chief state construction engineer. This last position he held from 1939 until his death. About the time Mr. Herzberg was selected as chief construction engineer, the State of Alabama undertook its largest highway expansion program, which consisted of improving existing highways, widening narrow bridges, constructing large bridges over major streams, as well as locating, designing, and constructing four-lane highways as entrances to the larger cities in the state and building a number of access roads to Army camps, air bases, and war plants.

During his twenty-two years of service with the State of Alabama, Mr. Herzberg accomplished much. His sincerity of purpose was appreciated by his colleagues. His work was characterized by thoroughness, keen insight, and tremendous energy.

Mr. Herzberg was married on July 8, 1914, to Katherine Harrington, of Mobile, Ala., and is survived by his widow.

Mr. Herzberg was elected a Member of the American Society of Civil Engineers on December 21, 1942.

ROBERT LESLIE HOLMES, M. Am. Soc. C. E.[1]

Died April 26, 1943

Robert Leslie Holmes was born at Clinton, La., on September 8, 1880, the son of James Alexander and Martha Louise (White) Holmes. He received his elementary education at Clinton Male and Female Academy and studied civil engineering by correspondence.

In March, 1900, Mr. Holmes entered the service of the Texas and Pacific Railway Company as chainman on the construction of a branch line in Louisiana. He advanced through the grades of rodman and instrumentman during the progress of this work, and, on its completion in November, 1903, was transferred to maintenance-of-way duties. In September, 1906, he was promoted to assistant engineer on construction and maintenance of way and in August, 1911, was advanced to the position of division engineer at Marshall, Tex. In August, 1915, he was promoted to assistant chief engineer at Dallas, Tex., and held this position until May, 1916, at which time he was placed in charge of the development and maintenance of water supply for the entire railway, receiving the title of engineer of water supply.

---

[1] Memoir prepared by R. J. Gammie, M. Am. Soc. C. E., and J. G. Roney, Assoc. M. Am. Soc. C. E.

At that time the water supply of the Texas and Pacific Railway Company was inadequate to meet the needs, and the quality was poor. Realizing the great importance of this phase of railroad work, Mr. Holmes began to make a careful study of the problem and zealously devoted himself to the task of developing various sources of supply sufficient to meet the requirements of the railway and choosing the correct methods of treatment to provide water of the proper quality. In this he was successful, for his efforts resulted in producing a system of water facilities second to none, and at the time of his death there was not one water station on the railway that had not been newly constructed or completely rebuilt under his direction.

The largest single project for which Mr. Holmes was responsible was the selection of the site for a large storage reservoir in the Davis Mountains of Texas and, in connection therewith, the design, location, and construction of an earth dam and a 37-mile pipe line to carry the water to points of distribution.

Concurrently with this work, over a period of many years, Mr. Holmes constructed a system of bank protection at several points on the Red River, where it closely parallels the railway. This was done by the installation of a series of jetty boxes, of a type of his own devising, at threatened points of erosion, resulting in the halting of the inroads of the river and obviating the necessity for expensive relocations of the railway. Mr. Holmes was so successful in this type of work that he was frequently consulted by other railways, state highway departments, and foreign countries in connection with similar problems.

In 1920 another project of considerable magnitude was delegated to Mr. Holmes. At this time the railway changed from coal to oil as a locomotive fuel, and Mr. Holmes was charged with the task of designing and constructing facilities for the storage and dispensing of fuel oil, with all the intricacies of unloading devices, heating and pumping equipment, and pipe lines.

In 1916 Mr. Holmes was elected to membership in the American Railway Engineering Association (A.R.E.A.) and for many years served on the Water Service, Fire Protection and Sanitation Committee.

In 1927, at the time of the devastating floods caused by the breaking of the levees of the Mississippi River and some of its tributaries and outlets in Louisiana, Mr. Holmes took an active part in rescue work. His efforts were recognized by the A.R.E.A., which presented him with a scroll bearing the following testimonial:

> "In appreciation of distinguished and meritorious services rendered in rescue and relief work during the Mississippi Valley Flood of 1927, this token of appreciation is awarded to R. L. Holmes on behalf of the American Railway Engineering Association."

On May 26, 1909, Mr. Holmes was married to Minnie Porter, who died on December 16, 1909. On October 16, 1912, he was married to Shirley Wade. Mrs. Holmes, together with two daughters, Shirley (Mrs. K. W. Krueger) and Edith, and a son, Robert Leslie Holmes, Jr., survives him.

Mr. Holmes was by faith a Presbyterian. He was a member of Marshall Lodge No. 22, A. F. and A. M., and Tancred Commandry in Dallas.

Although his entire career was spent in the service of the Texas and Pacific Railway Company, Mr. Holmes enjoyed a wide acquaintance and numbered his friends by the score. He was held in greatest esteem by every one who knew him, from the highest executive on the railway to the lowliest laborer. He earned the respect not only of his superiors, but of those upon whom he relied for assistance as well. The efficient operation of his department was due in a large measure to the kind and considerate treatment he accorded his subordinates.

Mr. Holmes was elected a Junior of the American Society of Civil Engineers on October 1, 1907; an Associate Member on February 6, 1912; and a Member on October 12, 1921.

WINFRED DEAN HUBBARD, M. Am. Soc. C. E.[1]

Died August 13, 1943

Winfred Dean Hubbard was born at Concord, Mass., on January 18, 1877. He was the son of George D. and Elmina (Taylor) Hubbard of Concord whose ancestors were Colonial settlers of that region.

Mr. Hubbard was graduated from the Massachusetts Institute of Technology at Cambridge, Mass., in 1898. From then until October, 1899, he was employed by the late Leonard Metcalf,[2] M. Am. Soc. C. E., consulting engineer, on design and construction of sewers and water supplies for various cities, including Concord. In January, 1900, he was appointed Superintendent of Water Works and Sewers of Concord.

He served in this capacity until March, 1906, when he was appointed an assistant engineer of the Board of Water Supply of the City of New York on reservoir construction work at Brown's Station, N. Y., on various surveys and details. He held this position until August, 1917, when he became principal assistant engineer on the Croton-Catskill Division of the Department of Water Supply, Gas and Electricity for New York, N. Y., which had charge of the distribution of all water delivered to that city. He was appointed division engineer of the latter department in April, 1931, which position he held until his death on August 13, 1943.

His work was varied and consisted of topographical surveys and the relocation of the Ulster and Delaware Railroad, together with new alinements of highways, the location and construction of the Olive Bridge Dam and adjoining dikes, and the construction and operation of the headworks of the Catskill aqueduct, with final inspection and maintenance of all gate-houses and appurtenances included in this extensive system.

---

[1] Memoir prepared by Sidney K. Clapp, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1110.

His work included the design and maintenance, together with the operation, of many sewage disposal plants throughout the Catskill-Croton developments, and the extension of the Staten Island and Long Island water supplies. The large territory of nearly 1,000 sq miles which Mr. Hubbard supervised included the watersheds of the Schoharie Creek, Esopus Creek, Kensico Reservoir, and Croton River developments, with aqueducts, pipe lines, control gates, and other fixtures north of the city line at Yonkers, N. Y.

During Mr. Hubbard's tenure in office with the Board of Water Supply and the Department of Water Supply he was able to acquire a widely diversified and an intimate knowledge of the details and operations of the water supply of New York City using one billion gallons of water per day—practically the largest water supply system in the world. He was consulted frequently on matters pertaining to storage and distribution of other water supplies, and, due to his wide experience, this advice was of considerable value.

Mr. Hubbard was joint author with the late W. Kiersted, M. Am. Soc. C. E., of a book entitled "Water Works, Management and Maintenance." The book attained a wide circulation among civil engineers and others connected with water works.

For more than thirty-five years Mr. Hubbard was a resident of Kingston, N. Y., where he was widely known. He was a member of the St. Johns Episcopal Church and a member of Masonic Lodge of Concord.

Mr. Hubbard was married on June 4, 1901, to Edna Thatcher, who died in November, 1934. Surviving are a daughter, Mary Thatcher Hubbard; two brothers, Frank S., and Walter; and a sister, Florence Priscilla (Mrs. A. J. Underwood). Both Mr. Hubbard and his wife are buried in the Sleepy Hollow Cemetery at Concord.

Mr. Hubbard was elected an Associate Member of the American Society of Civil Engineers on November 5, 1902, and a Member on April 13, 1936. He became a Life Member in 1937.

CLARENCE WALTER HUDSON, M. Am. Soc. C. E.[1]

Died May 10, 1943

Clarence W. Hudson was born at Manasquan, N. J., on May 20, 1867, the son of James B. and Rachel M. (Longstreet) Hudson. On his mother's side Mr. Hudson's genealogy extended back to the Revolutionary War. He was a member of Pennsylvania Society of the Sons of the Revolution.

Mr. Hudson prepared for college at Freehold Institute, Freehold, N. J.; entered Lehigh University at Bethlehem, Pa., in 1885; and was graduated in 1889 with the degree of Civil Engineer. That he was a good student is evidenced by his election to the Tau Beta Pi fraternity. That he was later an exceptionally able and successful civil engineer is recorded in this memoir and is attested indisputably by the honorary degree of Doctor of Engineering bestowed on him by his alma mater in 1925.

---

[1] Memoir prepared by Emil Diebitsch, M. Am. Soc. C. E.

During the first year after his graduation Mr. Hudson was employed as instrumentman and draftsman on the U. S. Hydrographic surveys for the Third District, Mississippi River, but thereafter he devoted himself to either teaching the basic principles of engineering or applying these principles to the design and construction of bridges and other framed structures. For one year he was with the New Jersey Steel and Iron Works of Trenton, N. J., as draftsman and field engineer on bridge work. The next year he was loaned to Lehigh University, where he was instructor in civil engineering. Then he returned to the New Jersey Steel and Iron Works in charge of a squad of draftsmen on bridge work. Mr. Hudson's reputation as an engineer was growing, and soon the Phoenix Bridge Company of Phoenixville, Pa., secured his services and employed him for fourteen years as designing engineer, in which capacity he designed practically all types of steel bridges.

In 1907, after eighteen years of experience, Mr. Hudson formed a partnership with the late Mansfield Merriman,[2] M. Am. Soc. C. E., of Lehigh University, and they established an office in New York, N. Y., as consulting engineers. Mr. Merriman withdrew from the firm after one year, but Mr. Hudson continued the business until his retirement in 1933.

Mr. Hudson's engineering advice and opinion were in steady demand. He was consultant for the Delaware, Lackawanna and Western Railroad Company on the Tunkhannock Viaduct and the Martins Creek Viaduct. At that time (1908) the Tunkhannock structure was the largest concrete bridge in the world. For the Lehigh and New England Railroad Company, he was consultant on eleven bridges in the anthracite coal region; for the Galveston (Texas) Causeway, Mr. Hudson advised on a concrete bridge one mile long; for the Palisades Interstate Park Commission of New York and New Jersey, he passed on four bridges. He was consultant for private firms on two toll bridges over the Ohio River, one at Steubenville, Ohio, and the other at Pomeroy, Ohio.

Among his most important work was the design and supervision of construction on three outstanding bridges costing, respectively, \$3,000,000, \$3,800,000, and \$3,100,000. The first was the Hill to Hill Bridge at Bethlehem, which welded the three boroughs of Bethlehem, South Bethlehem, and West Bethlehem into the City of Bethlehem. This bridge is of concrete-arch design, with ramps of easy grade feeding the traffic from the low-level streets to the main structure. It stretches in a magnificient roadway over the Lehigh River from the hilltop at the Moravian Seminary in Bethlehem to Fountain Hill in South Bethlehem. The Bethlehem bridge is a masterpiece of engineering design, solving an important municipal problem economically. The bridge was a great asset to a greater Bethlehem. The second bridge, the Washington Bridge at Providence, R. I., also a concrete-arch bridge, faced with granite, was outstanding in its structural stability and architectural beauty. The third structure which Mr. Hudson designed and for which he supervised construction was the Victory Bridge at Perth Amboy, N. J., for the State of New Jersey. This bridge is of steel-girder type.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1658.

From September, 1909, until June, 1927, Mr. Hudson was professor of Civil Engineering and in charge of the Department of Civil Engineering at the Polytechnic Institute of Brooklyn (N. Y.). Despite the duties imposed by so responsible and important a position, Mr. Hudson found time to write and publish three technical books.[3,4,5]

In the business world, Mr. Hudson's acumen and sound conservative judgment were appreciated. He was elected a trustee of Lehigh University in 1924 and held that office until 1932. In 1920–1922 he was a director of the Society. From 1933 to the day of his death he was a member of the United Engineering Trustees, Inc., which holds in trust all the properties and endowments of the four national engineering societies—American Society of Civil Engineers, American Society of Mechanical Engineers, American Institute of Electrical Engineers, and American Institute of Mining and Metallurgical Engineers.

Mr. Hudson was proud of his profession and enjoyed social meetings with his fellow men and fellow engineers. He was a member of the Engineers Club of New York City, Providence Engineering Society, Bethlehem Club, and Upper Montclair (New Jersey) Country Club. His favorite diversions were golf and fishing.

He was married on June 4, 1903, to Elizabeth Morgan at Phoenixville. He is survived by his widow and one son, James Hudson.

Mr. Hudson was elected an Associate Member of the American Society of Civil Engineers on November 7, 1894, and a Member on February 7, 1900.

DAVID EDWARD HUGHES, M. Am. Soc. C. E.[1]

DIED NOVEMBER 19, 1942

David Edward Hughes was born at Palmyra, Ohio, on September 21, 1861, the son of Evan and Ann Johns Hughes. His youth was spent in that locality —attending school, farming for his widowed mother, and working in sandstone quarries. At the age of sixteen, he went to Dunigan in northern California to live with an uncle.

Blessed with mental capability given to few, Mr. Hughes led an intensely active life. He continued his self-education by devoting every spare moment to study, exhausting book after book on both medicine and engineering; teaching in country schools and at Pierce College, College City, Calif.; and, finally, although still in his early twenties, becoming a professor of mathematics.

---

[3] "Deflections and Statically Indeterminate Stresses," by Clarence W. Hudson, John Wiley & Sons, Inc., New York, N. Y., 1911.

[4] "Notes on Plate-Girder Design," by Clarence W. Hudson, John Wiley & Sons, Inc., New York, N. Y., 1911.

[5] "Elements of Graphic Statics," by Clarence W. Hudson and Edward J. Squire, McGraw-Hill Book Co., Inc., New York, N. Y., 1923.

[1] Memoir prepared by H. M. McOuat, of San Pedro, Calif.

During this period, the die was cast and for the remainder of his life he followed the profession of civil engineering, gaining eminence. He was to a remarkable degree self-effacing, never striving for gain or credit but always for accomplishment, truth, honesty, and simplicity. Mr. Hughes was, without doubt, one of the finest applied mathematicians of his generation. He evolved, calculated, and published a table on the perfect transition ("Sickle"[3]) curve, or American spiral, which was used on the construction of the Chicago elevated railroad, as well as on other projects. Six-place functions were developed for this spiral at 10-min intervals between 0° and 30° and at 30-min intervals to 90°.

The intense activity of his younger years exhausted Mr. Hughes physically, and he sought opportunity to apply his rare combination of high technical ability and sound common sense in the out of doors. Accordingly, he accepted employment with the U. S. Engineer Department in 1893, remaining with that service until his retirement in 1932.

His first work for the War Department was on the improvement of Humboldt Bay, California, stabilizing the entrance by jetty construction. In 1902 he was transferred to the newly-created Los Angeles (Calif.) Engineer District and, for the following thirty years, was its ranking civilian engineer. He built jetties and fortifications at San Diego, Calif., and breakwaters and fortifications for Los Angeles-Long Beach (Calif.) Harbor. For a year he served as engineer for a board appointed by President Taft to determine the site and the design for San Carlos Dam, later Coolidge Dam, in Arizona. His profound writings on silting, as applied to this project, brought him an offer of a professorship in a state university. His was the guiding hand in the establishment of federal harbor lines along the Southern California coast. These lines determined the development of the harbors.

In his thorough manner, Mr. Hughes investigated tideland law and ownership and became an authority on the subject. Typically, he urged and aided in the litigation which restored the local tidelands to the people and resulted in municipally owned harbors. He experimented and wrote on *surge* and *seiche*. He evolved the idea of substitution of waste dredgings for costly stone in the strengthening of breakwaters, and the vast chain at San Pedro, or Los Angeles, Harbor, built and building from this design is a monument to him.

With all of this vast activity, he remained simple, approachable, and a friend to all. He aided both individuals and communities in time of trouble. He gave unsparingly of his time and substance to help the unfortunate and the ambitious. Many an engineer has had complex and involved problems simplified by Mr. Hughes. Many a young engineer was guided along not only the right technical but also the right ethical and moral road by this kindly, unselfish man.

He hated sham; he hated hypocrisy; he hated pretense. He was unswervingly honest, mentally as well as morally. With his passing a rare individual has gone on—a kindly, unpretentious, disarming soul housing the intellect of a giant.

---

[3] "The Sickle, or Perfect Railway Curve," by David E. Hughes, *Transactions*, Technical Soc. of the Pacific Coast, San Francisco, Calif., April, 1892, pp. 63–81.

Mr. Hughes was married to Lydia Wiklund in 1913 at Florence, Ariz. He is survived by his widow and a brother, Thomas R. Hughes.

Mr. Hughes was elected a Member of the American Society of Civil Engineers on September 6, 1905.

WILLIAM ROBERT KALES, M. Am. Soc. C. E.[1]

Died December 3, 1942

William Robert Kales was born in Chicago, Ill., on August 8, 1870, the son of Francis Henry and Ellen Parker (Davis) Kales. He received his early education in Chicago and was a student at Phillips Exeter Academy at Exeter, N. H., for three years, being graduated with the class of 1889. He then attended Massachusetts Institute of Technology at Boston, for four years, being graduated with the degree of Bachelor of Science in Mechanical Engineering in 1892. He was a member of Sigma Chi fraternity and was president of his class during his senior year, a position he held for life.

After graduation, Mr. Kales was employed in various positions in Worcester, Mass., Chicago, and Milwaukee, Wis., from 1892 to 1895, and then went to Cleveland, Ohio, with the Brown Hoisting and Conveying Machine Company, where he remained until 1899. While in Cleveland, he designed one of the earliest grab buckets, which proved very successful.

On October 1, 1895, he was married to Alice Gray of Detroit, Mich., daughter of John S. Gray. Mr. Gray was a prominent businessman in Detroit and was one of Henry Ford's backers in the founding of the Ford Motor Company and the first president of Mr. Ford's company. Mrs. Kales, four children, and eleven grandchildren survive Mr. Kales.

In 1899 Mr. Kales moved from Cleveland to Detroit and, with James T. Whitehead, founded the Whitehead and Kales Iron Works. The little business prospered and, in 1905, was incorporated as The Whitehead and Kales Company, specializing in the fabrication and erection of structural steel. Mr. Kales served as vice-president and chief engineer during the lifetime of Mr. Whitehead, and, on the latter's death in 1930, he assumed the presidency, which office he held until his death. His company fabricated and erected all the steel for the mammoth Willow Run bomber plant of the Ford Motor Company, for the two great Wright Aeronautical Company plants near Cincinnati, Ohio, as well as for many other large war plants.

Mr. Kales was a brilliant and thorough engineer. He designed many of the early structures fabricated by his company, and much of the erection equipment, adding very considerably to the art of erecting steel. His company once received a contract for prison cell construction, only to find that patents fully covered all known types of acceptable cells. After many busy days and nights, Mr. Kales invented a new type of lock which not only circumvented all the patents, but was a great improvement. Long hours meant nothing to him, and, when there was a big job to do, he begrudged the necessary time to eat and sleep. He was also a tremendous reader and student, and, because of a naturally fine memory, he retained all that he read for years. Study was a pastime, and, while in his middle sixties, he reviewed his college calculus with obvious enjoyment.

---

[1] Memoir prepared by George H. Fenkell, Hon. M. Am. Soc. C. E., and Neil C. McMath, M. Am. Soc. C. E.

He was long a prominent member of the old Detroit Engineering Society, serving as president from April 23, 1909, to April 15, 1910. The year 1924 saw many honors come his way, for it was then that he joined the Society, was made a life member of the Corporation of Massachusetts Institute of Technology, and was appointed by the Mayor of Detroit a member of the City Plan Commission. He had been a member of the Public Lighting Commission of the City of Detroit from 1906 to 1912. He belonged to the American Society of Mechanical Engineers and the new Engineering Society of Detroit.

During the first World War, Mr. Kales had charge of the manufacture and erection of steel hangars for the U. S. Air Corps. He was commissioned a Captain in the Fourth Air Park in the First Pursuit Group of the Air Service and served in France with the Army of Occupation until September, 1919. Following the war, he became a reserve officer with the rank of Lieutenant Colonel in the Corps of Engineers.

He was an Episcopalian and, at the time of his death and for many years before, was a vestryman at St. Paul's Cathedral. In 1936 he received the honorary degree of Doctor of Laws from Wayne University of Detroit.

He was a lover of all kinds of outdoor life and was always fascinated with boats, especially sailing vessels and yachts. He took several trips in his youth on the old lumber schooners operating out of Chicago; in his later years he enjoyed sailboat races both in the Great Lakes and at his summer home on Cape Cod (Massachusetts). He was also an enthusiastic hunter, and, only ten days before his death, he returned from a successful deer hunt in northern Michigan.

During the forty years that Mr. Kales was a resident of Detroit, he became connected intimately with every phase of its financial and commercial life. He played an important part in the development of Detroit's factories, banks and trust companies, homes, and parks and playgrounds, and was an earnest advocate of many civic improvements. He considered the old-fashioned type of education the one best fitted to develop the reasoning powers of man. Greek, Latin, and mathematics to him were essential parts of such an education. He traveled to many lands, becoming acquainted with many different nationalities, which helped him to a better understanding of the people and problems of the world.

No memoir of Mr. Kales could be complete without emphasizing his deep love for his family, his home, and his many friends, and his enduring faith in the Christian religion.

Mr. Kales was elected a Member of the American Society of Civil Engineers on April 7, 1924.

DAVID ANDREW KEEFE, M. Am. Soc. C. E.[1]

DIED MAY 17, 1943

David Andrew Keefe was born in Athens, Pa., on January 28, 1869, the son of Mary (Pyne) and Marcus Keefe, the second generation of sturdy stock from Cork, Ireland. With the exception of seven years, from 1888 to 1895, he spent his entire life in Athens. He was married to Clara Angela Wingerter of Wheeling, W. Va., on September 23, 1913. They had two children, a son and a daughter. A brief outline of Mr. Keefe's career appears in *Who's Who in America,* 1925 edition.

His elementary schooling was in the public schools of Athens, and his high school training in the Athens Academy. His technical and business education was obtained in night schools, the American School of Correspondence, and home study. He was a consistent and ardent student all his days, keeping up to date on the latest developments and applying them.

At the age of sixteen he worked in the shops of Kellog and Maurice (later the Union Bridge Company) in Athens. In 1888 he was employed by the Wisconsin Bridge and Iron Company as shop foreman, later serving as draftsman and designer. After three years with this firm, during which time he took technical courses in night school and under private tutors, he went with R. W. Hildreth and Company and, for four years, was on the 155th Street Viaduct and McCombs Dam Bridge in New York, N. Y.

On the completion of his New York assignment he returned to Athens where he practiced consulting engineering, confining his efforts to the design and construction of bridges, piers, and industrial plants; reports; and structural steel inspection. He was employed by Bradford County, Pennsylvania, as bridge engineer, by Luzerne and Carbon counties (Pennsylvania), and by the State of Pennsylvania in connection with the design and construction of twelve bridges over the Susquehanna River. He was retained by Yucatan, Central America, in connection with the ocean pier at Progresso. He was consultant in connection with bridges for the Lehigh Valley, the Scranton and Binghamton, and the Lehigh and New England railroad companies. The reconstruction of the Steel Pier and the construction of the Garden City Pier at Atlantic City, N. J., are examples of his work as are certain plants of the Union Carbide Company, the Ingersoll-Rand Company, and the Hudson Coal Company colliery at Scranton, Pa. Various towns and commercial interests in the Lehigh and Susquehanna valleys retained him to report on facilities. During his lifetime he was probably the most distinguished engineer of the Susquehanna River Valley.

Early in his life Mr. Keefe became interested in finance. His judgment in financial matters was so uniformly sound that, at the age of twenty seven, he was elected a director of the Athens National Bank and, in 1927, was made president of the Merchants and Mechanics National Bank in Sayre, Pa., resigning from the latter position in order to give more time to civic and personal affairs.

---

[1] **Memoir prepared by F. O. Dufour, M. Am. Soc. C. E.**

His hobbies were golf, travel, motoring, and civic matters. Mr. and Mrs. Keefe made a wedding trip around the world. Also, he took several cruises to the Caribbean and the Mediterranean and went to Ireland to see the old family homestead, still occupied by relatives.

Head of the tire rationing board and active in Red Cross, the Sayre Hospital, the Athens Library, county politics, and town improvements, he took his civic and social duties seriously as well as enthusiastically. He was a member and on the Board of Governors of the Shepards Hill Country Club in Sayre. He belonged to the Rotary Club, the Pennsylvania Society, and the Knights of Columbus. It may well be said of Mr. Keefe that he was an ideal engineer, citizen, and man.

Mr. Keefe was elected a Member of the American Society of Civil Engineers on December 3, 1913.

HARRY RAYMOND LEACH, M. Am. Soc. C. E.[1]

Died June 24, 1941

Harry Raymond Leach was born on July 14, 1891, in Saginaw, Mich., the son of Henry Morris and Kitty (Wilder) Leach. His engineering career began in 1909 when, at the age of eighteen, he went to work as rodman and instrumentman on surveys of the U. S. Indian Irrigation Service in the Pacific Coast states. After three years of field experience he entered the University of Michigan at Ann Arbor, from which he was graduated in 1916 with a degree in civil engineering. He remained in Ann Arbor, assisting H. W. King, M. Am. Soc. C. E., in the preparation of the *Handbook of Hydraulics* until January, 1917, when he was employed by Robert E. Horton, M. Am. Soc. C. E., consulting engineer, of Albany, N. Y.

In 1917, the United States entered the world conflict, and Mr. Leach returned to his home in Saginaw early in 1918 to offer his services to his country. While waiting to join the Army, he worked in the office of the District Highway Engineer. In September, 1918, he entered the Signal Corps of the U. S. Army. In the Meteorological Section of the Signal Corps he was trained in the technique of upper-air sounding and later was placed in charge of Meteorological Detachments at Jefferson Barracks, Missouri, and Camp Bragg, North Carolina.

In the spring of 1919, Mr. Leach returned to civil life and undertook a study of floods in Saginaw County, Michigan. In the summer of the same year he was retained by the Indian Irrigation Service to conduct a study of storage possibilities in the watersheds of the Uinta, White Rock, and Lakefork rivers in Utah—an investigation that required arduous explorations on horseback through the most rugged parts of the Uinta Mountains. When this work was completed, he conducted a four-month investigation of flood flows in the Saginaw River basin. In 1920 he returned to the Indian Irrigation Service, remaining with that organization until August, 1921, when he again became associated with Mr. Horton. The following thirteen years constituted one of the most productive periods of his life. As principal assistant to Mr. Horton, Mr. Leach participated, with distinction, in notable hydraulic and hydrologic investigations and in the design and construction of a number of water supply and hydroelectric projects.

---

[1] Memoir prepared by Howard L. Cook, M. Am. Soc. C. E.

In 1934 he was employed by the State of New Hampshire to supervise the establishment of recreational centers. For a short period, he was associated with an industrial concern. In December, 1935, he became affiliated with the Soil Conservation Service of the U. S. Department of Agriculture and thus was enabled to devote his full time to research in hydrology and hydraulics. In this connection he had hoped to bring into an orderly array the knowledge he had accumulated during the busy years of his professional practice, to fortify this with research, and thus to contribute a number of technical writings to engineering science. He had long contemplated such an undertaking but had hesitated to proceed without that thorough preparation demanded by his exacting personal standards. A long illness and his untimely death on June 24, 1941, prevented attainment of this goal. He was buried in Arlington National Cemetery in Fort Myer, Va.

In December, 1919, he was married to Helen Knudsen in Myton, Utah. His son, Harry King, and his daughter, Helen Karen, were a source of constant refreshment to him. He was deeply grateful to see them reach young manhood and womanhood before his death.

So wholeheartedly did Harry Leach devote himself to his daily tasks that he found little time for technical writing. This, coupled with the exceptionally high standards he set for his work, led him to defer the preparation of papers on many subjects in which he was intensely interested. Fortunately, he did make available to his fellow engineers the very useful procedure for calculating backwater curves that has come to be known as the "Leach Method." During his last illness, under a heavy handicap, he contributed to the volume, "Hydrology," in the Physics of the Earth Series sponsored by the National Research Council. These studies and the few other short papers he managed to prepare during his busiest years do not measure his capacity fairly; but they do indicate the magnitude of the loss suffered by the engineering profession with his death.

During his lifetime Harry Leach worked with many engineers throughout the United States and, to an unusual degree, was afforded opportunities to collaborate with workers in other fields of science. Regardless of the background of his co-workers, he invariably won their respect and affection. They found that he cheerfully carried more than his share of any task and willingly made greater sacrifices than were demanded of him. Moreover, they learned to place a high value on his opinions, not only because of his broad knowledge of engineering principles and methods, but even more because of his sound judgment—for, although engineers of equal erudition are not uncommon, one who possesses so great a store of common sense is not often found. Thus his fellow workers came to depend on him as does a workman on a tried and proved tool, only occasionally appreciating the full magnitude of their reliance.

His professional associates were not alone in cherishing his friendship, for he made true friends in all walks of life. Those who were privileged to know him loved him for his humanity, kindliness, and tolerance; admired his contempt of hypocrisy and superficiality; and sought his companionship because of his congeniality, his gruff yet kindly humor, his constant loyalty, and his quickness to appreciate the good work of others. He numbered among his friends poor men, rich men, laborers, executives, artists, industrialists, and, in fact, people in every circle into which his wide interests brought him. It is notable that so many who knew him have said, "I never knew a man who didn't like Harry Leach."

Mr. Leach was elected a Junior of the American Society of Civil Engineers on August 31, 1915; an Associate Member on May 28, 1923; and a Member on March 10, 1930.

LLOYD TEVIS McAFEE, M. Am. Soc. C. E.[1]

DIED JANUARY 1, 1942

Lloyd Tevis McAfee was born in San Francisco, Calif., on June 27, 1881, the elder son of Clark William and Elizabeth (Cooke) McAfee. His parents came to California from Louisville, Ky. Mr. McAfee attended the public schools of San Francisco and Belmont School (private) in San Mateo County.

At the age of twenty he began his engineering career, following the hard path of a young engineer without the advantage of college training. From 1901 to 1905 he was engaged progressively from rodman to chief of party on railroad surveys in Shasta County and on proposed hydroelectric projects in Kings County; on surveys for Western Pacific Railroad Company in Nevada and California; and on construction of transmission lines, ditches, tunnels, and flumes for the North Mountain Power Company in Trinity County.

He had a wonderful personality, friendly and genial, with vision, humor, and ability finely balanced. On the job he was known fondly as "Mac" by everybody from the laborer to the "Chief." He had the faculty of inspiring confidence in his ability and fairness, earning and holding the respect of his men and the contractors. He had a great love for the out of doors and a faculty for getting things done. All these qualities naturally led him into the construction field.

---

[1] Memoir prepared by N. A. Eckart and L. W. Stocker, Members, Am. Soc. C. E.

In 1905 he took over as construction engineer for the Ocean Shore Railroad Company in Santa Cruz and San Mateo counties. He often referred to the four years on this work until 1909 as happy and busy ones.

After the Ocean Shore Railroad was completed, Mr. McAfee entered the service of the City of San Francisco and spent the next nine years supervising various projects for the city, rebuilding its utilities and facilities after the disastrous fire of 1906. As assistant city engineer of San Francisco, he was engaged in construction of cisterns for the city's auxiliary high-pressure fire-protection system, storm and sanitary sewers, the Fort Mason pumping station, the high-pressure pipe system, and the Stockton Street and Twin Peaks tunnels.

In 1918 Mr. McAfee was appointed construction engineer on the O'Shaughnessy Dam for the Hetch Hetchy Water Supply, with which project he was to be identified intimately and actively for the remaining years of his life. To appreciate the type of work which he directed, a brief outline of this project should be included.

In 1913 Congress approved a grant of lands in the high Sierra to the City of San Francisco to permit development of an adequate water supply. With that authority to use public lands, the city has constructed a great system by which water is impounded and stored in reservoirs in the mountains and carried 137 miles through mountain tunnels and steel pipe to Crystal Springs reservoir on the San Francisco peninsula. The principal units for the impounding and conveyance of this water and for the incidental generation of electric energy are a standard-gage railroad, 68 miles long, to transport construction materials; a construction power plant of 3,300-kva capacity; a storage dam at Lake Eleanor; a larger storage dam at Hetch Hetchy, giving a reservoir capacity of 360,000 acre-ft; the aqueduct system, 137 miles long; a regulatory reservoir with penstocks; and an 80,000-kw power plant at Moccasin Creek; as well as a transmission line, 98 miles long. Expenditures on the Hetch Hetchy Project have amounted to more than $100,000,000. Mr. McAfee, as construction engineer until 1930, had direct charge in the field of several divisions of the project, embracing a wide range of types of construction.

On March 3, 1930, Mr. McAfee was promoted to the position of chief assistant city engineer (under the late M. M. O'Shaughnessy,[2] M. Am. Soc. C. E., then city engineer) with general supervision of all Hetch Hetchy work in the headquarters office and in the field, from the mountains to the sea. In 1932 he became manager and chief engineer of the Hetch Hetchy Water Supply, serving also as chief of the Bureau of Engineering of the newly created Public Utilities Commission. Mr. O'Shaughnessy then was appointed consulting engineer to the commission, which post he held until his death in 1934. In this capacity, the design and construction work of the San Francisco Airport, and later the construction of the $4,300,000 Exposition building program on Treasure Island, came under Mr. McAfee's jurisdiction. Until 1940 the work under his supervision excluded responsibility for operation of the electric power division of the Hetch Hetchy Project, but, in that year, his authority was extended by his appointment as manager and chief engineer of the combined Hetch Hetchy Water Supply, Power and Utilities Engineering Bureau. Thus, he assumed the management of the construction and operation of the Hetch Hetchy Power Project and the supervision of engineering work for the municipal railway.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1710.

Mr. McAfee's engineering career consisted for the most part in planning and building public works which were vital to the welfare of the community which he served.

He was married on July 2, 1907, to Emilie K. Meckel, who, with their three children, Katharine (Mrs. Daniel J. Roche), Harriet (Mrs. Victor Wilson), and Dorothy H. McAfee, survives him.

Mr. McAfee was elected a Member of the American Society of Civil Engineers on July 14, 1930.

CHARLES T. MAIN, M. Am. Soc. C. E.[1]

Died March 6, 1943

Charles T. Main was an engineer by profession. He liked the definition that a professional man is a man trained to service. Men loved him and trusted him. His outlook on life was simple and broad. His standards of conduct and ethics were high and rigidly applied to himself. When judging others, however, his great soul always remembered charity. He lived long and usefully.

Charles Thomas Main was born in Marblehead, Mass., on February 16, 1856, the son of Thomas Main, Jr., and Cordelia Green (Reed) Main. He was descended from Colonial stock. In 1883, he was married to Elizabeth F. Appleton. They had three children, Charles Reed, Alice Appleton, and Theodore.

Mr. Main attended the schools of Marblehead and then was graduated from Massachusetts Institute of Technology in Boston, in 1876. He taught at "Tech" for two years while taking advanced work. He gave his alma mater years of service as president of the Alumni Association, term and life member of the Corporation, and member of the Executive Committee. Also, he helped select several of her presidents.

He followed the principles of his forebears by working hard and long. One could not escape the impression that he worked without apparent effort, which may account for his long and useful life. Mr. Main did much original thinking on technical subjects, especially seeking underlying principles. The study of values relating to industrial subjects interested him, and he brought to this research the trained mind of the engineer and the dearly-bought wisdom of his ancestry.

---

[1] Memoir prepared by F. M. Gunby, M. Am. Soc. C. E.

Starting in 1891, he presented papers before several engineering and technical societies on this subject. These papers—on supply and cost of power, valuation of manufacturing properties, taxation, and other subjects vital to industry—were so clear and simple that they are still basic. In 1924 he presented a paper on power supply for industry before the First World Power Conference in London, England.[2] The method of valuing water power, by comparing costs of methods, which has been in common use for many years, was introduced by Mr. Main.

As a result of this work he received medals from Boston Society of Civil Engineers, National Association of Cotton Manufacturers, and American Society of Mechanical Engineers. Northeastern University in Boston, Mass., conferred on him the honorary degree of Doctor of Engineering. However, the quality of his work was more important to him than the reward.

After leaving Massachusetts Institute of Technology he spent twelve years in the mills at Manchester, N. H., and at Lawrence, Mass. At the age of twenty five, he was employed by the Lower Pacific Mills, designing and constructing mills and water power and steam power plants, the operation of which he later superintended. The effects of this experience were reflected throughout his subsequent work, giving him an unusually broad viewpoint on both engineering and manufacturing. He was adept at learning from books and from life.

After 1893, he specialized in consulting engineering—as a consultant and as a designer and supervisor of construction. He was charitable to other men, seeking their points of view. This contributed to his greatness as a consultant. In 1926, with associates, he incorporated the nationally-known business, bearing his name.

As a result of his own experience, his work and business fortunately covered a wide field. At first the basic interest of the firm was textile mills. By the time this field was slowing up, other work had been developed so that, when the textiles became almost negligible, other lines of industry—steam power, hydroelectric plants, etc.—were available.

Among the more important projects for which he acted as engineer were the Wood Worsted Mills, which, when built, was probably the largest single manufacturing building in the United States; water power and steam power and other plants for many manufacturing establishments; sugar refining, soap manufacturing, rope and jute plants; a system of hydroelectric plants for the Montana Power Company, totaling about a quarter of a million kilowatts; hydroelectric plant in Belgian Congo; manufacturing plants in Canada, India, and the Argentine; the Boston Parcel Post Building; and the foundations for buildings of Massachusetts Institute of Technology. During World War I his firm acted as architect-engineer on work for Camp Edwards, Cape Cod, Massachusetts; the Mississippi Ordnance Plant and Camp McCain in Mississippi; and the Holston Ordnance Works at Kingsport, Tenn. He often acted as consultant, expert witness, or arbitrator.

Mr. Main's philosophy about his business was simple:

---

[2] "Power for Textile Mills," by Charles T. Main, *Transactions*, The First World Power Conference, London, 1924.

> "Get the facts. Give to the client value received and more where possible. Pay employees all you can afford and put it in the pay envelope without frills. Recognize good work on the part of individuals. Develop young men. Give the client your best advice. Then, if he wants something else, help him to get it. If he wants something unsafe, he should go elsewhere. Be fair."

A conservative, he believed in keeping ample reserves from good times to tide over depressions. As a result he lived to send out cards in January, 1943, in commemoration of the fiftieth anniversary of the founding of his business.

Mr. Main felt his responsibilities as a citizen and was active in civic life. While in Lawrence, he and a group of outstanding men where drafted to run for office and clear up a political mess. For three years he was an alderman. In Winchester, Mass., his home, he was a member and chairman of the Water and Sewer Board for years. For the State of Massachusetts, he served on various commissions. He was a director of a savings bank and regularly attended its meetings.

A member of the Congregational Church, he did a great deal of work for the church but would never hold office, believing that he could be more effective in other ways. He performed various duties, among them heading committees to select a pastor and to build a parish house. He was a generous, but always anonymous, giver to charity, both in money and time.

A member of many worth-while social and scientific organizations, he was life member, honorary member, and president of the American Society of Mechanical Engineers; president and honorary member of the Boston Society of Civil Engineers; president and honorary member of the Engineers Club of Boston, which he helped to organize; and president of the American Institute of Consulting Engineers. At the time of his death he was the oldest member of the National Association of Cotton Manufacturers.

Mr. Main always regarded engineering as a profession which must be on a high ethical plane. When president of the Boston Society of Civil Engineers, he drafted the first code of ethics adopted by any engineering society in the United States. He also served on committees for codes of ethics for various engineering societies.

During World War I, he was adviser to the Construction Division of the Army, giving freely of his time. He refused to have his firm considered for engineering work because one of his young associates was in the Construction Division. The Secretary of War appointed him to several boards, principally in connection with construction matters. At the request of the French Government, the American Engineering Delegation, of which he was a member, visited France in 1919 to advise in connection with rehabilitation.

In World War II, his organization has been called on to do a large amount of important work which was a matter of particular satisfaction to Mr. Main. Although he could take no active part, he could look back on the broad problems of the previous war and give priceless advice.

As a man, he had stability of character and calm courage. He was gentle with the gentleness of great strength and poise. He was tactful, patient, and livable. He had imagination and a keen sense of humor and usually had a good story to make a point clear or to relieve a tense situation.

His pastor and friend said at his funeral

> "He was simple and modest, as all great souls are. He had the sunniest, winsomest, sweetest smile of any man I ever knew. Anyone who saw the playful quirk at the corners of his mouth and the kindling of the twinkle in his eye will never forget them. He was a landmark in our lives, like some tall tree on a hill-top. Now he is gone, rich in honors and ripe in years."

Mr. Main was elected a Member of the American Society of Civil Engineers on July 1, 1909.

JOHN WALTER MAY, M. Am. Soc. C. E.[1]

DIED AUGUST 8, 1942

John Walter May was born in Philadelphia, Pa., on August 18, 1884, the son of Frederick May and Mary (Rapp) May. His education was received in the public schools and later through extension courses.

The early years of Mr. May's professional life, 1900 to 1913, were spent with Albertson and Young Company and Lane Construction Company in the vicinity of Atlantic City, N. J., which was then expanding at a rapid rate. This work was highly diversified including, in addition to general building and equipment, the design and construction of water supply and sewage disposal systems, piers, sea walls, and reinforced concrete foundations above and below water. The greater number of these projects were important and gave Mr. May sound experience that proved to be of inestimable value later.

In 1916 he was employed by the George B. Clapp organization of Philadelphia as manager of construction. Here he continued to be interested in reinforced concrete. In this connection his most important work was on the construction of foundations and other structures for a five-and-one-half-mile extension of the Philadelphia Elevated System.

In 1918 Mr. May joined Day and Zimmermann Engineering and Construction Company (later United Engineers and Constructors, Inc.) first as construction supervisor and then as construction manager. The national scope of this company's activities, together with the great variety of work done, presented problems ranging from the normal to the extremely difficult.

In the industrial and general building field, Mr. May directed some thirty projects of different kinds. Among these were textile, cement, cold storage, coal cleaning, paper, and asphalt plants; colleges; hospitals; general service buildings; railroad stations; monumental buildings; and a penitentiary. However, his greatest activities and accomplishments were in the public utility field, supervising the construction of numerous steam power plants, transmission lines, substations, several hydro additions, and the revamping of 300 miles of high-speed electric railway. In all, he was connected with fifty three such projects, several of which are worthy of special note either for their pioneering or for nationally recognized aspects.

---

[1] Memoir prepared by Wallace E. Belcher, M. Am. Soc. C. E., and Rossiter W. Wilbraham, Chief Electrical Engr., United-Engrs. and Constructors, Inc., Philadelphia, Pa.

The 220,000-volt Conowingo transmission line involved an unusual river crossing with difficult cofferdam and foundation conditions. The construction details of the narrow-waisted transmission tower, which was originated for this job, and the methods and provisions for field assembly were greatly influenced by Mr. May. This tower revolutionized heavy transmission line design and construction. It was a definite contribution to the industry and has been used on every major project since then. The original model of this tower has been placed in the Franklin Institute at its request.

Revamping the 300 miles of the Ohio Electric Railway high-speed system presented the problem of changing the power supply from 25 cycles to 60 cycles, and required the building of new substations and the replacement of outmoded equipment in all others. The overhead transmission line and trolley systems had to be modernized and generally reconditioned, and, in the adjacent towns and villages that were supplied with light and power, all customers' meters and utilization devices had to be changed or readapted. Railway schedules and good customer relations had to be maintained under all conditions. The successful completion of this project was a tribute to both the ability for organization and to the good judgment of Mr. May.

The outstanding accomplishment of his professional life was the construction of the Deepwater Submarine Cable Crossing of the Delaware River in the vicinity of Wilmington, Del. Eight 66,000-volt submarine cables were laid in a dredged trench 10 ft deep and 20 ft wide at a maximum depth of 50 ft below mean low water. The length was 3,700 ft and the excavation, approximately 160,000 cu yd. Although it was necessary to maintain constant river traffic, the work was done continuously throughout a severe winter. The cable—the largest single length that could be manufactured—was but slightly greater than the distance across. Thus, the cable had to be laid to a degree of straightness in the trench that never before had been necessary or attempted. To do this, Mr. May developed methods of control for the dredges, maintained the cable-laying barge on a transit line, and laid all eight cables in the trench with one crossing of the river. Equally as important as the cable operation was the development of the first slotted-island-type cable terminal platform which allowed the cables to be terminated offshore. This was a pioneer project in every way and received national recognition. These design and construction methods made possible other similar installations on which Mr. May's construction methods were adopted.

Mr. May was an unusual man in many ways. He was of sterling character with a striking personality and sense of tolerance that won for him many sincere friends. His sound engineering judgment, organizational ability, and leadership commanded the respect of associates and employees as well as of contractors and clients. Mr. May successfully demonstrated the power of accomplishment by strong personal leadership, commencing at a time when "hard driving" was the general practice.

He was a member of the Engineers Club of Philadelphia and the Aronomink Country Club. He was always an active Roman Catholic.

In 1906 Mr. May was married to Mary T. Lane. He is survived by his widow; one son, John; and two daughters, Florence Claire (Mrs. Arthur Bender) and Mary Lane (Mrs. Hugh Clark).

Mr. May was elected a Member of the American Society of Civil Engineers on August 17, 1931.

JAMES LAFAYETTE PARKER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 10, 1942

James Lafayette Parker was born in Fayetteville, N. C., on April 6, 1881, the son of James F. and Elizabeth (Cromartie) Parker. He attended the North Carolina State College at Raleigh, and was graduated in 1902 with the degree of Bachelor in Engineering.

After graduation Mr. Parker was employed, for about four years, by various steel companies, obtaining valuable experience in structural detailing and designing. From 1906 to 1916, he was associated with Herbert C. Keith, M. Am. Soc. C. E., in New York, N. Y., as principal assistant engineer in charge of the design and preparation of plans for various kinds of bridges, including movable spans. After this he spent two years with the New York Central Railroad Company on preliminary designs for a high level bridge over the Hudson River near Albany, N. Y., to cost about $5,000,000. Also, he prepared design plans for a four-track lift span.

From August, 1918, to December, 1918, he served as Captain in the U. S. Corps of Engineers. After the war for a year and a half, Mr. Parker was with the Bureau of Public Roads (later Public Roads Administration) as bridge engineer for the Eighth District, with headquarters in Montgomery, Ala. Following this, he served as bridge engineer for the South Carolina State Highway Department in charge of the design and construction of bridges, costing several million dollars. He was the designer of the $550,000 Ashley River Bridge and directed construction on the $850,000 Santee River Bridge.

From 1924 to 1935, Mr. Parker was in private practice as consulting engineer at Charleston, S. C., specializing in the design and construction of road and bridge projects. In March, 1936, he was reinstated in the Public Roads Administration at Washington, D. C., as highway bridge engineer. Later he was promoted to senior highway bridge engineer, continuing in this capacity until his death.

Mr. Parker designed a proposed highway bridge across Chesapeake Bay near Baltimore, Md., and spent considerable time attempting to develop the project. Immediately before the outbreak of the second World War, there was renewed interest in this project from various sources.

---

[1] Memoir prepared by Nathan Wilson Morgan, M. Am. Soc. C. E.

An engineer of outstanding ability, Mr. Parker was thoroughly competent in the design of all types of bridge superstructures and foundations. He was an accurate analyst and possessed keen ability in sensing the most logical solution to bridge engineering problems. His thoroughness and insight often led him to discover defects in designs, particularly the failure to consider the action of certain natural forces acting upon the structure. Thus, his work was always thorough and never lacked the principle of conservatism. His associates found him a man of retiring disposition, forceful in his opinions, yet friendly and congenial.

Mr. Parker never was married. He was devoted to his family and made his home with his brother in Hyattsville, Md. He is survived by this brother, James F. Parker, and by a sister, Eunice (Mrs. Eunice Parker Chesnutt). He was a member of the Antioch Methodist Church of Parkersburg, N. C., which town was named for Mr. Parker's grandfather.

Mr. Parker was elected a Junior of the American Society of Civil Engineers on April 2, 1907; an Associate Member on May 3, 1910; and a Member on November 21, 1921.

HARRY TINKER POE, JR., M. Am. Soc. C. E.[1]

Died November 21, 1942

Harry Tinker Poe, Jr., the son of Hal T. and Dora (Sloan) Poe was born in Greenville, S. C., on November 15, 1882. He was educated at Clemson College, at Clemson, S. C., where he received in 1902 a Bachelor of Science degree in Electrical and Mechanical Engineering, and in 1903 a degree in Civil and Textile Engineering.

Although most of his life was spent in and around Greenville, prior to 1917 he was in responsible charge of projects in Texas, Pennsylvania, Georgia, Alabama, Tennessee, and Maryland. For a time he was a marketing specialist in Washington, D. C. From 1921 through 1933, he was president of Poe Construction Company of Greenville, a firm engaged in construction of water and sewer systems, roads, streets, etc., as well as in engineering, design, and construction of industrial work.

From 1934 through 1937 he was state traveling engineer; inspector for South Carolina; project engineer for Meeting Street Housing Project, Charleston, S. C.; and senior engineer, Power Division, Public Works Administration, Washington, D. C. From 1938 through 1940, Mr. Poe was principal engineer on the Santee-Cooper Power and Navigation Project in Charleston. At the time of his death he was employed by J. E. Sirrine and Company, Engineers, of Greenville, and was serving as resident engineer on a Greenville housing project.

During World War I, Mr. Poe was commissioned Major, Construction Department, Quartermaster Corps.

---

[1] Memoir prepared by J. E. Sirrine and William S. Lindsay, Members, Am. Soc. C. E.

In 1913, Mr. Poe was married to Julia May Swift. Five children were born to them—Hal T., Frank Swift, William Nelson, Ann, and Nell.

Both in his work and in his personal life, Mr. Poe's efficiency and cheerful disposition won him many friends and admirers.

Mr. Poe was elected an Associate Member of the American Society of Civil Engineers on February 5, 1908, and a Member on November 13, 1939.

WALTER PERCIVAL RICE, M. Am. Soc. C. E.[1]

Died August 21, 1941

Walter Pervical Rice was born on September 2, 1855, in Cleveland, Ohio, the son of Percy W. and Mary T. (Cutter) Rice and a grandson of the Hon. Harvey Rice, of Conway, Mass., who was a descendent of Edmund Rice of Burkhamstead, England. On September 2, 1903, he was married to Margaret Anderson Barteau.

Mr. Rice attended Lehigh University in Bethlehem, Pa., being graduated in 1876 with the degree of Civil Engineer. His practical experience, which began soon after his graduation, was so varied in character that it was distinctly instrumental in developing a very able engineer. In addition to his educational training, diversified experience on important projects did much to further his career.

Mr. Rice's professional work began and was continued during the period before civil engineering had reached the specialized stage which it later attained. A successful engineer in this period consequently was required to do much original study and was forced to draw upon his own ability and knowledge when developing design and construction methods for any important work, since descriptions of similar work were often not available. This situation was characteristic of the period and probably explains the ability of the successful pioneer engineers of the United States in overcoming obstacles and proving themselves capable men. Mr. Rice was definitely a pioneer. Through associations he both gave and received the benefits of valuable experience. He developed a quality of leadership greatly valued and appreciated by his associates and assistants, who all looked upon him as a master in his profession. Illustrative of the engineering work which received Mr. Rice's attention and direction was that on rivers and harbors prior to 1887 as an assistant to Col. John M. Wilson, federal officer in charge of waterway improvements along Lake Erie from Dunkirk, N. Y., to Detroit, Mich.

He was City Engineer of Cleveland from 1887 to 1890, and again from 1893 to 1895. From 1899 to 1901, he was Director of Public Works of Cleveland. During this time he directed many important surveys, road construction, bridges, sewers, river and harbor work, and other similar projects naturally accompanying engineering progress in a large growing city. Every important project under his direction was given his personal attention, and he took a special delight in working out both the general plans and many of the important details. He was assistant engineer on the Superior Street Viaduct in Cleveland—one of the large and important bridges of its time, completed in 1879 under the general direction of the late B. F. Morse,[3] M. Am. Soc. C. E., then City Engineer. (The viaduct, about 3,500 ft long, was replaced by a more modern structure in 1916.) It was a swing bridge over the Cuyahoga River with a westerly approach of stone arches and an easterly approach of steel and iron construction. During Mr. Rice's second term as City Engineer he sponsored and, in 1895, built the Columbus Road Bridge, a double-span swing bridge with approaches extending over a bend in the Cuyahoga River. This bridge, said to be the only one of its kind in the United States, was built to widen the channel and to facilitate both roadway and river traffic. In 1940, a major river improvement necessitated the removal of this bridge to permit the erection of a modern lift bridge of larger span and roadway width.

---

[1] Memoir prepared by Robert Hoffmann, Hon. M. Am. Soc. C. E.

At various times when not holding public office, Mr. Rice was in private practice under the firm name of Walter P. Rice Engineering Company. Among the projects on which Mr. Rice was engaged were a three-hinged concrete arch at Greenville, Ohio; a large masonry arch at Wheeling, W. Va.; a sea wall on Lake Erie (in which he introduced the use of submerged groins); and, in collaboration with the late F. C. Osborn,[4] M. Am. Soc. C. E., the steel arch bridge between Brooklyn and Brighton in the suburbs of Cleveland. The steel bridge has been replaced as part of a general improvement. He also served on special commissions and gave expert testimony in many courts.

Although busy with engineering work, Mr. Rice found time to participate in societies and organizations devoted to promoting professional affairs. He was a man of many activities and a student of many subjects. His interests and studies developed not only an engineer of high standing but a many-sided professional man who reflected credit upon the profession. He was a charter member of the Cleveland Civil Engineers' Club and an honorary member of the Cleveland Engineering Society (successor to the Civil Engineers' Club). In addition he belonged to the Permanent International Association of Navigation Congresses and to the Lehigh Club of Northern Ohio.

Always interested in "Cleveland's" early history and growth, at the time of his death he was president of Cleveland's "Early Settlers Association of the Western Reserve." For recreation, he enjoyed hunting, fishing, and yachting. He was an Episcopalian. In politics, he was a Democrat.

Mr. Rice was elected a Member of the American Society of Civil Engineers on March 6, 1889.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1884.

[4] For memoir, *ibid.*, Vol. LXXXVI (1923), p. 1687.

JAMES WALTER RICKEY, M. Am. Soc. C. E.[1]

Died April 19, 1943

In the last decade of the nineteenth century when James Walter Rickey was beginning his professional career, a tremendous development began in the science and application of hydroelectric power which history will record as a prominent feature of human progress in the twentieth century. His activities coincided with so many of the forward steps taken during that time that, without hesitation, he can be awarded a foremost place among the leaders of that movement. His keen interest in every possible further improvement rewarded those whom he served handsomely, and was a source of inspiration to those under him. He became a leader in the profession at an early age, and his activities took him to many foreign lands, where he met the foremost men of his profession.

James W. Rickey, the son of James and Rosaltha (Jones) Rickey, was born at Dayton, Ohio, on November 10, 1871. He received his early education at Minneapolis, Minn., advancing from its high schools to Rensselaer Polytechnic Institute in Troy, N. Y. He was graduated with the degree of Civil Engineer in 1894. His first engineering work was a brief assignment in the office of the eminent consulting hydraulic engineer, the late John T. Fanning, M. Am. Soc. C. E., of Minneapolis. Then he served in the office of a division superintendent of the Northern Pacific Railway Company from September, 1894, to July, 1896. In the fall of 1896, he became principal assistant hydraulic engineer with the Lake Superior Power Corporation, working on a 10,000-hp hydroelectric plant in Sault Sainte Marie, Ont., Canada, and a 40,000-hp plant in Sault Sainte Marie, Mich. (one of the giant plants of its day). In 1897 he became principal assistant engineer with the St. Anthony Falls Water Power Company at Minneapolis, under the late William de la Barre, M. Am. Soc. C. E. This association lasted ten years, during which time he designed the 3,500-hp Hennepin Island and several other hydroelectric plants. In these early years of his career he applied himself intensively to his chosen field of work and by arrangement with his company undertook privately the design of several other projects in the West and Middle West. This additional work occupied his eager mind into late hours of the night for months at a time.

During this period he developed a number of important inventions and improvements in the design of hydroelectric plants, including such widely adopted features as semicircular steel bulkheads for turbine flumes and a novel type of bear-trap dam.

During these early years while Mr. Rickey was getting his practical training, a new industrial giant was developing. The aluminum industry, which had been an important factor in the development of electro-chemical industries using Niagara River power, found the possibilities of further development at Niagara rather limited, and had taken over the old St. Lawrence River Power Company's hydro plant on the St. Lawrence River at Massena, N. Y., in the belief that its requirements had been met for some years to come. However, the industry was growing so rapidly that, since the necessity for greater additional power sources was becoming apparent, a consistent policy of heavy expansion of power facilities had to be adopted. In June, 1907, Mr. Rickey came to Massena as chief hydraulic engineer of the Aluminum Company of America (the old Pittsburgh Reduction Company). He immediately embarked on construction operations with the object of increasing the capacity of the original Massena plant through enlargement of the canal and installation of additional generating units.

---

[1] Memoir prepared by a committee of the Pittsburgh Section consisting of James P. Growdon, Chairman, and R. F. Ewald, Members, Am. Soc. C. E., and D. J. Bleifuss, Assoc. M. Am. Soc. C. E.

At the same time, looking to the future, plans were prepared for the development of the Long Sault Rapids on the St. Lawrence River, and initial steps were taken toward the organization of a company for the purpose of making that project a reality. This called for active participation in many involved negotiations with state, federal, Canadian, and international authorities and engineers. The Long Sault project is one of the world's great water power possibilities, and as such it was certain to challenge the creative imagination and vision of an engineer such as Mr. Rickey. This was one of his fondest dreams, and he may properly be credited with the economic foresight that led to the eventual adoption by the U. S. Army Engineers of the single-stage high-head development now generally favored. Mr. Rickey maintained his interest in this project long after it had passed from the Aluminum Company, and it may be taken as a tribute to his skill that his plans for this great project have been approved by many prominent engineers connected with it at one time or other.

When it became evident that immediate development of the Long Sault would be barred by political considerations, the Aluminum Company turned to other localities for much needed power, and the acquisition and development of those sites constituted Mr. Rickey's major activities for the next thirty years. In 1910 and 1911 Mr. Rickey made a thorough search for, and surveys of, power resources east of the Mississippi, much of this work antedating any government interest in hydro power and requiring original survey work and collection of field data. More than thirty undeveloped sites were investigated personally.

In 1911 the Aluminum Company, under his direction, began the purchase of property in the Little Tennessee River basin in Tennessee and North Carolina, with the object of creating therein a completely integrated hydroelectric development. As the plans grew, many new and interesting engineering problems were solved. Some of these solutions constitute real "firsts" in hydroelectric engineering. Mr. Rickey's proposal for the construction of overfall dams 185 ft and more in height, allowing for floods of more than 250,000 cu ft per sec, was a very bold project for 1914, but, since it was based on thorough investigations of models which disclosed for the first time the importance of the hydraulic jump in the control of erosion, the soundness of the project was proved.

The Little Tennessee projects will always have an interesting place in the history of hydroelectric development in America. A brief description of each follows:

Project	Location	Completed	Capacity (kw)	Head (ft)
Cheoah	At Tapoco, N. C.	1919	80,000	185
Santeetlah	Near Robbinsville, N. C.	1927	45,000	663
Calderwood	At Calderwood, Tenn.	1930	121,500	215
Glenville	Near Glenville, N. C.	1941	21,600	1,215
Nantahala	At Nantahala, N. C.	1942	43,200	1,005

The turbines for Cheoah were duplicates of those installed at the Narrows plant on the Yadkin River in North Carolina and constituted a step forward in size and power when they were ordered in 1916. The use of vibrators in placing concrete was begun in the final stages of construction of Santeetlah Dam in 1927, probably marking the first use of such equipment on dams. Glenville and Nantahala, with the highest heads east of the Rocky Mountains, were constructed after the termination of Mr. Rickey's active services, but the general plans for them were prepared during his active years.

Fontana (N. C.), under construction during 1942–1944 by the Tennessee Valley Authority, was originally a part of the company's Little Tennessee System. This great project, with its 477-ft dam and 202,500-kw power house, is being built substantially according to plans made during Mr. Rickey's active direction of the preliminary studies for this project.

In 1915 a partly completed development at the Narrows on the Yadkin River, begun by a French company in 1913, was taken over by the Aluminum Company of America, when the former company found itself unable to proceed as a result of the outbreak of World War I. Mr. Rickey made drastic revisions in the plans, nearly tripling the spillway capacity and simplifying the generating units along lines that are now practically standard but that constituted a marked step forward in contrast to the then standard equipment proposed by the French company. In addition, two other developments were built under Mr. Rickey's direction on the Yadkin. The complete schedule for this project follows:

Project	Location	Completed	Capacity (kw)	Head (ft)
Narrows	Near Badin, N. C.	1917	81,200	179
Falls	Near Badin, N. C.	1919	20,300	52
High Rock	At High Rock, N. C.	1927	33,000	59

During the early 1920's, Mr. Rickey made several extended foreign tours which gave him excellent opportunities to familiarize himself with the best practice in hydraulic engineering in all parts of the world and to add to his already wide list of acquaintances.

In 1925, the aluminum industry took an active interest in the development of the Saguenay River in Quebec, Canada, and, as chief engineer of Alcoa Power Company Limited, Mr. Rickey was active in the design and construction of Chute à Caron power house and dam, and preliminary design of the Shipshaw Project. In connection with this work, he was responsible for one of the most spectacular feats of engineering history—the construction "on end" of a massive 92-ft-long concrete dam, which was tipped into place for the purpose of diverting the Saguenay River into a new channel during the closure of the main dam. This "precast dam" which became generally known as the "Chute à Caron Obelisk" was hailed far and wide as one of the boldest engineering ventures ever undertaken. Chute à Caron Dam and its 260,000-hp power house, completed in 1931, were the first steps in a power development which was completed in 1943 with a capacity of 1,200,000 hp at the one site.[3]

The foregoing brief synopsis covers only the major projects with which Mr. Rickey was associated during his busy career. He acted as consultant on numerous smaller projects in Canada, northern New York, and North Carolina, and on the investigation of hydro power projects all over the world.

Throughout his career, he applied himself intensively to his professional work and allowed himself little in the form of diversion or hobbies. His standard reply to inquiries regarding his favorite hobby was "anything pertaining to hydraulic power development." His mind was highly inventive and he was constantly seeking unconventional and new ideas which offered opportunities for important savings for his clients. He had the courage of his convictions, and, once he was satisfied that the engineering fundamentals were sound, he did not hesitate to stake his name and reputation on the adopted plan, no matter how bold.

He took particular delight when members of his staff developed valuable new ideas and insisted that they receive all the credit whenever such ideas proved successful. He was a "man builder" in the real sense and his demands for performance were high. Engineers by the scores who were members of his staff at one time or another may be found in many parts of the United States. Invariably they recall their good fortune in having been associated with Mr. Rickey, their chief, counselor, and good friend who inspired them to better engineering and better citizenship.

A strong and active member of the Society, he valued his membership and urged his associates and employees to participate actively in developing professional solidarity. In his desire to see his company's engineers advance in professional standing, he encouraged them to write papers for the Society's publications under their own names, at the same time thus making the developments carried out under his direction known to the profession at large. His sociability, histrionic abilities, and widespread charities will always be remembered.

Practically his only nonprofessional activity in Pittsburgh's civic affairs was his service as director of the Pennsylvania Association for the Blind and of Pittsburgh's Workshop for the Blind. He derived much satisfaction in being of assistance to those who could not see.

On January 1, 1938, Mr. Rickey retired from active service as chief hydraulic engineer of the Aluminum Company of America, but continued as a consultant. With characteristic energy he took up hobbies which he had foregone during his more active days and achieved considerable fame as an amateur photographer in the few short years remaining to him. As would be expected, he started an active career in photographic research, which no doubt would have led to many improvements had he been permitted to continue.

---

[3] "Blasting a Precast Dam into Place," by C. P. Dunn, *Civil Engineering*, December, 1930, p. 159; also "Models Predict Behavior of Falling Dam," by Adolph J. Ackerman, *ibid.*, July, 1932, p. 415.

On January 24, 1899, Mr. Rickey was married to Lucy A. Mitchell at Minneapolis. In 1940 he and Mrs. Rickey, who survives him, moved from Pittsburgh, Pa., to Washington, D. C. In his well-deserved freedom from the heavy professional responsibilities of earlier years, he found keen enjoyment, devoting much of his time to his wide circle of friends and to his newfound hobby. It was therefore all the more distressing to see this happiness suddenly interrupted overnight by the loss of his sight. However, the courage and optimism which had been the foundation of his great career carried him forward, and, during the eighteen months preceding his death, he maintained a cheerful spirit which was an inspiration to all who visited him. Strangely enough, the type of helpful encouragement, which he at one time had given to blind friends in Pittsburgh, served him well to the end.

He was a member of Sigma Xi and Theta Nu Epsilon fraternities, the American Technical Committee of the International Committee on High Dams, Engineers Society of Western Pennsylvania, and Rensselaer Society of Engineers.

Mr. Rickey was elected an Associate Member of the American Society of Civil Engineers on September 3, 1902, and a Member on April 4, 1905.

JOHN WESLEY RIPLEY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 10, 1943

John Wesley Ripley was born in Sag Harbor, L. I., N. Y., on February 5, 1870, the son of John W. and Julia (Douglas) Ripley. One of his forebears, Capt. Joseph Ripley of the brig *Hope* was a skipper of the Sag Harbor whaling fleet which, prior to 1800, carried a greater tonnage of foreign commerce in square rigged ships than the City of New York (N. Y.).

From his seafaring ancestors, Mr. Ripley inherited a great love for the sea and for sailing. During his boyhood in Sag Harbor he worked before and after school during the winter making and setting out fish nets and clamming. He also was employed in the local shipyard and, as first mate on a small craft, carrying freight and sailing parties. Later he skippered his own boat, advertised as follows:

"Poquatuck
Sailing or Picnic Parties
will find the
Yacht Poquatuck
Both comfortable and safe
with prices moderate
John W. Ripley
Orders may be left with G. A. Halsey"

---

[1] Memoir prepared by R. T. Betts, M. Am. Soc. C. E., with the assistance of F. Douglas Ripley, Rate Engr., Long Island Lighting Co. Mineola, N. Y.

Mr. Ripley was a life member and for two years a Commodore of the Bayside, L. I., N. Y., Yacht Club, and his schooner-rig *Kumalong* was always listed in the club's annual Block Island race. After his boat came in second or third several times, he determined to win. A scale model of the boat was made, and the writer witnessed the trial of different sizes, shapes, and sets of sails with the aid of an electric fan. That year Mr. Ripley won the race.

He was graduated in 1893 from Cornell University at Ithaca, N. Y., with the degree of Civil Engineer. During the next five years he was a draftsman for the Groton (N. Y.) Bridge Company, Milliken Brothers, and Jackson Architectural Iron Works.

From January, 1898, to February, 1901, he was principal assistant engineer on the 125th Street viaduct in New York for the extension of Riverside Drive, being in full charge for several months of this rather unique and spectacular crossing of the valley on that part of Manhattan Island. From February, 1901, until 1909, Mr. Ripley was chief engineer, structural department, of John Monks and Sons of New York, in full charge of design and construction of the superstructures of many piers in New York City, particularly those on the East River between Fulton Street and Coenties Slip. During this same period he acted as consulting structural engineer for Copeland and Dole, Architects of New York, on the design and construction of several tall buildings in New York City. While with John Monks and Sons, The Monks-Ripley Pile Company was formed to promote the use of the "Ripley Combination Pile," a 3-in. reinforced concrete covering for wooden piles to be used in waters infested with wood borers. Mr. Ripley was granted a patent for this invention which has long since expired. The Ripley Combination Pile was used about 1908 in the construction of a pier in Port au Prince, Haiti—the first use of a protection of this kind for wooden piles.

In October, 1909, Mr. Ripley and Allan A. Robbins, M. Am. Soc. C. E., organized The Robbins-Ripley Company, engineers and contractors, for the design and construction of all types of waterfront improvements. The firm participated in the design and reconstruction of the bulkhead and pier structures for the entire Brooklyn (N. Y.) waterfront of the New York Dock Company and in the construction of similar structures for the Eastern Steamship Lines, Inc.; New York Central Railroad Company; Cunard White Star, Ltd.; Furness, Withy and Company, Ltd.; Delaware, Lackawanna and Western Railroad Company; Hoboken Dock Company; Submarine Base, New London, Conn.; Montauk Development Company; Larchmont Yacht Club; Beard's Erie Basin, Inc.; The Port of Embarkation, Brooklyn Army Base; Merchant Marine Academy, King's Point, L. I., N. Y.; and others. Also, this firm acted as engineers for the Eastern Steamship Lines, Inc., on the design and construction of a new terminal on the North River and for Cunard White Star, Ltd., on the restoration and rehabilitation of Pier 54, North River, after the disastrous fire of May, 1932. Mr. Ripley was vice-president and treasurer of the company from its inception until his death.

A man of short and sturdy stature, he had a fine, gentle, and friendly disposition. He was a welcome guest and a delightful host.

A friend of more than fifty years paid tribute to him in the following words: "If I were to be left on a desert island with one other person only, I should choose John Ripley."

He was active in the affairs of the First Methodist Church and the Young Men's Christian Association of Flushing, L. I., N. Y. At the time of his death, he was a director of the Long Island Bankers and the Seaboard Commercial Corporation, and a trustee of the Queens County Savings Bank.

On August 27, 1896, Mr. Ripley was married to Eleanor Orr in Stamford, N. Y. He is survived by his son, F. Douglas; a daughter, Eleanor (Ripley) Crosby; and a granddaughter, Eleanor Crosby.

Mr. Ripley was elected a Member of the American Society of Civil Engineers on May 1, 1907.

LOUIS CHARLES ROCKETT, M. Am. Soc. C. E.[1]

DIED APRIL 13, 1943

Louis Charles Rockett was born on February 10, 1889, at Chicago, Ill., the son of Henry George Rockett and Elizabeth Mary (Rowe) Rockett.

He received his early education in the grade and high schools of Chicago. He entered the University of Wisconsin at Madison, in 1911, and was graduated with the degree of Bachelor of Science in Civil Engineering in June, 1915.

From June, 1915, to November, 1917, Mr. Rockett was employed in the division office of the Chicago and North Western Railway Company in Chicago. From November, 1917, to June, 1918, he worked for the Wisconsin Highway Commission. From July, 1918, to January, 1919, he was with the U. S. Army on location and construction of railroads. From February, 1919, to July, 1922, he was again employed by the Wisconsin Highway Commission. From July, 1922, to the time of his death, he was in the Santa Fe (N. Mex.) branch office of the U. S. Public Roads Administration.

Mr. Rockett was active in civic affairs, a veteran of World War I, and a member of the vestry of the Episcopal Church. Prominent in the Masonic Lodge, he served as Worshipful Master of Montezuma Lodge No. 1 and as Worthy Patron of the Order of Eastern Star, Chapter No. 19. In addition, he was Past Grand Master of the Grand Lodge of New Mexico, Past Grand High Priest of the New Mexico Grand Chapter, R. A. M., Past Grand Commander of the New Mexico Grand Lodge, Knights Templar, and Inspector General, Honorary 33 Degrees of the Scottish Rite bodies.

Mr. Rockett was married on June 30, 1927, in Portland, Ore., to Lillian Sundberg. He is survived by his widow; two sons, John and Louis Charles; his mother, Mrs. E. M. Rockett; three sisters, Mrs. Mary Richardson, Mrs.

---

[1] Memoir prepared by a Committee of the New Mexico Section consisting of Berkeley Johnson, M. Am. Soc. C. E., Rufus H. Carter, Jr., Assoc. M. Am. Soc. C. E., and John O. Joerns, Jun. Am. Soc. C. E.

Lilly Kenney, and Mrs. T. R. Marchant; and a brother, William R. Rockett. Funeral services and burial were conducted in Santa Fe on April 15, 1943.

Mr. Rockett was elected an Associate Member of the American Society of Civil Engineers on June 16, 1924, and a Member on October 26, 1931.

EDWARD HENRY ROCKWELL, M. Am. Soc. C. E.[1]

Died May 26, 1943

The death of Edward Henry Rockwell brought to a close a long and diversified career, well balanced between service as a teacher in Tufts College (Medford, Mass.), Lafayette College (Easton, Pa.), and Rutgers University (New Brunswick, N. J.), and a wide activity in the practical field of his profession. Although retired, he gladly returned to carry a share of the teaching burden, abnormally increased by war conditions, in the Department of Civil Engineering at Lafayette College, where he had spent fifteen years in administration and teaching. For forty years he gave valuable instruction to students in civil and structural engineering. He died suddenly from heart failure at his home in Easton.

Mr. Rockwell was born in Worcester, Mass., on April 20, 1869, the son of Edward Munson and Martha Josephine (Smith) Rockwell. He was educated at Worcester Polytechnic Institute in Worcester, being graduated in 1890 with the degree of Bachelor of Science. His alma mater also awarded him the professional degree of Civil Engineer in 1920 and, in 1933, the honorary degree of Doctor of Engineering.

After a brief experience in drafting and shop inspection of bridges in Chicago, Ill., with the late G. S. Morison,[2] Past-President, Am. Soc. C. E., he returned home to engage in business with his father. For two years he was Sub-master in the Leominster (Mass.) High School. Following a half-dozen years of work with several Massachusetts firms as draftsman and superintendent of bridge construction, he resumed his teaching, first as an instructor of civil engineering in Tufts College from 1902 to 1903. His advancement was rapid. He became assistant professor the next year and full professor of civil and structural engineering three years later, continuing from 1918 to 1922.

While teaching at Tufts College at Medford, a suburb of Boston, Mass., he was often engaged as a consultant and director on many engineering projects in that vicinity. He helped to design the Haymarket Station of the Boston subway, the $1,000,000 Suffolk County Court House Extension, and a number of important bridges in the Boston district. He examined and reported upon two proposed designs for the Galveston (Tex.) Causeway Reconstruction, and was consulting engineer in 1917 on designs for reinforced-concrete ships, and also for new United States arsenals. In 1921 he was in charge of plans and construction for the $300,000 chemical laboratory for Tufts College. Before that building was completed, Professor Rockwell was called to Rutgers University to become Dean of the Engineering School and Professor of Civil Engineering, a position that he held with distinction until 1928. From 1928 until his retirement in May, 1942, he was Simon Cameron Long Professor of Civil Engineering at Lafayette College and director of the division of civil engineering.

---

[1] Memoir prepared by Albert H. Gilmer, Prof., Speech and Dramatic Art, Lafayette College, Easton, Pa.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LIV (1905), p. 513.

Professor Rockwell was widely recognized as a leading authority in the field of blasting effects of explosives, and his services were often in demand by owners and operators of quarries and mines, who employed him as an expert witness in court cases. Such professional service often took him afar—to midwestern states and even to the mines of Montana. Out of experience and knowledge in this field he wrote his authoritative volume, "Vibrations Caused by Blasting and Their Effect on Structures."[3]

He also contributed the section, "Reinforced Concrete Arch Highway Bridges,"[4] in the *American Highway Engineers' Handbook,* and wrote occasional bulletins for the Society for the Promotion of Engineering Education.

Professor Rockwell was a member of Phi Beta Kappa, Sigma Xi, Tau Beta Pi, and Lambda Chi Alpha fraternities, in the academic field; and, in the professional field, of the Society for the Promotion of Engineering Education, the American Association of University Professors, and the New Jersey Society of Professional Engineers (ex-president, 1928). He was a member of the Lehigh Valley Section of the Society (President in 1932). At one time he was Lieutenant-Commander, Civil Engineer Corps, U.S.N.R.

Professor Rockwell attributed his usual good health to his love of the out of doors, where he engaged actively in his favorite recreational sports—fly-fishing for bass and trout in the streams of New England, and golf until the week of his death. He was a member of the Country Club of Northampton County, upon whose golf links he was often to be found thoroughly enjoying the game which he played so well with his colleagues.

Both a theoretical and practical scientist of solid character and of many notable achievements, Professor Rockwell had a high pride in his profession and enjoyed teaching its principles and standards to the prospective engineers in the classroom and laboratory.

He was gentle, yet sturdy, and, in his personal relationships, friendly and tolerant, with a desire to understand the viewpoints of others and thus to be fair. He had the high regard, confidence, and hearty cooperation of his fellow workers. His career as a teacher, as a creative engineer, and as a wise counselor in affairs pertinent to his profession is a testimonial to the value of one engineer's share in the mass contribution of civil engineers to the times and places in which they live and labor as cooperative citizens in civilized communities.

Professor Rockwell was married twice. On October 29, 1891, at Worcester, he was married to Lena Hortense Warfield. The five children of that union, all of whom are living, are Grace Margaret (Mrs. Stanley R. Kingman), Dorothy (Mrs. Mark A. Burns), Doris, Lt.-Comdr. Donald Edward, and Rosamond Warfield (Mrs. William W. Cheney). Mrs. Rockwell died in February, 1933. In June, 1941, Professor Rockwell was married to Mrs. Nellie May Williamson Owens of Easton. Besides the children of his first marriage, ten grandchildren, and two great-grandchildren, some of whom are in the military service of their country, he is survived by a brother, Alfred C. Rockwell, and a sister, Ruth Rockwell.

---

[3] "Vibrations Caused by Blasting and Their Effect on Structures," by Edward Henry Rockwell, Hercules Powder Co., Wilmington, Del., 1934.

[4] "Reinforced Concrete Arch Highway Bridges," by Edward Henry Rockwell, *American Highway Engineers' Handbook,* Arthur H. Blanchard, Editor-in-Chief, 1st Ed., John Wiley & Sons, Inc., New York, 1919, p. 1437.

Professor Rockwell was elected an Associate Member of the American Society of Civil Engineers on June 1, 1909, and a Member on May 28, 1912.

WALTER ALEXANDER ROGERS, M. Am. Soc. C. E.[1]

Died January 3, 1944

Walter Alexander Rogers was born in Milwaukee, Wis., on January 19, 1868, the son of Martha (Ross) Rogers and Alexander Hamilton Rogers. He was the second of five children, all of whom were boys.

In 1882 the family moved to a farm in Wauwatosa, Wis., not far from Milwaukee, where in early boyhood Mr. Rogers learned the value of hard work in the daily tasks on the farm.

When he finished high school in Milwaukee in 1884, he entered the University of Wisconsin at Madison. After being graduated in 1888 with the degree of Civil Engineer, he was awarded the John Johnston Fellowship and remained at the university for a year to teach mathematics and to do research work in the comparatively new subject of concrete. At this time he organized the first cement and concrete testing laboratory at the university—one of the first of its kind in the United States.

After a short period with the Wisconsin Central Railway Company, in the spring of 1891 he was employed by the Northern Pacific Railway Company as division engineer at Livingston, Mont.

Mr. Rogers then returned to Milwaukee and was employed by the Chicago, Milwaukee and St. Paul Railroad Company as assistant engineer in the Bridge and Building Department in charge of the double tracking of twenty miles of line, with headquarters at Gurnee, Ill. In 1898 the railroad undertook its first track elevation in Chicago, Ill., and Mr. Rogers was made engineer of subways to handle the bridge construction on this work. In 1899 he was appointed engineer of permanent structures on the Milwaukee Road System. All masonry at that time was built by company forces, and he directed the operation of these forces. While he held this position, the use of concrete in place of stone masonry became a practice on the railroad through Mr. Rogers' continued research and efforts.

---

[1] Memoir prepared by Lester C. Rogers, M. Am. Soc. C. E.

On April 26, 1901, he resigned his position with the Chicago, Milwaukee and St. Paul Railroad Company, and, with the late Onward Bates,[2] Hon. M. and Past-President, Am. Soc. C. E., organized the Bates and Rogers Construction Company. From 1909 until 1937 he was president and active head of the company.

In the early days the firm built such structures as the Tennessee River Bridge for the Illinois Central Railroad Company; one of the early irrigation dams for the United States Reclamation Service at Minidoka, Idaho; and the bridges and tunnel lining for the Chicago, Milwaukee, St. Paul and Pacific Railroad through the Bitterroot Mountains in Montana. When the Grand Trunk Pacific Railway extended its main line from Edmonton, Alta., Canada, to Prince Rupert, B. C., Canada, in 1913, this company constructed all the bridges and lined a number of the tunnels on this pioneer project, in spite of severe weather and transportation problems. The firm constructed two locks and dams on the Ohio River for the War Department, and did a large amount of work for the railroads of the United States until 1917.

During World War I the Bates and Rogers Construction Company was chosen to build the army cantonment at Camp Grant, Rockford, Ill.; the railroad yards at the Hog Island (Pa.) Shipyard; the large government warehouses at Cumberland, Pa.; the shipyard and wooden ships at Millville, Fla.; and the large air nitrates plant at Toledo, Ohio.

During the twenty-five years between World War I and World War II, the company built power stations and dams for a number of the large power companies, and constructed such unusual projects as the Ontario Street Subway for the Cleveland (Ohio) Union Terminals. The firm specialized also in tunnels, placing the lining in the famous five-mile Connaught Tunnel of the Canadian Pacific Railway, constructing and lining the Musconetcong Tunnel for the Lehigh Valley Railroad Company, together with the construction, enlargement, and revision of a large number of tunnels for various railroads throughout the West. In 1937 Mr. Rogers was made chairman of the board of Bates and Rogers Construction Corporation. The firm continued its tunnel work, building Blue and Kittatinny Mountain tunnels for the Pennsylvania Turnpike.

When World War II began, Mr. Rogers again placed the organization and facilities of his firm at the disposal of the government, and, under his chairmanship, war projects worth more than $100,000,000—including the huge Kingsbury and Oak Ordnance plants and permanent bridges on the Alaskan Military Highway—were creditably completed. The company also had the major interest in a joint venture known as Quincy Barge Builders, which built large numbers of tank landing craft and floating dry docks for the Navy.

Mr. Rogers was greatly interested in the construction industry as a whole, and was one of the original group that organized the Associated General Contractors of America, and helped to put meaning into the slogan of that organization—"skill, integrity, and responsibility." He was the second president of this organization, and continued to take a major interest in the growth and standards of the industry throughout his life.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1484.

On July 1, 1891, he was married to Julia Margaret Cushing, of Wauwatosa, who had attended high school and the University of Wisconsin with him. They made their home in Livingston until the spring of 1892. In 1941 Mr. and Mrs. Rogers celebrated their Golden Wedding Anniversary in company with their family and many friends. Mr. and Mrs. Rogers were the parents of five sons and a daughter—the writer, Margaret G., Ross W. (deceased), Carl R., Walter C., and John W. Mr. Rogers is survived by his widow and five children.

Mr. Rogers resided on his beautiful farm near Glen Ellyn, Ill., with his wife and daughter. He took a great interest in community life, and was an active supporter of the Glen Ellyn Congregational Church, the Young Men's Christian Association, and kindred activities. For many years he was chairman of the board of George Williams College in Chicago, an institution specializing in the training of young men and women for Christian group work in community life.

In his business and home life he insisted on the highest standards of honesty and integrity. Walter Alexander Rogers will be best remembered for his kindly, tolerant spirit, his quiet perseverance in what he believed was right, and his willingness to lend a helping hand.

Mr. Rogers was elected a Junior of the American Society of Civil Engineers on September 10, 1891; an Associate Member on November 3, 1897; and a Member on April 4, 1900.

HARRY ROBINSON SAFFORD, M. Am. Soc. C. E.[1]

DIED APRIL 10, 1943

Harry Robinson Safford was born in Madison, Ind., on February 7, 1875, the son of James Broderick and Josephine (Branham) Safford. He was educated at Purdue University in Lafayette, Ind., from which he received the degrees of Bachelor of Civil Engineering in 1895, Civil Engineer in 1918, and Doctor of Engineering in 1920.

In 1895, immediately after graduation, Mr. Safford began his career in railroad engineering in the engineering department of the Illinois Central Railroad Company as roadmaster from 1900 to 1903; principal assistant engineer from 1903 to 1905; assistant chief engineer from 1905 to 1907; and chief engineer, maintenance of way, from 1909 to 1910. From 1910 to 1911 he was employed as assistant to the president of Edgar Allen Manganese Steel Company of Chicago, Ill. He was chief engineer of the Grand Trunk Railway Company from 1911 to 1918. In the latter year he was asked by the government to serve as assistant regional director of the U. S. Railroad Administration, which position he held until 1920. From 1920 to 1925 he worked for the Chicago, Burlington and Quincy Railroad Company, first as assistant to the president and then as vice-president. During part of this period (1921–1925), he also served as assistant to the president of the Colorado and Southern Lines, Fort Worth and Denver City Railway Company. In 1925 he was appointed vice-president of the Missouri Pacific Railroad Company, and executive vice-president of Gulf Coast Lines and International–Great Northern Railroad Company and subsidiaries, continuing in this capacity until his death.

---

[1] Memoir prepared by Robert J. Cummins, M. Am. Soc. C. E.

As evidence of his tremendous responsibilities, a list of the twenty-two railroad companies and railroad properties for which he was a director is given here: Asherton and Gulf Railway Company; Asphalt Belt Railway; Austin Dam and Suburban Railway Company; Beaumont, Sour Lake and Western Railway Company; Brownsville and Matamoros Bridge Company; Galveston, Houston and Henderson Railroad Company; Houston and Brazos Valley Railway Company; Houston Belt and Terminal Railway Company; Houston North Shore Railway; Iberia, St. Mary and Eastern Railway Company; International–Great Northern Railway Company; New Iberia and Northern Railroad Company; New Orleans and Lower Coast Railroad Company; New Orleans, Texas and Mexico Railway Company; Orange and Northwestern Railroad Company; Rio Grande City Railway Company; St. Louis, Brownsville and Mexico Railway Company; San Antonio Southern Railway Company; San Antonio, Uvalde and Gulf Railroad Company; San Benito and Rio Grande Valley Railway Company; Sugar Land Railway Company; and Western Townsite Company.

In 1920–1921 he served as president of the American Railway Engineering Association. At one time he was a member of the Western Society of Engineers, a director of Engineering Institute of Canada, and a member of the Advisory Council of the U. S. Department of Commerce.

For two successive terms in 1931 and 1932, he served as president of the Chamber of Commerce of Houston, Tex., bringing to that high office the splendid executive ability that had rightly earned for him such high honors in railroad circles. These years tried men's souls, and it is an index of the man that, during Mr. Safford's administration as president, the following meritorious projects were inaugurated by the Chamber of Commerce: (1) A survey of the city's water supply with special reference to additional available sources of supply; (2) the opening-up of foreign trade with a number of South American republics and the establishment of new consulates in Houston; (3) "Foreign Trade Week," an annual event in Houston, was first sponsored; and (4) the highway department took active part in state and federal highway activities, and many miles of new highways were planned.

As an Elder in the First Presbyterian Church of Houston, Mr. Safford gave public expression to the deeply religious convictions that actuated him in all his business relationships and sought to carry into practice the teachings of his Saviour.

His associates on the official board of that Church speak with loving tenderness of his loyalty and devotion as a ruling Elder and his zealousness for the cause he held so dear, never stopping to count the cost in weariness of body or spirit that he might serve worthily as a soldier of the Cross.

Busy man though he was with the countless calls on his time both at home and on the road, Mr. Safford nevertheless found time to serve for four years as president of the Sam Houston Area Council, Boy Scouts of America. From 1938 until his death he was a member of the National Executive Board of the Boy Scouts of America.

His dignified bearing coupled with his abounding love for boys enabled him to wield a powerful influence over the members of the 409 Scout Troops in the Houston area. Hundreds of Houston boys serving with the armed forces were challenged in their younger days to a nobler way of life by Mr. Safford's inspired leadership and his heart warming talks to the Scouts at the numerous jamborees during his term of office. Those who were present on such occasions will long remember the lasting impression made on participants and audience alike as this man of kingly bearing bestowed the honors conferred on the members of the various troops for tasks faithfully performed.

The fact that he found time to devote to such altruistic movements was a source of wonderment to his friends and business associates, but find time he did, and never in half-hearted fashion, but with all his heart and soul and mind.

His influence for good will reach on through the years, and his closest personal friends will long remember him, not so much as one of the outstanding railroad executives of his generation, but rather as a Christian business man whose life exemplified the "Golden Rule" in principle and practice, and who literally inspired hundreds of boys to the nobler way of life.

Mr. Safford was a Thirty-Second Degree Mason and Shriner. He belonged to Sigma Alpha Epsilon fraternity.

On December 12, 1900, he was married to Nell Gertrude Whittemore of Little Rock, Ark., who, with his son, Capt. Harry Robinson Safford, Jr., of the Ordnance Department, U. S. Army, survives him.

Mr. Safford was elected a Member of the American Society of Civil Engineers on July 1, 1908. He became a Life Member in January, 1943.

WILL JOSEPH SANDO, M. Am. Soc. C. E.[1]

DIED FEBRUARY 23, 1943

Will Joseph Sando was born at Scranton, Pa., on April 9, 1864, the son of Joseph William Sando and Mary (Grogan) Sando. His early life was spent in Scranton, where he attended the public schools. At the age of fifteen he began a five-year apprenticeship in the machinist trade, followed by two years as a mechanical draftsman with the Dickson Manufacturing Company of Scranton. From 1886 to 1889 he served as draftsman and shop inspector on power plant machinery and pumping engines for E. D. Leavitt of Cambridgeport, Mass.

---

[1] Memoir prepared by James L. Ferebee and Langdon Pearse, Members, Am. Soc. C. E.

In 1889 Mr. Sando became draftsman in charge of the drafting office of the U. S. Engineers at the William Cramp and Sons Shipyards at Philadelphia, Pa. The following year he returned to the employ of Mr. Leavitt with whom he remained until 1897. From then until 1900 he served as superintendent of pumping stations for the Metropolitan Water Board (Boston, Mass.). From 1900 through 1901, he was chief engineer and manager of the pumping engine department for the International Steam Pump Company (New York, N. Y.). In 1903 he was appointed engineer, pumping department, of the Burr-Hering-Freeman Commission on Additional Water Supply for the City of New York. From 1904 to 1910, he was employed by the Allis Chalmers Company of Milwaukee, Wis., as chief engineer and manager of the pumping engine department, and, from 1907 to 1910, also managed its hydraulic turbine department and studied installations abroad.

From 1910 until his death, Mr. Sando was engaged in practice as a consulting engineer on plans, specifications, inspection, tests, and valuation reports for water works pumping plants; high-pressure fire systems; steam, electric, and hydraulic power plants; cracking still plants for the production of gasoline; and mechanical equipment in sewage treatment plants. For the City of Boston, he prepared plans and specifications for the underground automatic electric sewage pumping station in Summer Street, the high-pressure fire service, the Albany Street aboveground automatic electric sewage pumping station, and the remodeling of equipment at the Calf Pasture Pumping Station. He served the Milwaukee Sewerage Commission as consulting mechanical engineer during the construction and early operating stages of the Jones Island plant, and, from time to time, advised The Sanitary District of Chicago (Ill.) on sewage pumping equipment.

At various times he was engaged as an expert by the following: The City of Cincinnati (Ohio) on litigation in connection with its water works pumping plant; the Chicago and North Western Railway Company on personal injury litigation; Summit (N. J.) and the Indianapolis (Ind.) Water Company on water rate litigation; and the Passaic Valley (New Jersey) Water Commission on the acquisition of property of the Passaic Consolidated Water Company. His advice was sought on pumping engines by Lowell, Mass.; the Alberfoyle Manufacturing Company at Chester, Pa.; and the Pittsburgh Plate Glass Company at Crystal City, Mo.

During World War I he was president of the Milwaukee Boiler Company and of the Monarch Boiler Arch Company. He was a member of a Milwaukee draft board.

Among his other activities, he was an advisory engineer for the Reconstruction Finance Corporation, Chicago Loan Agency, analyzing various loans under consideration.

As an engineer, Mr. Sando was a recognized expert on pumping machinery of various types and on steam practice. His judgment, based on broad experience in the manufacturing and designing field, was valued by his associates.

Mr. Sando was a member of the American Society of Mechanical Engineers, the American Institute of Consulting Engineers (1925–1937), and the Boston Society of Civil Engineers. In addition to his professional work, he was interested in social clubs, serving for twelve years as a director of the Milwaukee Club. He enjoyed bowling and golf and formerly was a member of the Milwaukee Country Club and Tripoli Country Club (Milwaukee). At one time he also belonged to the Engineers Club in New York City and the Engineers Club in Boston.

In 1889 he was married in Boston to Clara Smith Collins, who died in 1906. In 1928 he was married in Chicago to Eleanor Burke Williams, who, with a stepdaughter, Eleanor Burke Williams, survives him.

Mr. Sando was elected a Member of the American Society of Civil Engineers on August 31, 1909.

MARSHALL NEY SHOEMAKER, M. Am. Soc. C. E.[1]

DIED FEBRUARY 9, 1943

Marshall Ney Shoemaker was born in Manayunk, Philadelphia, Pa., on October 6, 1873, the son of Marshall Ney and Anna Anderson (Simpson) Shoemaker. His Quaker ancestors came from Kriegsheim, Bavaria (later Germany), in the seventeenth century and settled Shoemakertown (later Elkins Park, Pa.) and Germantown, Pa.

He attended the public schools in Manayunk and was graduated from the Grammar School. Shortly thereafter he secured a position with A. and P. Roberts Company at Pencoyd, Pa., as apprentice surveyor. After about two years of surveying, during which he received individual instruction in engineering from the late C. C. Schneider,[2] Past-President, Am. Soc. C. E., and the late Paul L. Wolfel,[3] M. Am. Soc. C. E., he was transferred to the drafting room to trace stress sheets and figure estimates. In 1895 he had advanced so far in his studies and had gained sufficient experience to be given the responsibility of making estimates and structural designs for bridges and buildings. When the American Bridge Company was formed in May, 1900, Mr. Shoemaker was made secretary to the vice-president, Engineering Department. In 1901 he acted as assistant to the chief engineer at Pencoyd, and, from then until 1903, he was engineer of the Eastern Division Drawing Room, stationed in New York, N. Y.

From 1903 to 1910 he was vice-president of the American Concrete Steel Company with offices in Newark, N. J. During that time he designed and supervised the construction of a number of schools and industrial buildings of reinforced concrete construction.

In 1910 Marshall Shoemaker opened an office in Newark as a consulting engineer and architect and practiced until he retired in 1930. His work was concerned largely with industrial buildings. He was also active in many civic matters and wrote a new building code for the City of Newark, which was in effect at the time of his death. He was a prominent appraiser of real estate values and was called on continually to testify in this connection.

---

[1] Memoir prepared by G. E. J. Pistor, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1665.

[3] For memoir, *ibid.*, Vol. LXXXIV (1921), p. 940.

The possessor of a most kindly disposition, Mr. Shoemaker always was ready to assist his fellow men, if they were worthy. He extended a cheerful welcome to all. To those who were fortunate enough to know him well he was a staunch friend.

Although Mr. Shoemaker obtained his education the hard way—through study at night and experience by day—he was very appreciative of the opportunities offered by the institutions of higher learning. He was always ready and willing to help the young man who, without help, could not attend college. In fact, he took a sincere interest in any young person capable and anxious to advance.

On November 14, 1907, at St. Judes Church in Philadelphia, Pa., Mr. Shoemaker was married to Mary Bernard, who survives him.

He was a charter member of the Kiwanis Club of Newark; a life member of Mosque Temple; and a member of Kane Lodge No. 55 F. and A. M., Royal Order of Jesters, and Odd Fellows of Roxborough (Pa.). Among the scientific societies to which he belonged were the American Society for Testing Materials and the American Institute of Architects.

Mr. Shoemaker was elected an Associate Member of the American Society of Civil Engineers on April 1, 1903, and a Member on January 3, 1911. He became a Life Member in 1938.

ARTHUR GREGG SINGER, M. Am. Soc. C. E.[1]

Died November 9, 1942

Arthur Gregg Singer was born in Philadelphia, Pa., on December 10, 1872, the son of Charles Arthur and Elizabeth (Gregg) Singer. He received his preparatory education in the public schools of Philadelphia and was graduated from the University of Pennsylvania at Philadelphia, in 1892, with the degree of Bachelor of Science in Civil Engineering. He received the degree of Civil Engineer from the same University in 1893.

Mr. Singer's boyhood was spent in an atmosphere filled with educational interests. His father, Charles, and an Uncle Edgar (Singer) lived in adjoining homes and both possessed wide reputations as educators in the public schools of Philadelphia. For many years, Charles was the principal of the Marshall Boys Grammar School of Frankford, and Edgar was an assistant superintendent of the public school system in Philadelphia. Although Edgar's oldest son also became an educator, Charles' only son, Arthur, turned to the engineering profession for his lifework.

---

[1] Memoir prepared by Thomas Buckley, M. Am. Soc. C. E.

The community in which Mr. Singer was born and raised has been noted for three quarters of a century for the number of its young men who have sought technical careers in municipal service. He was attracted early to this field of activity and, after graduating from the University, entered the Bureau of Surveys, Department of Public Works of Philadelphia, on September 27, 1893. His engineering career began when he was appointed second assistant in the Second Survey District of the Bureau of Surveys and a member of a field corps in charge of surveys for sewers, paving, conveyances, park improvements, and the establishment of the city plan. He held this position until promoted to first assistant on June 15, 1901. From 1901 to 1907, as first assistant in the Second Survey District, he was in responsible charge of surveys, plans, and designs required in connection with sewer, park, and boulevard construction, and city planning.

In 1906 the City of Philadelphia began a comprehensive program of grade crossing eliminations that continued for many years. A number of trunk-line railroads and numerous grade separations were involved, and it became necessary to expand the engineering forces of the Bureau of Surveys by the creation of a unit designated as the Grade Crossing Division. Because of his excellent service record Mr. Singer was transferred in July, 1907, to the newly organized Grade Crossing Division as assistant engineer, remaining until July, 1920.

During this time he was in responsible charge of construction work on the abolition of numerous grade crossings on various lines of the Philadelphia and Reading Railway Company, the Pennsylvania Railroad Company, and the Baltimore and Ohio Railroad Company in the City of Philadelphia. One of the most notable of these projects was the elimination of grade crossings on the Port Richmond branch of the Philadelphia and Reading Railway Company and the opening and underpassing of several important highways blocked by this railway. The work included grade separations at two important arterial highway intersections, the construction of five new underpasses, and the reconstruction of an existing underpass. The majority of the bridge structures carry a 24-track system. There was, in addition, a great volume of construction work on the approaches to these underpasses, including extensive grading, sewer construction, underpinning, and repaving.

Mr. Singer's services in the Grade Crossing Division included the design and supervision of railroad structures and track layouts, the design and construction of sewer systems, street improvements, underpinning, railroad yards, structures, abutments, and the preparation of specifications and plans. During the same period he was also in responsible charge of construction work on the widening and improvement of Delaware Avenue, which runs alongside the Delaware River, and constitutes Philadelphia's most important commercial and industrial highway.

In 1920 the program of grade crossing abolitions was completed, and, on July 1 of that year, Mr. Singer was appointed surveyor and regulator in the Bureau of Surveys, Department of Public Works, and assigned to the Second Survey District. It is of interest to record that this district which he directed was the same one in which he began his engineering career twenty-seven years earlier. As surveyor and regulator he had charge of city planning, plans and projects for parks, boulevards, and other municipal improvements, and the making of surveys for all classes of improvements for both public and private interests within the Second Survey District. On March 17, 1925, he was placed in charge of the Tenth Survey District, which division of the Bureau of Surveys was doing the largest volume of construction work ever performed by any survey district.

Grade crossing elimination activities were resumed in Philadelphia in 1926, and Mr. Singer resigned his position as district surveyor and regulator to accept the appointment of assistant division engineer in the Grade Crossing Division of the Department of Public Works. Upon his return to the work of eliminating grade crossings, Mr. Singer directed construction for abolishing grade crossings on the lines of both the Philadelphia, Germantown and Norristown Railroad Company and the Philadelphia, Germantown and Chestnut Hill branches of the Philadelphia and Reading Railway Company. A number of important highway underpasses were constructed under this project in the Manayunk, Germantown, and Chestnut Hill sections of the City of Philadelphia.

Mr. Singer resigned from municipal employment on December 15, 1932, retiring after a period of nearly forty years of continuous efficient and honorable service in the Department of Public Works of the City of Philadelphia. His record in the Bureau of Surveys and the Grade Crossing Division affords an excellent example of a career man in public service. In addition to his engineering attainments, Mr. Singer was endowed with a pleasing personality and maintained at all times an attitude of helpfulness and consideration in his relationships with others. His judgment in public affairs was firm and sound and, in all contractual relations, tempered by a sense of fairness that won universal respect. He was unswervingly loyal to the best interests of the city and a trustworthy servant of both the city and the public.

For a long period of his life Mr. Singer was an enthusiastic and highly competent cricket player. He was a member of the old Oxford Cricket Club and the Frankford Cricket Club, and participated in many important games in this field of sport. He was a member of the Engineering Alumni of the University of Pennsylvania, the Engineers Club of Philadelphia, and the Frankford Historical Society.

Being of an exceedingly industrious nature, Mr. Singer's greatest enjoyment was found in constructive labor. He succeeded his father, Charles Arthur Singer, as the secretary of the Decauter Building and Loan Association, and was interested in the development of sound building association activities. At the time of Mr. Singer's death the Decauter Association was about one hundred years old and the oldest organization of its kind in the United States.

On January 21, 1909, he was married to Edith Elizabeth Good in Olney (Philadelphia). He is survived by his widow; two sons, Charles Gregg and Arthur Gregg, Jr.; and his mother, a woman of remarkable qualities who had reached the notable age of 102 years at the time of her son's death.

Mr. Singer was elected a Member of the American Society of Civil Engineers on August 28, 1922.

CHARLES SLEEPER STERLING, M. Am. Soc. C. E.[1]

Died September 6, 1943

Charles Sleeper Sterling was born on August 29, 1878, at Watkins Glen, N. Y., the son of Daniel and Sara (Sleeper) Sterling. He was graduated in 1902 from Colgate University, at Hamilton, N. Y., with the degree of Bachelor of Philosophy. He was a member of Phi Beta Kappa and Phi Kappa Psi fraternities. He was registered in New York State as a Professional Engineer and Land Surveyor. Although his formal education did not lead to an engineering degree, it was not strange that he entered the engineering field as his father had long been identified with railroad construction and maintenance.

For the greater part of the period between 1902 and 1915 he was employed in railroad engineering work by the Pennsylvania and the Lehigh Valley railroad companies, and on the construction of an electric railway in New Jersey and railroad surveys in Cuba. The remainder of this period was spent as a teacher of mathematics at the Mercersburg (Pa.) Academy and the Morristown (N. J.) School for Boys, as engineer for the New York State Barge Canal Terminal Commission, and in private practice. From 1915 to early in 1918 he was engaged on highway work for the State of New York. In 1918 he was commissioned a Captain in the Engineer Reserve Corps, U. S. Army. On receiving his honorable discharge from the armed forces he resumed his work with the State of New York and in various engineering capacities was engaged on Barge Canal construction until the end of 1924.

In 1925 Mr. Sterling again entered private practice, and from then until his death he was associated with the late Frank M. Williams,[2] M. Am. Soc. C. E., and the writers in general consulting practice at Albany, N. Y.

Charles Sterling was a man of deep convictions. Once he had decided on a course of action charted as the result of his characteristic critical analysis of the facts, there was no deviation. Any thought of appeasement or compromise involving principles never entered his mind. He was a good engineer and displayed a keen interest in upholding the standards of his profession. He gave generously of his time and effort to many worthy causes. His unfailing interest in civil affairs, in the activities of his church, and, particularly, in the welfare of the less fortunate was most pronounced. Not only to his close friends but to a wide circle of acquaintances his strength of character and his unswerving determination to do what he felt was right made a deep and lasting impression.

Mr. Sterling was married in 1921 to Eugenie M. Spoor of Rochester, N. Y., who survives him.

Mr. Sterling was elected a Member of the American Society of Civil Engineers on July 11, 1927.

---

[1] Memoir prepared by Roy G. Finch and Edward H. Sargent, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1626.

GEORGE HENRY TINKER, M. Am. Soc. C. E.[1]

DIED OCTOBER 1, 1942

George Henry Tinker was born in Lawrence, Mass., on January 18, 1868, the son of William H. and Emma (Beale) Tinker. He was married to Ida A. Leek of Cleveland, Ohio, in 1906. Mrs. Tinker and her two daughters by a former marriage are dead.

Mr. Tinker was educated at the University of Nebraska, in Lincoln, being graduated with the degree of Bachelor of Civil Engineering in 1890. In 1897 he received the degree of Civil Engineer.

From 1890 to 1899, he was engaged in miscellaneous engineering work for city and county organizations in western sections of the United States. From 1899 to 1903 he was employed in engineering positions on western railroads. From 1903 to January, 1904, Mr. Tinker was with the Minnequa Water Company, in charge of location of a canal and of designing wood-stave pipe siphons.

From April, 1904, until he retired in 1939, he was in the employ of the New York, Chicago and St. Louis Railway Company. At first, he functioned as inspector and assistant engineer on important work for the railroad company. In 1907 he was promoted to bridge engineer. In his position of bridge engineer he redesigned and rebuilt many structures requiring strengthening and reconstruction, as well as new bridges in connection with grade-crossing eliminations and other locations.

Although Mr. Tinker's principal employment considered from the viewpoint of years of service, and possibly from the degree of importance, was as a bridge engineer, he acquired a varied experience in other important engineering lines—especially on work in the West during his early years as civil engineer.

He was outstandingly careful and conscientious in his work, which caused him to be held in great respect and esteem by his associates and other engineering representatives who admired his ability.

An insight into his character and activities is afforded by the following:[2]

> "Mr. Tinker was actively interested in engineering and scientific societies. He was a life member of the American Society of Civil Engineers and of the Cleveland Engineering Society, and had served on committees of both organizations for many years. Before his retirement he had also been an active member of the American Society for Testing Materials and served on several joint committees.
>
> "Mr. Tinker joined the American Railway Engineering Association in 1907, and from that date until his retirement, he took an active interest in all its work, seldom missing an annual meeting. He was a member of Committee 15 for many years, retaining his committee membership after his

---

[1] Memoir prepared by a Committee of the Cleveland Section consisting of Robert Hoffmann, Hon. M. Am. Soc. C. E., and Wendell P. Brown and B. R. Leffler, Members, Am. Soc. C. E.

[2] *Bulletin*, A. R. E. A., January, 1943.

retirement. His advice and counsel were always welcome and were of great benefit to the Association. He was a real student in matters pertaining to his profession."

For many years Mr. Tinker was secretary-treasurer of the Cleveland Section of the Society. He also gave much of his time and effort to promoting the welfare of other engineering organizations.

For recreation Mr. Tinker was interested in nature studies and incidental to them he developed the hobby of mountain climbing and hiking. He was a member of the Appalachian and of the Rocky Mountain climbers clubs and of the Cleveland Hiking Club.

Although not active in politics, he was interested in the Republican Party. His church affiliations were Protestant Episcopalian.

Mr. Tinker was elected a Member of the American Society of Civil Engineers on August 31, 1909.

ENRIQUE AUGUSTO TOUCEDA, M. Am. Soc. C. E.[1]

DIED OCTOBER 20, 1943

Enrique Augusto Touceda, the son of Gabriel Touceda and Maria Augustina Solis Touceda, was born on April 15, 1862, in Matanzas, Cuba. He received his early education in Cuba; later attended Georgetown University at Washington, D. C.; and was graduated from Rensselaer Polytechnic Institute at Troy, N. Y., in 1887 with the degree of Civil Engineer.

During his student days he was employed as a chemist and from 1887 to 1893 he was metallurgist and mechanical engineer for the Troy Steel and Iron Company. From 1893 to 1895 he was employed as a metallurgist and mechanical engineer by the Walter A. Wood Company of Hoosick Falls, N. Y., manufacturers of harvesting machinery. From 1895 to 1898 Mr. Touceda was chief metallurgist of the Troy Steel Products Company, a reorganization of the old Troy Steel and Iron Company. In 1899 he opened an office and laboratory as a consulting metallurgical engineer which he maintained actively until his death.

In 1906 Mr. Touceda was appointed professor of metallurgy by Rensselaer Polytechnic Institute, remaining for thirty years as an active professor. In 1937 he was appointed Emeritus Professor of Metallurgy and consultant to the department of metallurgy, which position he held at the time of his death. While engaged in teaching, Mr. Touceda was constantly employed as a consulting engineer and in his laboratory in an endeavor to improve metal products. As a result of his investigations, in 1912 he was appointed consulting engineer by the American Malleable Castings Association, later the Malleable Founders' Society. He continued in this capacity until his death.

---

[1] Memoir prepared by Ernest D. Hendricks, M. Am. Soc. C. E., in collaboration with Helen Callan, former Secretary to the deceased, and Charles P. Wagoner, Albany, N. Y.

Professor Touceda was the first recipient of the John A. Penton Gold Medal awarded by the American Foundrymen's Association (A.F.A.) for conspicuous service to the foundry industry. From June 21 to 23, 1922, he delivered an exchange address for the A.F.A. before the Institute of British Foundrymen at Birmingham, England. The subject of this paper was American methods of manufacture of malleable castings, and some data in connection with the finished products were also presented. Following the presentation of this paper he was awarded a gold medal by the Institute for outstanding contributions to the science of metallurgy. In addition, Professor Touceda contributed to the technical press and to the *Transactions* of the American Society of Mechanical Engineers, American Institute of Mining and Metallurgical Engineers, Society of Automotive Engineers, and A.F.A.

For many years Professor Touceda acted as a consultant for the Draper Corporation of Hopedale, Mass., and the York Manufacturing Company of York, Pa. Many other companies also called on him for advice and assistance in solving their problems. He considered the improvement in the metallurgical and mechanical properties of malleable cast iron as his greatest accomplishment.

For forty years Professor Touceda was a member of the American Society for Testing Materials. He also belonged to the Society of Automotive Engineers, American Institute of Mining and Metallurgical Engineers, American Chemical Society, Albany (N. Y.) Society of Engineers, Society of Engineers of Eastern New York, A. F. A., Army Ordnance Association, the Institute of British Foundrymen (honorary member), British Iron and Steel Institute, Technical Association of the Paper and Pulp Industry, Sigma Xi, University Club of Albany (charter member), Albany Country Club, the Union League Club of Chicago, and Chi Phi fraternity. Professor Touceda was a licensed professional engineer in New York State.

Although Professor Touceda devoted most of his time to metallurgy, he also investigated mining properties in Cuba, Mexico, Alaska, and Cobalt, Canada, for clients who recognized his honesty and ability.

He was a scientist who was constantly endeavoring to improve the materials used by the engineering profession. In collaboration with his son he discovered an alloy which is extensively used in war work. Although Mr. Touceda was not actively engaged in the practice of civil engineering, the products of his knowledge and laboratory experiments are constantly being used by civil engineers. His laboratory in Albany was utilized not only for the analysis of chemical and mineral substances but also for the purpose of instructing students in metallurgy and kindred subjects. He gained his outstanding success as a teacher in his alma mater.

Professor Touceda came from a Cuban family of patriots who were driven from Cuba during the revolutions preceding the Spanish-American War. After his graduation from Rensselaer Polytechnic Institute and prior to the liberation of the Cuban people from Spanish tyranny, Professor Touceda contributed to the fullest extent of his financial ability to the cause of the Cuban patriots. Following the Spanish-American War his philanthropy did not cease but was devoted to the needy whenever his help was solicited. However, he always observed the principle of anonymity—no one but himself and the recipient was aware of his gifts.

A charter member of the University Club of Albany, he was usually present every Saturday night to preside over the whisky-poker game. It was an educational privilege to hear his terse and wise remarks on the topics of the day and his descriptions of contemporaneous public characters whom he knew. Of a most unassuming and modest disposition himself, his gentle satire would deftly prick the bubble of publicity-seeking braggarts. His memory will long remain fresh in the minds of his friends, and his character has left an enduring impression that will never be erased.

On February 27, 1892, he was married to Mary Augusta Ritschell, of Troy. He is survived by his widow; two sons, Enrique G. Touceda, Lieutenant Commander, U. S. Navy, and Arturo A. Touceda; three grandchildren; and a sister, Mrs. Arturo Padro.

Professor Touceda was elected a Member of the American Society of Civil Engineers on January 3, 1900. He became a Life Member in January, 1933.

THOMAS THOMSON TOWLES, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 27, 1943

Thomas Thomson Towles, son of the late Dr. William Beverley Towles of "Lynnwood," Cumberland Court House, Va., and Mary Ellen Thomson of Fort Motte, S. C., was born on March 2, 1886, in Charlottesville, Va., where his childhood and early manhood were passed. His father was for many years a distinguished member of the faculties of the University of Virginia at Charlottesville and the University of Vermont at Burlington. Mr. Towles' grandfather, Dr. William B. Towles I, built "Lynnwood."

On both sides of his family, he was descended from early Colonial ancestry. About 1650 the Towles family emigrated from Yorkshire, England, to the Colony of Virginia, settling on the banks of the Rappahannock River at "Towles Point." (A part of the original homestead is standing, still owned by the family.) Mr. Towles' great-great-grandfather, Col. Oliver Towles, Fifth Regiment of the Line, Continental Army, was the first secretary of the Society of the Cincinnati in Virginia and one of the founders. Oliver Towles, Jr., his great-grandfather, was a Major in the War of 1812. His father, Dr. William Towles, left home at the age of sixteen to enlist in the Confederate Army and served with the Army of Northern Virginia for two years before the surrender of Gen. Robert E. Lee at Appomattox. After he was graduated in medicine at the University of Virginia, he became distinguished as professor of anatomy at that institution and at the University of Vermont. Mr. Towles' great-great-great-grandfather on the maternal side, Col. William Thomson of South Carolina, was cited for bravery and formally thanked by the Continental Congress in Philadelphia, Pa., on July 20, 1776, for his defense of Sullivan's Island with his regiment of Rangers (the Third Regiment in South Carolina) on June 28, 1776.

---

[1] Memoir prepared by Mrs. Thomas T. Towles with the considerable assistance of Miss Eleanor Towles, John L. Newcomb, M. Am. Soc. C. E., Douglas Freeman, Editor of *Richmond News,* Richmond, Va., and through the courtesy of H. E. Bralley, Asst. to the President, Stone & Webster Eng. Corp., New York, N. Y.

Mr. Towles' early education was obtained in Charlottesville at the preparatory school of Maj. Horace W. Jones, who had served on the staff of General Lee in the War Between the States.. He entered the University of Virginia at a very early age and completed three subjects—Latin, Greek, and Mathematics—of the four required for the degree of Master of Arts in the Classics by the time he was eighteen years old. Active in athletics at the University of Virginia, he was awarded two letters for two years of varsity football, and he also played tennis and baseball. He was a member of Delta Tau Delta fraternity. Between the time spent in the College of Arts at the University of Virginia and his study of engineering, for two years he was a teacher in the McGuire Preparatory School in Richmond, Va., and at the same time the coach of the football teams.

Following a year with the Bridge Department of the Virginia State Highway Commission as a draftsman on steel highway bridges, he was employed in Panama from 1912 through 1914 by the Isthmian Canal Commission as chief of party and designing draftsman on the machinery for the Miraflores Locks and on the general design of bridges, sewers, and water supply systems.

In September, 1914, he entered the University of Texas at Austin to complete his engineering work started at the University of Virginia. In June, 1915, he received from the University of Texas the degree of Bachelor of Arts; also, from the School of Engineering at that University, degrees in Civil Engineering and the degree of Bachelor of Science in Mathematics.

Upon the completion of this work, he was engaged for two years on maintenance and general construction for the Gulf, Colorado, and Santa Fe Railway Company, a branch of the Atchison, Topeka and Santa Fe Railway Company at Galveston, Tex. When the United States entered World War I, he took an active part in the design and construction of the Naval Aeronautic Station at Pensacola, Fla.

In December, 1918, Mr. Towles entered the employ of the Dupont Engineering Company, serving as construction engineer, division engineer in charge of civil engineering design, and estimator at the Wilmington (Del.) office. In 1921 he became associated with the Department of Public Works of the City of Richmond. For the next five years he was, successively, Chief of the Bureau of Design, Assistant Director of Public Works, Engineer in Charge of City Planning, Chief Engineer of the Zoning Commission, and Acting Director of Public Works.

In 1926 Mr. Towles joined Stone and Webster Engineering Corporation. During the following sixteen years he was engaged almost continuously on major engineering and construction projects and on important engineering investigations and reports, in many parts of the United States, until his retirement because of ill health in May, 1942. He then returned to his home in Charlottesville.

Mr. Towles was an able and cultured engineer of broad experience in a variety of fields of activity. He was also a diligent student of world conditions, historically, politically, and economically. To this study he brought a mind wise, tolerant, and kind.

He is survived by his wife, Alice Otis Bird Towles, to whom he was married on June 11, 1918, at Trinity Church in Galveston; a sister, Eleanor; and a brother, Oliver.

Mr. Towles was elected a Member of the American Society of Civil Engineers on June 19, 1922.

PAUL ANTHONY TROST, M. Am. Soc. C. E.[1]

DIED APRIL 13, 1937

Paul Anthony Trost, long associated with George F. Hardy, M. Am. Soc. C. E., in work for the paper and pulp industry, died on April 13, 1937. He was born on March 9, 1877, at Jersey City, N. J., the third child of Paul and Margaret Trost. After graduation from high school, he attended the U. S. Naval Academy at Annapolis, Md., for a time and was tutored privately in architecture and engineering until 1897. From then until late in 1898 he secured some practical experience and training under several architects and engineers in New York, N. Y., and vicinity.

In November, 1898, Mr. Trost obtained a position in the Engineering Department of the International Paper Company, whose offices were in New York City. He was assigned as assistant to one of that company's division engineers, W. R. Farnsworth, whose headquarters were in Turners Falls, Mass., and who had supervision over new construction and maintenance of the company's mills in Massachusetts, Vermont, and New Hampshire. Mr. Trost held this position until November, 1901, and then conducted a mapping business for several months.

Mr. Hardy was chief engineer of the International Paper Company from 1898 to late 1901, when he opened an office in New York City for general consulting and designing work in the pulp and paper industry. In March, 1902, Mr. Trost joined him as one of his principal assistants, remaining as such until his death. In the year 1918, while Mr. Hardy was New York district manager of Supply Emergency Fleet Corporation, Mr. Trost was also his principal assistant, returning with him to the pulp and paper industry when the war was over. In this association he was prominently identified with many plants in the United States and Canada and was highly esteemed in the industry and by his associates.

Mr. Trost was a licensed professional engineer, and a member of the American Society of Mechanical Engineers and the Engineering Institute of

---

[1] Memoir prepared from records on file at the headquarters of the Society and of the American Society of Mechanical Engineers.

Canada. He was a Thirty-Second Degree Mason and a Knight Templar, and belonged to the fraternal orders of the Elks and Odd Fellows.

His widow, Mildred C. (Smith) Trost, to whom he was married in 1918, survived him but has since died. A stepdaughter, Mildred E. Brush, several brothers, and a sister survive him.

Mr. Trost was elected a Member of the American Society of Civil Engineers on June 11, 1917.

HENRY ALDIS VARNEY, M. Am. Soc. C. E.[1]

Died April 10, 1941

Henry Aldis Varney was born in Gilmanton, N. H., on June 7, 1870, the eldest son of Edwin and Julia (Brett) Varney. His paternal ancestors were Quakers, having moved from Vermont to Gilmanton. His maternal ancestors were settlers in Massachusetts. His father was a surveyor in Newport, R. I.

In 1888 Mr. Varney entered the employ of the late Alexis H. French,[2] M. Am. Soc. C. E., a civil engineer engaged in private practice. This office performed the engineering services required by Brookline, Mass., and, from 1890 to 1894, Mr. Varney supervised the construction of the town's sewers and masonry bridges.

The Town of Brookline established an Engineering Department in 1894, appointing Mr. French, Town Engineer, and Mr. Varney became a member of the engineering staff as an assistant engineer engaged in general municipal engineering work. In 1901 he was appointed first assistant engineer, in charge of the office staff, as well as of park construction and part of the new sewer, surface water drain, and street construction. He served in this capacity until September, 1914, when he was appointed Acting Town Engineer.

For many years Mr. Varney served as consulting engineer for Brett and Hall, of Boston, Mass., and Montreal, Quebec, Canada. This firm specialized in city planning. As a city planner, Mr. Varney's thinking was always four-dimensional: physical, cultural, social, and esthetic, and in his designs the factors of space, color, and massiveness always were incorporated.

In 1915 he was appointed Town Engineer of Brookline.

Unlike most other municipalities, Brookline had a by-law, which established the Engineering Department and required that all engineering work for the various municipal departments, whether design, estimate, construction, study, or report, be under the supervision of the Town Engineer. Such demands required an engineer possessing qualifications of diversified knowledge and experience. Mr. Varney was an ardent student of modern engineering and possessed a keen perception of future development. Although his Yankee background had taught him thrift, he believed that the best engineering and construction methods and practices were the most economical. The result was that the Town of Brookline developed and was making use of the latest engineering practices long before municipalities much greater in size adopted them. He built up the personnel of the department so that its capabilities, integrity, devotion to duty, and achievements are, and might well be, the envy of other municipal engineering departments.

---

[1] Memoir prepared by Walter A. Devine, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 2148.

He was an indefatigable worker, fair, earnest, and honest in his policies and determinations. The two motivating thoughts which were always foremost in his mind were his chosen profession and the rendering to his town of the greatest possible achievement. His personal and professional uprightness was always an inspiration to the members of his staff.

Although Brookline always has been a strictly residential area, the town grew from a sparsely settled town to a large metropolitan residential community during the time that Mr. Varney served as Town Engineer. At times, the growth in many sections was exceedingly rapid, entailing extensive hydraulic, highway, park, sanitary, and structural design and construction and also landscaping. Among the projects designed and constructed under Mr. Varney's direction were a water works filtration system and high service storage reservoir, sanitary sewer and surface water drain systems in various sections of the town; all the bridge superstructures; highways; landscaping for parks, playground, and land about public buildings and several town service buildings; and the construction of one of the first municipal incinerators in New England. Brookline was one of the pioneers in the development and adoption of municipal zoning laws, and the Engineering Department, under Mr. Varney's guidance, made all the necessary studies and final plans. Mr. Varney had his department so well organized and balanced that these great demands upon it were not only met, but accomplished, with the exactitude and proficiency demanded of the best engineering offices.

From 1926 to 1940, as engineer and secretary of the Brookline Planning Board, Mr. Varney spent many hours beyond the demands made upon his time as Town Engineer in planning for the future needs of Brookline. His foresight and consequent able planning saved the town much valuable time and expense. He never deviated from any plan or course of action which he considered right, no matter what pressure was brought upon him. He possessed the courage of his convictions and yielded only in deference to better ones.

On July 1, 1940, after forty-six years of continuous service in the Brookline Engineering Department, Mr. Varney resigned from the department. He returned to Gilmanton, where he always had maintained a summer home. After a brief illness, he passed away on April 10, 1941.

Mr. Varney was married in 1899 to Lucia C. Pillsbury, daughter of Dr. S. B. Pillsbury, of Middletown, N. Y. She died at Brookline on February 20, 1940, after a long illness. Mr. Varney's great devotion to her, especially during her later years, was often the subject of comment among his many friends.

In 1897 Mr. Varney became a member, and in 1927 a senior member, of the American Society for Municipal Improvements. In 1940, in recognition of his forty-six years of service to the Town of Brookline, that society presented him with its veteran's award. He was also a member of the Boston Society of Civil Engineers, as well as Past Master of Brookline Lodge, A. F. and A. M.

Mr. Varney was elected a Member of the American Society of Civil Engineers on April 17, 1917.

WILLIAM WESLEY VARNEY, M. Am. Soc. C. E.[1]

Died July 30, 1943

William Wesley Varney died at the home of his son in Washington, N. J., on July 30, 1943. He was born in Boston, Mass., on September 17, 1864, the son of William Henry and Mary E. (Hoffman) Varney. He studied mechanical engineering for three years at Massachusetts Institute of Technology, at Boston, being a member of the Class of 1886. He was graduated from the University of Maryland Law School, at Baltimore, in 1893 and was admitted to practice in federal and state courts the same year.

His engineering career began as a draftsman for Cramp's Shipyard, Philadelphia, Pa. In 1889 he moved to Baltimore as draftsman in charge and superintendent of construction for the Columbian Iron Works, where he remained until 1892. From 1893 to 1899 he practiced as a consulting engineer. He served as city commissioner (city engineer) and president of the Board of Public Works of Baltimore from 1899 to 1900, and was a member of the Water Board from 1900 to 1903. He again engaged in the practice of consulting engineering from 1903 to 1917. He was Harbor Engineer from 1917 to 1918.

In his later years he testified as expert in patent cases and practiced patent law until shortly before his death. He filed patent applications on Television and Television in Natural Colors, in U. S. Patent Office, January 9, 1892, Serial No. 418,209, "System and Means of Transmitting Optical Impressions in Colors"; January 8, 1892, Serial No. 417,404, "System and Means for Transmitting Optical Impressions" (which covered the transmission of moving pictures in natural colors, from life as well as from films); and January 22, 1892, Serial No. 418,898, "Method and Means of Transmitting Optical Impressions Electrically and Reproducing the Same in Record Form."

He was a member of the American Society of Mechanical Engineers (member of the nominating committee in 1921, chairman in 1922, and ex-officio member in 1923); a Member of the Society of Naval Architects and Marine Engineers; and he served on the American Engineering Council for four years (including a year and a half as treasurer, and two years on the executive board). He was Past Grand Master of the Independent Order of Odd Fellows of Maryland.

---

[1] Memoir prepared by W. Watters Pagon, M. Am. Soc. C. E.

Mr. Varney was a reticent, silent man. He was extremely fond of boating, sailing what is known in the Chesapeake Bay as a "bugeye." He was a member of the Maryland Yacht Club (vice-commodore for two years), and the Corinthian Yacht Club (past-commodore for three years). He served on the Board of Trustees of the First Christian Church of which he was a loyal member. He was a member of the Engineers Club of Baltimore and was an active participant in club activities for many years.

He was married in 1899 to Edith McDonald, who died before him. Surviving are two sons, Dr. William H. Varney and John H. Varney.

Mr. Varney was elected a Member of the American Society of Civil Engineers on April 16, 1918.

ROBERT SPURR WESTON, M. Am. Soc. C. E.[1]

DIED JULY 29, 1943

Robert Spurr Weston was born of early New England Colonial ancestry on August 1, 1869, in Concord, N. H., the son of Lon and Martha Burtch (Greenman) Weston.

At the age of four or five, his parents established a home in Mystic, Conn., and two years later moved to Brockton, Mass., remaining until 1909. Mr. Weston's grandfather on his mother's side was the builder of ships in Mystic and founded the Village of Greenmanville (Connecticut). His father was the owner of the Tuck Manufacturing Company, the makers of fine tools, in Brockton. Following his marriage in 1909, Robert Weston made his home in Brookline, Mass.

In 1891 Mr. Weston was graduated from Amherst College at Amherst, Mass., with the degree of Bachelor of Science, and received the degree of Master of Arts in 1894. During the period from 1894 to 1896 he was a graduate student in chemistry, biology, and sanitary engineering at the Massachusetts Institute of Technology in Boston, and studied in Germany at the University of Berlin during the winter of 1896–1897. From 1913 to 1916 he was assistant professor of public health engineering at the Massachusetts Institute of Technology.

Mr. Weston's professional career began as a chemist. Chemistry was always a subject that he greatly enjoyed, and a field in which he won distinction. One of his earliest engagements was as assistant chemist in the important water filtration experiments and studies at Louisville, Ky., in 1895–1896. His fellow workers included J. W. Ellms, the late George W. Fuller,[2] and the late George A. Johnson,[3] Members, Am. Soc. C. E., whose names have since made sanitary engineering history.

In 1898–1899 he was associated with Mr. Fuller, then a consulting engineer in New York, N. Y.; and during 1899 he joined with the late William Wheeler, M. Am. Soc. C. E., of Boston, in private practice, as a consulting sanitary engineer. During the succeeding years his services were widely sought on many sanitary engineering problems.

---

[1] Memoir prepared by George A. Sampson, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1653.

[3] For memoir, *ibid.*, p. 1678.

On January 1, 1916, he entered a consulting engineering partnership with the writer, his principal assistant engineer, under the firm name of Weston and Sampson, with offices and laboratory in Boston. This association was terminated only by his death. Few engineering partnerships have continued for more than twenty-seven years without any change in the principals and with complete harmony. Mr. Weston was truly an ideal partner.

Mr. Weston served several states and many cities, towns, corporations, institutions, and individuals, in connection with water supply, water purification, sewerage, sewage treatment and disposal, industrial waste treatment, stream and harbor pollution, and other public health problems. During World War I, he was consulting engineer for the United States Shipping Board and the United States Housing Corporation. He was engaged as a sanitary expert in many important litigations, including those on the Chicago Drainage Canal, Delaware River (New Jersey versus New York), and the Connecticut River (resulting from the taking of the Ware and Swift rivers for the Boston metropolitan water supply).

Mr. Weston was coauthor of "Waterworks Handbook,"[4] published in 1916 with new editions in 1918 and 1927, and author of many technical papers dealing with public health subjects. He was always seeking new and better methods of solving perplexing problems, and was recognized by his associates as well abreast of the times. An eminent sanitary chemist has remarked that many of Mr. Weston's theories, although considered radical by some, were in fact ten years in the future. He was an early advocate of the filtration of water supplies to improve public health and their chemical treatment to prevent corrosion of the distribution systems. He was the first to establish the value of prechlorination of colored surface sources as an aid in coagulation and filtration. For his studies on lead poisoning and its prevention, in 1920 he was awarded the Dexter Brackett Memorial Medal by the New England Water Works Association. He also received the Desmond Fitzgerald Medal from the Boston Society of Civil Engineers.

Mr. Weston faithfully attended engineering, chemical, public health, water works, sewage works, and other meetings of associations to which he belonged, and contributed freely to discussion. An indefatigable reader of technical literature, he readily retained the important facts. In addition to his professional career, he greatly enjoyed social contacts, faithfully attended church, and was fond of music. He was kind, understanding, generous to a fault, and especially helpful to young chemists and engineers in whom he took a fatherly interest. One of his outstanding traits was fairness, and his constant purpose was to maintain high ethical standards in his profession. Mr. Weston was a man of large stature and commanding presence.

---

[4] "Waterworks Handbook of Design, Construction and Operation," compiled by A. D. Flinn, R. S. Weston, and C. L. Bogert, McGraw-Hill Book Co., Inc., New York, N. Y., 1916, 1918, and 1927.

In addition to the Society, Mr. Weston had long belonged to the American Institute of Consulting Engineers, American Institute of Chemical Engineers, American Chemical Society, American Public Health Association, American Public Works Association, American Academy of Arts and Sciences, American Water Works Association (honorary life member), New England Water Works Association (honorary member), New England Sewage Works Association, Boston Society of Civil Engineers, Boston Rotary Club, Boston City Club, and the Society of Mayflower Descendants. He was a former president of the New England Water Works Association, Boston Society of Civil Engineers, and Boston Rotary Club; vice-president of the American Institute of Consulting Engineers and the Howard Benevolent Society of Boston; chairman of the Sanitary Engineering Division of the Society; and director of the American Water Works Association. He served on many technical committees and gave freely of his time and energy.

On December 29, 1909, he was married to Josephine Fitz-Randolph, of Plainfield, N. J., who, together with a son, Joseph Fitz-Randolph, and a brother, Lawrence G. Weston, survives him. A daughter, Elizabeth, died on September 28, 1932.

Mr. Weston was elected an Associate Member of the American Society of Civil Engineers on February 5, 1902, and a Member on October 31, 1911.

DANIEL MERRICK WHEELER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 29, 1943

This is the life story of a man who practiced his profession actively as a civil engineer for more than seventy-five years.

Daniel Merrick Wheeler was born on July 3, 1846, in Rutland, Mass., on Wheeler Farm (acquired by an ancestor in 1756 and now owned by one of the fifth generation in succession).

Mr. Wheeler attended school at Worcester Academy in Worcester, Mass., and in 1867 and 1868 he was a special student at Massachusetts Institute of Technology at Boston. He was a member of the first class to be graduated from that institution. Early in 1943 he received the congratulations of the Alumni Association on the seventy-fifth anniversary of his graduation.

His first work was in Massachusetts where for ten years, 1869 to 1879, he was a member of the firm of Buttrick and Wheeler, Civil Engineers, of Worcester. In this capacity, he was, from 1869 to 1870, engineer in charge of surveys for the Lancaster Railroad Company; from October, 1870, to February, 1872, locating engineer for the Massachusetts Central Railroad Company; from 1872 to 1873, chief engineer for the Worcester and Shrewsbury Railroad Company; from 1873 to 1876, chief engineer for the Commonwealth of Massachusetts in connection with the building of a very large state hospital at

---

[1] Memoir prepared by Edmund R. Kinsey, M. Am. Soc. C. E.

Bloomingdale, Mass.; and, from 1876 to 1878, engineer on various railroad surveys and miscellaneous construction work around Worcester.

At the age of thirty two, Mr. Wheeler was appointed chief engineer of the Magdalena Railroad in the United States of Colombia, South America, where, from November, 1878, to December, 1880, he located and built a railroad with one half of its mileage either on trestles or in tunnels.

He then returned to the United States and within a month was made assistant engineer of the St. Louis-San Francisco Railway Company. In that position, from January, 1881, to January, 1884, he had charge of the construction of part of the main line of that railway through the Ozark Mountains in northwestern Arkansas, including the Boston Mountain tunnel and much heavy mountain work. He also located and built feeder lines in southwestern Missouri.

This was a period of almost feverish railroad building in the Middle West, and Mr. Wheeler moved from job to job, sometimes even overlapping, as fast as work was completed. His services were in great demand, and, in rapid succession, he held the following positions:

January, 1884, to March, 1885—assistant chief engineer of the Burlington, Cedar Rapids and Northern Railway Company (later a part of the Chicago, Rock Island and Pacific Railway Company), building railroads in Iowa, Minnesota, and Dakota Territory.

March, 1885, to April, 1886—assistant chief engineer of the St. Paul and Northern Pacific Railway Company, constructing a double-track railroad from St. Paul, Minn., to Minneapolis, Minn., with several branch lines (the work including the Westminster Tunnel in St. Paul, a double-track bridge across the Mississippi River at Minneapolis, and the Northern Pacific shops at Como, Minn.).

In 1886—chief engineer of the Duluth, Watertown and Pacific Railway Company (later part of the Great Northern Railway Company) with headquarters at Watertown, S. Dak. (then Dakota Territory); in the same year, chief engineer of the Wisconsin, Minnesota, and Pacific Railroad Company, building for the latter company a road from Eagle Lake, Minn., to Mankato, Minn.

January 1, 1887, to April, 1888—division engineer and later acting chief engineer of the Minneapolis, St. Paul and Sault Sainte Marie Railway Company, in charge of work from Minneapolis across Minnesota, Wisconsin, and Michigan to Sault Sainte Marie, Mich., and also from Minneapolis into North Dakota.

April, 1888, to February, 1892—chief engineer and superintendent of the Winona and Southwestern Railway Company with headquarters at Winona, Minn., building a railroad from Winona to Osage, Iowa, and in charge of operation; at the same time, chief engineer of the Winona Bridge Railway Company in charge of construction on the lower drawbridge over the Mississippi River at Winona (the draw span of which was 444 ft long, at that time the longest on the Mississippi River), and building a line from Marshland, Wis., to the Winona Bridge.

February, 1892, to September, 1893—in charge of construction of an extension of the Missouri, Kansas and Texas Railway Company from Boonville, Mo., to St. Louis, Mo.

The railroad building boom was nearly completed. Mr. Wheeler, who had worked on more than one thousand miles of railways, returned to Massachusetts and in 1894 was appointed state inspector of railroads by the Railroad Commission of Massachusetts. He remained in this capacity until July 3, 1916, when, because he was seventy years old, he was retired by provision of law. As a matter of fact, however, he was only seventy years young.

In April, 1917, Mr. Wheeler was appointed consulting engineer of the Berkshire Street Railway Company, operating street railways in Pittsfield, Mass., and other towns, and interurban lines reaching south into Connecticut, north into Vermont, and into the State of New York. In February, 1921, he became chief engineer of the Berkshire System, holding that position until his death on September 29, 1943. He worked actively until after his ninety-seventh birthday. Thus, although he was retired because of age when seventy, he continued for twenty-seven years longer in the practice of his profession.

During such a busy life there were naturally many interesting experiences. About two years after his graduation, Mr. Wheeler became dissatisfied with the engineer's chain then in common use, and he searched for some kind of improvement. A Worcester firm, making the women's hoop skirts of the period, used long, narrow steel bands in the process. Mr. Wheeler obtained some of this material, experimented in methods of using it, and finally adopted a method of putting spots of solder on the bands. On these spots the foot marks were accurately placed. Thus the steel tape in common use today was discovered.

When the Massachusetts Institute of Technology moved from Boston, Mr. Wheeler was accorded the honor of turning the key in the old building in which he had pursued his studies, and then joining the ceremonial procession to Cambridge.

It was natural for Mr. Wheeler to be deeply interested in the history of the New England states where his forebears participated actively in early settlements, in repelling attacks by Indians, in the French and Indian War, and in the Revolutionary War. Acting for the Massachusetts Historical Society, he rebuilt Fort Massachusetts on the ruins of the old fort on the Hoosac River near Williamstown, Mass. This fort figured prominently in the early Indian wars and in the Revolutionary War, and has been under four flags—Colonial, French, British, and United States.

Mr. Wheeler had a practice which is commended to all engineers—he kept diaries, and he kept them faithfully. He began this practice on January 1, 1867. Among other records, he wrote down the mileage traveled each day, at the end of each diary he noted the mileage traveled in the year, and from year to year he computed the total. There are seventy-seven diaries in this series; the last entry, made on August 31, 1943, indicated a grand total of 1,834,511 miles (approximate average of once around the earth for each of seventy-six years) of travel during his long and busy life.

In his professional practice and in his personal life Mr. Wheeler had one compelling motive—he sought to learn the right thing to do. Once satisfied on that point, nothing swerved him from the indicated course. However, the stern application to duty which he required of himself and those under him and the driving force of his executive capacity were in no way inconsistent with his kindly personality toward people, nor with his great love of beauty in nature and in art.

As a hobby he collected pictures of covered wooden bridges and had a surprisingly large collection. He also was an admirer of architectural beauty, particularly as expressed in old New England churches, of which he had a very fine collection of pictures.

Mr. Wheeler had a keen sense of humor, and as long as he lived he exchanged clippings of funny stories with correspondents. He was vigorous all his life and at ninety drove his own car and could outwalk many men forty years younger. The writer knows about that. He was a Christian gentleman who lived his religion seven days in each week and all his life.

Mr. Wheeler came from early New England stock, and his American genealogical record is virtually complete. Of sixty-four ancestors in the seventh generation back, he had records of fifty six, all of whom were born in England and came to America in the early part of the seventeenth century. His records of the succeeding generations which converged in him are complete. His American descent from the Wheeler line traces from Richard Wheeler, born in England in 1611, who was killed by Indians in 1675 at Lancaster, Mass., during King Philip's War. Fourteen other relatives also were massacred in this war.

Other lines of his forebears show many names familiar in New England history. Lest someone might think that dominant characteristics are always conveyed through male lines of descent, it may be noted that an explanation of Mr. Wheeler's long life may be drawn from the fact that his mother, Susan Halladay, lived to be ninety two; her mother, Susanna Adams, ninety two; her mother, Susanna Underwood, ninety eight; and her mother, Hannah Richardson, ninety five.

Mr. Wheeler was married to Arvilla Jane Putnam on November 19, 1868. As might be inferred from her name, Mrs. Wheeler too was from old New England stock, and her ancestral background contains many names familiar in early American history. She died on October 9, 1918.

Mr. Wheeler is survived by a son, Milton Merrick, and by three daughters, Inez Viola (Mrs. Edmund R. Kinsey), Clara Susan (Mrs. Charles F. Harris), and Grace Leota.

Thus ends the story of an all-American gentleman who used his excellent technical training to contribute much to the development of the United States during a long and useful life.

Mr. Wheeler was elected a Member of the American Society of Civil Engineers on March 6, 1901. He became a Life Member in January, 1926.

JAMES GILBERT WHITE, M. Am. Soc. C. E.[1]

DIED JUNE 2, 1942

James Gilbert White was born at Milroy, Pa., on August 29, 1861, the son of the Rev. J. W. White and Mary (Beaver) White. He was graduated from the Pennsylvania State College, at State College, in 1882 with the degree of Bachelor of Arts, and in 1884 received the degree of Master of Arts from the college. At the time of Mr. White's graduation, instruction at the Pennsylvania State College was largely in mathematics and physics. Since he desired to supplement his education, on receiving a fellowship in electrical engineering, he entered Cornell University, at Ithaca, N. Y., being granted the degree of Doctor of Philosophy in 1885. Leaving Cornell, Mr. White became an instructor in physics at the University of Nebraska, in Lincoln, where he taught from 1885 to 1887.

Mr. White was married in Lincoln on December 15, 1886, to Maud Mullon, who survives him, as do one son, J. Dugald White; a sister, Mary B. White; and a brother, John Beaver White.

In the spring of 1887, Mr. White organized the Western Engineering Company with Dugald C. Jackson, M. Am. Soc. C. E., and the late Harris J. Ryan, as associates. For the Western Engineering Company, Mr. White solved some electrical engineering problems in the development of steam and hydroelectric generating stations and electric railways. He also supervised a number of small but important installations. However, the Western Engineering Company was dissolved when his principal associates desired to return to the field of teaching. In 1890 he established the firm of J. G. White and Company, Inc., in New York, N. Y.

During the decade from 1890 to 1900, Mr. White directed many pioneer installations, including steam and hydraulic electric power plants, high tension transmission lines, and one of the first high-speed interurban electric roads—the Buffalo-Niagara Falls Electric Railway. Mr. White's work during this period brought him national and international distinction. English capitalists sought his advice and skill on the construction of a large amount of engineering work in Australia. In 1900 these connections led to the establishment of an English branch of the firm, J. G. White and Company, Ltd. Through that company, Mr. White supervised a large amount of power plant and electric railway and other electrical construction work, including an extensive underground conduit street railway system for the London County Council in London, England. During the next twenty years, Mr. White's English and American companies operated extensively in five continents. In addition to the more strictly electrical work, Mr. White was instrumental in forming the Waring White Building Company, Ltd., of London, which built such notable structures as the Ritz and Waldorf hotels in London, the Liverpool Cotton Exchange in Liverpool, England, the great Selfridge Store in

---

[1] Memoir prepared by Gano Dunn, M. Am. Soc. C. E.

London, and many important office and apartment buildings. Mr. White also established a Canadian subsidiary, the Canadian White Company, Ltd., which built important steam and hydroelectric plants and important office buildings in Canada.

In 1913 Mr. White formed The J. G. White Engineering Corporation to assume the functions previously exercised by the engineering and construction departments of the parent company, which since that date has restricted its activities to the investment banking field. The work of Mr. White's American companies is too extensive and too well known to be listed in this memoir, but the range of their activities covers practically all branches of the engineering and construction fields. Mr. White retired from active business in 1928.

Mr. White was distinctly a pioneer and a leader, a useful and inspiring citizen, as well as an eminent engineer. He was a warm friend and commanded the respect and affection of his associates. Throughout his life he maintained an active interest in his alma mater, the Pennsylvania State College, and for many years he was a trustee of that institution, at which he established loan funds for students, and contributed generously to an emergency building fund of the college. His technical affiliations were many, including membership in the American Institute of Electrical Engineers. During his long business career, he was a member of many clubs and organizations in New York and London.

Mr. White was elected a Member of the American Society of Civil Engineers on March 2, 1904.

HARRY JAMES WIGHT, M. Am. Soc. C. E.[1]

Died July 30, 1943

Harry James Wight, the son of Ambrose Elerson Wight and Mary (Casler) Wight, was born on January 31, 1881, in Ilion, N. Y. His early boyhood was spent in Ilion. In 1887 his family moved to Youngstown, Ohio, and, in 1889, to Chattanooga, Tenn. Mr. Wight lived in Chattanooga until 1891, and then went to Harriman, Tenn. In 1897 he moved to Knoxville, Tenn., where he resided at the time of his death. He was educated in the grammar and high schools of Youngstown, Chattanooga, and Harriman. His technical education was obtained through private instruction.

In the spring of 1902, he entered the employ of the Louisville and Nashville Railroad Company in the Engineering Department, advancing from rodman to instrumentman and finally to construction engineer. In 1908 he left the railroad to become engineer on construction of railroads and highways in Cuba for the William J. Oliver Company, a railroad contracting organization. Upon completion of this work, in eighteen months, Mr. Wight returned to the United States as engineer on railroad construction in North Carolina until 1915, when he accepted a position with the Chicago, Burlington, and Quincy Railroad Company as a valuation engineer. In 1918 he went to Washington, D. C., to do similar work for the Interstate Commerce Commission.

---

[1] Memoir prepared by Leonard C. Bailey, Assoc. M. Am. Soc. C. E.

Because it was urgent for Mr. Wight to be with his family in Knoxville, he resigned from the employ of the Interstate Commerce Commission in March, 1923, and became office engineer in the Department of Public Service of the City of Knoxville. In December, 1927, he was appointed city engineer of Knoxville, remaining in this position until he was promoted to Director of Public Service in August, 1933. During his tenure of office, Mr. Wight was in charge of one of the largest engineering and construction programs ever undertaken by the city. Many miles of streets, bridges, and sanitary and storm sewers were built. Consultation with many governmental agencies regarding expansion of their programs greatly augmented the activities of the department. Mr. Wight was efficient, accurate, and painstaking in every job undertaken; his men followed his leadership willingly and eagerly.

In January, 1938, he resigned as Director of Public Service and accepted employment as engineer of surveys for the Knoxville Housing Authority on the planning and construction of three large low-rent public housing projects.

In June, 1941, on completion of the housing projects, he was employed by the Tennessee Valley Authority as a cost engineer in the Construction and Maintenance Division. In this capacity, he handled construction costs on highways, bridges, sewers, and other types of work in connection with building Watts Bar Reservoir on the Tennessee River. At this time, Mr. Wight's health declined greatly, and he resigned about January, 1942, in order to recuperate. Although he regained some of his strength, he never fully recovered. Because of the man-power shortage, he returned to the city engineer's office in Knoxville as office engineer to assist on the type of work he knew so well, remaining until his death on July 30, 1943.

He is survived by his widow, Carrie Hoefer Wight, to whom he was married on November 23, 1927, and by his daughter, Harriet Jane Wight.

A tireless and energetic worker, his character, honesty, and consideration for his fellow men created loyalty in, and left an indelible impression on, all those who had the privilege of working with him.

He was a member of the First Methodist Church of Knoxville. He was a registered Professional Engineer of the State of Tennessee.

Mr. Wight was elected a Member of the American Society of Civil Engineers on October 17, 1938.

EDGAR KENNARD WILSON, M. Am. Soc. C. E.[1]

Died March 31, 1943

Edgar Kennard Wilson was born in Southington, Conn., on November 14, 1878, the son of the Rev. Joseph K. and Lucy Stone (Taylor) Wilson.

Mr. Wilson was educated at Worcester Academy at Worcester, Mass., and University of Maine at Orono. He belonged to Sigma Chi fraternity.

From 1898 to 1905 Mr. Wilson was engaged in surveying and construction, advancing through the usual grades, from chainman to chief of party. In 1905 he was assistant engineer in charge of construction for the Philadelphia and Western Railroad. From 1906 to 1909 Mr. Wilson worked with the engineering force of the Panama Canal, as transitman in charge of estimates on the Culebra Division and later as assistant in the office of the resident engineer on layouts and design of railroad yards, trestles, and a suspension bridge. After 1909 Mr. Wilson spent some time in private practice in Illinois, and as assistant engineer of the Water District in Portland, Me.

He joined The Pitometer Company, Engineers, in 1912 and continued to serve this firm with great skill and fidelity for thirty-one years until his death, except for five months during World War I. At this time he volunteered for service in the U. S. Army, and, after training at Camp Humphreys, Virginia, was commissioned Captain of Engineers.

In 1918 Mr. Wilson returned to The Pitometer Company, later becoming chief engineer and treasurer. He was engaged actively on hydraulic experiments for the development of the pitometer and on the design of extensions for water distribution systems in many cities throughout the United States.

His work also included the analysis of flow for the betterment of trunk-main conditions in city water works—a service to which he contributed much in conception of methods as well as organizing skill. Mr. Wilson was widely known among water works men in all parts of the United States for his knowledge of distribution problems. He was the author of a number of valuable papers[2] on water works subjects, which added greatly to his reputation. During his association with The Pitometer Company he won the esteem of his associates for his sterling character and professional skill. He also was vice-president and director of the Pitometer Log Corporation.

Mr. Wilson belonged to American Water Works Association, serving as chairman of the New York section; New England Water Works Association;

---

[1] **Memoir prepared by Edward S. Cole, M. Am. Soc. C. E.**

[2] For example, see the following papers by Mr. Wilson: "Keeping Water Works Maps and Field Records Up to Date," *Water Works Engineering,* Vol. 83, No. 16, July 30, 1930, pp. 1143–1144 and 1177–1178; "Maps and Field Records," *Journal,* Am. Water Works Assn., Vol. 23, No. 6, June, 1931, p. 809; "Trunk Main Surveys," *Journal,* Am. Water Works Assn., Vol. 24, No. 5, May, 1932, p. 669; "Why Valves?" *Journal,* Am. Water Works Assn., Vol. 26, No. 1, January, 1934, p. 111; "Condition of Mains in Typical American Cities," *Journal,* Am. Water Works Assn., Vol. 28, No. 9, September, 1936, p. 1304; "Carrying Capacity of Mains in Typical American Cities," *Water Works Engineering,* Vol. 89, No. 18, September 2, 1936, p. 1130; and "Finding Lost Water," *Engineering News-Record,* Vol. 118, No. 22, June 3, 1937, p. 819.

Montclair Society of Engineers; and Railroad Machinery Club. For several years he was on the New Jersey Mosquito Extermination Commission.

He was a devoted member of Union Congregational Church, Upper Montclair, N. J., which he served as trustee and custodian of property.

In 1909, he was married to Ida H. Pinkham of Portland. Mr. Wilson is survived by his widow and by his daughter, Elizabeth (Mrs. Dura S. Bradford), and two grandchildren. Of old New England stock on both sides of his family, he lived up to the best traditions of his patriot ancestors.

Mr. Wilson was elected an Associate Member of the American Society of Civil Engineers on May 28, 1923, and a Member on October 1, 1926.

SAMUEL LAMSON WONSON, M. Am. Soc. C. E.[1]

Died September 30, 1943

Samuel Lamson Wonson, son of Gardner Kidder Wonson and Harriet Augusta (Chase) Wonson was born at Gloucester, Mass., on July 1, 1877. The first of the Wonson family settled in Gloucester in 1720, having come from Scotland where the Wonson clan, of Scandinavian origin, had lived for several centuries.

Mr. Wonson's boyhood was spent in Gloucester where he attended the public schools. An episode characteristic of the young Sam is the following: On the opening day he was taken to school dressed according to the then current style for boys living on "the right side of the tracks." Upon returning home he announced in no uncertain terms that his clothes, of what a few years later would have been best described as a "Little Lord Fauntleroy outfit," were definitely out and that, before he would return to school, he must have trousers with patches on them like the other boys. This striving to be a good fellow among men remained with Sam throughout his life.

Sam's father died when the boy was only eight, but this did not prevent the completion of his schooling. At Harvard College, he was an outstanding student, being graduated in 1899, Magna Cum Laude, with the degree of Bachelor of Arts. Further evidence of his scholarship was his election to membership in Phi Beta Kappa—although modesty prevented his key from ever being displayed. After leaving Harvard, Mr. Wonson spent two years at the Massachusetts Institute of Technology, being graduated with the degree of Bachelor of Science in Civil Engineering in 1901. The following year was spent at M.I.T. as Assistant in the Department of Civil Engineering, teaching surveying. He then entered the service of the American Bridge Company where, between June, 1902, and April, 1904, he was a draftsman in the erecting department, and, from 1904 to 1907, Assistant Engineer. During this period he was in charge on the ground of the erection of a railroad bridge over Rio d'Agua in Colombia, South America.

---

[1] Memoir prepared by W. J. Burton and C. E. Smith, Members, Am. Soc. C. E.

This engagement was followed by one which took Mr. Wonson to Mexico City as Assistant Bridge Engineer of the Mexican Railway Company. The three years ending in July, 1910, were spent in the design and reconstruction of both substructures and superstructures for many of the bridges on that railroad.

In August, 1910, desiring to return to the United States, Mr. Wonson accepted an offer from C. E. Smith, M. Am. Soc. C. E., then Bridge Engineer of the Missouri Pacific, to become the latter's assistant in the very large program of bridge work being undertaken. In this work Mr. Wonson's training, supplemented by those traits usually associated with the combination of Scotch and New England ancestry, made him a very valuable man. Because much of its mileage was in the lower Mississippi Valley, the Missouri Pacific had a greater mileage of bridges and trestles than any other large railroad. Among these were some thirty draw spans over navigable (or theoretically navigable) streams. There were more than ten thousand bridge openings and some 150 miles of untreated timber trestles, all of which had been designed for light locomotives. The originally plentiful local timber supply had resulted in the use of untreated timber to such an extent that, when increasing timber prices compelled the change to more permanent construction, there was a near crisis, and great skill was required to make the best of the very limited funds then available.

Mr. Wonson became Bridge Engineer of the Missouri Pacific in 1913 and ten years later was promoted to Assistant Chief Engineer, under the late E. A. Hadley,[3] M. Am. Soc. C. E. About this time the work of bringing the railroad to a higher standard for efficient transportation was greatly accelerated, and Mr. Wonson's promotion brought with it the responsibility for all parts of the construction program. Each year this called for the study and design of a very considerable number of projects. Although many of these were relatively small and followed standardized practice and plans, a not inconsiderable number involved choices between possible alternates and thus much detailed study. No job was too small or too large for Mr. Wonson's personal careful study. He never avoided onerous details by merely passing these on to subordinates. Every conceivable contingency was considered and appropriately provided for. He naturally dealt with a great many contractors and among these, which included some of the best known in the United States, he was regarded as eminently fair, never taking advantage of mere technicalities but at the same time always looking out for the rights of his employer. He recognized that a reputation for fair dealing, when unforeseen contingencies arise, pays in the long run through lower contract prices.

An idea of the extent of planning and supervision under Mr. Wonson's direction is had from the fact that during his twenty years as Assistant Chief Engineer some sixty thousand separate improvement projects were completed at a total cost of about $150,000,000.

Mr. Wonson never was married. Shortly after moving to St. Louis, Mo., he brought his mother to live with him, and he was devoted to her. Even after her death in 1930, he maintained the home alone, as nearly unchanged as possible. His only avocation was his collection of United States postage stamps, with which he spent many long evenings.

---

[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1547.

He was an active member of the American Railway Engineering Association, taking part in the work of two committees—those on Economies of Railway Location and Operation, and Waterways and Harbors. He was also a member of the St. Louis Engineers' Club, The Circle Club, and Delta Upsilon fraternity. He was a Baptist, being a member, and at one time a deacon, of the Second Baptist Church in St. Louis. His funeral in St. Louis was followed by burial in the Wonson family lot in Mt. Pleasant cemetery at Gloucester. His nearest surviving relatives are cousins and an aunt in Massachusetts.

Mr. Wonson was elected an Associate Member of the American Society of Civil Engineers on October 5, 1909, and a Member on October 11, 1920.

EDWARD LOWREY WOODRUFF, M. Am. Soc. C. E.[1]

Died April 1, 1943

Edward Lowrey Woodruff was born on October 8, 1851, in Buffalo, N. Y., the son of Gen. Israel Carle Woodruff, Corps of Engineers, U. S. Army, and Caroline A. (Mayhew) Woodruff.

With two brothers and three sisters his early years were spent in Washington, D. C. There he was educated at the Emerson Institute and the George Washington University (then known as Columbian College). At nineteen years of age he left the latter school to attend the Columbia University School of Mines, New York, N. Y., during the years 1870 and 1871.

During the crucial years of the Civil War his father was second in command to Gen. J. G. Barnard of the City of Washington, and some of Mr. Woodruff's most vivid recollections were of that period. His boyhood services then were much the same as those of Boy Scouts of the present day, filling canteens and performing other similar tasks for the soldiers constantly passing through Washington. His pleasantest memories included meeting Abraham Lincoln and playing games with the president's sons, Tad and Willie. Still a boy at the close of the war, he was introduced to Gen. Ulysses S. Grant and Gen. Robert E. Lee.

During an unbroken professional career of fifty-two years Mr. Woodruff's engineering and architectural talents were devoted to the public service of his country. This doubtless was a natural heritage, for his family history is replete with distinguished public service. His mother was a direct descendant of Thomas Mayhew, 1593–1682, "Patriarch to the Indians, the Worshipful Governor & Chief Magistrate of the Island of Martha's Vineyard, Proprietary of Martha's Vineyard, Nantucket & the Elizabeth Isles, and Lord of the Manor of Lisburg in North America" (see "Thomas Mayhew," by Lloyd C. M. Hare). His father's family home was in Trenton, N. J., and his forebear, Aaron Dickinson Woodruff, was that state's attorney for twenty-four years. Mr. Woodruff's father and two brothers pursued distinguished careers in the U. S. Army.

---

[1] Memoir prepared by R. R. Tinkham, M. Am. Soc. C. E.

Immediately on leaving school, in 1872, Edward L. Woodruff launched a career in the civil service of the federal government as noteworthy as the military service of his brothers. He was employed as assistant superintendent of construction by the U. S. Lighthouse Service, and in that capacity, and successively as superintendent of construction and assistant civil engineer, served in the 3d Lighthouse District, New York, for eighteen years. Early manifestation of those talents that were to earn for him eminence as a lighthouse engineer was rewarded by a call to Washington during the successive summer periods of 1875, 1876, and 1877 as special assistant to Joseph Henry, the great physicist, who was Director of the Smithsonian Institution from its foundation in 1852 until his death in 1878, and who at that time was also chairman of the Lighthouse Board, the administrative agency of the Lighthouse Service. It was Mr. Woodruff's privilege to assist Joseph Henry in experimental research on lighthouse illuminants and sound fog signals that was to revolutionize lighthouse practice and equipment.

During the period of his service in New York he designed, constructed, and equipped eight major light stations in the waters of Connecticut, New York, and New Jersey. Notable among these were Race Rock, Stratford Shoals, and Stepping Stones, all of which stand today, with every evidence of permanence, as monuments to his professional integrity. Concurrently many engineering tasks of less import commanded his equally meticulous attention, including sea-wall construction and minor lights and other aids to navigation. An interesting item not long before his departure from New York was his design and construction of a power house and electric plant on Bedloe's Island for lighting the Statue of Liberty.

With the seemingly inexhaustible energy that remained an outstanding characteristic to the end of his life, Mr. Woodruff occupied his own time outside his official duties to meet a discriminating clientele for his architectural talents, and between the years 1876 and 1890 in New York he designed and prepared the plans for many public and private buildings, including two hotels, a boat club, an amusement park, schoolhouses, the chapel of St. John's Church, and some thirty-six residences and seaside homes, from New Jersey to Massachusetts.

In the spring of 1890 Mr. Woodruff accepted the opportunity for greater responsibilities in his chosen career by transferring to Detroit, Mich., which then was headquarters for the Lighthouse Service on the Great Lakes. There, as the principal assistant engineer, he was closely associated with some of the outstanding officers of the Corps of Engineers, including the late Gen. O. M. Poe and the late Gen. William Ludlow,[2] Members, Am. Soc. C. E., and under their direction was responsible for all design and construction of lighthouses and other aids to navigation on Lake Michigan, Lake Superior, Lake Huron, Lake St. Clair, and connecting waters. During the next nine years he designed and directed the construction of eighteen major light stations in those waters, together with appurtenances and minor aids to navigation, constituting a majority of such structures then extant on the Lakes.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1700.

In February, 1899, the growing magnitude of the work on the Great Lakes having made a division necessary, Mr. Woodruff transferred to the newly created 9th District with headquarters at Milwaukee, Wis. It was there, between 1899 and 1908, that he performed some of his most outstanding accomplishments as a lighthouse engineer. Notable among his lighthouses of this period were Racine Reef and White Shoals, both distinctive and monumental in architecture, and both exceptional engineering structures on subaqueous foundations—Racine Reef in 30 ft and White Shoals in 20 ft, subject to the unique forces of winter ice packs and storm seas of Lake Michigan. These, too, may be added to his many monuments that will stand far into the future.

In 1910 the Congress enacted statutory reorganization of the Lighthouse Service by creating the Bureau of Lighthouses in the Department of Commerce, superseding the Lighthouse Board, which since 1852 had functioned with notable credit to the Army, Navy, and civil officers who had directed its activities.. This threefold responsibility under transitory officials had become outmoded, and the new organization was designed to eliminate much duplication and overlapping. Quoting from the Army and Navy Register for March 4, 1939, regarding this transition: "The end sought justified the means—the United States system of lighthouses, lightships, buoys and ranges is now unsurpassed in all the maritime world." To this record Mr. Woodruff was an important contributor.

Between 1908 and 1910 Mr. Woodruff indulged in a well-earned recess from the exacting demands of his career, the first real vacation of his life. During that period he visited Europe, the first of many such visits to England and the Continent since an early voyage to England with his father when they were entertained by the Master (Duke of Edinburgh) and Brethren of Trinity House, the governing agency of the British Lighthouse Service, a visit that indubitably influenced his subsequent career. Afterward he sojourned in Los Angeles, Calif., where he renewed the private practice of architecture he had undertaken in spare time during those earlier years in New York. However, this was but an interlude.

In the reorganization of the Lighthouse Service in 1910, Mr. Woodruff was selected for one of the principal executive posts in the Service—that of Lighthouse Inspector (later Superintendent of Lighthouses) in charge of the 11th District. Pending the transition from residuary officers of the Lighthouse Board administration to the completed personnel reorganization of the Bureau of Lighthouses, Mr. Woodruff served as Superintendent (chief engineering officer) of the 18th District in San Francisco, Calif. In harness once more, he devoted his talents with renewed energy and enthusiasm to the problems of lighthouse engineering on the California coast. It was at this time he designed the lighthouse which may be said, perhaps, to have become the most notable of his career, the Los Angeles Harbor Light Station, constructed at the seaward extremity of San Pedro west breakwater. It is a bas-relief replica of this lighthouse which adorns the façade of the monumental Department of Commerce building in Washington, D. C., as symbolic of the Lighthouse Service. Years later, for his design of this lighthouse, Mr. Woodruff received the Phebe Hobson Fowler Architectural Award (3d prize) from the American Society of Civil Engineers, in 1929—a belated but fitting tribute to the completion of his active professional career.

In 1912 Mr. Woodruff assumed his post as Lighthouse Inspector in charge of the 11th District with headquarters in Detroit, having under his jurisdiction all aids to navigation and their operating appurtenances in the United States waters of Lake Superior, Lake Huron, Lake St. Clair, and their connecting and tributary waterways to the mouth of the Detroit River. Thus he returned to fields essayed two decades earlier replete with products of his own genius; but now he was the "boss," and drawing upon his wealth of knowledge and experience and progressive thought he devoted himself during the next twelve years to perfecting and reimplementing this ancient service to mariners and to directing the activities and developing the talents of the younger engineers of his staff whose feet were on the lower rungs of the ladder he had surmounted. This was a period of accelerated progress in lighthouse engineering, which witnessed the introduction of automatic flashing lights and modern fog signals to multiply the aids to the navigator, and culminated in the radiobeacon which first became a world reality in the United States in 1921. In all of this Mr. Woodruff had an active part, and in the closing years of his career he was instrumental in extending the system of aids to navigation on the Great Lakes to meet the needs of that vast inland commerce which had attained a tonnage greater than that of any other waterway of the world.

In 1924 Mr. Woodruff retired from active service at the age of 72, twice having received executive extensions of service beyond the statutory retirement age of 70 in order that he might complete important undertakings he had initiated. His career as a lighthouse engineer has no parallel in the United States. It is more nearly comparable with those of successive generations of the Stevenson family in Scotland whose services as lighthouse engineers for the Commissioners of Northern Lights in Great Britain have been made famous by the chronicles of Robert Louis Stevenson.

Remarkable as was his professional career, Mr. Woodruff was equally devoted to his family, for whom he had a deep though unobtrusive pride. On August 25, 1875, he was married to Mariana Bateman in Portland, Me. They had three children, Grace L. (the Lady Ashfield, wife of Lord Ashfield, London, England), Josephine H. (the late Mrs. C. Austin Whitehouse, New York), and Leslie B. (Public Service Corporation, New Jersey). He leaves seven grandchildren—two daughters of Lord and Lady Ashfield, three daughters of Mr. and Mrs. Whitehouse, and a son and daughter of Mr. and Mrs. L. B. Woodruff. There are thirteen great-grandchildren. Mrs. Woodruff died in 1937 shortly after they had celebrated their sixty-first wedding anniversary, terminating a remarkably long and close companionship. In 1925 they had returned to the scenes of their early days together to make their home in Flushing, New York City, where they resided during the many years remaining to them.

Mr. Woodruff was catholic in his tastes and interests. Of wiry physique and possessed of limitless nervous energy, he remained physically active until the day of his death, keenly interested in all that transpired about him. He was fond of books and was widely read, often emphasizing a point by classic quotation. He was a member of the Protestant Episcopal Church in which he was an active layman for many years. In Detroit he was an active member of the Detroit Yacht Club and an Honorary Commodore. A deep interest in the early history of his country led him to affiliate with a number of patriotic societies, to membership in which his lineage entitled him. These included Society of Colonial Wars, Colonial Lords of the Manor in America, National Society of Puritan Descendants, and the Military Order of the Loyal Legion. He was a member of the National Geographic Society, and a charter member of the Society of American Military Engineers. During World War I he served by Executive Order under the Navy and was awarded the Victory Medal.

To those of us who had the good fortune of personal association with Mr. Woodruff he was a man of charming personality, a thorough gentleman of innate dignity, kindly and tolerant, possessed of a fine sense of humor, but one who, on occasion, could be roused to righteous wrath by any who sought unscrupulous advantage or unethical and devious influence; for he was imbued with high ethical standards, and with an unswerving integrity to his profession and loyalty to his trust. The beneficence of his personal influence on his subordinate associates was immeasurable. He was at once a mentor and a director, of rare ability and success. He maintained his interest in these men and his friendship for them to the end of his life. When the Lighthouse Service was merged with the U. S. Coast Guard in 1939 by Executive Order, a considerable number of the lighthouse engineers who then were commissioned as officers in the Coast Guard were of Mr. Woodruff's tutelage, and all occupy positions of trust and responsibility. In his later years Mr. Woodruff could contemplate with much satisfaction the part he had taken in the training of these men, and on their part he was regarded with highest esteem and affection.

Mr. Woodruff was elected a Member of the American Society of Civil Engineers on October 3, 1900. He became a Life Member in 1925.

EDWARD RUSSELL ALLEN, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 26, 1941

Edward Russell Allen was born in Towson, Md., on November 22, 1905, the youngest of five children of the late State Senator Newton D. R. Allen and Rose (Heathcote) Allen.

---

[1] Memoir prepared by Carlton M. Soule and Joseph K. Knoerle, Members, Am. Soc. C. E.

Following graduation from the Towson High School in 1922, Mr. Allen entered the Engineering College of the University of Maryland at College Park. He was graduated with the degree of Bachelor of Science in Civil Engineering in June, 1926. While at the university, he was particularly interested in military training activities and in his senior year was Captain of the Reserve Officers Training Corps Company which received the award for being the best drilled. He was an excellent lacrosse player and was named to a defense position on the all-American team.

After graduation from the University of Maryland, Mr. Allen entered the employ of the J. E. Greiner Company, Consulting Engineers, of Baltimore, Md., and remained with that firm until his untimely death. His first assignment was as assistant resident engineer on the construction of the General U. S. Grant Highway Bridge across the Ohio River at Portsmouth, Ohio. This bridge was a wire cable suspension bridge with a main span of 700 ft and side spans of 350 ft.

This project was completed in February, 1927, and Mr. Allen returned to the firm's Baltimore office where, as designer and draftsman, he worked on different types of fixed and movable bridges, piers, bulkheads, and buildings. By January, 1930, he had demonstrated his ability to such an extent that he was appointed assistant chief draftsman and placed in charge of design and preparation of plans for many of the projects on which the firm was engaged. These included the Howard Street Bridge in Baltimore, which later was awarded honorable mention in the annual bridge competition conducted by the American Institute of Steel Construction (A.I.S.C.).

In October, 1930, Mr. Allen was transferred to field work and during the next twelve months served as resident engineer in charge of the construction of a cantilever bridge across the Ohio River at Ashland, Ky. This bridge, with a main span of 739 ft and anchor arms of 431 ft, presented an interesting construction problem since pneumatic steel caissons were used in placing the main river piers.

Upon completion of this bridge, Mr. Allen went to Cumberland, Md., where for the next six months he acted as assistant resident engineer on the construction of the Thomas W. Koon Dam near that city. This dam was built to supplement the existing water supply of the City of Cumberland and was the first dam upon which, to compensate for the varying moisture content of the sand and rock, adjustments in the amount of water used for concrete mixing were continuously made by mechanical means.

When he returned to Baltimore in March, 1932, Mr. Allen was promoted to chief draftsman. In this position, which he held until March, 1936, he was in responsible charge of the design of all structures. Many of the reports which the firm made in connection with the repairs of existing bridges were also his responsibility.

From March, 1936, to January, 1938, Mr. Allen was resident engineer on the construction of the Twenty-ninth Street Bridge over Jones Falls in Baltimore, the complete design for which he had previously made. This bridge is a large highway structure of multiple reinforced concrete arches and is faced throughout with masonry. Its general appearance received much favorable comment.

Following the construction of the Twenty-ninth Street Bridge, Mr. Allen resumed the duties of chief draftsman for a few months. In May, 1938, he was appointed project engineer on the Susquehanna River Bridge, at Havre de Grace, Md.—one of the four included in the "Primary Bridge Program" of the State Roads Commission of Maryland. This bridge is a four-lane highway structure 7,613 ft long between abutment bearings and carries the Governor Nice Highway over both the east and west channels of the Susquehanna River. Since the bridge was to be a toll structure and financed by the sale of revenue bonds, the engineering report on the project preliminary to financing was of greater scope than would have been required for a toll-free crossing. When the engineering report had been completed, Mr. Allen prepared the contract, plans, and specifications; and, after contracts had been let, he supervised construction. The bridge, completed in October, 1940, was awarded first prize by the A.I.S.C. for the most beautiful steel bridge built in the United States in 1940. Mr. Allen's part in the conception and successful conclusion of this project was substantial, and his associates and many friends greatly regret that he was not living when the award was made.

Immediately after the completion of the Susquehanna River Bridge, Mr. Allen was placed in full charge of the engineering work for the construction of the Fort George G. Meade Cantonment in Maryland. This project involved the construction of temporary housing, roads, and utilities for approximately 25,000 persons. The defense program was just starting and speed was essential. Mr. Allen gathered together an organization of 214 men and in the four months before his death, in an automobile accident on January 26, 1941, such substantial progress was made that in February troops were quartered at the cantoment in large numbers. His contribution to the success of this vital project was recognized by the erection of a bronze plaque at the foot of General's Hill in the cantonment reading as follows:

"E. Russell Allen
1905–1941
Project Engineer for
Cantonment 1940–1941
In Commemoration of His
Unfaltering Loyalty
& Inspiring Leadership"

Mr. Allen's death at the early age of thirty five ended a career full of promise. He had just completed one large project and was successfully directing another of magnitude and importance. Greater opportunities and achievements were undoubtedly before him. His death was therefore a loss not only to his associates but also to the profession to which he had just begun to contribute significantly.

In 1931 Mr. Allen was married to Frances Somerville Houck of Towson. He was the father of one son, Francis Somerville, and one daughter, Melinda. He was devoted to the children and spent as much time with them as his duties permitted. In addition to his widow and children, Mr. Allen is survived by his mother; two brothers, Herschel H., M. Am. Soc. C. E., and Wendell D.; and one sister, Miriam (Mrs. Roland Wegner).

Mr. Allen was elected a Junior of the American Society of Civil Engineers on November 11, 1929, and an Associate Member on May 13, 1935.

HAROLD JAMES MANNING BAKER, Assoc. M. Am. Soc. C. E.[1]

DIED OCTOBER 2, 1943

The illness that removed Harold James Manning Baker from the Seattle District office, U. S. Engineer Department, terminated a connection begun in July, 1903, which was continued without interruption for forty years. He was the oldest employee of the district in point of service, and it can readily be understood that the loss of one with his strong, fine personality is deeply felt by all his friends and associates. In a sense Harold Baker was the office. His were the long, unbroken years of service; he supplied the thread of continuity to its events and occurrences; and he remembered incidents and recalled details out of the past. District engineers changed with the years and their terms of office. Others in the office sooner or later also moved. Harold Baker was the man who did not change—the steady, persistent, reliable, constant figure who calmly and quietly worked from one season to the next. His first engagement in 1903 was as surveyman. Thereafter he rose by successive steps to his final grade of principal engineer. From 1919 to the time of his death he was the principal civilian engineer in the district.

Mr. Baker was born on April 14, 1878, at Lowell, Mass., the son of the late Rev. Brookes O. Baker, M.D., of Sheerness, Kent, England, and Alice H. (Carleton) Baker, of Lowell and Haverhill, Mass. Through his mother, he was descended from John Carleton who settled in Haverhill in 1643. Nine generations of this pioneer's descendants are known to have lived on Lowell Avenue in Haverhill where John Carleton made a home three centuries ago.

Mr. Baker spent his early childhood in Honolulu, Hawaii; his boyhood in San Francisco, Calif., and in Port Townsend, Wash., where his father was on the medical staff of the Marine Hospital and also rector of St. Paul's Church for several years. He was graduated from the University of Washington at Seattle in 1900, receiving the degree of Bachelor of Science in Civil Engineering. He continued there in postgraduate work the following year and in 1901 and 1902 was a special student in electrical and hydraulic engineering at Cornell University in Ithaca, N. Y. His first engineering work from July, 1902, to January, 1903, was as rodman and instrumentman with the Constructing Quartermaster's office at Port Townsend, in charge of the topographical survey for the location of the post buildings at Fort Casey and Fort Worden in Washington. For a few months he was with the Spring Valley Water Company of Port Townsend as levelman and transitman. Then, in July, 1903, began the long engagement with the U. S. Engineer Department, during which Mr. Baker spent several years on fortification work. Later he was in responsible charge of all river and harbor work in the Puget Sound, Grays Harbor, Willapa Harbor, Columbia River, and Alaskan areas.

---

[1] Memoir prepared by a Committee consisting of Homer M. Hadley, Assoc. M. Am. Soc. C. E., Samuel De Moss, M. Am. Soc. C. E., and E. I. Pease, Asst. Chf., Civ. Works Div., Seattle Dist. Office, U. S. Eng. Div., Seattle, Wash.

In 1928 Mr. Baker was detached from regular river and harbor work and placed in charge of the preparation of a series of reports covering improvements—power, irrigation, flood control, and navigation—of the principal rivers in the Seattle Engineer District. By far the most difficult part of this assignment was the report on the Columbia River, which has been widely used by engineers throughout the United States, producing much favorable comment. Typical of such comment is the following excerpt from a paper by Henry Earle Riggs, Past-President and Hon. M. Am. Soc. C. E., in referring to the necessity for sound engineering reports prior to government expenditures: "No better example of such a study can be found than that on the Columbia River made by the Corps of Engineers after four years of study.[2]

Harold Baker was a man of high principles and firm convictions and governed his life according to these standards, quietly and undeviatingly. His relationships with subordinates were always friendly, cordial, and courteous. His few corrections and criticisms were always impersonal and were voiced because "we are trying to accomplish best results." Egotism and petty vanity had no part in his make-up. For his parents he had a great love and respect, and it was undoubtedly due to his father's influence that he developed the keen interest he had in medicine. He always found delight and pleasure in mathematics and the niceties of its accomplishments. A book that he treasured was an autographed copy of "The Lowell Hydraulic Experiments,"[3] by the late J. B. Francis,[4] Hon. M. Am. Soc. C. E., given him when he was a student at Cornell University by Mr. Francis' sister. Alertness and vigor always characterized his mind. He was a man of fine spirit and character whose loss is deeply regretted by a host of friends in all walks of life.

Mr. Baker was a member of Sigma Nu fraternity and was a Scottish Rite Mason.

He is survived by his widow, Ella M. Baker, and by two cousins, Marjorie Carleton Pratt and Brenda Carleton Simmons.

Mr. Baker was elected a Junior of the American Society of Civil Engineers on October 7, 1902, and an Associate Member on March 4, 1908. He became a Life Member in January, 1943.

---

[2] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 687.

[3] "The Lowell Hydraulic Experiments," by J. B. Francis, D. Van Nostrand Co., Inc., New York, N. Y., 1883.

[4] For memoir, see *Transactions*, Am. Soc. C. E., Vol. XIX (1893), p. 74.

EDWARD CROSBY BEBB, Assoc. M. Am. Soc. C. E.[1]

DIED AUGUST 7, 1942

Edward Crosby Bebb was born in Seward, Ill., on October 28, 1872, the son of Edward and Harriot Chase (Crosby) Bebb. He was graduated from the University of Wisconsin, at Madison, in the class of 1896 with the degree of Bachelor of Science in Civil Engineering.

In 1906 he was married to Fay Kent of Washington, D. C. He is survived by his widow, a son, Capt. Edward Kent Bebb, Jun. Am. Soc. C. E., and two daughters, Caroline (Mrs. John Williams), and Louise Harriot Bebb.

After graduation from college, Mr. Bebb entered Government Service in the U. S. Geological Survey, transferring in 1903 to the U. S. Bureau of Reclamation, where he served, with distinction, on many important western projects, until 1920.

During World War I he served as a Captain in the armed forces.

In 1920 he entered the service of the Federal Power Commission, where he remained until his death. As engineer for this Commission he passed on many of the largest power developments of the United States.

Ed Bebb was a very modest and retiring man; a kinder or more considerate man never lived. He had a tremendous fund of good "horse sense," with plenty of courage and an exceptional sense of justice for fellow workers and contractors under his supervision. These qualifications, plus his well-trained engineering mind, made him one of the truly great civil engineers of his time.

Mr. Bebb was elected an Associate Member of the American Society of Civil Engineers on November 6, 1907. He became a Life Member in 1942.

JULIUS MONTGOMERY BISCHOFF, Assoc. M. Am. Soc. C. E.[2]

DIED DECEMBER 5, 1943

Julius Montgomery Bischoff, son of Julius G. D. and Helen (Cronenbold) Bischoff, was born at St. Louis, Mo., on February 15, 1881. He was graduated from Washington University at St. Louis in 1904 with the degree of Bachelor of Science in Civil Engineering. During the summers of 1902 and 1903, he worked as a draftsman with the Terminal Railroad Association of St. Louis and with the Louisiana Purchase Exposition in St. Louis.

Upon graduation in June, 1904, Mr. Bischoff spent about one year as inspector and draftsman for the City of St. Louis. Thereafter, his career was identified with railway engineering:

---

[1] Memoir prepared by Francis T. Crowe, Hon. M. Am. Soc. C. E.

[2] Memoir prepared by H. Austill, M. Am. Soc. C. E.

1905—rodman for the St. Louis Southwestern Railway Company at Tyler, Tex.;

July, 1905, to August, 1907—transitman and resident engineer for Morey-Faulhaber Construction Company on the construction of the Apalachicola Northern Railroad, in charge of all trestle construction on 20 miles of line;

August, 1907, to January, 1909—assistant engineer for Edward Flad, M. Am. Soc. C. E., consulting engineer, of St. Louis, on the construction of a steel viaduct with reinforced concrete approaches for the Manufacturers Railway Company;

January, 1909—assistant engineer for the Terminal Railroad Association of St. Louis;

May, 1911—resident engineer with the Cuban Central Railways, Ltd., with headquarters at Sagua la Grande, Santa Clara Province, Cuba;

June, 1912—chief construction engineer for Cuban Central Railways, Ltd.;

September, 1914—assistant engineer for the Terminal Railroad Association of St. Louis;

March, 1918—principal assistant engineer of the St. Louis-East St. Louis Terminal District for the U. S. Railroad Administration; and

March, 1920, until his death—office engineer with the Terminal Railroad Association of St. Louis.

Mr. Bischoff was married to Elizabeth Butler in Tyler, on June 27, 1908. He is survived by his widow and three children, Julius Wiley, Elizabeth, and Eugene Butler.

He was a member of Kappa Sigma fraternity, the Railway Club of St. Louis, Missouri Society of Professional Engineers, and an enthusiastic supporter of the Railway Young Men's Christian Association.

Mr. Bischoff was particularly fond of tennis and regularly joined his friends on the court where he played a splendid game. In fact, he died suddenly while standing on the tennis court.

Mr. Bischoff was elected an Associate Member of the American Society of Civil Engineers on January 4, 1910.

ARTHUR BONIFACE, Assoc. M. Am. Soc. C. E.[1]

DIED JULY 18, 1943

Arthur Boniface was born at Hastings, England, on September 6, 1878. After attending public school and preparatory school there, he completed a course in the British Naval School at Portsmouth, majoring in higher mathematics. He served with the British Navy from 1898 to 1902. Interested in engineering work, he realized the limitations of his naval service and in 1902 resigned and came to the United States.

---

[1] Memoir prepared by Fred Lavis, M. Am. Soc. C. E. (former Trustee and Mayor of Scarsdale, N. Y.).

He was employed as a draftsman by the Western Electric Company in 1902 and thereafter served in various capacities with the Rapid Transit Subway Company of New York, N. Y., as draftsman and designing engineer. He later became assistant engineer, then resident engineer in charge of the office. Following the opening of the first subway he was engaged for several years in designing and preparing plans and estimates for many of the new routes subsequently built in New York City. His studies included the development of plans for a belt line subway for passenger and freight traffic entering and leaving New York City, and for a large union freight yard in the Newark (N. J.) Meadows.

In 1914 he was made resident engineer in charge of construction work in lower Manhattan of Section I of the Seventh Avenue Subway. The work on this section—one of the most difficult along the route—included the underpinning of the elevated railroad, the underpinning of approximately 4,000 lin ft of buildings, many of them high, relocating and rebuilding all of the subsurface structures, as well as the construction of the subway proper involving excavation both through rock and in some places through quicksand.

Arthur Boniface came to Scarsdale, N. Y., in 1907 when the population of the Town was slightly more than 1,000. He became interested in civic matters and on March 19, 1918, was elected a Village Trustee. He was elected Village President in 1921, and in 1922 he resigned as President of the Village and was appointed Village Engineer. His principal work had been his service as Engineer and Manager of the Village of Scarsdale, a service which extended over twenty years until his death deprived the community of a faithful and competent servant.

He served under changing Boards of Trustees (which included women as well as men) and under various and varying Presidents and Mayors, through the period when a rapidly developing suburban community expanded and grew from a population of 3,000 to more than 13,000. He built a new efficient municipal plant for these people.

Although at first his title was Village Engineer, he was actually Village Manager from the time of his appointment, and later his title was changed to that of Village Manager. His work involved not only solving important engineering problems of street construction, sewers, drainage, water supply, and waste and garbage disposal in a tightly restricted residential community but also dealing with many different people—many interested only in the development of some particular section and others from communities whose background, training, customs, and personal idiosyncrasies were different from the Scarsdale tradition of deep-rooted civic consciousness.

At one time he set forth his philosophy with respect to public work:

> "If a citizen presents a request, never turn it down, no matter what its nature. Investigate every complaint however trivial its character. Endeavor always to recognize the other man's viewpoint in all matters of public service, and never assume an arbitrary position without having a very good reason for so doing. Always remember the men in the various departments whose loyalty and service are just as important as the man's who may be directing them."

Scarsdale has over 95% of its area zoned for one-family residences, and problems of zoning were at times complicated and troublesome. The proper proportioning of assessments for street construction, sewers, and water supply in the new developments required the continuous exercise of good judgment.

Waste and garbage disposal in a wholly residential community with no waste land (a good proportion developed with high-class properties) presented difficult problems in which the human element was an important factor. These have been solved satisfactorily. Arthur Boniface went to Italy in 1926 to study the Beccari System of Garbage Disposal, and after his return he spent several years in research work to ascertain why the Beccari System was not entirely successful in the United States. He discovered that the difficulty was with the different composition of American garbage as compared with that produced in Italy. He then concentrated his efforts on the development of the "Boniface Method of Treating Garbage" and received Letters Patent for this method of treating garbage on August 4, 1936.

He dealt as successfully with the Mayors, Trustees, and people as he did with the technical engineering problems. In this he was a true engineer, using the human forces with which he had to deal for the benefit of the community. He brought his engineering knowledge to bear on all these problems with such good judgment and wise planning that Scarsdale has a low funded debt, low costs of maintenance, an efficient municipal plant, and a well organized operating personnel.

With the entry of the United States in the present war and the immediate need for adequate and trained civilian defense forces to meet emergencies, he turned, with characteristic energy, to the organization of the Scarsdale War Council and, as Deputy County Director of Civilian Protection, to the training and establishment of all the Civilian Protection Units within the provisions and framework of the New York State War Emergency Act. These forces were quickly and effectively created, and their smooth functioning has been commended by the County of Westchester, New York State, and U. S. Army authorities.

Although defense problems consumed a great part of his time during the period of war emergency, he was also able to make a splendid beginning on preliminary studies of Scarsdale postwar problems. A tentative schedule of Postwar Public Works for Scarsdale was prepared by him and, with the approval of the Village Board, was filed with the Temporary State Commission for Postwar Public Works Planning.

Author Boniface was never afraid to say "No" when occasion required, but he made few enemies and had many friends. He eschewed politics. Perhaps the greatest tribute that can be paid to him is that among those who knew him he was universally respected. He was a servant of the community and a wise counselor.

Among the organizations to which he belonged were the New England Water Works Association, the Westchester County Water Commission (chairman), the National Society of Professional Engineers, the New York State Society of Professional Engineers, and the Engineers Club of New York.

He was married to S. Marlene Talmage in New York City on September 5, 1907. They had four children—Arthur, Gwendolyn, Marlette, and John, all of whom survive.

Mr. Boniface was elected a Junior of the American Society of Civil Engineers on January 2, 1906, and an Associate Member on October 4, 1910.

DALTON MUNROE BROWN, Assoc. M. Am. Soc. C. E.[1]

Died May 16, 1943

Dalton Munroe Brown was born in Nashua, N. H., on January 13, 1900. He attended public schools in Milford, N. H., and Dartmouth College in Hanover, N. H., being graduated in 1923 with the degree of Bachelor of Science. In 1925, after studying for two years in the Thayer School of Civil Engineering at Dartmouth College, he received the degree of Civil Engineer. During summer vacations, Mr. Brown worked for the Illinois State Highway Department on inspection work.

For the greater part of his engineering career, Mr. Brown was on the engineering staffs of contractors' organizations. With Kenn-Well Contracting Company of New York, N. Y., he was assistant construction engineer on the erection of the East River Power Station for the New York Edison Company. He was employed by Thomas Crimmins Contracting Company as field engineer on construction for the Hunt's Point Gas Plant and additions to the Hell Gate Power Plant. Turning to subway construction, he worked for the Marcus Contracting Company as field engineer on the building of the northerly section of the Nassau Street Subway and later on similar engineering construction for the Marcy-Lafayette Avenue section of the Independent Subway in Brooklyn, N. Y. After a year with the Work Projects Administration as assistant to the project engineer on the development of the New York City park system, he joined the field organization staff of Spencer, White and Prentis, Inc., as field engineer. In this capacity he was engaged on the construction of the Sixth Avenue Subway from Eighth Street to Eighteenth Street in Manhattan, New York, and on the installation of deep foundations for the New York City Criminal Courts Building.

When the national defense program was begun in 1940, Mr. Brown joined the staff of Dry Dock Contractors, builders of the Philadelphia and Norfolk naval dry docks, and was assigned to field work at Norfolk Dock No. 8, which was completed shortly before his death.

Dalton Brown was extremely modest, a hard working and loyal engineer, and a staff man who exhibited efficiency in construction. In addition to his devotion to his family and his profession, Mr. Brown gave much of his talent to the Boy Scouts of America (B.S.A.). As a child he attained the grade of

---

[1] Memoir prepared by Herbert M. Hale, M. Am. Soc. C. E.

Eagle Scout. While he was in New York, he was associated with the B.S.A., and for several years before his death he was chairman, Troop Committee, Troop No. 8, B.S.A., in Brooklyn.

On October 27, 1939, at Brooklyn, N. Y., Mr. Brown was married to Alice E. Gunnerson, who survives him, together with a son, Lawrence D. Brown.

Mr. Brown was elected a Junior of the American Society of Civil Engineers on April 18, 1927, and an Associate Member on February 27, 1933.

KNOX FOLSOM BURNETT, Assoc. M. Am. Soc. C. E.[1]

Died August 13, 1943

Knox Folsom Burnett was born on August 19, 1903, at Lincoln, Nebr., the only son of Edgar Albert and Nellie E. (Folsom) Burnett. His father was associated with the faculty of the University of Nebraska, at Lincoln, from 1899 until his death in 1941, and for the last fourteen years of his life was Chancellor of the University.

Knox F. Burnett was educated in the public schools of his native city. He attended the University of Nebraska where he majored in architectural engineering, being graduated in 1924 with the degree of Bachelor of Arts. Immediately after graduation, he obtained a position with John Latenser and Sons, Architects, at Omaha, Nebr., as a structural draftsman and thus started on a brief, but eventful and successful engineering career.

After staying one-half year on his first job, and another eight months with Engineering Service Company of Omaha, where he gained experience in mechanical and electrical work, in 1925 Mr. Burnett became structural engineer for Davis and Wilson, Architects and Engineers of Lincoln. It was soon discovered, however, that, besides his ability as a structural designer, Mr. Burnett also had an aptitude as a specification writer and as an inspector and supervisor of field work. He learned how the efforts of several contractors on the same project can be coordinated for best results. Consequently, he was entrusted with the execution of several important contracts both by the firm and by others. He supervised the construction of the $500,000 First Plymouth Church for the Church Building Committee and acted as superintendent for the City of Lincoln on the construction of a large pumping plant.

His association with Davis and Wilson lasted until 1932. During those seven years, Mr. Burnett collaborated in the design, and in the supervision of the construction of numerous important building projects, among which the best known are Morrill Hall, Andrews Hall, and Women's Dormitories of the University of Nebraska, Stuart Theatre and Office Building, and Gold and Company Store Building, all at Lincoln.

---

[1] Memoir prepared by Eugene E. Halmos, M. Am. Soc. C. E.

During 1932 and 1933, Mr. Burnett was superintendent for the Olson Construction Company of Lincoln, on the construction of an overhead viaduct at Falls City, Nebr.

Mr. Burnett was engineer inspector with the government on Public Works Administration (PWA) work in Nebraska from November, 1933, until July, 1934, when he joined the engineering staff of the Platte Valley Public Power and Irrigation District at North Platte, Nebr. This District was organized to construct a ten-million-dollar irrigation and hydroelectric project on the North Platte River. In the course of two years, Mr. Burnett advanced from structural designer to office engineer of the organization, coordinating the work of the several departments as well as that of the numerous contractors on the project. The design of the project was prepared under the direction of Parsons, Klapp, Brinckerhoff and Douglas, Consulting Engineers of New York, N. Y., and the contact which Mr. Burnett made with this firm and particularly with John P. Hogan, Past-President and M. Am. Soc. C. E., had a profound effect on his future.

Early in 1937, Colonel Hogan, who at that time was chief engineer of the New York World's Fair of 1939, engaged Mr. Burnett as exhibit engineer on the engineering staff of the Fair. In this capacity, Mr. Burnett conducted all dealings between the Fair Corporation and the exhibitors with reference to space occupancy, type and construction of buildings and displays, use of utilities, etc. This assignment required considerable technical and diplomatic skill, both of which he possessed and successfully applied. After the completion of the construction work, Mr. Burnett remained for two years with the operating organization of the Fair as chief exhibit engineer and general manager of utilities. In recognition of his tactful help to the Rumanian delegation at the Fair, he was made a Knight of the Crown of Rumania by King Carol and also received a decoration from the Lebanese Government.

In December, 1940, Mr. Burnett joined the staff of Caribbean Architect-Engineer, composed of Voorhees, Walker, Foley and Smith, architects, and Parsons, Klapp, Brinckerhoff and Douglas, engineers, to whom the government of the United States entrusted the planning of Army bases at Trinidad and at several of the British Windward Islands. Mr. Burnett first served as liaison engineer between the field and the New York office and later as resident engineer at Antigua, B. W. I. Upon completion of this assignment, he returned to the United States in December, 1942, and shortly thereafter became seriously ill. He lingered for several months, even trying to work for brief periods, but never recovered and passed away on August 13, 1943. His career was thus abruptly terminated when it already gave promise of future distinction. He was buried in the family plot at Lincoln.

Knox F. Burnett was of medium height, robustly built, and indefatigable in both physical exercise and office work. He knew how to take orders from his superiors and how to execute them. For his engineering and executive ability, resourcefulness, and dependability, he was held in high regard by his employers and by literally hundreds of exhibitors at the World's Fair. He demanded hard and loyal work from his subordinates but never asked them to do more than he was willing to do himself; in this manner, he acquired their admiration and respect. While he was intensely devoted to his duties, Mr. Burnett was a very pleasant and sociable man in his private life; he was a gracious host and a much sought-for companion. He was a member of the University Club of Lincoln.

He was married to Lela Mae Randall in 1929, who survives him. Also surviving are his mother, and one son, Knox Robert Burnett.

Mr. Burnett was elected an Associate Member of the American Society of Civil Engineers on October 29, 1934.

DEWITT CLINTON FENSTERMAKER, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 6, 1943

DeWitt Clinton Fenstermaker was born in Basil, Ohio, on December 27, 1873, the son of Samuel Fenstermaker and Barbara (Hoshor) Fenstermaker.

He was graduated from high school, and, in addition, obtained a technical education by night courses and private instruction while working as a draftsman and engineer in Columbus, Toledo, and Cleveland, Ohio.

He entered railroad service in 1890 as a rodman on the Toledo and Ohio Central Railway, serving until 1898 as rodman, instrumentman, drafstman, and assistant engineer on construction and maintenance work.

From July, 1898, to July, 1899, during the Spanish-American War, he served in Company A, Second U. S. Volunteer Engineers, and in Company 15, U. S. Volunteer Signal Corps. From October, 1899, to April, 1901, he was an assistant engineer in the Department of Havana, Cuba, and the Department of Pinar del Rio, Cuba, on highway construction and topographical surveys.

From April, 1901, to May, 1902, he served as assistant engineer on construction for the Lake Erie, Alliance and Wheeling Railroad Company. From May, 1902, to January, 1903, he was draftsman for the Forest City Steel and Iron Company of Cleveland on design and detail structural work. From January, 1903, to 1908 he was employed as resident engineer, division engineer, and chief engineer of the Louisiana Railway and Navigation Company on the location and construction of 191 miles of new line. During 1909 he was city engineer of Tulsa, Okla. In 1910 and 1911 he served as designing engineer for the Lehigh and New England Railroad Company on construction work in eastern Pennsylvania. From 1912 to 1921 he was resident engineer, assistant engineer, and district engineer for the Chicago, Milwaukee, St. Paul and Pacific Railroad Company in responsible charge of construction and maintenance work.

In April, 1921, he was granted a leave of absence to act as chief engineer for the contracting firm of Peterson, Shirley and Gunther, which was constructing 52 miles of railroad for the Atlantic Fruit Company in Oriente Province, Cuba. After completing this work, he resumed service with the

---

[1] Memoir prepared by W. H. Penfield, M. Am. Soc. C. E.

Chicago, Milwaukee, St. Paul and Pacific Railroad Company as principal assistant engineer on February 1, 1922. In 1923 he was granted a leave of absence to become chief engineer of the Montana Railroad and the Wyoming North and South Railroad companies in charge of the construction of 320 miles of railroad for these companies in Montana and Wyoming. He then returned to the Chicago, Milwaukee, St. Paul and Pacific Railroad Company as district engineer on April 1, 1929, continuing in this capacity until November 30, 1937, when he retired because of ill health.

Mr. Fenstermaker was always very active in civic matters in Itasca, Ill., working for the betterment of the village by giving it the benefit of his engineering ability and experience. He prepared a zoning ordinance which was accepted by the village, and also served as building commissioner—all without compensation.

He was married to Elsie Buckingham of Hopedale, Ohio, on December 23, 1901, and is survived by his widow, one brother, and four sisters.

The McKinley Camp, Spanish War Veterans, conducted the funeral services. He was cremated at Acacia Cemetery, Chicago, Ill., and the ashes were interred at the Greenwood Cemetery near Hopedale.

Mr. Fenstermaker was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908. He became a Life Member in January, 1943.

GEORGE ADDISON FIELD, Assoc. M. Am. Soc. C. E.[1]

Died September 3, 1943

George Addison Field, the son of Harry and Virginia (Townsend) Field, was born on November 8, 1899, in Calvert, Robertson County, Tex.

He was educated in the public schools of Calvert and attended the School of Civil Engineering of the University of Texas at Austin.

In 1922 Mr. Field was employed by the Texas Highway Department as equipment engineer and in 1925 was made division engineer at Lubbock, Tex. In 1929, in association with the writer, he formed the contracting firm of Field Brothers. This company built numerous highways, bridges, and irrigation projects in west and southwest Texas. One important achievement was a 90-mile canal in Maverick County, constructed in conjunction with the W. E. Callahan Construction Company.

Wishing to aid the national defense effort in 1940 Mr. Field gave up his private business to become superintendent of construction on the Naval Air Base at Corpus Christi, Tex., for Brown and Root, W. S. Bellows Construction Company, and Columbia Construction Company. In 1941 he went to Houston, Tex., as project manager for Brown and Root on construction of shipyard facilities for the Houston Shipbuilding Corporation, virtually building the Irish Bend Shipyard. In 1942 he planned the construction organization of the Cactus Ordnance Works in Dumas, Tex. During the same year he directed construction on the Big Spring (Tex.) Bombardier School and the Concho and Goodfellow airfields at San Angelo, Tex. In 1943 he organized construction and operating personnel for the San Jacinto Water Project in Harris County, Texas. In less than three years he supervised more than $150,000,000 worth of construction work.

---

[1] Memoir prepared by Hugh T. Field, Assoc. M. Am. Soc. C. E.

Mr. Field was a member of Sigma Alpha Epsilon fraternity, and a member and former director of the Texas Highway Branch of the Associated General Contractors of America.

On January 4, 1938, in San Antonio, Tex., he was married to Margaret Myrick Morris. He is survived by his widow and a son, George, Jr.

Mr. Field was elected an Associate Member of the American Society of Civil Engineers on December 22, 1930.

IRVIN GITTLESON, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 11, 1943

Irvin Gittleson was born in New York, N. Y., on December 14, 1902, the son of Philip and Maria (Melisov) Gittleson. He attended Virginia Polytechnic Institute at Blacksburg.

For the greater part of his professional life, Mr. Gittleson was a structural draftsman. His early engineering training was in the drafting room of the Virginia Bridge and Iron Company at Roanoke, where he spent the four years from 1922 to 1926. From 1926 to 1931 he worked in the office of Waddell and Hardesty in New York, and, thereafter for shorter periods, he was a member of the engineering staff for Port of New York Authority; Triborough Bridge Authority; Ash, Howard, Needles and Tammen; and Parsons, Klapp, Brinckerhoff and Douglas. In this work he came in contact with many leaders in the profession.

His many friends were genuinely shocked by his death at such an early age. He was a neat and fast workman, and in all of his associations he left the impression of industry and congeniality. Undoubtedly, many of the large bridges in the New York area contain numerous details of his devising or suggesting.

Among Mr. Gittleson's hobbies were gardening and the collection of original oil paintings.

On February 24, 1929, in New York, Mr. Gittleson was married to Emily Schoham. He is survived by his widow and a son, Laurance.

Mr. Gittleson was elected a Junior of the American Society of Civil Engineers on December 26, 1933, and an Associate Member on October 14, 1935.

---

[1] Memoir prepared by E. L. Macdonald, M. Am. Soc. C. E.

DAN A. HULICK, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 4, 1943

Dan A. Hulick was born in Lyons, N. J., on March 22, 1882, the son of John V. and Anna (Manning) Hulick.

He was graduated from the Summit, N. J., high school in 1898. In 1900 he was employed by F. S. Tainter,[2] M. Am. Soc. C. E., consulting engineer, as axman. So great was his enthusiasm and ambition that twelve years from that time he was made assistant engineer on the construction of country estates, roads, bridges, dams, and water and sewer systems. Two projects on which he was employed were Carnegie Lake in Princeton, N. J., and Peachtree Heights Park in Atlanta, Ga.

From 1912 to 1916 Mr. Hulick was engaged in private practice, chiefly on surveys, subdivisions, general construction of homes, development of estates, and water and sewer systems. In 1916 he helped to lay out and build the town of Kingsport, Tenn. As resident engineer, he directed the building of the Payne Aviation Field at West Point, Miss., in 1918. Upon completion of this project, he became field engineer for Westinghouse, Church, Kerr and Company, working on the construction of a nitrate plant and on the village plan for Muscle Shoals, Ala. He was commended by the chief engineer for this work. In 1919 Mr. Hulick returned to private practice in Atlanta and Greenville, S. C., doing general engineering work on business buildings, private homes, roads, bridges, and sewer and water systems.

Mr. Hulick was appointed city engineer of Greenville in 1925, and served in this capacity for ten years. During this time he directed all municipal improvements, including a $500,000 paving program, design and construction of bridges, sewer extensions, etc. Energetic by nature, he accomplished a great deal during his tenure of office, completing projects of which Greenville may well be proud. He was honest and dependable, and his records prove his meticulous observance of detail.

In 1935 Mr. Hulick became affiliated with the Works Progress Administration, first in Greenville County, then in Walterboro, S. C., and later in Charleston, S. C. He returned to Greenville County in 1938 as county engineer, rebuilding bridges, roads, etc., and preparing plans for a 110-mile road surfacing program. In 1939 he went to Washington, D. C., as sanitary engineer for the United States Housing Authority. Because of failing health, he returned to the South, hoping to regain his strength. However, he never recovered, and died at his home in Gladstone, N. J., four years later.

He is survived by two sisters, Molly (Mrs. D. M. Wolfe), and Georgia (Mrs. J. D. Stantial).

---

[1] Memoir compiled from information supplied by the family, by friends, and from data on file at the Headquarters of the Society.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1849.

Except for his high school course, Mr. Hulick received no formal education, but he was a prolific reader, completing the *Encyclopædia Britannica*, making notes in the volumes, and cross-referencing them. Intensely interested in old Indian trails, much of his knowledge of terrain—which served him well in engineering—was a result of his frequent long walks in search of paths, fords, etc., referred to in Indian song and legend. He had a noteworthy collection of Bibles, an entire shelf of his bookcase being filled with various editions. An eager student of South Carolina history, he was better informed on this subject than many a native South Carolinian.

He was a member of the South Carolina Society of Engineers, the National Society of Professional Engineers, and the Society of American Military Engineers.

Mr. Hulick was elected an Associate Member of the American Society of Civil Engineers on March 11, 1940.

ALEXANDER ANDREW LAIRD, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 18, 1943

Alexander Andrew Laird was born at Topeka, Kans., on August 16, 1884, the son of Alexander and Mary (Lowrie) Laird. His parents were born and married in Scotland. His preliminary education was received in the public schools of Topeka, and he was graduated from the local high school.

Following his graduation from high school, Mr. Laird was employed in September, 1902, by the engineering department of the Atchison, Topeka and Santa Fe Railway Company, successively as rodman, chainman, transitman, apprentice in the mechanical department, draftsman, and bridge inspector. While with this company, he enrolled in a civil engineering course under a private tutor and also pursued, diligently, correspondence and extension work in this field.

In November, 1908, Mr. Laird left the railway company to accept a position as assistant engineer with Burns and McDonnell, Consulting Engineers, of Kansas City, Mo., in charge of surveys for, and design and construction of, water works, sewers, and electric light plants. In March, 1914, he was employed in a similar capacity for four months by Black and Veatch, Consulting Engineers, also of Kansas City. He then became assistant engineer on the valuation of railroad properties for the Kansas City Southern Railway Company and the Texas and Pacific Railway Company. This work was terminated in May, 1917, shortly after the entry of the United States into World War I.

In February, 1917, Mr. Laird was commissioned a First Lieutenant in the Engineer Officers Reserve Corps of the United States Army and was ordered to active duty in May, 1917. He was promoted to Captain in the Corps of

---

[1] Memoir prepared by L. R. Dail, Assoc. M. Am. Soc. C. E.

Engineers in August, 1917, and assigned as Topographical Officer with the 315th Engineers, 90th Division, and served during all operations of his regiment in the United States, France, and Germany, including the major engagements at Saint-Mihiel and Meuse-Argonne in France. This period of active service terminated in July, 1919. His military service had a profound influence on him, and for the remainder of his life he was a keen student of military engineering. On February 5, 1924, he was commissioned Major, Engineer Officers Reserve Corps, and, on February 4, 1933, was promoted to the rank of Lieutenant-Colonel in the Reserve Corps.

In July, 1919, Mr. Laird accepted a position as resident engineer for the State Highway Commission of Kansas, in charge of federal aid road projects in Allen County, Kansas. In April, 1924, he became chief engineer and general superintendent for a contracting firm on a project which included 8 miles of grading, structures, and approximately 35,000 sq yd of concrete paving.

From December, 1925, to April, 1931, he was field engineer for the Portland Cement Association. He then was appointed engineer with the Road Supply and Metal Company in charge of design, promotion, and sales for drainage structures and water conduits.

From November, 1935, to January, 1936, he was consulting engineer to the Works Progress Administration on designs and estimates for water, sewer, and swimming pool projects, and on emergency airports. In January, 1936, he assumed full charge on the construction of a number of earth dams for the Soil Conservation Service of the U. S. Department of Agriculture. This work was sponsored by the Kansas Fish and Game Commission and involved the moving of approximately 10,000,000 cu yd of earth and the placing of 150,000 sq yd of rock riprap and 15,000 lin ft of sheet piling. The work was performed by enrollees of the Civilian Conservation Corps. An average of 4,000 men including supervisory personnel was engaged on these projects.

From December, 1937, to March, 1938, he worked on the design and construction of an office and warehouse building in Oklahoma City, Okla. In March, 1939, he moved to Phoenix, Ariz., where he entered private practice, investigating mining and industrial properties. In February, 1940, he became industrial engineer for the Arizona Tractor and Equipment Company of Phoenix.

In September, 1940, Mr. Laird was ordered to active duty as Lieutenant-Colonel, U. S. Army, and was assigned to the Arizona State Selective Service Headquarters as Liaison Officer for the Undersecretary of War as adviser on occupational deferments. He was promoted to Colonel in December, 1940, and served efficiently in this capacity until striken with the short illness which terminated in his death on May 18, 1943.

Mr. Laird had a splendid and upright character. Although he was a man of exemplary habits, reserved and reticent, as his associates came to know him they esteemed him highly and learned to value his sound judgment. He was not a man to be shaken from what he thought was the right course. He was an exceptionally diligent and energetic man and his refusal to ease up when his health became impaired undoubtedly shortened his life.

Mr. Laird was married on June 19, 1912, in Kansas City, Mo., to Lottie Defenbaugh. He is survived by his widow and a daughter, Mary E. He was a devoted husband and father. In later years his main outside interests centered around his home. He loved to work in the yard, principally growing flowers. Much of his time, however, was devoted to studying military engineering in connection with his standing in the Engineer Officers Reserve Corps.

He was a member of the Masonic Order, a Shriner, and belonged to the Society of American Military Engineers.

Mr. Laird was elected an Associate Member of the American Society of Civil Engineers on July 6, 1920.

JAMES HENRY MINER, Assoc. M. Am. Soc. C. E.[1]

Died May 31, 1943

James Henry Miner was born in Phillipston, Mass., on November 24, 1877, the son of Frank Henry Miner and Ellen (White) Miner. He was a descendant of Thomas Miner, who came to America in 1628, and later founded the town of Stonington, Conn. When Mr. Miner was five years old, the family moved to Warrensville, Ohio, where he received his education in the district school. In 1896, he was graduated from the University School of Cleveland, Ohio. In the fall of the same year, he entered Cornell University at Ithaca, N. Y., being graduated from Cornell with the degree of Bachelor of Science in Civil Engineering in 1900. As editor and business manager of the *Cornell Era* and other college papers, he practically paid for his college education.

In 1904 Mr. Miner entered the Bureau of Reclamation as an assistant engineer on the Uncompahgre Project (Colorado), advancing to field engineer on the construction of the Gunnison tunnel (six miles in length) and then to office engineer. In 1910 he was assigned as project manager in local charge of construction on the Grand Valley Project. Diversion works in the Grand River for this project included the largest rolling crest installation in the United States at that time. In 1917 the Bureau of Reclamation appointed Mr. Miner project manager on the investigation and reconstruction of the King-Hill Carey Act Project in Idaho.

During World War I he served with the American International Shipbuilding Corporation at Hog Island, Pa., as inspector and expediter on the construction of steel hulls. Following this, he joined Dwight P. Robinson and Company as engineer on estimates for new work, cost control of jobs, and studies of hydroelectric projects on the Hudson, Genessee, Santee, Des Moines, Ottawa, and St. Lawrence rivers; as structural engineer on the Marblehead Lime Company plant; and as supervising engineer in the New York (N. Y.) office on the construction of the thirty-six-story Warwick Hotel, the American

---

[1] Memoir prepared by Fred J. Sharkey, Assoc. M. Am. Soc. C. E.

Consulate at Seville, Spain, and five dams in Ceara, Brazil, for the Brazilian Bureau of Reclamation.

From 1928 to 1932, Mr. Miner was supervising engineer for Dwight P. Robinson in Argentina on the construction of the five-and-one-half-mile Lacroze subway in Buenos Aires. From 1932 to the early part of 1934, he was resident engineer for United Engineers and Constructors, Inc. This firm acted as agent for the Treasury Department in building a central heating plant and distribution system to replace independent plants in some fifty public buildings in Washington, D. C.

In April, 1934, Mr. Miner returned to the Bureau of Reclamation as office engineer. He later became assistant supervising engineer on the Columbia Basin Project, remaining until his death.

Mr. Miner is survived by his widow, Olga (Lacher) Miner, to whom he was married in 1906; four children, Helene (Mrs. Thomas W. Hopper), Jean (Mrs. H. A. O'Connell), Florence (a nurse in the U. S. Army), and James H., Jr. (Aviation Cadet, U. S. Naval Aviation); six grandchildren; two sisters, Ethel (Mrs. Fred Muggleston) and Grace (Mrs. Allen Dicks); and one brother, Warren Miner. Mr. Miner's eldest son, Warren, died in 1919.

His kindly humor, his innate courtesy, his sound judgment, and his deep and constructive interest in the problems and details of his work will be sorely missed by a host of friends and associates.

Mr. Miner was elected a Junior of the American Society of Civil Engineers on March 31, 1903, and an Associate Member on November 8, 1909.

ARTHUR LELAND RICHARDSON, Assoc. M. Am. Soc. C. E.[1]

Died March 11, 1943

Arthur Leland Richardson was born in Goldendale, Wash., on April 21, 1879. He was the son of the late Jacob Richardson and Anna (Bell) Richardson. His father was county surveyor of Klickitat County, Washington, for several years. Previous to that time, he was a deputy government surveyor.

Mr. Richardson spent his early life in Klickitat County. The first school he attended was in a log cabin. Later he completed the course in the grade school at Goldendale and in the Academy at the same location. He decided to be an engineer and continued his studies, tutored by his uncle, Edwin Richardson, who at one time was surveyor general of the State of Washington. Mr. Richardson's first job, in engineering, was as a rodman with the Union Pacific Railroad Company, surveying for a railroad location in the vicinity of Madras, Ore.

In April, 1898, Mr. Richardson offered his services to his government and was accepted in Company H, Oregon Volunteers, Second Infantry. He served as a private in the Philippines during the Spanish-American War and received an honorable discharge in September, 1899.

---

[1] Memoir prepared by F. W. Panhorst, M. Am. Soc. C. E.

From 1900 to 1908 he was county engineer of Klickitat County, in charge of road and bridge work. As an engineer in private practice from 1909 to 1911, he designed and built water systems for Forest Grove, Willamina, Toledo, and Joseph, Ore. Continuing the same type of work, from 1911 to 1913 he was associated with the late J. L. Stannard,[2] M. Am. Soc. C. E., on water works systems at Carlton, Cornelius, Beaverton, Falls City, and Hillsboro, Ore., and at Woodland and Olympia, Wash., and constructed many other similar projects in the Northwest. As a general contractor, in 1914 and 1915 he engaged in the construction of bridges and roads. In 1915 he returned to his chosen profession and until 1924 was assistant engineer, Maintenance of Way Department, Southern Pacific Railroad Company.

The latter part of Mr. Richardson's life was spent with the Bridge Department of the Division of Highways, State of California. From 1924 until his retirement in 1941 he was in charge of construction of many and various bridges throughout the State of California, leaving many monuments to his ability as a bridge builder.

The greatest monuments and reminders left by Mr. Richardson are not of stone and steel, but rather the many young engineers whose knowledge and character were greatly increased and influenced by association with him and by training received from him. He took a great interest and pride in assisting, teaching, and encouraging the many young construction engineers who embarked on their engineering careers under his tutelage. The greatest compliment to be paid a man is to state that he was truly unselfish. The many young engineers fortunate enough to have been associated with him and who gained such information and advice not found in books can testify to his great unselfishness. He will be missed greatly by his many associates but, due to his influence, his character will be perpetuated in many engineers.

Continual failing health caused Mr. Richardson's retirement in January, 1941, and, after almost twenty years of the unsettled life of the construction engineer, he settled down on a beautiful ranch near Yamhill, Ore., to enjoy the remaining years in thoughtful and peaceful reminiscence of his many engineering travels and attainments; but, as with most active engineers, the time allotted to retirement was short and ended with his death on March 11, 1943.

In November, 1902, Mr. Richardson was married to Delia V. Laughlin of Yamhill. He is survived by his widow and by a daughter, Ferhn C. (Mrs. R. J. Muckle).

Mr. Richardson was elected an Associate Member of the American Society of Civil Engineers on January 14, 1929.

---

[2] For memoir, see *Transactions*, Am. Soc. C. E. Vol. 96 (1932), p. 1558.

WILLIAM SHERMAN VAN LOAN, Assoc. M. Am. Soc. C. E.[1]

Died February 24, 1943

William Sherman Van Loan, the son of William and Angeline (Sherman) Van Loan, was born in Poughkeepsie, N. Y., on November 30, 1879. He received his early education in the public schools of that city and his college education at the University of West Virginia in Morgantown.

Mr. Van Loan was employed as draftsman with the firm of Alden and Harlow, architects, of Pittsburgh, Pa., from 1899 to 1901. From 1901 to 1903 he was inspector on the erection of a malleable steel plant at McKees Rocks, Pa., and of a foundry at Swissvale, Pa., under Armin Schotts of Pittsburgh. From 1903 to 1906 he served as an inspector for the U. S. Naval Academy in Annapolis, Md., in charge of building foundation construction.

After completing this work, Mr. Van Loan was employed, successively, on several large public building projects, including the Public Library in Utica, N. Y., for Carrare and Hastings, architects, and on apartment house and residence construction for William Crawford, builder, of New York, N. Y. During the period 1910 to 1917, he was construction engineer for the Supervising Architect, Treasury Department, Washington, D. C., directing the construction and repairs of public buildings in the eastern part of the United States.

During World War I, Mr. Van Loan was Captain in the 7th Regiment of Engineers, Fifth Division. He served sixteen months in France, taking part in the Saint-Mihiel and Meuse-Argonne offensives. He was promoted to the rank of Major, Engineer Reserve Corps, on December 3, 1919.

After the war he returned to the office of the Supervising Architect at Washington, D. C., and was again engaged in supervising the construction and repairs of public buildings. In 1924 he joined the staff of Frank A. Spangenberg, architect, of Buffalo, N. Y., on the construction of buildings, churches, and schools in and around Buffalo until 1927. From 1927 until his death, Major Van Loan was employed by the Board of Education, City of Buffalo, as Supervisor of Construction of all Buffalo public school buildings and in charge of all maintenance and repairs.

Major Van Loan was a great lover of the out of doors—an ardent fisherman and an enthusiastic golfer. He was interested in the boy scout movement and, after acting for several years as scoutmaster of Troop 83, Church of Good Shepherd in Buffalo, he was made chairman of the Camp Committee for the Buffalo Council, Boy Scouts of America. In 1937 he was awarded the Silver Beaver "for distinguished service to boyhood," by the Buffalo Council, Boy Scouts of America.

On January 14, 1903, he was married at Annapolis to Katherine Mason Hodges, who survives him. Also surviving are two children, Capt. William

---

[1] Memoir prepared by William T. Huber and William G. Seyfang, Members, Am. Soc. C. E.

H. Van Loan and Katharine Mason Van Loan (Mrs. Lawrence Moses), and four grandchildren.

Major Van Loan was elected an Associate Member of the American Society of Civil Engineers on May 25, 1931.

ALBERT WILLARD WALKER, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 15, 1943

Albert Willard Walker was born at Marlboro, Mass., on March 17, 1882, the son of Joseph and Francena (Woods) Walker, and reared on a farm in the vicinity. He attended grade and high school for the normal periods, and was graduated from the Massachusetts Institute of Technology in Boston in 1905, with the degree of Bachelor of Science in Sanitary Engineering.

During intermittent periods from 1899 to 1904, he was with the Metropolitan Water and Sewage Board of Boston, as rodman and instrumentman.

On June 7, 1905, through civil service examination, Mr. Walker entered the service of the Bureau of Reclamation as an engineering aide. He was employed continuously in that branch of the federal service until his death at Great Falls, Mont. He was first assigned to the Belle Fourche Project in South Dakota, where he soon was placed in charge of surveys and estimates and also served as inspector on the construction of the Belle Fourche Dam—an earth-fill structure built by contract at a cost of $1,250,000. Upon completion of this work he was resident engineer in charge of location and construction of many parts of the irrigation system of this project.

Transferred to the Milk River Project in Montana, in June, 1912, Mr. Walker was assistant engineer on the construction of the headquarters office building at Malta, Mont., and on the design of Dodson North Canal structures. In July, 1913, he was transferred to the Huntley Project in Montana, as engineer in charge of investigations, design, and construction by government forces of an extensive drainage system consisting mainly of deep tile drains, costing about $500,000. Thus he entered a field in which much of his future life was to be spent and in which ultimately he excelled by virtue of his understanding of soils and the people who work soils.

On March 14, 1916, he was transferred to the Grand Valley Project, Grand Junction, Colo. From that date until January, 1917, he conducted the investigations and prepared the plans for the drainage of the Grand Valley Drainage District—a cooperative project with an estimated cost of $1,500,000. Thereafter he conducted a similar investigation and made plans, prepared specifications, and supervised the construction of the drainage system for the Grand Valley Project. This program cost about $300,000.

From August, 1921, until February, 1925, Mr. Walker was employed principally on the Newlands Project in Nevada, as engineer in charge of all phases of a $700,000 drainage program involving deep open drain construction with heavy dragline equipment. He was also engaged at odd times as consultant on drainage for other reclamation projects and on special investigations in the San Luis Valley in Colorado. During the remainder of 1925 he was special assistant on soil and drainage matters to reclamation adjustment boards examining the Minidoka, King Hill, Boise, Klamath, and Newlands projects. When this work was completed, he returned to the Newlands Project to work on a small drainage program and to make the investigations, plans, and estimates for the Spanish Forks extension of the project, a 36,000-acre development. In addition, he assumed the duties of project superintendent in December, 1926, retaining this position until May, 1929. During this latter period he directed further drainage work costing $250,000 and the $150,000 Truckee Canal reconstruction, and made a series of investigations for reservoir sites in the Truckee River Basin.

---

[1] Memoir prepared by Irwin B. Hosig, Engr., Bureau of Reclamation, Denver, Colo., and H. P. Wangen, Secretary, Greenfields Irrig. Dist., Fairfield, Mont.

He was transferred in May, 1929, to the position of construction engineer on the Sun River Project, Montana, and in December of that year also assumed the duties of superintendent. Operation and maintenance of much of this project were transferred to the Greenfields Irrigation District in 1931, and, under a cooperative arrangement with the Bureau of Reclamation, Mr. Walker also served as manager of the district. He retained both positions until his death. During this period the project was extended from 56,000 to 99,000 irrigable acres, the principal existing canals and reservoirs were enlarged and improved, and drainage work was performed as necessary and possible—all at a total cost of about $2,500,000. As the canal system was extended to large areas of virgin land, qualified farm families were selected under his direction and permitted to establish homestead residences on the land. The project grew from 400 to 1,050 farms; and its population, from 1,400 to 3,700 persons. To be known as a guiding hand in such growth is much.

Although diligently devoted to the important official tasks with which he was entrusted, Mr. Walker at all times maintained a participating interest in the civic and social activities of the community. His sound judgment and solution of the perplexing problems of his fellow men were respected and accepted by the many who turned to him for advice. He was a man of simple tastes, exercising extreme patience and a calm demeanor under the most trying situations.

To those in the Bureau of Reclamation who knew him, and to the people of the community in which he resided, he was a person rich in character. That character obviously had its beginnings in the habits of sobriety, thrift, and industry necessary to a New England farm home and in the simplicity, honesty, and directness of approach existing in a solid rural community. His life was a beautiful demonstration of the application of such a character without change to a professional career in the public service. For it his associates admired and loved him.

He was married on December 2, 1908, at Belle Fourche, S. Dak., to Grace Eva Wilkinson, who died on December 8, 1926. Three children of that marriage, Russell W., Leland J., and Katherine M. (Mrs. L. R. Jourdonnais) survive him. On September 5, 1932, he was married at Seattle, Wash., to Esther McArdle who also survives him.

Mr. Walker was a member of Fairfield Montana lodge No. 127, A. F. and A. M. and a charter member and first president of the Fairfield Lions Club.

Mr. Walker was elected an Associate Member of the American Society of Civil Engineers on May 3, 1910.

CARL ROBERT WEIDNER, Assoc. M. Am. Soc. C. E.[1]

Died August 20, 1941

Carl Robert Weidner was born on March 21, 1883, at Buffalo, N. Y., the son of Karl Gustav and Clara Francis (Neuman) Weidner. He attended Cornell University at Ithaca, N. Y., and was graduated in 1904 with the degree of Civil Engineer.

During college vacations Mr. Weidner gained a varied experience. In 1901 he was employed by F. K. Wing, M. Am. Soc. C. E., of Buffalo, on water works and sewerage construction. In 1902 he was rodman on maintenance of way for the Erie Railroad Company. In 1903 he served for two months as assistant engineer with D. Fairchild, civil engineer, of Buffalo, in charge of construction of 2 miles of electric railway. In 1904 he worked for several months as draftsman at the Bureau of Filtration, Pittsburgh, Pa.

After his graduation from Cornell, from October, 1904, to October, 1908, Mr. Weidner was assistant engineer for the Pipe Line Department of The Prairie Oil and Gas Company at Independence, Kans. He was in charge of surveys for, and at times directed, construction of pipe lines, pumping stations, reservoirs, and steel tanks in Kansas and Oklahoma for the transportation and storage of crude petroleum. This was pioneer work in a pioneer section of the United States.

From October, 1908, to March, 1909, he traveled and studied in Europe.

During the spring of 1909 he was engaged as instructor in civil engineering at Pennsylvania State College at State College. For the next seven years, as instructor in hydraulic engineering, he taught at the University of Wisconsin in Madison, conducting research and writing several bulletins for the university. These bulletins included "Theory and Test of an Overshot Water Wheel"[2] and "The Diaphragm Method for the Measurement of Water in Open Channels of Uniform Cross-Section."[3] He was also joint author of a bulletin on "An Investigation of the Air Lift Pump."[4]

---

[1] Memoir prepared by L. E. Davis, Engr., Sinclair Refining Co., Pipe Line Dept., Independence, Kans.

[2] "Theory and Test of an Overshot Water Wheel," by Carl R. Weidner, *Bulletin No. 529,* Univ. of Wisconsin, Madison, 1913.

[3] "The Diaphragm Method for the Measurement of Water in Open Channels of Uniform Cross-Section," by Carl R. Weidner, *Bulletin No. 672,* Univ. of Wisconsin, Madison, 1914.

[4] "An Investigation of the Air Lift Pump," by G. J. Davis, Jr., and Carl R. Weidner, *Bulletin No. 450,* Univ. of Wisconsin, Madison, 1911.

During summer vacations, Mr. Weidner accept various positions. In 1912 he served as chief hydrographer for the Knoxville Power Company in charge of stream gaging and stream flow computations for a 400,000-hp water power project. In 1915 he was with the Miami Conservancy District at Dayton, Ohio, on flood control work.

Leaving the academic field, in 1916 Mr. Weidner was engaged to make an inventory and appraisal of the property of The Prairie Pipe Line Company, which had been formed in 1915. In the same year he was made chief engineer of the company, and he continued to serve in this capacity with this company and its successor, Sinclair Refining Company, Pipe Line Department. During these years he had charge of surveys for, and design of, many miles of pipe lines. The organization expanded its facilities from approximately 6,000 miles of laid pipe of all sizes in 1915 to approximately 14,000 miles in 1931. During this time 274 pumping stations were built, and many of the trunk line stations rebuilt and enlarged.

The first major project of the company was an inventory of equipment to adjust investment records in accordance with the Uniform System of Accounts of the Interstate Commerce Commission. This work was completed in 1922, in spite of the interruption of World War I and the heavy volume of construction. During these years Mr. Weidner assisted in developing pipe line accounting procedure in accordance with the Uniform System of Accounts prescribed by the Interstate Commerce Commission. For many years he studied the depreciation of pipe line property, and participated in the settlement of several tax cases involving depreciation.

Carl Weidner was privileged to be identified with oil pipe line transportation during one of its rapid stages of development. He fostered many technical advances, among them: Welding in pipe line construction in 1921; and prevention of corrosion of buried pipe lines by use of protective coatings in 1925.

An active member of the American Petroleum Institute (A.P.I.), he belonged to the original Corrosion Committee; he was the author of a paper on "Protecting and Reconditioning Underground Pipe Lines"[5] (presented at a Group Session Symposium on Corrosion at the annual meeting of the A.P.I. in 1926); and he was joint author of a paper on "Relation of Pipe Line Currents and Soil Resistivity to Corrosion"[6] (presented at the 1931 meeting). In addition, he was active in organizing cooperative tests on pipe coatings, conducted between 1930 and 1940 by the Pipe Coating Manufacturers and the A.P.I. under the direction of the Bureau of Standards, and he supervised the installation of several cathodic units for pipe protection by electrical drainage. From 1934 to 1937 the valuation of the oil pipe lines of the Sinclair Refining Company by the Interstate Commerce Commission demanded much of his time and effort. From 1934 until his death he served on the Engineers-Accountants Committee on Valuation of Pipe Lines of the A.P.I. and made important contributions to the solution of valuation and depreciation problems. In 1939 he edited a report on the history, construction, and operation of oil pipe lines for the Committee on Uniform Methods of Oil Accounting of the A.P.I. Mr. Weidner contributed generously to the standardization and research program of the A.P.I. His work on the committees on Corrosion, Standardization of Line Pipe, Evaporation Losses, and Valuation of Pipe Lines was notable. A clear thinker and writer, he is remembered by fellow committee members for his capacity for work, for his obvious ability, and for his eagerness to cooperate with others. He was held in high esteem and his death was a great loss.

---

[5] "Protecting and Reconditioning Underground Pipe Lines," by C. R. Weidner, *Bulletin*, Am. Petroleum Inst., Vol. VIII, No. 6, January 31, 1927, p. 352.

[6] "Relation of Pipe Line Currents and Soil Resistivity to Corrosion," by C. R. Weidner and L. E. Davis, *Proceedings*, Am. Petroleum Inst., Section IV, December, 1931, p. 36.

The following Resolution in Memoriam was adopted by the Central Committee on Pipe Line Transportation of the Division of Production of the A.P.I. at the Annual Convention on November 5, 1941:

> "Mr. Weidner was very active in the American Petroleum Institute work and at the time of his death was a member of the Committee on Valuation of Pipe Lines; the Central Committee on Pipe Line Transportation; the Topical Committee on Pipe Line Technology; the Correlating Committee on Measuring, Sampling, and Testing Crude Oil; the Central Committee on District Activities; the Topical Committee on Vocational Training; Engineer's Sub-Committee on Tanks; Mid-Continent District Committee on Pipe, as well as chairman of the Mid-Continent District of Group 3, Regional Sub-Committee on Pipe Lines and chairman of the Mid-Continent Sub-Committee on Tanks. Mr. Weidner was also Chairman of a Special Reviewing Committee for the vocational training course, 'Pipe Line Practices,' and had practically completed writing the text at the time of his passing.
>
> "His constructive leadership and kindly personality will be greatly missed in Institute work and by those privileged to know him."

In addition to his technical ability, Mr. Weidner had other talents. In his early years, he was an excellent violinist. A strong man physically, he excelled in swimming, tennis, and golf. He was a considerate employer and encouraged those working under his direction to study and advance. Although he was dignified and reserved, he was a very pleasant companion with a wide range of interesting conversational topics, and he was tolerant of the views and opinions of others. At a meeting of pipe line executives he once gave a fine address on "The Spirit of the Organization."

In addition to the Society and A.P.I., Mr. Weidner belonged to Sigma Xi; Oklahoma Society of Professional Engineers; Independence Lodge No. 780, Benevolent and Protective Order of Elks; and Fortitude Lodge No. 107, A. F. and A. M., affiliated Masonic bodies of the York rite in Independence, and Mirza Temple Shrine in Pittsburg, Kans. He was a member of the Independence Country Club and had served as president. He contributed liberally to many civic causes.

Carl Weidner was never married. He is survived by his mother, Mrs. Clara Francis Weidner; a sister, Martha Emma (Mrs. G. E. Reister); and a brother, Paul G.

Mr. Weidner was elected a Junior of the American Society of Civil Engineers on February 28, 1905, and an Associate Member on April 5, 1910.

ROBERT LEE WHITEHEAD, JR., Assoc. M. Am. Soc. C. E.[1]

Died January 16, 1943

Robert Lee Whitehead, Jr., was born in Amherst, Va., on October 23, 1890. A family biography reveals the following:

> "He was a son of Robert Lee Whitehead of Amherst County, Virginia, and Fanny Lea Zogbaum of Germantown, Pa.
>
> "The Whitehead family is an old Virginia family. Edgar, the emigrant, came to Virginia from England during the reign of Charles II and settled on a tract of land granted him near York River. A descendant was a soldier in the Revolution.
>
> "The family later moved to Lynchburg and Amherst and Nelson Counties, Virginia, where they were prominent in the welfare and development of their community. Robert Lee Whitehead, Sr., was educated at Lehigh University [Bethlehem], Pa., and at the time of his marriage was Chemist and Manager for a Baltimore Electric Refining Company."

Mr. Whitehead was graduated from the Episcopal High School in Alexandria, Va., and received his college training at Washington and Lee University at Lexington, Va. He was an active member of the Episcopal Church, serving as a Vestryman at the time of his death. During World War I, he was a Second Lieutenant in the Field Artillery, U. S. Army.

After World War I until 1920, Mr. Whitehead was employed as an instrumentman by Larson and Fox at Perth Amboy, N. J. From 1920 to 1921, he served as instrumentman for the late C. F. Mebus,[2] M. Am. Soc. C. E., consulting engineer of Philadelphia, Pa. From 1921 to 1922, he was assistant superintendent for F. L. Grant, Inc., contractor, of Clarksburg, W. Va. In 1922 he went to work as instrumentman for the North Carolina State Highway Commission, remaining with the Highway Commission continuously until his death in January, 1943. From 1924 to 1943, Mr. Whitehead was resident engineer and senior resident engineer on all types of highway construction.

Mr. Whitehead was married to Janie Jackson of Dunn, N. C., in 1926. In addition to his widow, he is survived by his mother, one sister, and one brother.

Mr. Whitehead was a capable engineer and will be missed in the Highway Department. Well liked by the men with whom he worked, he had a pleasing manner and a pleasant approach. Those who came in close contact with him cherish the memory of his friendship.

Mr. Whitehead was elected an Associate Member of the American Society of Civil Engineers on November 27, 1933.

---

[1] Memoir prepared by Berrien W. Davis, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Vol. 106 (1941), p. 1620.

GUY SMITH WOOD, Assoc. M. Am. Soc. C. E.[1]

Died January 30, 1943

Guy Smith Wood was born at Altona, Clinton County, N. Y., on August 8, 1888, the son of Eugene and Florence (Mott) Wood. During his early childhood his parents moved to Indian Lake, N. Y., where he received his grammar school education. He first attended high school at North Creek, N. Y., and later, when a high school was started in Indian Lake, he went there, being graduated with the first class in 1908.

He was graduated from Rensselaer Polytechnic Institute in Troy, N. Y., with the degree of Civil Engineer in 1912. At the Institute he was elected to both the Tau Beta Pi and Sigma Xi honor societies.

Immediately after graduation Mr. Wood entered the service of the New York State Public Service Commission as junior engineer and assistant engineer—first, on preliminary and location surveys and later on construction of subways in New York, N. Y., until May, 1917. From May, 1917, to November, 1918, he served as assistant engineer for the Degnon Contracting Company on subway construction work in New York City. After this he was employed for a short period by Ford, Bacon and Davis on the construction of a wartime manufacturing plant with an accompanying housing development in the State of Virginia.

In June, 1919, he entered the employ of the International Paper Company, remaining with this company until his death. For the first two years of his service with the company he was resident engineer on the construction of boiler houses and extensive paper mill alterations at Palmer, N. Y., and Glens Falls, N. Y., and also at Berlin, N. H. He was then called to the New York City office, where—with the exception of about a year as resident manager on the construction of a paper mill at Three Rivers, Quebec, Canada—he was in charge of layout, designs, and specifications for three of the largest newsprint mills ever constructed in Canada.

Mr. Wood's practical experience at the mills, together with a natural aptitude for layout and design, made him a particularly valuable man to his employers during this period of expansion. His ability to impart his knowledge to others and his patience with the younger designers made him well liked by those with whom he worked.

The period of newsprint mill expansion in Canada was completed about 1931, and, from 1931 to 1935, Mr. Wood was assistant manager of a subsidiary of the International Paper Company, the Montague Machine Company with shops at Turners Falls, Mass. Besides doing a general foundry and machine shop jobbing business on paper mill and water power machinery, this company built a very successful new type of pulpwood grinder which was sold to outside firms.

---

[1] Memoir prepared by Harry M. Nelson, Assoc. M. Am. Soc. C. E.

In the fall of 1935, Mr. Wood became manager of the Herkimer Mill of the International Paper Company at Herkimer, N. Y., manufacturing leather board and molded pulp products. He continued as manager of this plant until late in 1942 when it was temporarily closed.

Mr. Wood was a member of the Masons and also took an active interest in his college alumni society and in local civic affairs. His hobbies were fishing and hunting in the Adirondack Mountains.

On June 4, 1913, Mr. Wood was married to Edna Stansfield. He is survived by his widow and a daughter, Dorothy (Mrs. Harold Waldron).

Mr. Wood was elected an Associate Member of the American Society of Civil Engineers on May 19, 1924.

EDWIN PARK ARNESON, Jun. Am. Soc. C. E.[1]

DIED JANUARY 1, 1943

Edwin Park Arneson, son of the late Edwin P. Arneson[2], M. Am. Soc. C. E., and Anita (Park) Arneson, was born in San Antonio, Tex., on June 6, 1917.

He attended the public schools of San Antonio, being graduated from high school at the age of fifteen. After finishing high school, he took special work at Westmoreland College, San Antonio, for a year. In September, 1934, he entered the Agricultural and Mechanical College of Texas, at College Station, and was graduated in June, 1938, with the degree of Bachelor of Science in Civil Engineering. He held the rank of Captain in Field Artillery in the Cadet Corps at the time of his graduation, and was commissioned a Second Lieutenant in the Reserve Corps of the U. S. Army.

After graduation, Mr. Arneson was employed by the State Highway Department of Texas, Bridge Division, on tracing, detailing, designing, estimating, and checking work. In August, 1939, he went into the field as instrumentman on the construction of a bridge across the Colorado River. In April, 1940, he became cartographer for the Pan American Airways, with headquarters at Brownsville, Tex. While on this assignment, much of his work was in the Spanish-American countries of North America and Central America. His knowledge of the Spanish language, and his ability to translate, write, and speak it, enabled him to render valuable service. In June, 1941, Mr. Arneson left Pan American Airways, and later reentered the employ of the State Highway Department of Texas as resident engineer. Thus, he returned to his mother's home in San Antonio. He continued with the State Highway Department until he was called into the armed service of the United States.

---

[1] Memoir prepared by A. P. Rollins, M. Am. Soc. C. E.

[2] For memoir see *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1878.

Mr. Arneson was called to service as a First Lieutenant in the Field Artillery of the U. S. Army in October, 1941. In February, 1942, he was transferred to the Signal Corps and assigned to duty in Alaska. In March, 1942, at his own request, he was transferred to the Army Air Corps and qualified as a pilot in October, 1942. His first assignment was as instructor pilot of multiengine planes. On January 1, 1943, he was directing a cross-country flight of students near Stockton, Calif., when one of the students made a forced landing along a highway. Lieutenant Arneson, feeling responsible for the safety of his pupils, undertook to land in an open space near where the student landed, but far enough from the highway to safeguard other traffic. The open space proved to be a peat bog into which his P-38 immediately sank. He was killed instantly, and another life was placed upon the altar of patriotism as a sacrifice of war.

Lieutenant Arneson came from a notable lineage. His father was an outstanding engineer, being a Director of the Society at the time of his death in 1938, and an esteemed citizen. His mother, highly accomplished in music and reading, continues to be a leader in church and civic affairs in San Antonio, giving freely of her talents.

Lieutenant Arneson was gifted and versatile. He was skilled in music and art. He was proficient in research and possessed, in full measure, the essential fundamentals of engineering. His life, finished so soon, promised much, and his talents, so needed in the times to come, would have been of great benefit to his country. His death leaves an almost unbearable sorrow in the hearts of the loved ones who survive him, but the thought that father and son are reunited is comforting.

He is survived by his mother, Mrs. Anita Park Arneson; a sister, Lily Anita; and a brother, Richard Owen.

Lieutenant Arneson was elected a Junior of the American Society of Civil Engineers on February 13, 1940.

RICHARD ALESTER COLEMAN, Jun. Am. Soc. C. E.[1]

Died December 16, 1942

Richard Alester Coleman was born on May 6, 1920, at Savannah, Ga., and died from complications following an appendectomy on December 16, 1942, at Newport News, Va. He attended grade school in Baltimore, Md., junior high school in Washington, D. C., and the McKinley Technical High School of Washington, D. C., from which he was graduated when he was sixteen. He then entered Worcester Polytechnic Institute at Worcester, Mass., on a competitive scholarship awarded by the Washington alumni of Worcester Polytechnic Institute.

---

[1] Memoir prepared by E. F. Hewins, Chargeman, Eng. Technical Div., Newport News Shipbuilding and Dry Dock Co., Newport News, Va.

During his freshman year at Worcester Polytechnic Institute, he was secretary of his class. In 1938 he won the Class of 1879 prize for the best technical essay presented by an undergraduate. He organized the Debating Society, served as its president for two years, and in 1939 won the Wilfred Peel prize for the presentation and defense of an idea. Throughout his four years in college he achieved first honors, and, although barely twenty when he was graduated in 1940, he was the top ranking member of his class. He received the degree of Bachelor of Science "with high distinction" and was a recipient of the Graduates' Aid prizes. He was elected to Tau Beta Pi and Sigma Xi fraternities and was also a member of Masque and of Lambda Chi Alpha fraternity.

After graduation Mr. Coleman took a special short course in ship construction at Massachusetts Institute of Technology in Cambridge, before accepting a position as hull draftsman for the Newport News Shipbuilding and Dry Dock Company. Early in 1941 he entered partnership with Robert F. M. Pyle, M. Am. Soc. C. E., specializing in engineering surveys and construction. In June, 1941, he returned to the Newport News shipyard as a member of the technical division of the Engineering Department. Although a newcomer in the field of marine engineering, his excellent foundation in, and grasp of, mathematics and engineering fundamentals soon brought him in contact with some of the most interesting and difficult problems connected with shipbuilding. In addition to his regular work, he taught a night class in marine engineering for the University of Virginia extension course. So favorable was the impression he had made that, at the time of his death, Charles F. Bailey, formerly engineering director at the shipyard, said, "He came to be one of the outstanding younger men in the Engineering Department. The Institute [Worcester] has lost one of her brilliant and fine young graduates."

Mr. Coleman is survived by his father and mother, Mr. and Mrs. D. P. Coleman; his wife, Taimi Swan, to whom he was married on May 23, 1941, in South Hill, N. C.; and infant son, Richard Alester Coleman, Jr. (born on July 8, 1943).

Mr. Coleman was elected a Junior of the American Society of Civil Engineers on October 9, 1940.

NATHAN ELIAS ELKIN, Jun. Am. Soc. C. E.[1]

Died June 22, 1943

Nathan Elias Elkin was born in New York, N. Y., on April 15, 1911, the son of Joseph J. Elkin and Mary (Goodman) Elkin. His education was obtained in New York City—at Public School No. 11 (Queens); Stuyvesant High School; and New York University. He received the degree of Bachelor of Science in Civil Engineering from the latter institution in 1932, and then took postgraduate work at Columbia University.

---

[1] Memoir prepared by Joseph J. Elkin, President, Woodcrest Construction Co., Inc., New York, N. Y.

He followed the tradition of his family by entering the civil engineering profession, since his father and his uncle, Charles Goodman, M. Am. Soc. C. E., are both civil engineers. Nathan Elkin's apprenticeship was served with the Woodcrest Construction Company, Inc., on three sections of the Triborough Bridge, on much of the Grand Central Parkway in New York City, on Floyd Bennett Airport on Long Island, N. Y., and on the Mamaroneck Sewage Treatment Plant at Mamaroneck, N. Y.

His progress after graduation was extremely rapid, based upon his quick grasp of engineering construction. He became partner in the firm of M. D. Lieb, Inc., and while thus associated built some of the most difficult construction jobs in the East. Among them were intricate bridge repairs to the Manhattan Bridge; Pelham Bay Elevated line underpinning, for the Westchester Avenue Bronx underpass on the Hutchinson River Parkway Extension; barracks and mess halls for the United States Army; and Public Markets for the City of New York.

His work stands today as a monument to him among all those who knew him, from simple worker to section engineer. His keen executive ability and quick engineering eye led to decisions of sterling worth. He was a young man of the highest character and integrity, and his passing is deeply felt by all who knew him. In his profession his fine reputation far surpassed his years and will long be remembered. He was not married.

Mr. Eikin was elected a Junior of the American Society of Civil Engineers on November 28, 1932.

ROLLAND FRANCIS LANG, Jun. Am. Soc. C. E.[1]

DIED OCTOBER 7, 1943

Rolland Francis Lang was born in Kranzburg, S. Dak., on September 27, 1914, the son of Frank Eugene Lang and Luella (Hughes) Lang. He completed his high school training at Brookings, S. Dak., and entered South Dakota State College, at Brookings, in 1933. He enjoyed an active and productive undergraduate period and was graduated with the degree of Bachelor of Science in Civil Engineering in 1937. He made a superior record as a student and at the same time found time for many extracurricular activities, being active in the Student Chapter of the Society, Scabbard and Blade (national military fraternity), Blue Key (national campus leadership fraternity), and Delta Pi Chi (a local honorary engineering fraternity).

Upon graduation from college, Rolland Lang accepted employment in the vicinity of Brookings and served in various capacities on surveying operations and on some highway research under way at his alma mater. In the fall of 1937 he accepted a half-time instructorship in the Civil Engineering Department of South Dakota State College and devoted the remainder of his time to graduate work in civil engineering. In 1939 he became a full-time instructor. His graduate work was complete except for the final draft of his thesis, and he would have made outstanding contributions to the literature of engineering if he had had the opportunity to complete his investigations of the many problems which interested him. During summer vacations he served in a professional capacity in connection with street improvements and building operations in the City of Brookings.

---

[1] Memoir prepared by H. B. Blodgett, Assoc. M. Am. Soc. C. E.

In the fall of 1940 Rolland Lang accepted a position of greater responsibility with the Michigan College of Mining and Technology at Houghton, where he served as an instructor in civil engineering.

During his undergraduate days he had been active in the Reserve Officers Training Program and had maintained his commission after his graduation. In July, 1942, he was called to active duty as a Second Lieutenant and was subsequently assigned to an Engineer Amphibian Regiment. He received special training in the United States and was then sent to Australia and later to New Guinea. On October 7, 1943, Rolland Francis Lang was instantly killed in action, at the age of 29. A posthumous award of the order of the Purple Heart was made to his parents in November, 1943. He is survived by his parents, Mr. and Mrs. Frank E. Lang; and three sisters, Luella, Marjorie, and Geraldine.

Lieutenant Lang was an accomplished amateur photographer, a skilful draftsman, an inspiring teacher, and an able engineer. In addition to his Society membership, he belonged to the American Road Builders' Association.

Lieutenant Lang was elected a Junior of the American Society of Civil Engineers on May 13, 1940.

THOMAS ALBERT McTEER, Jun. Am. Soc. C. E.[1]

DIED JUNE 25, 1943

Thomas Albert McTeer was born on September 23, 1920, at McClellanville, S. C., the son of Clyde M. and Caroline (Morrison) McTeer.

Mr. McTeer attended South Carolina public schools at Bluffton, Summerton, and McClellanville, and was graduated from the McClellanville High School. He then studied civil engineering at Clemson College, in Clemson, S. C., being graduated in 1942. At Clemson he received honors during his sophomore year and was given high honors as a junior. He was elected to Tau Beta Pi fraternity, and was a member and vice-president of the Society's Student Chapter. As a member of the advanced Reserve Officers Training Corps, he was rated expert at the summer camp. During his senior year he was a member of the crack Senior Platoon, which gave exhibition drills at all football games and other celebrations.

---

[1] Memoir prepared by Clyde M. McTeer, Appeals Referee for the South Carolina Unemployment Compensation Commission, Columbia, S. C.

After receiving his degree in civil engineering and being commissioned a Reserve Officer in the United States Army, he was called for active duty on June 16, 1942. He entered training at Camp Blanding, Florida, with the 111th Engineer Battalion. Soon afterward he moved with this unit to Camp Edwards, Massachusetts, and passed the winter in New England on maneuvers. After five months of service he was promoted to First Lieutenant.

In March, 1943, Lieutenant McTeer was sent overseas to North Africa with his training unit—the 111th U. S. Engineer Battalion, 36th Division, Fifth Army. On June 25, 1943, he died from a gunshot wound. Of him his Commanding Officer wrote as follows: "He was a splendid officer, a brilliant engineer and the possessor of a winsome and loyal nature, which endeared him to all his associates, both officers and men."

He is survived by his father, a veteran of World War I; his mother; a brother, Harry; and his grandparents, Mr. and Mrs. Henry T. Morrison.

Lieutenant McTeer was elected a Junior of the American Society of Civil Engineers on September 9, 1942.

ERNEST FREDERICK HARTMAN, Affiliate Am. Soc. C. E.[1]

Died September 30, 1942

With the death of Ernest Frederick Hartman, a lifelong worker and advocate of conservation of lumber and timber resources through wood preservative treatment was lost to the building industry. Before his death at Chittenden, Vt., when he was sixty three, Mr. Hartman realized his ambition to obtain general acceptance of fireproofed wood as a building material through improved processes of chemical pressure treatment. In the wartime expansion program, millions of board-feet of treated lumber and timber are under fabrication and construction as well as millions more already incorporated in defense plants. Fire-retarded wood as a building material is especially useful for the protection of long span timber trusses, roof-deck planking, scaffolding, and accessory construction units, in order to conserve the use of more critical materials in essential defense work and as a fire-resistive substitute for unprotected steel construction where fire hazard is involved.

In cooperation with other pioneer workers and technical consultants, including Rudolph P. Miller, William J. Krefeld, and the late Albin H. Beyer,[2] Members, Am. Soc. C. E., T. R. Truax, Henry Klein, and Ira H. Woolson, in the development of processes and adaptations of old methods of treating lumber and timber to render it fire-resistive, he devoted his life to the establishment of definite specification standards and commercial values for fire-retarding, preservation, and consequent conservation of this important building material. Largely through his efforts, national recognition finally was accorded to fireproofed and flameproofed wood as a construction material by fire underwriter and other insurance authorities.

---

[1] Memoir prepared by George E. Strehan, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1499.

About 1891, when he was twelve years old, Mr. Hartman came to the United States from Germany and settled in Milwaukee, Wis., with his parents. About this time the U. S. Naval authorities adopted the use of fireproofed wood decking on warships, treated by the original process of the Bachert Electric Fireproofing Company of New York (N. Y.). Before this time, similar processes had been advocated by the navies of England, France, Sweden, Denmark, and Russia. In 1899, based on the work of Messrs. Woolson and Miller, the use of fireproofed wood was extended to the building requirements of the New York City code for the treatment of combustible wood trim and finished flooring in building structures. The requirements and specification-acceptance tests then developed by these pioneers are still the basis of present New York code regulations, and the application of fireproofed wood has been expanded to include the treatment of structural units.

Mr. Hartman was educated in the Milwaukee public schools and, as a young man, saw service with the Milwaukee National Guard in the Spanish-American War. Early in the present century he became associated with Max Bachert in the study of fireproofing processes and later served as midwestern representative of the Carbolineum Wood Preserving Company on the manufacture and sale of high-boiling creosotes and anthracene oils. In 1906, he moved east to open a New York office for that concern, and his progressive studies and efforts resulted in the establishment of a successor to that company under the name of the Protexol Corporation of which he was president. By acquiring all the United States patent rights to the Bachert process and by introducing refined improvements, he succeeded in combining fireproofing and preservative treatments for many practical and commercial uses.

Primarily through overzealous commercial promotion of competing companies, the fireproofed wood industry had been experiencing a rather haphazard and questionable existence. Mr. Hartman practiced extensive commercial and research activities, studied all foreign developments diligently, and maintained active correspondence with leading authorities on the subject. By such persistent efforts, he kept alive an interest in the true possibilities of fireproofed wood which had, up to the past decade, failed to receive recognition from American building and insurance supervising authorities. Originating in the laboratory of the Protexol Corporation, aided by research work of the Columbia University (New York) Department of Civil Engineering and the laboratories of the U. S. Bureau of Standards, the Forest Products Service, and the National Board of Fire Underwriters, this work eventually resulted in the acceptance of fireproofed wood as an approved building material.

Simultaneously with commercial activities, Mr. Hartman devoted much time to general research and study of the properties of wood and methods of wood preservation, fire-retardation, and flameproofing; and he developed many of his own formulas and processes. He accumulated what is probably the most complete and extensive library available on these subjects. Among other activities, the Protexol Corporation conducts a fire testing station operated under the auspices of New York building authorities for official investigation of new materials and devices and methods of fire-resistive building construction. A considerable part of these investigations concern the use of fireproofed wood. Continuing the work begun by Mr. Klein in the development of the "Flaimproof" fire door of one-hour and one-and-one-half-hour fire resistance under standard time-temperature control, the Protexol Testing Laboratories have applied treated wood successfully in fire-retarding partitions, doors, and fire stops in building construction to meet a wide range of standard time-temperature test requirements of building code and insurance regulations. This work is being continued by his surviving sons at the Kenilworth (N. J.) plant, assisted by Rollin C. Bastress as technical expert.

Mr. Hartman was an active member of the American Wood Preservers Association from 1911 until his death in 1942, serving as a member of the committee on "Preservatives" for twenty years. In 1936, he was elected chairman of the newly-established "Fireproofing Committee" (committee No. 9) and continued in that capacity during the last years of his life. He belonged to committee C–5 of the American Society for Testing Materials and was interested especially in the subcommittee for developing standards and test specifications for "Fireproofed Wood," under the general chairmanship of Mr. Miller. Also, he was a member of the National Fire Protection Association and The American Electric Railway Engineering Association.

During the last thirty years much of his leisure was devoted to the hobby of flower, shrub, and tree cultivation, and he became an expert botanist and authority on plant life—particularly on trees and their by-products. He was also a pteridologist of some repute and maintained extensive gardens at both his home in Amityville, Long Island, N. Y., and at the Kenilworth plant. In the wild-flower garden at his home he successfully grew fifty-six species of wild Vermont ferns of which he was quite proud. Visitors to either place were rewarded by the floral displays and never failed to carry away gift specimens of some favorite species for enjoyment and propagation. In addition to being a member of the Park Department at Amityville, he also was interested in local Boy Scout activities and, for many years, served as chairman of Troop 83, Boy Scouts of America. Holding membership in the Green Mountain Club, in Rutland, Vt., his love of flowers frequently drew him to that state, where, in later years, he spent his summers and where he eventually died from a chronic heart ailment. It was fitting that he was laid to rest in a little cemetery in the hills of Vermont with a simple white oak cross preserved and fireproofed by the process he himself had perfected.

Mr. Hartman is survived by his widow, Florence (Nelke) Hartman, to whom he was married on February 1, 1914; by two sons, Carl F. and Frederick A. Hartman; and by his mother, Mrs. Lydia Eirmann. A third son, Richard Hartman, was killed in combat with the Japanese in December, 1941, at Manila, Philippine Islands, in the first action of World War II.

Mr. Hartman was elected an Affiliate of the American Society of Civil Engineers on January 4, 1910.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 109

1944

Subject Index, Page 1597
Author Index, Page 1608

Titles of papers are in quotation marks when given with the author's name.

VOLUME 109

SUBJECT INDEX

ADDRESSES

"Chart for the Future." Address prepared for the Annual Convention, Cleveland, Ohio, July 19, 1944. Malcolm Pirnie. 1429.

(*Seventy-fourth Annual Convention canceled by request of Office of Defense Transportation.*)

AERATION

"Aeration of Spillways." G. H. Hickox. (With Discussion.) 537.

AIR COMPRESSION

Air consumption, power tools, Queens Midtown Tunnel. 719.

ALLOYS

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

ALUMINUM

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

APPARATUS

Apparatus for triaxial compression tests. 384.

Consolidation apparatus. 391.

ARCHES

"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." Richard M. Hodges. 913. Discussion: C. D. Geisler, Arthur G. Hayden, Bernard L. Weiner, Alfred L. Parmé, J. Charles Rathbun, Maurice Barron, Phillips H. Lovering, Jaroslav Polivka, A. A. Eremin, and Richard M. Hodges. 942.

BASCULE BRIDGES

See **BRIDGES**

BEAMS

"A Symmetrically Loaded Base Slab on an Elastic Foundation." Stanley U. Benscoter. (With Discussion.) 763.

"Strength of Slender Beams." George Winter. 1321. Discussion: T. McLean Jasper, C. M. Goodrich, J. P. Wadleck, H. D. Hussey, and George Winter. 1350.

BEARING CAPACITY

Properties of cohesionless soils affecting settlements bearing capacity. 482.

BIBLIOGRAPHIES

Beams and columns. 1346.

Beams and plates on yielding foundations. 787.

Bearing capacity of soils. 418.

Meteorology and flood flows. 347.

Model-prototype conformity. 112.

Procedure for deriving infiltration curves. 1309.

Sedimentation. 654, 1068.

Sewage disposal. 822.

Soil testing. 1181.

BRIDGES

"Development of the Chicago Type Bascule Bridge." Donald N. Becker. 995. Discussion: Alonzo J. Hammond, Clinton D. Hanover, Jr., Armour T. Granger, C. B. McCullough, and Donald N. Becker. 1026.

"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." Richard M. Hodges. (With Discussion.) 913.

BUILDINGS

"Application of Soil Mechanics in Designing Building Foundations." A. Casagrande and R. E. Fadum. (With Discussion.) 383.

CEMENT

Composition and fineness. 528.

CHANNELS

"Flow Around Bends in Stable Channels." C. A. Mockmore. (With Discussion.) 593.

"The Hydraulic Jump in Sloping Channels." Carl E. Kindsvater. (With Discussion.) 1107.

See also **WATER, FLOW OF, IN OPEN CHANNELS**

CIVIL ENGINEERING

See **ENGINEERS AND ENGINEERING**

CLAY

"Application of Soil Mechanics in Designing Building Foundations." A. Casagrande and R. E. Fadum. (With Discussion.) 383.

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. (With Discussion.) 1400.

"Relation of Undisturbed Sampling to Laboratory Testing." P. C. Rutledge. (With Discussion.) 1155.

COLUMNS

"Strength of Slender Beams." George Winter. (With Discussion.) 1321.

CONCRETE

"Concrete Reservoirs of the Vertical-Beam Type." C. Maxwell Stanley. (With Discussion.) 567.

"Strains, Stresses, and Shear in Engineering Problems." Silas H. Woodard. (With Discussion.) 491.

Surface waterproofing of concrete reservoirs. 587.

See also **REINFORCED CONCRETE** .

CONSTRUCTION

Queens Midtown Tunnel. 701.

CONSTRUCTION PLANT

See **CONTRACTORS' PLANT**

CONTRACTORS' PLANT

Hazards of fabrication and erection. 281.

COSTS

Cost of construction, vertical-beam type of reservoir. 583.

Costs of sewage treatment in Chicago. 835.

Relative cost of reservoir projects. 592.

"The Queens Midtown Tunnel." Ole Singstad. 679.

DAMS

"Conformity Between Model and Prototype." A Symposium. (With Discussion.) 1.

See also **RESERVOIRS**

DISCHARGE COEFFICIENTS

For Norris spillway. 72.

DRAINAGE

"A Method of Computing Urban Runoff." W. I. Hicks. (With Discussion.) 1217.

DRILLING

Correlation of seismic and drilling results. 210.

DRY DOCKS

"Conformity Between Model and Prototype (Symposium): A Dry-Dock Suction Chamber." C. B. Patterson. 29.

EARTH PRESSURES

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. (With Discussion.) 1400.

ENGINEERING GLOSSARIES

Meteorological terms. 349.

ENGINEERS AND ENGINEERING

"Chart for the Future." Address prepared for the Annual Convention, Cleveland, Ohio, July 19, 1944. Malcolm Pirnie. 1429.

(*Seventy-fourth Annual Convention canceled by request of Office of Defense Transportation.*)

"Development of the Chicago Type Bascule Bridge." Donald N. Becker. (With Discussion.) 995.

EROSION

"Sedimentation in Reservoirs." Berard J. Witzig. (With Discussion.) 1047.

FAILURES

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. (With Discussion.) 1400.

FATIGUE FORMULAS

Fatigue failure. 287.

FINANCE

"Organizing and Financing Sewage Treatment Projects." Samuel A. Greeley. (With Discussion.) 248.

FLOODS

"Primary Rôle of Meteorology in Flood Flow Estimating." Merrill Bernard. (With Discussion.) 311.

"Rôle of the Land During Flood Periods." W. W. Horner. (With Discussion.) 1269.

FLOW OF WATER

See **WATER, FLOW OF**

FLUMES

Comparison of observed Parshall flume discharges. 162.

"The Hydraulic Jump in Sloping Channels." Carl E. Kindsvater. (With Discussion.) 1107.

FOUNDATIONS

"A Symmetrically Loaded Base Slab on an Elastic Foundation." Stanley U. Benscoter. (With Discussion.) 763.

"Application of Soil Mechanics in Designing Building Foundations." A. Casagrande and R. E. Fadum. (With Discussion.) 383.

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. (With Discussion.) 1400.

"Seismic Subsurface Exploration on the St. Lawrence River Project." E. R. Shepard and Reuben M. Haines. (With Discussion.) 194.

GEOLOGY

"Seismic Subsurface Exploration on the St. Lawrence River Project." E. R. Shepard and Reuben M. Haines. (With Discussion.) 194.

GLOSSARIES

See **ENGINEERING GLOSSARIES**

HEALTH

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

HIGHWAYS

"The Queens Midtown Tunnel." Ole Singstad. (With Discussion.) 679.

HYDRAULIC JUMP

"The Hydraulic Jump in Sloping Channels." Carl E. Kindsvater. 1107. Discussion: Joe W. Johnson, Karl R. Kennison, J. C. Stevens, C. J. Posey, Jerome Fee, Frank S. Bailey, G. H. Hickox, and Carl E. Kindsvater. 1121.

See also **WATER, FLOW OF**

HYDRAULICS

"Conformity Between Model and Prototype." A Symposium. 1. Discussion: A. E. Niederhoff, C. I. Grimm, Joe W. Johnson, V. L. Streeter, Glen N. Cox, Graham Walton, H. A. Einstein, K. G. Tower, R. J. Pafford, Jr., Edward H. Schulz, F. T. Mavis, D. C. McConaughy, L. Standish Hall, A. J. Gilardi, Fred W. Blaisdell, A. R. Thomas, Gilbert H. Dunstan, Robert F. Kreiss, J. H. Douma, Marvin J. Webster, Martin E. Nelson, and Jacob E. Warnock and H. G. Dewey, Jr., Martin E. Nelson and James J. Hartigan, J. C. Stevens and R. B. Cochrane, Edward Soucek, and Frederick R. Brown. 118.

"Effect of Turbulence on Sedimentation." William E. Dobbins. (With Discussion.) 629.

"Flow Around Bends in Stable Channels." C. A. Mockmore. (With Discussion.) 593.

"Flow Characteristics at Rectangular Open-Channel Junctions." Edward H. Taylor. (With Discussion.) 893.

See also **WATER, FLOW OF**

HYDROGAPHS

"A Method of Computing Urban Runoff." W. I. Hicks. (With Discussion.) 1217.

HYDROLOGY

Hydrologic cycle functionalized. 312.

INSTRUMENTS

Hydrostatic pressure measuring device. 1415.

Installation of piezometers, manometers, dial gages, and point gages. 8.

Water measuring instruments. 174.

LAND

"Rôle of the Land During Flood Periods." W. W. Horner. 1269. Discussion: L. K. Sherman, Raphael G. Kazmann, W. G. Hoyt and W. B. Langbein, Charlie M. Moore, Waldo E. Smith, P. B. Rowe, C. S. Jarvis, and W. W. Horner. 1295.

LAW

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

"Organizing and Financing Sewage Treatment Projects." Samuel A. Greeley. (With Discussion.) 248.

LEVEES

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. 1400. Discussion: Ralph B. Peck, K. Terzaghi, T. A. Middlebrooks, D. P. Krynine, and Kenneth E. Fields and William L. Wells. 1414.

LOCKS

"Conformity Between Model and Prototype" (Symposium):

"Hydraulic Conditions to Lock and Dam No. 4." William J. Hopkins. 103.

"Pickwick, Wheeler, and Guntersville Locks." Martin E. Nelson and James J. Hartigan. 35.

MEMOIRS OF DECEASED MEMBERS
See name of member in Author Index, also Contents page x.

METEOROLOGY
"Characteristics of Heavy Rainfall in New Mexico and Arizona." Luna B. Leopold. (With Discussion.) 837.
"Primary Rôle of Meteorology in Flood Flow Estimating." Merrill Bernard. 311. Discussion: Edgar Dow Gilman, Clarence S. Jarvis, Ivan E. Houk, W. G. Hoyt, L. R. Beard, Robert E. Kennedy, L. K. Sherman, C. H. Eiffert, Gail A. Hathaway, L. Standish Hall, and Merrill Bernard. 352.

MODELS
"Conformity Between Model and Prototype." A Symposium. (With Discussion.) 1.

MOHR DIAGRAM
Mohr's theory illustrated by Mohr's circle of stress. 505.

PIPE
"Determination of Kutter's *n* for Sewers Partly Filled." C. Frank Johnson. (With Discussion.) 223.

POSTWAR PLANNING
"Chart for the Future." Address prepared for the Annual Convention, Cleveland, Ohio, July 19, 1944. Malcolm Pirnie. 1429.
(*Seventy-fourth Annual Convention canceled by request of Office of Defense Transportation.*)

PRECIPITATION
See **RAINFALL**

PRESSURES
See **EARTH PRESSURES**

PUMPS AND PUMPING
"Conformity Between Model and Prototype (Symposium): A Dry-Dock Suction Chamber." C. B. Patterson. 29.

RAINFALL
"A Method of Computing Urban Runoff." W. I. Hicks. (With Discussion.) 1217.
"Characteristics of Heavy Rainfall in New Mexico and Arizona." Luna B. Leopold. 837. Discussion: Lawrence Pratt, L. L. Harrold and A. J. Dickson, James Girand, Clarence S. Jarvis, Paul V. Hodges, Edgar E. Foster, R. W. Davenport, A. K. Showalter, Walter B. Langbein, Emil P. Schuleen, and Luna B. Leopold. 867.
"Primary Rôle of Meteorology in Flood Flow Estimating." Merrill Bernard. (With Discussion.) 311.
"Rôle of the Land During Flood Periods." W. W. Horner. (With Discussion.) 1269.

REINFORCED CONCRETE

"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." Richard M. Hodges. (With Discussion.) 913.

See also **CONCRETE**

RESERVOIRS

"Concrete Reservoirs of the Vertical-Beam Type." C. Maxwell Stanley, Jr. 567. Discussion: Lewis A. Schmidt, Jr., Victor H. Cochrane, R. E. Koon and J. C. Gearhart, Gustavo Pérez Guerra, and C. Maxwell Stanley, Jr. 577.

"Sedimentation in Reservoirs." Berard J. Witzig. (With Discussion.) 1047.

See also **DAMS**

REVETMENT

See **LEVEES**

RIVER POLLUTION

See **WATER POLLUTION**

RIVERS

"Rôle of the Land During Flood Periods." W. W. Horner. (With Discussion.) 1269.

RIVETED JOINTS

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. 1359. Discussion: Jonathan Jones, and Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. 1397.

RIVETS AND RIVETING

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

RUNOFF

"A Method of Computing Urban Runoff." W. I. Hicks. 1217. Discussion: C. S. Jarvis, S. W. Jens, W. W. Horner, and W. I. Hicks. 1254.

"Primary Rôle of Meteorology in Flood Flow Estimating." Merrill Bernard. (With Discussion.) 311.

"Rôle of the Land During Flood Periods." W. W. Horner. (With Discussion.) 1269.

See also **RAINFALL**

SAFETY AND HEALTH

For construction of Queens Midtown Tunnel. 731.

SANITATION

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

SEDIMENTATION

"Effect of Turbulence on Sedimentation." William E. Dobbins. 629. Discussion: A. M. Gaudin, John S. McNown, H. A. Einstein, Thomas R. Camp, Garbis H. Keulegan, C. W. Thornthwaite, A. A. Kalinske, E. R. Van Driest, R. Schuhmann, Jr., H. B. Southwood, L. Standish Hall, and William E. Dobbins. 657.

"Sedimentation in Reservoirs." Berard J. Witzig. 1047. Discussion: Joe W. Johnson, John W. Stanley, Stafford C. Happ, Thomas H. Means, Carl B. Brown, C. S. Jarvis, Hugh Stevens Bell, Harry F. Blaney, A. L. Sonderegger, L. Standish Hall, and Berard J. Witzig. 1072.

SEISMOLOGY

"Seismic Subsurface Exploration on the St. Lawrence River Project." E. R. Shepard and Reuben M. Haines. 194. Discussion: R. Woodward Moore, Arthur J. Grier, F. W. Lee, L. Don Leet, William L. Shannon and Winthrop A. Wells, and E. R. Shepard and Reuben M. Haines. 213.

SEWAGE DISPOSAL

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

"Organizing and Financing Sewage Treatment Projects." Samuel A. Greeley. 248. Discussion: Milton P. Adams, Hal F. Smith, Herbert Moore, Francis H. Kingsbury, A. Brinton Carson, and Samuel A. Greeley. 264.

"Sludge Drying Developments at Chicago, Ill." Lloyd M. Johnson. 979. Discussion: C. W. Gordon, Paul Hansen, Langdon Pearse, and Lloyd M. Johnson. 988.

SEWAGE TREATMENT

See **SEWAGE DISPOSAL**

SEWAGE WORKS

See **SEWAGE DISPOSAL**

SEWERAGE

"Determination of Kutter's *n* for Sewers Partly Filled." C. Frank Johnson. 223. Discussion: Thomas R. Camp, Richard G. Coulter, C. E. Ramser, and C. Frank Johnson. 240.

SHEAR

"Strains, Stresses, and Shear in Engineering Problems." Silas H. Woodard. (With Discussion.) 491.

SILT

"Sedimentation in Reservoirs." Berard J. Witzig. (With Discussion.) 1047.

SIMILARITY, THEORY OF

See **MODELS**

SLABS

"A Symmetrically Loaded Base Slab on an Elastic Foundation." Stanley U. Benscoter. 763. Discussion: Robert O. Jameson, L. J. Mensch, Alfred L. Parmé, A. A. Eremin, A. E. Cummings, Harris Epstein and James R. Ayers, Jr., and Stanley U. Benscoter. 777.

SNOW

Contribution of snow melt to storm runoff. 339.

SOILS

"Application of Soil Mechanics in Designing Building Foundations." A. Casagrande and R. E. Fadum. 383. Discussion: Jacob Feld, Leonard Zeevaert, Gregory P. Tschebotarioff, Guthlac Wilson, W. H. McAlpine, Karl Terzaghi, D. P. Krynine, O. G. Julian, A. A. Eremin, D. M. Burmister, Abraham Woolf, Lazarus White, George L. Freeman, Soil Mechanics Committee of the Los Angeles Section—Trent R. Dames, Chairman, Frederick J. Converse, Sterling S. Green, and Paul Baumann—and A. Casagrande and R. E. Fadum. 417.

"Pendleton Levee Failure." Kenneth E. Fields and William L. Wells. (With Discussion.) 1400.

"Relation of Undisturbed Sampling to Laboratory Testing." P. C. Rutledge. 1155. Discussion: Benjamin K. Hough, Jr., F. M. Van Auken, Jacob Feld, Raymond F. Dawson, Hamilton Gray, Karl Terzaghi, D. P. Krynine, D. M. Burmister, and P. C. Rutledge. 1184.

SPILLWAYS

"Aeration of Spillways." G. H. Hickox. 537. Discussion: Joe W. Johnson, J. W. Howe and Claud C. Lomax, Jr., J. C. Stevens, and G. H. Hickox. 557.

"Conformity Between Model and Prototype." A Symposium. (With Discussion.) 1.

STEEL

"Physical Properties that Affect the Behavior of Structural Members." Wilbur M. Wilson. 279. Discussion: Jonathan Jones, A. B. Kinzel, Almon H. Fuller, C. F. Goodrich, R. L. Moore, Leon S. Moisseiff, Fred L. Plummer, A. A. Eremin, and Wilbur M. Wilson. 290.

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

STRAIN

See **STRESS AND STRAIN**

STREAM FLOW

See **WATER, FLOW OF, IN OPEN CHANNELS**

STRENGTH OF MATERIALS

"Physical Properties that Affect the Behavior of Structural Members." Wilbur M. Wilson. (With Discussion.) 279.

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

STRESS AND STRAIN

"Physical Properties that Affect the Behavior of Structural Members." Wilbur M. Wilson. (With Discussion.) 279.

"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore. (With Discussion.) 1359.

"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." Richard M. Hodges. (With Discussion.) 913.

"Strains, Stresses, and Shear in Engineering Problems." Silas H. Woodard. 491. Discussion: C. M. Goodrich, Maurice P. van Buren, Joseph A. Wise, David B. Hall, Roberto Contini, I. Nelidov, Bernard L. Weiner, Richard W. Albrecht, Duff A. Abrams, C. A. P. Turner, Mark W. Huggins, A. A. Eremin, and Silas H. Woodard. 501.

"Strength of Slender Beams." George Winter. (With Discussion.) 1321.

STRUCTURAL STEEL

See **STEEL**

STRUCTURES, THEORY OF

"A Symmetrically Loaded Base Slab on an Elastic Foundation." Stanley U. Benscoter. (With Discussion.) 763.

"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." Richard M. Hodges. (With Discussion.) 913.

"Strains, Stresses, and Shear in Engineering Problems." Silas H. Woodard. (With Discussion.) 491.

"Strength of Slender Beams." George Winter. (With Discussion.) 1321.

TESTS

Sample disturbance and consolidation test results for soils. 1157.

Triaxial compression test. 1174.

Unconfined compression test. 1169.

TRAFFIC

Tunnel highways, connections, and traffic, Queens Midtown Tunnel. 683.

TUNNEL LINING

"The Queens Midtown Tunnel." Ole Singstad. 679.

TUNNELS

"The Queens Midtown Tunnel." Ole Singstad. 679. Discussion: Thomas W. Fluhr, Howard L. King, Francis V. Wagner, William J. Wilgus, S. A. Thoresen, H. G. Cruthers, Leo Geenens, Charles E. Conover, Edward Levy, Robert F. Legget, Orrin L. Brodie, A. A. Eremin, Arthur S. Tuttle, and Ole Singstad. 716.

TURBULENCE

"Effect of Turbulence on Sedimentation." William E. Dobbins. (With Discussion.) 629.

VENTILATION

Queens Midtown Tunnel. 699.

WATER, FLOW OF, IN OPEN CHANNELS

"Conformity Between Model and Prototype (Symposium): Grand Tower, Mississippi River." Frederick R. Brown. 100.

"Effect of Turbulence on Sedimentation." William E. Dobbins. (With Discussion.) 629.

"Flow Around Bends in Stable Channels." C. A. Mockmore. 593. Discussion: J. M. Robertson, Joe W. Johnson and M. A. Selim, I. Nelidov, S. M. Woodward and J. W. Howe and C. J. Posey, and C. A. Mockmore. 619.

"Flow Characteristics at Rectangular Open-Channel Junctions." Edward H. Taylor. 893. Discussion: Harold K. Palmer, G. H. Hickox, J. C. Stevens, C. J. Posey, I. B. Hosig, L. Standish Hall, Brooks T. Morris, and Edward H. Taylor. 903.

"Rôle of the Land During Flood Periods." W. W. Horner. (With Discussion.) 1269.

"The Hydraulic Jump in Sloping Channels." Carl E. Kindsvater. (With Discussion.) 1107.

WATER, FLOW OF, IN PIPES

"Determination of Kutter's *n* for Sewers Partly Filled." C. Frank Johnson. (With Discussion.) 223.

WATER, FLOW OF, OVER DAMS AND WEIRS

"Aeration of Spillways." G. H. Hickox. (With Discussion.) 537.

WATER POLLUTION

"Illinois Water Litigation, 1940–1941." Langdon Pearse. 799. Discussion: Willem Rudolfs, L. R. Howson, A. M. Buswell, Lloyd M. Johnson, H. P. Ramey, and Langdon Pearse. 824.

WATERPROOFING

Surface waterproofing of concrete reservoirs. 587.

WATER STORAGE

"Concrete Reservoirs of the Vertical-Beam Type." C. Maxwell Stanley. (With Discussion.) 567.

WATER SUPPLY

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

WATERWAYS

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

WATER WORKS

"Illinois Water Litigation, 1940–1941." Langdon Pearse. (With Discussion.) 799.

WEIRS

See **WATER, FLOW OF, OVER DAMS AND WEIRS**

AUTHOR INDEX

ABRAMS, DUFF A.
Strains, stresses, shear. 520.

ADAMS, MILTON P.
Sewage treatment finace. 264.

ALBRECHT, RICHARD W.
Strains, stresses, shear. 517.

ALLEN, EDWARD RUSSELL
Memoir of. 1558.

ALLIN, THOMAS DAVID
Memoir of. 1459.

ALVORD, JOHN WATSON
Memoir of. 1447.

ARNESON, EDWIN PARK
Memoir of. 1587.

AYERS, JAMES R., JR.
Base slab. 793.

BAILEY, FRANK S.
Sloping channels. 1141.

BAILEY, THOMAS WARD
Memoir of. 1461.

BAKER, HAROLD JAMES MANNING
Memoir of. 1561.

BARRON, MAURICE
Skewed rigid frames. 962.

BAUMANN, PAUL
Building foundations. 459.

BEARD, L. R.
Meteorology and flood estimating. 361.

BEBB, EDWARD CROSBY
Memoir of. 1563.

BECKER, DONALD N.
"Development of the Chicago Type Bascule Bridge." 995.

BELL, HUGH STEVENS
Sedimentation. 1088.

BENFIELD, BERNARD
Memoir of. 1464.

BENSCOTER, STANLEY U.
"A Symmetrically Loaded Base Slab on an Elastic Foundation." 763.

BERNARD, MERRILL
"Primary Rôle of Meteorology in Flood Flow Estimating." 311.

BISCHOFF, JULIUS MONTGOMERY
Memoir of. 1563.

BLAISDELL, FRED W.
Model-prototype conformity. 157.

BLANEY, HARRY F.
Sedimentation. 1089.

BOLTON, REGINALD PELHAM
Memoir of. 1465.

BONIFACE, ARTHUR
Memoir of. 1564.

BOWRON, CHARLES EDWARD
Memoir of. 1470.

BRODIE, ORRIN L.
Queens tunnel. 737.

BROWN, CARL B.
Sedimentation. 1080.

BROWN, DALTON MUNROE
Memoir of. 1567.

BROWN, FREDERICK R.
"Conformity Between Model and Prototype (Symposium): Grand Tower, Mississippi River." 100.

BURMISTER, D. M.
Building foundations. 444.
Undisturbed sampling. 1204.

BURNETT, KNOX FOLSOM
Memoir of. 1568.

BUSWELL, A. M.
Illinois litigation. 828.

CAMP, THOMAS R.
Kutter's *n* for sewers. 240.
Turbulence and sedimentation. 660.

CANNON, SYLVESTER QUAYLE
Memoir of. 1472.

CARSON, A. BRINTON
Sewage treatment finance. 271.

CASAGRANDE, A.
"Application of Soil Mechanics in Designing Building Foundations." 383.

CAYE, WOOLSEY MOORMAN
Memoir of. 1474.

COCHRANE, R. B.
"Conformity Between Model and Prototype: A Symposium: Pressure Heads on Bonneville Dam." 77.

COCHRANE, VICTOR H.
Vertical-beam reservoirs. 577.

COLEMAN, RICHARD ALESTER
Memoir of. 1588.

CONOVER, CHARLES E.
Queens tunnel. 729.

CONSTANCE, EDWARD CARTWRIGHT
Memoir of. 1477.

CONTINI, ROBERTO
Strains, stresses, shear. 509.

CONVERSE, FREDERICK J.
Building foundations. 459.

COOK, PAUL DARWIN
Memoir of. 1478.

COULTER, RICHARD G.
Kutter's *n* for sewers. 243.

COX, GLEN N.
Model-prototype conformity. 127.

CRUTHERS, H. G.
Queens tunnel. 727.

CUMMINGS, A. E.
Base slab. 787.

DAMES, TRENT R.
Building foundations. 459.

DAVENPORT, R. W.
New Mexico and Arizona. 877.

DAVISON, GEORGE STEWART
Memoir of. 1437.

DAWSON, RAYMOND F.
Undisturbed sampling. 1190.

DEWEY, H. G., JR.
"Conformity Between Model and Prototype (Symposium): Hydraulic Structures." 3.

DICKSON, A. J.
New Mexico and Arizona. 868.

DOBBINS, WILLIAM E.
"Effect of Turbulence on Sedimentation." 629.

DOUMA, J. H.
Model-prototype conformity. 170.

DUNSTAN, GILBERT H.
Model-prototype conformity. 168.

EIFFERT, C. H.
Meteorology and flood estimating. 369.

EINSTEIN, H. A.
Model-prototype conformity. 134.
Turbulence and sedimentation. 660.

ELKIN, NATHAN ELIAS
Memoir of. 1589.

EPSTEIN, HARRIS
Base slab. 793.

EREMIN, A. A.
Base slab. 785.
Building foundations. 443.
Queens tunnel. 747.
Skewed rigid frames. 968.
Strains, stresses, shear. 533.
Structural properties. 302.

FADUM, R. E.
"Application of Soil Mechanics in Designing Building Foundations." 383.

FEE, JEROME
Sloping channels. 1138.

FELD, JACOB
Building foundations. 417.
Undisturbed sampling. 1189.

FENSTERMAKER, DEWITT CLINTON
Memoir of. 1570

FIELD, GEORGE ADDISON
Memoir of. 1571.

FIELDS, KENNETH E.
"Pendleton Levee Failure." 1400.

FLUHR, THOMAS W.
Queens tunnel. 716.

FOSTER, EDGAR E.
New Mexico and Arizona. 876.

FREEMAN, GEORGE L.
Building foundations. 454.

FULLER, ALMON H.
Structural properties. 295.

GALBREATH, WILLIAM OTTO
Memoir of. 1481.

GALLOWAY, JOHN DEBO
Memoir of. 1451.

GAUDIN, A. M.
Turbulence and sedimentation. 657.

GEARHART, J. C.
Vertical-beam reservoirs. 580.

GEENENS, LEO
Queens tunnel. 728.

GEISLER, C. D.
Skewed rigid frames. 942.

GILARDI, A. J.
Model-prototype conformity. 154.

GILMAN, EDGAR DOW
Meteorology and flood estimating. 352.

GIRAND, JAMES
New Mexico and Arizona. 871.

GITTLESON, IRVIN
Memoir of. 1572.

GOODRICH, C. F.
Structural properties. 296.

GOODRICH, C. M.
Slender beams. 1350.
Strains, stresses, shear. 501.

GORDON, C. W.
Sludge drying. 988.

GRANGER, ARMOUR T.
Chicago bridges. 1031.

GRAY, HAMILTON
Undisturbed sampling. 1193.

GREELEY, SAMUEL A.
"Organizing and Financing Sewage Treatment Projects." 248.

GREEN, STERLING S.
Building foundations. 459.

GRIER, ARTHUR J.
Subsurface exploration. 214.

GRIMM, C. I.
Model-prototype conformity. 119.

HAINES, REUBEN M.
"Seismic Subsurface Exploration on the St. Lawrence River Project." 194.

HALL, DAVID B.
Strains, stresses, shear. 506.

HALL, L. STANDISH
Flow at channel junctions. 908.
Meteorology and flood estimating. 376.
Model-prototype conformity. 150.
Sedimentation. 1093.
Turbulence and sedimentation. 677.

HALLIHAN, JOHN PHILIP
Memoir of. 1483.

HAMMOND, ALONZO J.
Chicago bridges. 1026.

HANOVER, CLINTON D., JR.
Chicago bridges. 1028.

HANSEN, PAUL
Sludge drying. 989.

HAPP, STAFFORD C.
Sedimentation. 1078.

HARROLD, L. L.
New Mexico and Arizona. 868.

HARTIGAN, JAMES J.
"Conformity Between Model and Prototype (Symposium): Pickwick, Wheeler, and Guntersville Locks." 35.

HARTMAN, ERNEST FREDERICK
Memoir of. 1592.

HARTMANN, E. C.
"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." 1359.

HATHAWAY, GAIL A.
Meteorology and flood estimating. 370.

HAWLEY, GEORGE PRINCE
Memoir of. 1484.

HAYDEN, ARTHUR G.
Skewed rigid frames. 944.

HAYS, JOHN COFFEE
Memoir of. 1486.

HERZBERG, WILLIAM
Memoir of. 1489.

HICKOX, G. H.
"Aeration of Spillways." 537.
"Conformity Between Model and Prototype (Symposium): Spillway Coefficients, Norris Dam." 68.
Flow at channel junctions. 904.
Sloping channels. 1141.

HICKS, W. I.
"A Method of Computing Urban Runoff." 1217.

HODGES, PAUL V.
New Mexico and Arizona. 874.

HODGES, RICHARD M.
"Simplified Analysis of Skewed Reinforced Concrete Frames and Arches." 913.

HOLMES, ROBERT LESLIE
Memoir of. 1490.

HOPKINS, WILLIAM J.
"Conformity Between Model and Prototype (Symposium): Hydraulic Conditions to Lock and Dam No. 4." 103.

HORNER, W. W.
"Rôle of the Land During Flood Periods." 1269.
Urban runoff. 1260.

HOSIG, I. B.
Flow at channel junctions. 906.

HOUGH, BENJAMIN K., JR.
Undisturbed sampling. 1184.

HOUK, IVAN E.
Meteorology and flood estimating. 359.

HOWE, J. W.
Spillway aeration. 559.
Stable channels. 626.

HOWSON, L. R.
Illinois litigation. 826.

HOYT, W. G.
Meteorology and flood estimating. 360.
Rôle of land. 1299.

HUBBARD, WINFRED DEAN
Memoir of. 1492.

HUDSON, CLARENCE WALTER
Memoir of. 1493.

HUGGINS, MARK W.
Strains, stresses, shear. 533.

HUGHES, DAVID EDWARD
Memoir of. 1495.

HULICK, DAN A.
Memoir of. 1573.

HUSSEY, H. D.
Slender beams. 1354.

JAMESON, ROBERT O.
Base slab. 777.

JARVIS, CLARENCE S.
Meteorology and flood estimating. 353.
New Mexico and Arizona. 874.
Rôle of land. 1312.
Sedimentation. 1086.
Urban runoff. 1254.

JASPER, T. McLEAN
Slender beams. 1350.

JENS, S. W.
Urban runoff. 1255.

JOHNSON, C. FRANK
"Determination of Kutter's *n* for Sewers Partly Filled." 223.

JOHNSON, JOE W.
Model-prototype conformity. 120.
Sedimentation. 1072.
Sloping channels. 1121.
Spillway aeration. 557.
Stable channels. 623.

JOHNSON, LLOYD M.
Illinois litigation. 830.
"Sludge-Drying Developments at Chicago, Ill." 979.

JONES, JONATHAN
Steel and aluminum joints. 1397.
Structural properties. 290.

JULIAN, O. G.
Building foundations. 438.

KALES, WILLIAM ROBERT
Memoir of. 1497.
KALINSKE, A. A.
Turbulence and sedimentation. 672.
KAZMANN, RAPHAEL G.
Rôle of land. 1298.
KEEFE, DAVID ANDREW
Memoir of. 1499.
KENNEDY, ROBERT E.
Meteorology and flood estimating. 363.
KENNISON, KARL R.
Sloping channels. 1123.
KEULEGAN, GARBIS H.
Turbulence and sedimentation. 667.
KINDSVATER, CARL E.
"The Hydraulic Jump in Sloping Channels." 1107.
KING, HOWARD L.
Queens tunnel. 716.
KINGSBURY, FRANCIS H.
Sewage treatment finance. 269.
KINZEL, A. B.
Structural properties. 292.
KOON, R. E.
Vertical-beam reservoirs. 580.
KREISS, ROBERT F.
Model-prototype conformity. 168.
KRYNINE, D. P.
Building foundations. 435.
Levee failure. 1424.
Undisturbed sampling. 1201.
LAIRD, ALEXANDER ANDREW
Memoir of. 1574.
LANG, ROLLAND FRANCIS
Memoir of. 1590.
LANGBEIN, WALTER B.
New Mexico and Arizona. 879.
Rôle of land. 1299.
LEACH, HARRY RAYMOND
Memoir of. 1500.

LEE, F. W.
Subsurface exploration. 216.

LEET, L. DON
Subsurface exploration. 219.

LEGGET, ROBERT F.
Queens tunnel. 737.

LEOPOLD, LUNA B.
"Characteristics of Heavy Rainfall in New Mexico and Arizona." 837.

LEVY, EDWARD
Queens tunnel. 731.

LOMAX, CLAUD C., JR.
Spillway aeration. 559.

LOVERING, PHILLIPS H.
Skewed rigid frames. 966.

McAFEE, LLOYD TEVIS
Memoir of. 1502.

McALPINE, W. H.
Building foundations. 426.

McCONAUGHY, D. C.
Model-prototype conformity. 148.

McCULLOUGH, C. B.
Chicago bridges. 1035.

McNOWN, JOHN S.
Turbulence and sedimentation. 657.

McTEER, THOMAS ALBERT
Memoir of. 1591.

MAIN, CHARLES T.
Memoir of. 1504.

MAVIS, F. T.
Model-prototype conformity. 144.

MAY, JOHN WALTER
Memoir of. 1507.

MEANS, THOMAS H.
Sedimentation. 1079.

MENSCH, L. J.
Base slab. 780.

MIDDLEBROOKS, T. A.
Levee failure. 1421.

MINER, JAMES HENRY
Memoir of. 1576.

MOCKMORE, C. A.
"Flow Around Bends in Stable Channels." 593.

MOISSEIFF, LEON S.
"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." 1359.
Structural properties. 299.

MOORE, CHARLIE M.
Rôle of land. 1304.

MOORE, HERBERT
Sewage treatment finance. 268.

MOORE, R. L.
"Riveted and Pin-Connected Joints of Steel and Aluminum Alloys." 1359.
Structural properties. 297.

MOORE, R. WOODWARD
Subsurface exploration. 213.

MORRIS, BROOKS T.
Flow at channel junctions. 910.

NELIDOV, I.
Stable channels. 625.
Strains, stresses, shear. 511.

NELSON, MARTIN E.
"Conformity Between Model and Prototype (Symposium): Pickwick, Wheeler, and Guntersville Locks." 35. Discussion. 177.

NIEDERHOFF, A. E.
Model-prototype conformity. 118.

PAFFORD, R. J., JR.
Model-prototype conformity. 140.

PALMER, HAROLD K.
Flow at channel junctions. 903.

PARKER, JAMES LAFAYETTE
Memoir of. 1509.

PARMÉ, ALFRED L.
Base slab. 784.
Skewed rigid frames. 954.

PATTERSON, C. B.
"Conformity Between Model and Prototype (Symposium): A Dry-Dock Suction Chamber." 29.

PEARSE, LANGDON
"Illinois Water Litigation, 1940–1941." 799.
Sludge drying. 991.

PECK, RALPH B.
Levee failure. 1414.

PÉREZ GUERRA, GUSTAVO
Vertical-beam reservoirs. 587.

PIRNIE, MALCOLM
"Chart for the Future." 1430.

PLUMMER, FRED L.
Structural properties. 301.

POE, HARRY TINKER, JR.
Memoir of. 1510.

POLIVKA, JAROSLAV
Skewed rigid frames. 967.

POSEY, C. J.
Flow at channel junctions. 906.
Sloping channels. 1135.
Stable channels. 626.

PRATT, LAWRENCE
New Mexico and Arizona. 867.

RAMEY, H. P.
Illinois litigation. 831.

RAMSER, C. E.
Kutter's *n* for sewers. 245.

RATHBUN, J. CHARLES
Skewed rigid frames. 960.

RICE, WALTER PERCIVAL
Memoir of. 1511.

RICHARDSON, ARTHUR LELAND
Memoir of. 1577.

RICKEY, JAMES WALTER
Memoir of. 1513.

RIPLEY, JOHN WESLEY
Memoir of. 1517.

ROBERTSON, J. M.
Stable channels. 619.

ROCKETT, LOUIS CHARLES
Memoir of. 1519.

ROCKWELL, EDWARD HENRY
Memoir of. 1520.
ROGERS, WALTER ALEXANDER
Memoir of. 1522.
ROWE, P. B.
Rôle of land. 1310.
RUDOLFS, WILLEM
Illinois litigation. 824.
RUTLEDGE, P. C.
"Relation of Undisturbed Sampling to Laboratory Testing." 1155.
SAFFORD, HARRY ROBINSON
Memoir of. 1524.
SANDO, WILL JOSEPH
Memoir of. 1526.
SCHMIDT, LEWIS A., JR.
Vertical-beam reservoirs. 577.
SCHUHMANN, R., JR.
Turbulence and sedimentation. 676.
SCHULEEN, EMIL P.
New Mexico and Arizona. 884.
SCHULZ, EDWARD H.
Model-prototype conformity. 143.
SELIM, M. A.
Stable channels. 623.
SHANNON, WILLIAM L.
Subsurface exploration. 219.
SHEPARD, E. R.
"Seismic Subsurface Exploration on the St. Lawrence River Project." 194.
SHERMAN, L. K.
Meteorology and flood estimating. 365.
Rôle of land. 1295.
SHOEMAKER, MARSHALL NEY
Memoir of. 1528.
SHOWALTER, A. K.
New Mexico and Arizona. 878.
SINGER, ARTHUR GREGG
Memoir of. 1529.
SINGSTAD, OLE
"The Queens Midtown Tunnel." 679.

SMITH, HAL F.
Sewage treatment finance. 266.

SMITH, WALDO E.
Rôle of land. 1305.

SONDEREGGER, A. L.
Sedimentation. 1091.

SOUCEK, EDWARD
"Conformity Between Model and Prototype (Symposium): Meter Measurements of Discharge, University Dam." 86.

SOUTHWOOD, H. B.
Turbulence and sedimentation. 676.

STANLEY, C. MAXWELL, JR.
"Concrete Reservoirs of the Vertical-Beam Type." 567.

STANLEY, JOHN W.
Sedimentation. 1075.

STERLING, CHARLES SLEEPER
Memoir of. 1532.

STEVENS, J. C.
"Conformity Between Model and Prototype (Symposium): Pressure Heads on Bonneville Dam." 77.
Flow at channel junctions. 905.
Sloping channels. 1125.
Spillway aeration. 563.

STEVENS, JOHN FRANK
Memoir of. 1440.

STREETER, V. L.
Model-prototype conformity. 124.

TAYLOR, EDWARD H.
"Flow Characteristics at Rectangular Open-Channel Junctions." 893.

TERZAGHI, KARL
Building foundations. 427.
Levee failure. 1416.
Undisturbed sampling. 1196.

THOMAS, A. R.
Model-prototype conformity. 167.

THORESEN, S. A.
Queens tunnel. 725.

THORNTHWAITE, C. W.
Turbulence and sedimentation. 669.

TINKER, GEORGE HENRY
Memoir of. 1533.

TOUCEDA, ENRIQUE AUGUSTO
Memoir of. 1534.

TOWER, K. G.
Model-prototype conformity. 139.

TOWLES, THOMAS THOMSON
Memoir of. 1536.

TROST, PAUL ANTHONY
Memoir of. 1538.

TSCHEBOTARIOFF, GREGORY P.
Building foundations. 422.

TURNER, C. A. P.
Strains, stresses, shear. 529.

TUTTLE, ARTHUR S.
Queens tunnel. 749.

VAN AUKEN, F. M.
Undisturbed sampling. 1186.

VAN BUREN, MAURICE P.
Strains, stresses, shear. 502.

VAN DRIEST, E. R.
Turbulence and sedimentation. 674.

VAN LOAN, WILLIAM SHERMAN
Memoir of. 1579.

VARNEY, HENRY ALDIS
Memoir of. 1539.

VARNEY, WILLIAM WESLEY
Memoir of. 1541.

WADLECK, J. P.
Slender beams. 1351.

WAGNER, FRANCIS V.
Queens tunnel. 717.

WALKER, ALBERT WILLARD
Memoir of. 1580.

WALTON, GRAHAM
Model-prototype conformity. 131.

WARNOCK, JACOB E.
"Conformity Between Model and Prototype (Symposium): Hydraulic Structures." 3.

WEBSTER, MARVIN J.
Model-prototype conformity. 173.

WEIDNER, CARL ROBERT
Memoir of. 1582.

WEINER, BERNARD L.
Skewed rigid frames. 946.
Strains, stresses, shear. 514.

WELLS, WILLIAM L.
"Pendleton Levee Failure." 1400.

WELLS, WINTHROP A.
Subsurface exploration. 219.

WESTON, ROBERT SPURR
Memoir of. 1542.

WHEELER, DANIEL MERRICK
Memoir of. 1544.

WHITE, JAMES GILBERT
Memoir of. 1548.

WHITE, LAZARUS
Building foundations. 452.

WHITEHEAD, ROBERT LEE, JR.
Memoir of. 1585.

WIGHT, HARRY JAMES
Memoir of. 1549.

WILGUS, WILLIAM J.
Queens tunnel. 724.

WILSON, EDGAR KENNARD
Memoir of. 1551.

WILSON, GUTHLAC
Building foundations. 424.

WILSON, WILBUR M.
"Physical Properties That Affect the Behavior of Structural Members." 279.

WING, S. P.
"Conformity Between Model and Prototype (Symposium): Boulder Dam Outlet Works." 59.

WINTER, GEORGE
"Strength of Slender Beams." 1321.

WISE, JOSEPH A.
Strains, stresses, shear. 503.

WITZIG, BERARD J.
"Sedimentation in Reservoirs." 1047.

WONSON, SAMUEL LAMSON
Memoir of. 1552.

WOOD, GUY SMITH
Memoir of. 1586.

WOODARD, SILAS H.
"Strains, Stresses, and Shear in Engineering Problems." 491.

WOODRUFF, EDWARD LOWREY
Memoir of. 1554.

WOODWARD, S. M.
Stable channels. 626.

WOOLF, ABRAHAM
Building foundations. 448.

WORCESTER, JOSEPH RUGGLES
Memoir of. 1456.

ZEEVAERT, LEONARD
Building foundations. 419.